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Higher Symmetries in (Non-)Compact String Models

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Motivation

M/string theory on singular special holonomy spaces X:

- Non-compact spaces X → Geometric engineering of supersymmetric quantum field theories (SQFTs):

- Build dictionary:{operators, symmetries}⟺{geometry,topology}
- Focus: higher-form global symmetries ⟺ topology (associated with ``flavor'' branes)

- Compact spaces X → Quantum field theory (QFT) w/ gravity → Higher-form symmetries gauged or broken Physical consistency conditions → swampland program

Higher-form symmetries in (S)QFT - active field of research

[Gaiotto, Kapustin, Seiberg, Willet, 2014],…

Higher-form symmetries & geometric engineering

[Del Zotto, Heckman, Park, Rudelius, 2015],…
[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],…
[M.C., Dierigl, Lin, Zhang, 2020],..
[Apruzzi,Bhardwaj, Oh, Schäfer-Nameki, 2021],…
[M.C., Dierigl, Lin, Zhang, 2021],…
[M.C., Heckman, Hübner, Torres, 2203.10102],
[Del Zotto, Garcia Etxebarria, Schäfer-Nameki, 2022],…
[Hübner, Morrison, Schäfer-Nameki,Wang, 2022],…
[Heckman, Hübner, Torres, Zhang, 2023],…
[M.C.,Heckman, Hübner, Torres, Zhang, 2023],..

Higher-form symmetries & and compact geometry

[M.C., Dierigl, Lin, Zhang, 2020, 2021,2022],…
[M.C., Heckman, Hübner, Torres, 2307.1023],…

Goals

- Identify geometric origin of higher-form symmetries (0-form, 1-form & 2-group) for M-/string theory on non-compact & compact special holonomy (Calabi-Yau) spaces
  Punchline:
  - Higher form symmetries via cutting & gluing of singular boundary of non-compact X^{loc}
  - The fate of higher-form symmetries (gauged or broken) via gluing of X^{loc} to compact X

- Examples: M-theory on(non-)compact
  - Calabi-Yau n-folds (n=2, some n=3)
  - For elliptically fibered CY n-folds, dual to F-theory confront results with those, obtained via resolutions & arithmetic properties of elliptic curves

Outline

- Introduction: Defect Group in geom. engineering

- Defect group and higher-form symmetries → Topology of flavor symmetry group

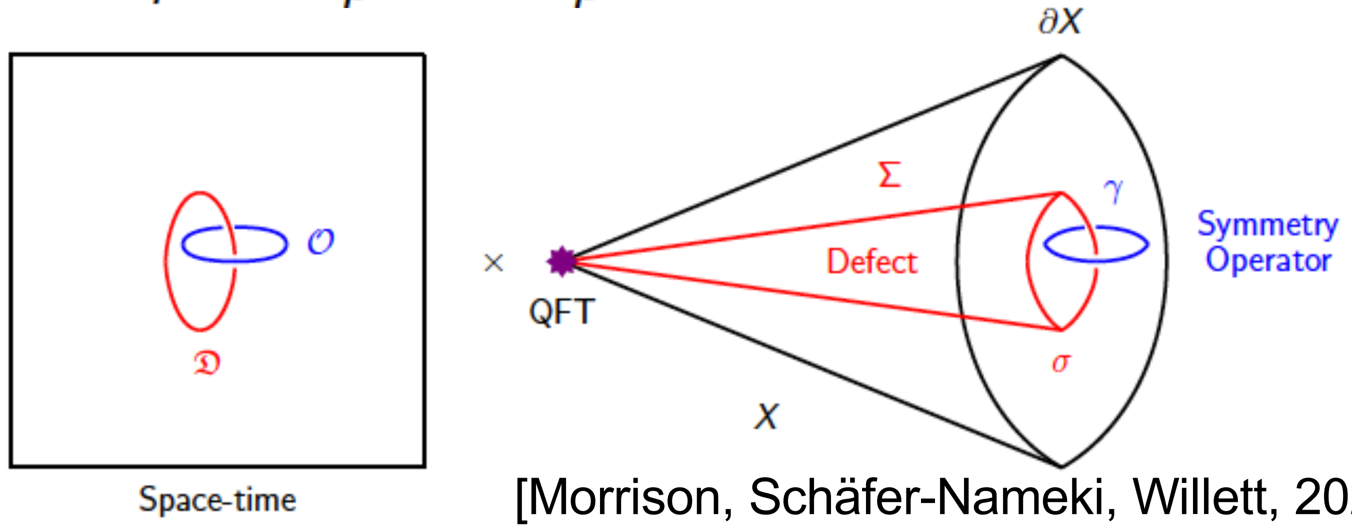
- Compact examples → fate of higher-form symmetries

