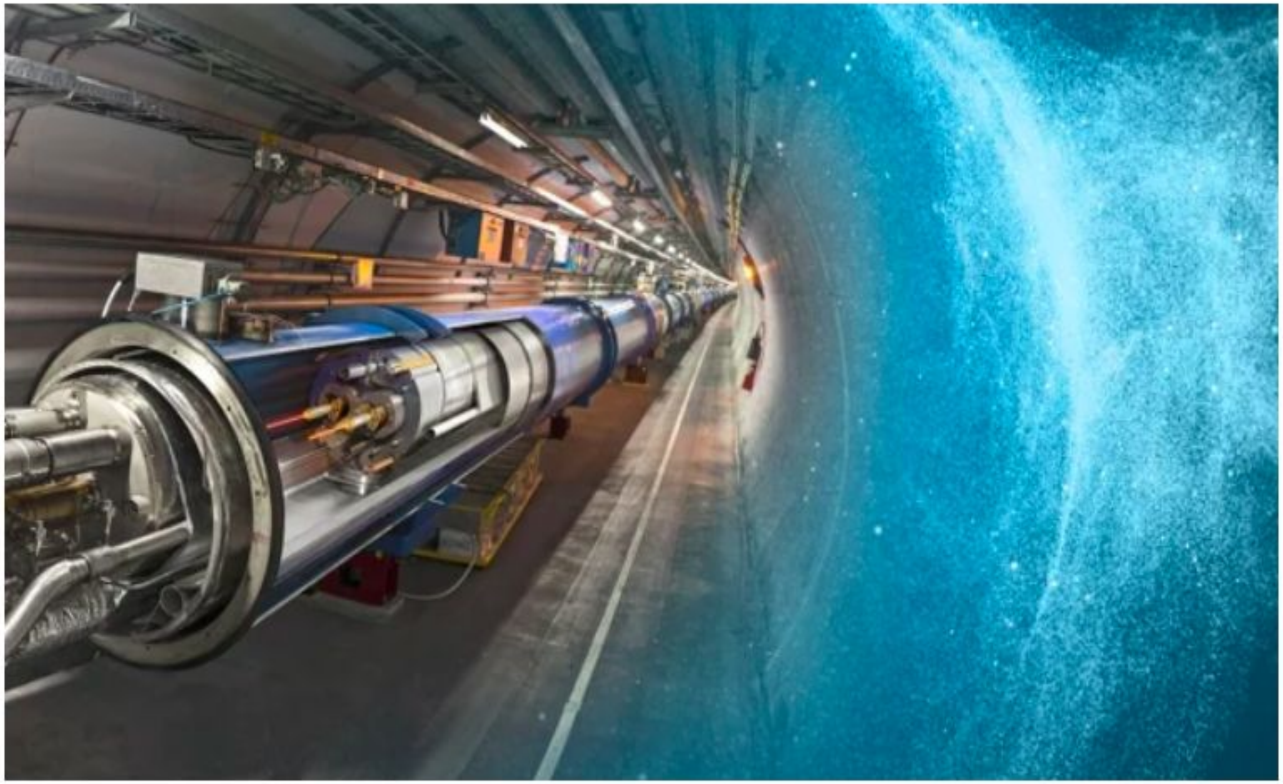
Introduction to the Standard Model

Summer Student Lecture 2023 – Part I



Alvaro Lopez Solis

Deutsches Elektronen Synchrotron

24th-26th July 2023

HELMHOLTZ ASSOCIATION

Many thanks to Sarah Heim and Thorsten Kuhl for their lectures and help

DESY

Preface

- Concentrating on *general concepts and a broad overview.*

- This lecture cannot replace a university course on particle physics

- Very different level of knowledge: Hard to devise a course that fits all....
  → Some parts more interesting to beginners, some more to the advanced
  - You will see many formulas ! Don't panic and don't focus on them. They are there to support a physics explanation or just to give you a glimpse of what is the theory behind all these concepts (i.e. show that something exists even if we won't talk in depth about it).

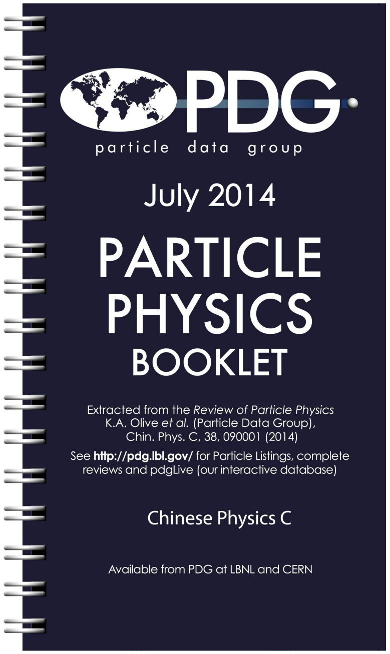
- **Please do ask questions!**

- **If you have any further question, my email is : alvaro.lopez@desy.de**

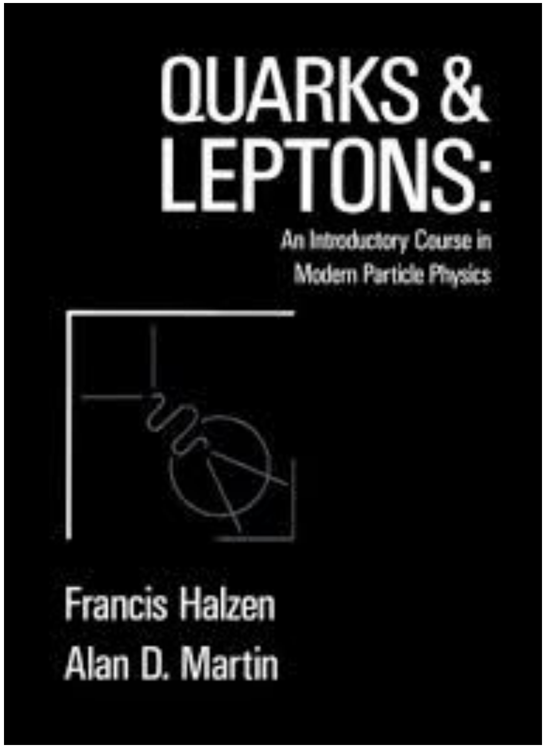
Literature

- http://pdg.lbl.gov/

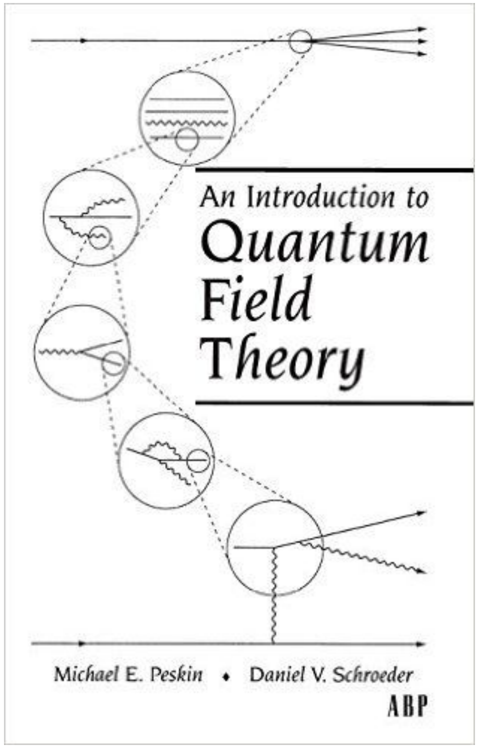
The summary



The book



The BIBLE



Content

- **0) Introduction**
  - **- What is the Standard Model?**
  - **- Coupling constants, masses and charges**
  - **- Units and scales**

- **1) Interactions**
  - **- Relativistic kinematics**
  - **- Symmetries and conserved quantities**
  - **- Feynman diagrams**
  - **- Running couplings and masses**

- 2)Quantum electrodynamics
  - - Test of QED: Magnetic momentum of the muon
  - - Test of QED: High energy colliders

Content

- 3) Strong Interaction: Quantum-Chromodynamics
  - A short history of hadrons and quarks
  - DIS and gluons
  - QCD and its properties

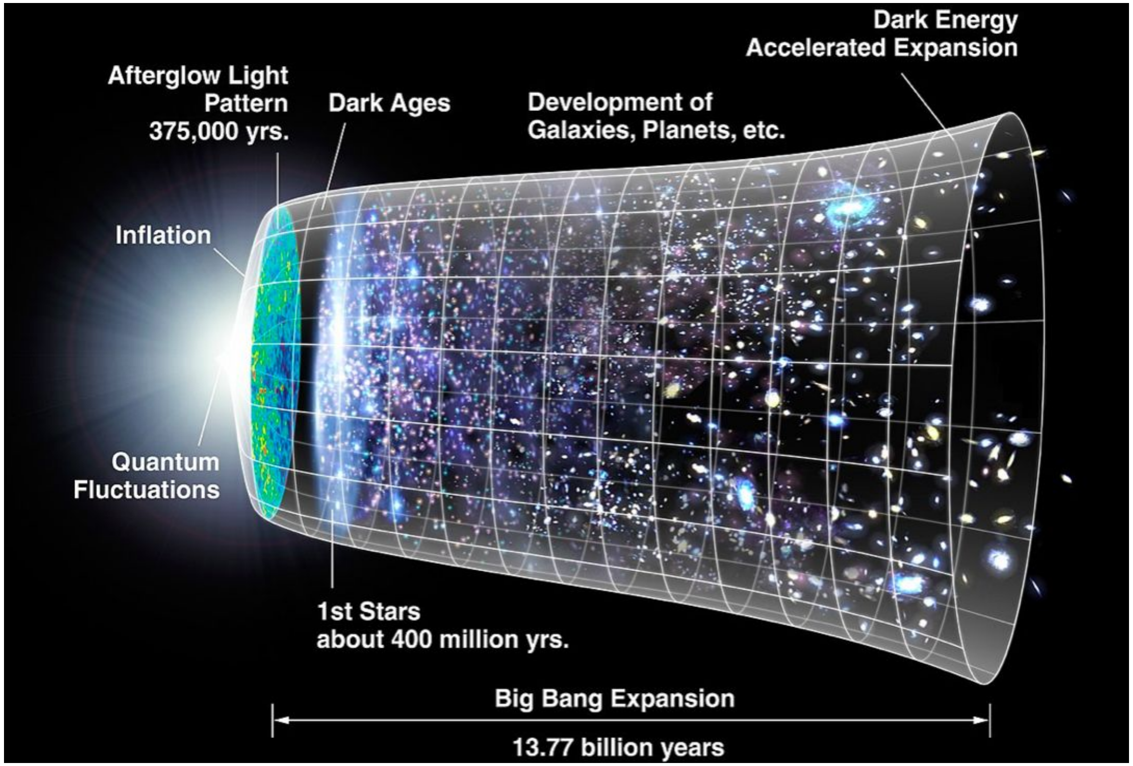
- 4) Electroweak interactions
  - Discovery of electroweak bosons
  - Tests of angular distributions
  - Feynman rules
  - Handed-ness of electroweak interactions
  - More tests of the electroweak SM

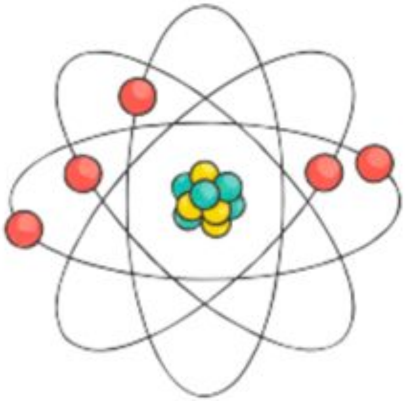
- 5) The Higgs
  - Why was it predicted?
  - How was it found?

Introduction

Introduction to the Standard Model

From atoms to the galaxies and clusters, the history of our Universe and its evolution is determined by the presence of particles and the interactions amongst them





During these days, we will try to summarize our current knowledge of the most basic components of Nature and their interactions

Let’s start easy: what is a particle in QM ?

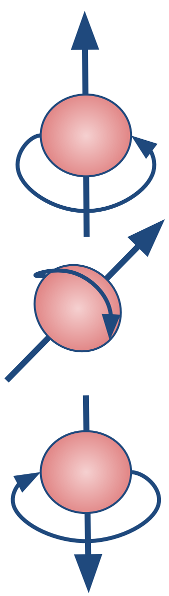
In your quantum mechanics courses, a particle is described by a wave-function.

$$|\psi\rangle = \Psi(x, p, t; S, S_Z; L, L_z, ..)$$

Momentum (or position) and time

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx/\hbar} dx$$

Spin

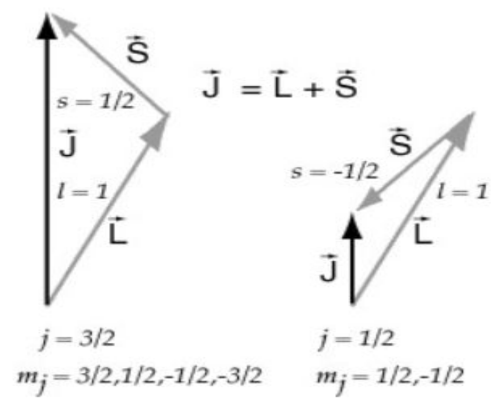


$$|\uparrow\rangle$$

$$\frac{1}{\sqrt{2}} |\uparrow\rangle \pm \frac{1}{\sqrt{2}} |\downarrow\rangle$$

$$|\downarrow\rangle$$

Angular momentum



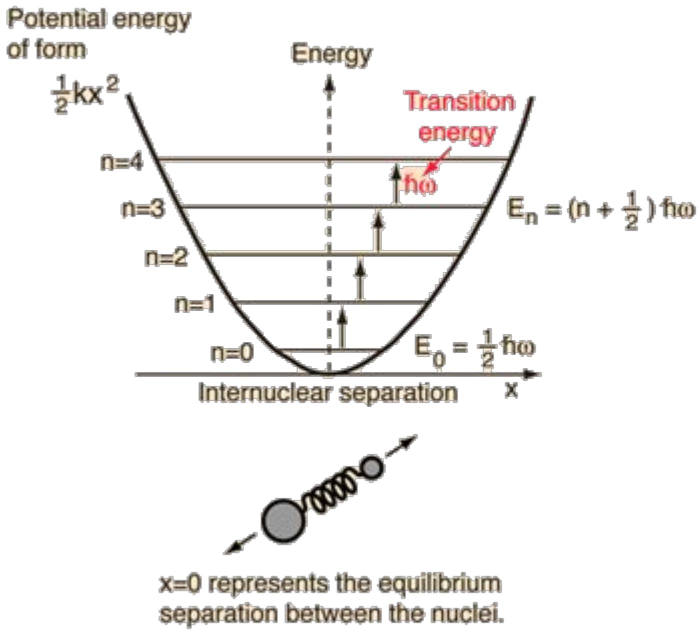
\+ other properties

Let's start easy: what is a particle in QM ?

The evolution of the wave-function is governed by Schroedinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Harmonic oscillator of 1 particle system



Hamiltonian

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2$$

$$\hat{H} = \left(\hat{a}^\dagger\hat{a} + \tfrac{1}{2}\right)\hbar\omega$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{X} + \frac{i}{\sqrt{2m\omega\hbar}}\hat{P}$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\hat{X} - \frac{i}{\sqrt{2m\omega\hbar}}\hat{P}$$

Creation and annihilation

$$\hat{a}|n\rangle = c_n|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = d_n|n+1\rangle$$

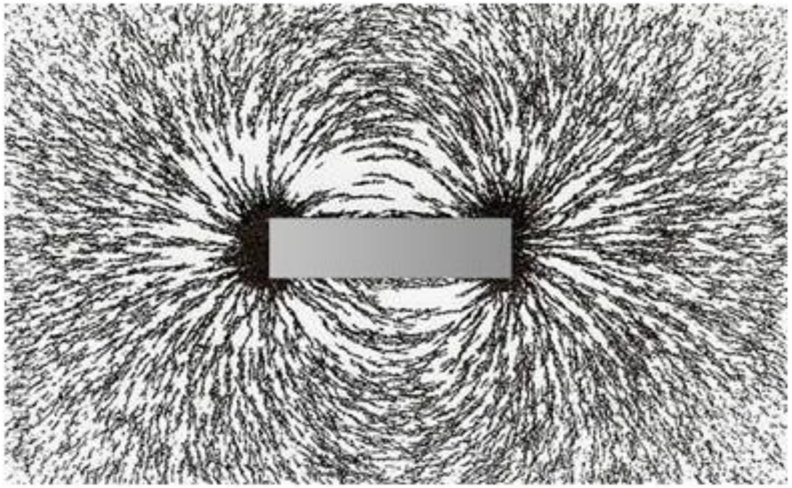
$$\hat{H}|n-1\rangle = E_{n-1}|n-1\rangle = (E_n - \hbar\omega)|n-1\rangle$$

$$\hat{H}|n+1\rangle = E_{n+1}|n+1\rangle = (E_n + \hbar\omega)|n+1\rangle$$

Energy is quantized. Ground state of minimal energy and excited states.

