ADS model of CB100

MADMAX Analysis week

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HELMHOLTZ

Overview

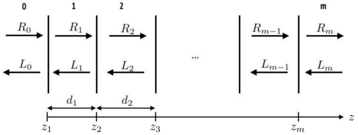
Booster theory

Most simple 1D simulation

- Uniform electric field with right and left moving parts R_r and L_r propagated by $k = \omega n_r$ in every region r

$$E_r = R_r \exp(+ikz) + L_r \exp(-ikz) + E_{a,r}$$

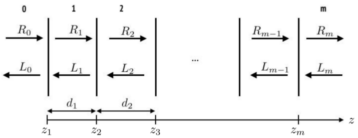
- Couples perfectly to uniform axion induced E-field E_a
- Easily solved by eg transfer matrix formalism
- Can calculate reflection, transmission and boostfactor



1D booster schematic

Most simple 1D simulation

- Uniform electric field with right and left moving parts R_r and L_r propagated by $k = \omega n_r$ in every region r



1D booster schematic

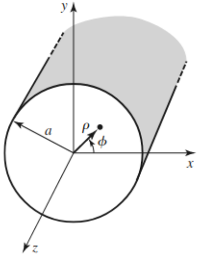
$$E_r = R_r \exp(+ikz) + L_r \exp(-ikz) + E_{a,r}$$

- Couples perfectly to uniform axion induced E-field E_a
- Easily solved by eg transfer matrix formalism
- Can calculate reflection, transmission and boostfactor

In the real world, R_r and L_r are functions of (x, y, z) and propagation is influenced by boundary conditions!

Circular waveguides

- Determine fields by solving Maxwells equations with circular boundaries
  $\rightarrow$ Infinite solutions (modes)
- Number of propagating modes constrained by size and frequency
  $\rightarrow$ CB100: $\sim$80 modes allowed ("overmoded system")
- Rotationally symmetric
  $\rightarrow$ arbitrary rotation of vector field (polarisation)
- CB100 is a stack of circular waveguides



Circular waveguide

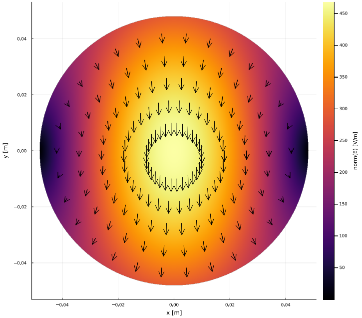
Circular waveguide

TE solution

Two main quantities (for our case):

- Propagation constant: $\beta = \sqrt{k^2 - \frac{p'_{nm}}{r}^2}$
- Impedance: $Z_{TE} = \frac{kZ_0}{\beta}$

with $Z_0 = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \simeq Z_{\text{freespace}} \sqrt{\frac{1}{\epsilon_r}}$, and p'_{nm} the roots of the derivatives of the bessel functions (can be looked up).



Transverse TE11 field inside circular waveguide

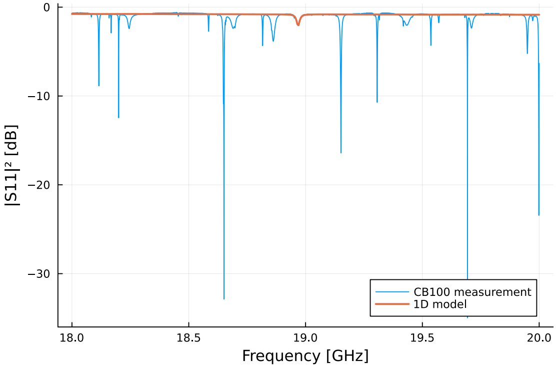
Advanced 1D simulation

- Consider single circular waveguide mode (TE11)
- Propagation constant β and impedance Z_{TE} from Maxwells equation
- Reflection by impedance
- Can also be solved by eg transfer matrix formalism

Verification of single mode assumption

By reflectivity measurement:

- Different modes form resonances at different frequencies due to different β
- Resonances show up in reflectivity measurement
- On resonance, other modes are suppressed
- No other resonances in region of interest can be seen
  $\rightarrow$ Single mode approximation valid



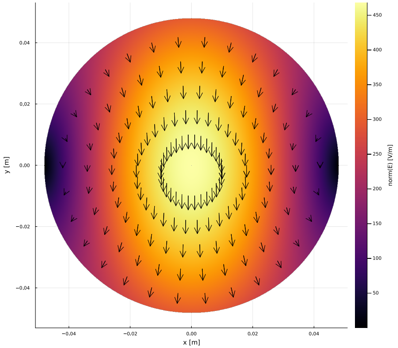
Reflectivity of CB100

Boostfactor determination

- ”Real” power boostfactor needs to consider coupling to Axion induced field
- Coupling by overlap integral, assuming homogeneous B-field in y-direction, y-polarized E-field and constant Axion field:

$$\eta = \frac{\left|\int \mathbf{E}_{TE11} \cdot \hat{y} dA\right|^2}{A \int |\mathbf{E}_{TE11}|^2 dA} \simeq 0.84 \qquad (1)$$

$$\rightarrow \beta^2_{real} = \beta^2_{1D} \cdot \eta^2$$



TE11 field

Summary

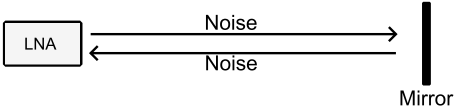
1D boostfactor calculation

- TE11 only resonant mode at boostfactor
  $\rightarrow$ single mode simulation valid
- Use propagation constant β and impedance Z_{TE11} of circular waveguide
- Formfactor needed for ”real” boostfactor (considering the field shape)

