LUXE ECAL-P readout/calibration optimization

Aleksander Filip Żarnecki

FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

August 2, 2023

Outline

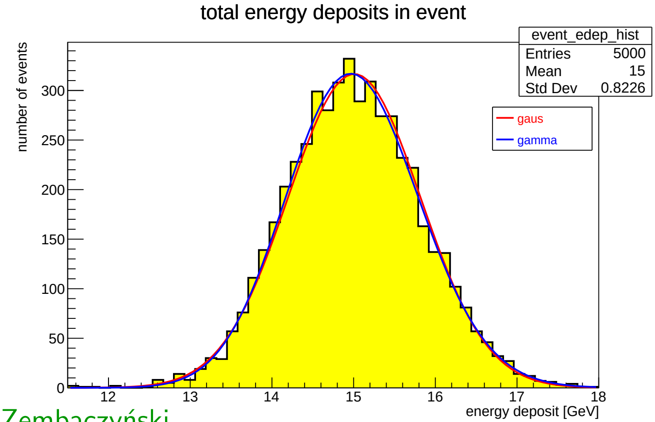
Outline

Optimization goals

Response ditribution

Distribution of the ECAL-P response to positrons with fixed energy, as obtained from the GEANT 4 simulation.

15 GeV positrons

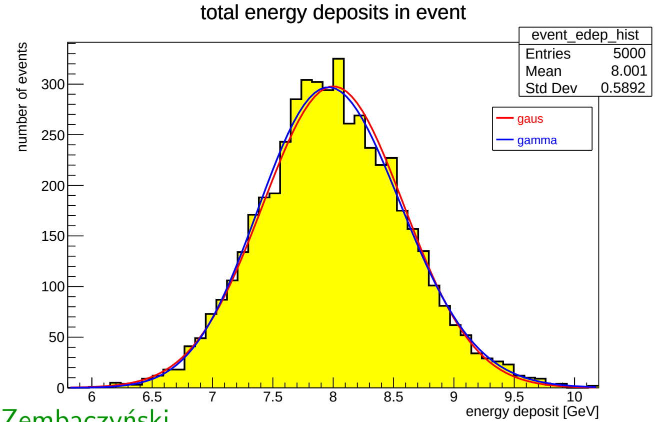


Results from Kamil Zembaczyński

Response ditribution

Distribution of the ECAL-P response to positrons with fixed energy, as obtained from the GEANT 4 simulation.

8 GeV positrons

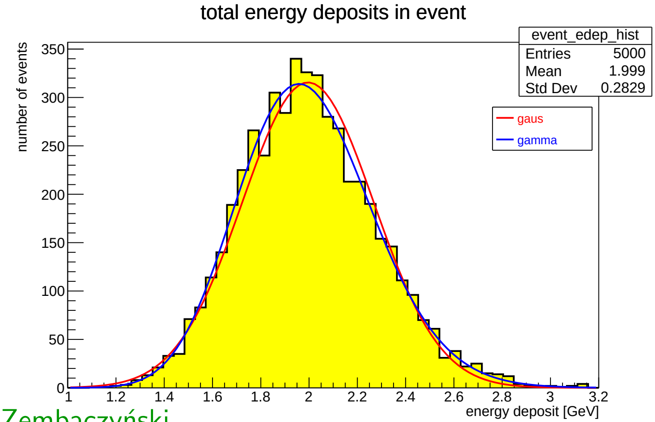


Results from Kamil Zembaczyński

Response ditribution

Distribution of the ECAL-P response to positrons with fixed energy, as obtained from the GEANT 4 simulation.

2 GeV positrons



Results from Kamil Zembaczyński

Optimization goals

Parameters

Distribution of the calorimeter response S, for given beam energy E, can be described by the Gamma distribution, which has two parameters.

Convenient choice for the optimization is:

- average response:

$$\bar{S} = \langle S \rangle$$

- response spread:

$$\sigma_S = \sqrt{\langle (S - \langle S \rangle)^2 \rangle} = \sqrt{\langle S^2 \rangle - \langle S \rangle^2}$$

Ideal calorimeter

For the perfect energy measurement we expect:

$$\bar{S} \rightarrow E \qquad \text{and} \qquad \sigma_S \rightarrow 0$$

Optimization goals

Expected resolution

For sampling calorimeter, assuming statistical fluctuations dominate:

$$\sigma_S \sim \sqrt{E} \qquad \Rightarrow \qquad \frac{\sigma_S}{\bar{S}} = \frac{a}{\sqrt{E}}$$

where a is a resolution parameter; simulation indicates $a \approx 0.2\sqrt{\text{GeV}}$ for ECAL-P.

