A Background Independent Algebra in Quantum Gravity

Edward Witten

Hamburg, November 9, 2023

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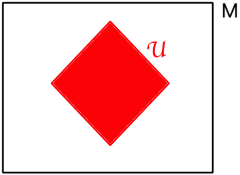

There is a problem in trying to do this in the presence of gravity, because spacetime fluctuates and it is in general hard to say what we mean by the region $\mathcal{U}$.

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Following Unruh (1976) and many others, I will model an observer by a timelike worldline γ (which I will take to be a geodesic) and I will assume that what the observer can measure are the quantum fields along the worldline, which make up an algebra that I will call $\mathcal{A}(\gamma)$.

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In a general diamond-like region

Jensen, Sorce, and Speranza (2023).

Stationary spacetimes:

Kudler-Flam, Leutheusser, and Satishchandran (2023).

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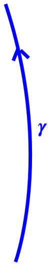

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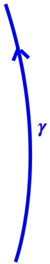

The worldline is parametrized by proper time τ. The observer measures along γ, for example, a scalar field ϕ, or the electromagnetic field $F_{\mu\nu}$, or the Riemann tensor $R_{\mu\nu\alpha\beta}$, as well as their covariant derivatives in normal directions.

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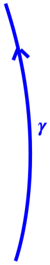

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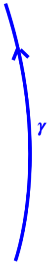

So $\phi(\tau)$ isn't by itself a meaningful observable: we need to introduce the observer's degrees of freedom and define τ relative to the observer's clock.

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In a minimal model, we equip the observer with a Hamiltonian $H_{\rm obs} = mc^2 + q$, and a canonical variable $p = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}q}$. However, it turns out that it is better to assume that the observer energy is bounded below, say $q \geq 0$ (so m is the observer's rest mass). We then only allow operators that preserve this condition, so for example $e^{-\mathrm{i}p}$, which does not preserve $q \geq 0$, should be replaced with $\Pi e^{-\mathrm{i}p}\Pi$, where $\Pi = \Theta(q)$ is the projection operator onto $q \geq 0$.

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where $H_{\rm bulk}$ is (any) gravitational constraint operator that generates a shift of τ along the worldline. An operator that commutes with $\widehat{H}$ is invariant under a spacetime diffeomorphism that moves the observer worldline forward in time, together with a time translation of the observer's system.

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How do we find operators that commute with $\widehat{H} = H_{\mathrm{bulk}} + H_{\mathrm{obs}}$? Since

$$[H_{\mathrm{bulk}}, \phi(\tau)] = -\mathrm{i}\dot{\phi}(\tau),$$

we need

$$[q, \phi(\tau)] = \mathrm{i}\dot{\phi}(\tau),$$

which we can achieve by just setting

$$\tau = p$$

or more generally

$$\tau = p + s$$

for a constant s.

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In addition to these operators (with ϕ possibly replaced by any local field along the worldline such as the electromagnetic field or the Riemann tensor) there is one more obvious operator that commutes with $\widehat{H}$, namely q itself. So we define an algebra $\mathcal{A}_{\rm obs}$ that is generated by the $\widehat{\phi}_s$ as well as q.

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To get background independence, we have to think of $\mathcal{A}_{\rm obs}$ as an operator product algebra, rather than an algebra of Hilbert space operators. The algebras for different M's are inequivalent representations of the same underlying operator product algebra.

In the absence of gravity, we would characterize the objects $\phi(\tau)$ by their universal short distance singularities:

$$\phi(\tau)\phi(\tau') \sim C(\tau - \tau' - \mathrm{i}\epsilon)^{-2\Delta} + \cdots .$$

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(1) is positive, in the sense that $\langle \mathcal{O}\mathcal{O}^\dagger \rangle \geq 0$ for all $\mathcal{O} \in \mathcal{A}_{\rm obs}$

(2) is consistent with all universal OPE relations.

If M is any spacetime in which the observer is found, $\mathcal{H}$ is the Hilbert space that describes the fields in M together with the observer, and $\Psi \in \mathcal{H}$, then

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is a state of the algebra $\mathcal{A}_{\rm obs}$, by that definition. Though these definitions make sense for any M, they are most interesting when, because of black hole or cosmological horizons, the part of the universe that the observer can see does not include a complete Cauchy hypersurface.