Noise theory

Noise interference

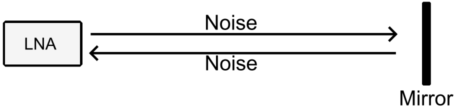
- Noise: random power source with zero average
- Random phase → no interference



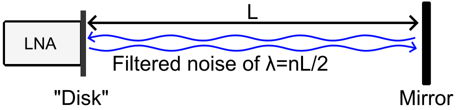
LNA noise with mirror

Noise interference

- Noise: random power source with zero average
- Random phase → no interference
- LNA badly matched
  → resonance between LNA and mirror
  → Resonator acts as filter creating coherance



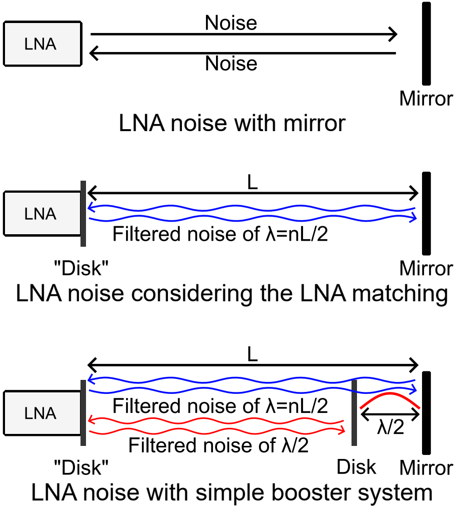
LNA noise with mirror



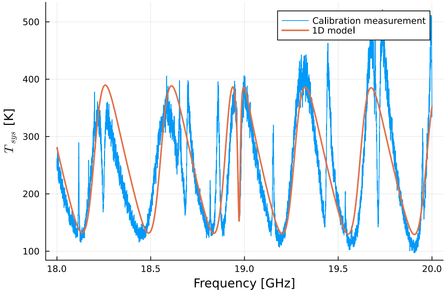
LNA noise considering the LNA matching

Noise interference

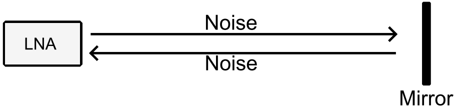
- Noise: random power source with zero average
- Random phase $\rightarrow$ no interference
- LNA badly matched
  $\rightarrow$ resonance between LNA and mirror
  $\rightarrow$ Resonator acts as filter creating coherance
- Adding a booster, combines several filters
  $\rightarrow$ leads to peaks/dips in ”main” interference pattern



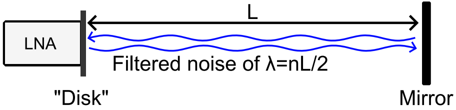
Noise interference



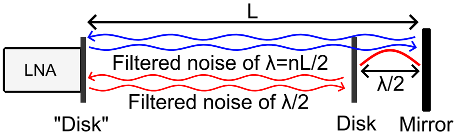
Noise measurement (2023)



LNA noise with mirror



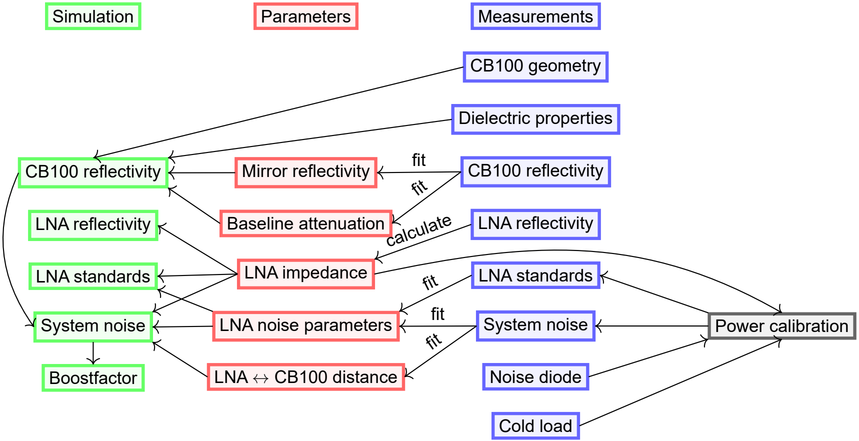
LNA noise considering the LNA matching



LNA noise with simple booster system

ADS model

Parameter overview



Reflectivity simulation

Geometry

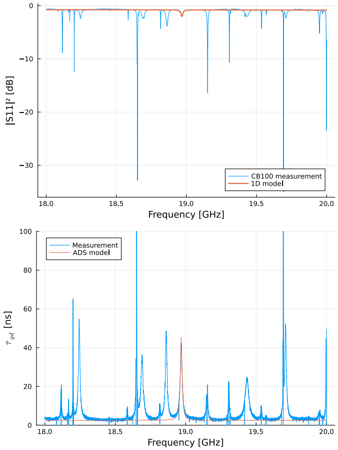
- Disk spacings (taper to mirror):
  1. $13.028\,\text{mm}$
  2. $12.189\,\text{mm}$
  3. $12.208\,\text{mm}$
  4. $8.2745\,\text{mm}$
- Disk thickness: $0.976\,\text{mm}$

Disk material properties

- $\epsilon_r = 9.36$
- $\tan\delta = 2 \times 10^{-5}$

Free ("fit") parameters

- Mirror reflectivity: $R = 0.99955$
- Baseline attenuation: $A = 0.0058\nu/c_0$ dB

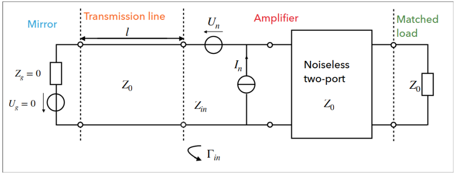


Reflectivity measurement compared to ADS simulation

LNA

LNA noise model

- LNA noise characterized by three parameters:
  - Voltage noise U_n
  - Current noise I_n
  - Their phase correlation ϕ_{corr}
- ”Mirror” (short) can be replaced by open or match
- Short bypasses I_n
- Open bypasses U_n
- Transmission line corresponds to internal length