Normalized response parameters

To compare detector response at different energies, we can use scaled quantities:

- relative calibration shift:

$$\delta_S = \frac{\bar{S} - E}{a\sqrt{E}} \qquad \to 0 \quad \text{for perfect calibration}$$

- relative resolution:

$$\delta_\sigma = \frac{\sigma_S}{\bar{S}} \cdot \frac{\sqrt{E}}{a} \qquad \to 1 \quad \text{for expected resolution}$$

Figure of merit

We can introduce two terms in the optimization procedure, for two optimization goals:

- best linearity:

$$F_S = \sum_E w_E \, \delta_S^2(E)$$

- best resolution:

$$F_\sigma = \sum_E w_E \, \delta_\sigma^2(E)$$

where weights w_E can be added to take the expected energy distribution into account.

Final optimization goal can be then defined as:

$$F = f \cdot F_S + (1 - f) \cdot F_\sigma$$

where f defines the relative weight of linearity vs resolution in the minimization procedure

Outline

Signal reconstruction

We assume that the calorimeter response is calculated as a weighted sum of signals from individual calorimeter layers:

$$S = \sum_i c_i \cdot s_i$$

where $i = 1 \ldots N = 20$ numbers the calorimeter layers.

For the averaged signal values we can write: $i, j = 1 \ldots N$

$$\langle S \rangle = \sum_i c_i \cdot \langle s_i \rangle$$

$$\langle S^2 \rangle = \sum_{i,j} c_i \cdot c_j \cdot \langle s_i s_j \rangle$$

Linearity

Relative response shift:

$$\delta_S(E) = \frac{\bar{S} - E}{a\sqrt{E}} = \frac{1}{a\sqrt{E}} \left[\left(\sum_i c_i \cdot \langle s_i^E \rangle \right) - E \right]$$

Linearity

Relative response shift:

$$\delta_S(E) = \frac{\bar{S} - E}{a\sqrt{E}} = \frac{1}{a\sqrt{E}} \left[\left(\sum_i c_i \cdot \langle s_i^E \rangle \right) - E \right]$$

Figure of merit for linearity:

$$F_S = \sum_E w_E \, \delta_S^2(E) = \sum_E \frac{w_E}{a^2 E} \left[\left(\sum_i c_i \cdot \langle s_i^E \rangle \right) - E \right] \cdot \left[\left(\sum_j c_j \cdot \langle s_j^E \rangle \right) - E \right]$$

Linearity

Relative response shift:

$$\delta_S(E) = \frac{\bar{S} - E}{a\sqrt{E}} = \frac{1}{a\sqrt{E}} \left[\left(\sum_i c_i \cdot \langle s_i^E \rangle \right) - E \right]$$

Figure of merit for linearity:

$$F_S = \sum_E w_E \, \delta_S^2(E) = \sum_E \frac{w_E}{a^2 E} \left[\left(\sum_i c_i \cdot \langle s_i^E \rangle \right) - E \right] \cdot \left[\left(\sum_j c_j \cdot \langle s_j^E \rangle \right) - E \right]$$

Minimum is found by calculating derivatives: $i, k = 1 \ldots N$

$$\frac{\partial F_S}{\partial c_k} = \sum_E 2 w_E \, \delta_S(E) \, \frac{\partial \delta_S(E)}{\partial c_k} = \sum_i c_i \left(\sum_E \frac{2\, w_E}{a^2 E} \langle s_i^E \rangle \langle s_k^E \rangle \right) - \sum_E \frac{2\, w_E}{a^2} \langle s_k^E \rangle$$

$\Rightarrow$ set of linear equations for extracting values of c_i...

Resolution

Expected resolution:

$$
\begin{aligned}
\sigma_S^2 &= \langle S^2 \rangle - \langle S \rangle^2 \\
&= \sum_{i,j} c_i \; c_j \; \langle s_i s_j \rangle - \left(\sum_i c_i \langle s_i \rangle \right) \cdot \left(\sum_j c_j \langle s_j \rangle \right) \\
&= \sum_{i,j} c_i \; c_j \; \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)
\end{aligned}
$$

Resolution

Expected resolution:

$$
\begin{aligned}
\sigma_S^2 &= \langle S^2 \rangle - \langle S \rangle^2 \\
&= \sum_{i,j} c_i \; c_j \; \langle s_i s_j \rangle - \left(\sum_i c_i \langle s_i \rangle \right) \cdot \left(\sum_j c_j \langle s_j \rangle \right) \\
&= \sum_{i,j} c_i \; c_j \; \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)
\end{aligned}
$$

