holography

chaos $\hookrightarrow$ quantum gravity

gravitational μ-states

GR

Late time chaos in 2d gravity

WPC Symposium, Nov. 23

Alexander Altland (Cologne),
Julian Sonner (Geneva),
Boris Post, Jeremy van der Hayden, Erik Verlinde (Amsterdam)

quantum chaos review

semiclassical chaos in gravity

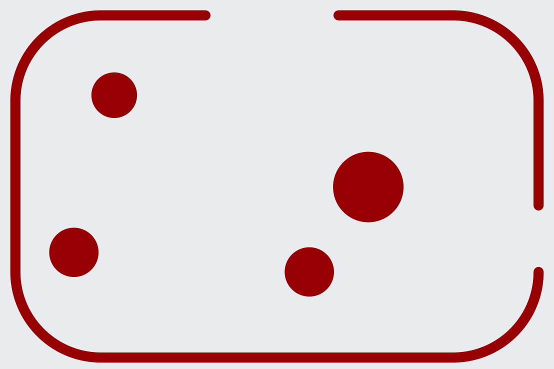
quantum chaos in gravity

arXiv:2008.02271, SciPost. 22
arXiv:2204.07583, SciPost 23

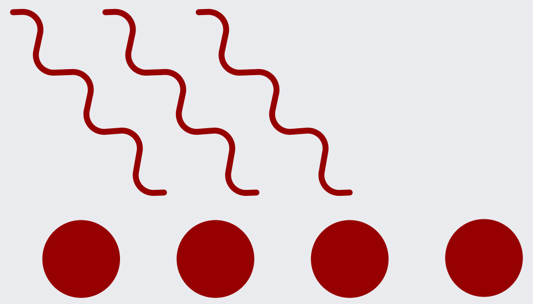
chaos (review)

The ergodic phase of quantum chaos

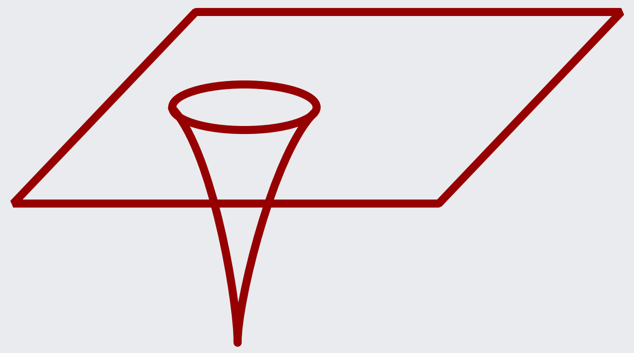
Realized in



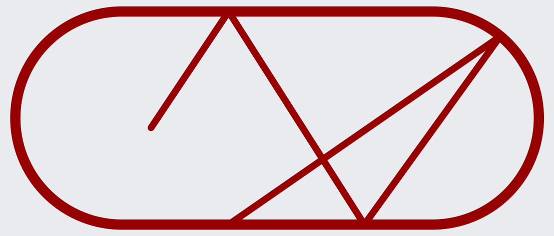
cond-mat



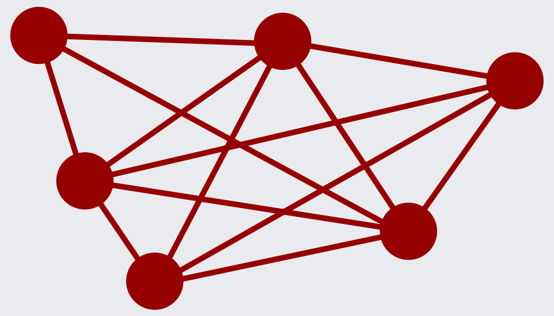
AMO



gravity



few body quantum chaos

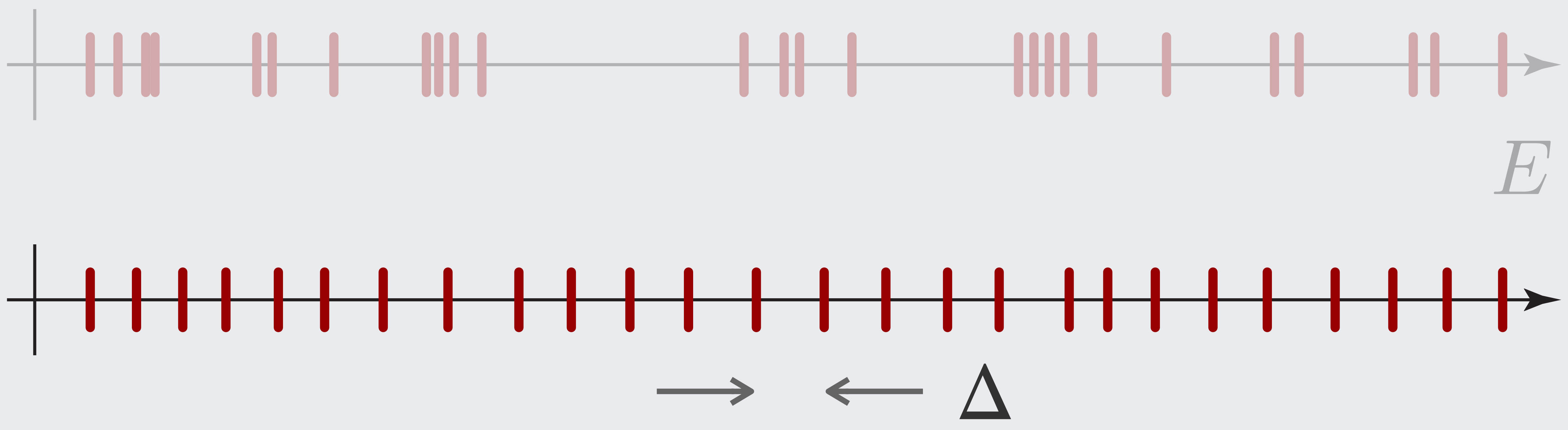


strongly correlated systems

The ergodic phase of quantum chaos

- states uniformly Gaussian distributed in Hilbert space (max. entropy, cf. ETH)

- spectrum shows high degree of order



integrable

chaotic

$$\boxed{s = \frac{\pi\omega}{\Delta}}$$

quantitatively described by

$$R_2(\omega) \equiv \frac{1}{\Delta^2}\langle \rho(E+\omega)\rho(E)\rangle_c \longrightarrow R_2(s) \longrightarrow \left\{ \begin{array}{l} \text{10 different antilinear symmetry classes:} \\ \text{A, AI, AII, AIII, BDI, C, CI, CII, D, DIII} \end{array} \right.$$

aa,
Zirnbauer , 97

Spectral correlation function

$$\frac{c_1}{s^2} + \frac{c_2}{s^3} + \ldots$$

deep quantum

(non-analytic in s^{-1})

semiclassical

(expansion in s^{-1})

Spectral form factor (favored by holography community)

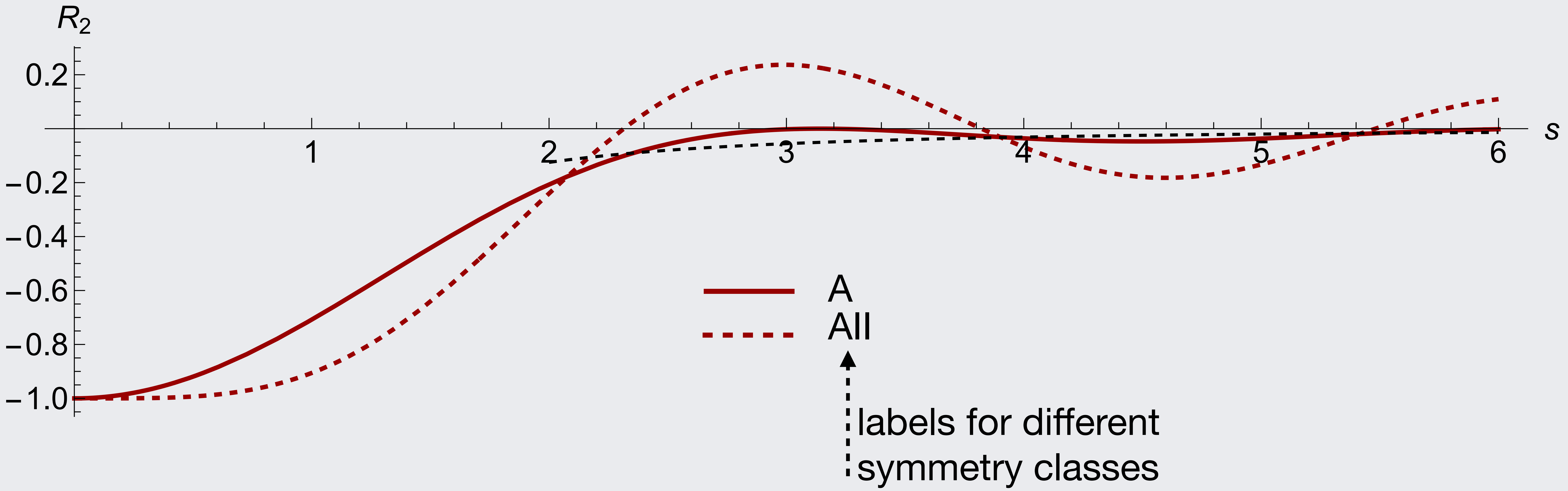
semiclassical

(expansion in τ)

deep quantum

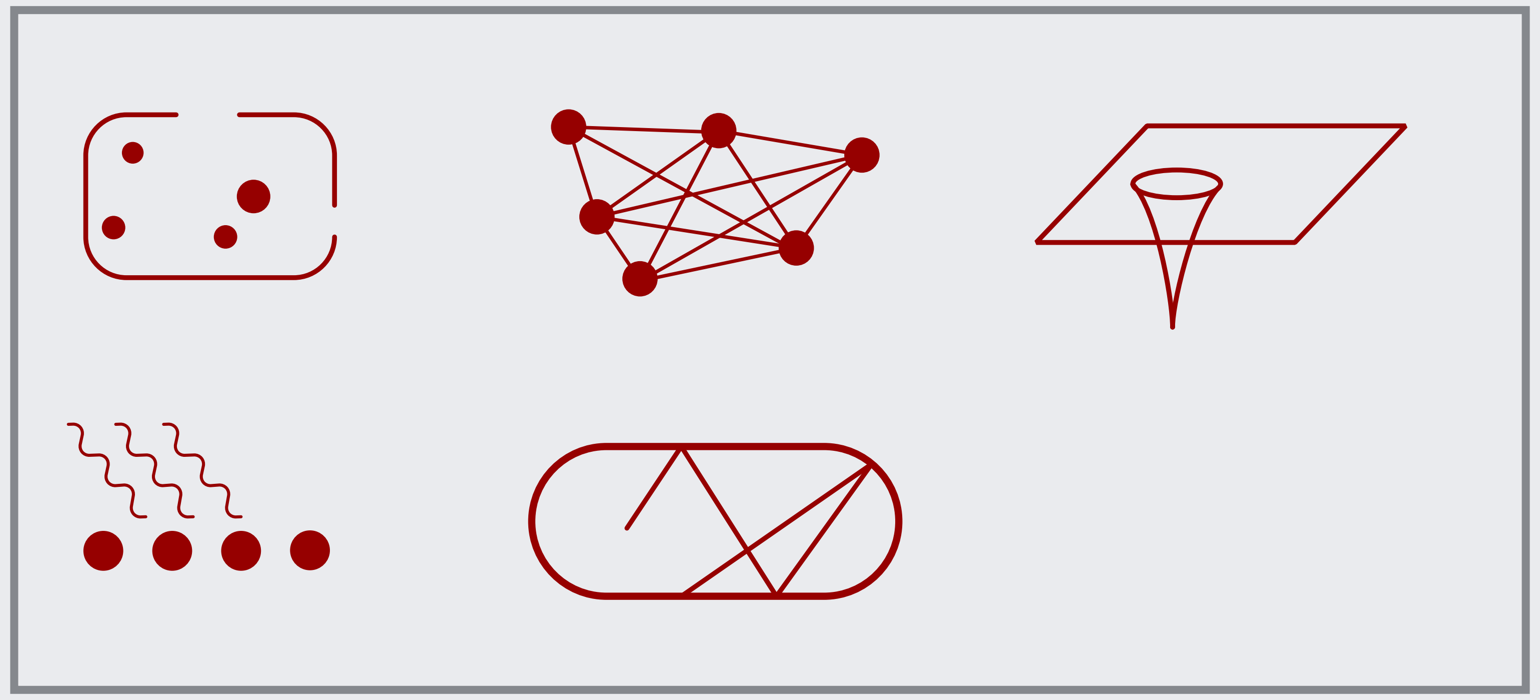
(non-analytic in τ)

Spectral universality



$$R_2(s) = -\frac{\sin^2(s)}{s^2}$$ A (GUE)

$$R_2(s) = \frac{\sin^2(2s)}{(2s)^2} - \frac{d}{d(2s)}\frac{\sin(2s)}{2s}\int_0^1 \frac{\sin(2st)}{t}$$ AII (GSE)



Shown by a large class of chaotic systems.