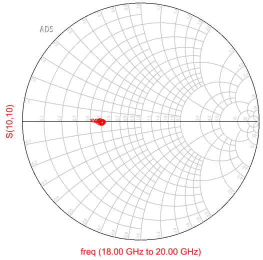


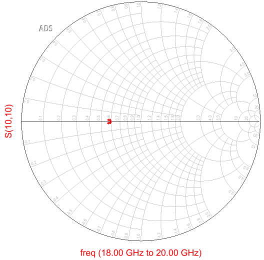
LNA noise model

LNA deembedding

"The old way"

- Noise at gain point
  $\rightarrow$ deembed internal length
- Internal length added to move impedance to real number
- $L_{2022} = 30.5\,\text{mm}, \epsilon_r = 1.4$
- $L_{2023} = 31.95\,\text{mm}, \epsilon_r = 2.1$
- $Z_{2022} = 25\,\Omega$
- $Z_{2023} = 29\,\Omega$



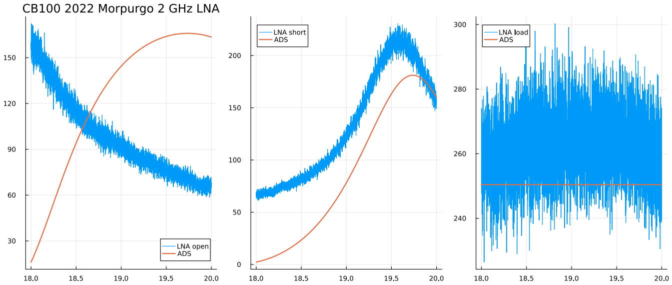


Deembedded smith charts 2022 (top) and 2023 (bottom)

LNA standards

2022

- Noise parameters matched to standard measurements:
  - $V_{\text{noise}} = 520\,\text{pV}$
  - $I_{\text{noise}} = 16.5\,\text{pA}$
  - $\phi_{\text{corr}} = 180\,\text{deg}$

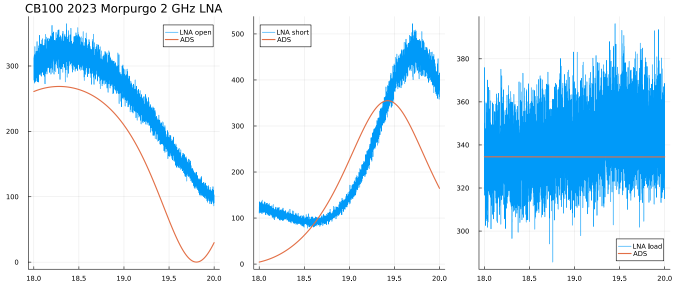


Noise standards 2022

LNA standards

2023

- Noise parameters matched to standard measurements:
  - $V_{\text{noise}} = 700\,\text{pV}$
  - $I_{\text{noise}} = 21\,\text{pA}$
  - $\phi_{\text{corr}} = 290\,\text{deg}$

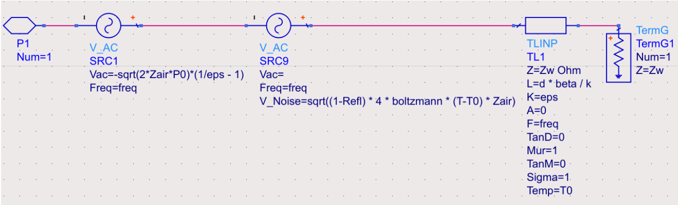


Noise standards 2023

System noise simulation

Thermal emissions

- At temperatures $> 0\,$K, everything emits radiation due to thermal vibrations
- Spectrum defined by black body radiation
- Emissivity $\Leftrightarrow$ absorption (conservation of energy)
- Black body microwave approximation: $V_n = \sqrt{4kTBZ}$
  $\rightarrow$ thermal emissions of mirror: $V_\text{mirror}/\sqrt{B} = \sqrt{(1-\Gamma)\cdot 4kTZ}$
- Interested in longitudinal transmissions
  $\rightarrow$ emissivity of disks & mirror considered

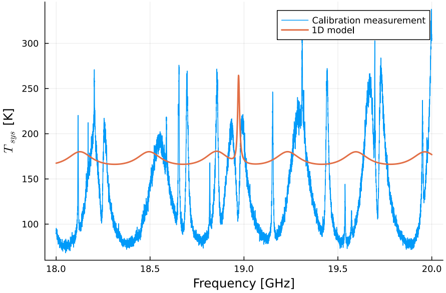


ADS implementation of mirror with thermal emissions

System noise

2022

- New parameters:
  - Temperature $T = 20\,°$C
  - Connection LNA $\leftrightarrow$ CB100 $L_{con} = 131.55\,$mm



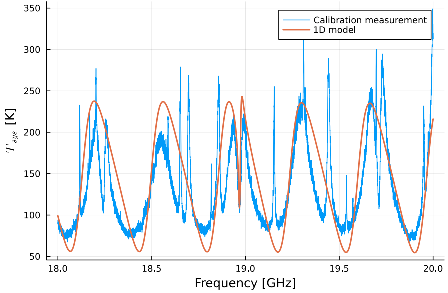
System noise 2022

System noise

2022

> New parameters:

- Temperature $T = 20\,°$C
- Connection LNA $\leftrightarrow$ CB100 $L_{con} = 131.55$ mm
- Changing ϕ_{corr} to from 180 to 325