Let’s start easy: what is an interaction in QM ?

Interactions between particles, via classical continuous fields.



[Classical field]

Continuous

Mainly describing forces

Following field equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

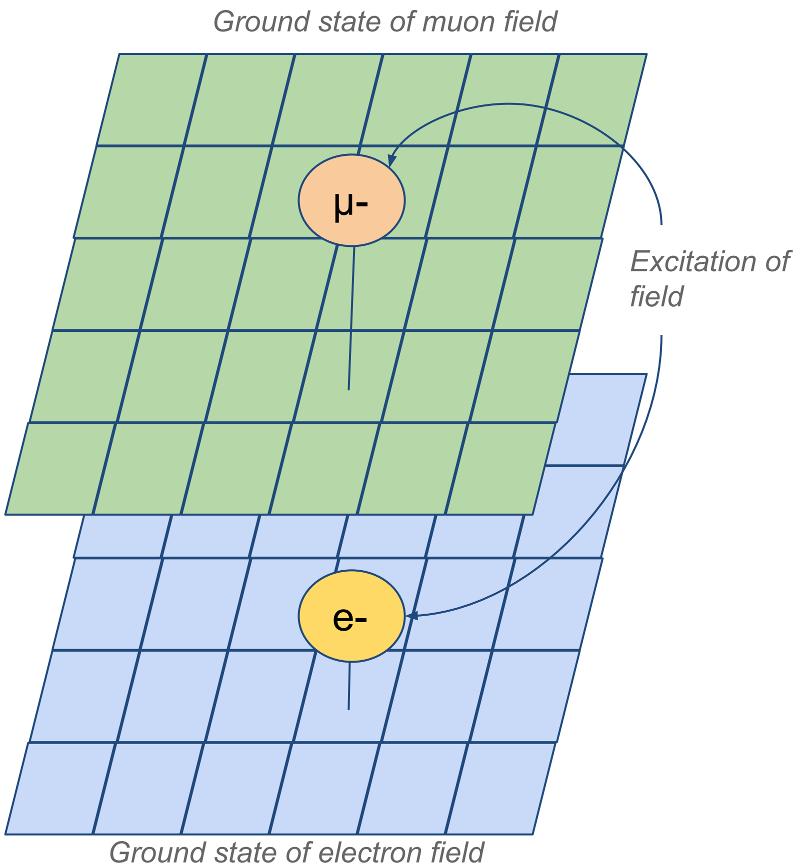
Let’s start easy: what is a particle ?

In modern particle physics, particles are just an excited states of an underlying quantized field !

$$|\psi(\mathbf{x})\rangle = \phi(\mathbf{x})\,|0\rangle$$

$$\phi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega_p}}\left(a_p + a^\dagger_{-p}\right)e^{i\mathbf{p}\cdot\mathbf{x}}$$

Free scalar field for particle creation at position x



- One field → one particle type
- Matter fields and force fields
- Discrete (quantized), not continuous

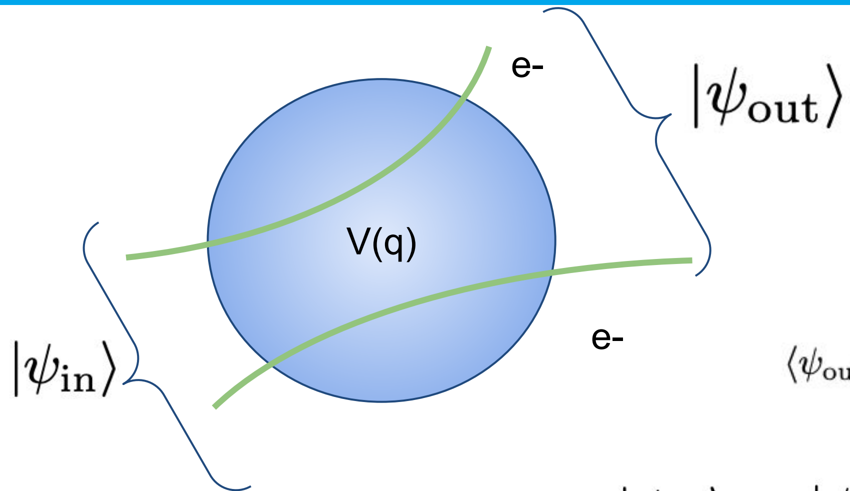
Bosonic fields (integer spin)

$$\left[a_p, a^\dagger_{p'}\right] = i\,(2\pi)^3\,\delta^3\,(p - p')$$

Fermionic fields (semi-integer spin)

$$\left\{a_p, b^\dagger_{p'}\right\} = i\,(2\pi)^3\,\delta^3\,(p - p')$$

Let’s start easy: what is an interaction between particles ?



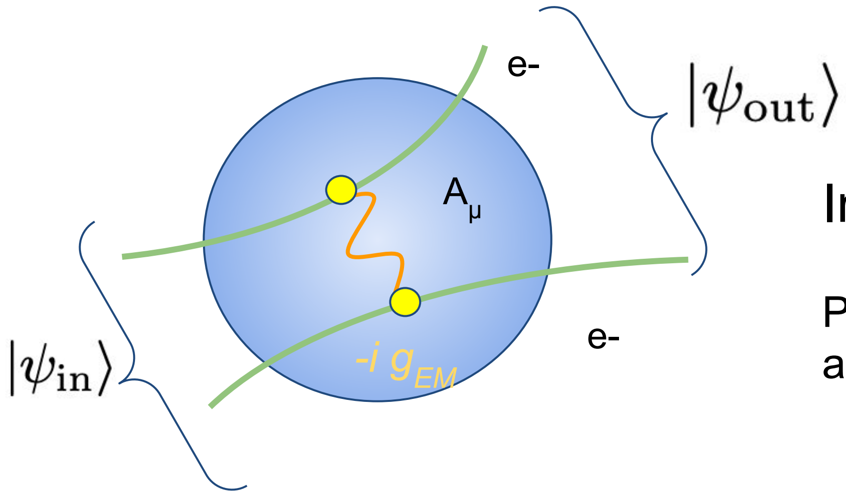
Interactions in QM

In QM, encoded in potential terms in Hamiltonian

$$\langle\psi_{\text{out}}|\,\hat{H}\,|\psi_{\text{in}}\rangle = \langle\psi_{\text{out}}|\,H_0 + H_I\,|\psi_{\text{in}}\rangle = \langle\psi_{\text{out}}|-\frac{h^2}{2m}\nabla^2 + V(q)\,|\psi_{\text{in}}\rangle$$

Probability of interaction so state $|\psi_{\text{in}}\rangle$!= $|\psi_{\text{out}}\rangle$?

$$\boxed{\mathcal{T} \propto |\langle\psi_{\text{out}}|\,V(q)\,|\psi_{\text{in}}\rangle|^2}$$



Forces/interactions in QFT

Interaction are exchanges of “force” fields

Possible between fields that have a coupling/charge associated to this force field in Lagrangian

Probability of interaction so state $|\psi_{\text{in}}\rangle$!= $|\psi_{\text{out}}\rangle$?

$$\boxed{i\mathcal{M} \propto \left|g_{\text{EM}}^2\,\langle\psi_{\text{out}}|\,A_\mu\,|\psi_{\text{in}}\rangle\right|^2}$$

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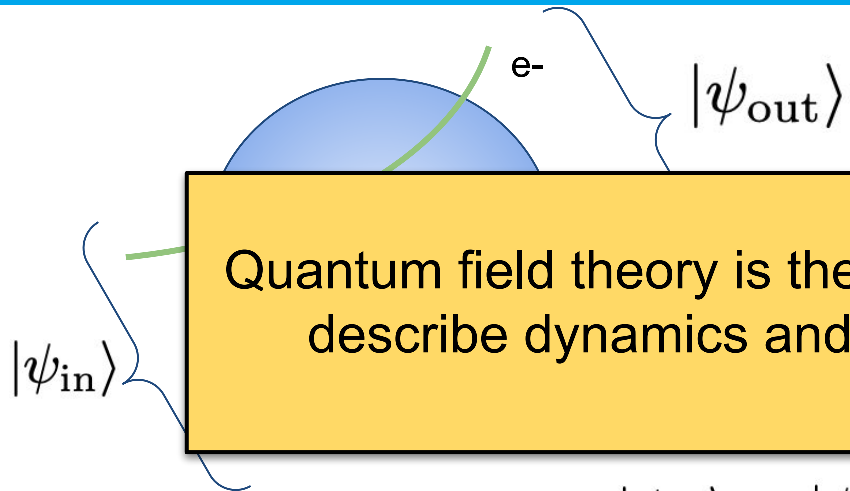

Interactions in QM

In QM, encoded in potential terms in

Quantum field theory is the theoretical framework we use to describe dynamics and interactions of different fields

$+ V(q) |\psi_{\text{in}}\rangle$

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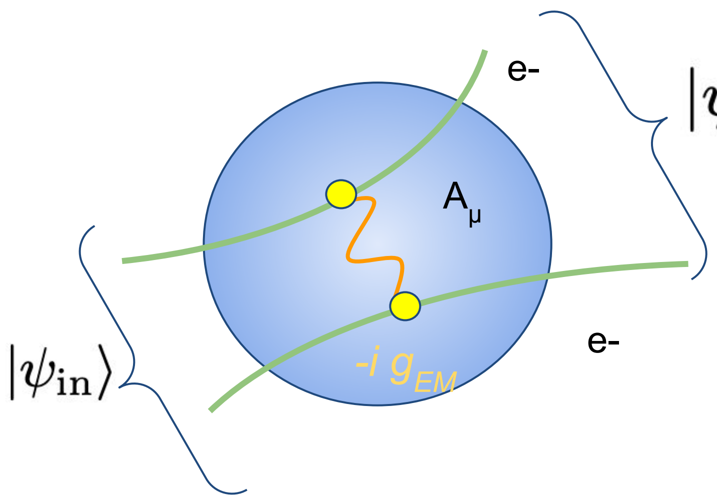

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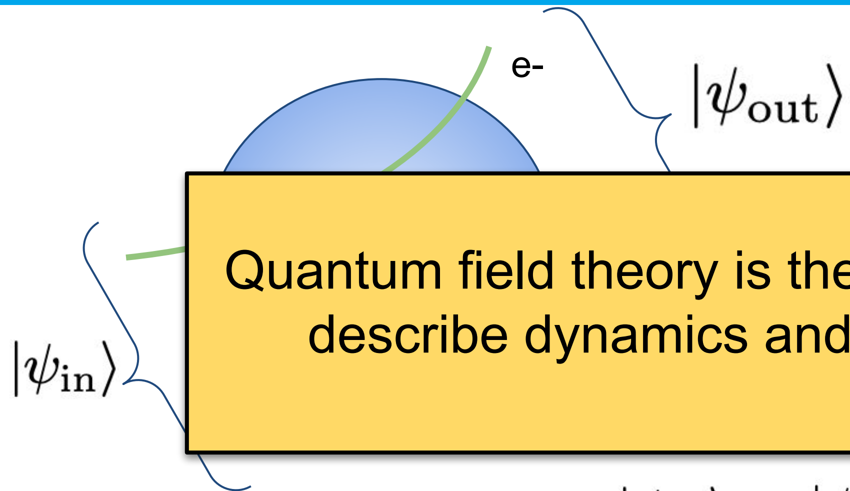

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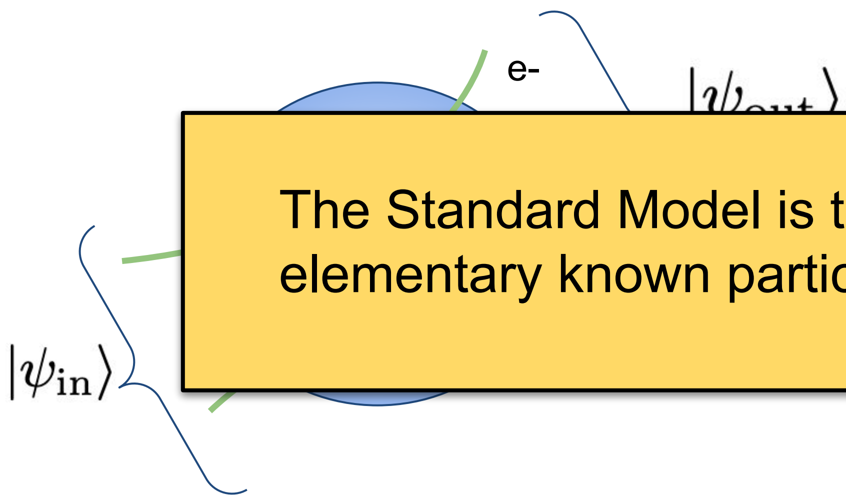


Forces/interactions in QFT

The Standard Model is the theory that describes all elementary known particles and their interactions.

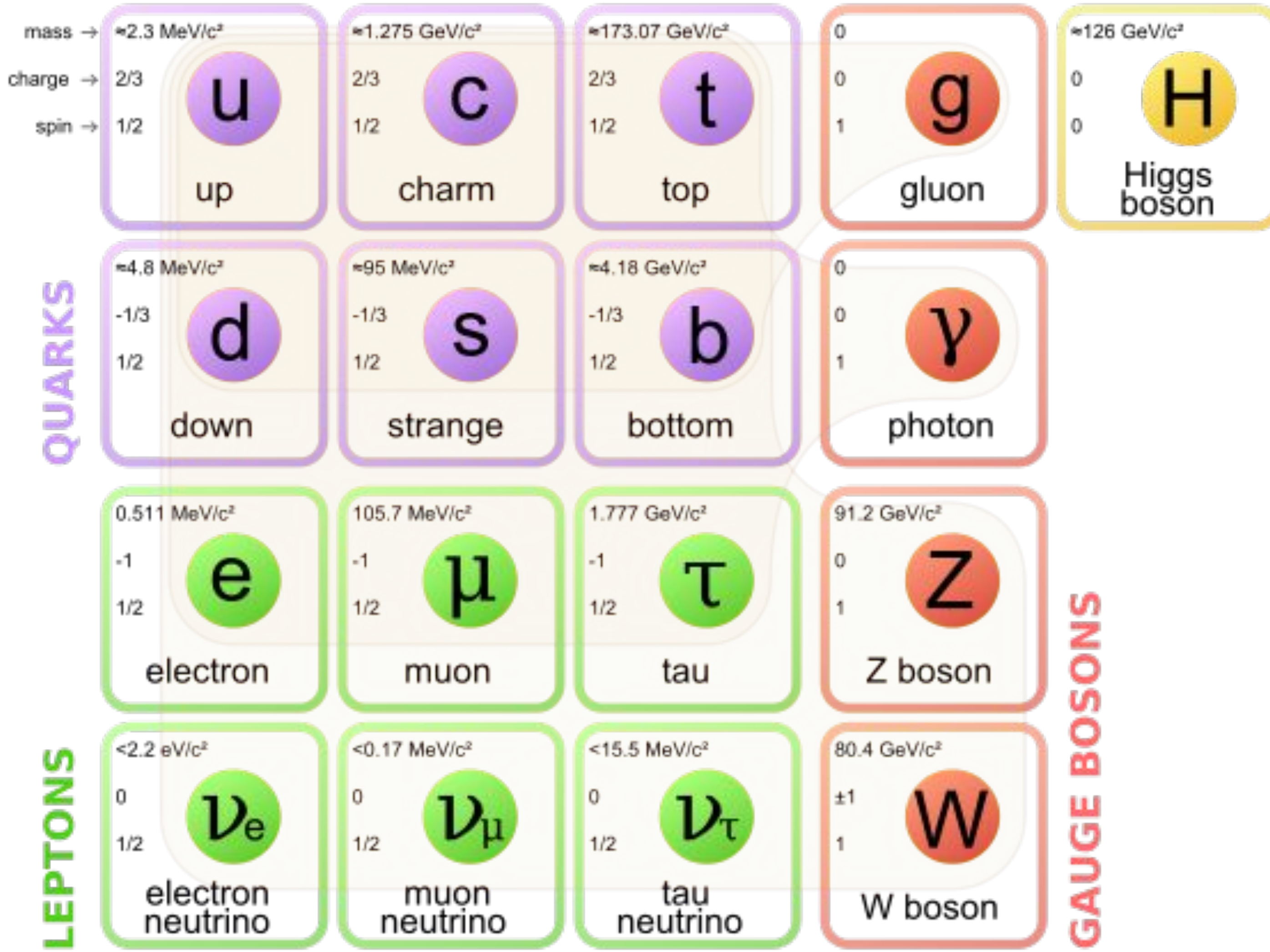
fields

/charge

Probability of interaction so state $|\psi_{\text{in}}\rangle$!= $|\psi_{\text{out}}\rangle$?

$$i\mathcal{M} \propto \left|g_{\text{EM}}^2 \,\langle\psi_{\text{out}}|\, A_\mu \,|\psi_{\text{in}}\rangle\right|^2$$

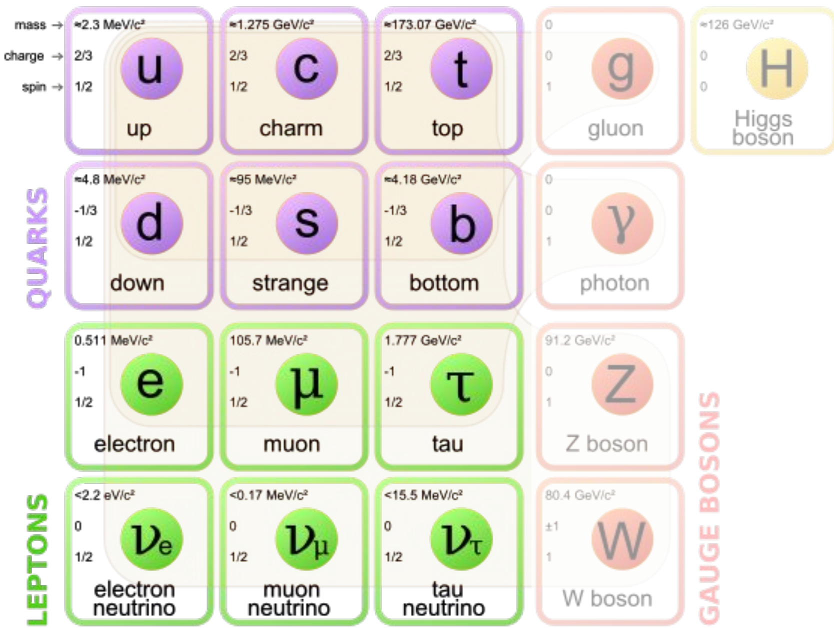
What is the Standard Model?