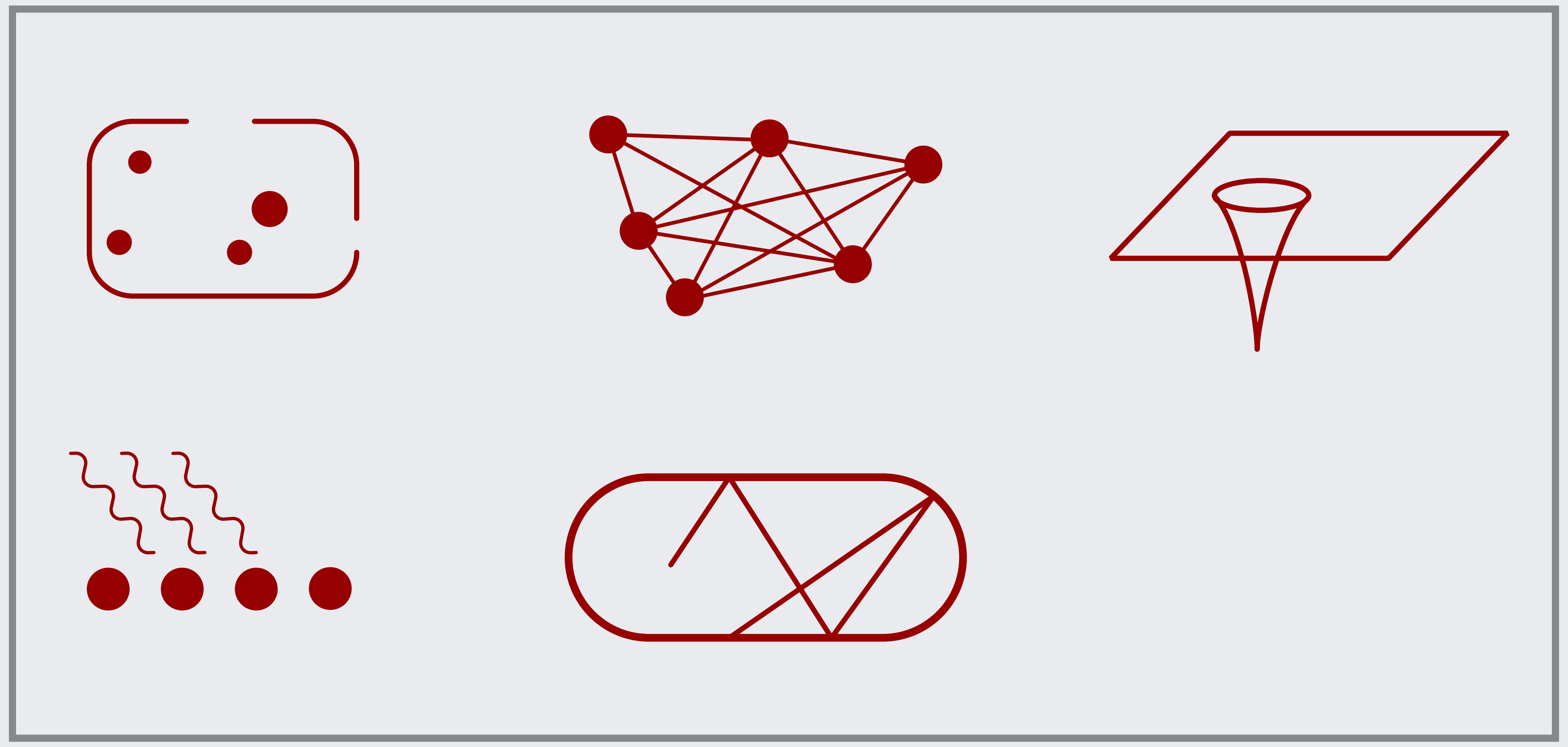
Q: How can this level of universality be understood?

Understanding universal spectral correlations

Constructive approach (aka Bohigas–Gianonni–Schmit (GGS) conjecture)

RMT $\boxed{H = \{H_{\mu\nu}\}, \qquad \mathrm{var}(H_{\mu\nu}) = \mathrm{const}\,.}$ Wigner 1955

$\downarrow$



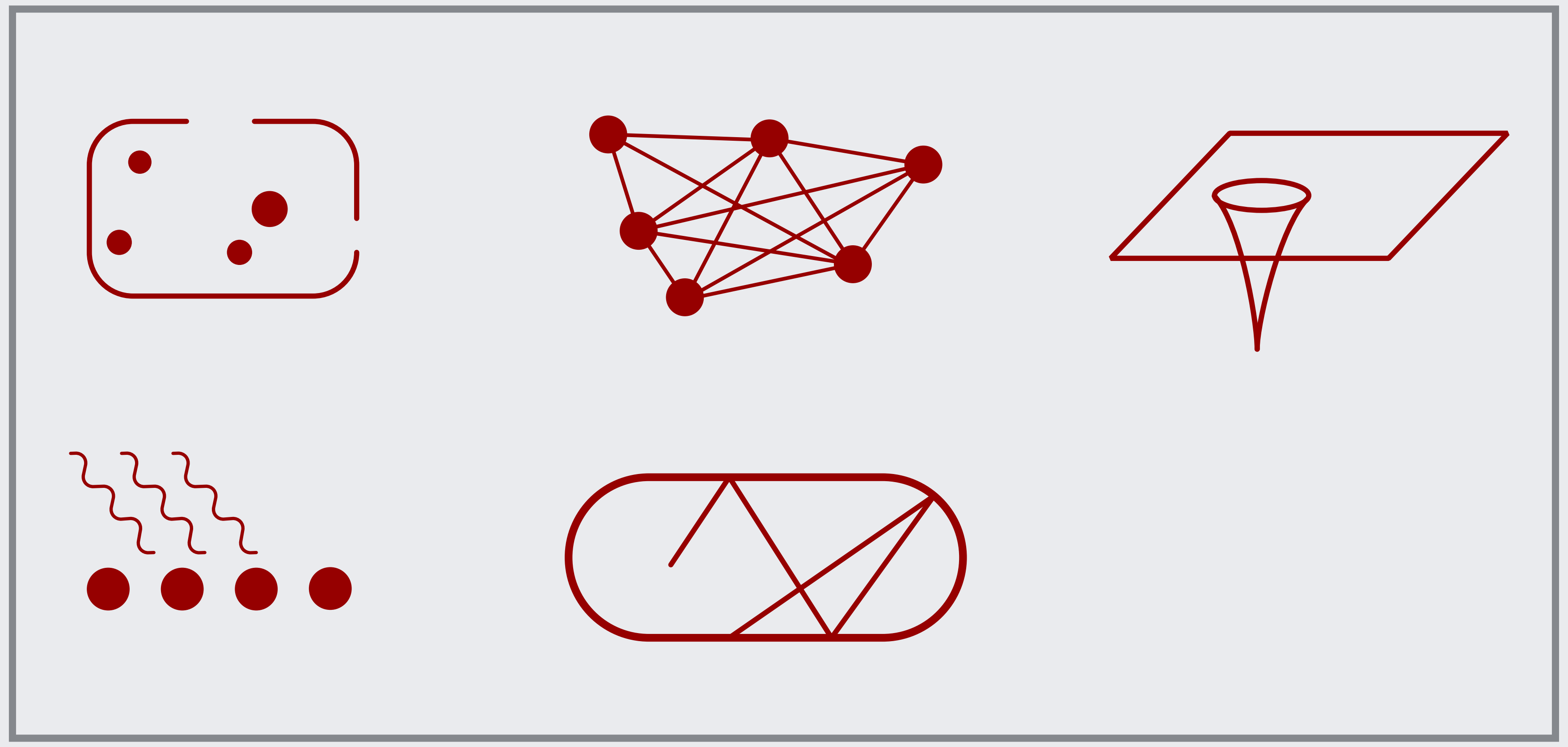
chaotic systems in cond-mat, AMO, …, gravity.

$\xrightarrow[\text{long times}]{\text{low energies}}$ $\boxed{\text{universal correlations}}$

cf. “Ising model of magnetism”

Understanding universal spectral correlations

Conceptual approach



chaotic systems in cond-mat, AMO, …, gravity.

$\xrightarrow[\text{long times}]{\text{low energies}}$ **effective theory**

Wegner 1980
Efetov 1981

cf. “ϕ^4 theory of magnetism”

Understanding universality (from matrix theory)

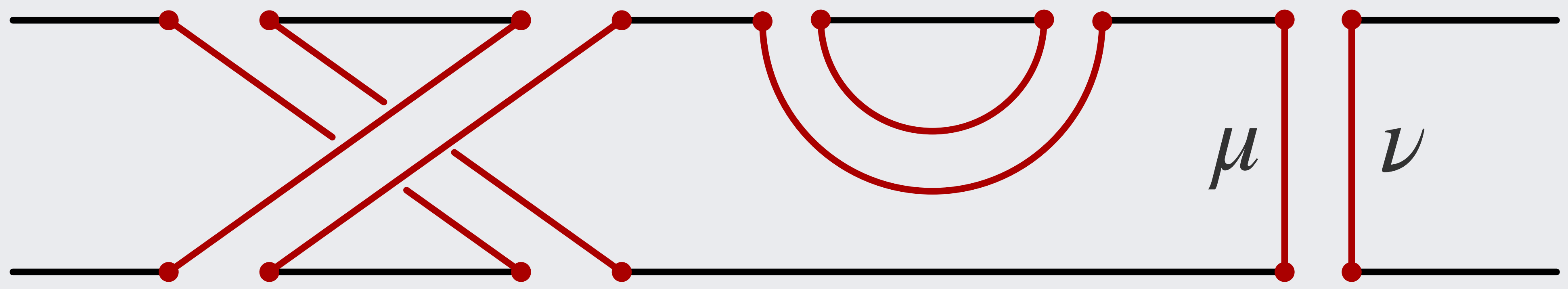
$$H = \{H_{\mu\nu}\} \longrightarrow \left\langle \mathrm{tr}\frac{1}{E+i0-H}\ \mathrm{tr}\frac{1}{E'-i0-H}\right\rangle, \qquad E-E' \sim s\Delta$$