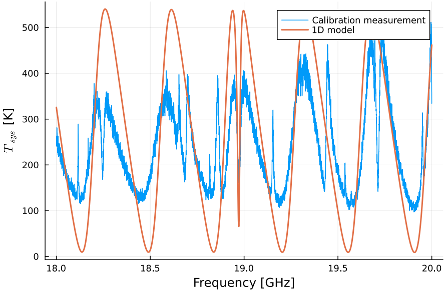


System noise 2022

System noise

2023

- New parameters:
  - Temperature $T = 20\,°\text{C}$
  - Connection LNA $\leftrightarrow$ CB100 $L_{con} = 134.7\,\text{mm}$



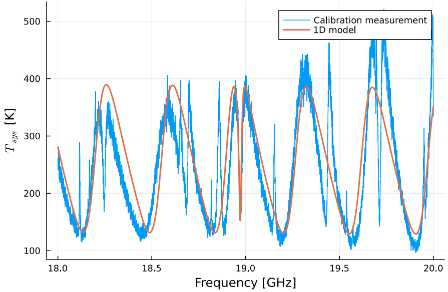
System noise 2023

System noise

2023

> New parameters:

- Temperature $T = 20\,°$C
- Connection LNA $\leftrightarrow$ CB100 $L_{con} = 134.7\,$mm
- Changing ϕ_{corr} to from 290 to 333



System noise 2023

Analysis flow

Optimal case

1. Fit booster reflectivity measurement to get mirror reflectivity and baseline attenuation
2. Fit LNA standard measurements to get LNA noise parameters
3. Fit booster noise measurement to get connection length from LNA to CB100
   $\rightarrow$ Model now spits out a boostfactor

Analysis flow

Suboptimal case

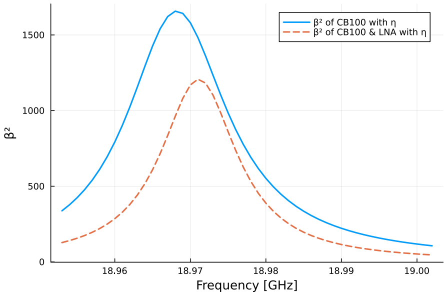
1. Fit booster reflectivity measurement to get mirror reflectivity and baseline attenuation
2. Fit LNA standard measurements to get LNA noise parameters
3. Fit booster noise measurement to get connection length from LNA to CB100 **as well as adjust LNA noise parameters**
   $\rightarrow$ Model now spits out a boostfactor

Parameter overview

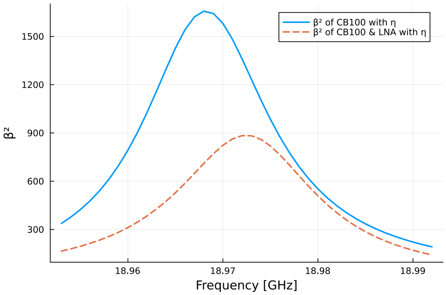
Complete model parameter overview

Year	R_{mirror}	A [dB]	Z_{LNA} [Ω]	L_{int} [mm] ($\epsilon_r = 2.1$)	V_n [pV]	I_n [pA]	ϕ_{corr}	L_{con} [mm]	T [°C]
2022 (LNA)	0.99955	$0.0058\nu/c_0$	25	24.9	520	16.5	**180**	131.55	20
2022 (booster)	0.99955	$0.0058\nu/c_0$	25	24.9	520	16.5	**325**	131.55	20
2023 (LNA)	0.99955	$0.0058\nu/c_0$	29	31.95	700	21.0	**290**	134.7	20
2023 (booster)	0.99955	$0.0058\nu/c_0$	29	31.95	700	21.0	**333**	134.7	20

Boostfactor



Boostfactor 2022



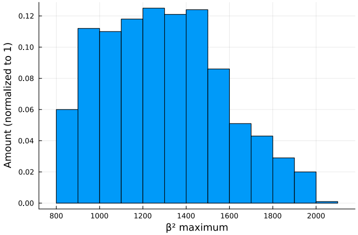
Boostfactor 2023

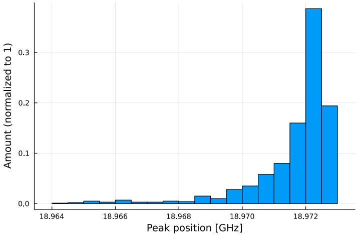
Uncertainties

- MonteCarlo error study (1000 samples) of β^2

Uncertainties considered

I_{noise} [pA]	V_{noise} [pV]	ϕ_{corr}	L_{con} [mm]	T [°C]
21 ± 1.05	700 ± 35	333 ± 50	134.7 ± 0.5	20 ± 5