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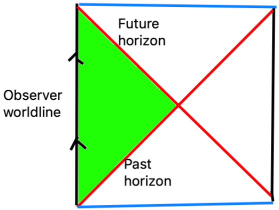
where $\mathrm{d}\Omega^2$ is the metric of a round sphere of dimension $d = D - 1$ and unit radius. At time t, the sphere has radius $R \cosh t$, so it grows exponentially toward either the past or the future.

This spacetime can be described by a Penrose diagram:



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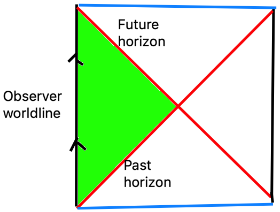

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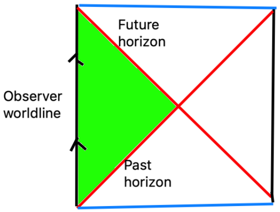

We can pick coordinates so that the worldline γ of the observer is the left boundary of the figure. The green region is called a static patch, because it is invariant under a particular de Sitter generator H that advances the proper time of the observer.

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(1) Time translation symmetry:

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(2) The KMS condition:

$$\langle\Psi_{\rm dS}|\phi(\tau)\phi'(0)|\Psi_{\rm dS}\rangle=\langle\Psi_{\rm dS}|\phi'(0)\phi(\tau-{\rm i}\beta)|\Psi_{\rm dS}\rangle.$$

To understand the KMS condition: for an ordinary thermal system with Hamiltonian H, define the thermal density matrix $\rho = \frac{1}{Z} e^{-\beta H}$ and time-dependent operators $A(t) = e^{\mathrm{i}Ht} A(0) e^{-\mathrm{i}Ht}$, $B(t) = e^{\mathrm{i}Ht} B(0) e^{-\mathrm{i}Ht}$.

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$$\langle A(t) B(0) \rangle_\beta = \mathrm{Tr}\, \rho A(t) B(0) = \frac{1}{Z} \mathrm{Tr}\, e^{-\beta H} e^{\mathrm{i}Ht} A(0) e^{-\mathrm{i}Ht} B(0)$$

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$$= \mathrm{Tr}\,\rho B(0) e^{\mathrm{i}H(t+\mathrm{i}\beta)} A(0) e^{-\mathrm{i}H(t+\mathrm{i}\beta)} = \langle B(0)A(t+\mathrm{i}\beta)\rangle_\beta.$$

Including gravity and the observer, we define a special state in which the observer energy has a thermal distribution at the de Sitter temperature

$$\Psi_{\rm max} = \Psi_{\rm dS} e^{-\beta_{\rm dS} q/2} \sqrt{\beta_{\rm dS}},$$

and we replace operators $\phi(\tau)$ by "gravitationally dressed" operators $\widehat{\phi}_s = \Pi \phi(p+s) \Pi$.

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and we replace operators $\phi(\tau)$ by "gravitationally dressed" operators $\widehat{\phi}_s = \Pi \phi(p+s) \Pi$. Then a straightforward computation shows that

(1′) We still have time-translation symmetry

$$\langle \Psi_{\rm max} | \widehat{\phi}_s \widehat{\phi}'_{s'} | \Psi_{\rm max} \rangle = \langle \Psi_{\rm max} | \widehat{\phi}'_{s+c} \widehat{\phi}'_{s'+c} | \Psi_{\rm max} \rangle, \quad c \in \mathbb{R}.$$

Including gravity and the observer, we define a special state in which the observer energy has a thermal distribution at the de Sitter temperature

$$\Psi_{\rm max}=\Psi_{\rm dS}e^{-\beta_{\rm dS}q/2}\sqrt{\beta_{\rm dS}},$$

and we replace operators $\phi(\tau)$ by "gravitationally dressed" operators $\widehat{\phi}_s=\Pi\phi(p+s)\Pi$. Then a straightforward computation shows that

(1′) We still have time-translation symmetry

$$\langle\Psi_{\rm max}|\widehat{\phi}_s\widehat{\phi}'_{s'}|\Psi_{\rm max}\rangle=\langle\Psi_{\rm max}|\widehat{\phi}'_{s+c}\widehat{\phi}'_{s'+c}|\Psi_{\rm max}\rangle,\quad c\in\mathbb{R}.$$

(2′) The KMS condition simplifies:

$$\langle\Psi_{\rm max}|\widehat{\phi}_s\widehat{\phi}'_{s'}|\Psi_{\rm max}\rangle=\langle\Psi_{\rm max}|\widehat{\phi}'_{s'}\widehat{\phi}_s|\Psi_{\rm max}\rangle.$$

Statement (1′) is fairly obvious, and statement (2′) can be proved by a simple contour deformation argument, though I reluctantly decided not to show the proof (see arXiv:2308.03663).