Relative resolution modified to obtain a linear problem:

$$
\delta_\sigma^2 = \frac{\sigma_S^2}{\bar{S}^2} \cdot \frac{E}{a^2} \approx \frac{\sigma_S^2}{a^2 E} \quad \text{for proper calibration: } \bar{S} \to E
$$

Resolution

Figure of merit for resolution:

$$F_\sigma = \sum_E w_E \, \delta_\sigma^2(E) = \sum_E \frac{w_E}{a^2 E} \sum_{i,j} c_i \, c_j \left(\langle s_i^E s_j^E \rangle - \langle s_i^E \rangle \langle s_j^E \rangle \right)$$

Analytic optimization

Resolution

Figure of merit for resolution:

$$F_\sigma = \sum_E w_E \, \delta^2_\sigma(E) = \sum_E \frac{w_E}{a^2 E} \sum_{i,j} c_i \, c_j \left(\langle s_i^E s_j^E \rangle - \langle s_i^E \rangle \langle s_j^E \rangle \right)$$

Partial derivatives:

$$\frac{\partial F_\sigma}{\partial c_k} = \sum_E 2 w_E \, \delta_\sigma(E) \, \frac{\partial \delta_\sigma(E)}{\partial c_k} = \sum_i c_i \left[\sum_E \frac{2 \, w_E}{a^2 E} \left(\langle s_i^E s_k^E \rangle - \langle s_i^E \rangle \langle s_k^E \rangle \right) \right]$$

Resolution

Figure of merit for resolution:

$$F_\sigma = \sum_E w_E \, \delta_\sigma^2(E) = \sum_E \frac{w_E}{a^2 E} \sum_{i,j} c_i \, c_j \left(\langle s_i^E s_j^E \rangle - \langle s_i^E \rangle \langle s_j^E \rangle \right)$$

Partial derivatives:

$$\frac{\partial F_\sigma}{\partial c_k} = \sum_E 2 w_E \, \delta_\sigma(E) \, \frac{\partial \delta_\sigma(E)}{\partial c_k} = \sum_i c_i \left[\sum_E \frac{2 \, w_E}{a^2 E} \left(\langle s_i^E s_k^E \rangle - \langle s_i^E \rangle \langle s_k^E \rangle \right) \right]$$

Minimum condition for the total FoM:

$$\frac{\partial F}{\partial c_k} = f \cdot \frac{\partial F_S}{\partial c_k} + (1-f) \cdot \frac{\partial F_\sigma}{\partial c_k} = 0$$

f – relative weight of linearity FoM

$$\Rightarrow \sum_i c_i \sum_E \frac{2 \, w_E}{a^2 E} \left[(1-f) \langle s_i^E s_k^E \rangle + (2f-1) \langle s_i^E \rangle \langle s_k^E \rangle \right] = \sum_E \frac{2 \, w_E}{a^2} \, f \, \langle s_k^E \rangle$$

Analytic optimization

Implementation

Calibration factors for all layers, c_i, can be found by solving a set of linear equations.
One can write it in a symbolic form:

$$\mathbb{A} \cdot \vec{c} \;=\; \vec{B}$$

where matrix $\mathbb{A}$ and vector $\vec{B}$ can be calculated from single layer averages, $\langle s_i^E \rangle$ and $\langle s_i^E s_j^E \rangle$.

These averages can be calculated only once (from MC event samples)
and then use to test different optimization strategies $\Rightarrow$ extremely fast!

Implementation

Calibration factors for all layers, c_i, can be found by solving a set of linear equations.
One can write it in a symbolic form:

$$\mathbb{A} \cdot \vec{c} \quad = \quad \vec{B}$$

where matrix $\mathbb{A}$ and vector $\vec{B}$ can be calculated from single layer averages, $\langle s_i^E \rangle$ and $\langle s_i^E s_j^E \rangle$.

These averages can be calculated only once (from MC event samples)
and then use to test different optimization strategies $\Rightarrow$ extremely fast!