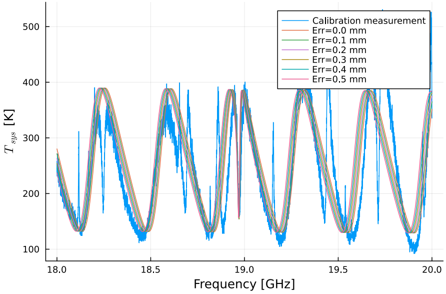




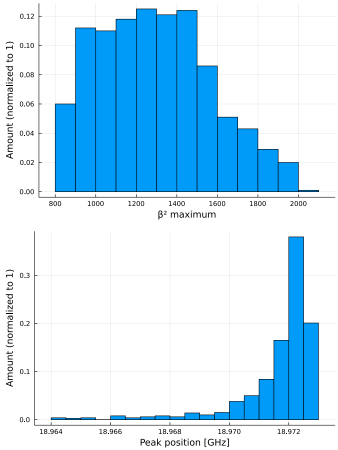
β^2 uncertainties

Uncertainties 2023

System noise



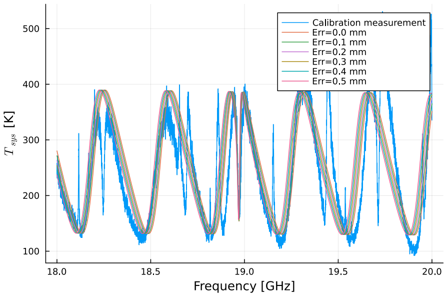
Noise difference for different L_{con}



$$L_{\text{con}} = (135.7 \pm 0.5)\ \text{mm}$$

Uncertainties 2023

System noise



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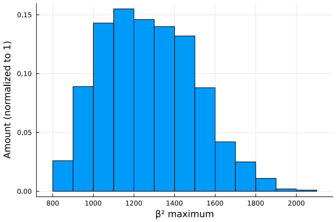
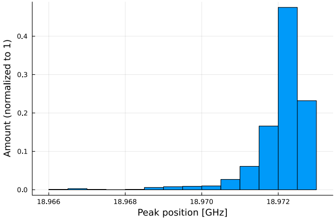
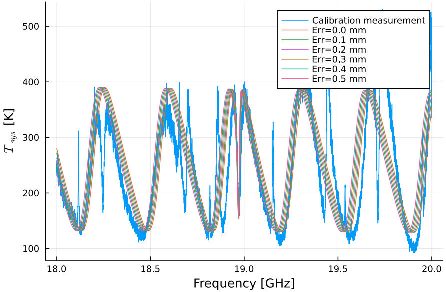




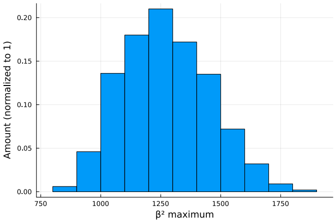
$$L_{\text{con}} = (135.7 \pm 0.4) \text{ mm}$$

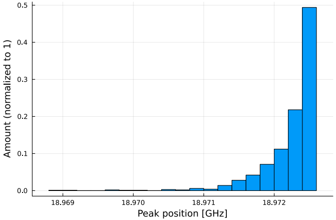
Uncertainties 2023

System noise



Noise difference for different L_{con}

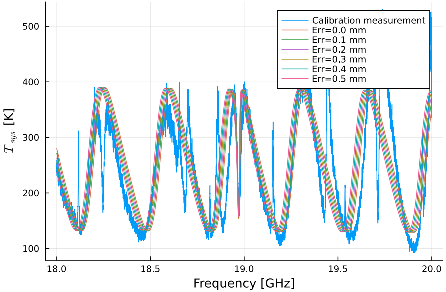




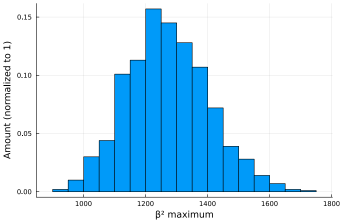
$$L_{\text{con}} = (135.7 \pm 0.3)\ \text{mm}$$

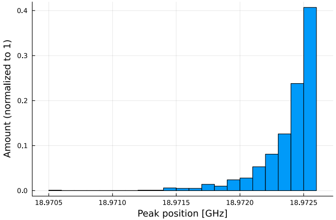
Uncertainties 2023

System noise



Noise difference for different L_{con}

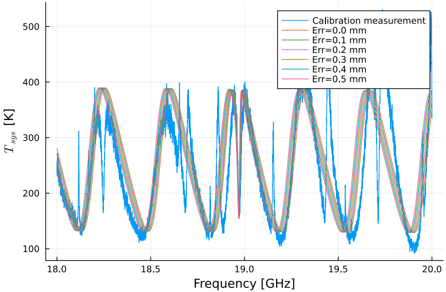




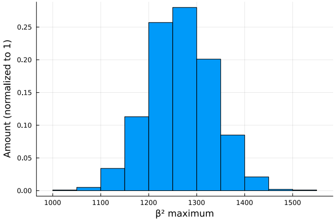
$$L_{\text{con}} = (135.7 \pm 0.2)\ \text{mm}$$

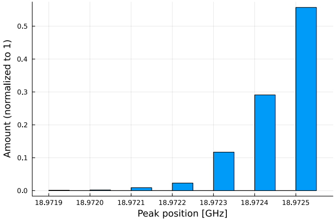
Uncertainties 2023

System noise



Noise difference for different L_{con}

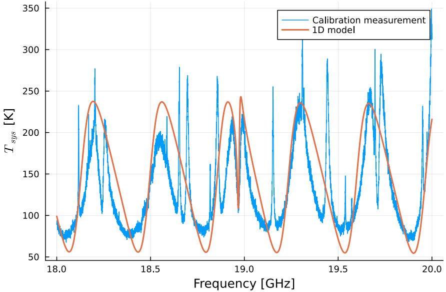




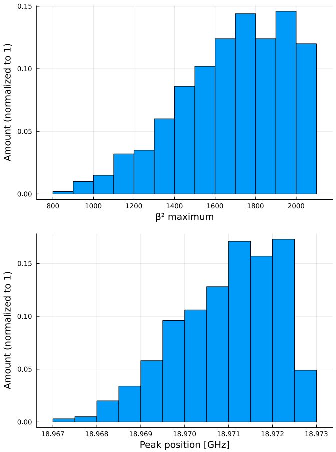
$$L_{\text{con}} = (135.7 \pm 0.1)\ \text{mm}$$