- hep-th $\rightarrow$ topology
- cond-mat $\rightarrow$ correlations

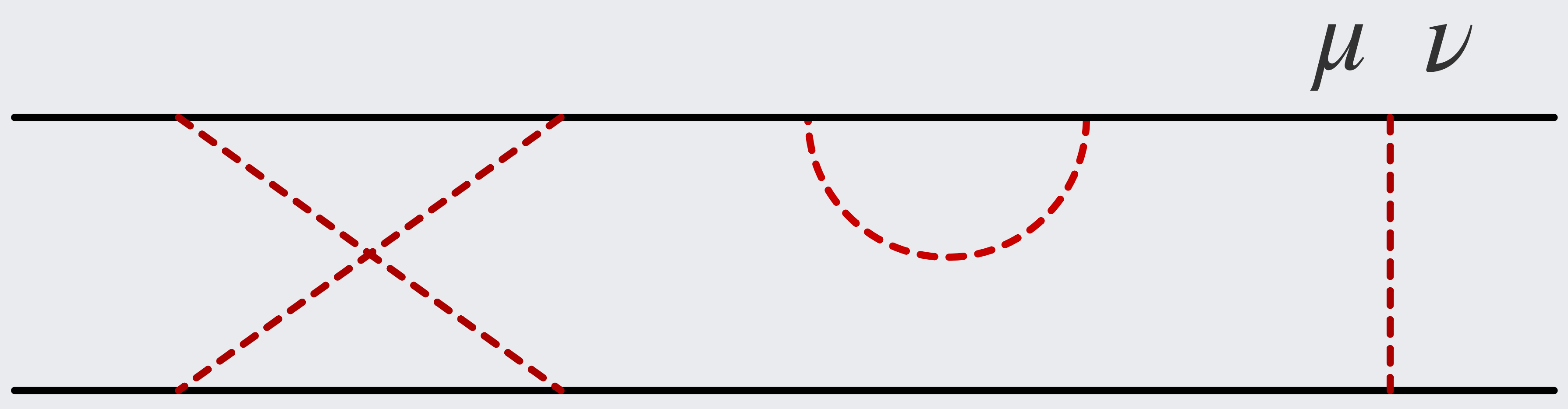
$$\downarrow$$

$\boxed{\text{expansion in } H}$

$$\downarrow$$



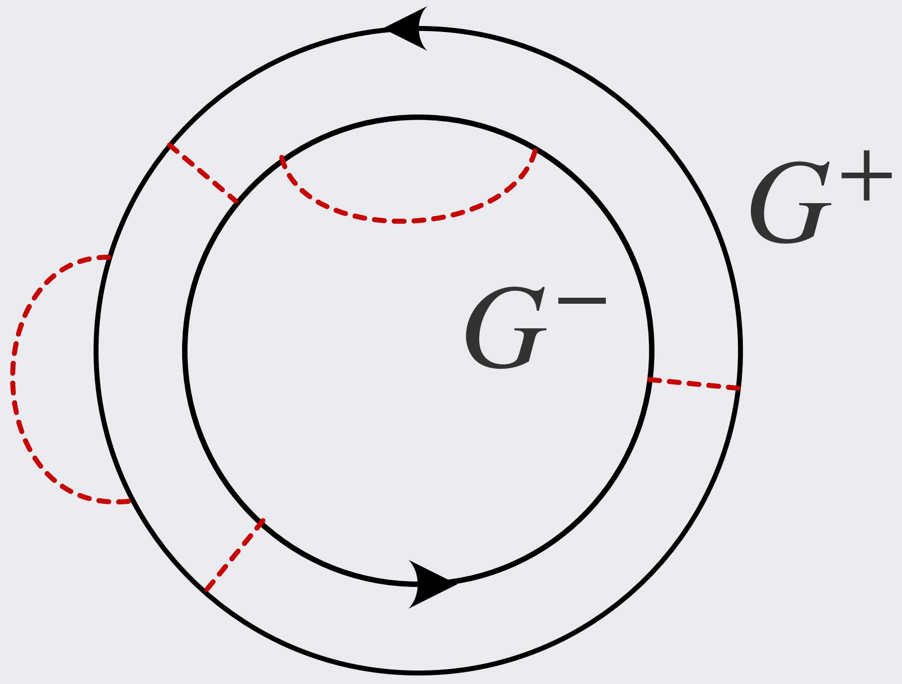
hep-th: “double line notation”



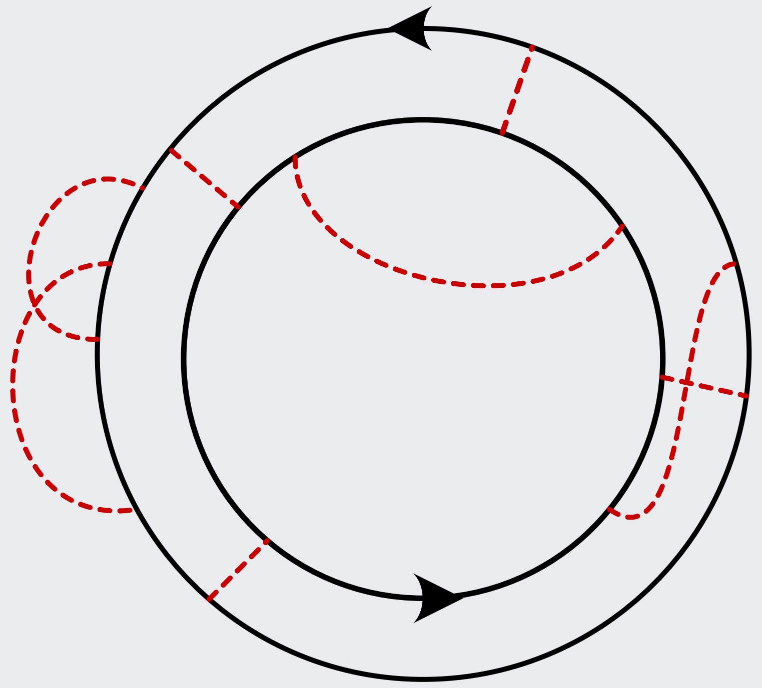
cond-mat: “impurity diagram notation”

Perturbative expansion – topological perspective

$g = 0$



$g = 1$



$\vdots$

topological expansion for 2-point function:

$$\langle G^+ G^- \rangle = \sum_g D^{-2g} R_{g,2}$$

topological recursion (symbolically):

Eynard-Orantin, 07

$$R_{g,n} = \mathscr{F}(R)$$

$$R = \{R_{h,n} \,|\, n = 1,2,3; h \leq g\}$$

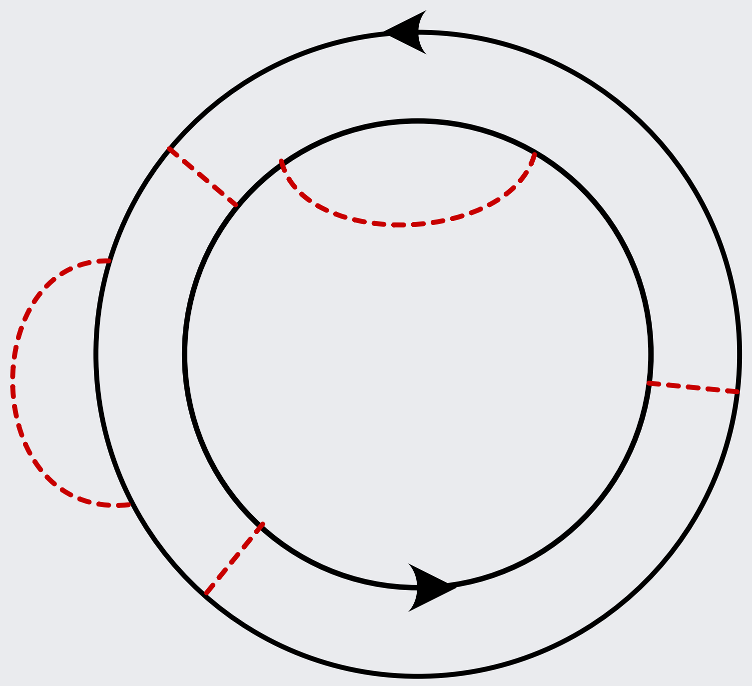
Disclaimer: for ensembles with T-invariance (AI, AII, CI, ⋯) need non-orientable surfaces

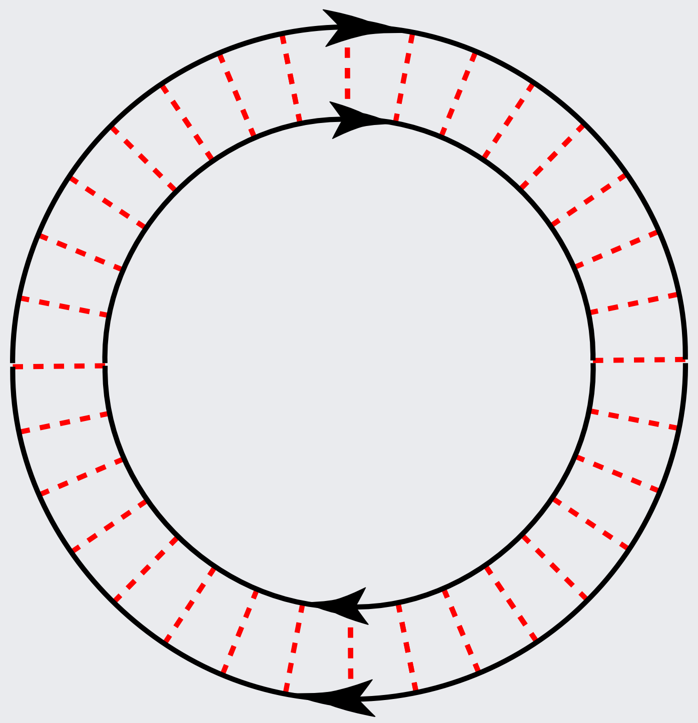
Brezin & Neuberger, 90
Stanford & Witten, 19

