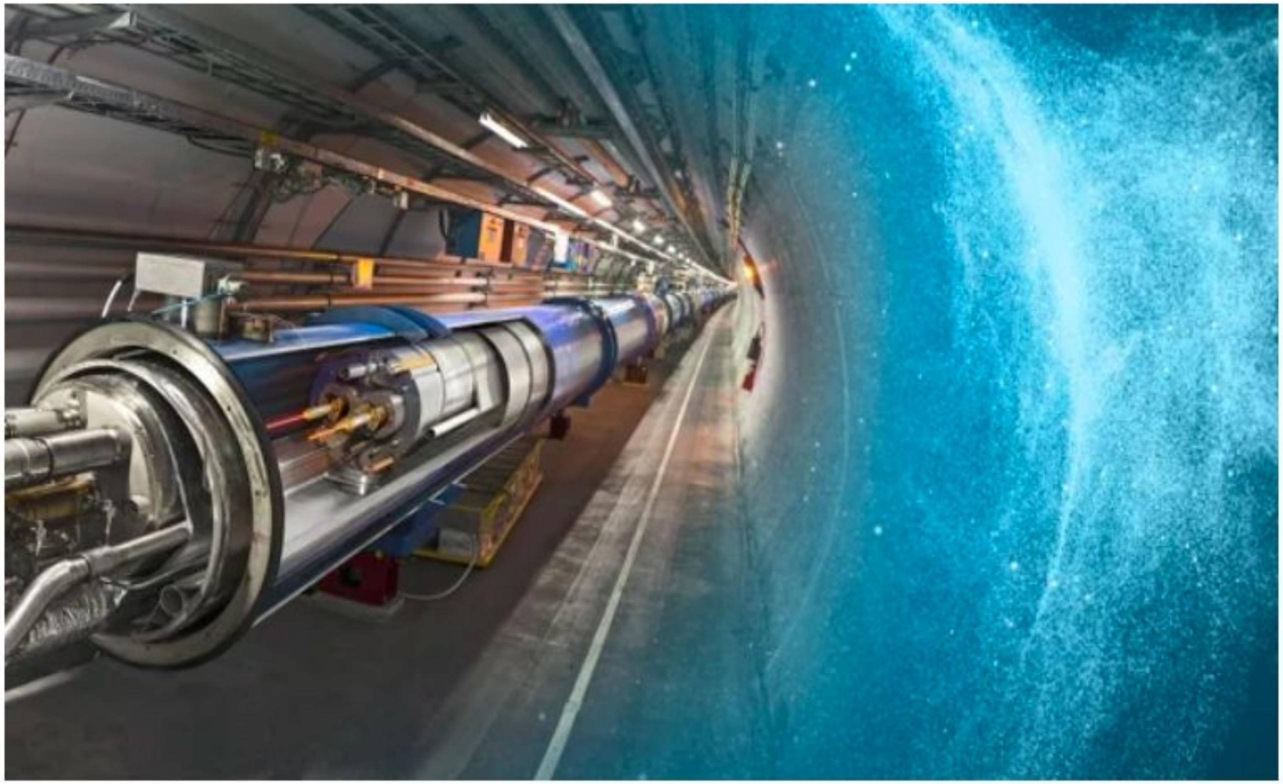
Introduction to the Standard Model

Summer Student Lecture 2023 – Part II



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Deutsches Elektronen Synchrotron

24^{th}-26^{th} July 2023

HELMHOLTZ ASSOCIATION

Many thanks to Sarah Heim and Thorsten Kuhl for their lectures and help

DESY

Content

> 0) Introduction
  - What is the Standard Model?
  - Coupling constants, masses and charges
  - Units and scales

> 1) Interactions
  - Relativistic kinematics
  - Symmetries and conserved quantities
  - Feynman diagrams
  - **Running couplings and masses**

> **2)Quantum electrodynamics**
  - **Test of QED: Magnetic momentum of the muon**
  - **Test of QED: High energy colliders**

Content

> **3) Strong Interaction: Quantum-Chromodynamics**
>   **- A short history of hadrons and quarks**
>   **- DIS and gluons**
>   **- QCD and its properties**

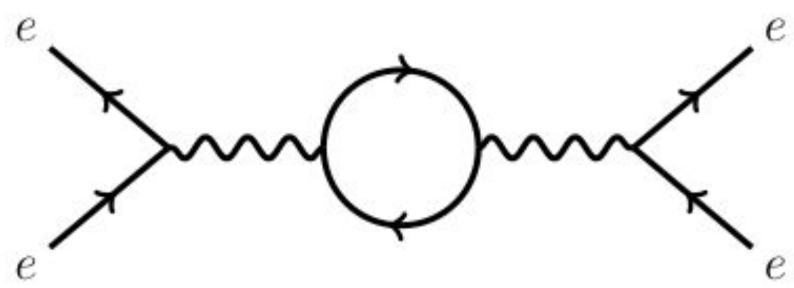
> 4) Electroweak interactions
>   - Discovery of electroweak bosons
>   - Tests of angular distributions
>   - Feynman rules
>   - Handed-ness of electroweak interactions
>   - More tests of the electroweak SM

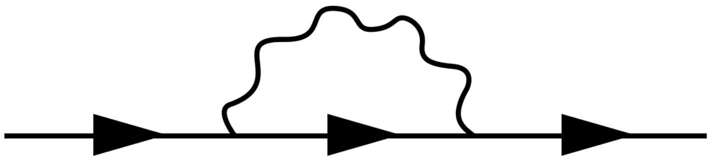
> 5) The Higgs
>   - Why was it predicted?
>   - How was it found?

Running couplings and masses !

Couplings, masses and Feynman diagrams

- When calculating the contribution of certain Feynman diagrams to the cross-section of an interaction, divergenes (infinities) appear.
  - According to the laws of statistics, we have to integrate over all possibilities.
  - → We have to integrate over all possible momenta the virtual particles in loops → Infinity !

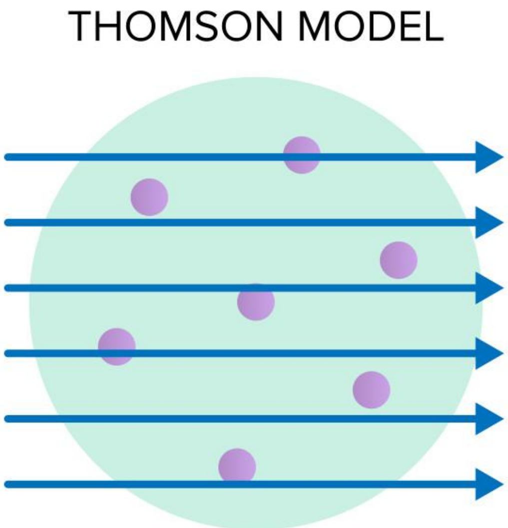


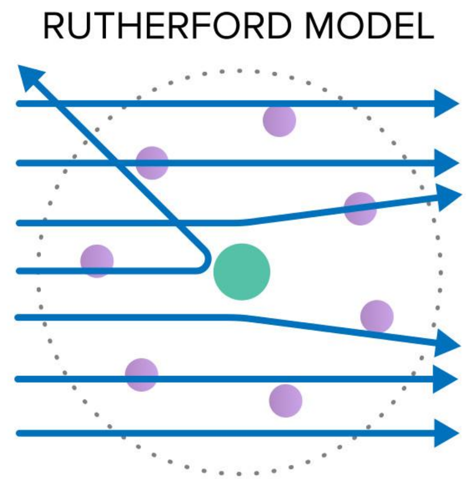


- What does it mean when you have a virtual particle creation close to infinity? If you are just making a collision at LHC, is it true that the possibility of creating virtual particles with infinity momenta will affect you?

Let’s go back to the Heisenberg principle

$$\Delta p \, \Delta x \geq \hbar c \approx 200 \, \mathrm{MeV \, fm}$$



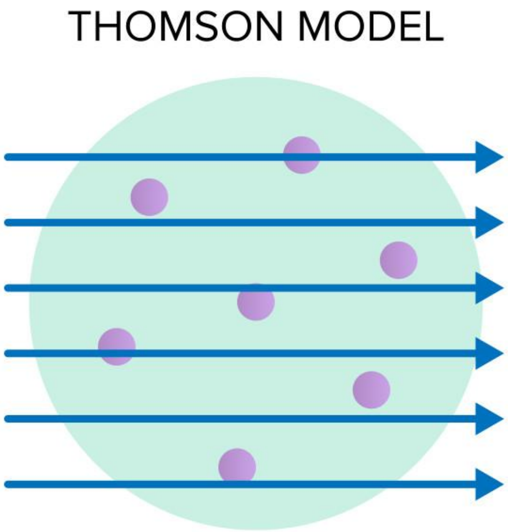


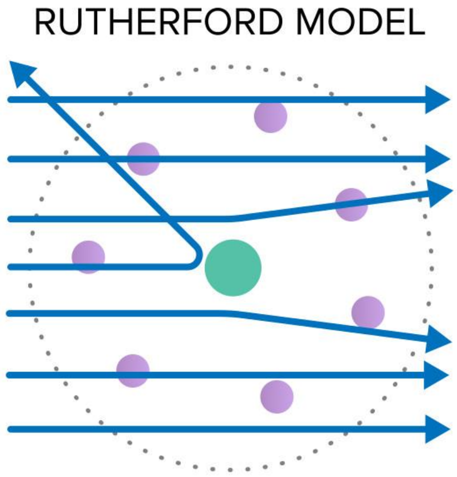
- If I want to have a good resolution of the atom structure (1fm), at least the momentum transfer between the incoming particle and the atom has to be beyond 200 MeV.

- If I want a better resolution, momentum exchange has to be higher !

Let's go back to the Heisenberg principle

$$\Delta p \, \Delta x \geq \hbar c \approx 200 \, \text{MeV fm}$$





> Conversely, diagrams with virtual particles with very high momentum (energy) are diagrams that are happening in very short scales (times)

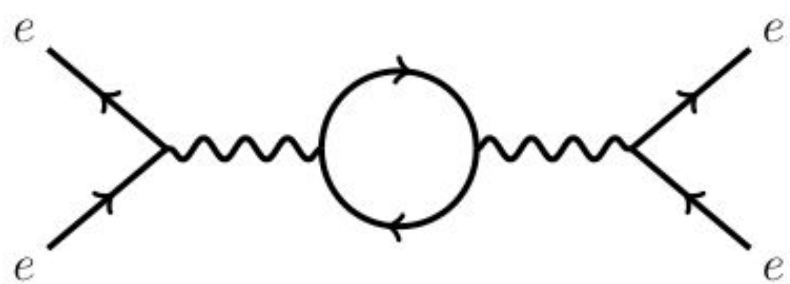
>If my interaction is happening at momentum 200 GeV and therefore can structures and processes at x = 1 am

Can my measurements be affected by things happening at $\Delta x \sim 0$?

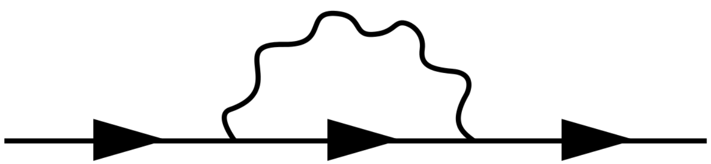
-> the assumption is that not ! $\rightarrow$ Energy cut-off Λ

Effective couplings and masses

- Regularization and renormalization: the procedure on which I take these divergences and cut-offs and absorb them into definition of the physical quantities
  - Some of these infinities might cancel out due to symmetries or because similar contributions but opposite sign.



Loop diagrams etc: correction to the interaction coupling (i.e. g_{EM}) → Effective coupling



Self-interaction: correction to the mass of the particle → Effective mass

Message to take home: the observed mass and coupling strength in an interaction depends on the energy exchanged in the interaction

Quantum electrodynamics
QED

Quantum electrodynamics in a nutshell

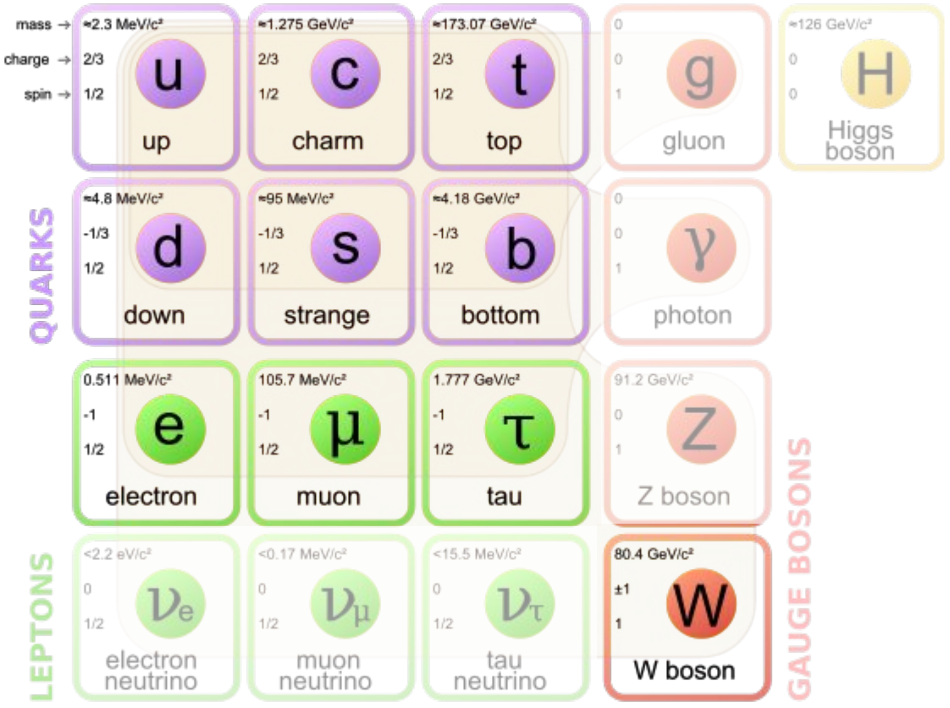
- Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction
  - Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

Quarks:
Up/charm/top : q = ⅔
Down/Strang/Bottom: q = -⅓

Anti-quarks:
Exactly opposite sign charge of their anti-particle

Leptons:
Electron/muon/tau : q = -1
Neutrinos: q = 0

Anti-leptons:
Exactly opposite sign charge of their anti-particle



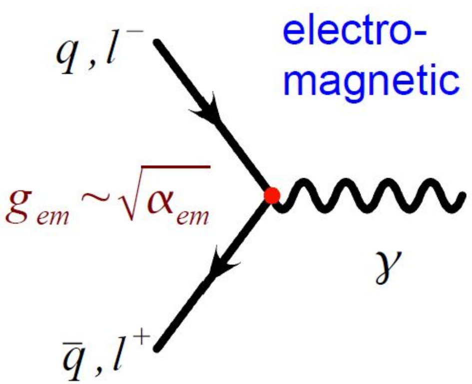
W-bosons: q = +-1 (particle-anti-particle)

Quantum electrodynamics in a nutshell

- Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction
  - Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

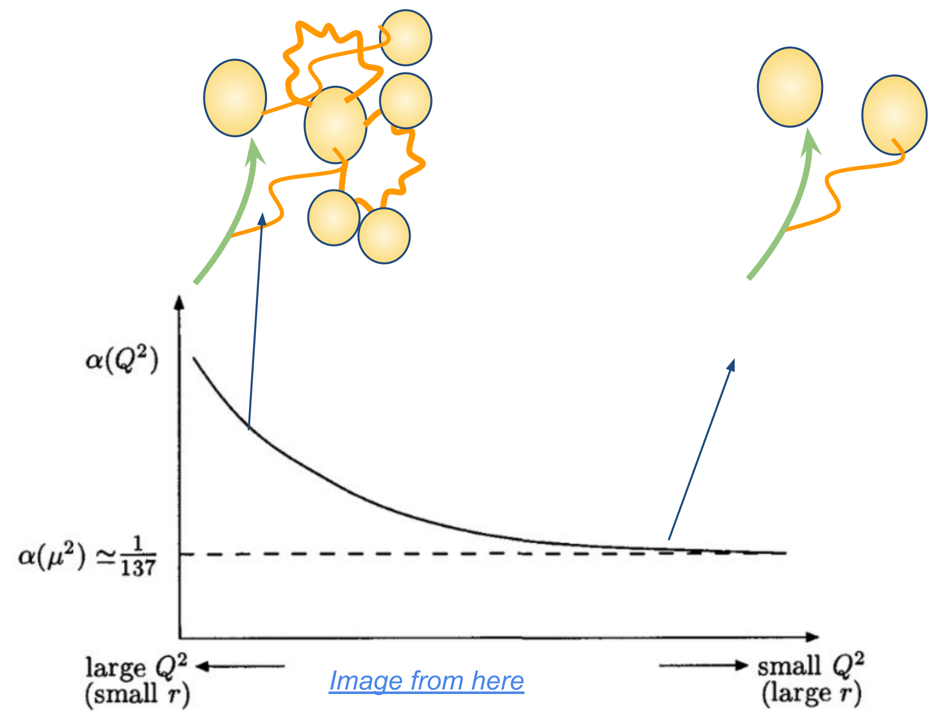
Coupling constant

Universal (not different coupling depending on the particle)
At low energies, fine structure constant



$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi} = \frac{1}{137}$$



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Electromagnetic Interactions: Tests of QED

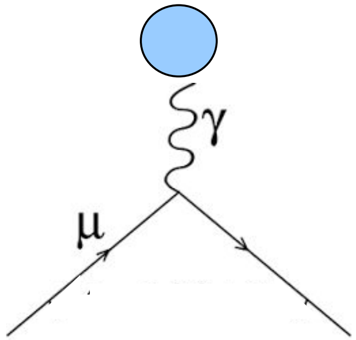
- Electromagnetism is very well-known at low energies and for a high range of energies.