To avoid systematic bias (towards lower c_i values), resulting from the modified δ_σ^2 definition, additional constraint is added: implemented using Lagrange multiplier

$$\sum_E \left(\sum_i c_i \langle s_i^E \rangle \; - \; E \right) \quad = \quad 0$$

Outline

Test setup

Results presented are based on the MC samples produced in Nov 2021, available at
`/nfs/dust/luxe/group/MCProduction/SinglePositron/elaser_positron`

- files: `mc21.singlePositron_*.G4gun.SIM.se0002.root`
- e-laser configuration, positron gun at $z = -7.4$ m
- positron energy range: from 2.0 GeV to 15.0 GeV, with 0.5 GeV step
- 5000 events per file

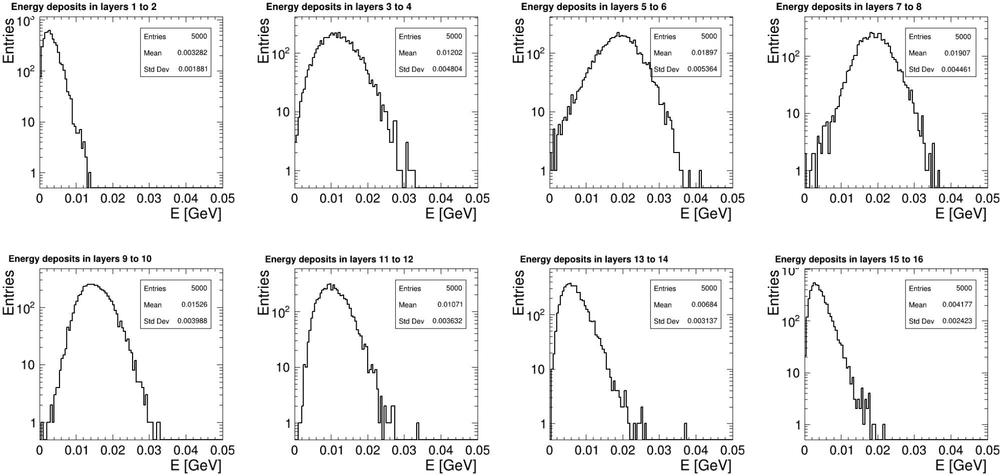
For better stability: consider calibration of layer pairs

⇒ Calorimeter with 10 double layers (deposits summed in layer pairs)

⇒ 10 average values $\langle s_i^E \rangle$ and 100 average products $\langle s_i^E s_j^E \rangle$ calculated for each energy

Performance tests

Monte Carlo sample analysis

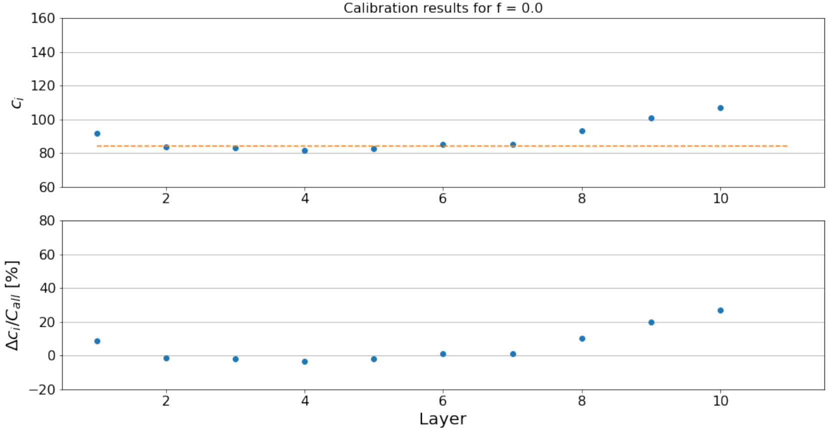


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 8 GeV.