Condition (2′) tells us that if, for any $\mathbf{a} \in \mathcal{A}_{\mathrm{obs}}$, we define

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$$\mathrm{Tr}\, \mathbf{ab} = \mathrm{Tr}\, \mathbf{ba}, \quad \mathbf{a}, \mathbf{b} \in \mathcal{A}_{\rm obs}.$$

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This function has the property that $\mathrm{Tr}\,\mathbf{a}^\dagger \mathbf{a} > 0$ for all $\mathbf{a} \neq 0$, meaning in particular that it is "nondegenerate." Note that if $\Psi_{\rm max}$ is normalized to $\langle \Psi_{\rm max} | \Psi_{\rm max} \rangle = 1$, then

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Although Tr has the algebraic properties of a trace, it is not actually a trace in any representation of the algebra $\mathcal{A}_{\mathrm{obs}}$. We do have a Hilbert space representation of $\mathcal{A}_{\mathrm{obs}}$, namely it acts on $\mathcal{H}_{\mathrm{dS}} \otimes L^2(\mathbb{R}_+)$, where $\mathcal{H}_{\mathrm{dS}}$ is the Hilbert space that describes quantum fields in de Sitter space and $L^2(\mathbb{R}_+)$ is the Hilbert space of the observer.

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Let Ψ be any state in $\mathcal{H}_{\rm dS}$ and consider the function $\mathbf{a} \to \langle\Psi|\mathbf{a}|\Psi\rangle$, $\mathbf{a} \in \mathcal{A}_{\rm obs}$.

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Rather as in ordinary quantum mechanics, we expect ρ to be a positive element $\rho \in \mathcal{A}_{\mathrm{obs}}$ with $\mathrm{Tr}\,\rho = 1$. For example, let us find the density matrix of the state Ψ_{max}.

The definition of the trace makes it clear that the density matrix of the state $\Psi_{\max}$ is $\sigma_{\max} = 1$, since to satisfy

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This means that $\Psi_{\rm max}$ is "maximally mixed," similar to a maximally mixed state in ordinary quantum mechanics whose density matrix is a multiple of the identity.

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In fact, in the example of de Sitter space, the algebra obtained this way is of Type II_1, as follows from the existence and properties of the trace. (This is discussed in the CLPW paper, where this construction was originally motivated in a different way.)

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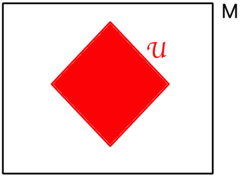

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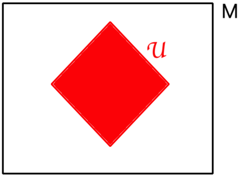

and then one can try to define an entropy $S(\rho) = -\mathrm{Tr}\,\rho \log \rho$. But as was first shown by R. Sorkin (1983), this doesn't work: the entropy of any region is ultraviolet divergent.

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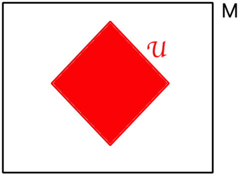

and then one can try to define an entropy $S(\rho) = -\mathrm{Tr}\,\rho\log\rho$. But as was first shown by R. Sorkin (1983), this doesn't work: the entropy of any region is ultraviolet divergent. An abstract explanation is that in the absence of gravity, the algebras $\mathcal{A}_\mathcal{U}$ are of Type III (as first found by Araki in 1964).

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$$S_{\mathrm{gen}} = \frac{A}{4G} + S_{\mathrm{out}},$$

where A is the area of the horizon – the black hole or cosmological horizon – and S_{out} is the ordinary entropy of matter and radiation outside the black hole horizon or (in the cosmological case) in the region of spacetime visible to an observer.

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where A is the area of the horizon – the black hole or cosmological horizon – and $S_{\rm out}$ is the ordinary entropy of matter and radiation outside the black hole horizon or (in the cosmological case) in the region of spacetime visible to an observer. In the case of de Sitter space, assuming we are interested in the portion of spacetime visible to an observer in a given static patch, $S_{\rm out}$ would be the entropy of matter and radiation inside that static patch.