>picture from wikipedia

What is the Standard Model?

> Fields described by the Standard Model can be classified as



Matter fields

These are the fields of which most of the traditional particles (protons, neutrons, electrons) that we know are composed

- Leptons: don't interact with strong force
- Quarks: interact with strong force
- Spin ½
- 3 Generations or flavours

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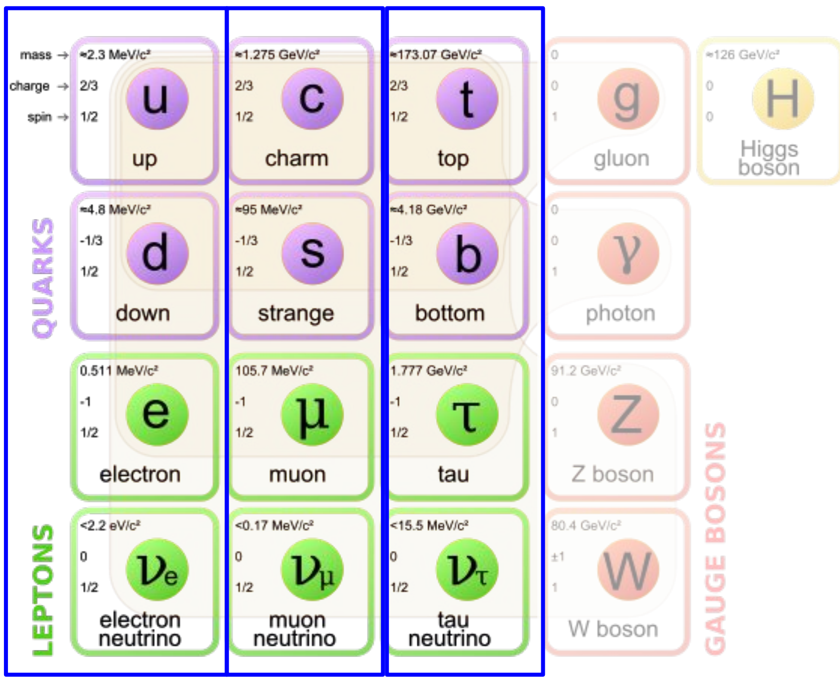

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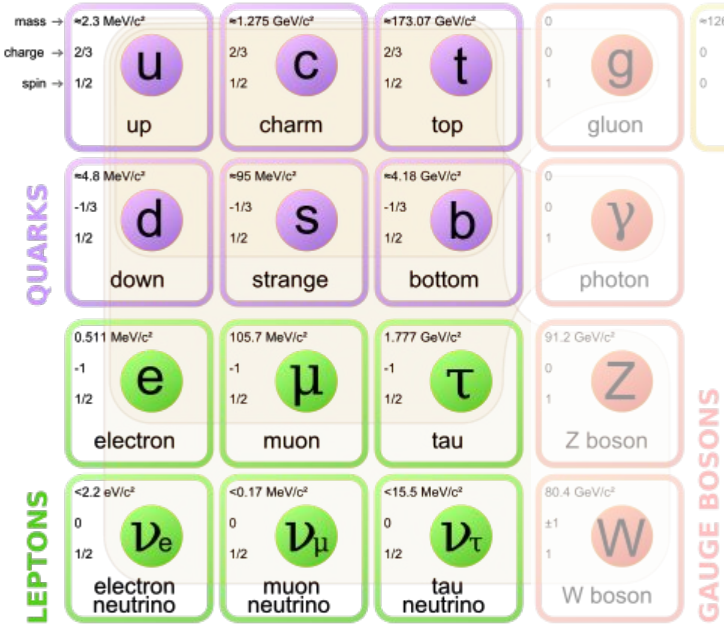

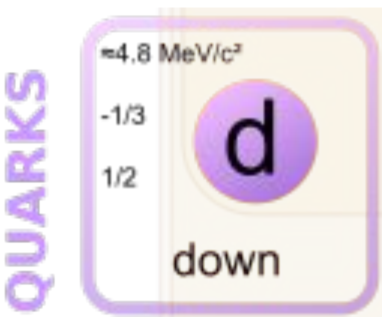

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Particle properties

- Baryon number
- Lepton number
- Electric charge, weak isospin and color
- Mass

+Isospin, strangeness, charm, bottomness, topness

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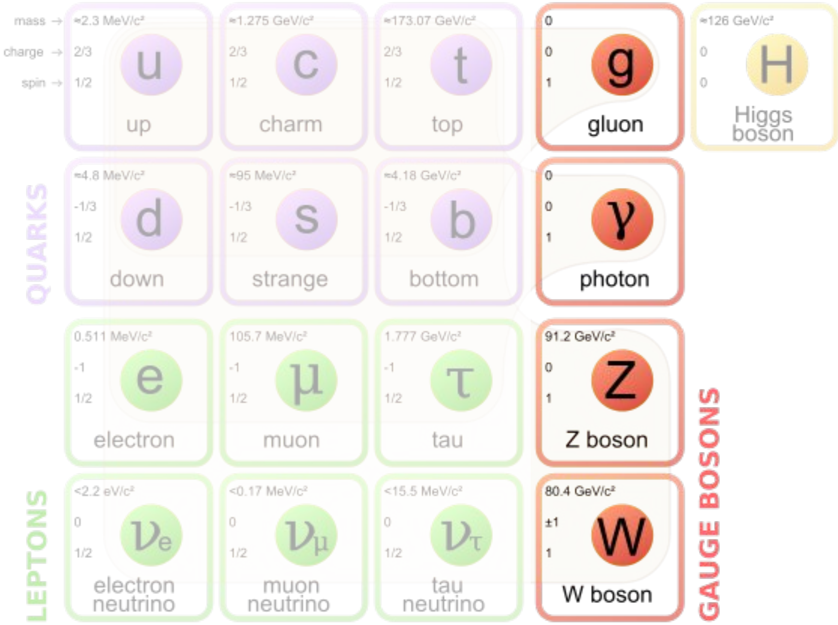

Force fields

These are the fields exchanged in the interactions between matter fields

- Gauge bosons
- Spin 1
- Mediator of interactions between particles that are charged under the treated interaction

Three forces currently described by the SM

What is the Standard Model?

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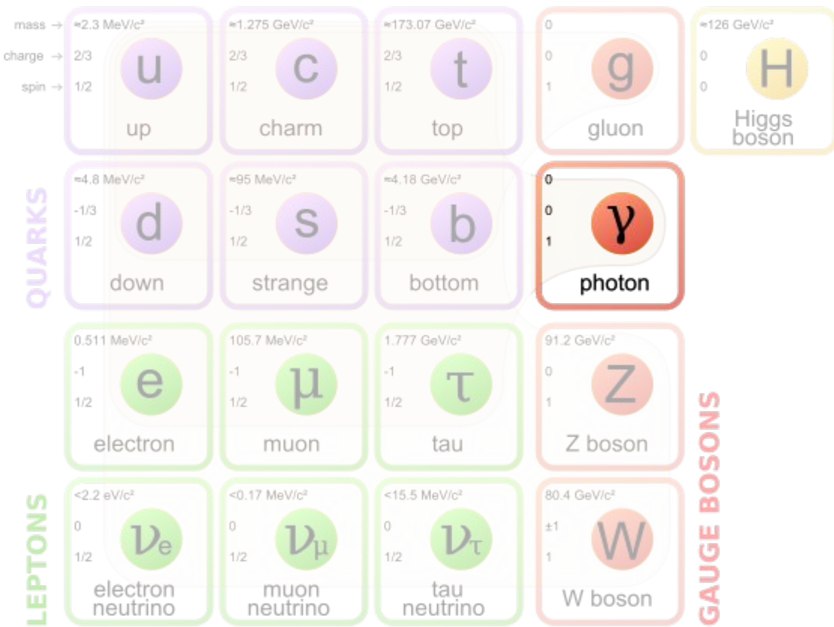

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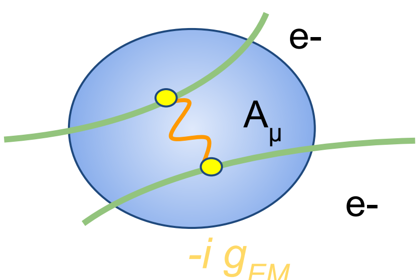
Electromagnetic interaction

Gauge boson: photon (A_μ)

Possible between fields that possess electromagnetic charge (g_{EM})

Quantum electrodynamics (QED)

Three forces currently described by the SM



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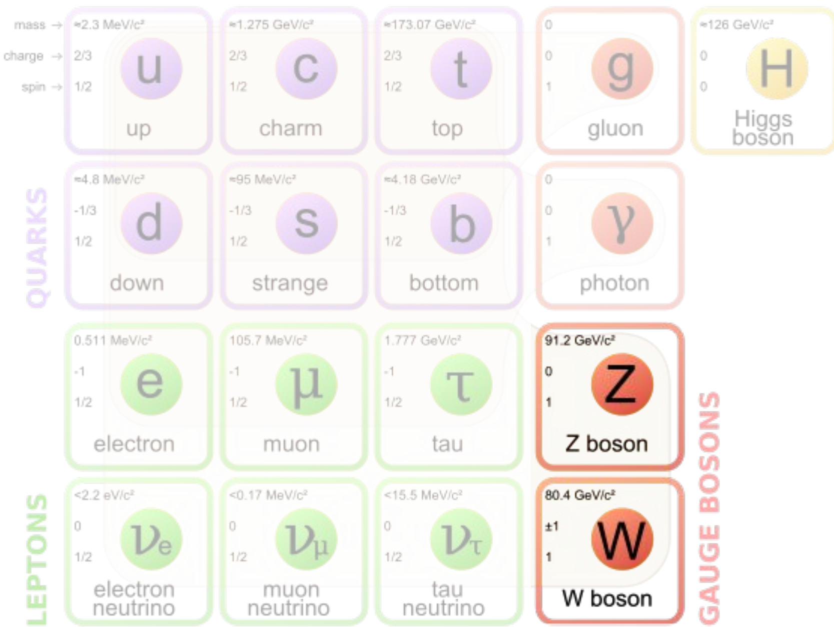

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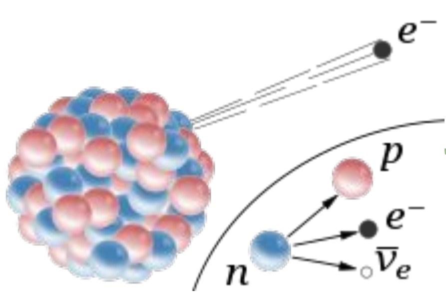
Three forces currently described by the SM

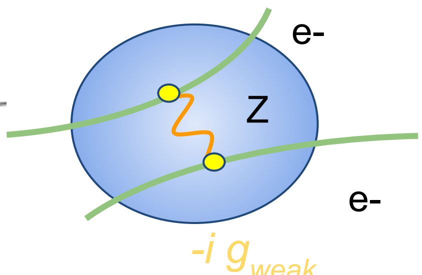
Weak interaction

Gauge bosons: W^+, W^-, Z^0 (W_μ)

Possible between fields that possess Isospin and/or hypercharge (g_{weak})

GSW mechanism





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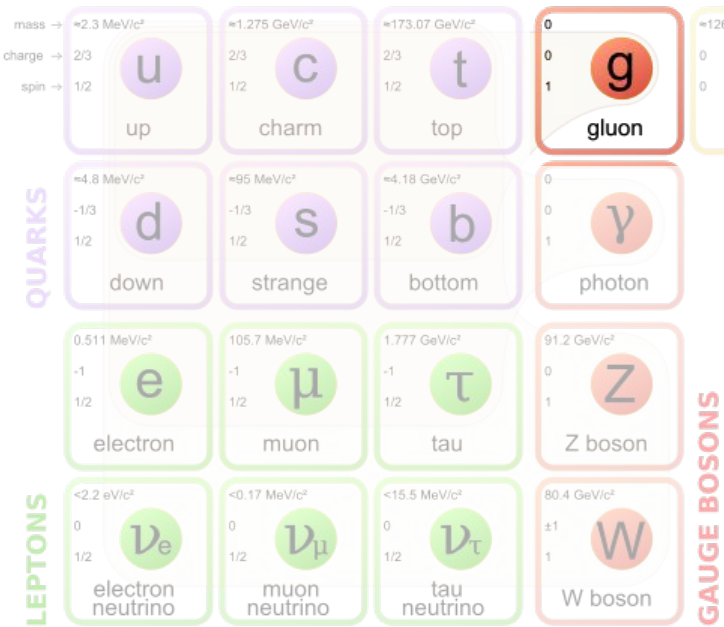

Force fields

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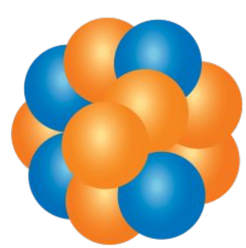
Three forces currently described by the SM

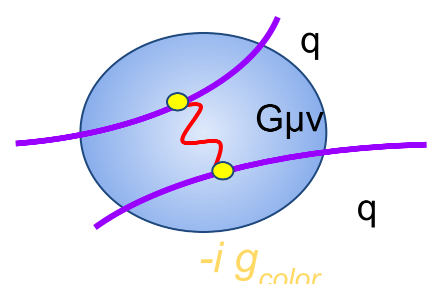
Strong interaction

Gauge bosons: gluons

Possible between fields that possess color charge (g_{color})

Quantum chromodynamics (QCD)





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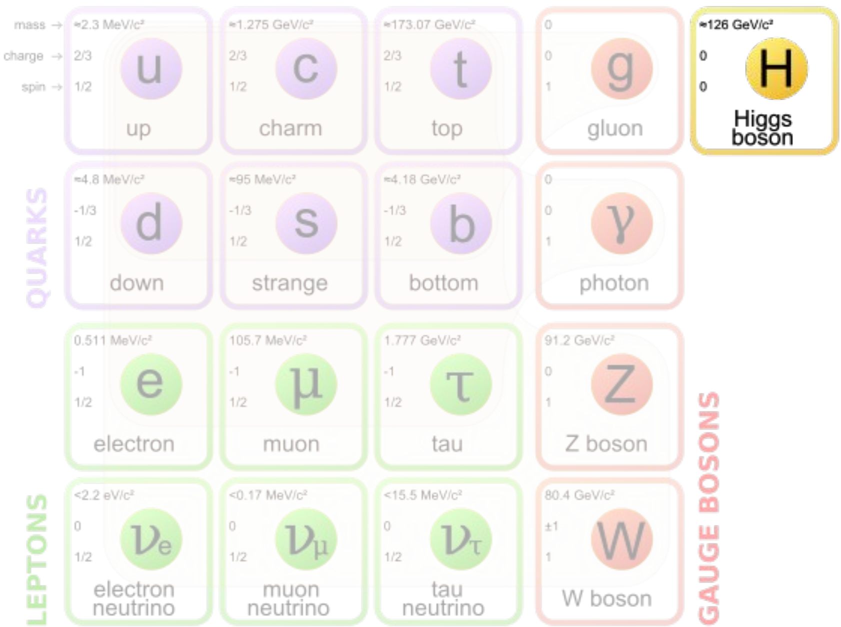

Higgs field

Particles in the Standard Model obtain masses via interaction with the Higgs field.