$f = 0$

resolution optimization only

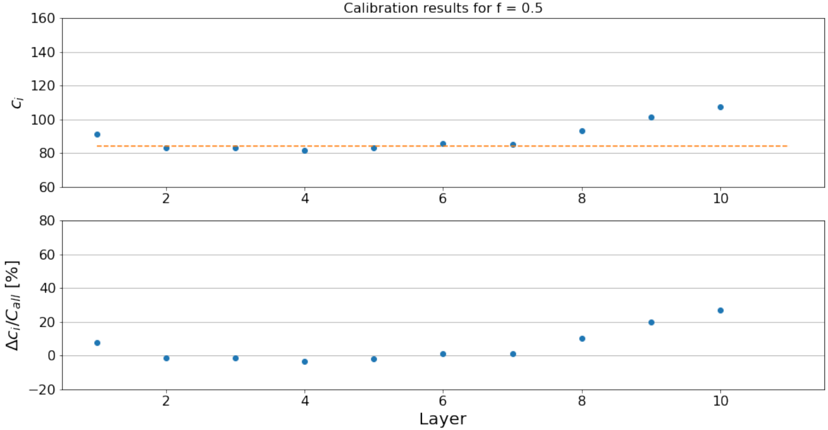


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.5$

balanced optimization

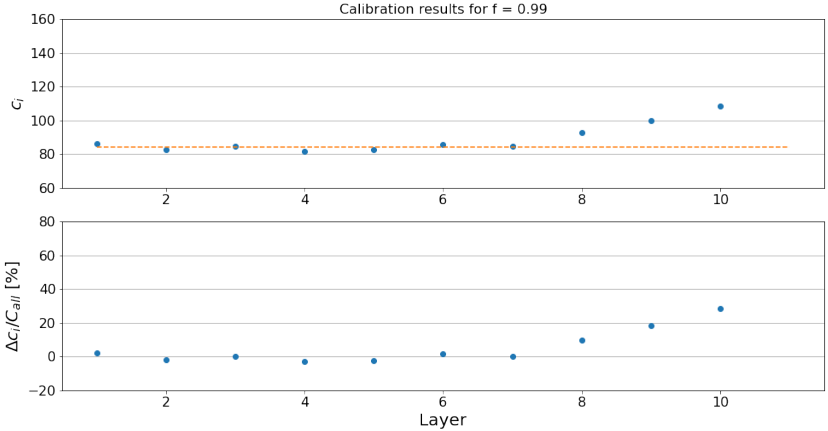


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.99$

priority on linearity

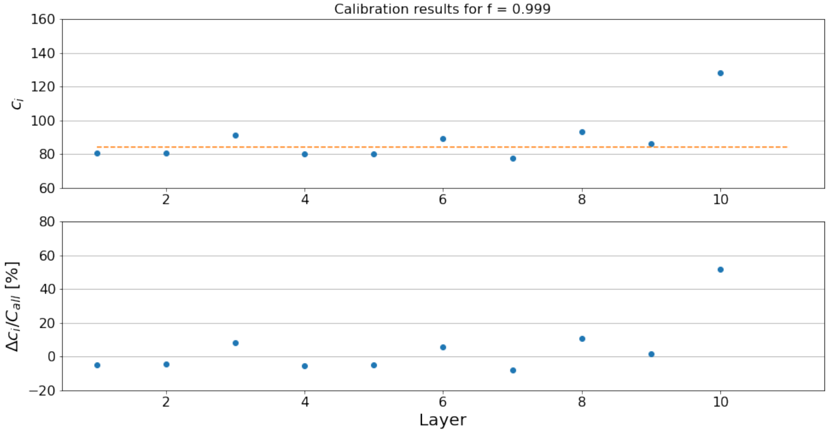


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.999$

unstable for $f \to 1$



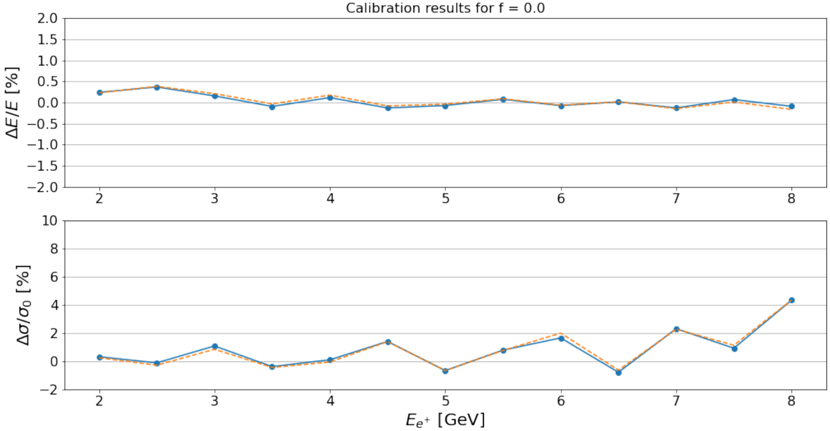
Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 8 GeV.