- Quantum effects can only be validated at experiments with high precision
  $\rightarrow$ cross-sections at high energies !

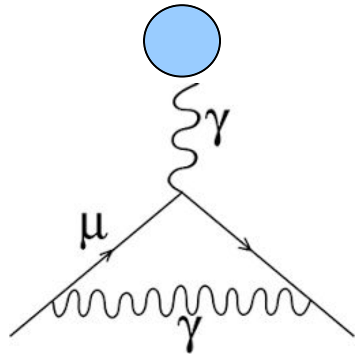
Magnetic moment of charged particles

Magnetic moment due to charged body with angular momentum and/or spin

$$\mu = -g\frac{e}{2m}s = -\frac{g}{2}\frac{e}{2m}$$



Dirac theory predicts g=2

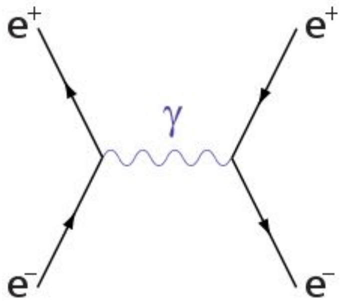


Quantum corrections

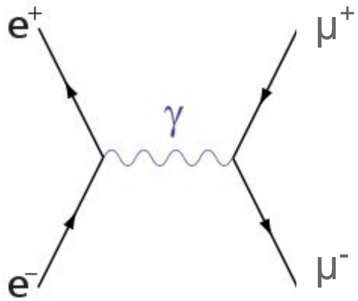
Test of QED at high energy

Verify cross-section predictions at high energies
QED running coupling
QED lepton coupling universality

$\sigma(e^+e^- \rightarrow e^+e^-)$

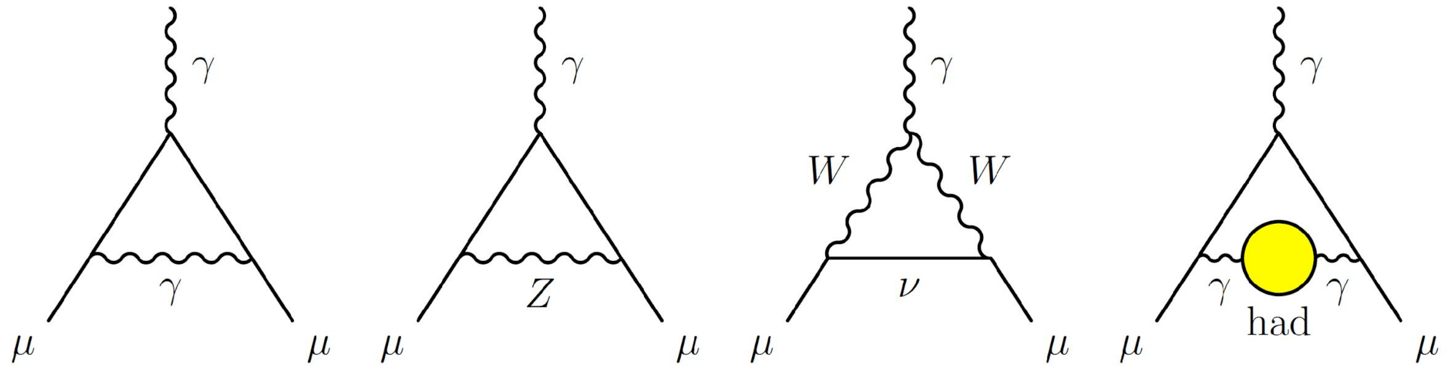


$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

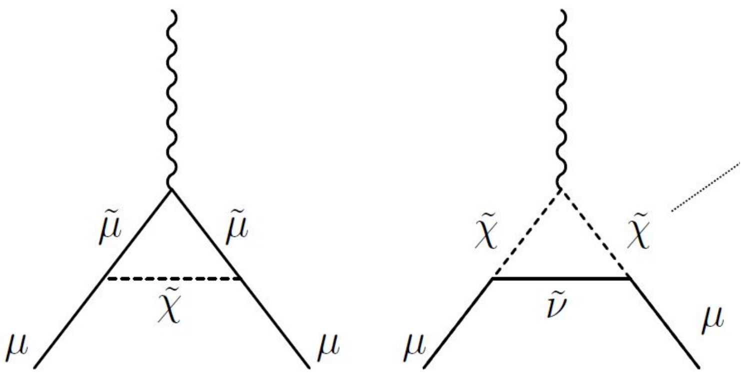


Tests of QED: muon magnetic momentum

- Muon magnetic moment with larger corrections due to QCD and EWK

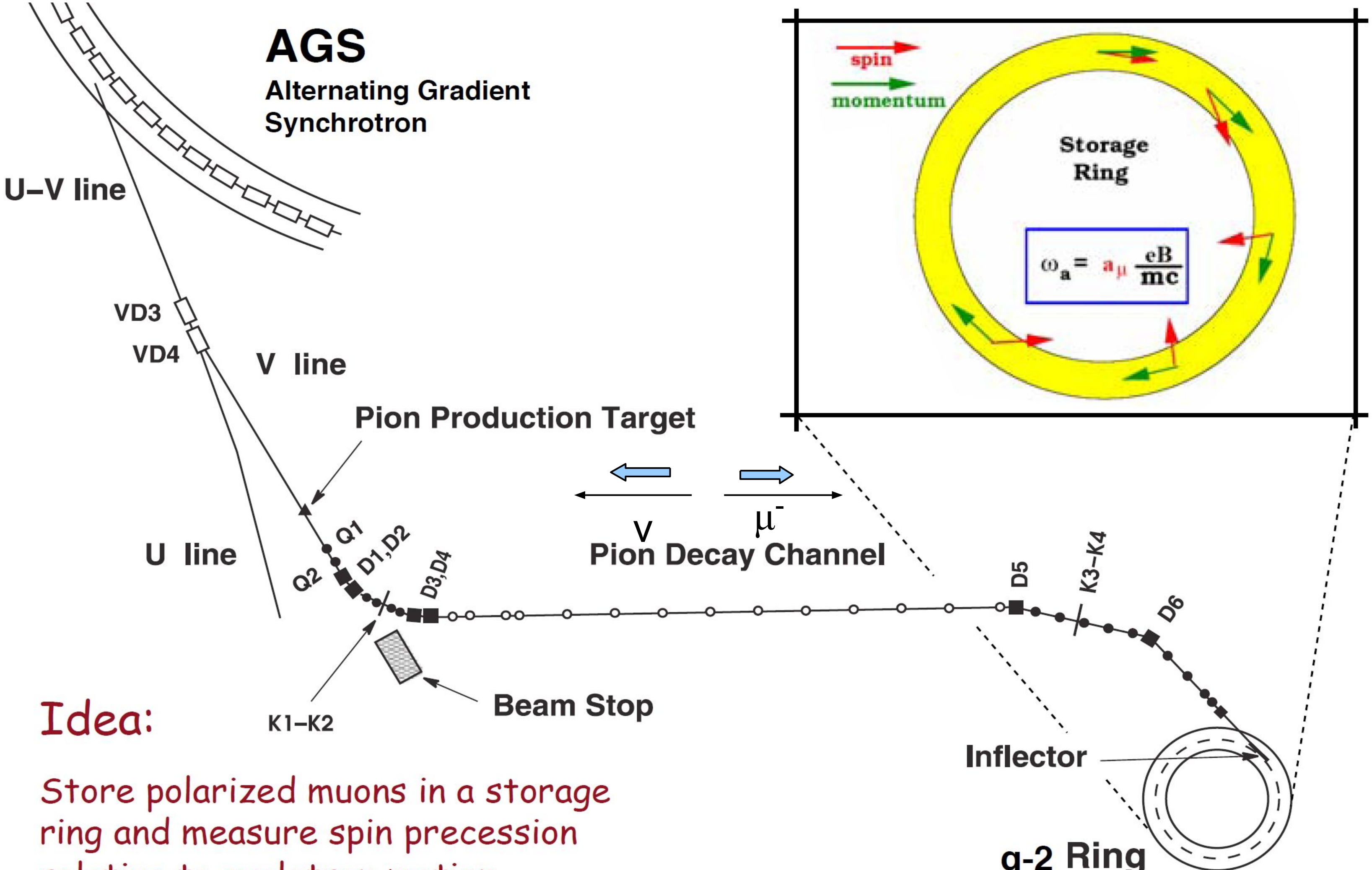


- New physics ?



$$\lambda_{\mathrm{NP}} = \left(\frac{m_\mu^2}{M_{\tilde{\chi}}^2}\right)$$

Tests of QED: magnetic momentum for muons



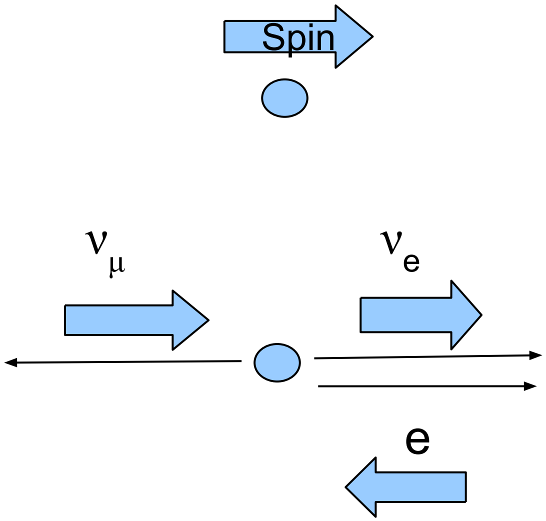
Idea:

Store polarized muons in a storage ring and measure spin precession relative to cyclotron motion

$$\vec{\omega}_{a_\mu} = \vec{\omega}_S - \vec{\omega}_C = -g_\mu \frac{Qe\vec{B}}{2m} - (1-\gamma)\frac{Qe\vec{B}}{\gamma m} + \frac{Qe\vec{B}}{\gamma m} = -\left(\frac{g_\mu - 2}{2}\right)\frac{Qe}{m}\vec{B} = -a_\mu \frac{Qe}{m}\vec{B}$$

Tests of QED: magnetic momentum for muons

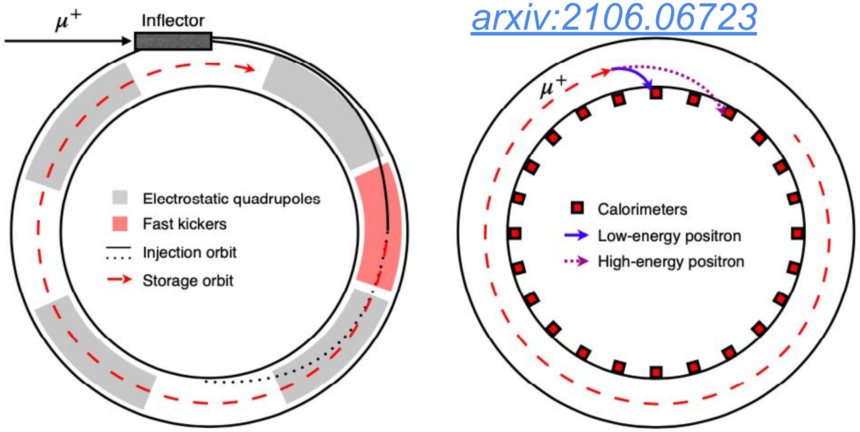
Muon-Decay in Rest

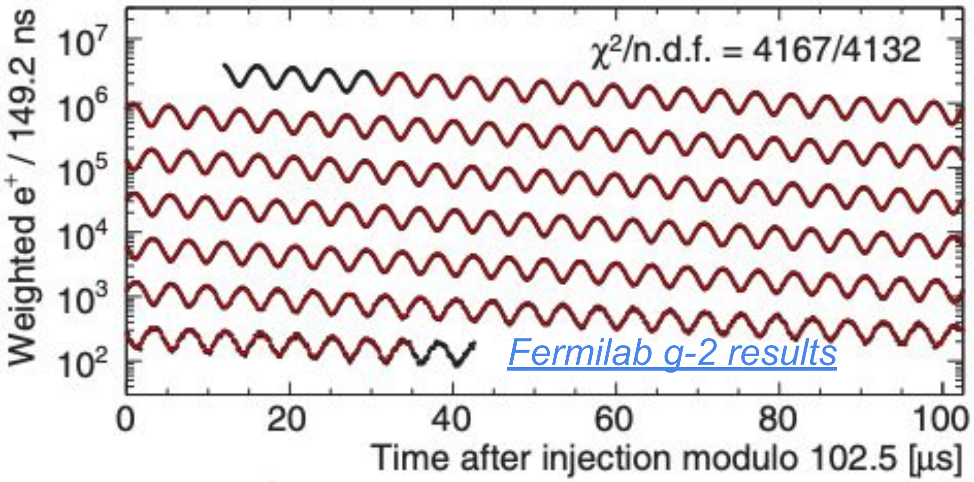


Electrons from Muon decay prefer to flight in Muon spin direction
→ Electron energy gives information of muon spin

Count electrons above a Energy threshold vs time at fixed position

arxiv:2106.06723





Fermilab g-2 results

$$N(t) = N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)]$$

Tests of QED: magnetic momentum for muons

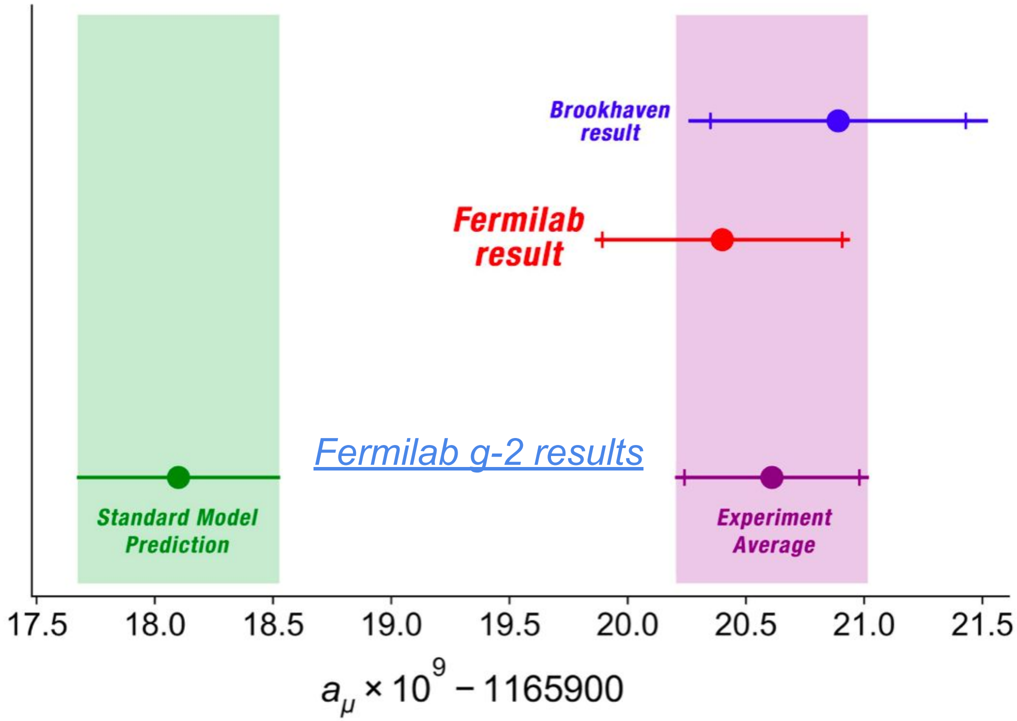
- Results with precision to 10^{-11}
- Combined measurements of BNL and Fermilab disagree **by 4.2 standard deviations of Standard Model prediction**

Reference 1:

Probability that the SM can explain this is 2.7×10^{-5}

Reference 2:

Discovery in HEP is claimed if 5σ



e.g. corrections from Higgs: $a_\mu^{\mathrm{EW}}[\text{2-loop}] = -41.2(1.0) \times 10^{-11}$