Uncertainties 2022

System noise



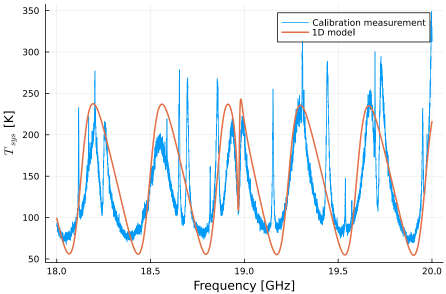
Noise difference for different L_{con}



$$L_{\text{con}} = (131.55 \pm 0.5)\text{ mm}$$

Uncertainties 2022

System noise



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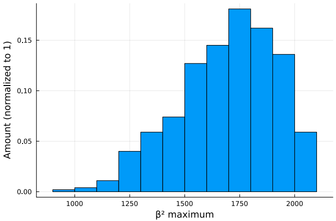
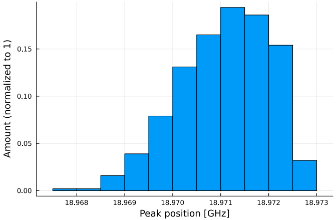
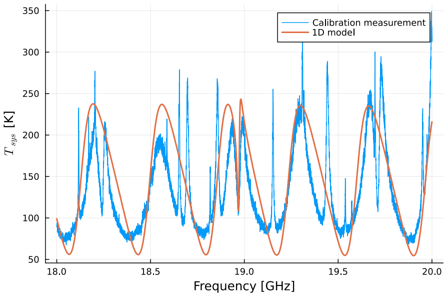




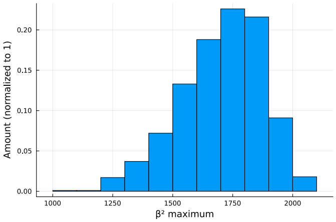
$$L_{\text{con}} = (131.55 \pm 0.4)\ \text{mm}$$

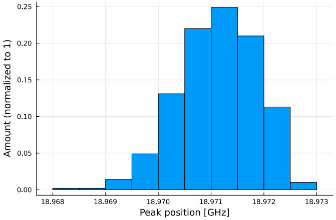
Uncertainties 2022

System noise



Noise difference for different L_{con}

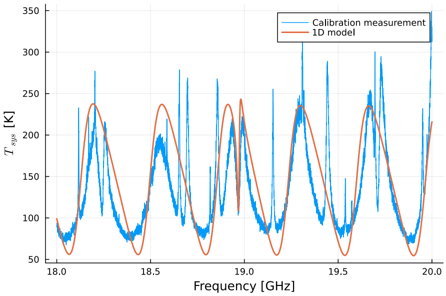




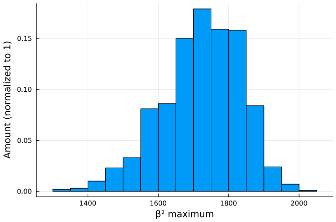
$$L_{\text{con}} = (131.55 \pm 0.3)\ \text{mm}$$

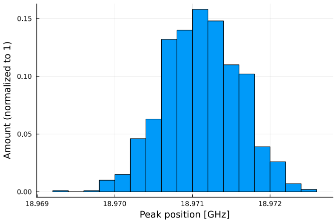
Uncertainties 2022

System noise



Noise difference for different L_{con}

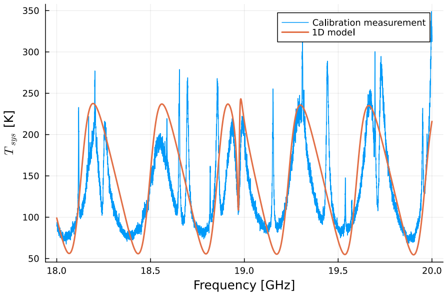




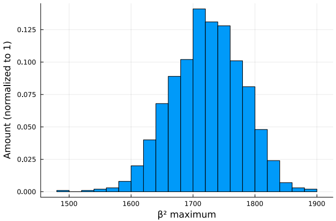
$L_{\text{con}} = (131.55 \pm 0.2)$ mm

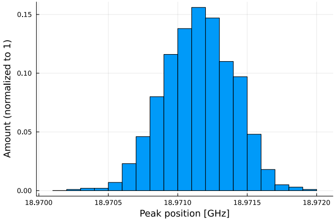
Uncertainties 2022

System noise



Noise difference for different L_{con}





$$L_{\text{con}} = (131.55 \pm 0.1)\ \text{mm}$$

Discussions/TODO

Uncertainties

Boostfactor

- Dominant error source: LNA $\leftrightarrow$ CB100 connection

How to determine errors?

- **Current status**
  - Currently, only error on noise parameters and connection length considered
  - Errors estimated from intuition
- **The proper way (?)**
  - All parameters have uncertainties!
    - Geometry from measurements?
    - LNA parameters from fit?
    - What about Y-factor calibration? ($\rightarrow$ our dataset is uncertain)
  - Dicussion: How to propagate them?

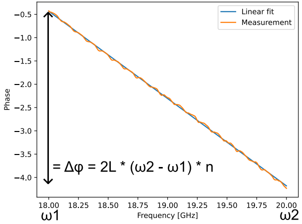
Fitting procedure

- Currently, fits are mostly manual
- Automation with Julia implemented
- Main issue: cost function?
  $\rightarrow$ Model doesn’t fit all peaks, cut them out?
- Very slow due to ADS