$f = 0$

little improvement w.r.t. uniform calibration (dashed line)

deviation from $\frac{20\%}{\sqrt{E}}$ $\Rightarrow$

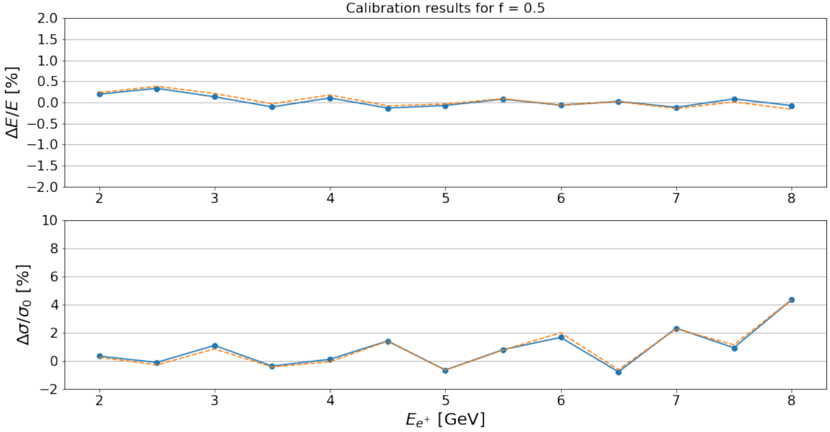


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.5$

stable resolution

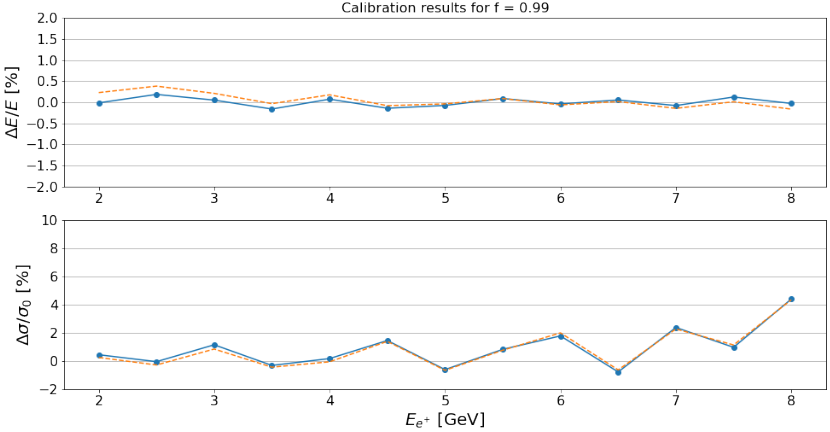


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.99$

stable resolution
improved linearity

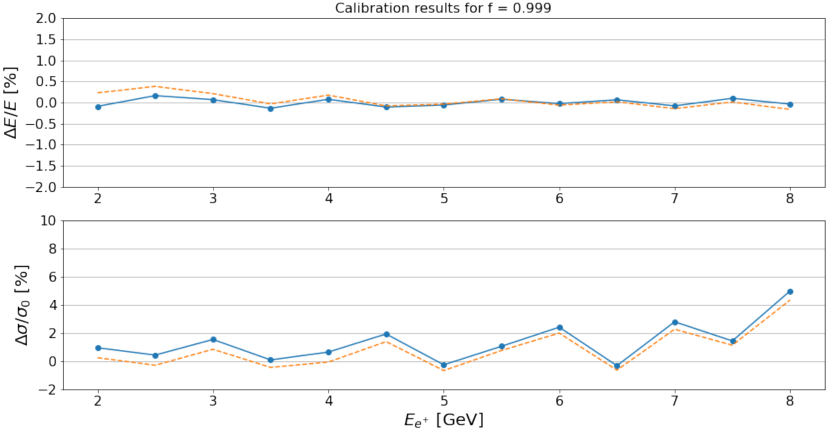


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 8 GeV.

$f = 0.999$

resolution gets slightly worse

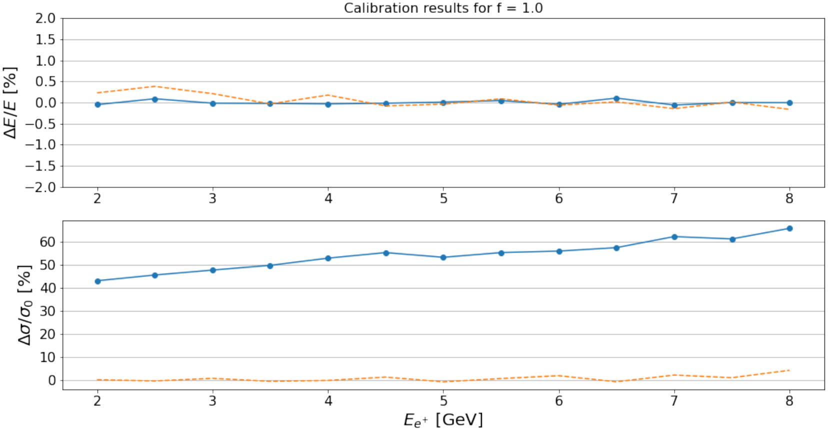


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 8 GeV.

