MAGO Report

Cavity status, first measurements and simulations

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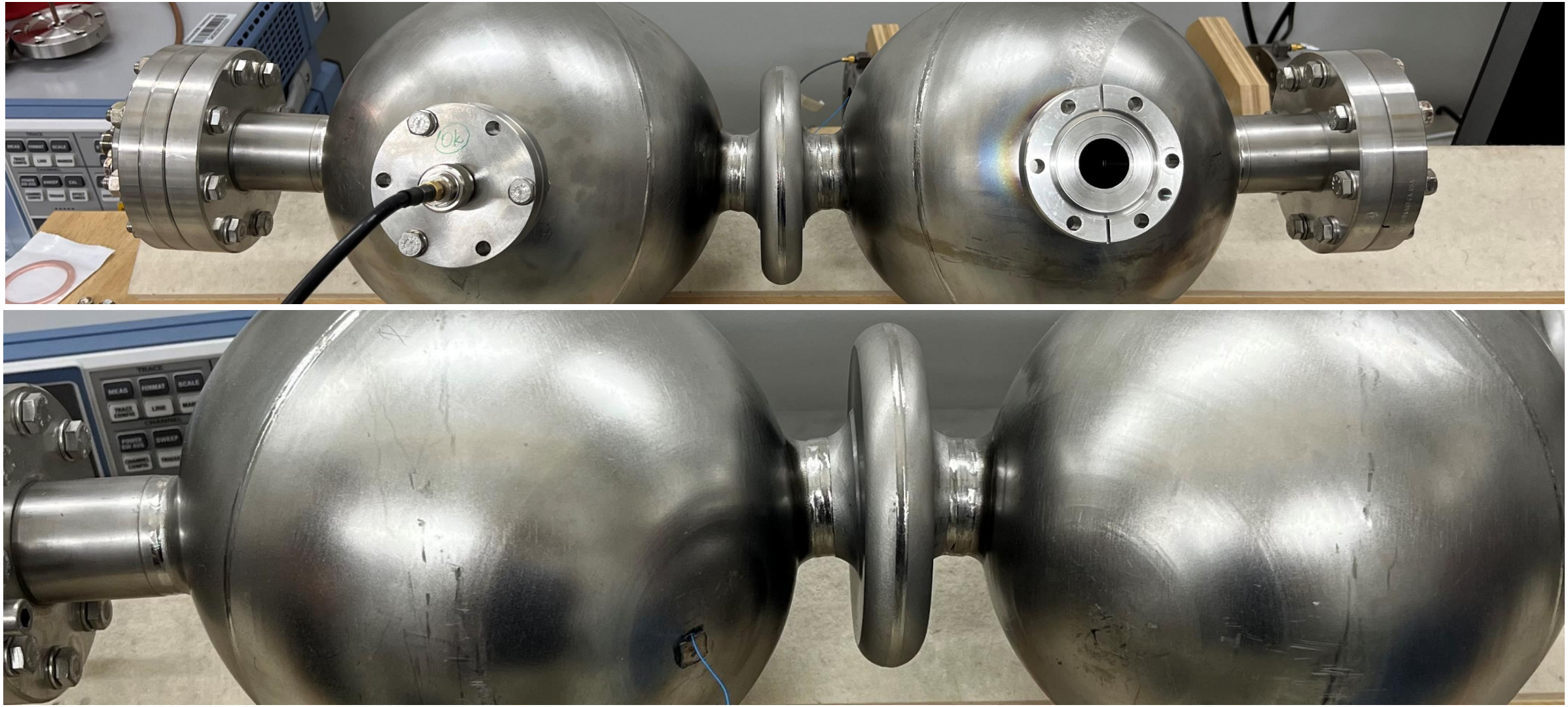
HELMHOLTZ

DESY.