- Excited quanta : Higgs boson
- Spin 0
- The strength of the coupling between the Higgs field is the reason of the mass hierarchy in SM
  - Larger coupling → higher mass of the particle.

What is the Standard Model?

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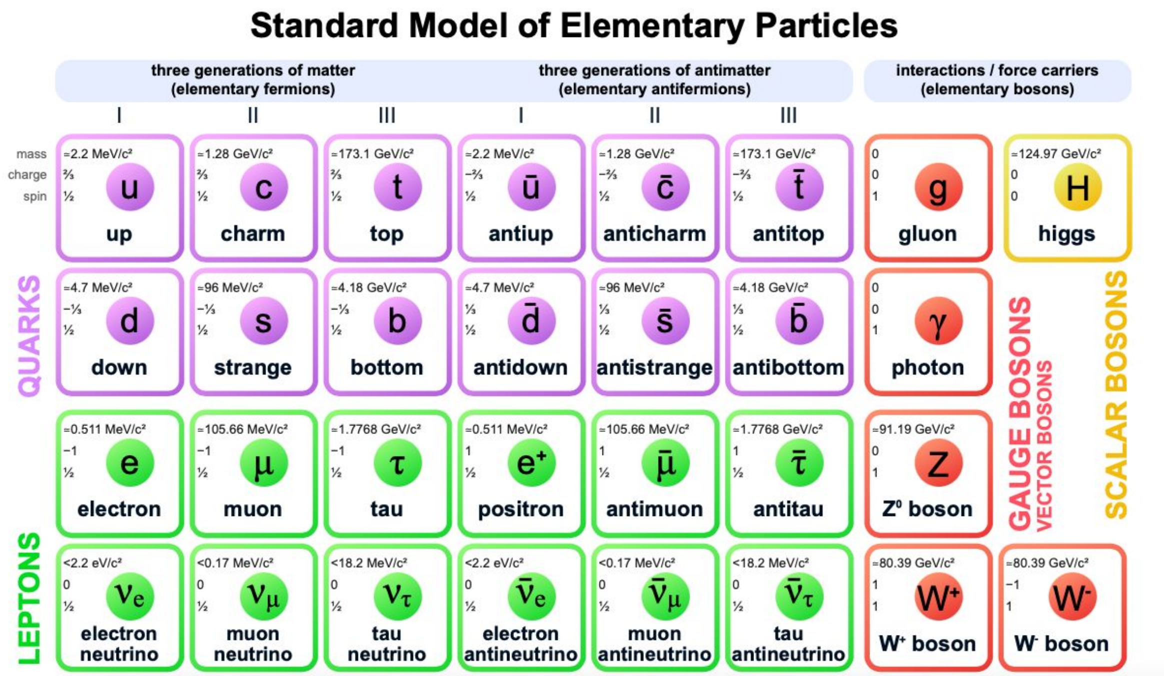


Particles and anti-particles

Each of the particles in the SM has its anti-particle

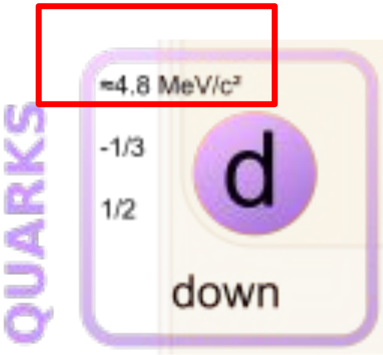
- Same mass
- Same flavour but opposite lepton or baryon number
- Same spin
- Color → anti-color ; negative electric charge →positive electric charge

Units and scales

> "natural units" → $c = 1 \quad \hbar = 1$
  (masses, energies and momenta measured in GeV)

Conventional Mass, Length, Time Units, and Positron Charge in Terms of $\hbar = c = 1$ Energy Units

Conversion Factor	$\hbar = c = 1$ Units	Actual Dimension
$1\ \text{kg} = 5.61 \times 10^{26}\ \text{GeV}$	GeV	$\dfrac{\text{GeV}}{c^2}$
$1\ \text{m} = 5.07 \times 10^{15}\ \text{GeV}^{-1}$	GeV^{-1}	$\dfrac{\hbar c}{\text{GeV}}$
$1\ \text{sec} = 1.52 \times 10^{24}\ \text{GeV}^{-1}$	GeV^{-1}	$\dfrac{\hbar}{\text{GeV}}$
$e = \sqrt{4\pi\alpha}$	—	$(\hbar c)^{1/2}$



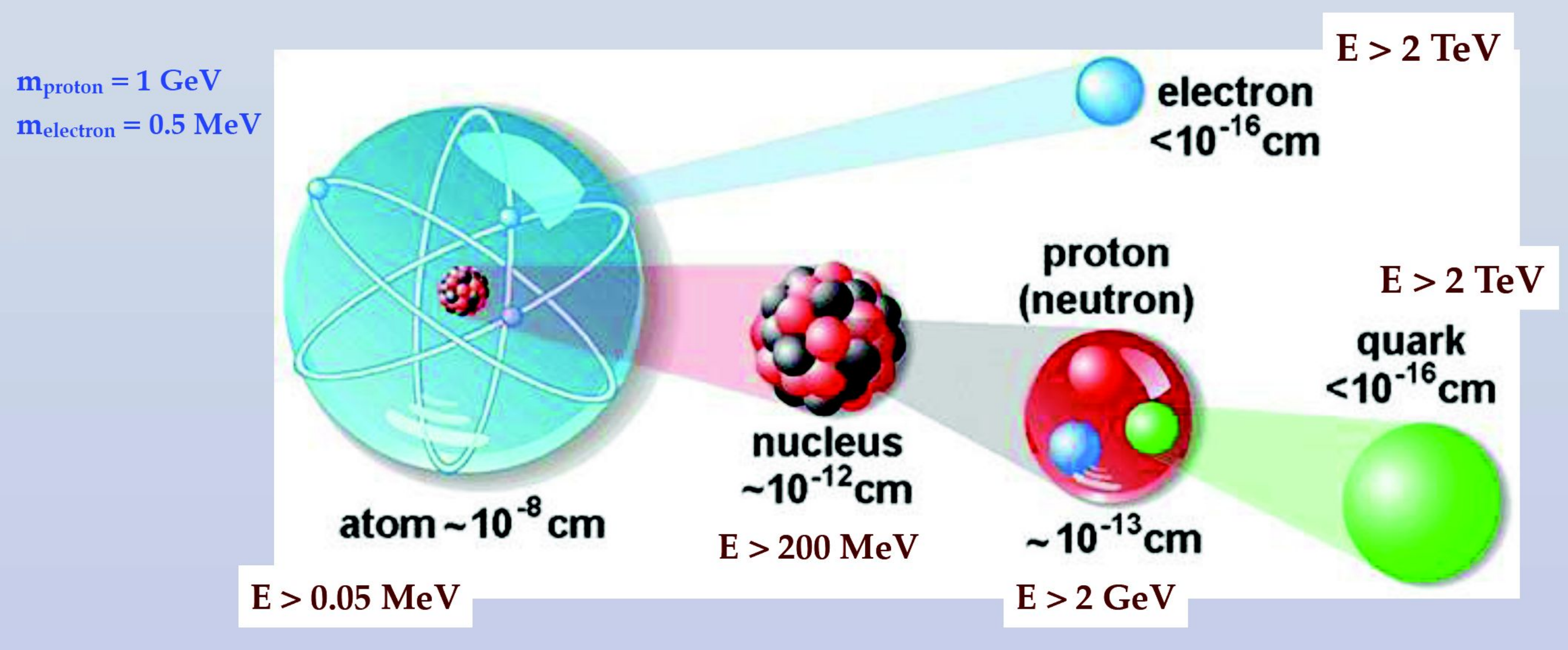
Some Useful Conversion Factors

$1\ \text{TeV} = 10^3\ \text{GeV} = 10^6\ \text{MeV} = 10^9\ \text{KeV} = 10^{12}\ \text{eV}$
$1\ \text{fermi} \equiv 1\ \text{F} = 10^{-13}\ \text{cm} = 5.07\ \text{GeV}^{-1}$
$(1\ \text{F})^2 = 10\ \text{mb} = 10^4\ \mu\text{b} = 10^7\ \text{nb} = 10^{10}\ \text{pb}$
$(1\ \text{GeV})^{-2} = 0.389\ \text{mb}$

[Taken from:
Quarks and Leptons:
An Introductory Course in Modern Particle Physics
Francis Halzen/Alan D. Martin]

$\hbar c \sim 197\ \text{MeV fm}$

Units and scales



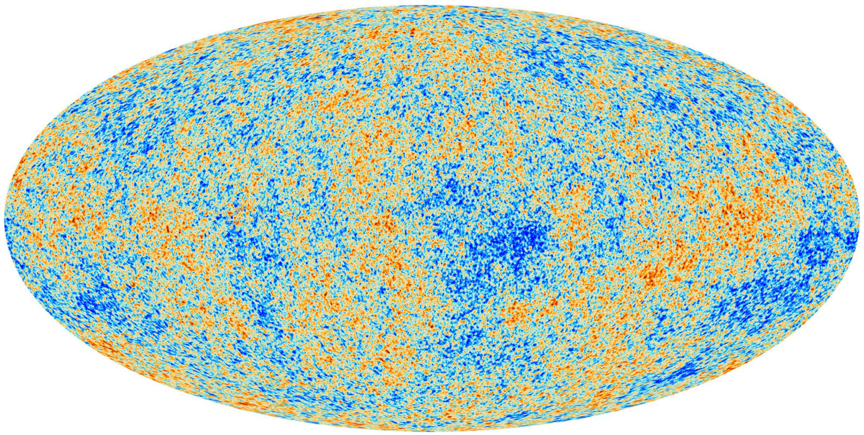
$$
\begin{aligned}
1\,\mathrm{eV} &= 1.602 \cdot 10^{-19}\,\mathrm{J} \\
\mathrm{keV} &= 10^{3}\,\mathrm{eV} \\
\mathrm{MeV} &= 10^{6}\,\mathrm{eV} \\
\mathrm{GeV} &= 10^{9}\,\mathrm{eV} \\
\mathrm{TeV} &= 10^{12}\,\mathrm{eV}.
\end{aligned}
$$

$$
\begin{aligned}
m_e &= 511\,\mathrm{keV} \\
m_p &= 938\,\mathrm{MeV} \\
m_n &= 939\,\mathrm{MeV} \\
E_e(\mathrm{LEP}) &= 104.5\,\mathrm{GeV} \\
E_p(\mathrm{Tevatron}) &= 980\,\mathrm{GeV} \\
E_p(\mathrm{LHC}) &= 7\,\mathrm{TeV}.
\end{aligned}
$$

Basic blocks of the Standard Model: conserved quantities, interactions, Feynman !

We have the Standard Model. Why did we do that ?

> We have formulated how fields interact in a consistent way. Why ?



Ey SM! Nice to meet you ! I like history. Tell me the history of this light ?

You know that photons at high energy cannot travel too far, right ?! Someone cooled them down to 3000 K at one point

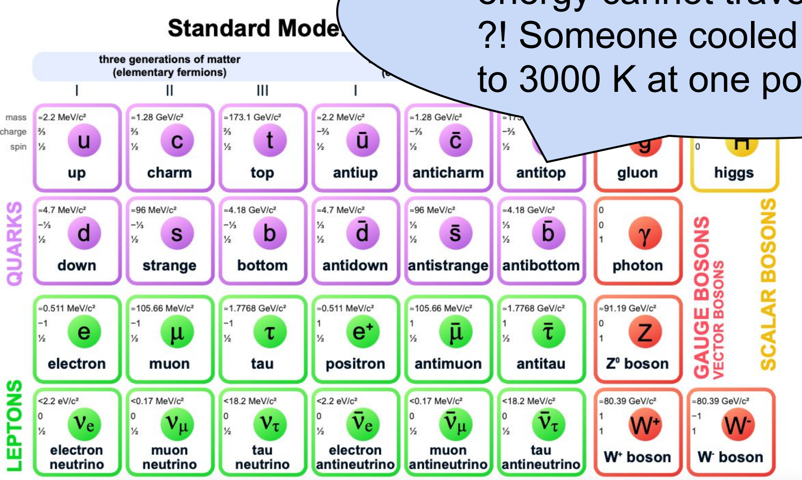


Image from LibreText

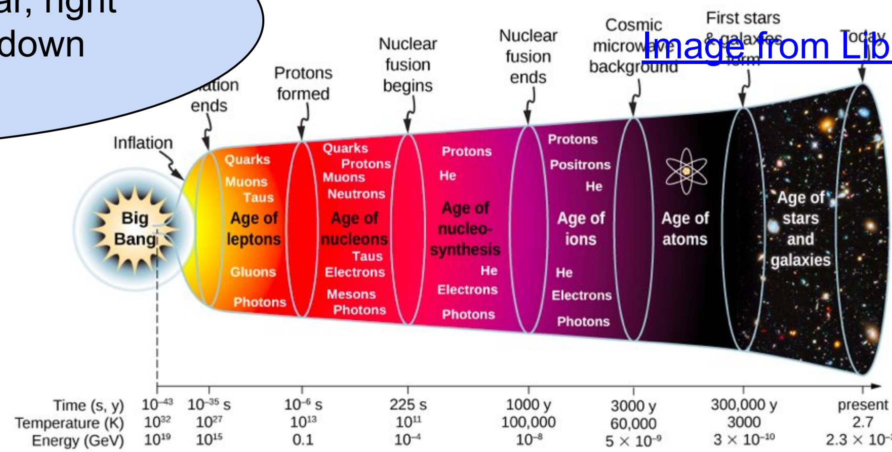


Figure 11.8.2: An approximate timeline for the evolution of the universe from the Big Bang to the present.