$f = 1.0$

best linearity but very poor resolution

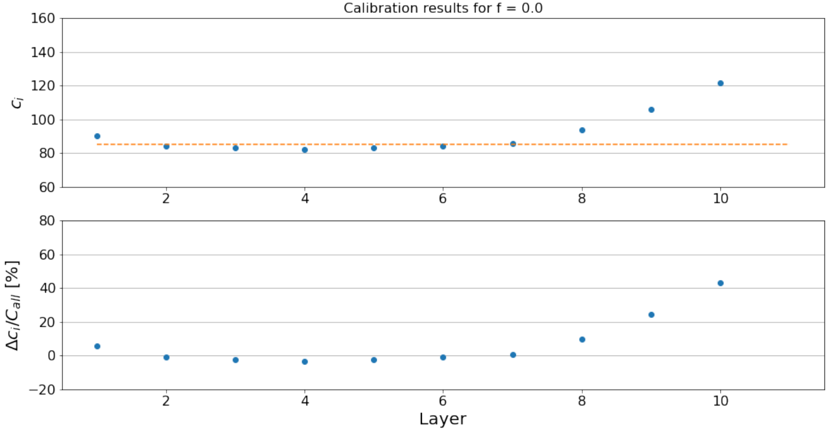


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 15 GeV.

$f = 0$

resolution optimization only

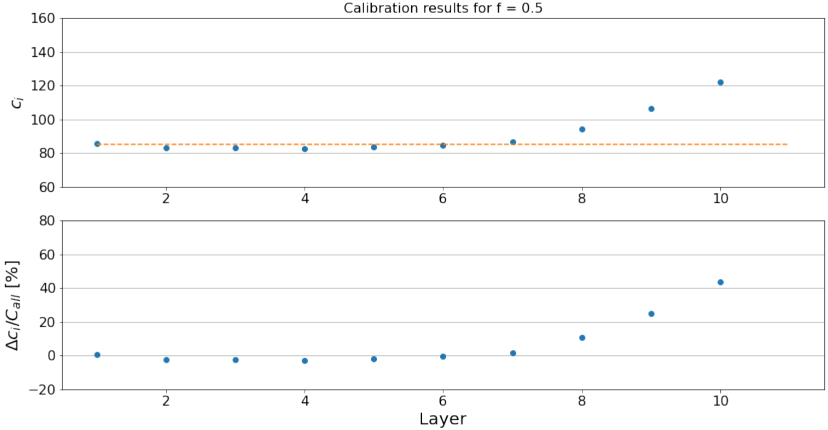


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.5$

balanced optimization

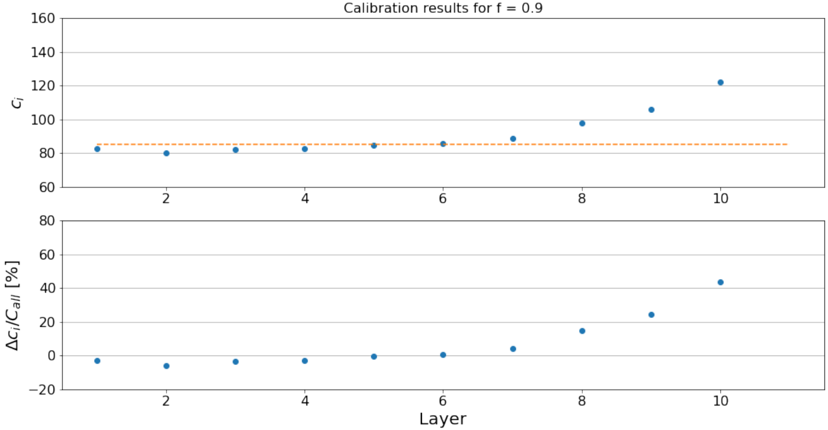


Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.9$

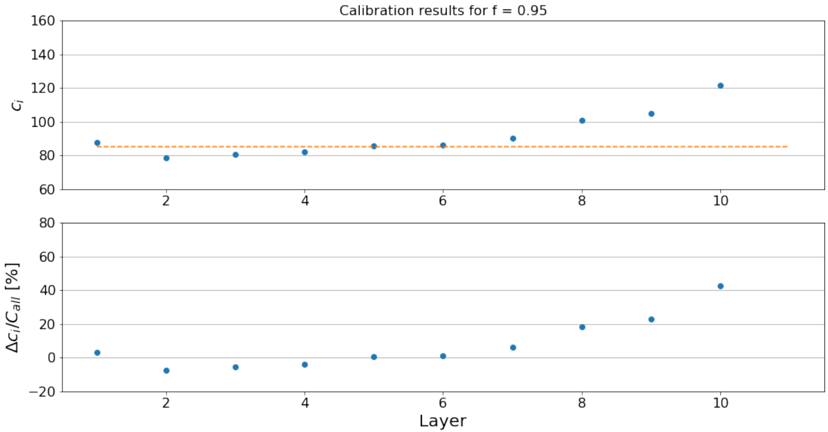
more focus on linearity



Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 15 GeV.