- Concluding remarks

I. Defect group for M-theory on non-compact X

- Defect Group for extended *p*-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$

- Schematically:



[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020]

w/ M2, M5 in X wrapping relative cycles

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X,\partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$

[p-dim el. operators in SCFT]

Focus on (Wilson)lines: p=1 el. operators

$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X,\partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$

[p-dim mag. operators in SCFT]

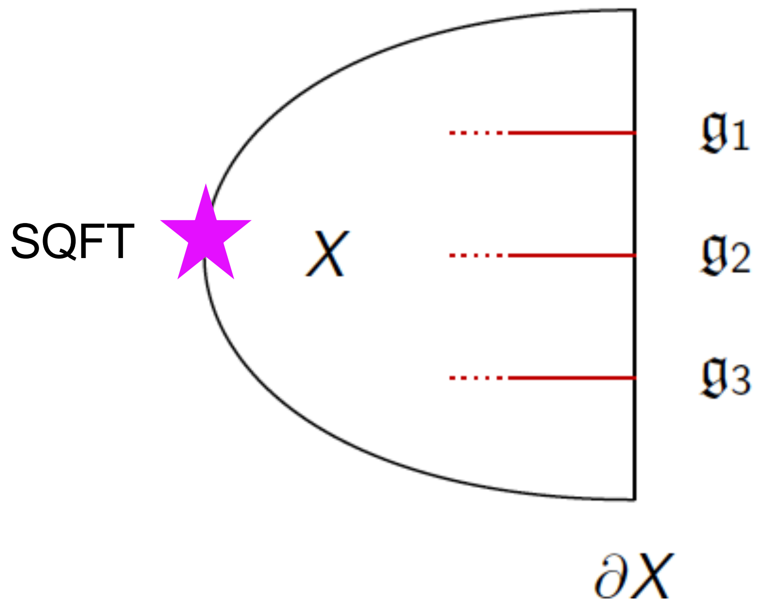
Focus on torsional cycles

- Focus on defect ops.; symmetry ops.c.f. [Heckman, Hübner, Torres, Zhang 2022,.. M.C., Donagi,Heckman, Hübner, Torres, Zhang 2023]

II. Geometrizing Topology of Flavor Group

Non-compact ADE loci ≡ flavor branes → flavor symmetries

Naïve flavor symmetry $\widetilde{G}_F$ (simply connected w/ Lie Algebra $\mathfrak{g}_i$)