Perturbative expansion — correlation perspective

Berry, 85
Altshuler & Shklovskii, 86

$g = 0$

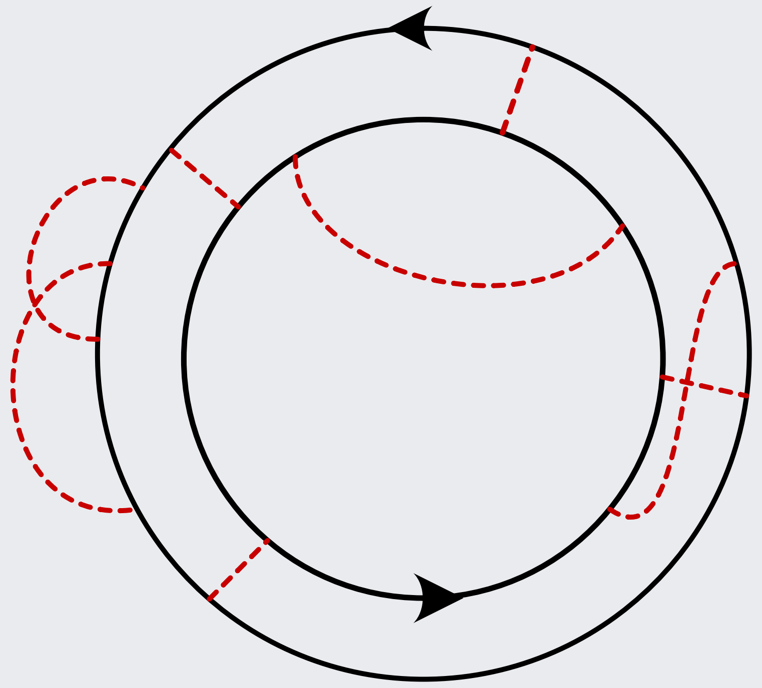


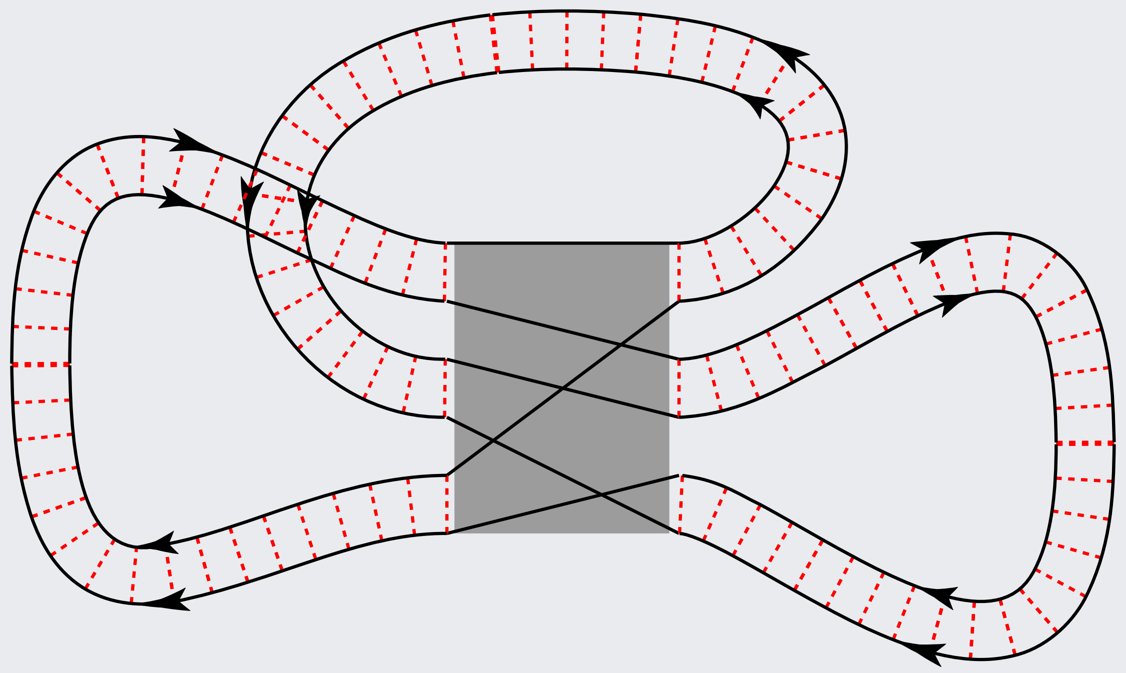


s^{-2}

cf. Aleiner & Larkin, 96
cf. Sieber & Richter, 01

$g = 1$

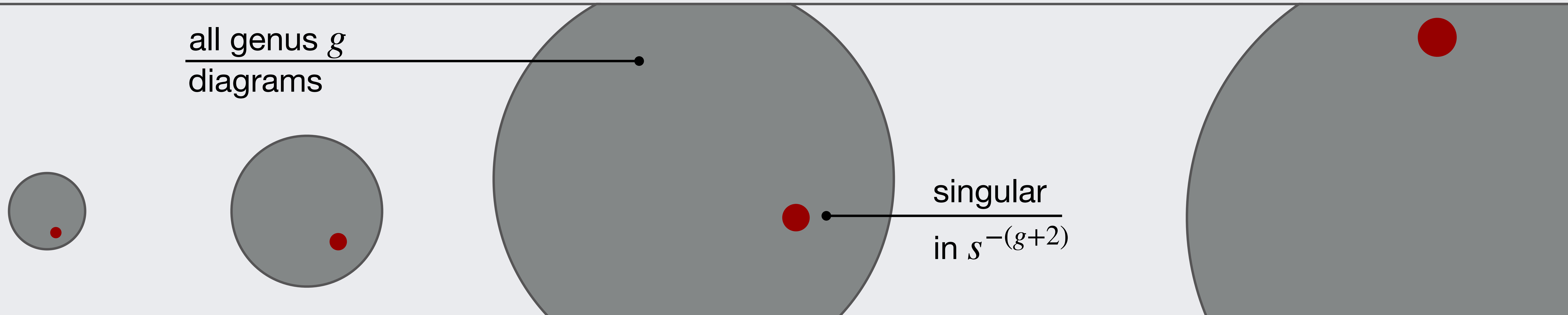




s^{-3}

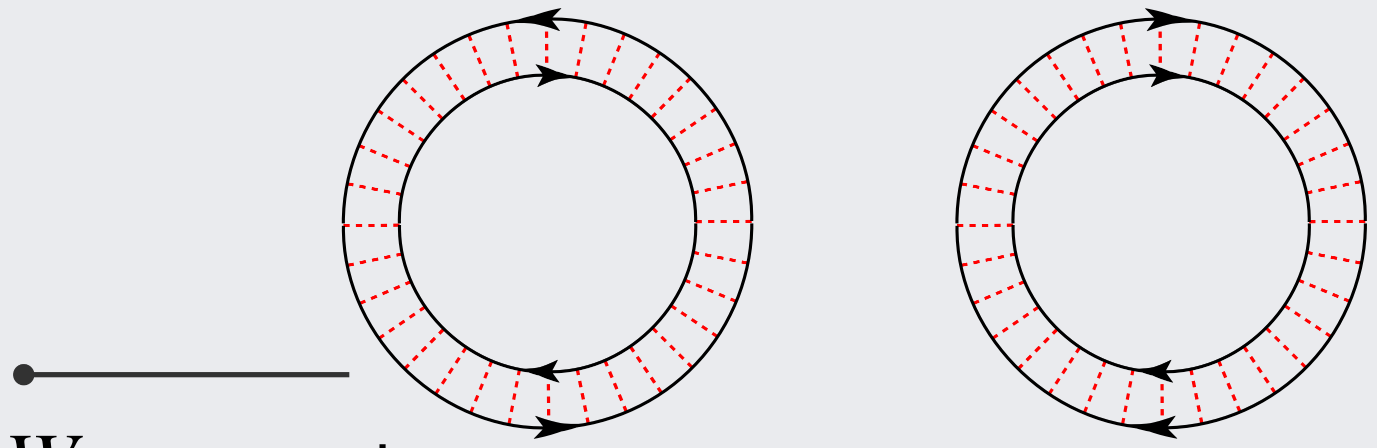
$\vdots$

$s^{-(g+2)}$ diagrams:
small subset of $R_{g,2}$

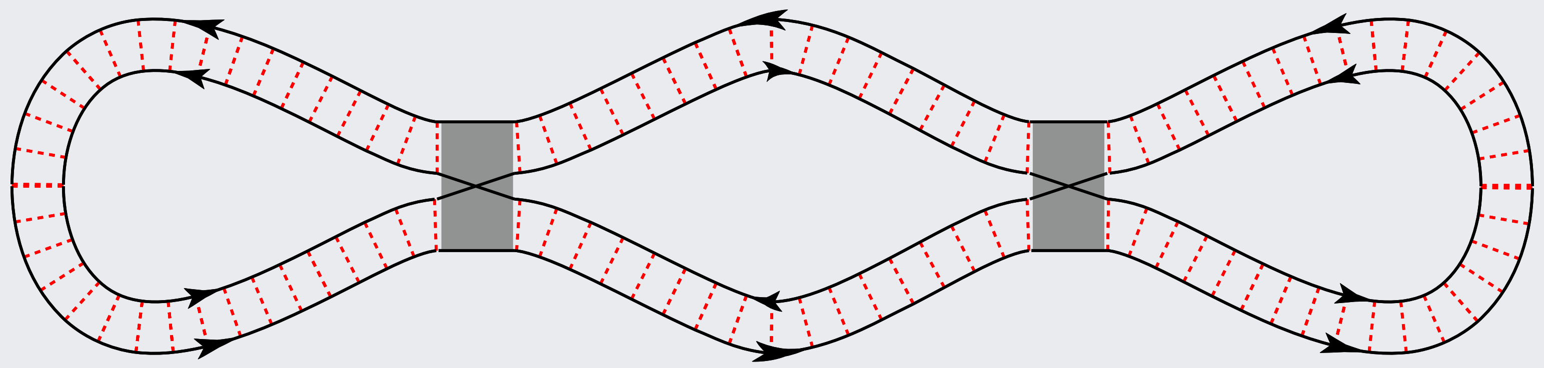


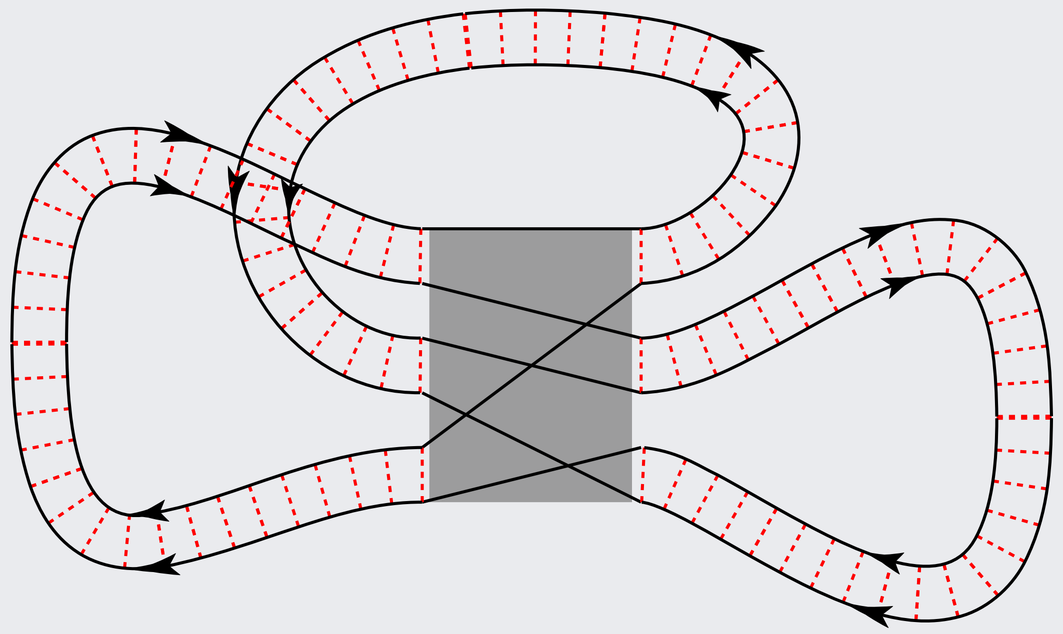
Perturbative expansion — (mean) field theory

$$Q = T\tau_3 T^{-1}, \quad T = \exp(W)$$



W-propagator
$\sim s^{-1}$





$$\boxed{Z = \int_{X(n|n)} dQ\, e^{is\,\mathrm{tr}(Q\tau_3)}}$$

Wegner 1980
Efetov 1981

effective theory of
ergodic quantum chaos

- Q: low dimensional (flavor) matrices
- diagrams: loop expansion
- full integration: correlations beyond perturbation theory

semiclassical chaos in gravity

Holography background

The holographic principle: d-dimensional gravitational systems cast $(d-1)$-dimensional holographic shadows.*

Black holes are chaotic systems.

't Hooft 93, Susskind 95

> 2015 search for a simple dimensional holographic correspondence between 1-dimensional quantum chaotic boundary theory and 2-dimensional gravity theory.

Kitaev 15, …

* classic example: gravity in $\mathrm{AdS}_5 \times S^5$ $(d=5) \rightarrow \mathcal{N}=4$ super Yang-Mills $(d=4)$

Maldacena 97

Sachdev-Ye-Kitaev Model (15)

A model of *N* randomly interacting *Majorana* fermions

$$\boxed{\hat{H} = \sum_{ijkl}^{N} \underbrace{J_{ijkl}}_{\text{random}} \hat{\chi}_i \hat{\chi}_j \hat{\chi}_k \hat{\chi}_l, \qquad \{\hat{\chi}_i, \hat{\chi}_j\} = 2\delta_{ij}}$$

SYK model

Kitaev 15

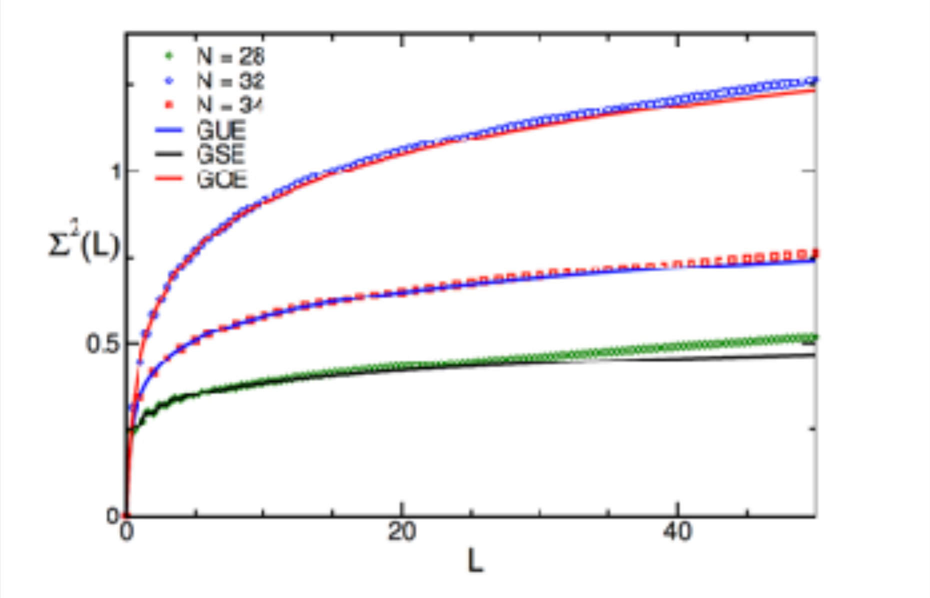
cf. Sachdev, Ye 90

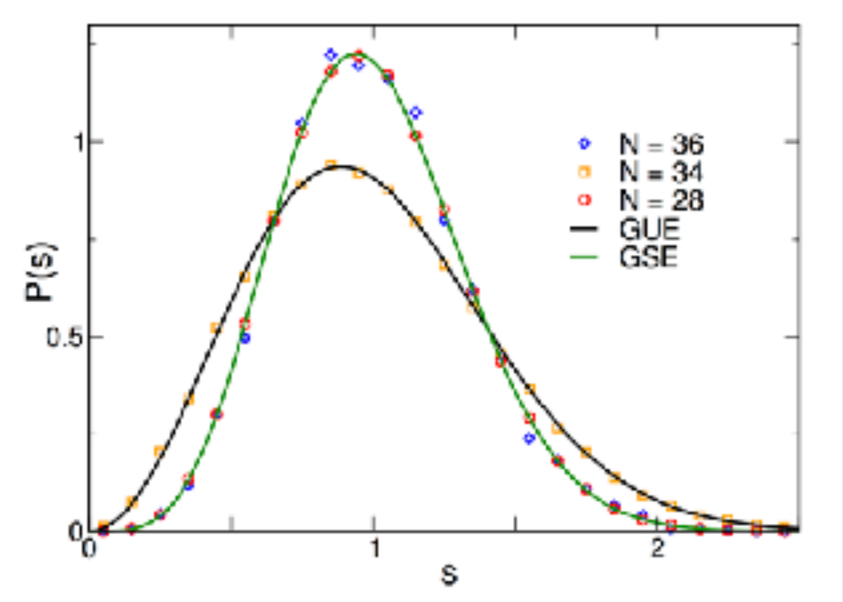
cf. Bohigas, French,
Weidenmüller, …
early 70s