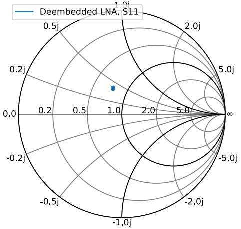
LNA deembedding

New proposal

- Internal length calculated by phase difference $\Delta\phi$ between ω_1 and ω_2:
  $L = \frac{c_0 \cdot \Delta\phi}{2 \cdot (\omega_2 - \omega_1)\sqrt{\epsilon_r}}$
  $L_{2023} = 30.87$ mm (before: $L_{2023} = 31.95$ mm)
- Results in complex impedance:
  $Z_{\text{LNA,2023}} = (39.34 + 19.38i) \pm (1.29 + 1.75i)\ \Omega$
  $Z_{\text{LNA,2022}} = (24.41 + 2.47i) \pm (1.45 + 0.78i)\ \Omega$
  $\rightarrow 2\,\%$ uncertainty on calibration factor



LNA S11 phase with linear fit (2023)

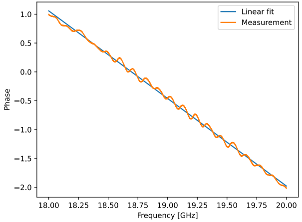


Resulting deembedded S11 (2023)

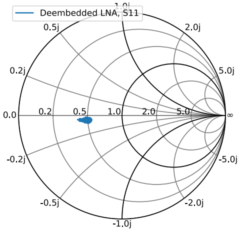
LNA deembedding

New proposal

- Internal length calculated by phase difference $\Delta\phi$ between ω_1 and ω_2:
  $L = \frac{c_0 \cdot \Delta\phi}{2 \cdot (\omega_2 - \omega_1)\sqrt{\epsilon_r}}$
  $L_{2022} = 25.01\,\text{mm}$ (before: $L_{2022} = 20.33\,\text{mm}$)
- Results in complex impedance:
  $Z_{\text{LNA,2023}} = (39.34 + 19.38i) \pm (1.29 + 1.75i)\ \Omega$
  $Z_{\text{LNA,2022}} = (24.41 + 2.47i) \pm (1.45 + 0.78i)\ \Omega$
  $\rightarrow 2\,\%$ uncertainty on calibration factor



LNA S11 phase with linear fit (2022)



Resulting deembedded S11 (2022)

LNA deembedding

Discussion

- Internal length will matter when we have precise L_{con} measurement
  $\rightarrow$ might tell us which approach is better
- LNA transmission: $1-\Gamma$? Or $1-|\Gamma|$? Or just 1?
- Fit to standards: analytical model?
  $\rightarrow$ easy if there was no internal length
  - Basic idea (short): $T_{LNA} = e^{i\omega n d_{int}} \cdot -1 \cdot e^{i\omega n d_{int}} \cdot \Gamma$
    $V_{LNA} = \frac{V_n}{1-T_{LNA}}$
    $P = |V_{LNA} \cdot \text{S21}|^2/(2 \cdot 50)$
  - Mixing of V_n and I_n more complicated

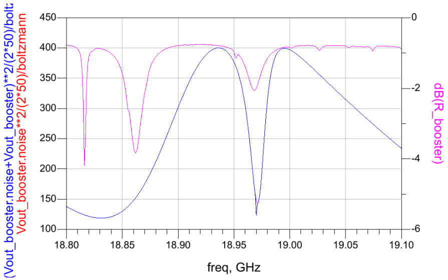
Axion signal

Signal properties:

- $\Delta\nu = (v_{\text{virial}}/c)^2 \cdot \nu \simeq 10\,\text{kHz}$
- $\tau_c = 1/\Delta\nu \simeq 100\,\mu\text{s}$

Power detected:

- Sum of Axion signal and noise: $P \propto |E_n + E_a|^2$
- Both signals have a random phase offset!
  $\rightarrow P \propto |e_n e^{i\phi_n} + e_a e^{i\phi_a}|^2 = |E_n(1 + \frac{e_a}{e_n} e^{i\delta\phi})|^2$
- $P_{avg} \rightarrow \int_0^{2\pi} |E_n(1 + \frac{e_a}{e_n} e^{i\delta\phi})|^2 d(\delta\phi) = 2\pi\left(e_a^2 + e_n^2\right)$



Simulated Axion signal

Summary ADS

Done

- Full chain from measurement to boostfactor with some manual work

Preliminary results

- $\beta^2_{2023,\eta} \simeq 875(200)$
- $\nu_{\beta,2023} \simeq 18.9725(10)$ GHz
- $\beta^2_{2022,\eta} \simeq 1225(200)$
- $\nu_{\beta,2022} \simeq 18.9710(10)$ GHz

TODO

- Automated fits?
- Solve LNA model analytically?
- Uncertainties!
  - Determine uncertainties on all parameters
  - Define error propagation procedure

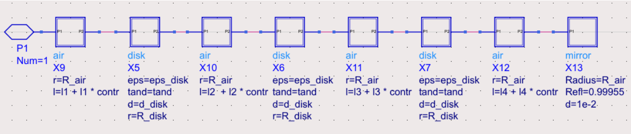
ADS implementation

Booster implementation

Overview

Booster is a one-port device which consists of:

- Air parts
- Disk parts
- Mirror



Booster in ADS

Booster implementation

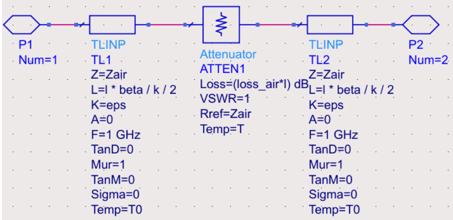
Air

Air consists of:

- Transmission line with impedance and length from waveguide theory
- Attenuator for losses

Losses scale by real element length!
$\rightarrow$ therefore manually scaled and not put into the TLINP, which has a ”virtual” length