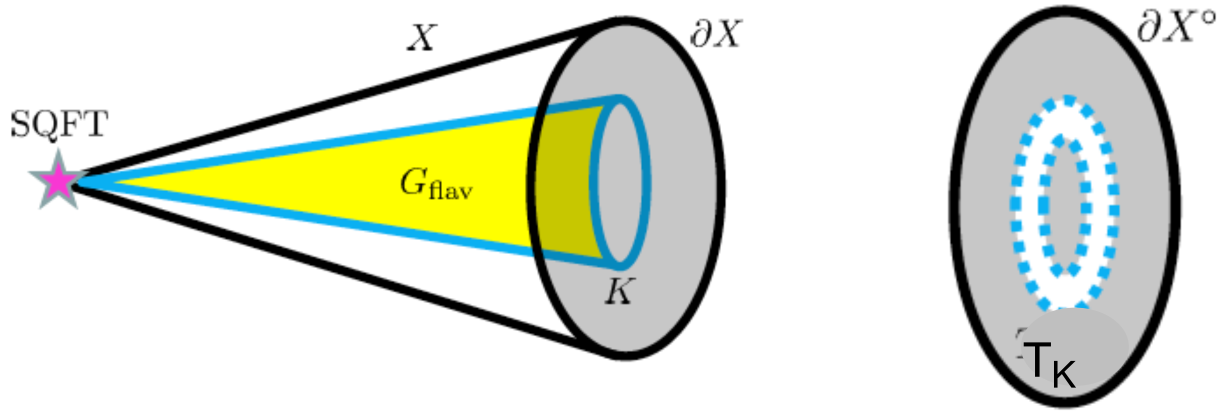
[From now on only ADE's in ∂X]

$$\widetilde{G}_F = \widetilde{G}_1 \times \widetilde{G}_2 \times \widetilde{G}_3 \times \ldots$$

(Flavor Wilson) lines → fix topology of favor symmetry G_F
from singular boundary topology

III. Boundary geometry of flavor branes:

- Singular non-compact space X w/ [Example: $X \cong \mathbb{C}^2/\Gamma_i$]
  $K = \cup_i K_i$ - ADE loci (of flavor branes) in the boundary ∂X
- Define a smooth boundary $\partial X^\circ = \partial X \setminus K$
  & a tubular region T_K (excise K)
- Locally $T_K \cap \partial X^\circ \cong \cup_i K_i \times S^3/\Gamma_i$

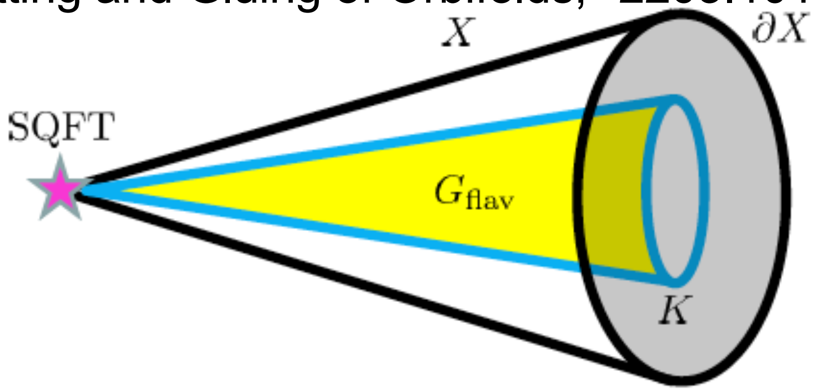


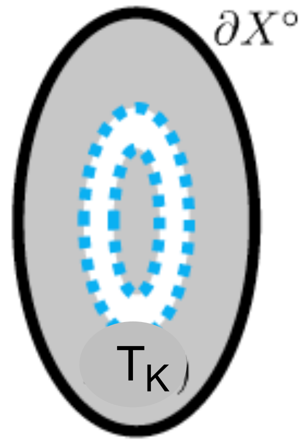
- Naïve flavor center symmetry:

$$Z_{\widetilde{G}_F} = \text{Tor}\, H_1(T_K \cap \partial X^\circ) \cong Z_{\widetilde{G}_1} \oplus Z_{\widetilde{G}_2} \oplus Z_{\widetilde{G}_3} \oplus \ldots$$

Boundary geometry of true flavor center symmetry Z_{G_F}

M. C., J. J. Heckman, M. Hübner and E. Torres: "0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds," 2203.10102





[Mayer, 1929], [Vietoris, 1930]

- Key: Mayer-Vietoris sequence in homology for singular boundary

$$\partial X = \partial X^\circ \cup T_K$$

true flavor center naïve flavor center

$Z_{\text{0-form}}$:

$$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

1-form:

$$0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to H_1(\partial X^\circ) \oplus H_1(T_K) \to H_1(\partial X) \to 0$$

naïve 1-form true 1-form

$$Z_{G_F} = \ \mathrm{Ker}\left(\iota_1 : Z_{\widetilde{G}_F} \cong H_1(\partial X^\circ \cap T_K) \to H_1(\partial X^\circ) \oplus H_1(T_K)\right)$$

When the bottom sequence does not split → 0-form and 1-form intertwined: 2-group!