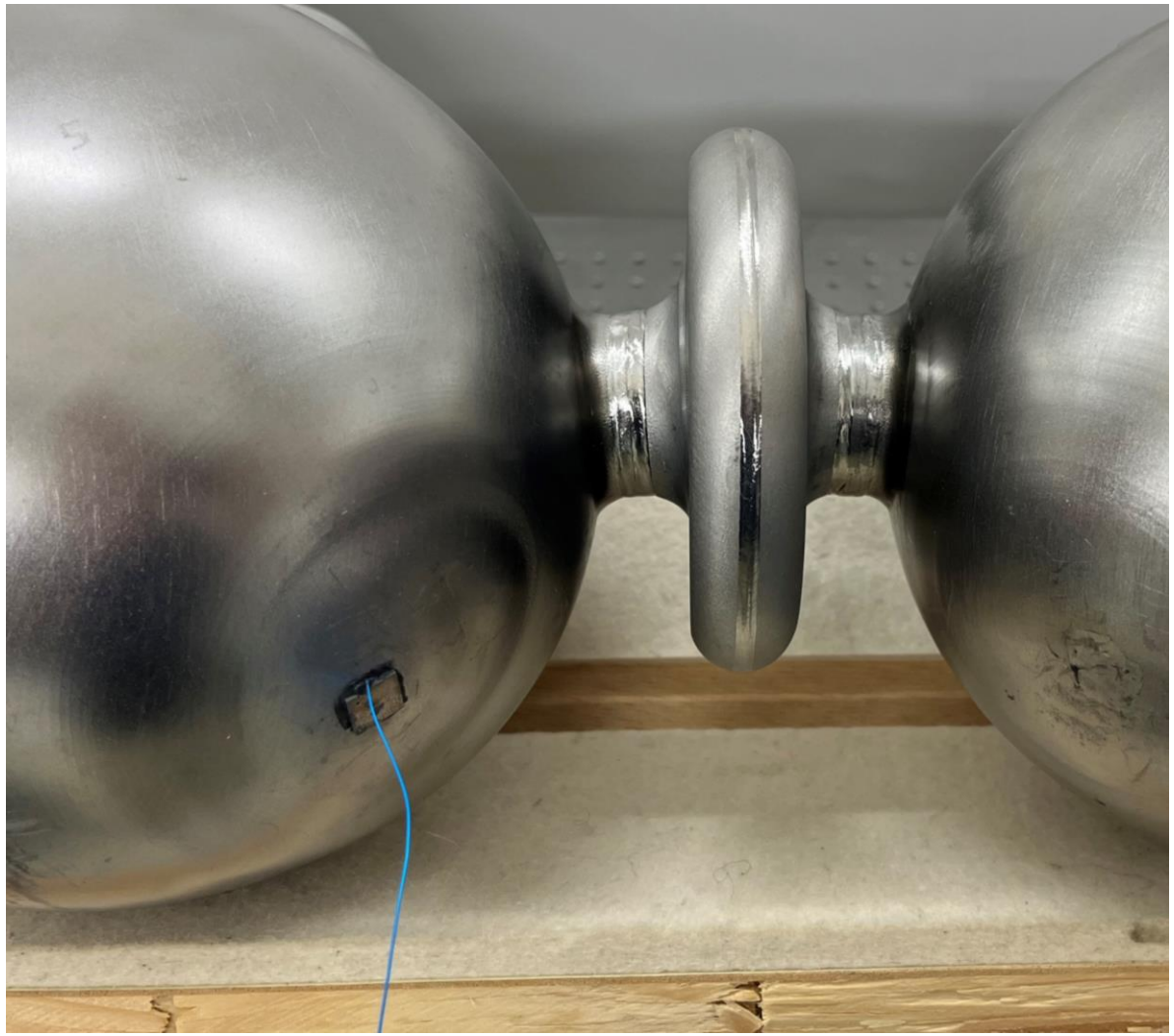
MAGO Cavity

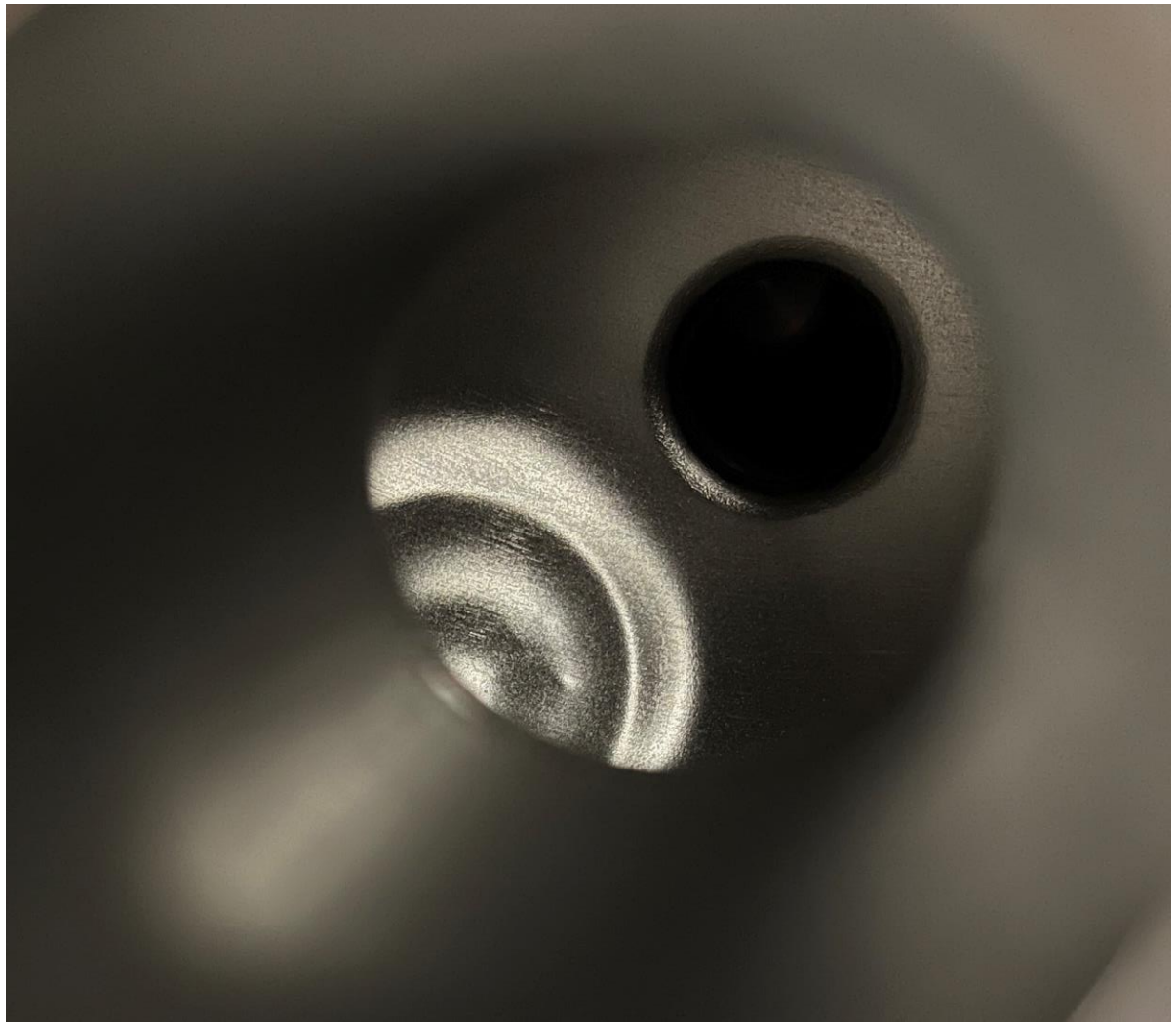
Significant bend and torsion in coupling module



MAGO Cavity

Spheroidal shape deformation





Analytic solutions for spheroidal cavities ($m = 0$)