- ☑ one-dimension (quantum mechanics)
- ☑ hard quantum chaos
- ☑ (weakly broken) conformal invariance

SYK – quantum chaos

ergodic spectral correlations witnessed by correlation functions

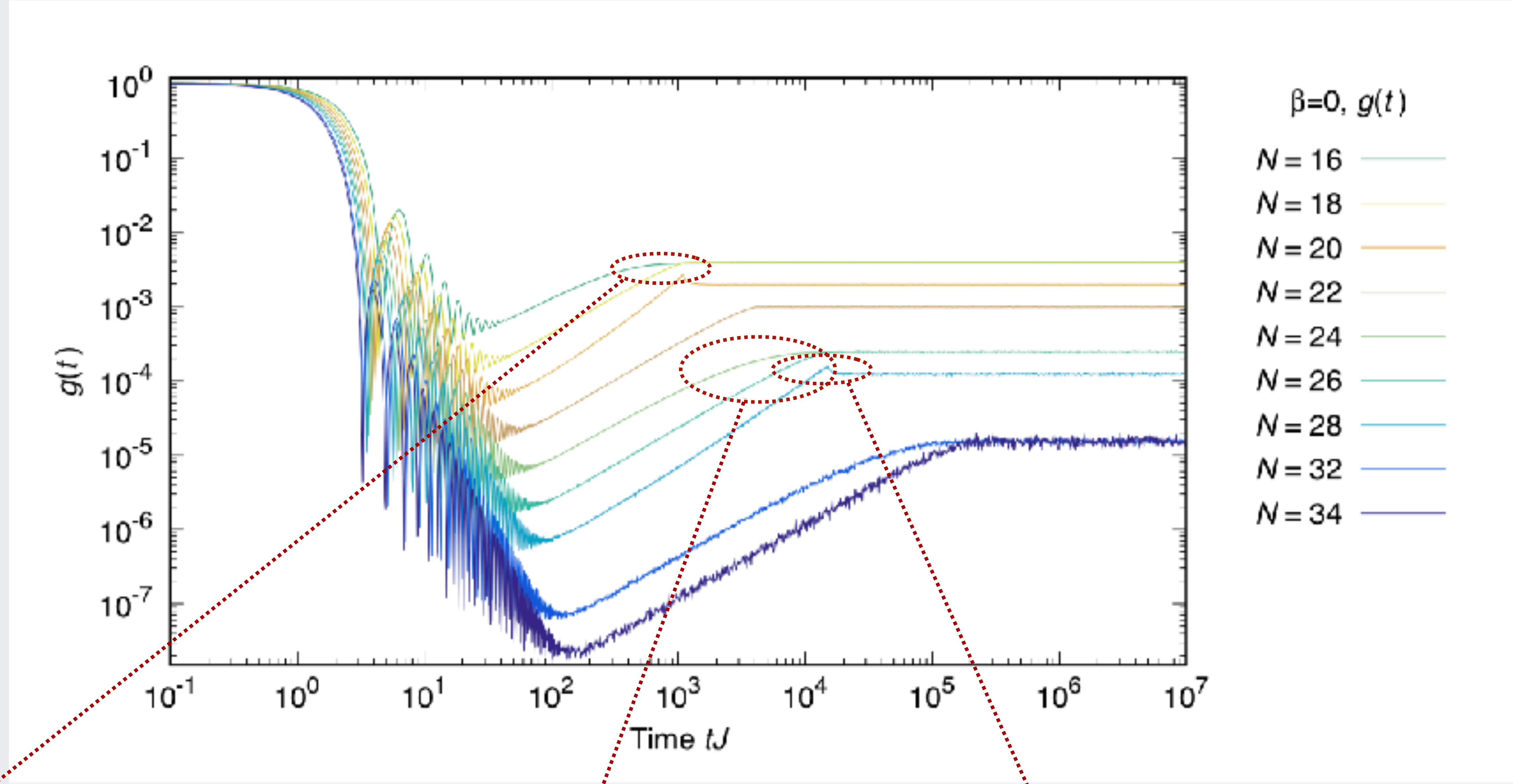




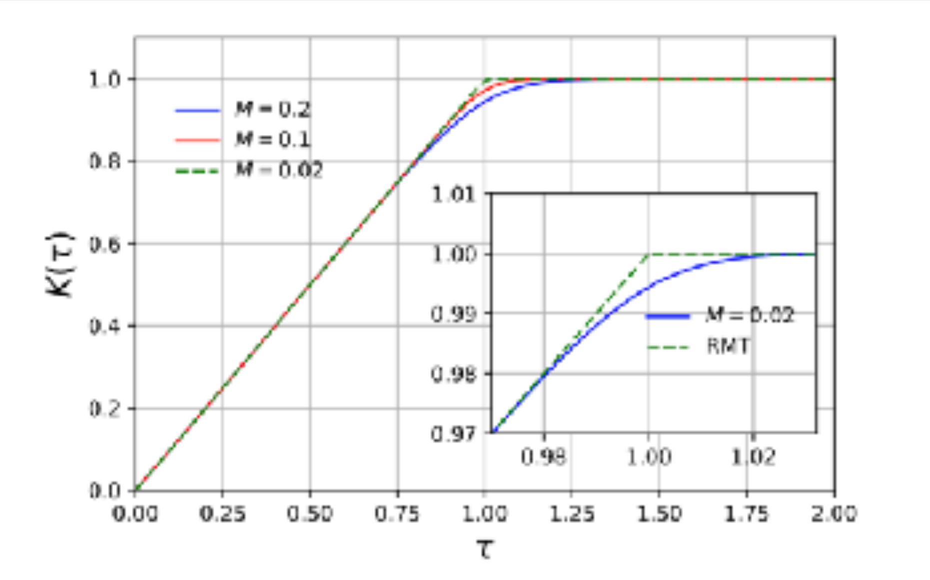
Verbaarschot, Garcia-Garcia, 16

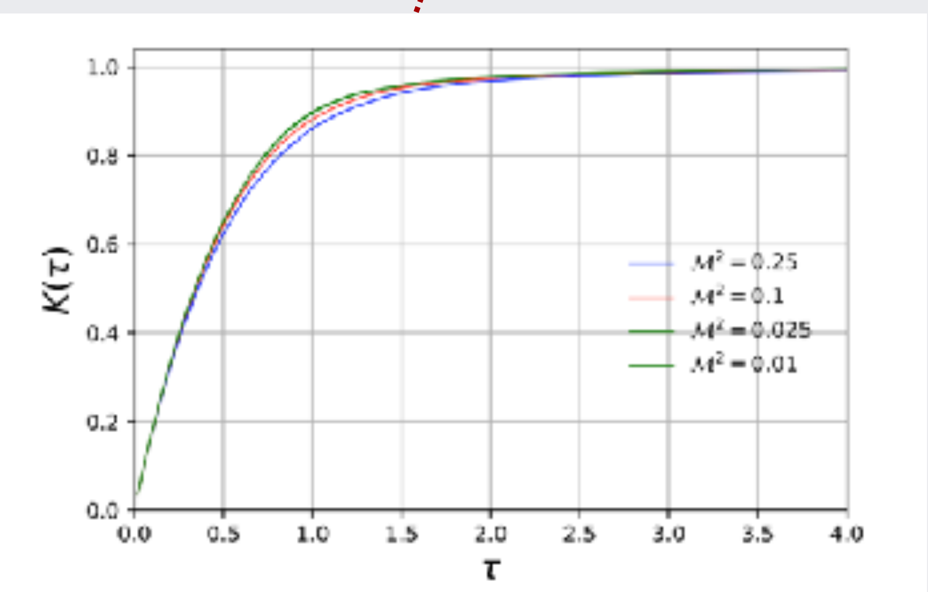
effective field theory identified

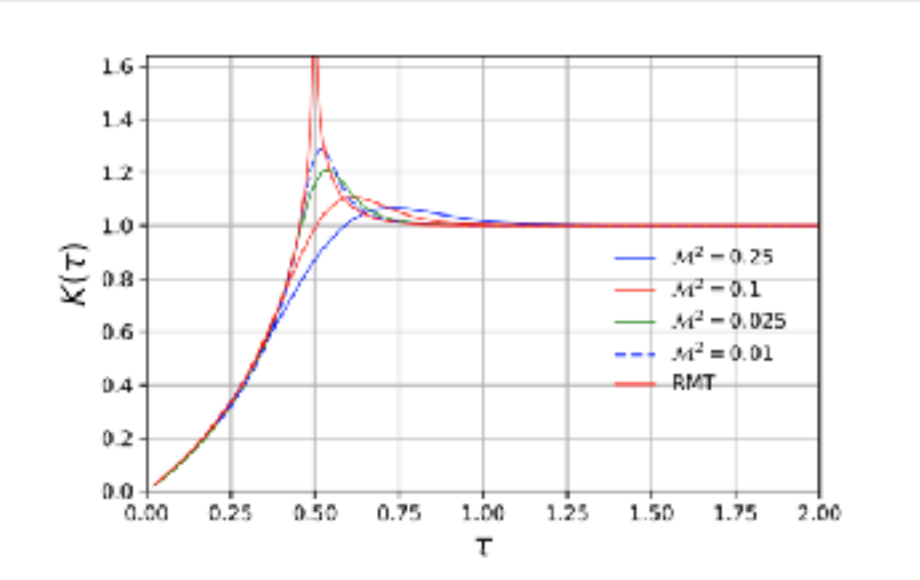
Bagrets, aa, 18



Cotler et al. 17







aa, Micklitz, Verbaarschot unpublished

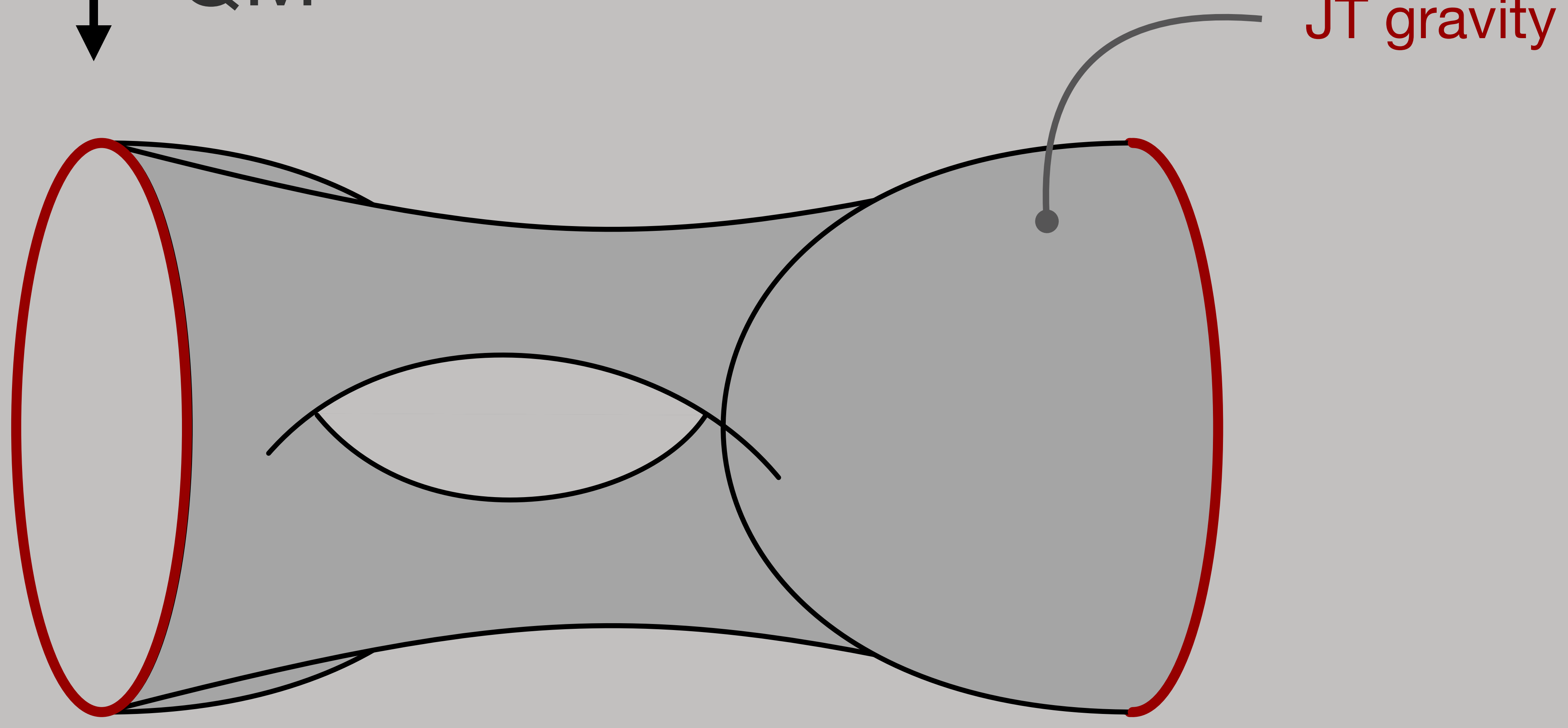
SYK holographic correspondence

$$\left\langle \; G = \frac{1}{z-H} \qquad\qquad G = \frac{1}{z-H} \; \right\rangle_{\text{disorder}}$$

$\Big\uparrow$ imag. time FT

$$\left\langle \; Z = \operatorname{tr} e^{-\beta H} \qquad\qquad Z = \operatorname{tr} e^{-\beta H} \; \right\rangle$$

$\Big\downarrow$ imag. time QM



holographic principle