Examples: non-compact orbifolds, elliptically fibered CY's, G_2, etc.

[Benini, Cordova, Hsin, 2019], [Lee, Ohmori, Tachikawa, 2021], .

IV. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- For elliptically fibered geometries via M/F-theory duality:

- Non-Abelian group algebras – ADE Kodaira classification

group topology → Mordell-Weil torsion

[Aspinwall, Morrison, 1998], [Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

- Abelian groups → Mordell-Weil ``free'' part

[Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

- Total group topology → Shoida map of Mordell-Weil

[M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

Digression: F-theory on elliptically fibered Calabi-Yau fourfolds
w/specific elliptic fibration (F_{11} polytope)
led to D=4 N=1effective theory

[M.C., Klevers, Peña, Oehlmann, Reuter, ’15]

Standard Model gauge group $\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$

w/ gauge group topology
(geometric - encoded in Shioda Map of MW)

[M.C., Lin, ’17]

⬇ w/ toric bases B_3 (3D polytopes)

[M.C., Halverson, Lin, Liu, Tian, ’19, PRL]

Quadrillion Standard Models (QSMs)
with 3-chiral families & gauge coupling unification

[gauge divisors – in class of *anti-canonical divisor* $\overline{K_C}$]

Further efforts: determination the exact matter spectra
(including vector pair & # of Higgs pairs)
→ Studies of (limit) root bundles on matter curves

[Bies, M.C., Donagi,(Liu), Ong, ’21,’22,’23]

[No time, but the latest status in [Bies, M.C., Donagi, Ong, 2307.02535]

How to relate these results, encoded in resolution & arithmetic structure of elliptic curve, due Mordell-Weil, to higher-form symmetries and singular geometry?

Some progress already on the fate of higher-form symmetries for compact models:

- Global symmetries, including higher-form ones should be gauged or broken [No Global Symmetry Hypothesis]

…[Harlow, Ooguri '18]

- In 8D N =1 Supergravity:
quantified conditions under which no anomalies due to gauged 1-form symmetry
[magnetic version – preferred polarization!]

[M.C., Diriegl, Ling, Zhang, '20]

- True for all 8D N=1 string compactifications (beyond F-theory) via (refined) string junction construction

[M.C., Diriegl, Ling, Zhang, '22]

& work in progress for ``frozen-singularities''

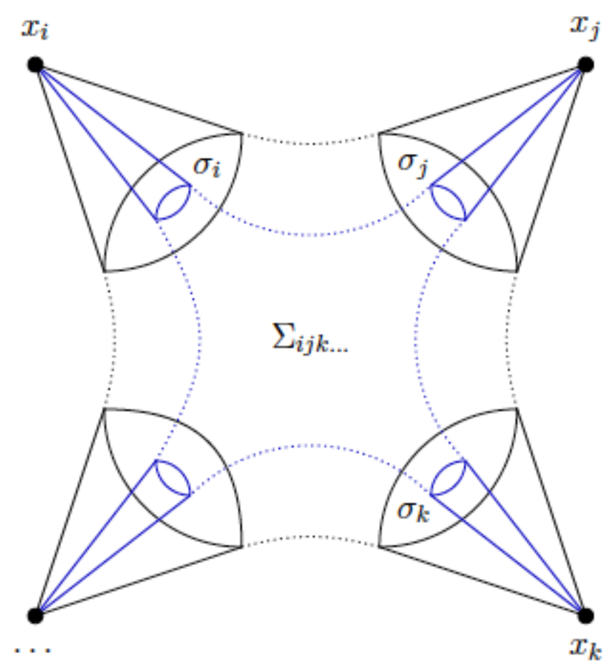
Fate of higher-form structures via cutting and gluing