Tests of QED: results for the g-2 electron

- Similarly, tests were performed for the electron

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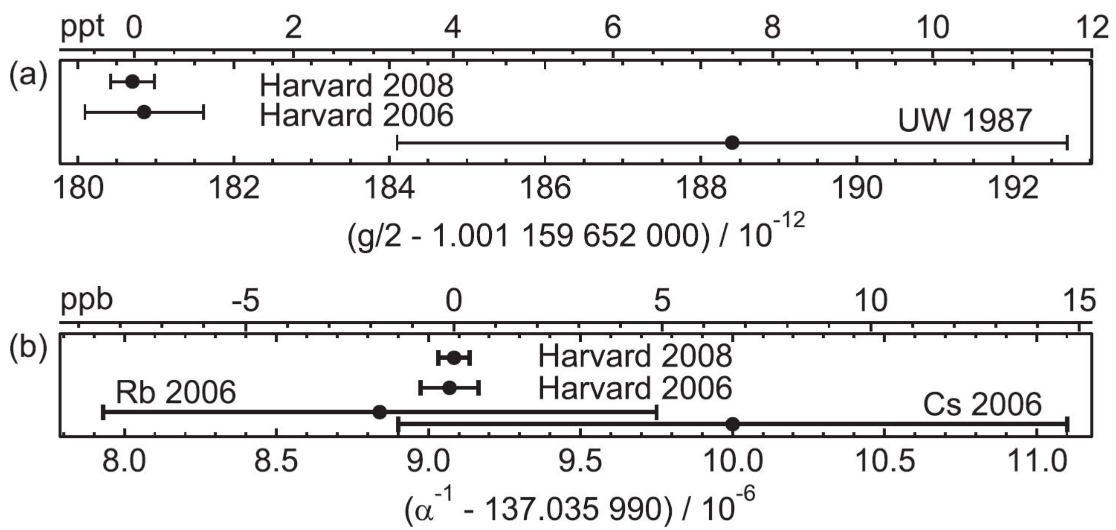
Phys. Rev. Lett. **100**, 120801

Reference 1:

Probability that the SM can explain this is 0.19



Reference 2:

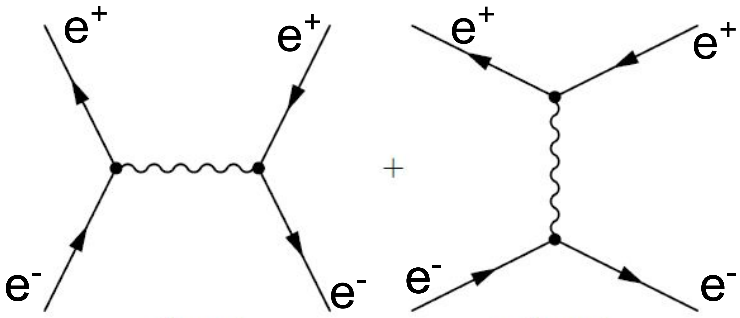
To even consider that there is some evidence of BSM, HEP needs ≥ 2 σ - 3 σ

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.5\,(8.1) \times 10^{-13}$$

Difference of around 1.3 σ

Bhabha scattering: ee → ee

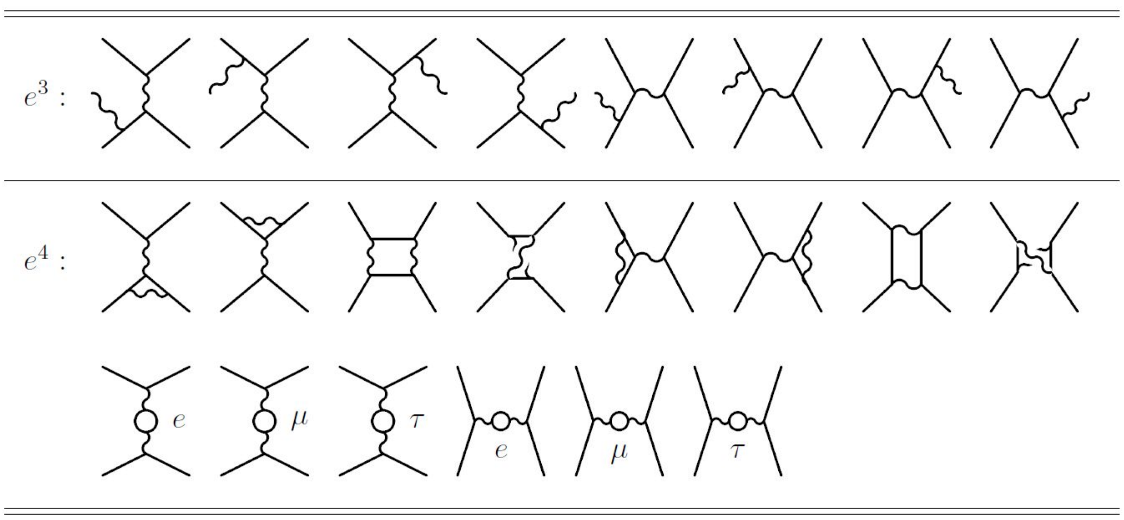
> High-energy colliders probe the following processes:



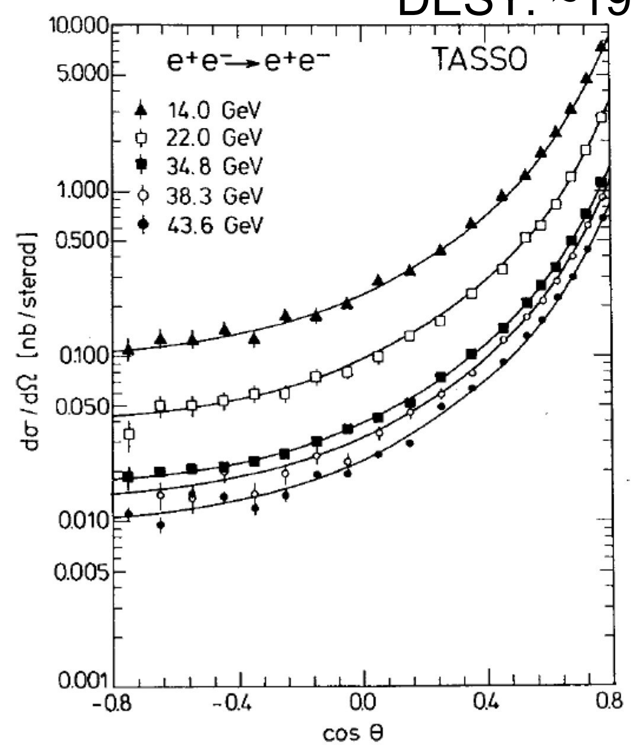
s-channel $\qquad$ t-channel

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{4s}\left(\underbrace{\frac{t^2+s^2}{u^2}}_{t\text{-channel}} + \underbrace{\frac{2t^2}{us}}_{\text{interference}} + \underbrace{\frac{t^2+u^2}{s^2}}_{s\text{-channel}}\right)$$

$$= \frac{\alpha^2}{4s}\left(\frac{3+\cos^2\vartheta}{1-\cos\vartheta}\right)^2 .$$

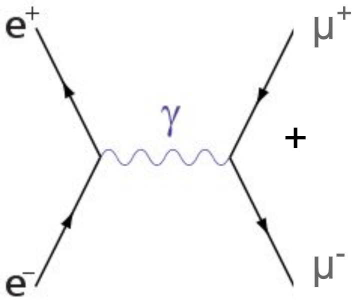


DESY: ~1979



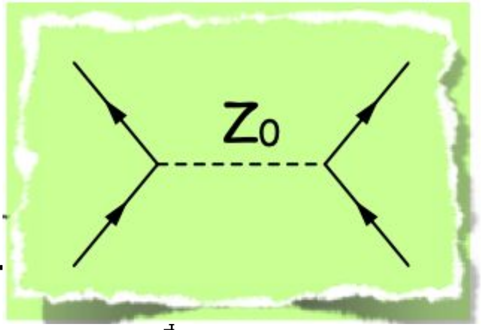
Electromagnetic Interactions: Lepton pair production

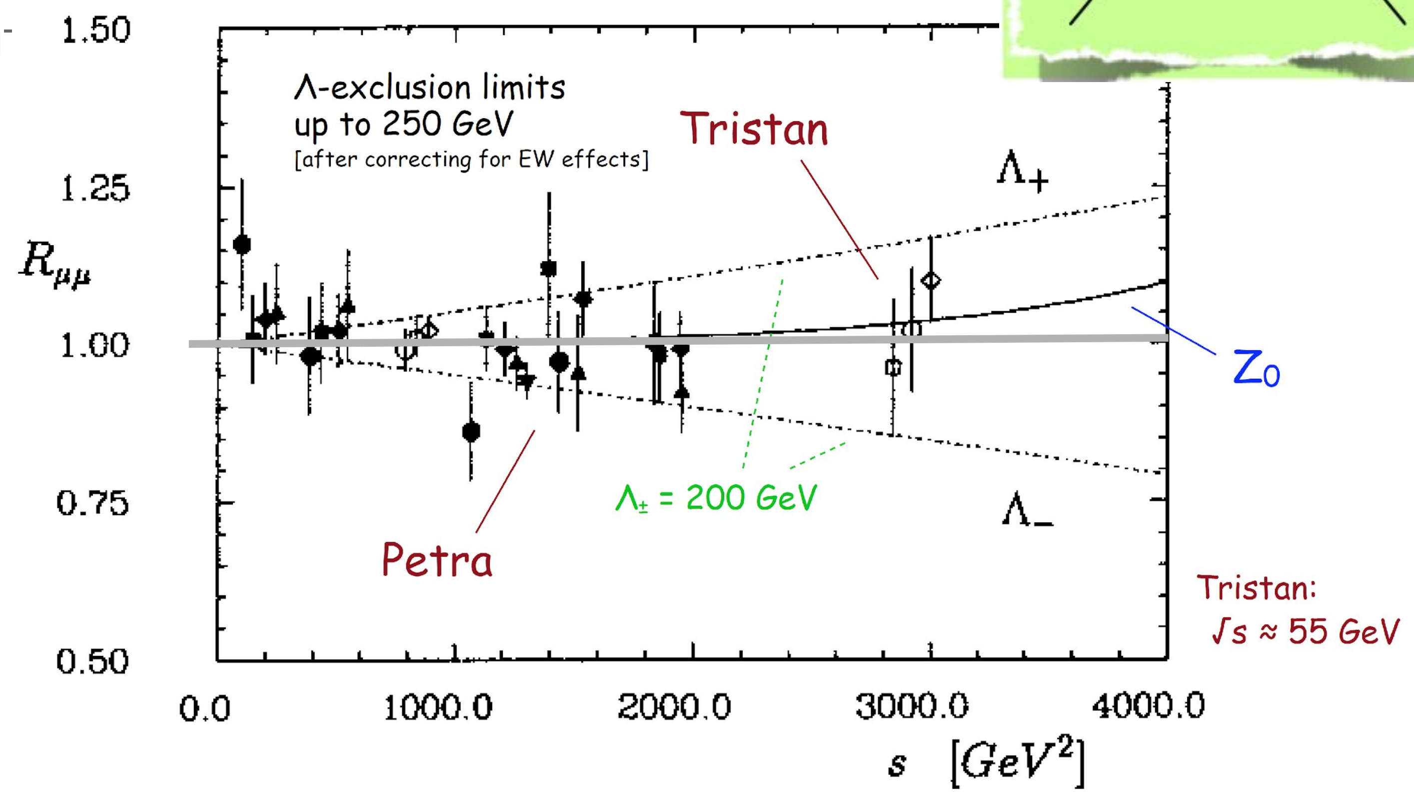
QED process



+ corrections

$$R_{\mu\mu} = \frac{\sigma_{\text{meas}}^{e^+e^-\to\mu^+\mu^-}}{\sigma_{\text{QED}}^{e^+e^-\to\mu^+\mu^-}},$$

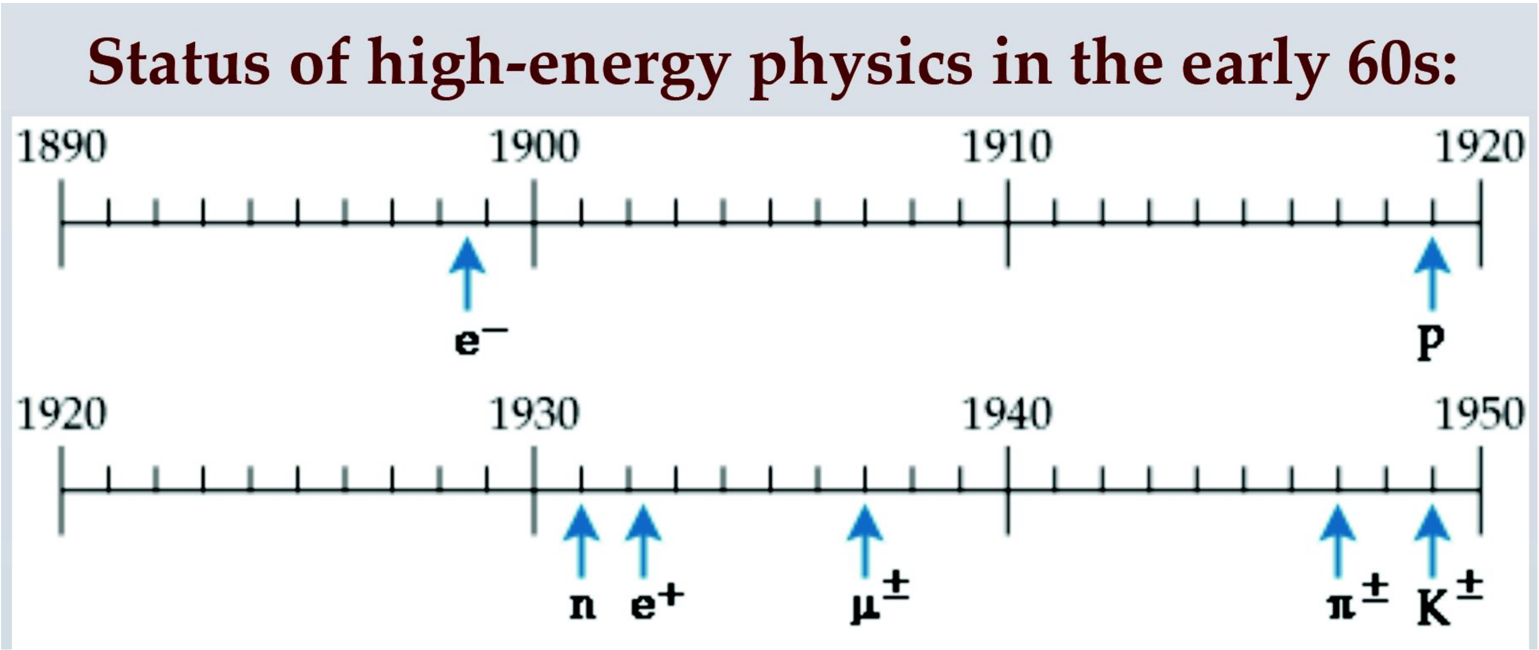


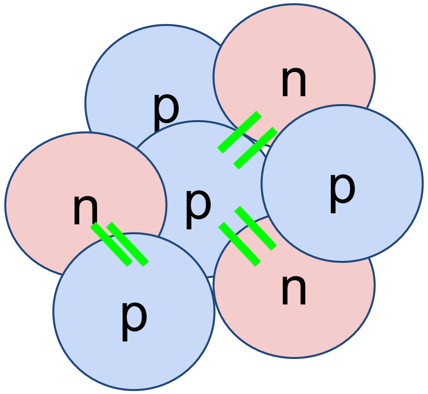


First indication of a unified EW force → Tomorrow's lecture

Quantum chromodynamics
QCD

A historical perspective: the strong force

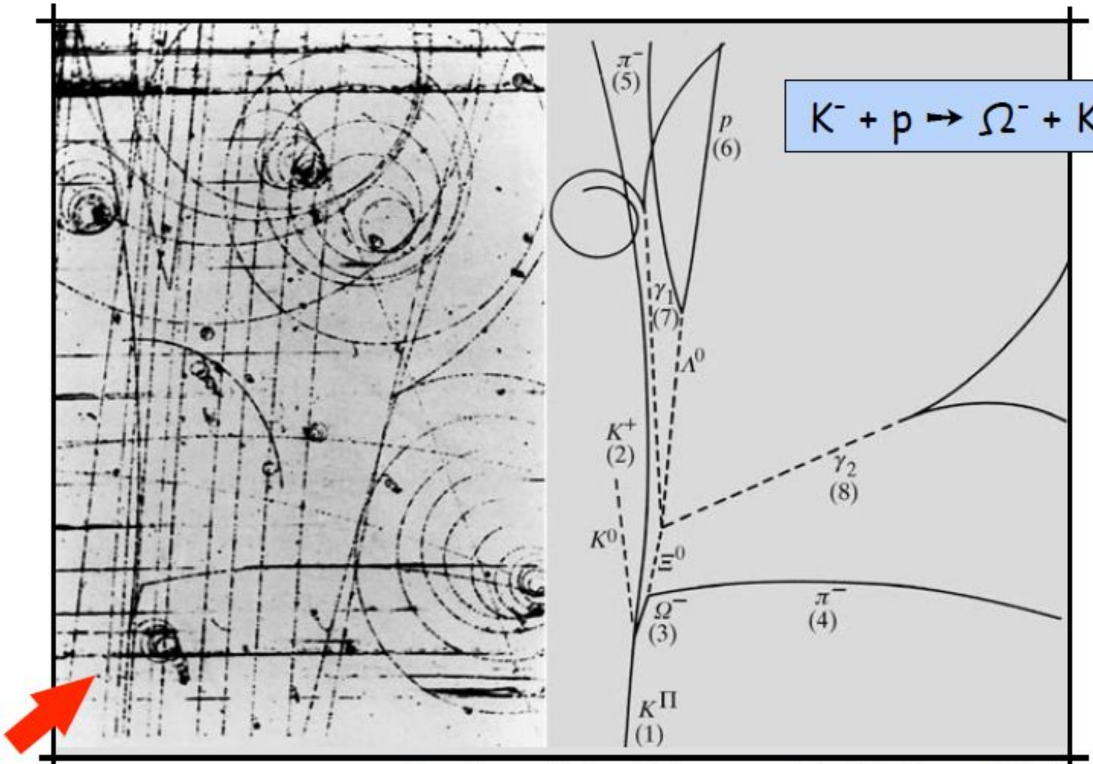




- In early 20th century, limited knowledge of particles.
- Atomic nuclei composed of protons (+1 charge) and neutrons.
- How do they hold together ? → **Strong force** !