- Temperature set on attenuator
  $\rightarrow$ emissivitiy from losses



Air implementation

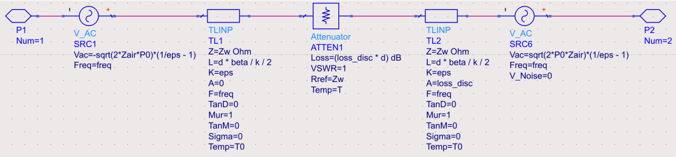
Booster implementation

Disk

- Basically the same as Air
- Additional Axion emissions

$$V_{ac} = -\sqrt{2 Z_{air} P_0}\left(\frac{1}{\epsilon_r} - 1\right)$$

(peak voltage)

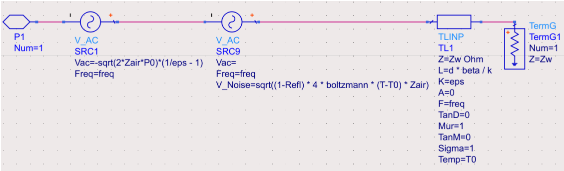


Disk implementation

Booster implementation

Mirror

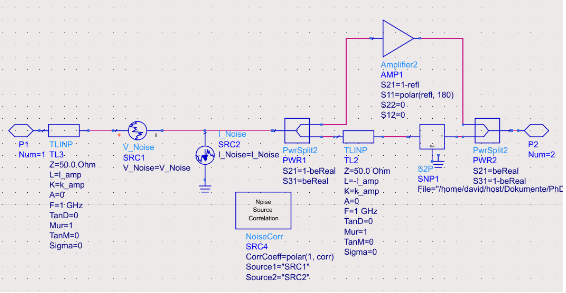
- Mirror simulated with high ϵ_r on a TLINP
- ϵ_r from reflectivity
- Emissivitiy from reflectivity
- Manual emissivitiy as losses are not set on TLINP
- Thermal voltage from Pozar: $V_n/\sqrt{B} = \sqrt{4k_BTZ}$ (Microwave approximation)
- V_n is RMS, but only one side of mirror into booster, do they cancel out?



Mirror implementation

LNA implementation

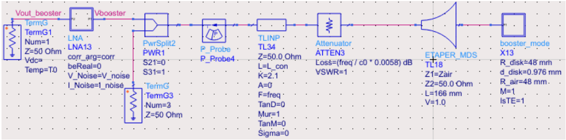
- Power splitter used for conditionally using either measurement or ideal amp
- TLINP with negative length for deembedding internal length
- No gain on ideal amp, due to calibrated data



LNA implementation

Complete chain

- ETAPER_MDS to go from $50\,\Omega$ to waveguide impedance
- Attenuator to fit reflectivity baseline
- Power probe to determine boostfactor
- Power splitter to look at system without LNA



Booster chain implementation

Usage

Preparation

1. Clone gitlab repository
2. Download Morpurgo data
3. Create directory with ”ADS_data” and ”ADS_sim” directories inside
4. Use ”convert_to_ads.ipynb” to fill ”ADS_data” directory
5. Add ”params.dscr” to ADS_data

Usage

ADS setup

1. Open ADS project
2. Open ”setup” schematic
3. Change ”data_dir” variable to directory containing ADS_data and ADS_sim
4. Open ”booster_chain” or ”lna_tests’ schematic
5. Click the simulate button (gears in the toolbar)

Backup

Power calibration

Y-factor

- Use y-factor method to find calibration factor to go from P [W] to T [K]
- Big difference between receiver chain and SA
- Freq dependencies:

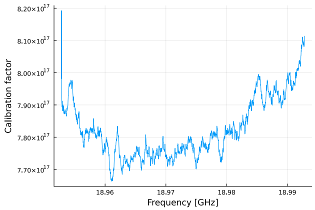
$$Y(\nu) = \frac{N_{on}(\nu)}{N_{off}(\nu)}$$

$$T_e(\nu) = \frac{T(\nu)_{hot} - Y(\nu)T_{cold}}{Y(\nu) - 1}\frac{1-\Gamma(\nu)}{1+\Gamma(\nu)}$$

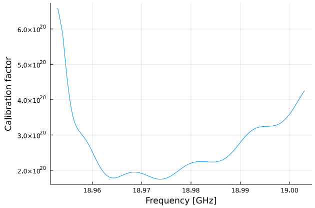
$$T_{load,amp}(\nu) = T_e(\nu)(1+\Gamma(\nu)) + T_{cold}(1-\Gamma(\nu))$$

$$C(\nu) = \frac{T_{load,amp}(\nu)}{N_{off}(\nu)}$$

  - Currently Y & Γ is averaged
  - Noise measurements need be smoothed a lot!
  - Y is very close to unity ($\sim$1.09)



Calibration factor 2023 (50 MHz)



Calibration factor 2022 (50 MHz)

Power calibration

Y-factor

- Use y-factor method to find calibration factor to go from P [W] to T [K]
- Big difference between receiver chain and SA
- Freq dependencies:

$$Y(\nu) = \frac{N_{on}(\nu)}{N_{off}(\nu)}$$

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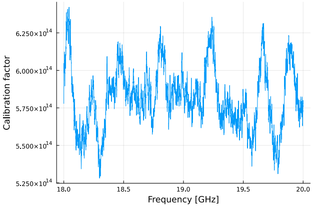
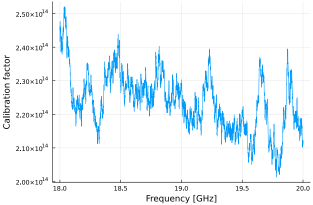


Calibration factor 2023 (2 GHz)



Calibration factor 2022 (2 GHz)

Shortcomings of ADS

- Lacking Documentation
- Slow simulation
- Bad integration

$\rightarrow$ **Alternatives?**

scikit-rf

- Well supported OpenSource python package
- In case of doubt, source available to understand what is happening
- Experience in fermilab group (esp noise simulation)