M. C., Heckman, Hübner, Torres,
Generalized Symmetries, Gravity, and the Swampland, 2307.1023

- Decompose X → $\cup_i X_i^{loc}$ into local models X_i^{loc}

  Converse: Glue $\{X_i^{loc}\}$ to X ⬌ Couple $\{SQFT_i\}$ to a resulting QFT & include gravity

  Schematically:



Defect Operators:
M2-/M5-brane wrapped on cones

Σ_{ijk} – two-chain glues defects

- Some relative Cycles in X_i^{loc} survive & compactify →
  (Some) defects in $SQFT_i$ become dynamical - ``gauged''
  → determine the gauge group in compact models

Fate of higher-form structures in compact geometries (continued):

- Quantify: Mayer-Vietoris long exact sequence (Relates X and X^{loc})

$$\dots \xrightarrow{\jmath_n} H_n(X) \xrightarrow{\partial_n} H_{n-1}(\partial X^{\mathrm{loc}}) \xrightarrow{\imath_{n-1}} H_{n-1}(X^\circ) \oplus H_{n-1}(X^{\mathrm{loc}}) \to \dots$$

w/ respect to the covering: $X = X^{\mathrm{loc}} \cup X^\circ\,, \qquad X^\circ = X \setminus X^{\mathrm{loc}}$

w/ total local model: $X^{\mathrm{loc}} = \coprod_i X_i^{\mathrm{loc}}$

- Example of K3 Surfaces → Mayer-Vietoris short exact sequence:

Extra $U(1)$'s	Massive Matter	Local Model Defect/Symmery Ops	Emergent/Broken Symmetries

$$0 \to H_2(X^\circ) \to H_2(X) \to H_1(\partial X^{\mathrm{loc}}) \to H_1(X^\circ) \to 0$$

$$H_1(\partial X^{\mathrm{loc}}) \cong \oplus_i H_1(\partial X_i^{\mathrm{loc}}) \qquad H_1(X^\circ) \cong (\mathrm{Tor}\, H_2(X))^\vee$$

Decomposition of compact two-cycles into a sum of relative cycles associated with each local model ∂_2: $H_2(X) \to \oplus_i H_1(\partial X_i^{loc})$

Elliptically fibered K3s: torsional cycles associated w/ Mordell-Weil decompose into relative cycles of local patches → complementary geometric results!

V. Concluding Remarks

- Systematic determination of higher symmetries & gauge groups in (non-)compact string models via cutting and gluing techniques
- Analysis of compact models, beyond elliptically fibered K3, c.f., all T^4/Γ_i orbifolds:

$$T^4/\mathbb{Z}_2: \quad G = \frac{(SU(2)^{16}/\mathbb{Z}_2^5) \times U(1)^6}{\mathbb{Z}_2^6}$$

$$T^4/\mathbb{Z}_3: \quad G = \frac{(SU(3)^{9}/\mathbb{Z}_3^3) \times U(1)^4}{\mathbb{Z}_3^3}$$

$$T^4/\mathbb{Z}_4: \quad G = \frac{(SU(4)^{4}/\mathbb{Z}_4 \times \mathbb{Z}_2^2) \times SU(2)^6 \times U(1)^4}{\mathbb{Z}_4^2 \times \mathbb{Z}_2^2}$$

$$T^4/\mathbb{Z}_6: \quad G = \frac{([SU(6) \times SU(3)^4 \times SU(2)^5]/\mathbb{Z}_3 \times \mathbb{Z}_2) \times U(1)^4}{\mathbb{Z}_6^3 \times \mathbb{Z}_2}$$

& higher dimensional CY's: some T^6/Γ_i orbifolds (subtleties)…

- Open questions:
  Fate of 2-group → ``dissolved'' in global models
  Non-invertible symmetries → fusion algebra of TFT, starting w/local models…

Work in progress w/ R. Donagi, J. Heckman, M. Hübner, E. Torres, H. Zhang

Thank you!