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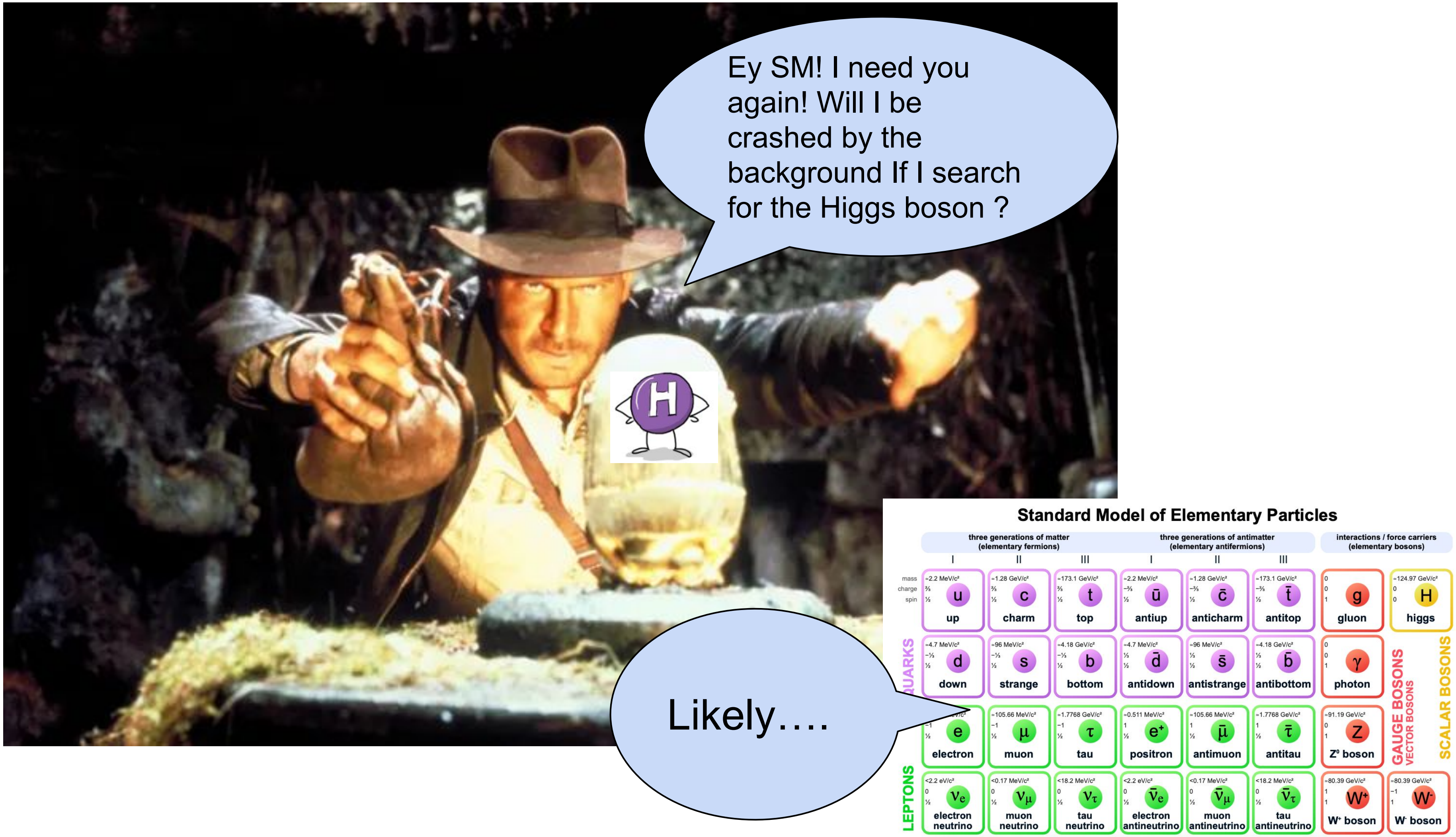

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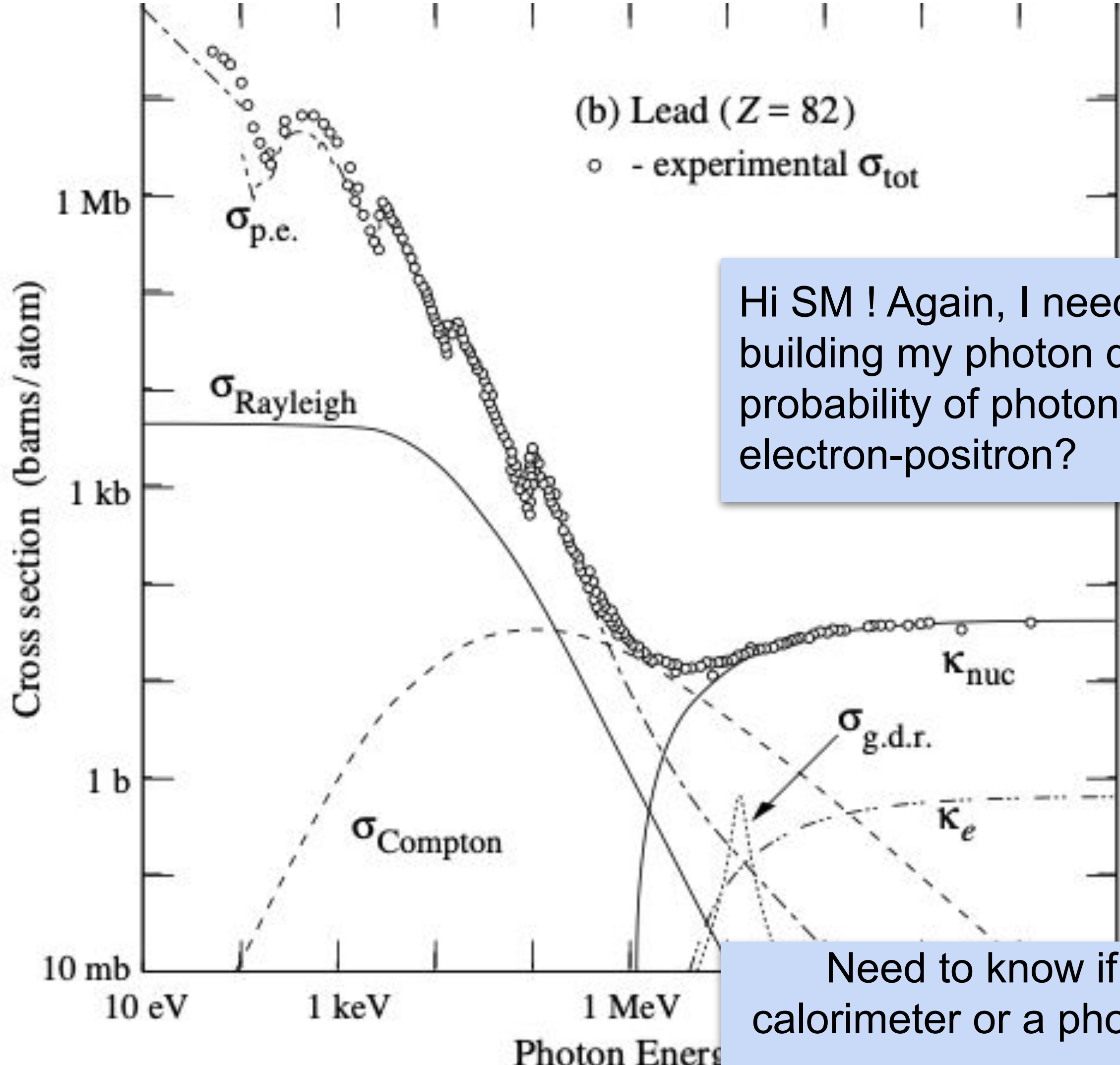
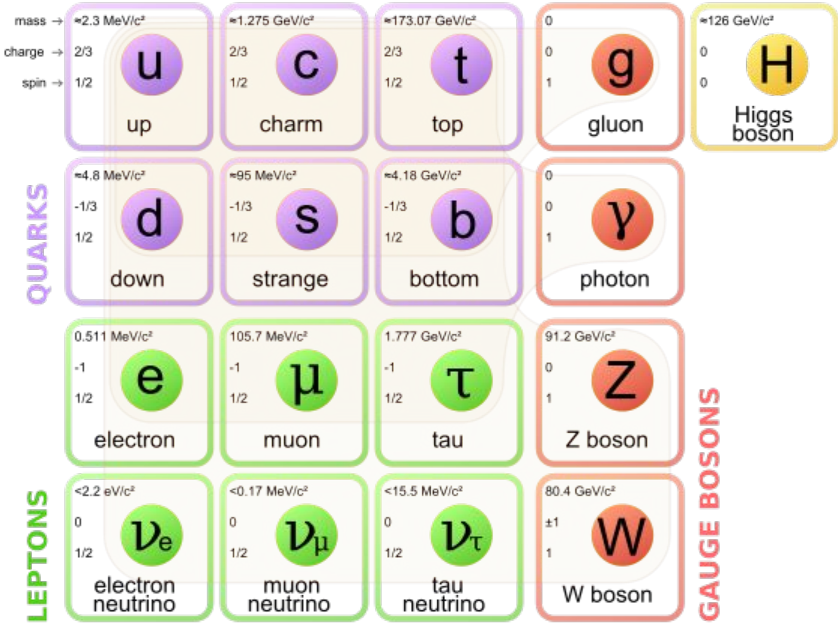


Hi SM ! Again, I need your magic. I am building my photon detector. What is the probability of photons to decay into electron-positron?

Need to know if I need a calorimeter or a photo-multiplier.

We have the Standard Model. Why did we do that ?

>We have formulated how fields interact in a consistent way. Now, what ?



Knowledge of what interactions we can expect from every particle.

Predictions + measurements of masses and couplings.

Predictions of how likely an interaction is to happen.

…….

Deviations from the Standard Model predictions → New Physics !

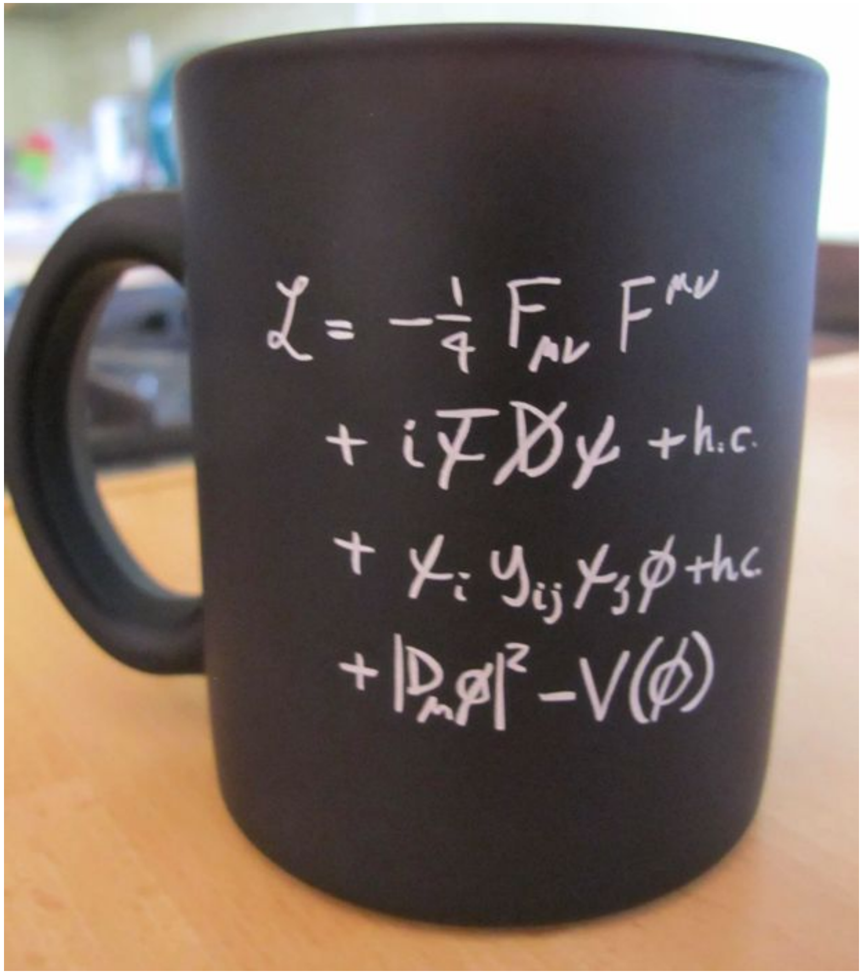
New particles (DM..) or forces (super-QED ?) → New Physics !

How do we calculate probabilities ? The big SM monster

Lagrangian formalism

SM describing dynamics of fields and interactions -> Using a lagrangian

- Kinematics of spin-1 bosons
- Kinematics of fermions.
- Interactions between fields
- Kinematics of spin-0 fields (no interactions)



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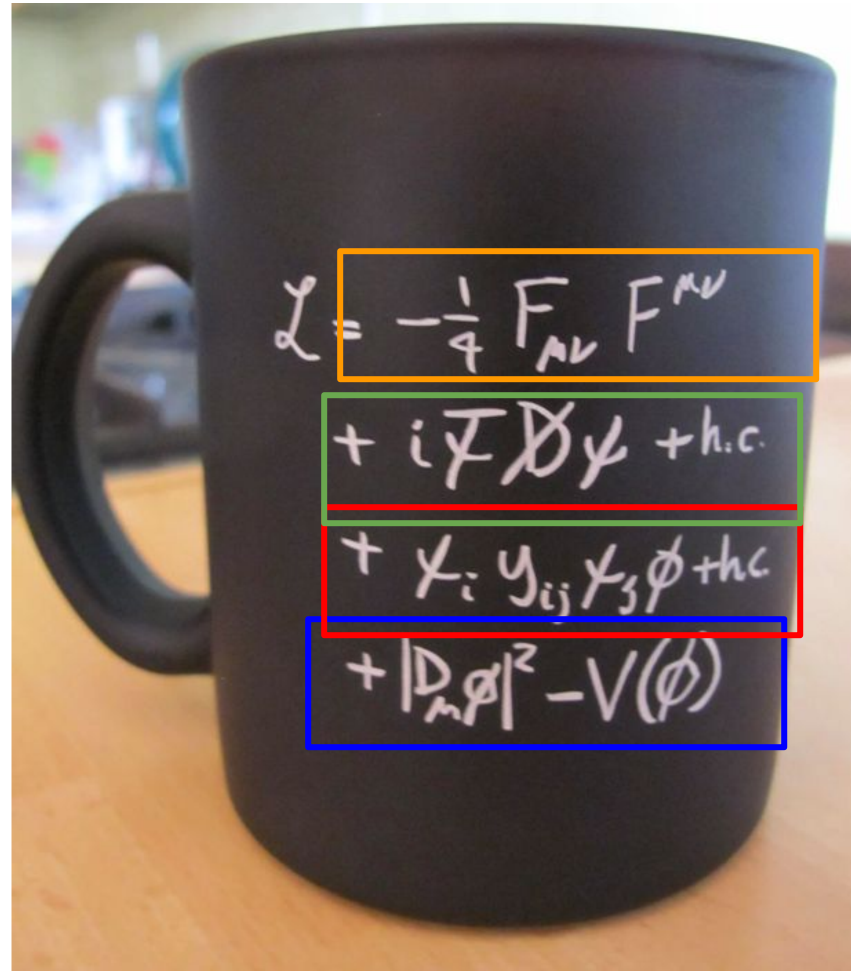

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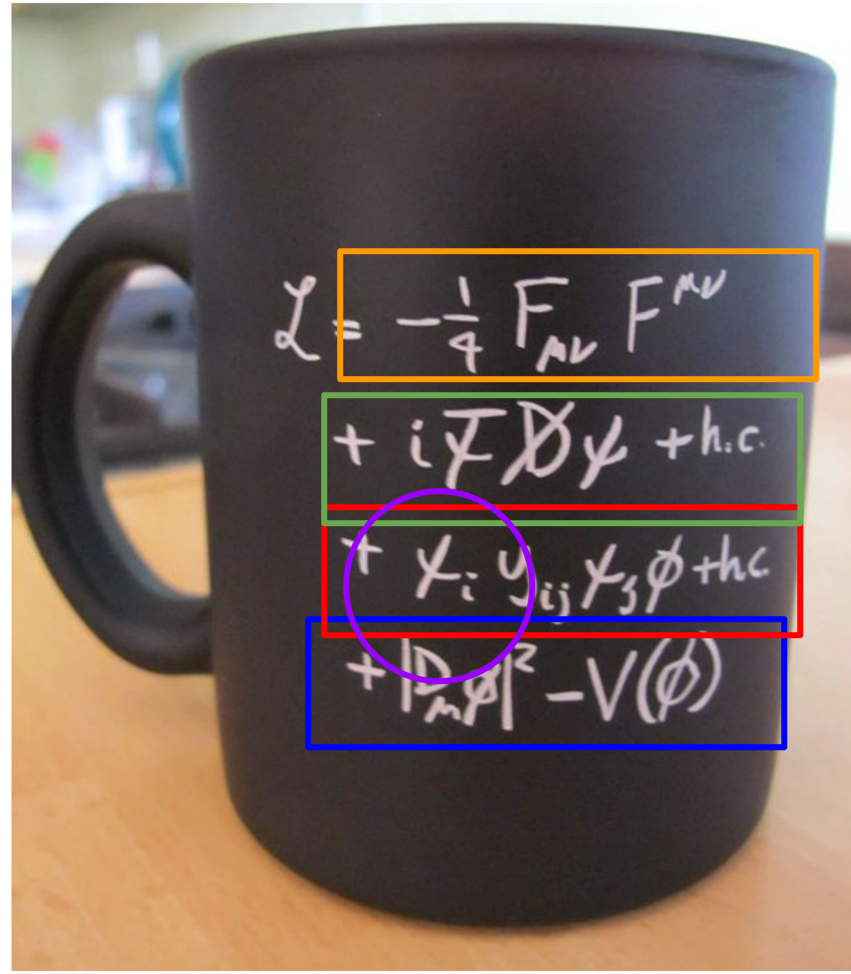
Coupling between different fields.