A historical perspective: the particle zoo

- However, in the 50's and 60's, experiments in bubble and spark chambers were showing the creation of new particles → Particle Zoo

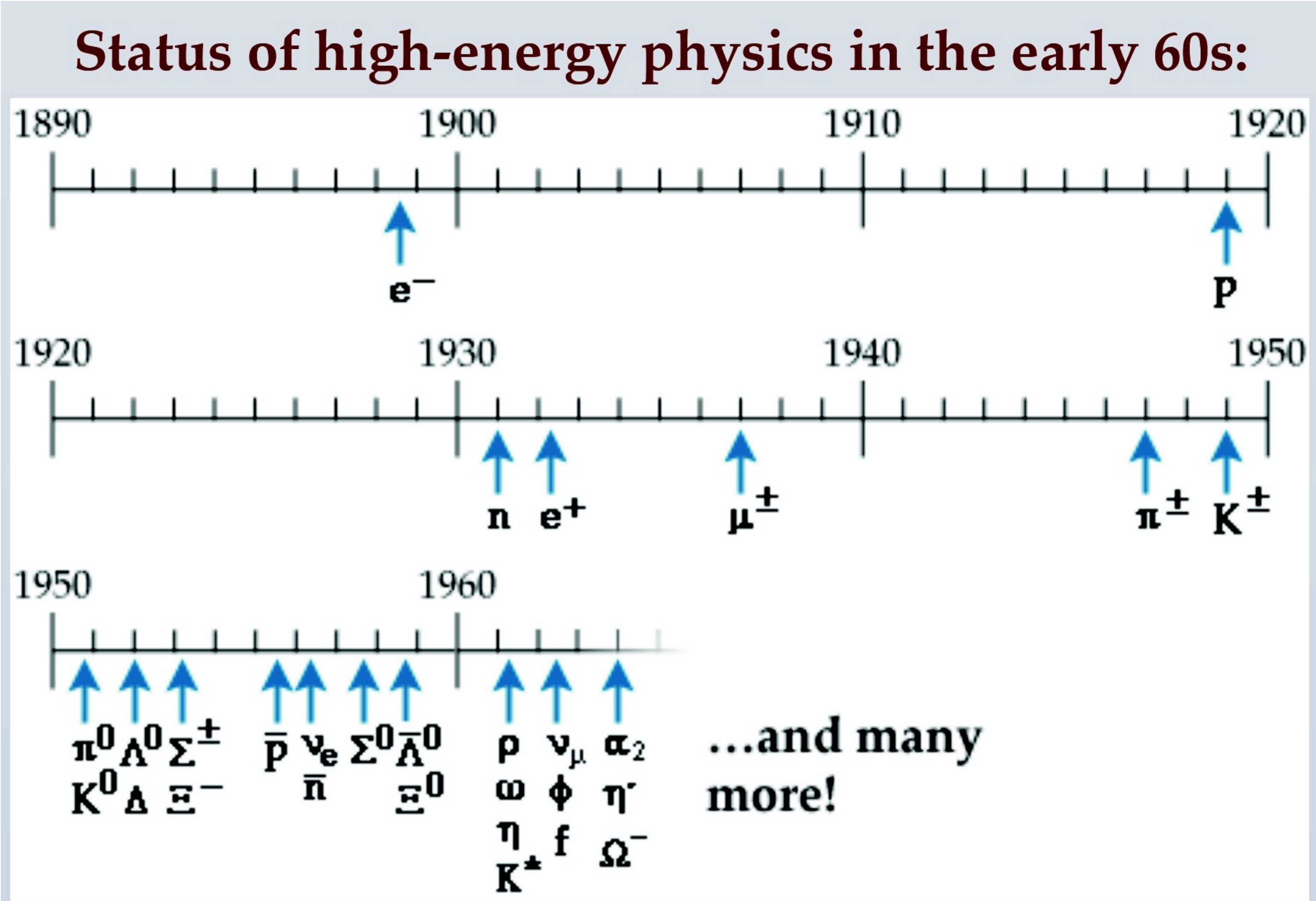


$K^- + p \rightarrow \Omega^- + K^+ + K^0$

Brookhaven National Laboratory, 5 GeV K⁻-Mesons [1964]

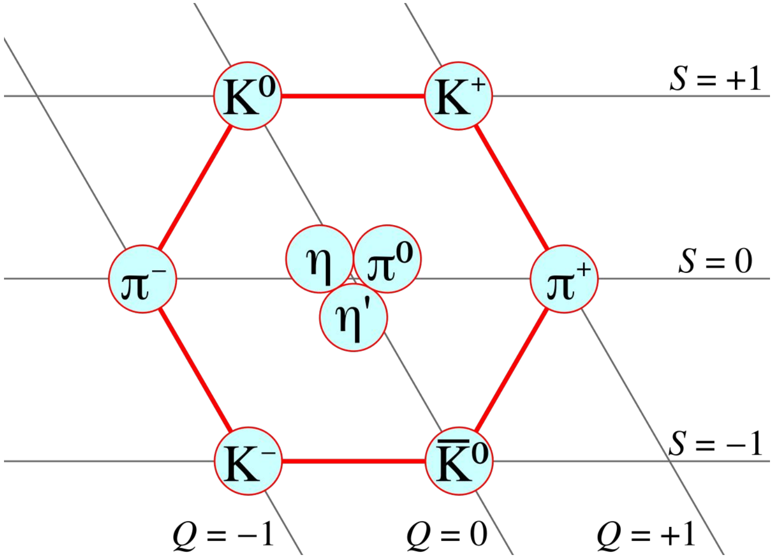
- Particles with integer spin (0,1)
  - Mesons: pions, kaons, …
- Particles with half-integer spin (½, 3/2)
  - Baryons: protons, neutrons …

A historical perspective: the particle zoo



> New particles similar in properties to protons and neutrons → Not alone anymore

Classification of new particles. Any pattern in this zoo ?



- Physicists tried to order this zoo based on properties of the particles
- Strangely, several properties are shared by them
  - Some particles have very similar masses, spin with different charges.
  - Thought initially to be elementary particles

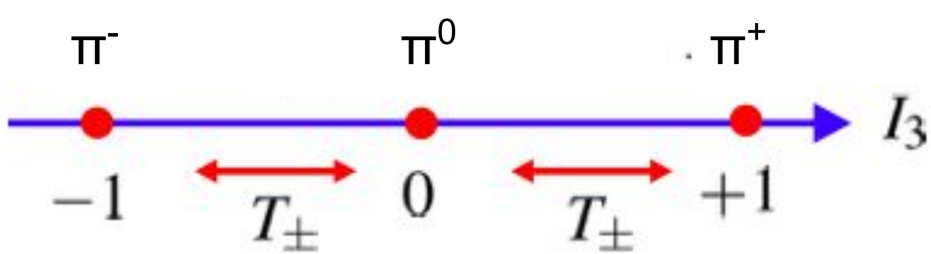
Isospin, strangeness and baryon decuplet

$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57039 \pm 0.00018$ MeV (S = 1.8)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

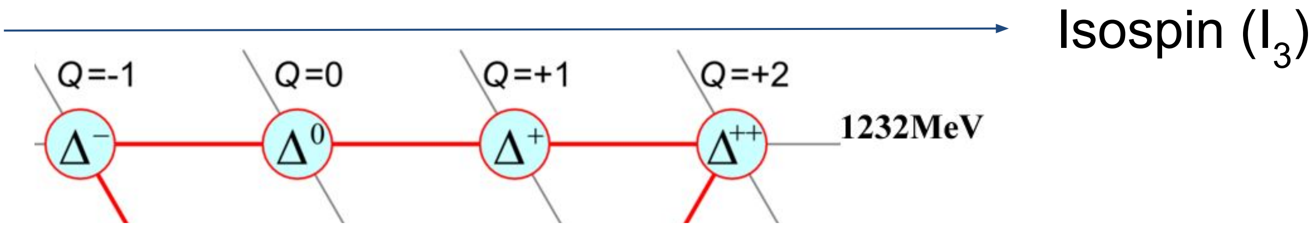
π^0 $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9768 \pm 0.0005$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.43 \pm 0.13) \times 10^{-17}$ s (S = 1.2)
$c\tau = 25.3$ nm



- They can be ordered by mass and electric charge
  - If mass similar but different particles, a symmetry is conserved → Isospin (e.g. pions)

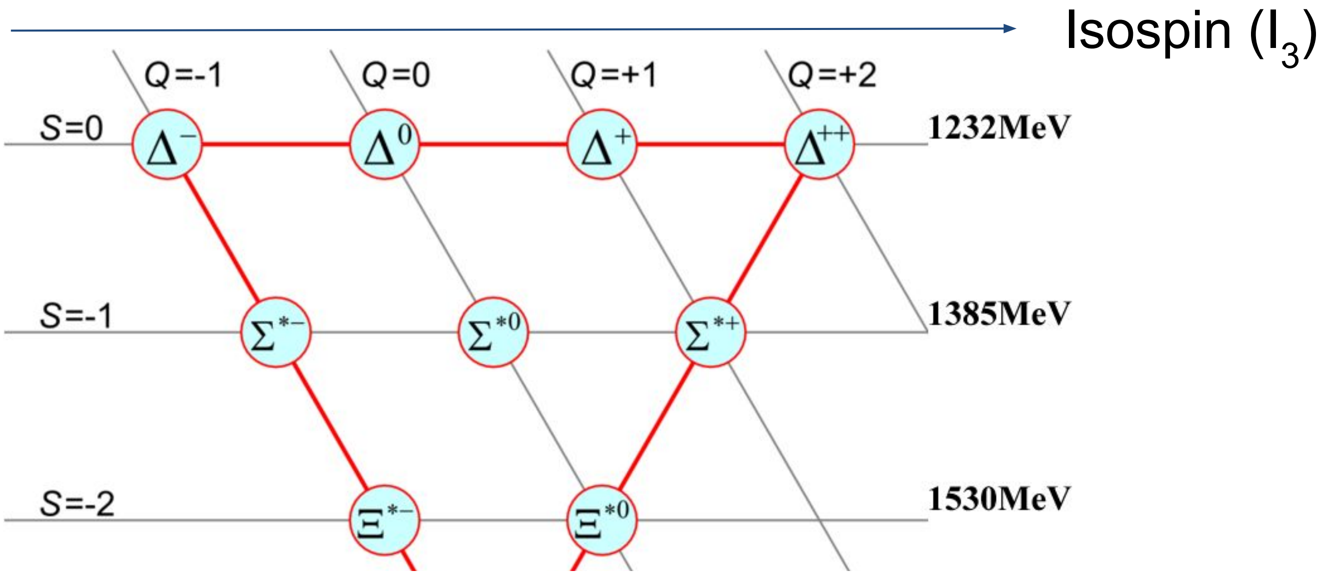
Isospin, strangeness and baryon decuplet



Let's take for instance the baryons with spin 3/2

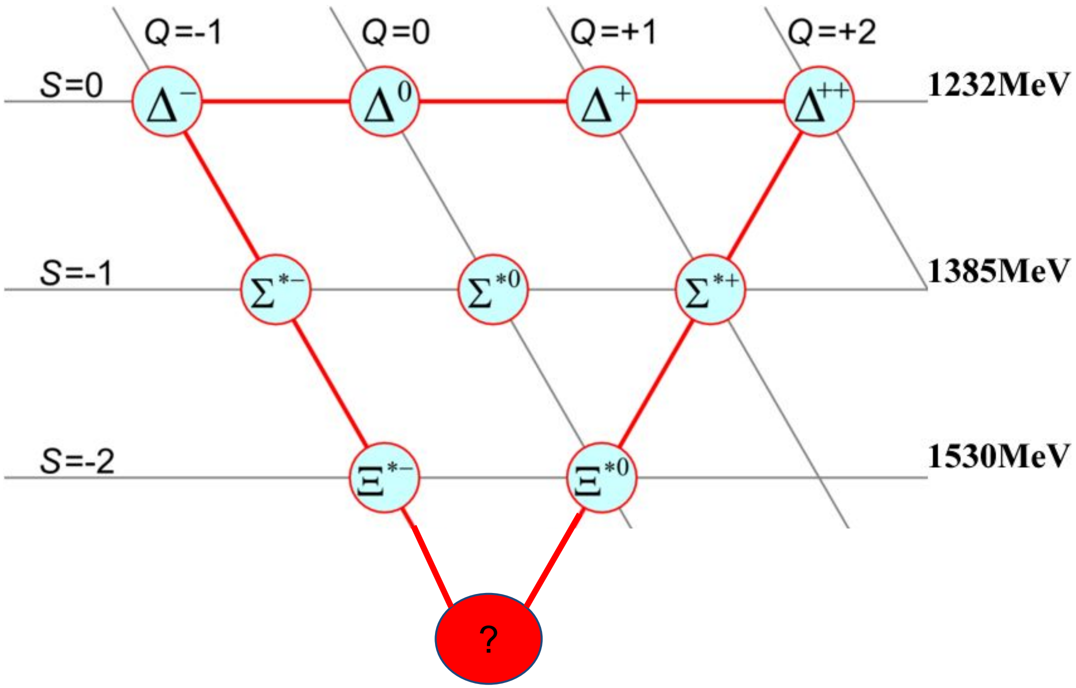
- They can be ordered by mass and electric charge
  - If mass similar but different particles, a symmetry is conserved → Isospin (e.g. pions or spin 3/2)

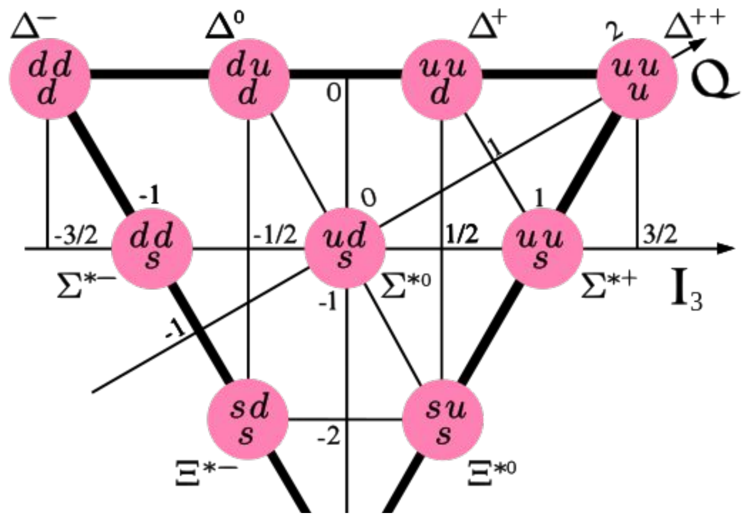
Isospin, strangeness and baryon decuplet



- Another set of spin 3/2 showed different masses and longer lifetime than other counterparts
  - Another quantum number called strangeness
  - Symmetrical patterns appear if have their strangeness plotted against their electric charge.