Eigenfrequency calculation and E-field distribution

Maxwell eq. in prolate spheroidal coordinates

$x = f\sqrt{(\xi^2 - 1)(1 - \eta^2)}\cos\phi$ $y = f\sqrt{(\xi^2 - 1)(1 - \eta^2)}\sin\phi$ $z = f\xi\eta$	$2f$ distance of focal points $-1 \le \eta \le 1;\ 0 \le \xi \le \infty;$ $0 \le \phi \le 2\pi$

$$\nabla \times E = -\partial_t B \qquad \nabla \times H = \partial_t D$$

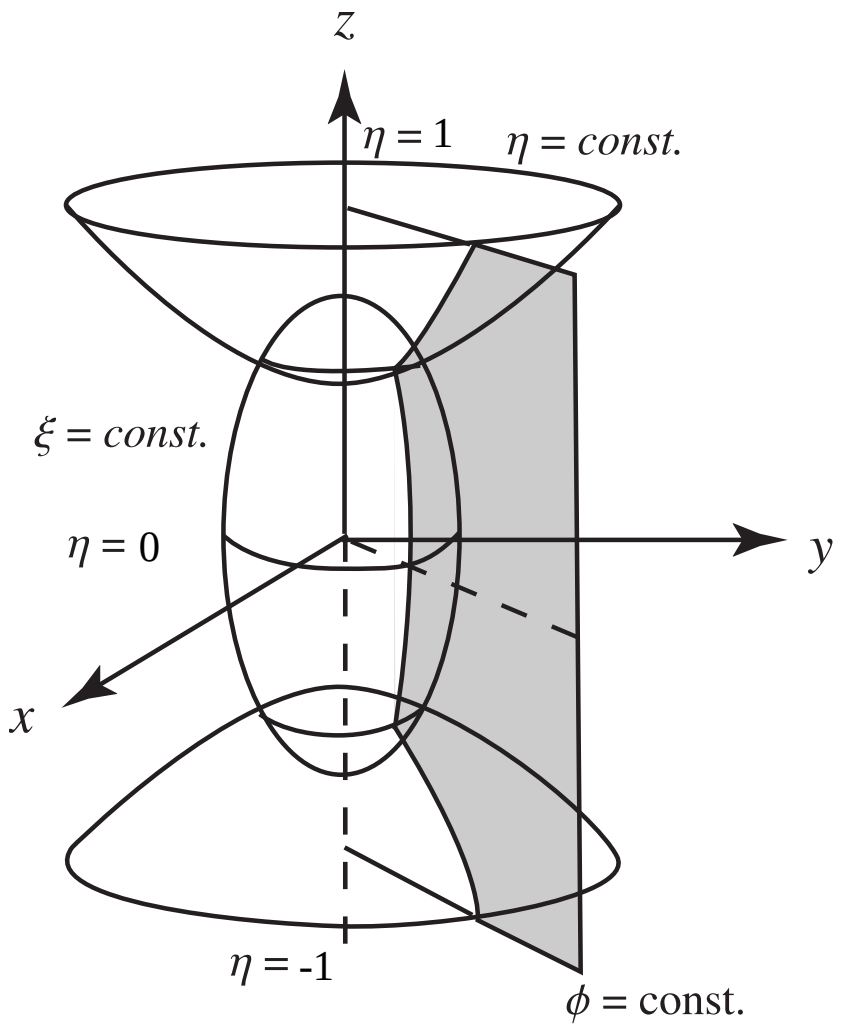
- Limited to modes with $m = 0$ (Qn for ϕ), the non-zero components for the TM modes are

$$H_\phi = B_n C_n R_{ln}(kf, \xi) S_{ln}(kf, \eta)$$

$$E_\xi = \frac{A}{i\omega\epsilon\sqrt{1 - \eta^2}} R_{ln}(kf, \xi)[B_n C_n \sqrt{1 - \eta^2}\,\partial_\eta S_{ln}(kf, \eta) + f S_{ln}(kf, \eta)]$$

$$E_\eta = \frac{A}{i\omega\epsilon} B_n C_n \partial_\xi \left[R_{ln}(kf, \xi)\sqrt{\xi^2 - 1}\right]$$

- Swap E_ϕ for H_ϕ, $-H_\xi$ for E_ξ and $-H_\eta$ for E_η to obtain the TE mode field distributions.