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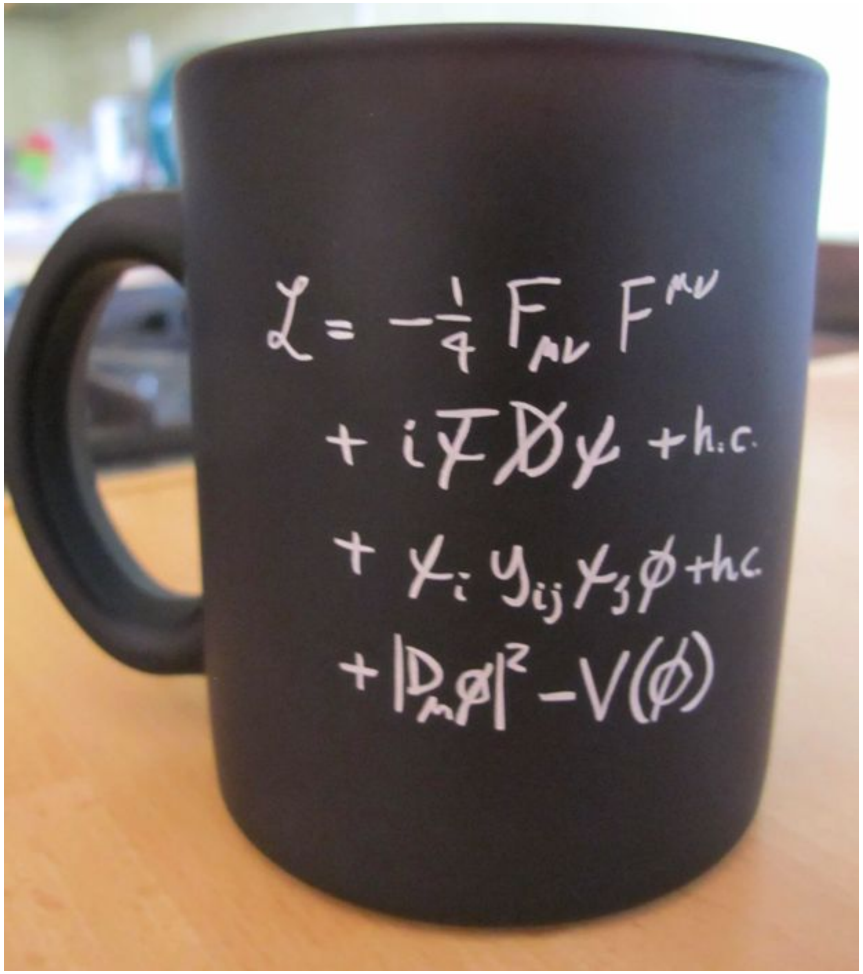


Euler-Lagrange

Equations of motion

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))} \right) - \frac{\partial \mathcal{L}}{\partial \phi(x)} = 0$$

Klein-Gordon eq.: kinematics spin-0 fields

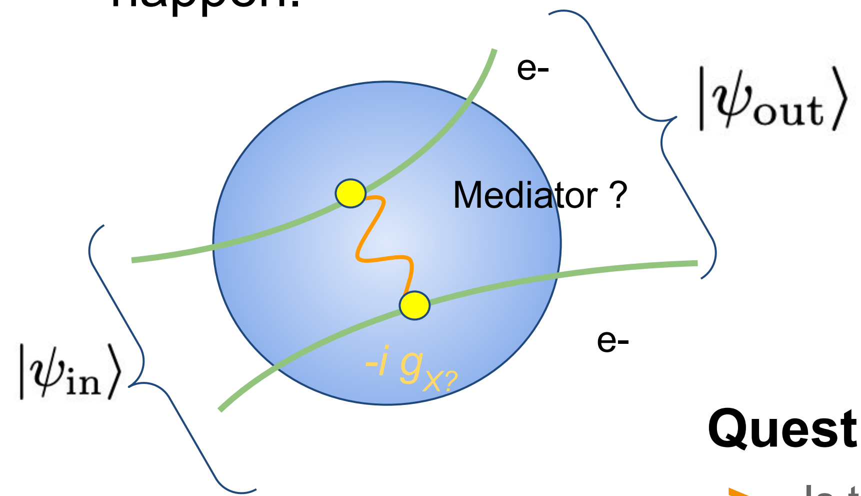
$$\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi(x) \phi(x) = 0$$

Dirac equation: kinematics spin ½ field

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

Probability that two particles interact (e.g electrons through QED)?

- The strength of an interaction depends on the probability of a process to happen.



$$i\mathcal{M} \propto \left| g_{\text{EM}}^2 \langle \psi_{\text{out}} | A_\mu | \psi_{\text{in}} \rangle \right|^2$$

Questions for a possible interaction:

- Is the process kinematically available? Do we have enough energy in the interaction to produce a mediator?
- Are there symmetries of my interaction that wouldn’t allow me to have this interactions ?
  - E.g. everything seems fine, but spin of the system is not preserved in the interaction.
- Do my fields have a coupling in the lagrangian?
  - How large is the numerical value of this coupling?

Interactions and conserved quantities: symmetries

- From Quantum mechanics: Symmetry connected to conserved quantity
- Different interactions conserve different quantities

quantity	interaction			invariance
	strong	elm.	weak	
energy	yes	yes	yes	translation in time
momentum	yes	yes	yes	translation in space
angular momentum	yes	yes	yes	rotation in space
P (parity)	yes	yes	no	coordinate inversion
C (charge parity)	yes	yes	no	charge conjugation (particle ↔ anti-particle)
T (time parity)	yes	yes	no	time inversion
CPT	yes	yes	yes	
lepton number	yes	yes	yes	
baryon number	yes	yes	yes	
isospin	yes	no	no	

\+ Flavour : conserved by strong and electromagnetic. Not by Weak interaction.

Interactions and conserved quantities: Relativistic kinematics

- particle quantities are described as 4-vectors:

  energy/momentum: $p=(E,\vec{p})$

  time/space: $x=(t,\vec{x})$

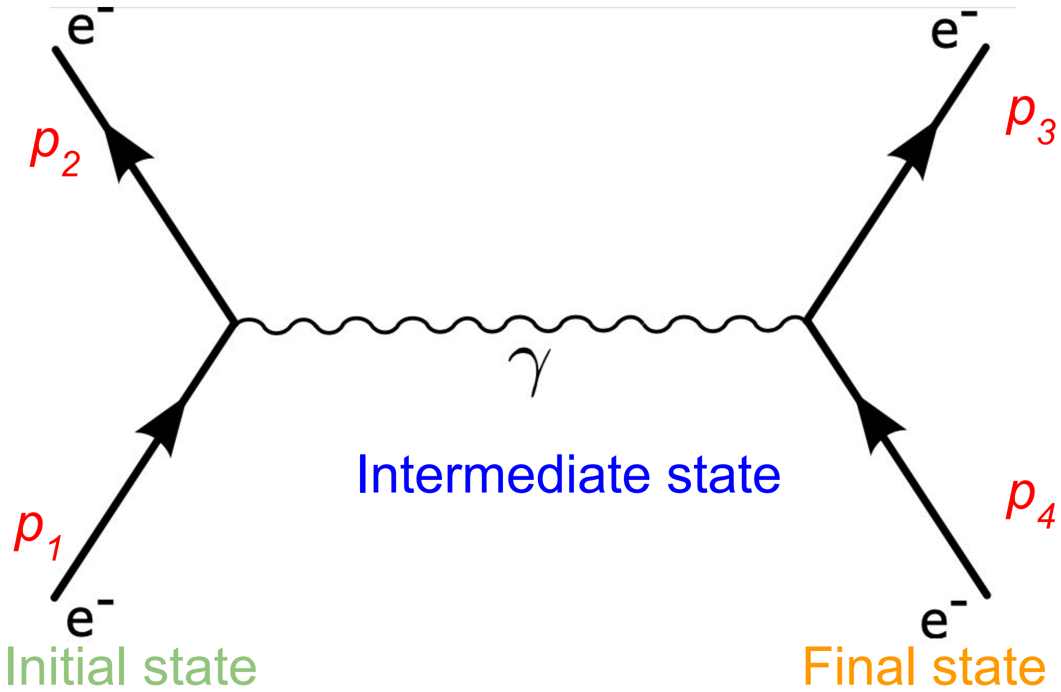
- product of 4-vectors is invariant: $p_1 \cdot p_2=(E_1 \cdot E_2-\vec{p}_1 \cdot \vec{p}_2)=\mathrm{constant}$

  → can use the “easiest” reference frame for calculations

- special case: $p \cdot p=(E^2-\vec{p} \cdot \vec{p})=(E_0^2-0)=m^2$

  with $\beta=v/c$ and $\gamma=1/\sqrt{1-\beta^2}$: $E=\gamma m$ and $|\vec{p}|=\beta\gamma m$

Interactions and conserved quantities: Relativistic kinematics



Pair-annihilation/creation

Particle-antiparticle annihilate and produce alternative field

Scattering

Interacting particles exchange a force field

Decay

Particle decays into several subparticles

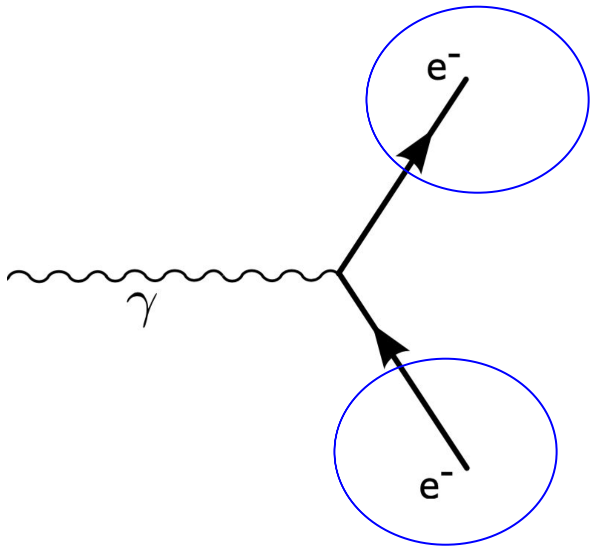
Radiation

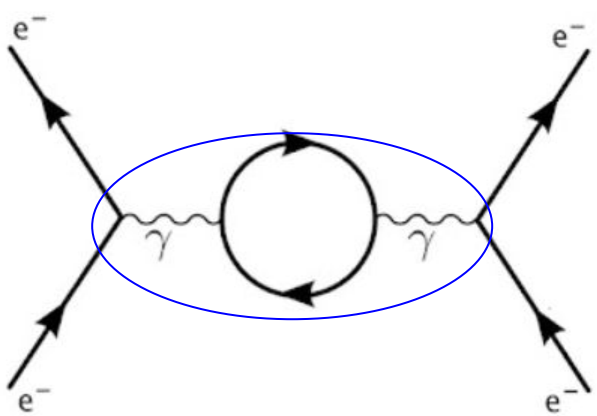
Emission of a final state particle

Kinematically allowed ? → Conservation of total 4-momentum between initial and final states

$$p_1 + p_2 = p_3 + p_4 \qquad \begin{pmatrix} E_1 + E_2 \\ \vec{p_1} + \vec{p_2} \end{pmatrix} = \begin{pmatrix} E_3 + E_4 \\ \vec{p_3} + \vec{p_4} \end{pmatrix}$$

Kinematics: on-shell (real) and off-shell (virtual) particles





On-shell particle or real particle

Particles produced in the collision and satisfying momentum-mass relation

$$p \cdot p = m^2$$

Off-shell particle or virtual particle

Intermediate particles in the interaction. Conserve momentum and energy in the vertices. Quantum fluctuation, possible for very short time thanks to Heisenberg principle.

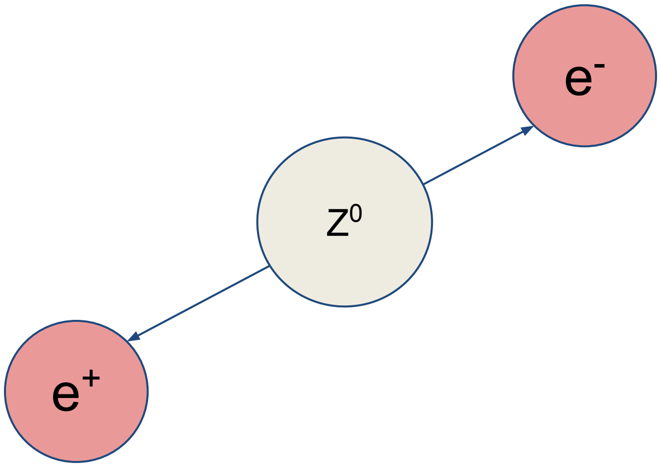
$$p \cdot p \neq m^2$$

$$\Delta E \, \Delta t \geq \hbar \approx 6.6 \cdot 10^{-22} \, \mathrm{MeV\,s}$$

Probability of interaction, higher if on-shell particles (resonance)

Kinematically allowed: decay example

Decay of on-shell Z-boson



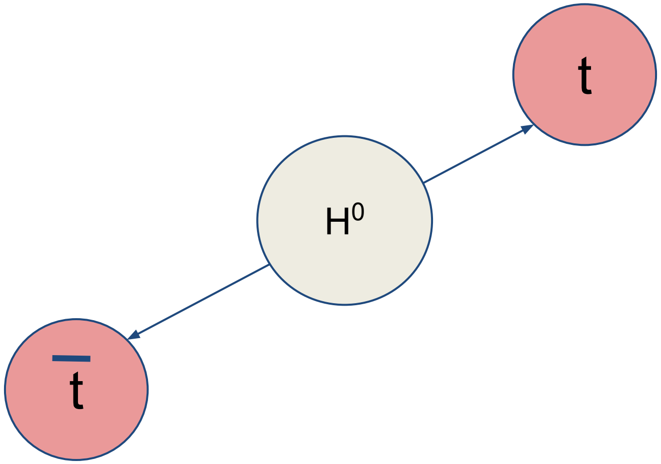
Mass of Z-boson ~ 91 GeV

Mass of electron/positron = 0.511 GeV

m_Z >> 2 x m(electron-positron)

Allowed !

Decay of Higgs boson to top quarks



Mass of Higgs boson ~ 125 GeV

Mass of top/anti-top = 175 GeV

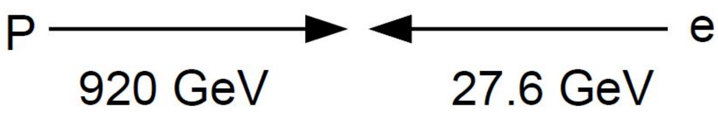
Top-quark has mass (highest mass in SM) → Interaction H-t exists

m(H) << 2 x m(top)

Not allowed !

Relativistic kinematics: additional concepts

- centre-of-mass energy of a collider, e.g. HERA



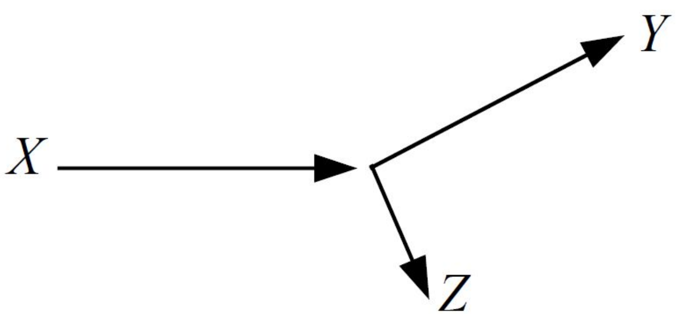
$$p_P = (E_P, \vec{p}_P) = (E_P, 0, 0, E_P)$$

$$p_e = (E_e, \vec{p}_e) = (E_e, 0, 0, -E_e)$$

$$s = (p_P + p_e)^2 = (E_P + E_e)^2 - (E_P - E_e)^2 = 4 E_P E_e \approx 10^5 \,\mathrm{GeV}^2$$

$$\Rightarrow \quad \sqrt{s} = 318 \,\mathrm{GeV}$$

- decay of a particle $X \rightarrow Y Z$:



$$\begin{aligned} M_X^2 &= (p_X)^2 = (p_Y + p_Z)^2 \\ &= m_Y^2 + m_Z^2 + 2\, p_Y p_Z \\ &= m_Y^2 + m_Z^2 + 2(E_Y E_Z - \vec{p}_Y \cdot \vec{p}_Z) \end{aligned}$$

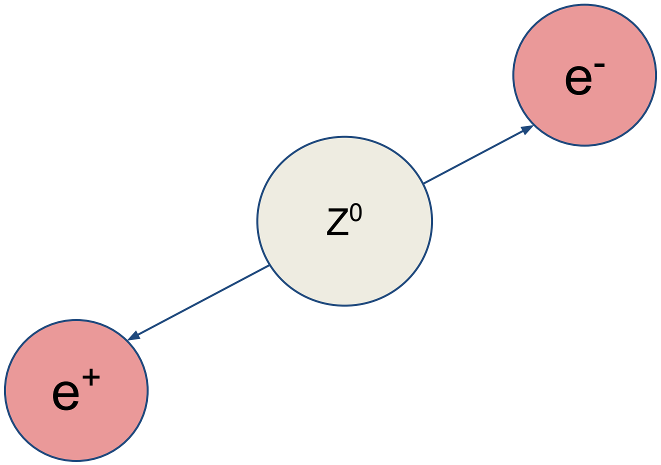
and $E_Y^2 = m_Y^2 + |p_Y|^2, \; E_Z^2 = m_Z^2 + |p_Z|^2$

$\Rightarrow$ if daughter particle types are known (or their masses are negligible), mass of decaying mother particle can be reconstructed from the momenta of the daughters (“invariant mass”)

Symmetries of the interaction: conserved quantities

> Interactions, apart from 4-momentum, conserve properties of the initial and final state.