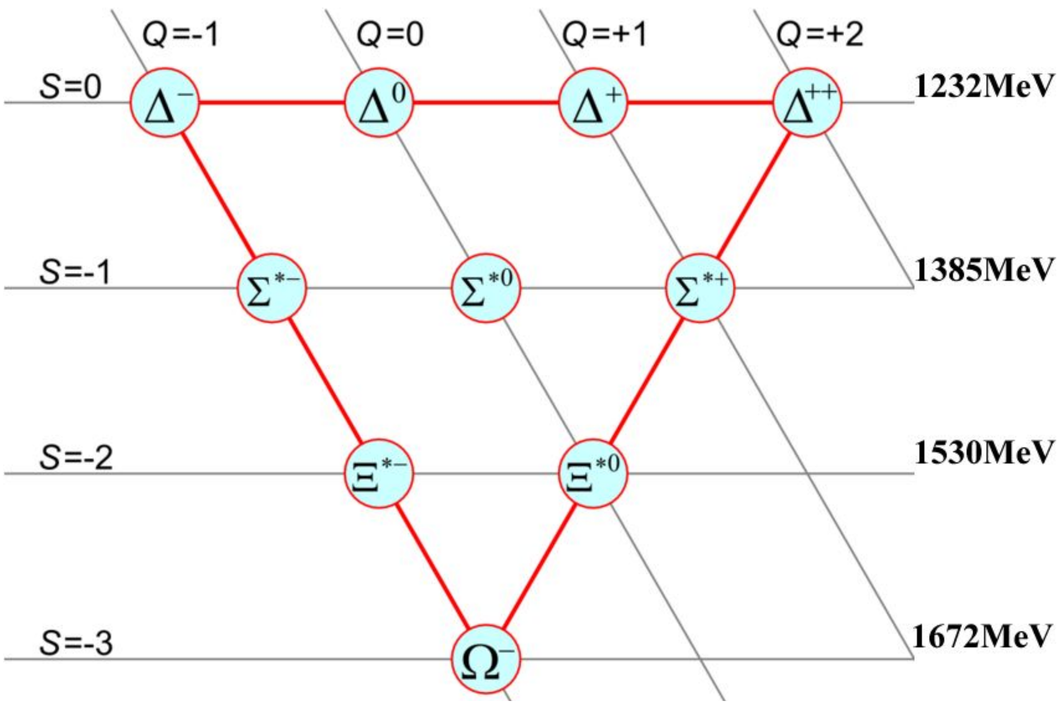
Isospin, strangeness and baryon decuplet

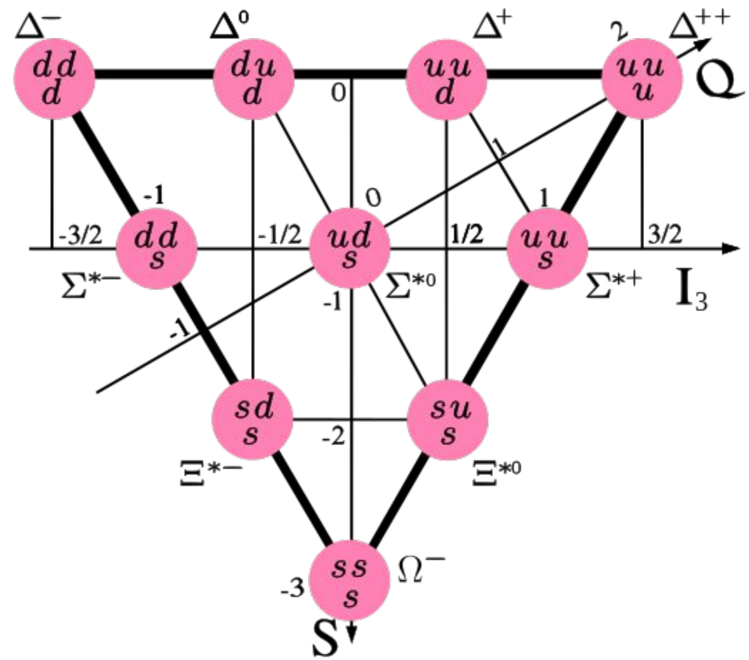




- > Lead to definition of these particles as composed particles → Quarks!
  - 3 quarks: u, d and s → spin ½
  - s carries the quantum number strange.
  - u,d are the particles carrying the isospin ½ and -½
- > Prediction of the Ω^- with 3 s-quarks, no isospin and negative charge ..

Isospin, strangeness and baryon decuplet





- Lead to the definition of the quarks as constituents of these particles
  - 3 quarks: u, d and s → spin ½
  - s carries the quantum number strange.
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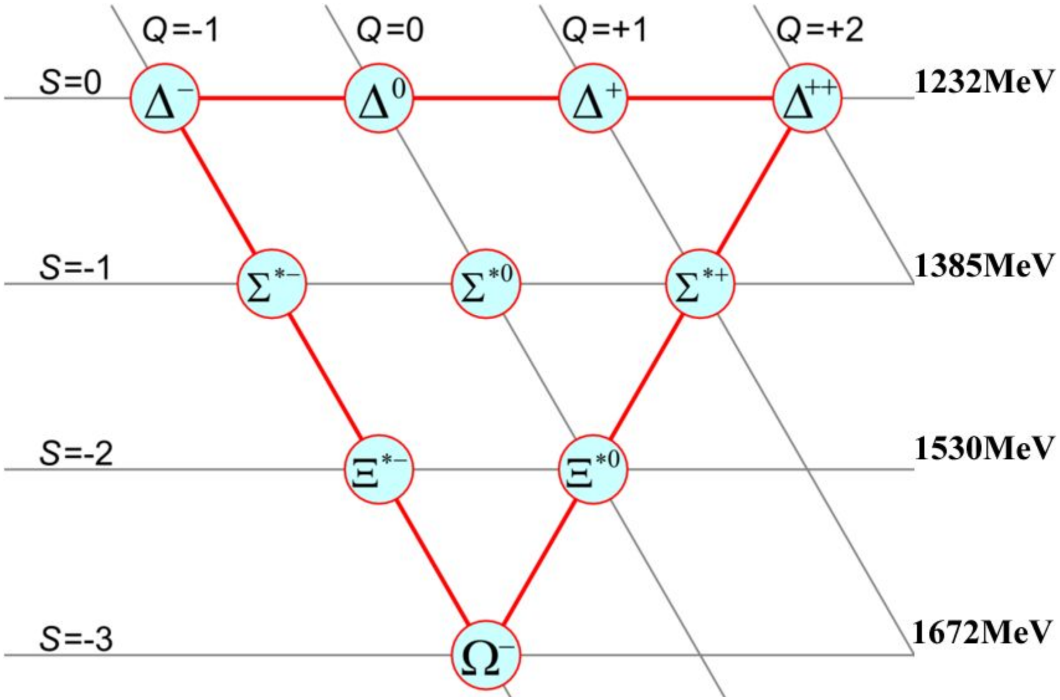
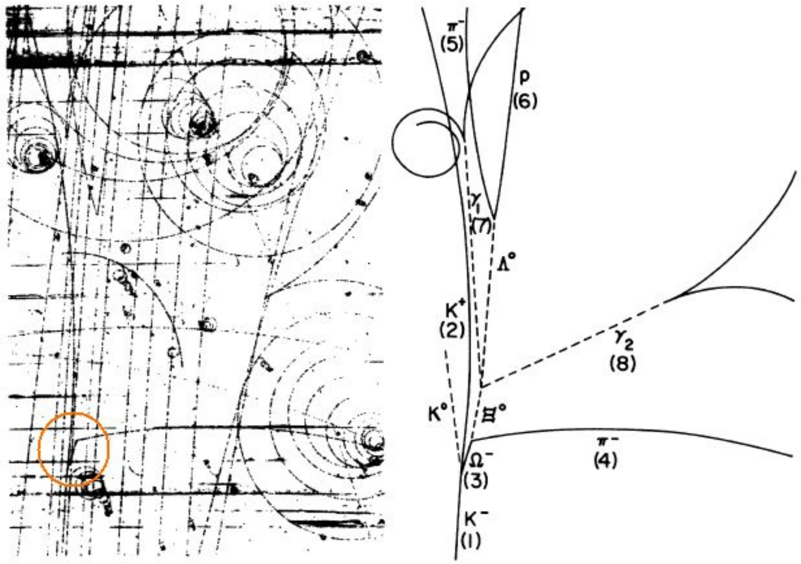
- Prediction of the Ω^- with 3 s-quarks, no isospin and negative charge ..
  - Success !!! (1964 link)

Isospin, strangeness and baryon decuplet





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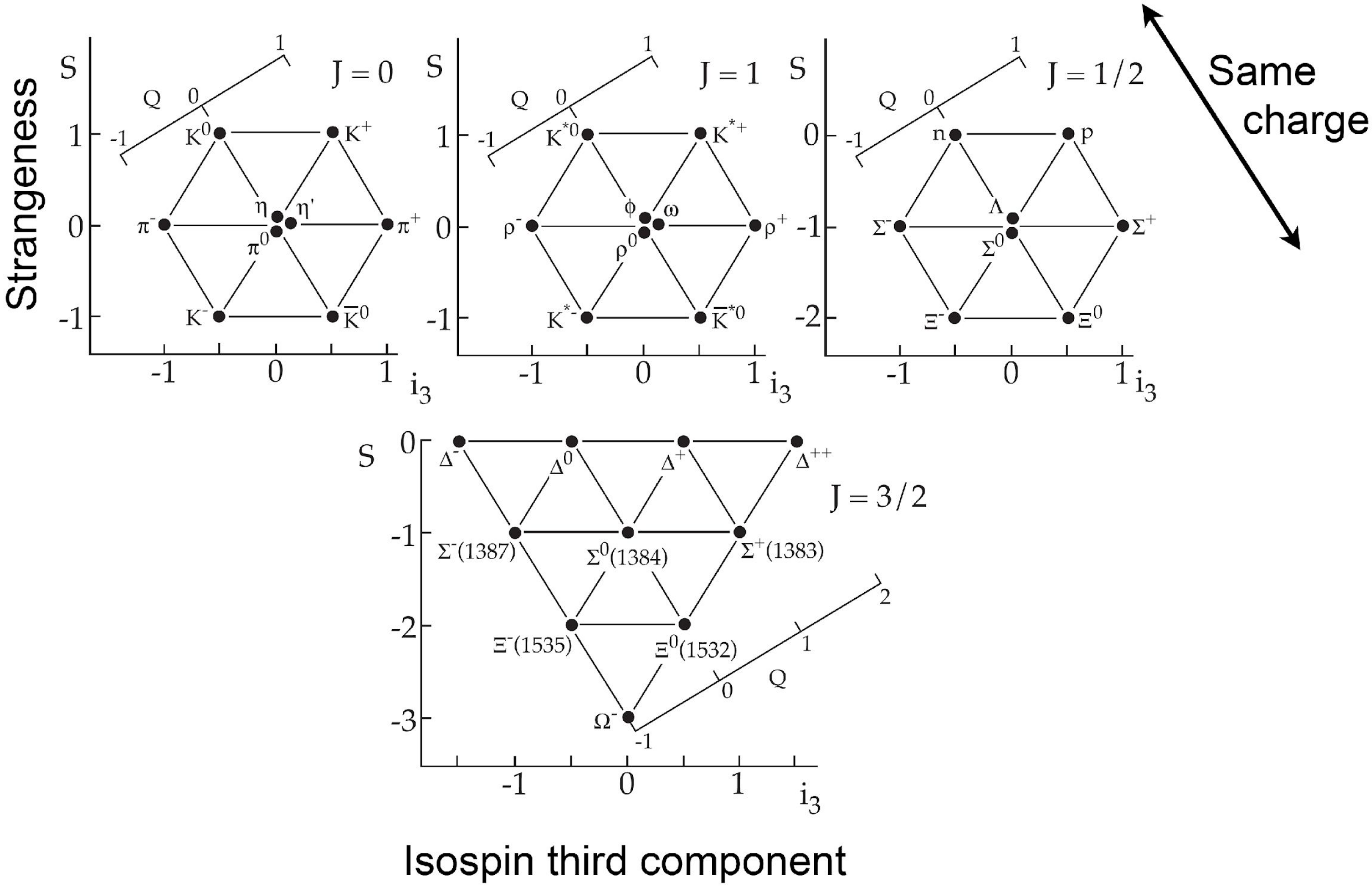
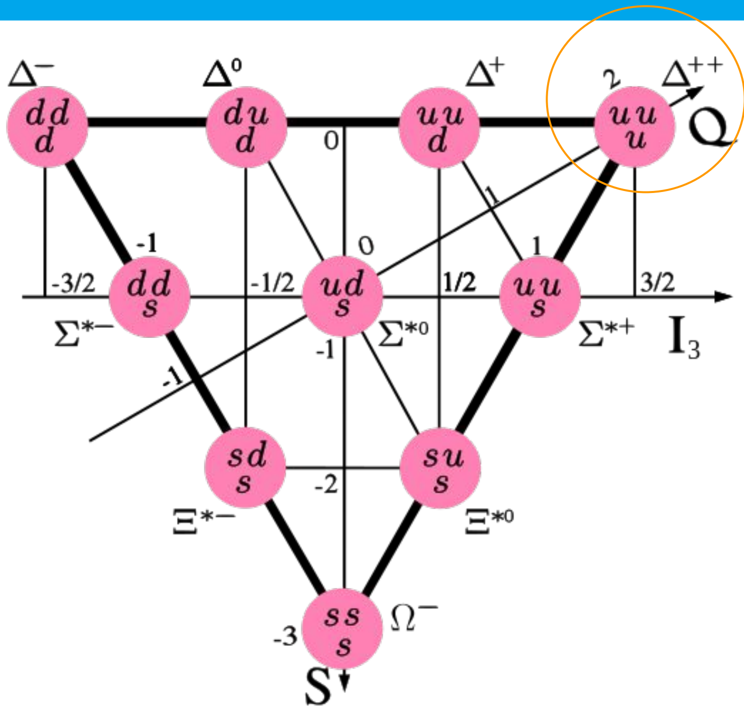
Quarks



Strangeness
S
J = 0
J = 1
J = 1/2
J = 3/2
Q
K^0 K^+ π^- η η' π^0 π^+ K^- $\bar{K}^0$
K^{*0} K^{*+} ρ^- ϕ ω ρ^0 ρ^+ K^{*-} $\bar{K}^{*0}$
n p Σ^- Λ Σ^0 Σ^+ Ξ^- Ξ^0
Δ^- Δ^0 Δ^+ Δ^{++}
$\Sigma^-(1387)$ $\Sigma^0(1384)$ $\Sigma^+(1383)$
$\Xi^-(1535)$ $\Xi^0(1532)$
Ω^-
i_3
Same charge
Isospin third component

A story of spin and color



In the quark model, Δ++ posed a problem.

This is a particle with spin 3/2, with spin I_3 component +- 3/2 composed by same particles

Symmetric wave-function but it is a fermion !
Violation of Pauli principle !

$$|\Delta^{++}\rangle = |u_{\uparrow}\, u_{\uparrow}\, u_{\uparrow}\rangle$$

Solution: another quantum number → Color !

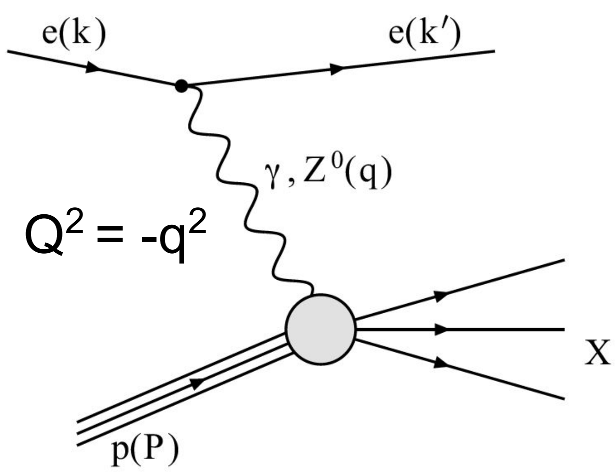
$$|\Delta^{++}\rangle = |u,\uparrow,g\rangle + |u,\uparrow,r\rangle + |u,\uparrow,b\rangle$$

Initially postulated 3 values of color: red, blue and green.

Summary so far and next step

Organized the particle zoo as being composed of 3 different quarks.
Introduced isospin, strangeness and color
Explained charge, masses, spin and lifetimes of some particles.

But we have never seen a single spin ½ particle alone. Where are they ?
Answer to this question came from deep inelastic scattering measurements on the proton! (SLAC 1969) $e + p \rightarrow e + X$



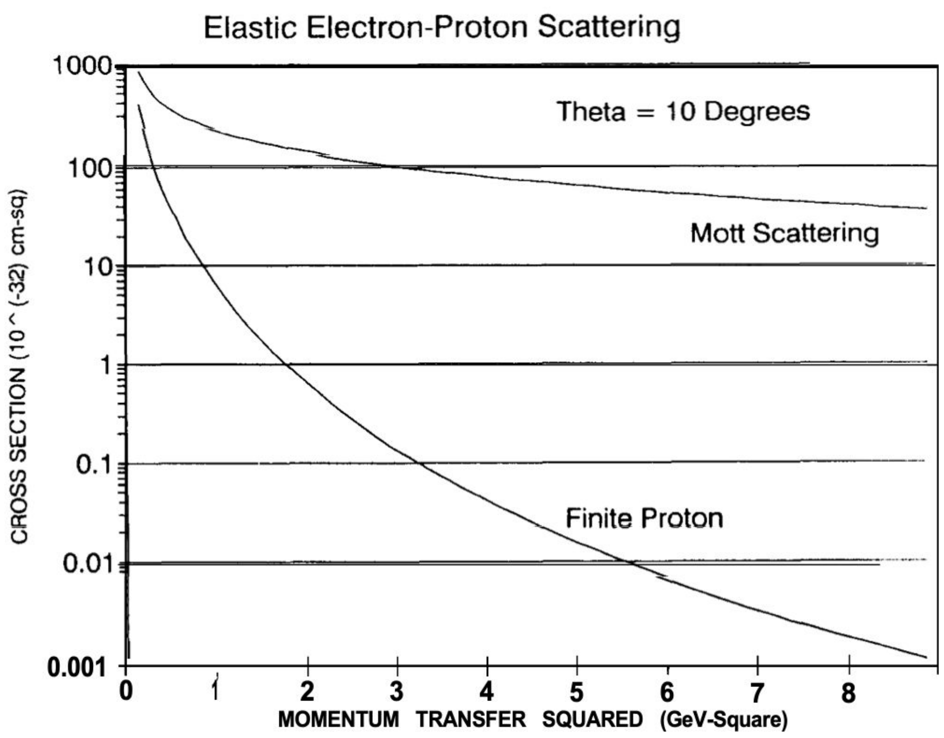
$Q^2 = -q^2$

At low e-beam energies (low Q^2)
Resolution is close to the size of the proton

At medium e-beam energies (medium Q^2)
Electron and scattering can start seeing the internal structure

At high e-beam energies (high Q^2)
Can see the structure of the proton!