Analytic solutions for spheroidal cavities ($m = 0$)

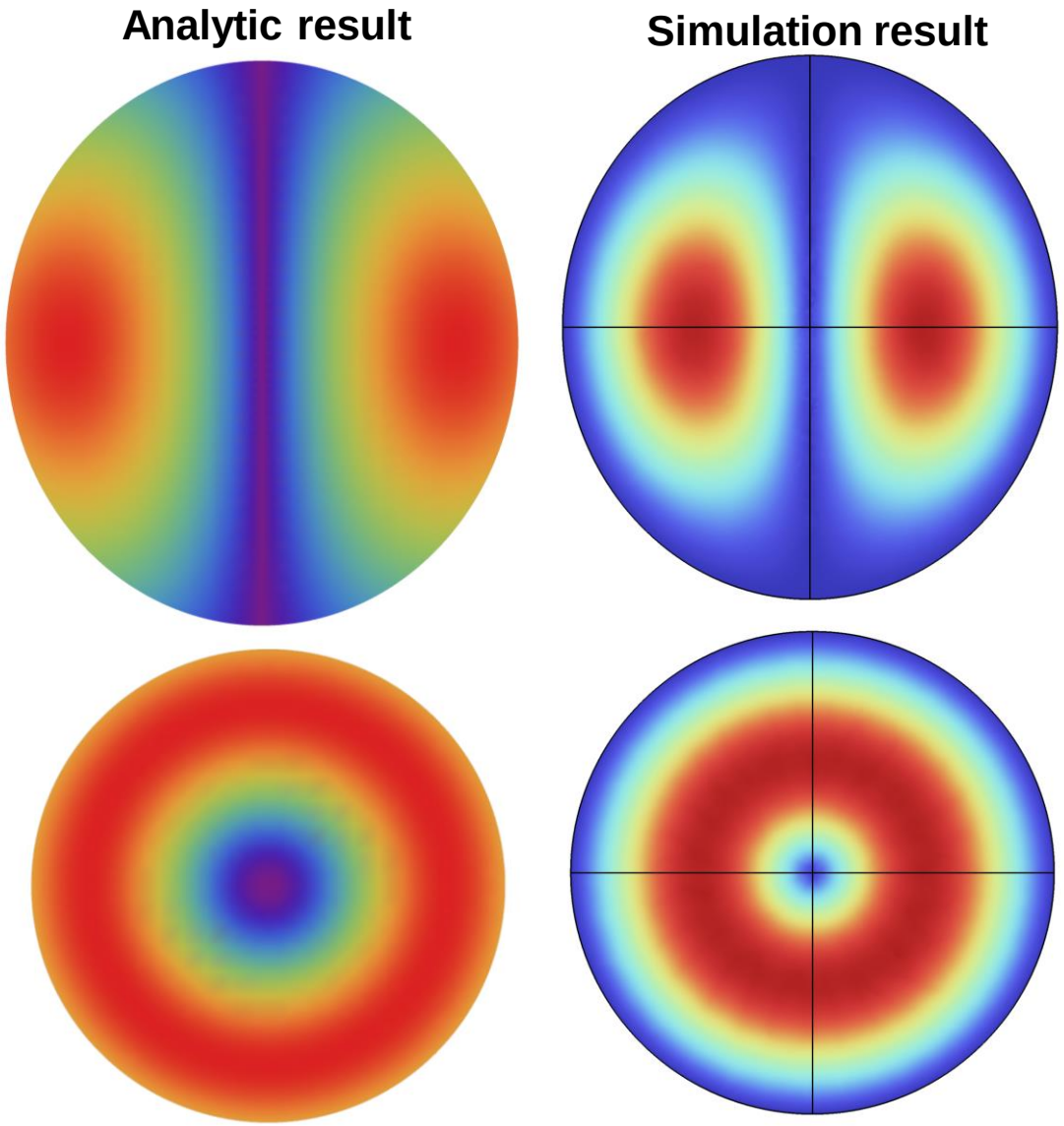
Eigenfrequency calculation and E-field distribution