“euclidean wormhole”

JT gravity

Jackiw, 83
Teitelboim, 85

2d Einstein–Hilbert action coupled to dilaton field

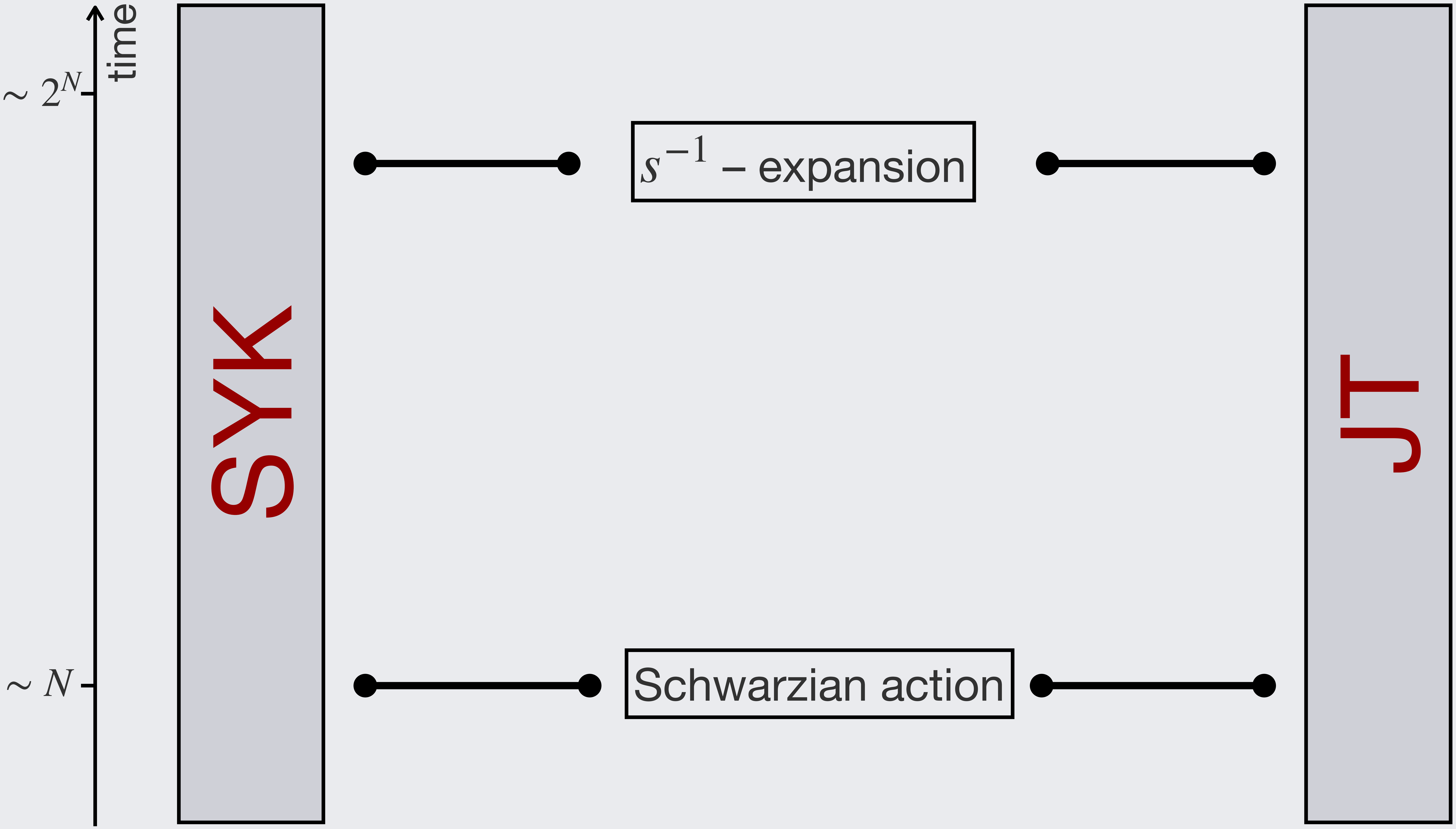
dilaton field

negative cosmological constant

$$S = \frac{1}{16\pi G}\int \sqrt{g}\,\phi(R + \Lambda) + \ldots$$

curvature

Jackiw Teitelboim gravity



Eynard, Orantin, 08
Mirzhakani, 06
Shenker *et al. 19*
Stanford & Witten 19

Kitaev, Suh,
Stanford,
Maldacena, aa,
Bagrets, …
2015+

The gravitational path integral

JT partition sum

topological action

Schwarzian action

$$Z = \sum_g e^{-S_0 g} \int (\text{moduli space}) \int (\text{boundary wiggles})\, e^{-\int_{\text{boundary}} \mathscr{K}\phi}$$

genus expansion

these
parameters

"an integral over geometries …"

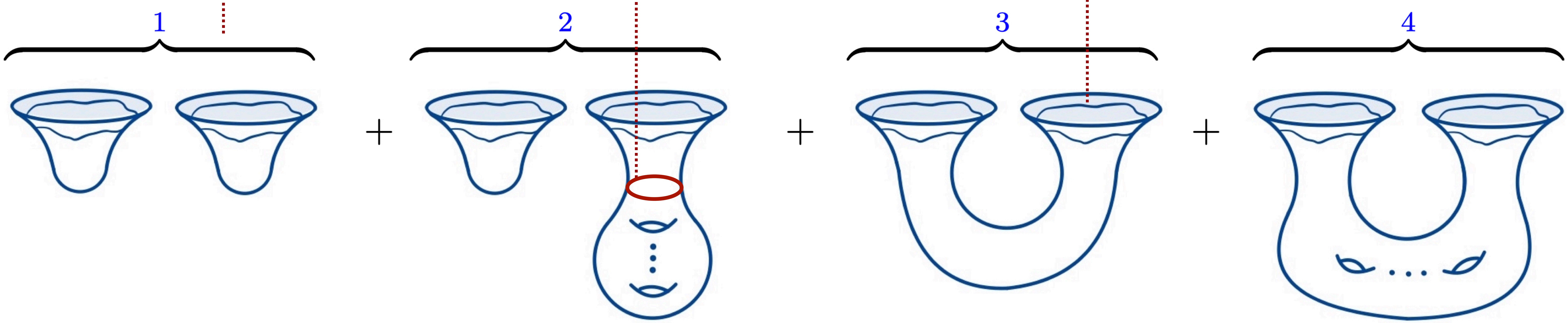


figure: Post *et al.* 22

Gravity topological recursion

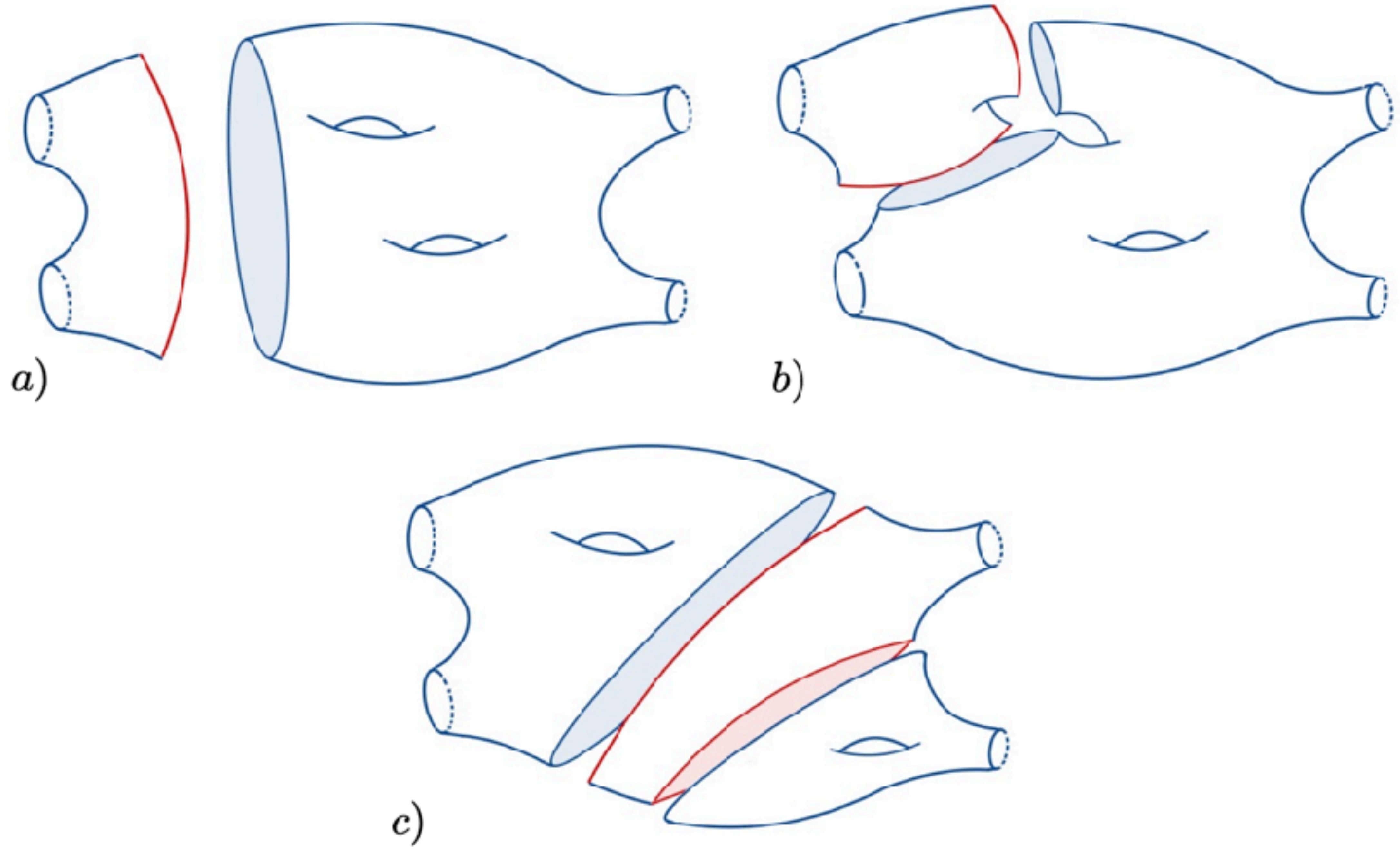
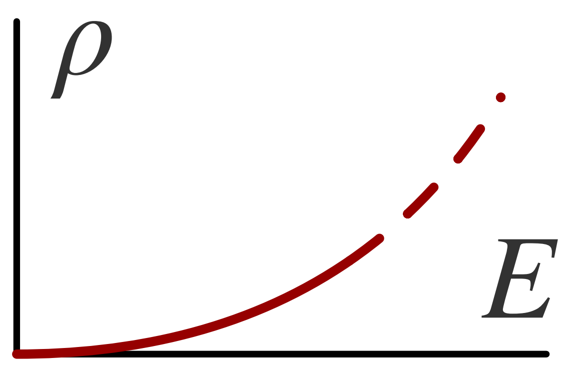