Decay of on-shell Z-boson



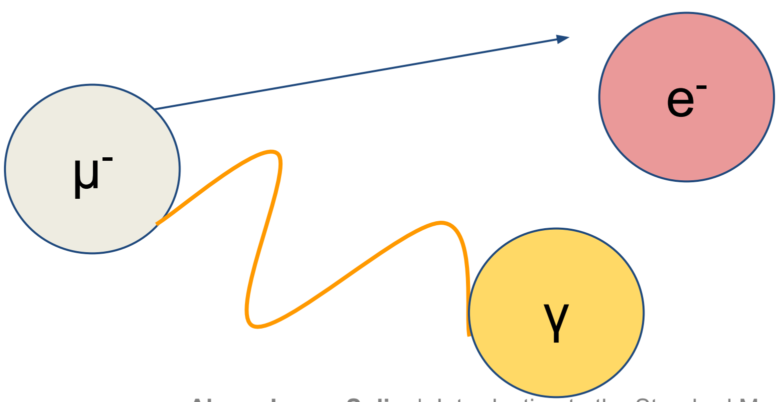
Electric charge of the Z^0 = 0

Electric charge of electron = -1

Electric charge of positron = +1

$$\sum_{\text{initial}} q_i = \sum_{\text{final}} q_j = 0$$

Muon into electron and photon



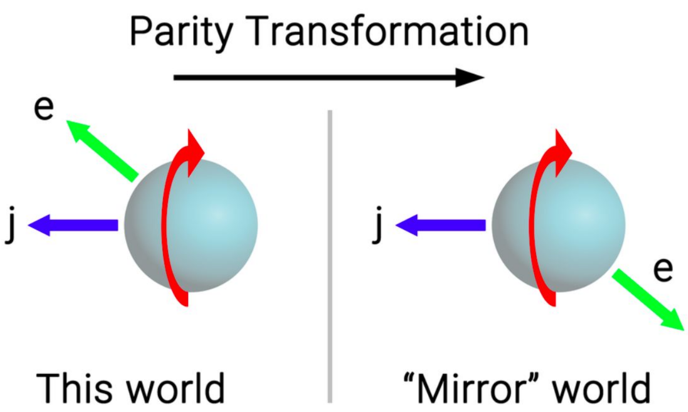
I can clearly write a lagrangian term for this (theory would allow me)

Lepton flavour is violated and SM don't violate flavour

Other conserved properties of an interaction?

Parity

Mirror the coordinates of the particle.
Changes sign of momentum, coordinates
Spin doesn't change sign.



Time reversal

If I revert the time, would the interaction take place in the same way ?

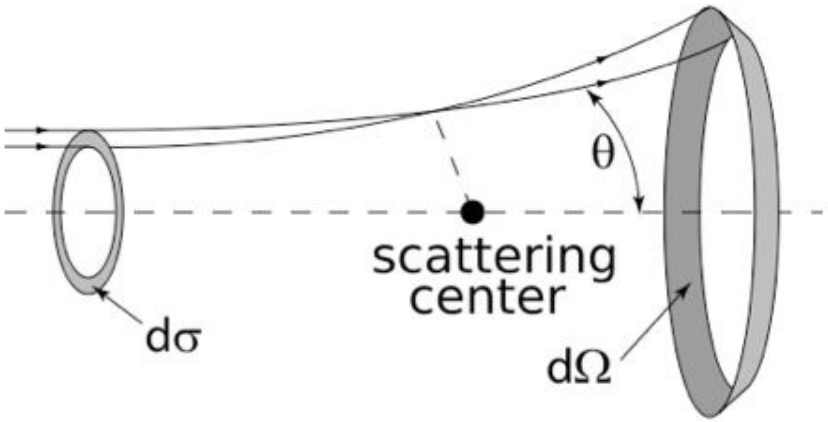
Charge conjugation

Change a particle by its anti-particle

And combinations: CP, CPT ?

Cross-section of interaction

> If my interaction is not violating any symmetry or kinematics, time to make some calculations !



Cross-section

Encodes the probability of the interaction to happen

Depends on:

- Type and geometry of the interaction
- Incident particles 4-momentum.
- Quantum amplitude of the interaction.

General cross-section formula for processes such that: *A + B → C +D +E + …. (Peskin, Schroeder)*

$$d\sigma = \underbrace{\frac{1}{2E_A 2E_B |v_A - v_B|}}_{\text{Momenta and energy of input particles (A,B)}} \underbrace{\left(\prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right)}_{\text{Momenta of outgoing particles}} \times \underbrace{|\mathcal{M}(p_A, p_B \to p_f)|^2}_{\text{Amplitude of interaction (here's where the SM magic happens)}} \underbrace{(2\pi)^4 \delta^4(p_A + p_B - \sum_f p_f)}_{\text{4-momentum conservation}}$$

How to calculate the amplitude of the interaction?

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \boxed{\bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda} - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + \boxed{igs_w A_\mu}\left(\boxed{-(\bar{e}^\lambda\gamma^\mu e^\lambda)} + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right).
\end{gathered}
$$

$$e^+e^- \to \mu^+\mu^-$$

One of most simple interactions (in QED)

- Annihilation of electron-positron, virtual photon, and pair-creation

Fermion fields

$$
\begin{aligned}
\psi(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s\left(a^s_p\, u^s(\mathbf{p})\, e^{-iP\cdot x} + b^{s\dagger}_p\, v^s(\mathbf{p})\, e^{+iP\cdot x}\right) \\
\psi^\dagger(x) &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s\left(a^{s\dagger}_p\, u^{s\dagger}(\mathbf{p})\, e^{iP\cdot x} + b^s_p\, v^{s\dagger}(\mathbf{p})\, e^{+iP\cdot x}\right)
\end{aligned}
$$

Boson fields

$$\phi(\mathbf{x}, t_0) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}}\left(a_{\mathbf{p}}(t_0)e^{i\mathbf{p}\mathbf{x}} + a^\dagger_{\mathbf{p}}(t_0)e^{-i\mathbf{p}\mathbf{x}}\right)$$

Photon field

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_{\mathbf{p}}}}\sum_s\left(a^s_{\mathbf{p}}\epsilon^s_\mu(p)e^{-ipx} + a^{s\dagger}_{\mathbf{p}}\epsilon^{s*}_\mu(p)e^{ipx}\right)$$

Each step : creation, annihilation for final prob

$$\mathcal{M} \propto \langle 0|T\phi_I(x)\phi_I(y)\phi_I(z_1)\cdots\phi_I(z_{4n})|0\rangle$$

How to calculate the amplitude of the interaction?

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h \left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M \left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2 \alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \\
\partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \\
\partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \\
\partial_\mu\bar{X}^-X^-) - \tfrac{1}{2}gM\left(\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + igMs_w\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0\right) .
\end{gathered}
$$

$$
\psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s \left(a^s_p\, u^s(\mathbf{p})\, e^{-iP\cdot x} + b^{s\dagger}_p\, v^s(\mathbf{p})\, e^{+iP\cdot x}\right)
$$

$$
\psi^\dagger(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s \left(a^{s\dagger}_p\, u^{s\dagger}(\mathbf{p})\, e^{iP\cdot x} + b^s_p\, v^{s\dagger}(\mathbf{p})\, e^{+iP\cdot x}\right)
$$

Boson fields

$$
\phi(\mathbf{x}, t_0) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_\mathbf{p}}}\left(a_\mathbf{p}(t_0)e^{i\mathbf{px}} + a^\dagger_\mathbf{p}(t_0)e^{-i\mathbf{px}}\right)
$$

Photon field

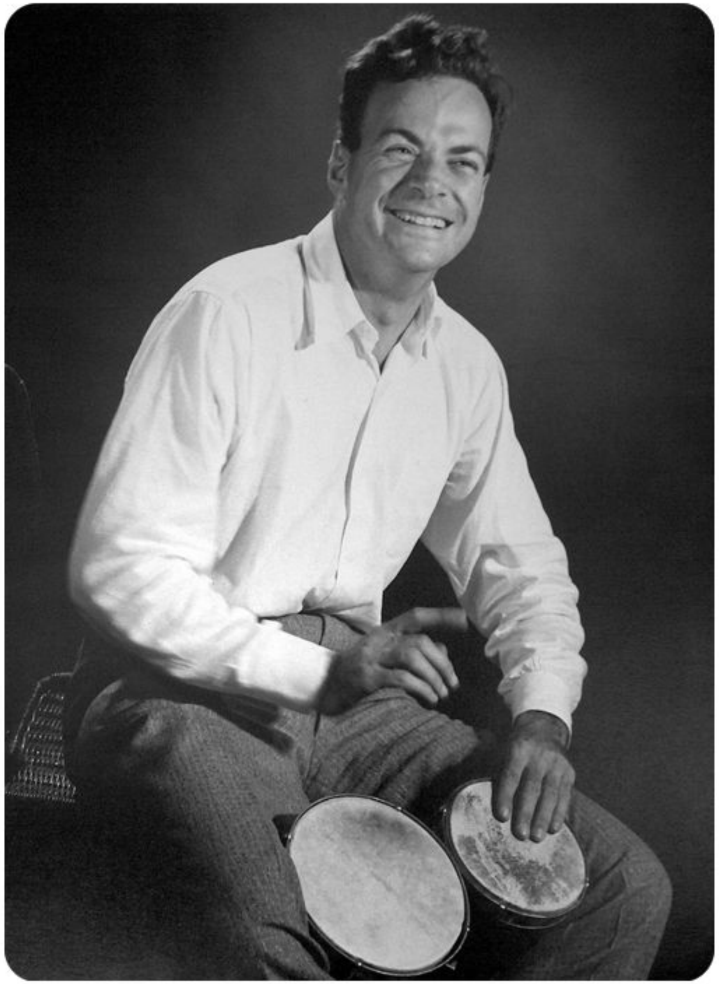
$$
A_\mu(x) = \int \frac{d^3p}{(2\pi)^3\sqrt{2E_\mathbf{p}}}\sum_s \left(a^s_\mathbf{p}\epsilon^s_\mu(p)e^{-ipx} + a^{s\dagger}_\mathbf{p}\epsilon^{s*}_\mu(p)e^{ipx}\right)
$$

Each step : creation, annihilation for final prob

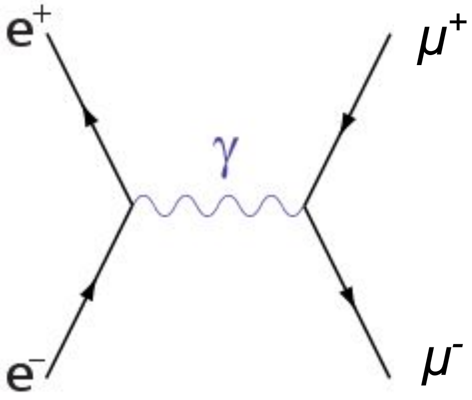
$$
\mathcal{M} \propto \langle 0|T\phi_I(x)\phi_I(y)\phi_I(z_1)\cdots\phi_I(z_{4n})|0\rangle
$$

DESY

Reason why particle physicists LOVE Feynman



[Feynman diagrams]

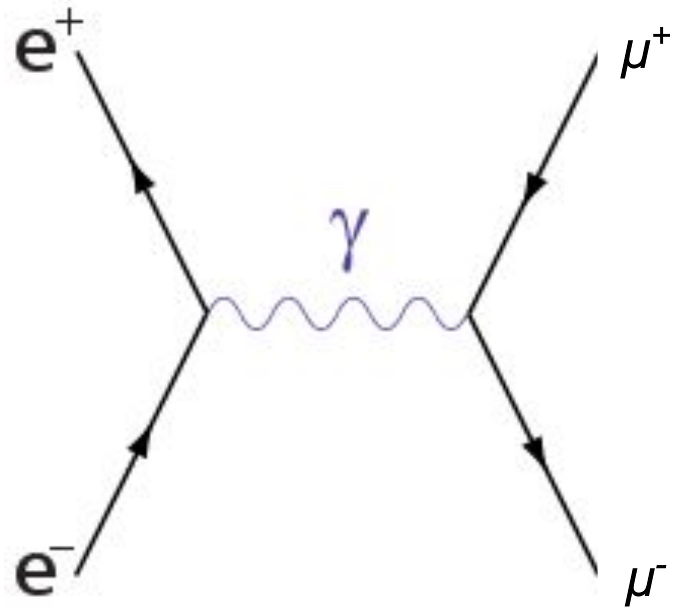


Because these are not just visual representations of the process

They are recipes on how to build the equation of each possible interaction leading to your process.

[Feynman rules]

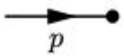
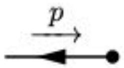
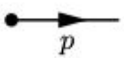
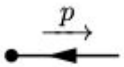
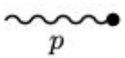
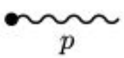
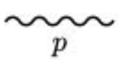
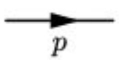
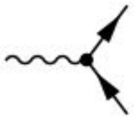
Feynman rules: example QED



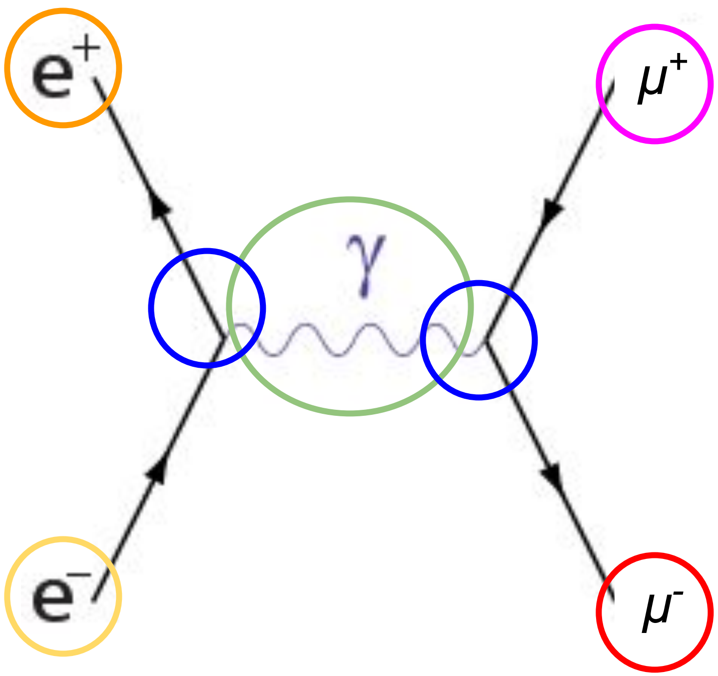
$$= \bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k')$$

Which is what we would've arrived with doing it the hard way

$$\langle 0|T\phi_I(x)\phi_I(y)\phi_I(z_1)\cdots\phi_I(z_{4n})|0\rangle$$

1. Incoming fermion  $= u^s(p)$
2. Incoming antifermion  $= \bar{v}^s(p)$
3. Outgoing fermion  $= \bar{u}^s(p)$
4. Outgoing antifermion  $= v^s(p)$
5. Incoming photon  $= \epsilon^{\mu}$
6. Outgoing photon  $= \epsilon^{\mu*}$
7. Photon propagator  $= \dfrac{-ig^{\mu\nu}}{p^2+i\epsilon}$
8. Fermion propagator  $= \dfrac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$
9. Vertex  $= -ie\gamma^{\mu}$
10. Impose 4-momentum conservation at each vertex.
11. Integrate over momenta not determined by 10.: $\displaystyle\int\frac{d^4p}{(2\pi)^4}$
12. Figure out the overall sign of the diagram.