Imposing boundary conditions at $\xi = \xi_0$

- For TE modes $E_\xi|_{\xi_0} = 0$, thus

$$R_{ln}(kf, \xi)\Big|_{\xi=\xi_0} = 0$$

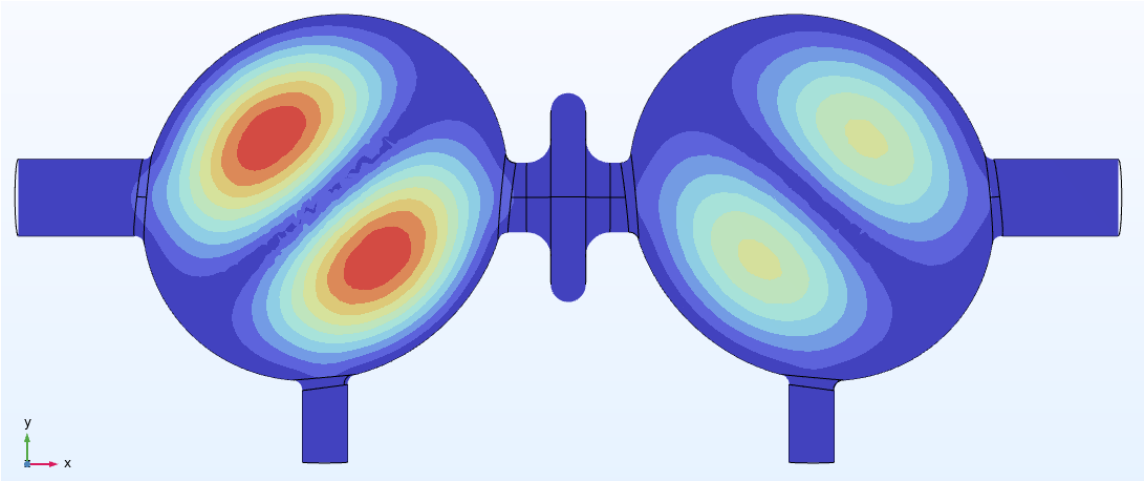
- Gives the first TE eigenvalue $k = 44.15$ and thus $\nu_{E0} = 2.1064\ GHz$
- Simulation of a spheroidal cavity in COMSOL yields $\nu_{E0} = 2.1061\ GHz$
- All calculations and simulations are done for a spheroid with $a = 0.11\ m$ and $b = 0.1\ m$
- Need to work out amplitudes for the modes