Deep inelastic scattering



Rutherford scattering

Elastic spin-less electron scattering against point-like proton

Mott scattering

Inelastic electron scattering against point-like proton

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}}$$

Inelastic electron scattering against composite proton

Form factors describing the charge distribution inside proton.

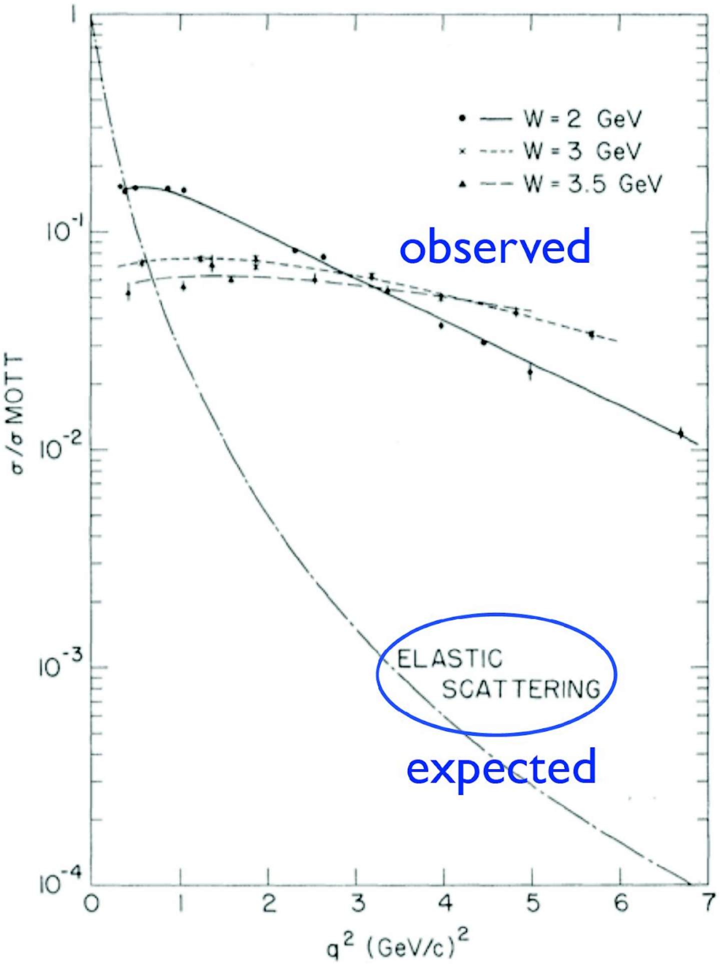
$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}} \left[W_2(x,q^2) + W_1(x,q^2)\tan^2\frac{\theta}{2} \right]$$

W_2 is the form factor of the electric charge

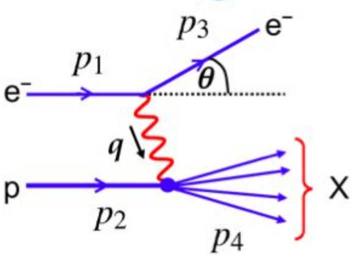
x : momentum fraction of the charge inside proton

Q,q: momentum exchange in scattering

Scattering experiments



M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969)



$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$

W = M_X

Early results from SLAC (1969)

$E = 7$ - 17.7 GeV

$\theta = 10°$

Elastic cross section falls off rapidly due to the proton not being point-like

Inelastic: $W > M$

Ratio to Mott cross section nearly flat in Q^2

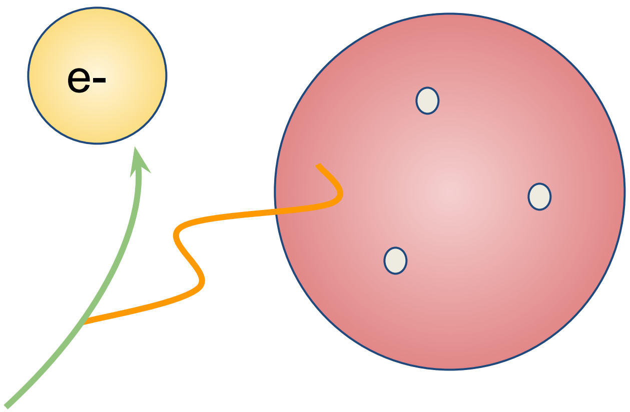
Q^2 dependence becomes weaker for increasing W

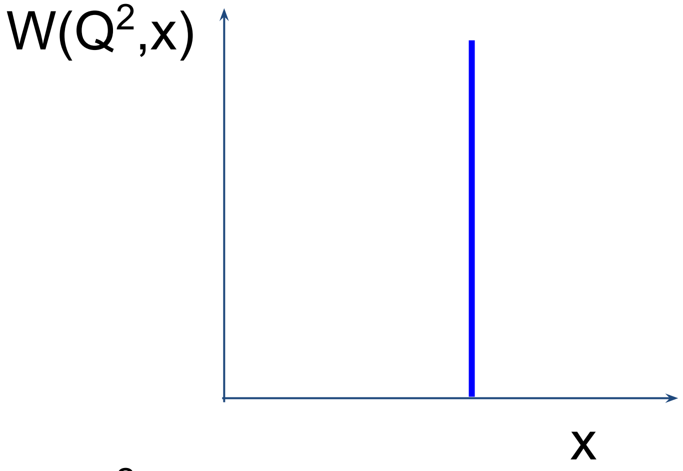
Proton a composite particle!

Deep inelastic scattering: Form Factors

In this interaction, electron momentum and cross-section will be sensitive to proton charge Form Factors → Measurement

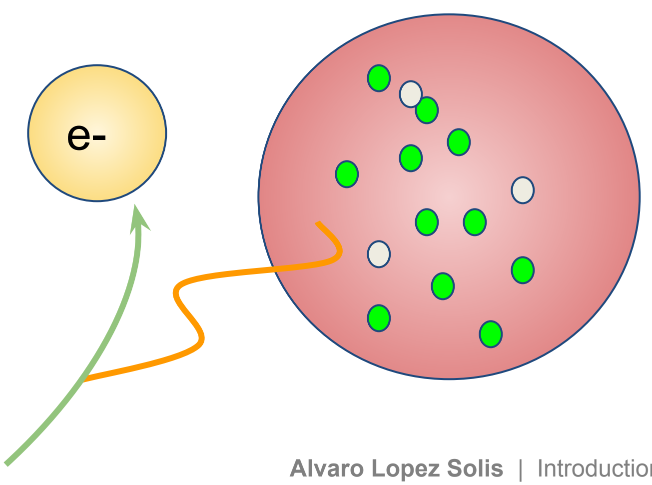
Spatial Fourier transform of the charge distribution inside the proton $W(q^2,x)$

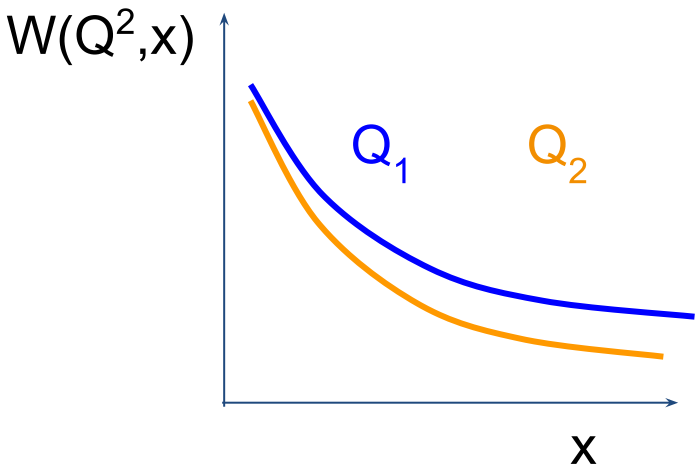




Point-like particles

We will see the same charge → Scaling



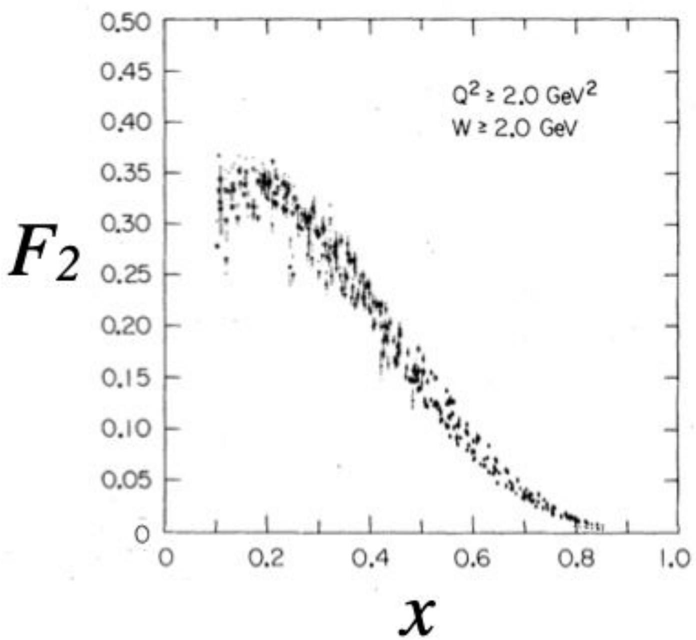


Distribution of particles

More or less charges carrying different momentum of the proton
Different numbers observed at different energies

Mass of the proton → The gluon !

[Experimental studies of the neutron and proton electromagnetic structure functions]



$$\int_0^1 F_2^p(x)\mathrm{d}x \approx 0.18 \quad \text{and} \quad \int_0^1 F_2^n(x)\mathrm{d}x \approx 0.12$$

$$\Rightarrow \boxed{f_u = 0.36 \ \text{and} \ f_d = 0.18}$$

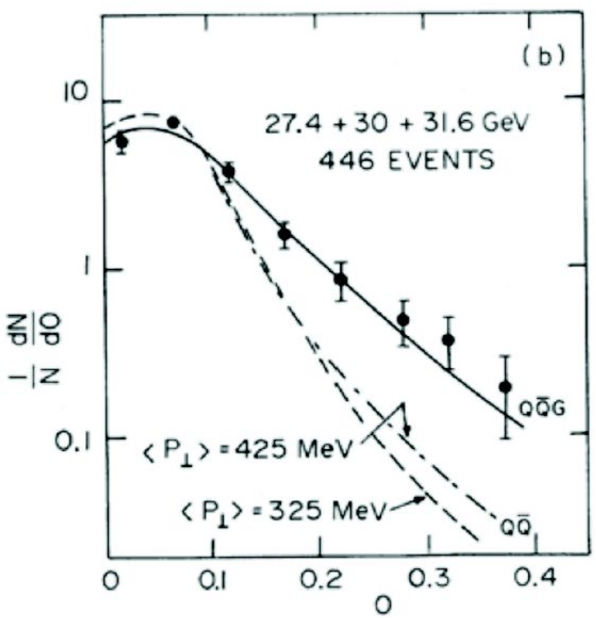
~ 50%. Where is the rest of the proton momentum ???

New proton component → Gluon !

Discovered at PETRA in 1979

Hadrons composed of quarks and gluons → partons !

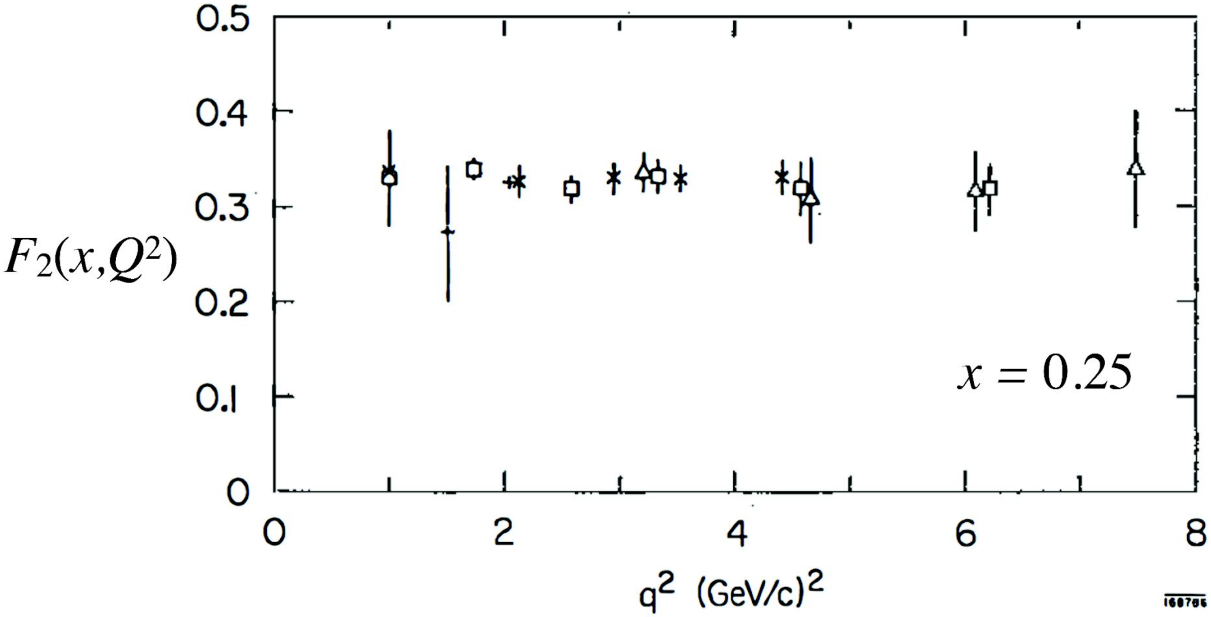
Quark Parton Model success



D. P. Barber (Mark-J), Phys.Lett.B89, 139(1979)

Scattering experiments: Scaling and asymptotic freedom

J.T. Friedman, H.W. Kendall, Ann. Rev. Nucl. Sci. 22, 203 (1972)



Scaling!

Independence of the structure functions of Q^2: $F_i(x,Q^2) = F_i(x)$

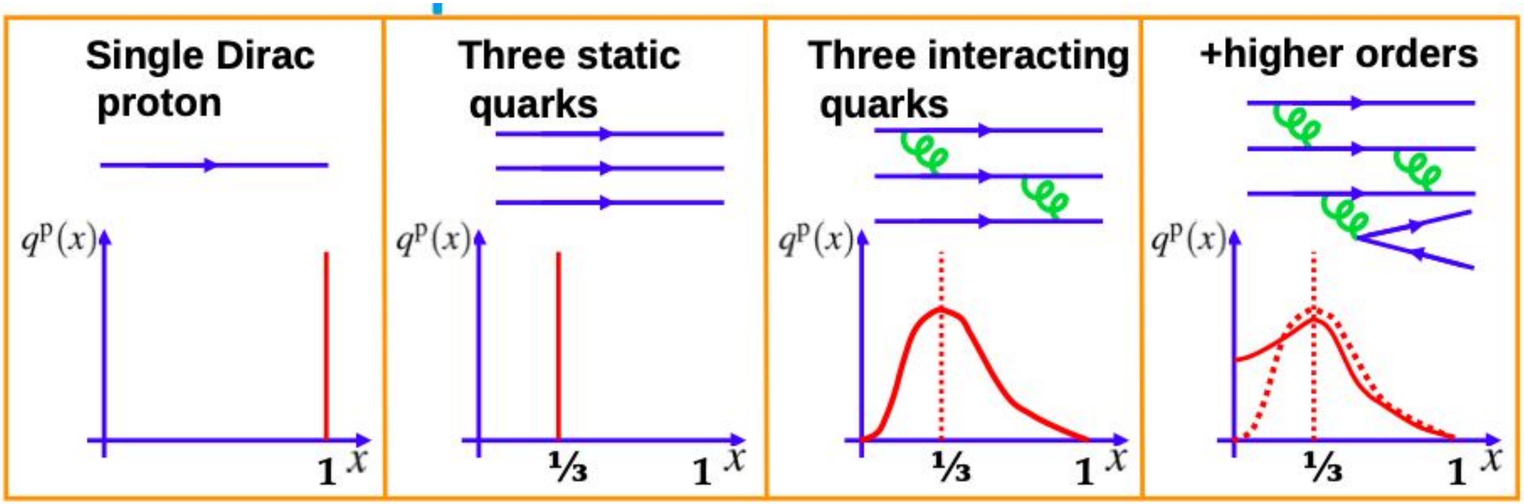
J.D. Bjørken predicted scaling for $Q^2 \to \infty$ as x stays fixed.
Scaling is obtained using Gell-Mann's current algebra in the quark model.

Scattering from point-like constituents of the proton!