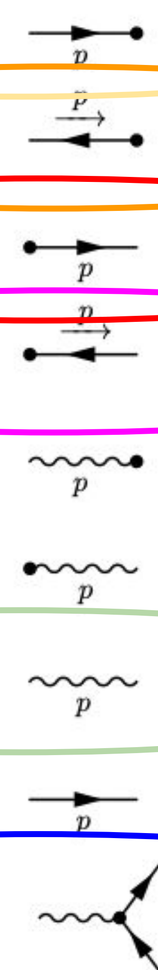
Feynman rules: example QED



$$= \bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k')$$

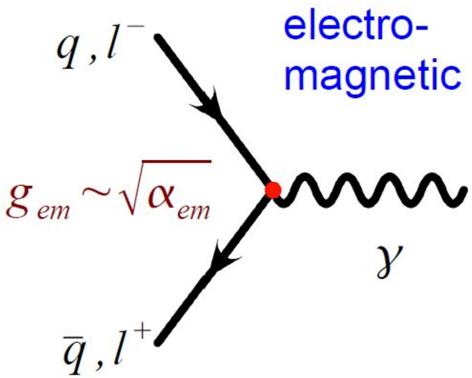
Which is what we would've arrived with doing it the hard way

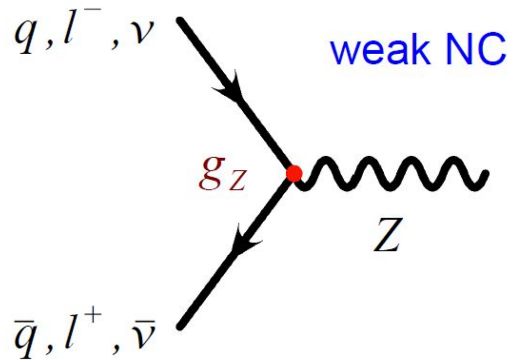
$$\langle 0|T\phi_I(x)\phi_I(y)\phi_I(z_1)\cdots\phi_I(z_{4n})|0\rangle$$

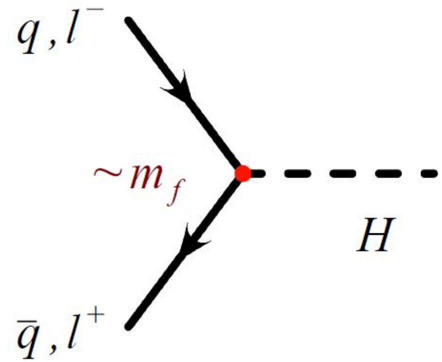


1. Incoming fermion $= u^s(p)$
2. Incoming antifermion $= \bar{v}^s(p)$
3. Outgoing fermion $= \bar{u}^s(p)$
4. Outgoing antifermion $= v^s(p)$
5. Incoming photon $= \epsilon^{\mu}$
6. Outgoing photon $= \epsilon^{\mu *}$
7. Photon propagator $= \frac{-ig^{\mu\nu}}{p^2 + i\epsilon}$
8. Fermion propagator $= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$
9. Vertex $= -ie\gamma^{\mu}$
10. Impose 4-momentum conservation at each vertex.
11. Integrate over momenta not determined by 10.: $\int \frac{d^4p}{(2\pi)^4}$
12. Figure out the overall sign of the diagram.

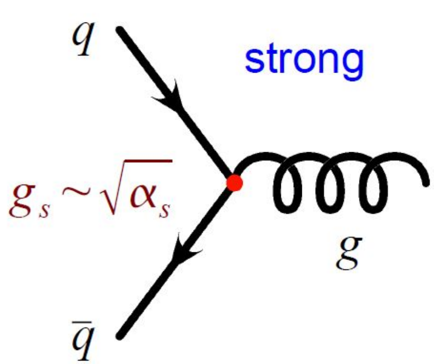
Feynman rules: different for each force

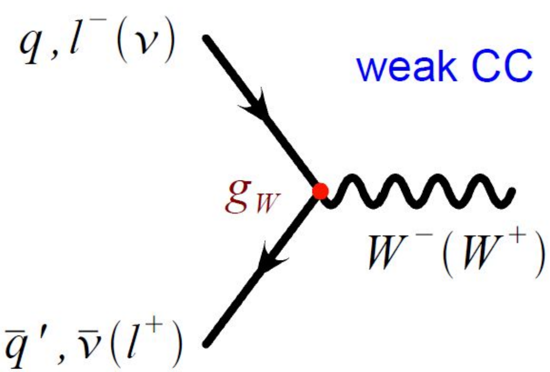






neutrinos???

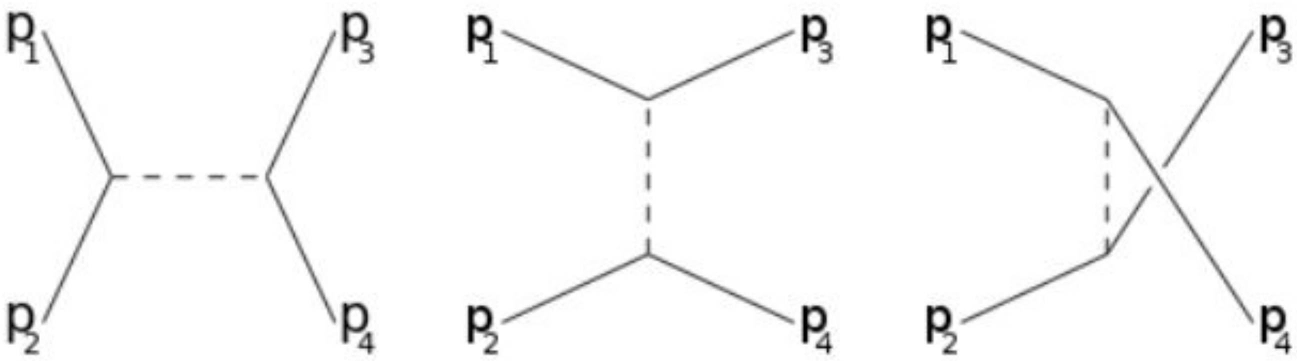




- mixes generations for quarks
- specific helicity structure

always 2 fermions
and 1 boson

Channels and Mandelstam variables



s-channel t-channel u-channel

Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_4 - p_2)^2$$
$$u = (p_1 - p_4)^2 = (p_3 - p_2)^2$$

Leading order and high order diagrams

When collision of particles happen, several kind of Feynman diagrams might appear

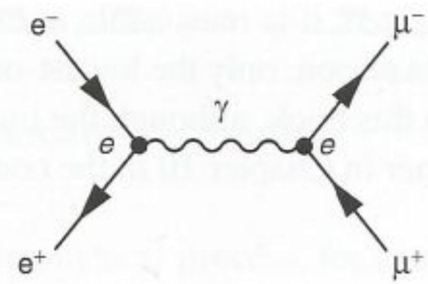


Fig. 6.1 The lowest-order Feynman diagram for the QED annihilation process $e^+e^- \to \mu^+\mu^-$.

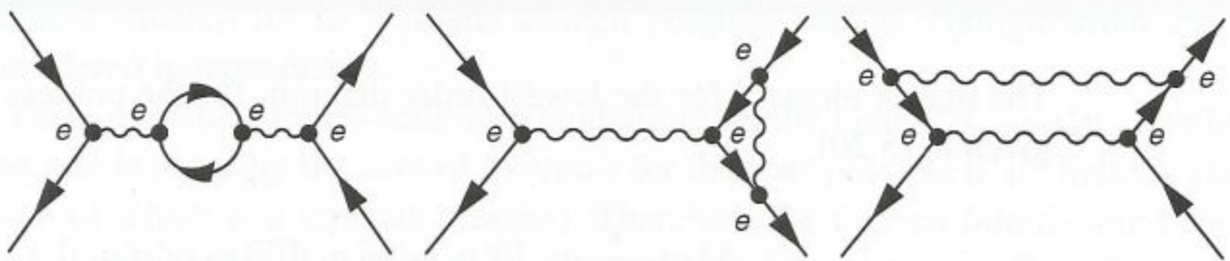


Fig. 6.2 Three of the $\mathcal{O}(\alpha^4)$ Feynman diagrams contributing the QED annihilation process $e^+e^- \to \mu^+\mu^-$.

All of these diagrams contribute to the total amplitude of the process to happen !

But the more vertices, the lower is the contribution to the total amplitude (given couplings are < 1)

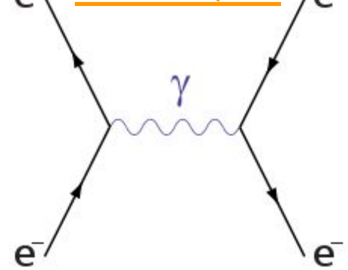
Leading order and high order diagrams

When collision of particles happen, several kind of Feynman diagrams might appear

Leading order (LO) diagrams

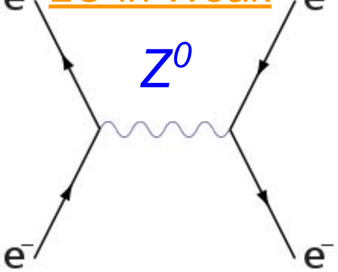
Diagrams with less number of vertices possible to allow the interaction by a certain force

LO in QED

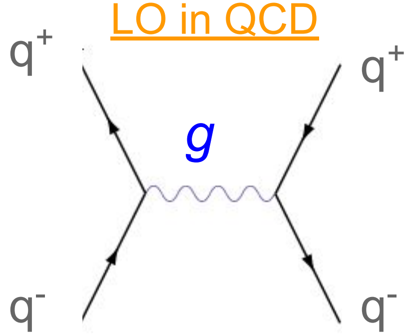


$$M^2 \propto \alpha_{em}^2$$

LO in Weak



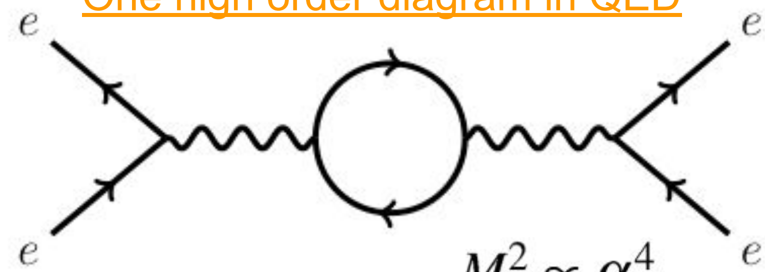
LO in QCD



Higher order (NLO, NNLO,...)

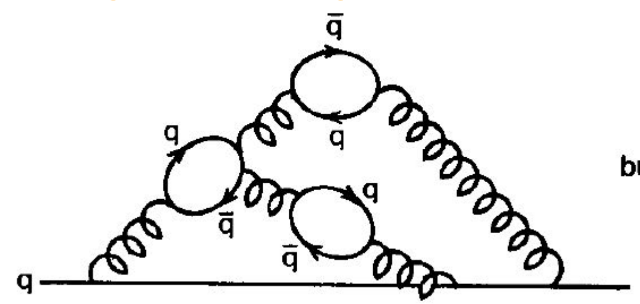
Diagrams with larger number of vertices

One high order diagram in QED



$$M^2 \propto \alpha_{em}^4$$

One high order diagram in QCD



All of these diagrams contribute to the total amplitude of the process to happen !

But the more vertices, the lower is the contribution to the total amplitude (given couplings are < 1)

How to measure a cross section

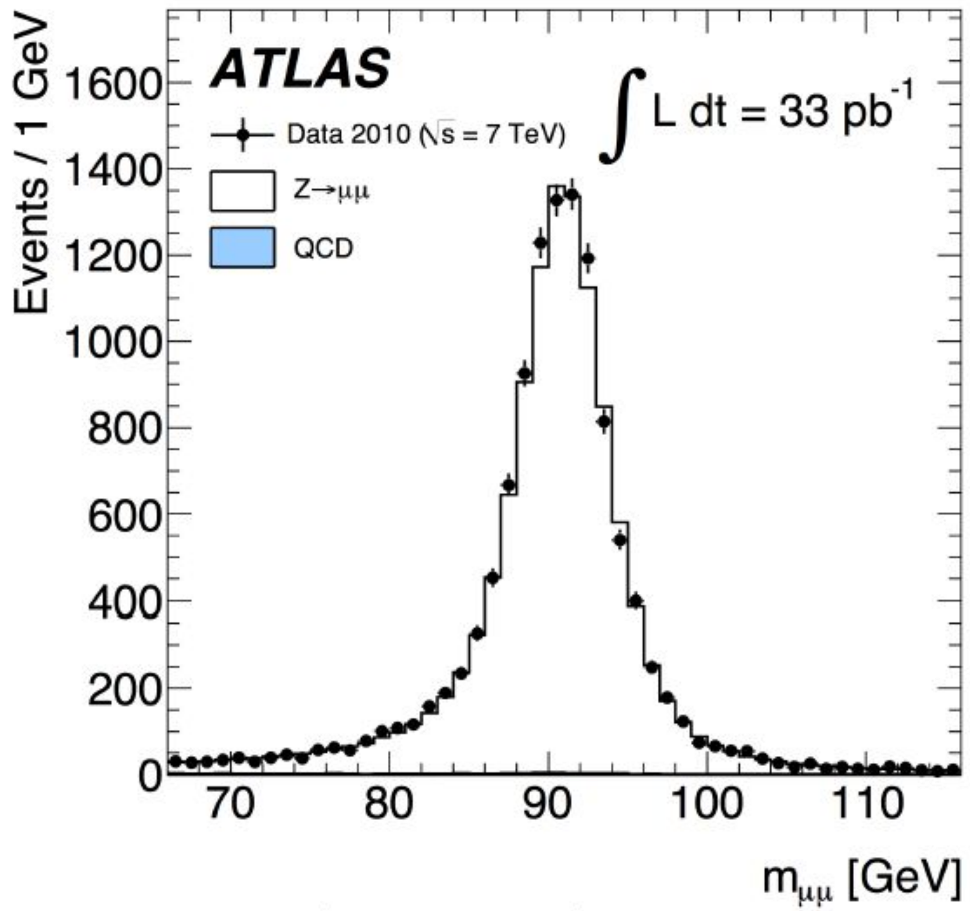
- (simple) example:

  $Z^0 \to \mu^+\mu^-$ from ATLAS

- identify muons in the detector, select events with at least 2 muons of opposite charge, calculate the invariant mass, fill the mass into a histogram

- determine how much background

  - use the prediction from a MC simulation (works if background is well known)
  - fit a function of the form f(m) = signal(m) + bkg(m) to the data (works if expected shapes are known)



$$\sigma = \frac{(N_{\mathrm{evt}} - N_{\mathrm{Bkg}})}{\epsilon\ BR\ \int L\, dt}$$

Feynman diagrams: example of eμ scattering

- Measurements:

$$\text{Cross section} = \frac{W_{fi}}{(\text{initial flux})}(\text{number of final states})$$

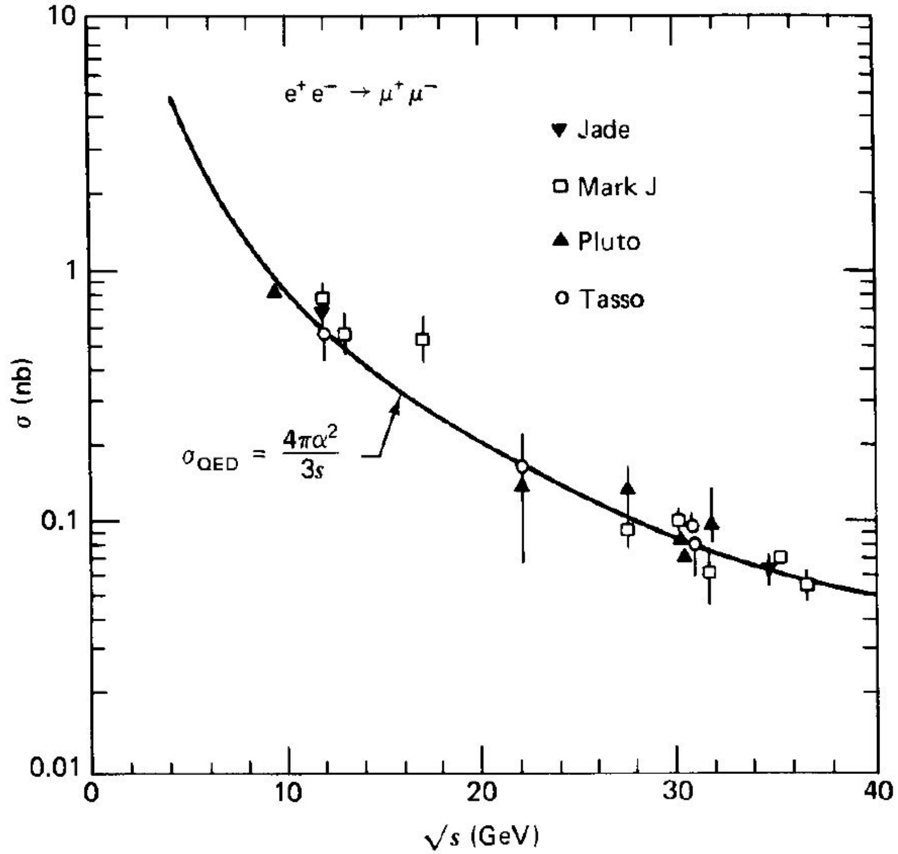


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

- Need to integrate over full solid angle here and momenta