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is an entropy has passed many tests. For example, in the presence of a black hole the usual second law of thermodynamics is not true: $S_{\rm out}$ will go down, as Bekenstein said, if I toss a cup of tea into a black hole. But it turns out that there is a generalized second law (Wall, 2011): $S_{\rm gen}$ is always increasing.

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Although there has been plenty of evidence that $S_{\rm gen}$ behaves like an entropy, the sense in which it actually is an entropy has been unclear. What I have told you gives at least a partial answer to this question, because it is possible to show (CLPW 2022) that entropy defined as I have explained agrees *up to an additive constant independent of the state* with the usual generalized entropy

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for a suitable class of semi-classical states $\Psi \in \mathcal{H}_{\mathrm{dS}}$. An additive constant is missing because entropy defined this way is really a sort of renormalized entropy, from which an infinite constant has been subtracted. (For more on entropy of a state of a Type II algebra, see Longo and EW (2021). As we will see, the missing constant is the entropy of the maximum entropy state, where is 0 with the definitions I have given, but equals $A_{\mathrm{dS}}/4G$ in more standard approaches.)

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$$S(\rho) < 0.$$

So Ψ_{max} is a state of maximal possible entropy.

Thus, the system consisting of an observer in a static patch in de Sitter space has a state of maximum entropy

$$\Psi_{\rm max} = \Psi_{\rm dS} e^{-\beta_{\rm dS} q/2} \sqrt{\beta_{\rm dS}},$$

consisting of empty de Sitter space with a thermal distribution of the observer energy.

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consisting of empty de Sitter space with a thermal distribution of the observer energy. Why did this happen?

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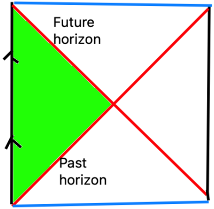

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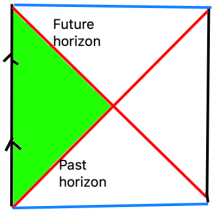
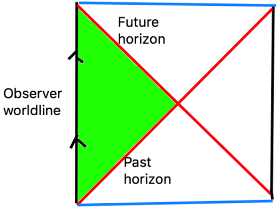


Since the static patch spontaneously evolves to become empty in the far future, the Generalized Second Law seems to imply that an empty static patch must have maximum entropy.

Concerning this argument,



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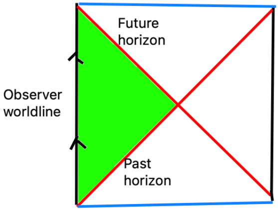

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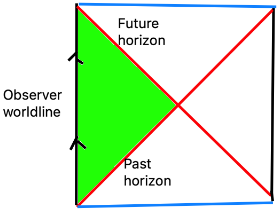
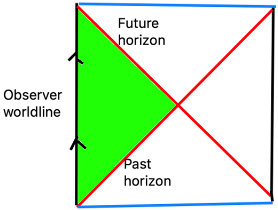


we note that in our discussion we've defined the static patch by the presence of the observer, so by definition the observer doesn't leave the static patch even in the far future. But we can expect that in the far future the static patch will be empty except for the presence of the observer, and that the observer will be in thermal equilibrium with the bulk quantum fields, and that is what we see in the state $\Psi_{\max}$.

Concerning this argument,



we note that in our discussion we've defined the static patch by the presence of the observer, so by definition the observer doesn't leave the static patch even in the far future. But we can expect that in the far future the static patch will be empty except for the presence of the observer, and that the observer will be in thermal equilibrium with the bulk quantum fields, and that is what we see in the state $\Psi_{\rm max}$. So the maximum entropy state that we found is the one suggested by Bousso's argument.

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Note that Bousso made his argument without having any precise explanation of why or in what sense the generalized entropy $S_{\rm gen}$ – which he claimed is maximized by empty de Sitter space – actually is an entropy. We have recovered Bousso's claim with a precise definition of entropy as the von Neumann entropy of a state of an algebra.

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It might come as a surprise that empty de Sitter space has maximum entropy. If we put matter and radiation in the static patch, will not this increase the entropy of the static patch? It turns out that if we put matter and radiation in the static patch, then the gravity of the matter and radiation causes the area of the cosmological horizon to become *smaller*, and this *reduces* the generalized entropy $A/4G + S_{\rm out}$.

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