Quarks ! And at high energies, they behave like free particles

Parton density functions

Form factors describing the charge distribution inside proton → PDFs



Instagram picture of a proton

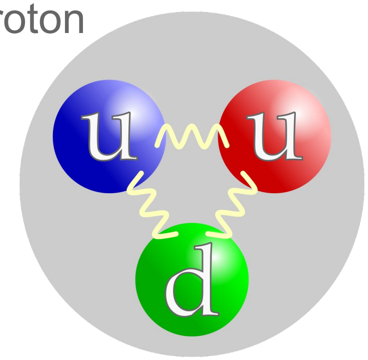
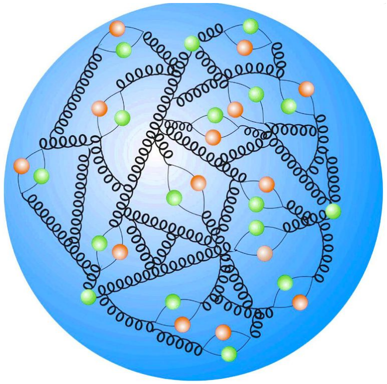


image from official HEP instagram (wikipedia)

Real picture: Parton density functions



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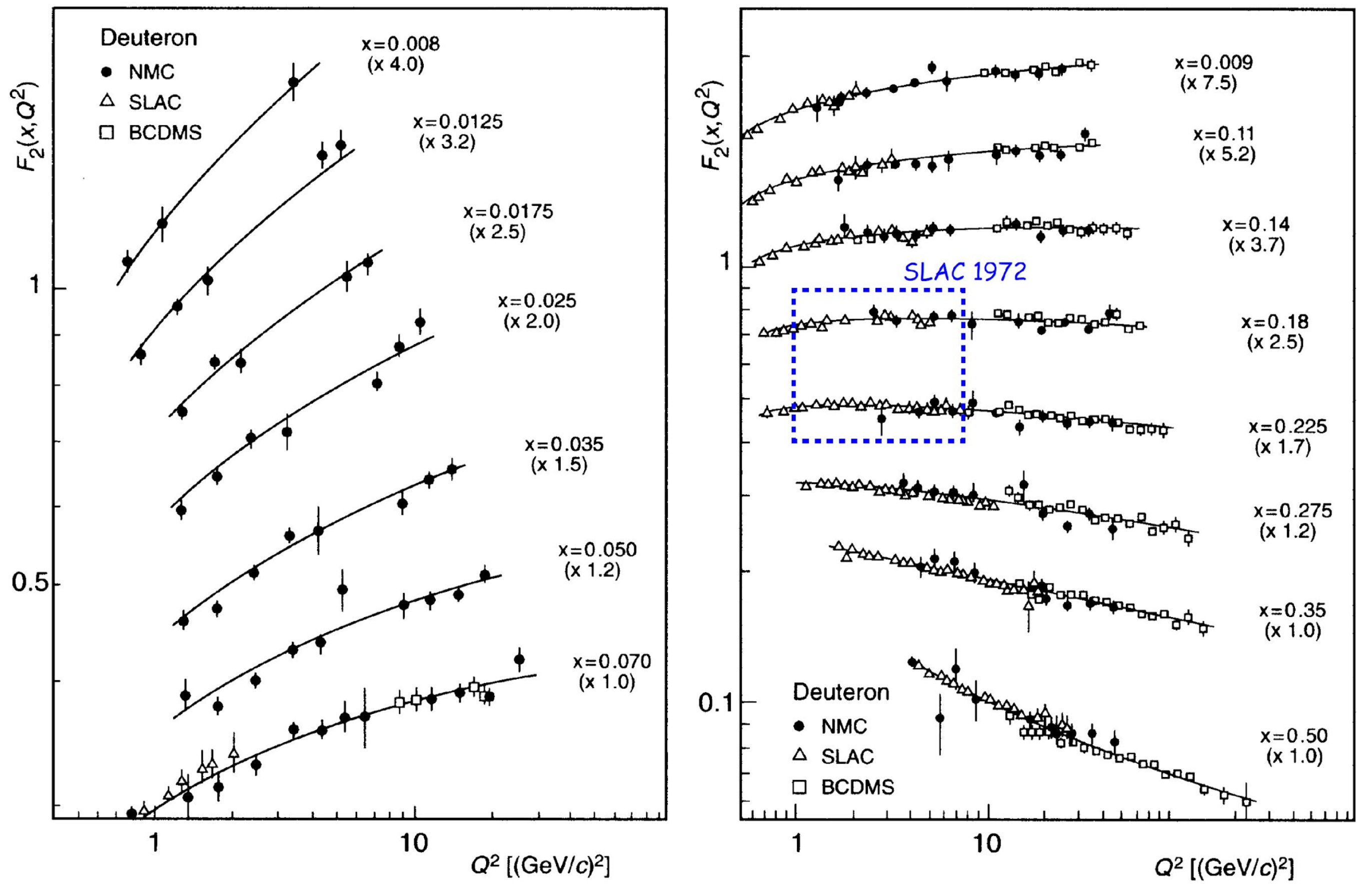
Scaling violation: Parton density functions

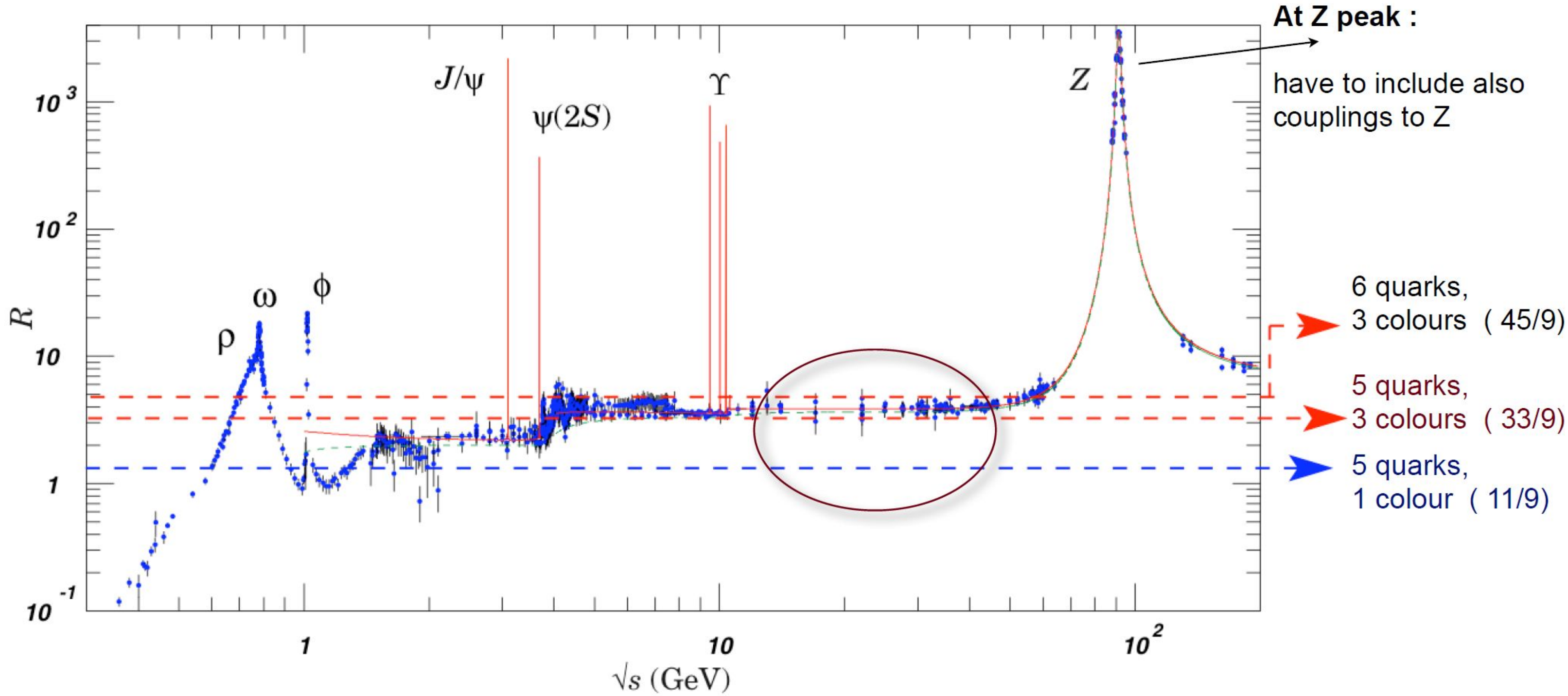
$$F_2(x, Q^2) = \sum e_q^2 x q(x, Q^2)$$



Scaling violation → small, only appears in QFTs with asymptotic freedom → Might the strong force be explained as a perturbation theory (like QED) but with asymptotic freedom ?

Tests of QCD: Hadron pair production (how many colors?)

> Prediction for Ratio: $R = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}} = N_c \sum_q e_q^2$



$$R = N_c \sum_q e_q^2 = N_c \left[\underbrace{\left(\frac{2}{3}\right)^2}_{u} + \underbrace{\left(-\frac{1}{3}\right)^2}_{d} + \underbrace{\left(-\frac{1}{3}\right)^2}_{s} + \underbrace{\left(\frac{2}{3}\right)^2}_{c} + \underbrace{\left(-\frac{1}{3}\right)^2}_{b} \right] = N_c \frac{11}{9}.$$

c-quark discovered in 1974

b-quark discovered in 1977

t-quark discovered in 1995

Summary so far

- Organized the particle zoo as being composed of 3 different quarks.
- Introduced isospin, strangeness and color
- Explained charge, masses, spin and lifetimes of some particles.
- DIS experiments showed that quarks are not only mathematical constructs but components of the hadrons
- DIS showed that the charged constituents of *p* follow a structure form $F(x,Q^2)$ → parton density functions.
- If interaction with proton, results weakly dependent of the energy of the collision → asymptotic freedom (pointlike particles moving as)
- DIS showed that not all the proton mass can be explained only by only quark composition → gluon ! → discovery at PETRA → Identified as the mediator of the strong interaction.
- Ratio σ(e-e+ → hadrons)/σ(e-e+ →μ+μ-) at increasing energies showed not 3 quarks but 6 in total

Quantum chromodynamics

Quantum Chromodynamics

- Gauge theory invariant under transformations in the color space
- Kinematics and interactions of particles charged under the strong force → color charge

This is the perturbative story of quarks and gluons !

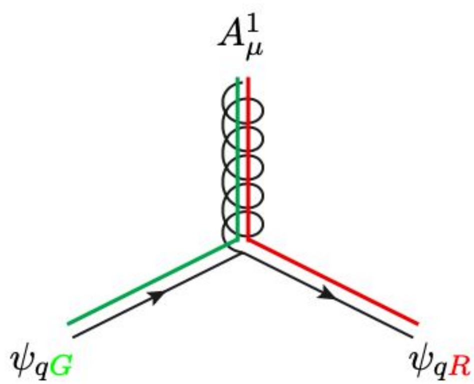
Quarks and anti-quarks

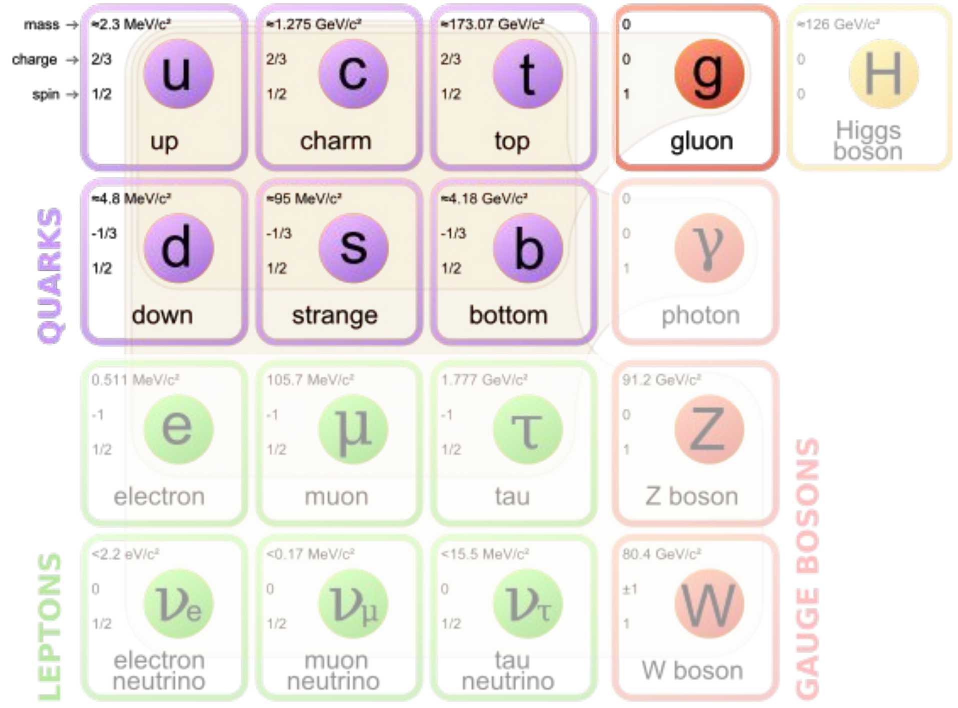
Exactly one single color, can be either of the three available

Gluons:

Two colour numbers

qqg vertex



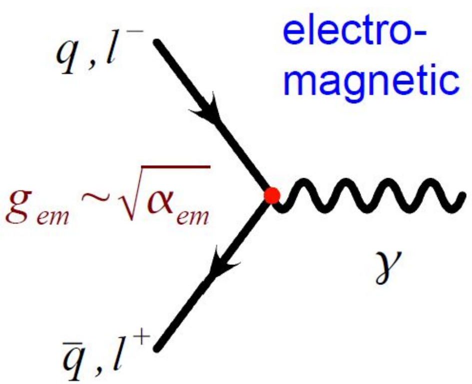


> Gauge theory invariant under transformations in the color space

> Kinematics and interactions of particles charged under the strong force → color charge

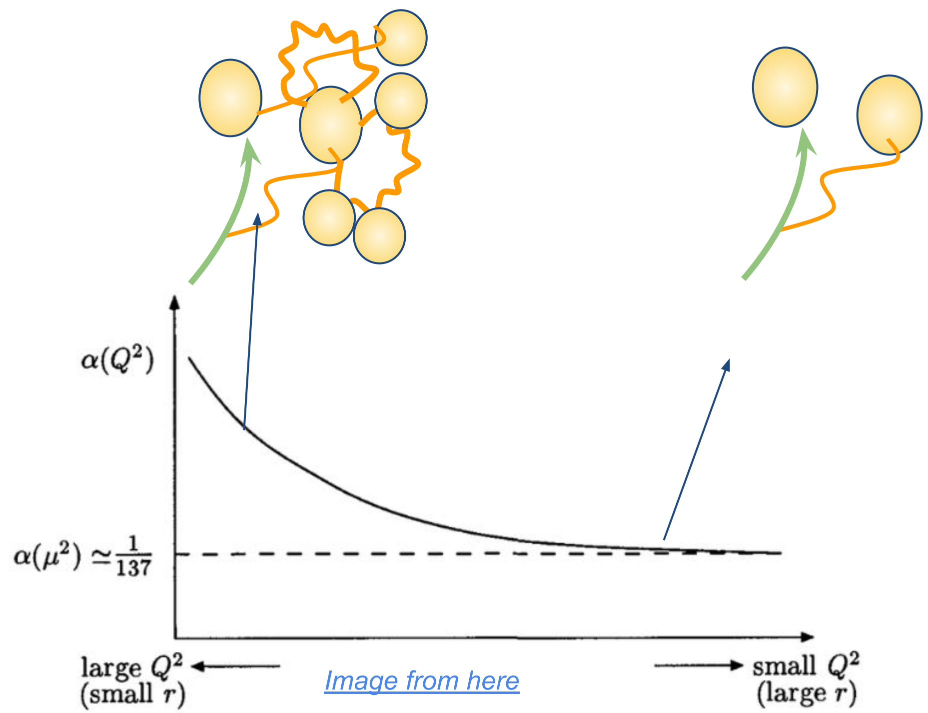
Coupling constant (Reminder QED)

Universal (not different magnitude of coupling depending on the particle or color)
Similar to the electromagnetic coupling, expressed in terms of an α constant ….