MAGO RF modes

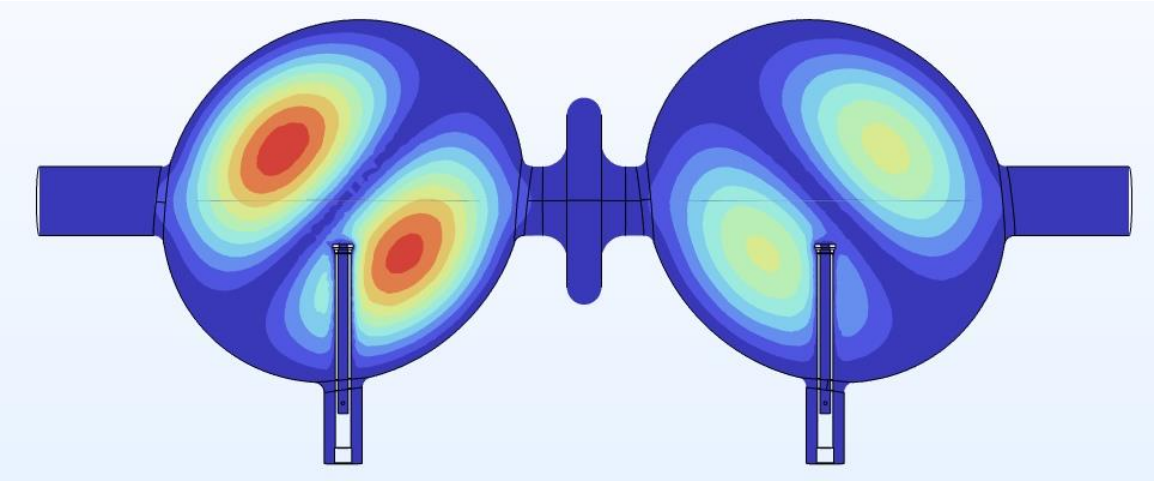
Simulation

Without antennas



- Lowest TE mode with $m = 0$ has frequency $\nu_{E0} = 2.1033\ GHz$

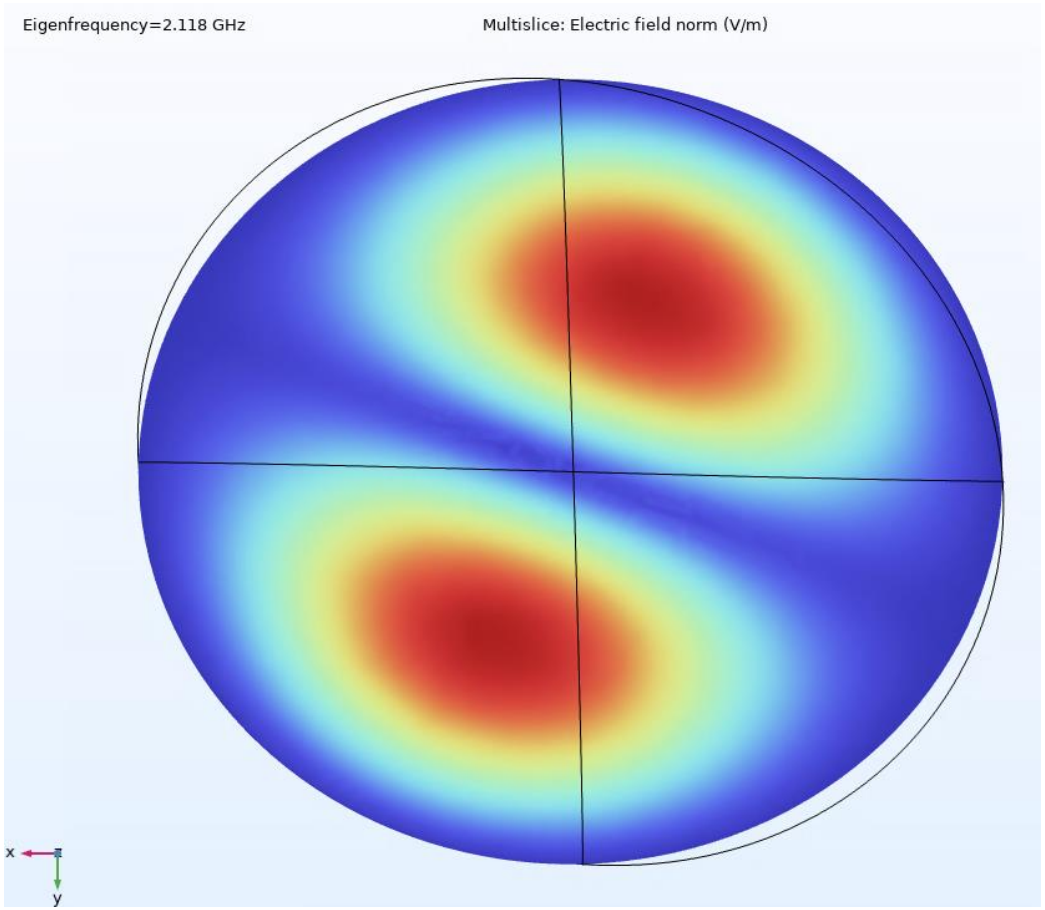
With antennas

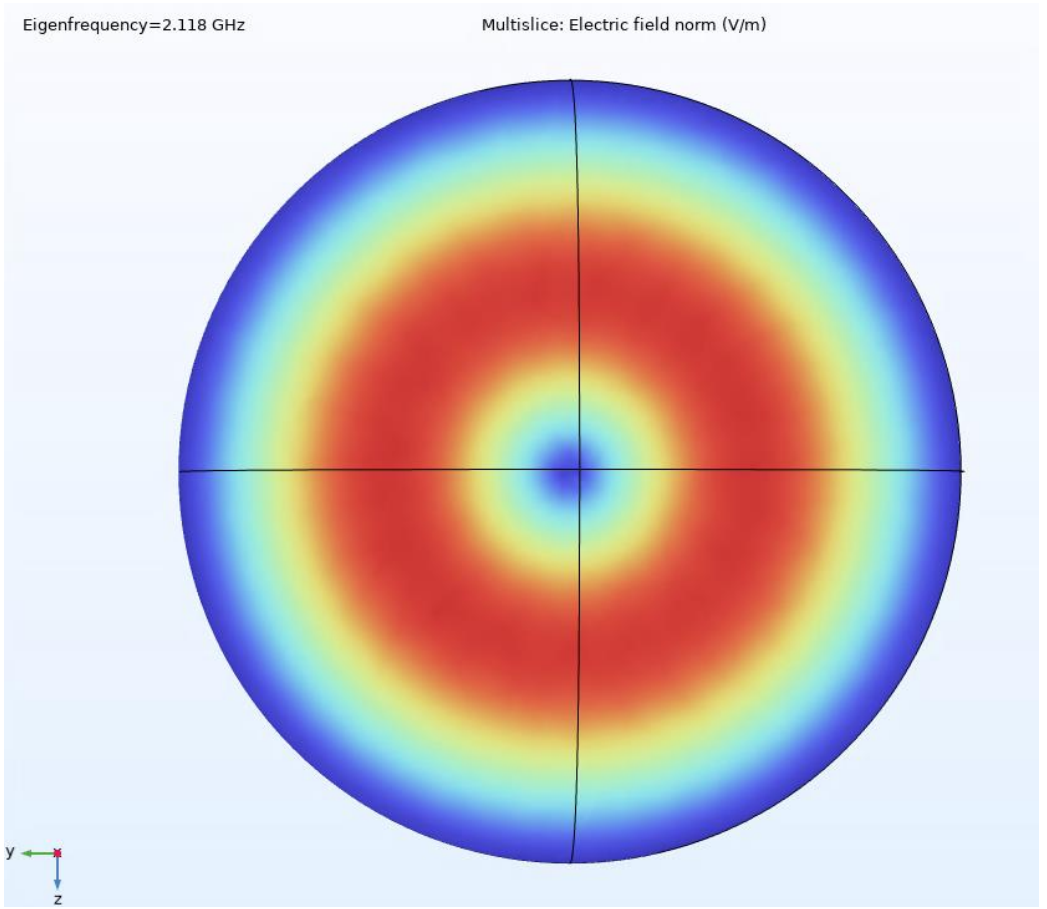


- With rod antennas inside each spheroid the frequency is $\nu_{E0} = 2.1037\ GHz$

Spheroidal Cavity Modes with Deformation

Shape perturbation via excitation of mechanical eigenmodes





For some arbitrary excitation of a mechanical mode with $f_{mech} = 4.096\ kHz$ the RF mode frequency shifts from $f_{E0} = 2.103\ GHz$ to $f'_{E0} = 2.118\ GHz$.

MAGO S-parameters

Rod antenna S-parameter measurement