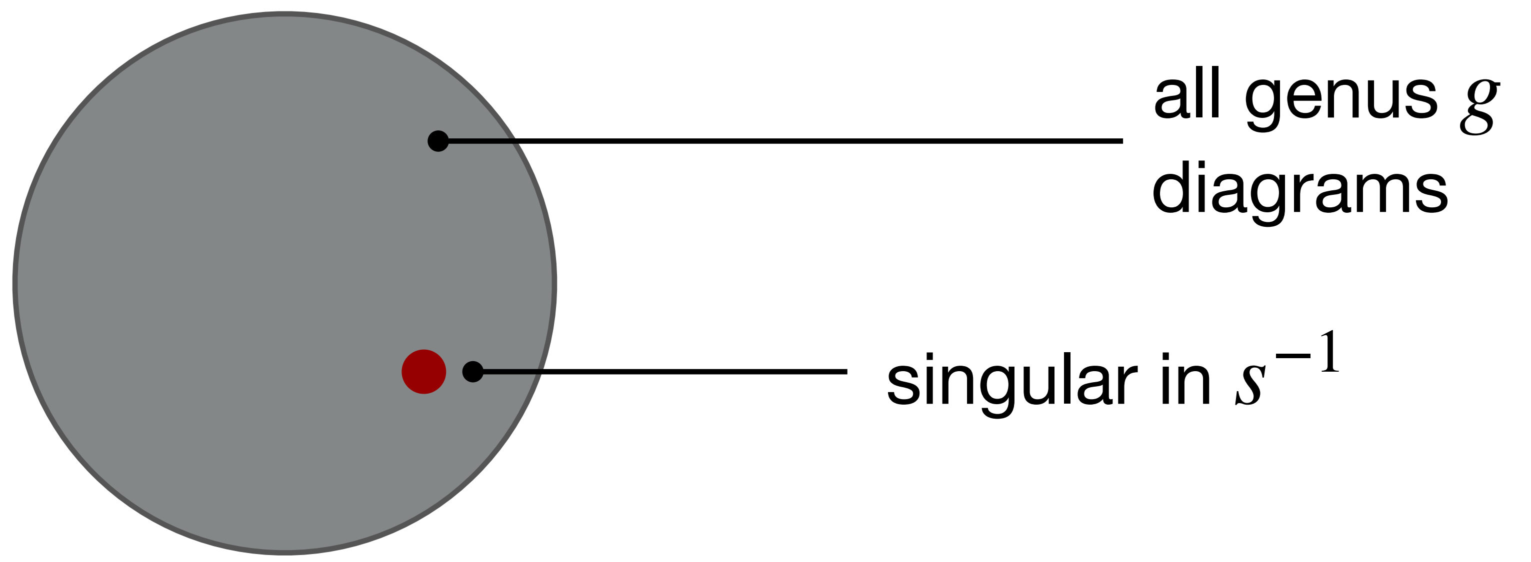
figure: Post *et al*. 22

I: Formulate recursive relation for topological expansion of JT — Mirzhakani, 06

II: compare to topological recursion of matrix model with spectral density matching that of SYK.



→ perfect match — Shenker *et al.* 19



Expansion of JT partition sum captures perturbative expansion of spectral correlations

Gravity topological recursion

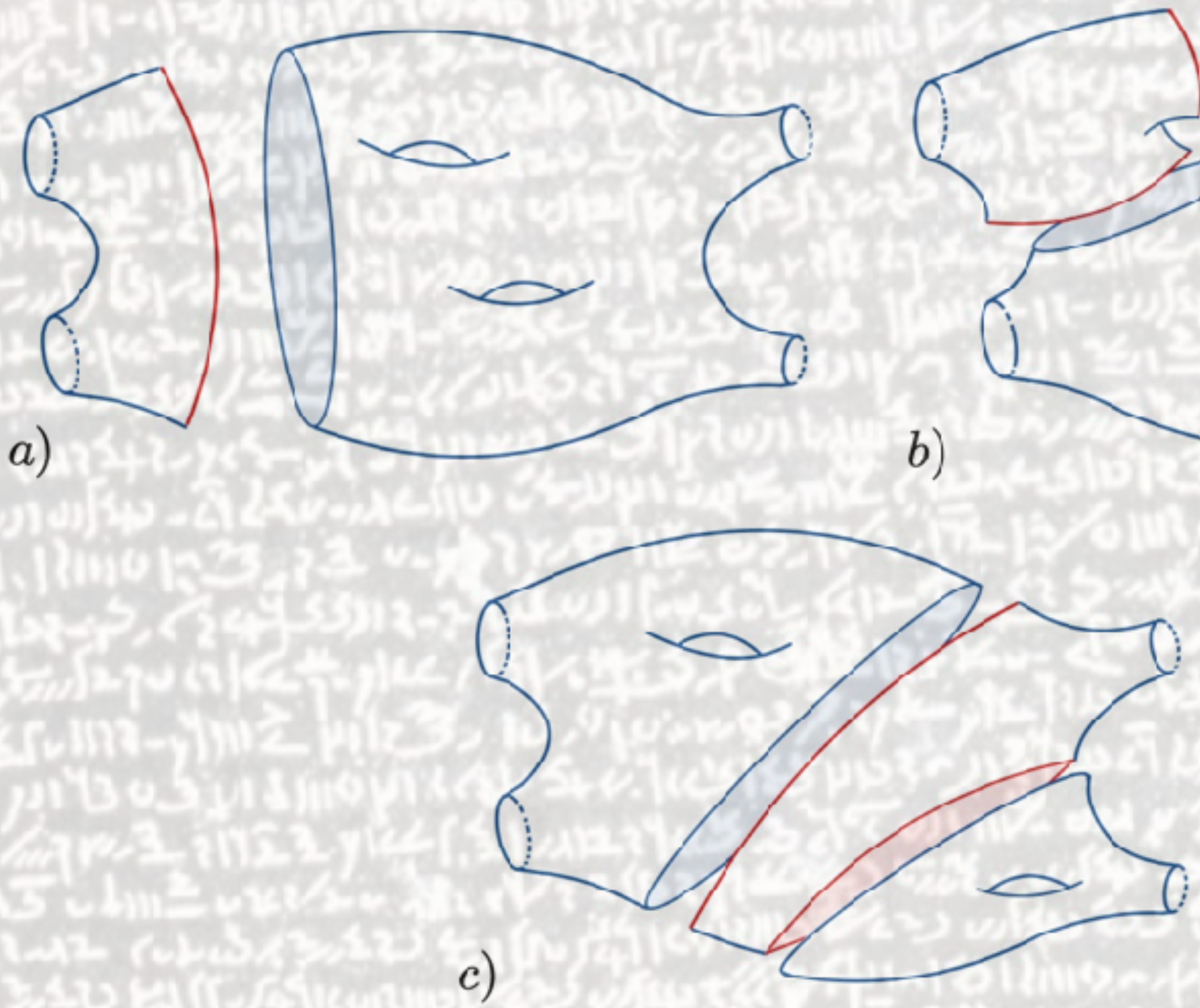
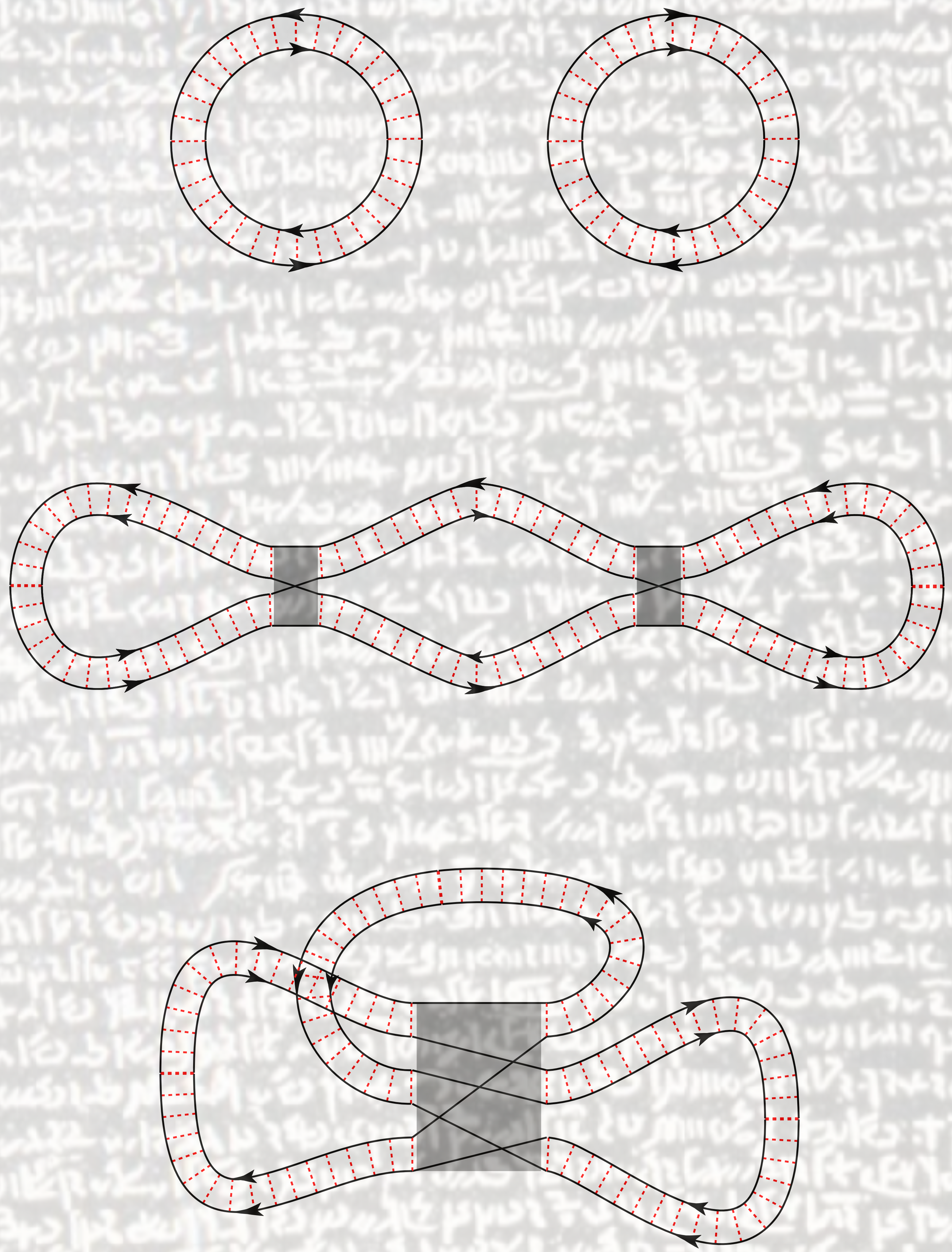
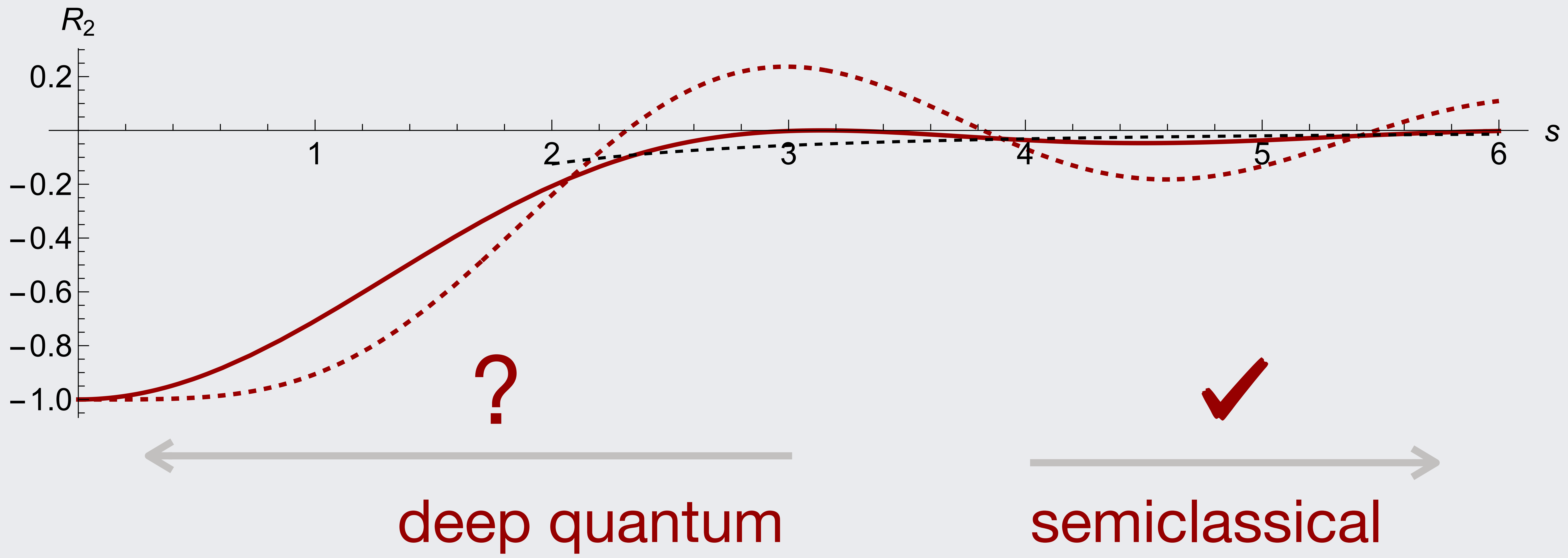