$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi} = \frac{1}{137}$$



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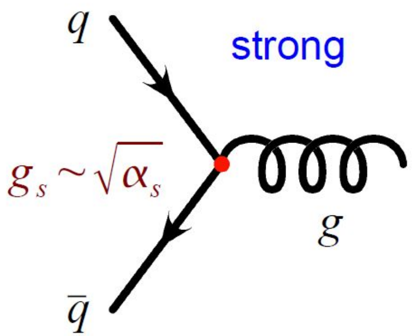
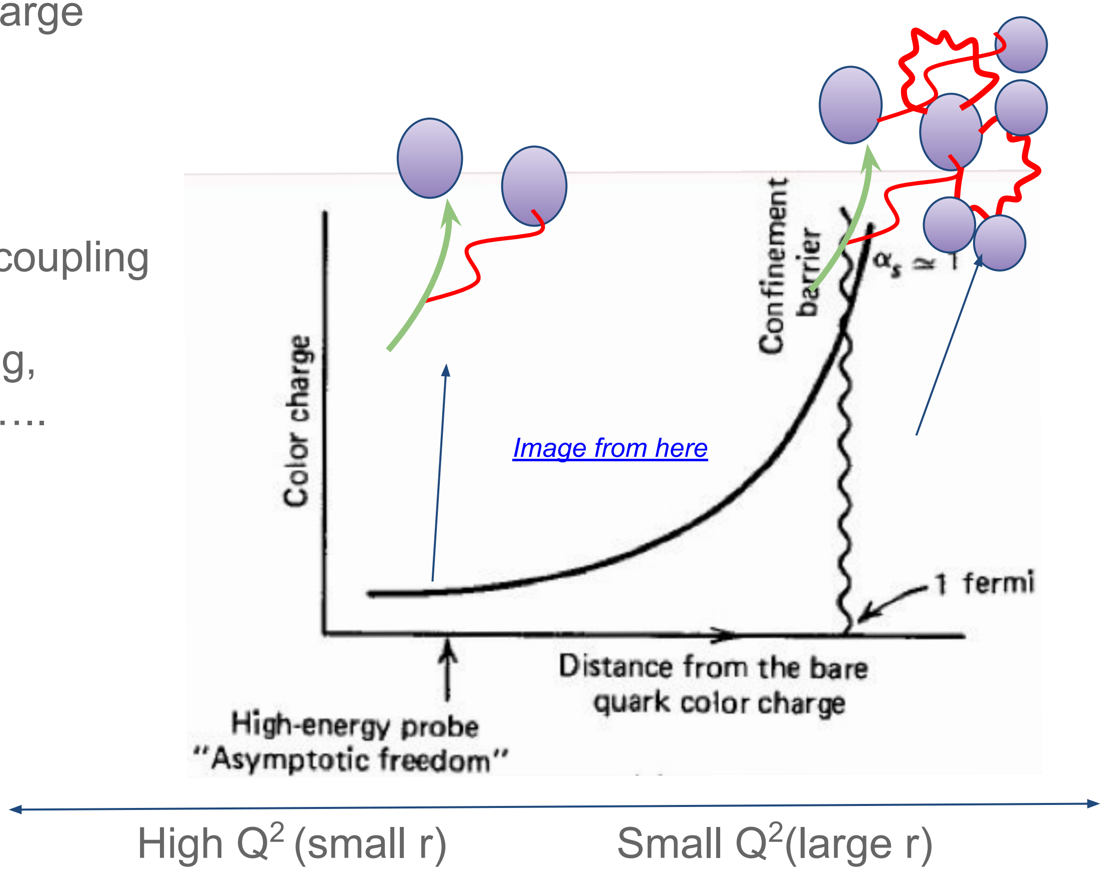
Quantum Chromodynamics

- Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction
  - Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

Coupling constant

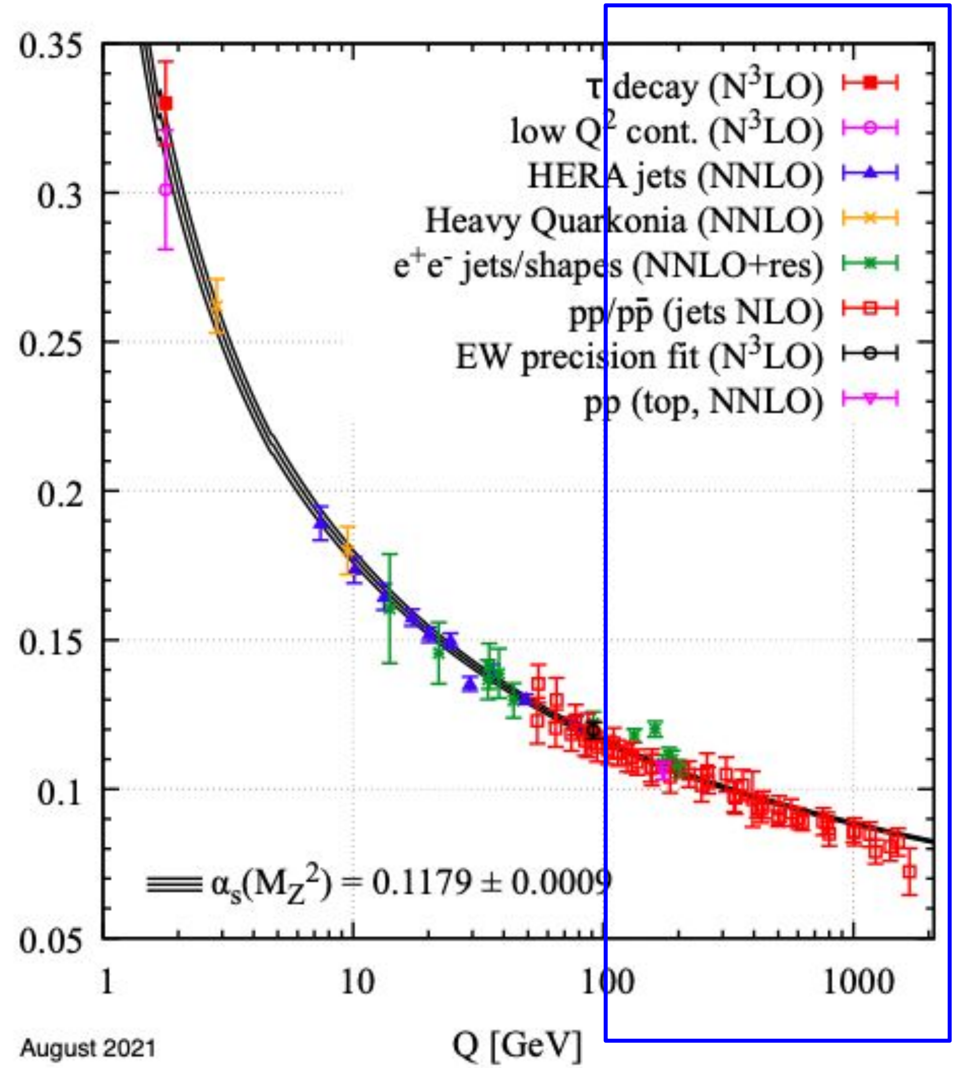
Universal (not different magnitude of coupling depending on the particle or color)
Similar to the electromagnetic coupling, expressed in terms of an α constant …..
BUT ! Different evolution with Q^2





Asymptotic freedom

In QCD interactions with high energy, coupling tends to zero
→ At high energies, quarks and gluons would behave as if the strong force doesn't exist

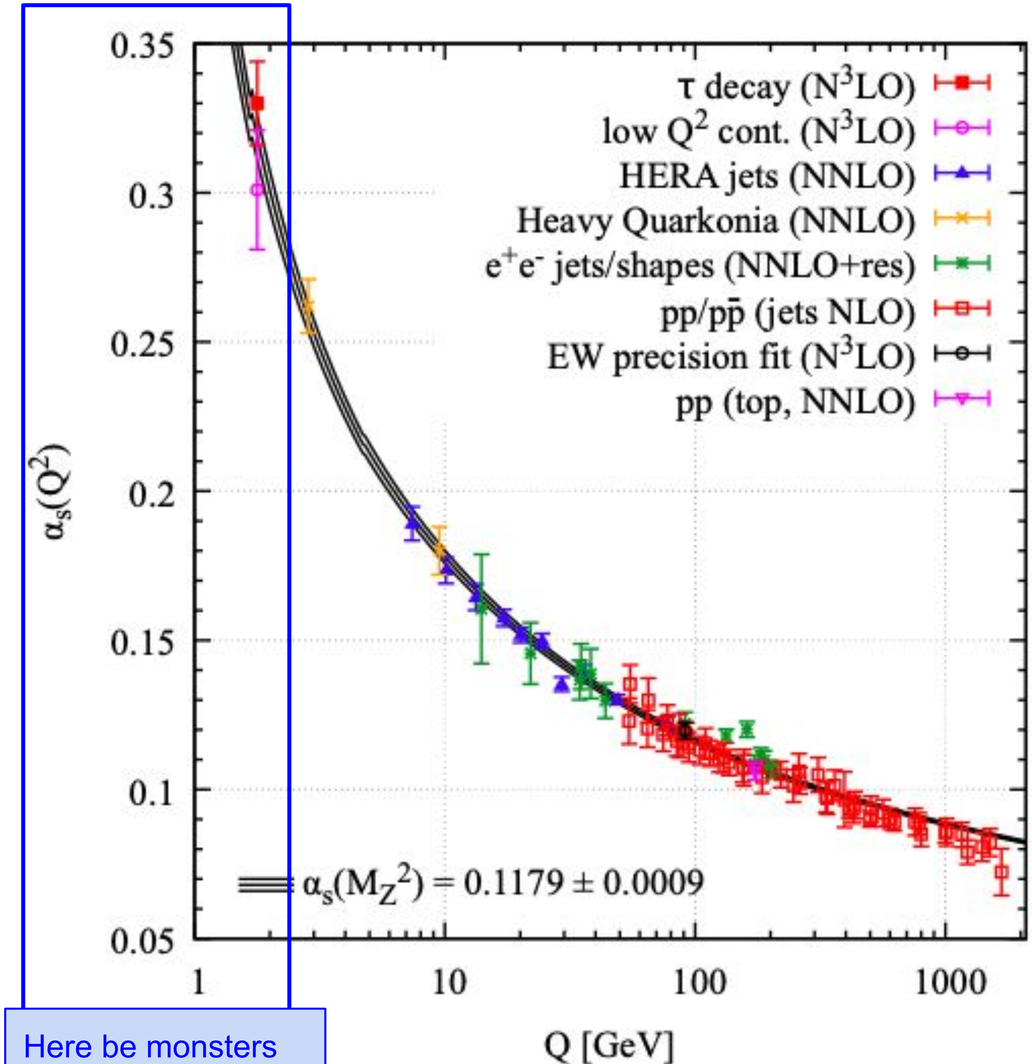


QCD at low energies

At low energies, coupling increases to very high values

- At Λ ~ 200 MeV → non-perturbative QCD

Now you know why we won't see a quark alone!



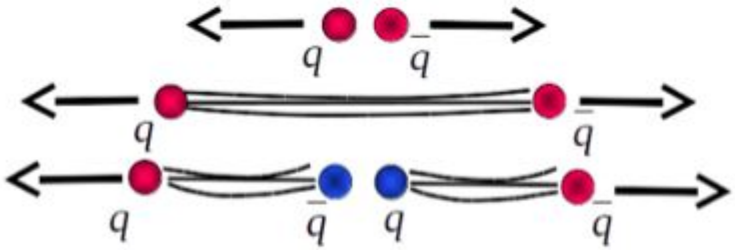
Here be monsters

QCD at low energies: some consequences

Confinement

Quarks and/or gluons move away

- Energy between partons become high
- Spontaneous creation of pair quark-anti-quark
- Hadronization (shower of particles)



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Parton density functions

Charge distribution within hadrons. Production and annihilation of partons constant.

- If high energy interaction with proton, interaction with a distribution of particles → PDFs

Instagram picture of a proton

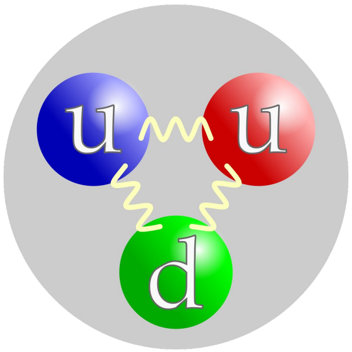
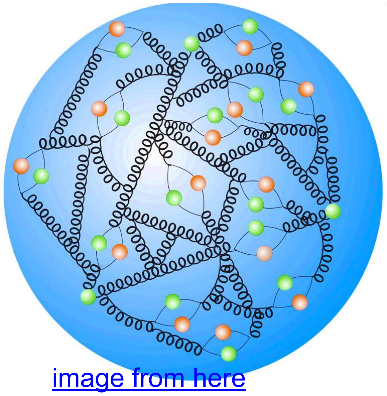


image from official HEP instagram (wikipedia)

Real picture: Parton density functions



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SM predictions fail here → Can only know through experiments by now

And many more interesting QCD features

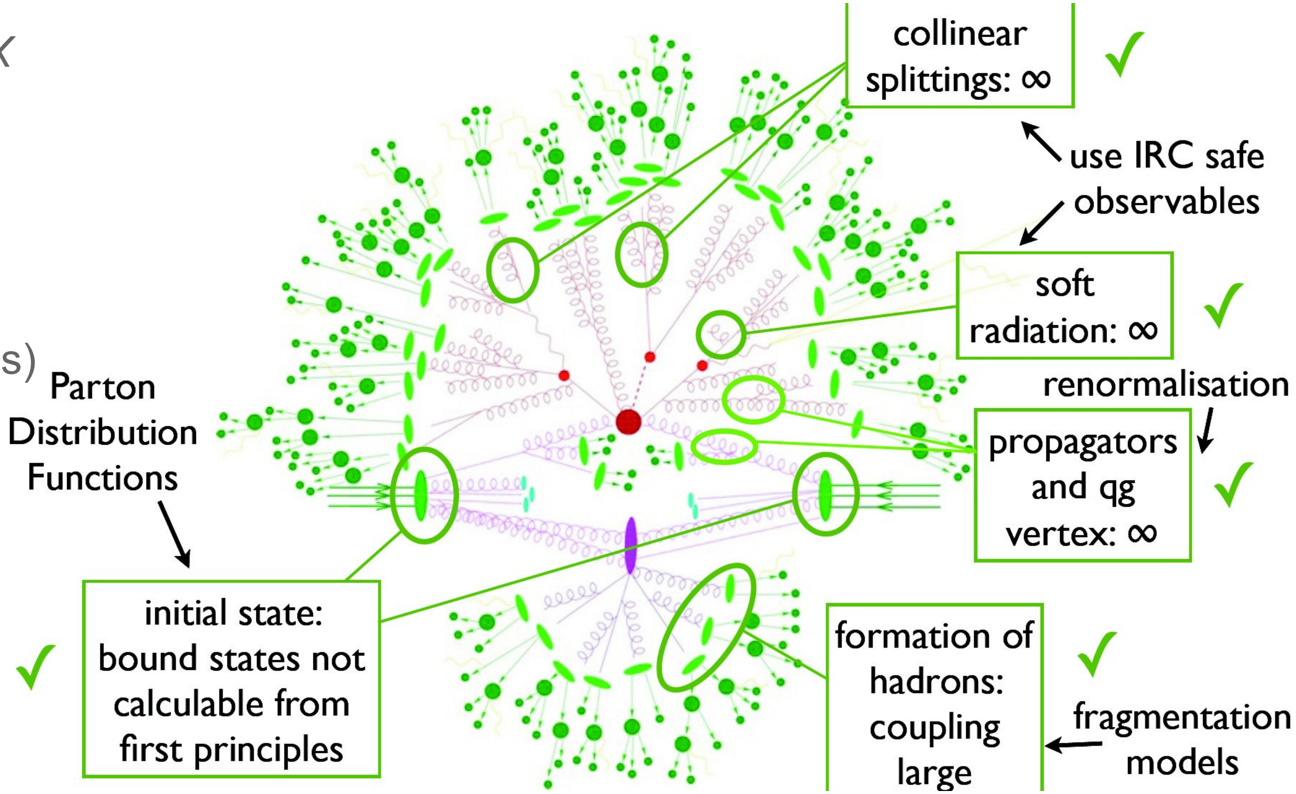
Collisions at LHC: *pp* → *XX*

Perturbative QCD

- Cross-section
- Infrared radiation

Non-perturbative QCD

- Proton structure (PDFs)
- Parton shower
- Hadronization
- Jets



$$\sigma_{PP\to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q}\to X}(\alpha, Q^2)$$

To know more and better of perturbative QCD → Lectures of Andreas Maier

Measuring QCD properties: Differential jet cross-section

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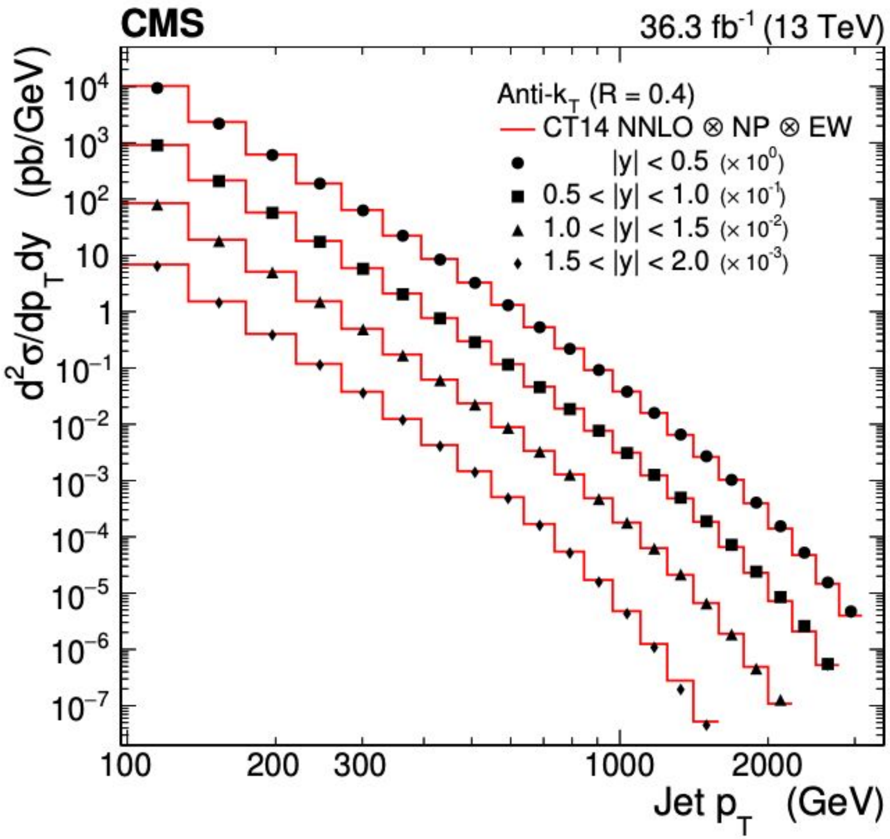

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