figure: Post *et al.* 22





Q: Where is the non-perturbative quantum chaos?

A: In string theory.

quantum chaos in gravity

KS field theory

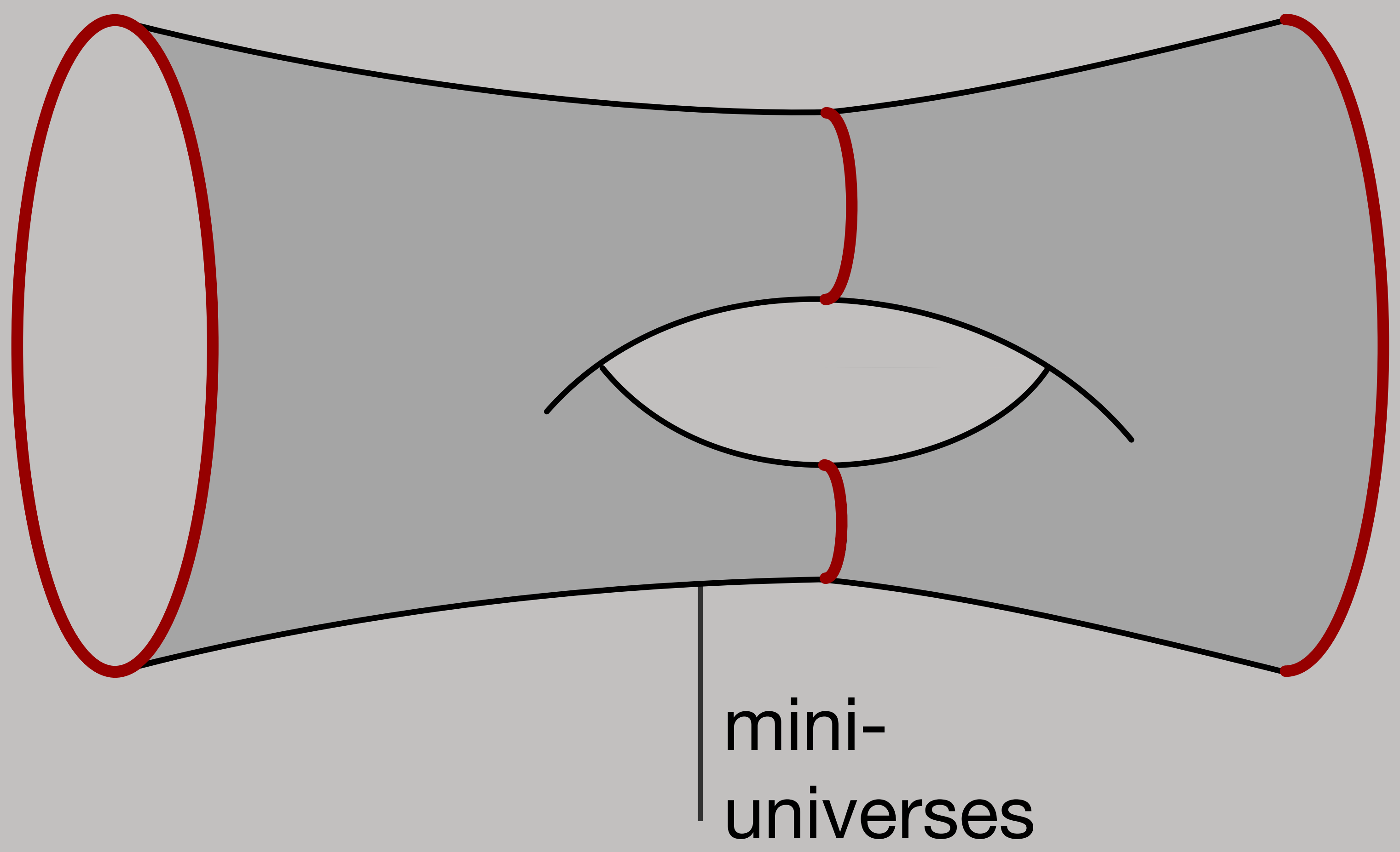
- Think of JT partition sum as asymptotic expansion of some quantum theory

- Need quantum field theory of scattering of universes or closed strings $\rightarrow$

KS field theory obtained by dimensional reduction from $6d$ Kodaira-Spencer theory

- A $2d$ CFT of chiral boson with with cubic nonlinearity

- perturbation theory in nonlinearity $\rightarrow$ JT partition sum.



Kodaira, Spencer 58
Bershadsky *et al. 93*

Post *et al. 22*

chaos from KS

Introducing boundary sources (flavor probe branes) and taking low energy limit can show:

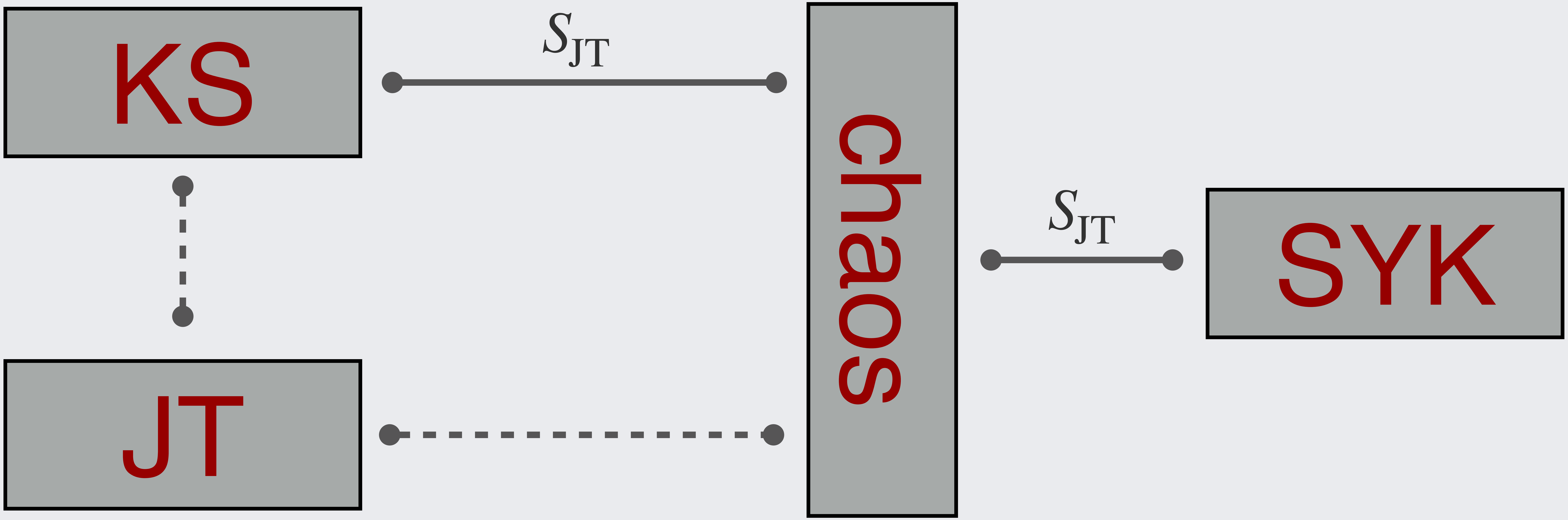
aa, Sonner, *et al.* 22

$$\left\langle : e^{\phi(z_1)} e^{-\phi(z_2)} e^{\phi(z_3)} e^{-\phi(z_4)} : \right\rangle_{\mathrm{KS}}$$

Late time physics of KS theory $\rightarrow$ ergodic phase of quantum chaos.

Fine structure of bulk spectrum resolved.

conclusions



perturbation theory/info on μ-structure lost

non-perturbative

It turns out that even the incomplete knowledge we have of string theory nevertheless conveys important new principles of physics.

~~Many~~ Some of these new principles are ~~counter-~~intuitive.

I will try to explain some of these new principles and use examples to illustrate them.

Vafa, 23

Chaos from KS

Consider correlation function

$$\left\langle e^{+\phi(\epsilon_1^+)}\, e^{-\phi(\epsilon_2^+)}\, e^{+\phi(\epsilon_1^-)}\, e^{-\phi(\epsilon_2^-)} \right\rangle_{\mathrm{KS}}$$

- CFT perspective: $e^{\phi(z)}$ is primary field
- Chaos perspective: $e^{\pm\phi(z)}$ holographically dual to $\det(z-H)^{\pm 1}$
- String theory perspective: $e^{\pm\phi(z)}$ insertion of compact/non-compact probe brane at energy z.

Chaos from KS

With $(\epsilon_1^+, \ldots, \epsilon_2^-) \equiv (x_1, \ldots, x_4) \equiv X$,

$$\left\langle e^{+\phi(x_1)}\, e^{-\phi(x_2)}\, e^{+\phi(x_3)}\, e^{-\phi(x_4)} \right\rangle_{\mathrm{KS}}$$

energy-
like

super Vandermonde

normal ordering

$$= \int dY\, e^{\frac{i}{\lambda} X\cdot Y}\, \Delta(Y) \left\langle :\, e^{+\phi(y_1)}\, e^{-\phi(y_2)}\, e^{+\phi(y_3)}\, e^{-\phi(y_4)}\, : \right\rangle_{\mathrm{KS}}$$

diagonal elements
of supermatrix

$$= \int dY\, e^{\frac{i}{\lambda} X\cdot Y}\, \Delta(Y)\, e^{-\Gamma(Y)}$$

Chaos from KS

$$\left\langle : e^{+\phi(x_1)}\, e^{-\phi(x_2)}\, e^{+\phi(x_3)}\, e^{-\phi(x_4)} : \right\rangle_{\mathrm{KS}} = \int_{\mathrm{GL}(2|2)} dA\, e^{\lambda^{-1}(i\,\mathrm{str}(AX)-\Gamma(A))}$$

$$\longrightarrow \boxed{\int_{\mathrm{A}_{2|2}} dQ\, e^{\frac{i}{\lambda}\,\mathrm{str}(QX)}}$$

Efetov 81

- (Statistics of) micro-spectrum fully resolved
- non-perturbative duality to SYK
- a chaotic string theory
- geometric signatures of chaos field theory — Altshuler-Andreev saddles, non-compactness, $\cdots$ — afford string interpretation (not discussed here.)