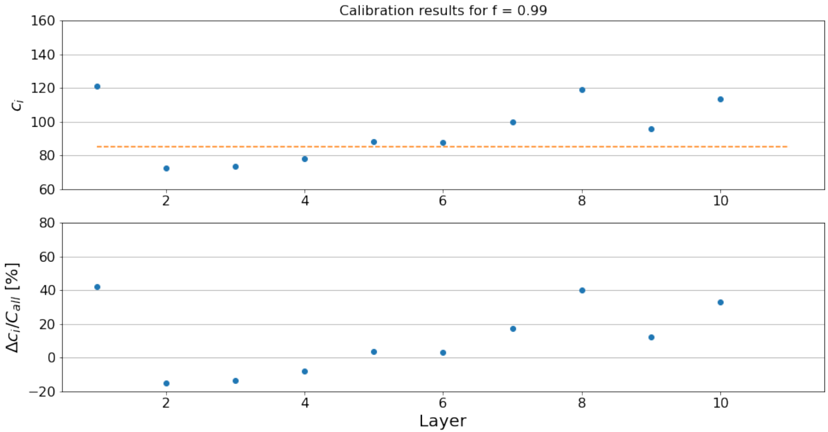
$f = 0.95$
priority on linearity



Impact of relative weight f

Calibration factors based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.99$
starts to be unstable



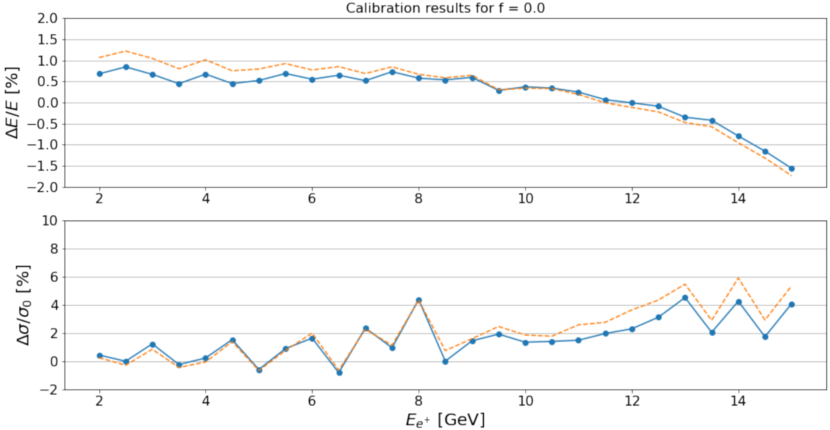
Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 15 GeV.

$f = 0$

visible improvement w.r.t. uniform calibration (dashed line)

deviation from $\frac{20\%}{\sqrt{E}}$ $\Rightarrow$

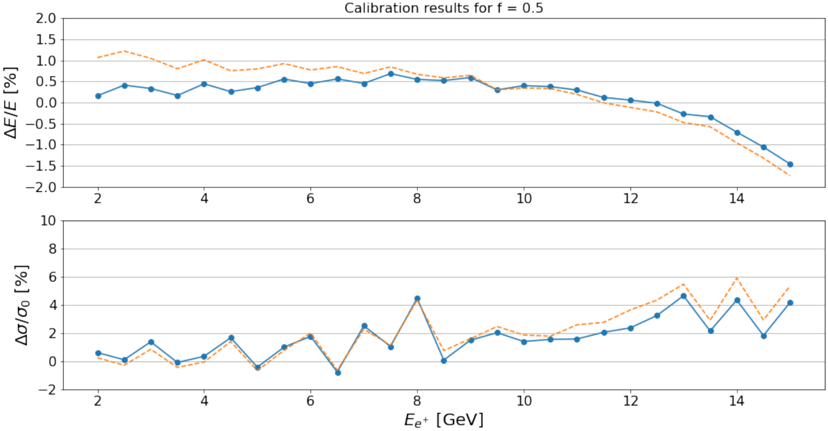


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.5$

stable resolution
improved linearity

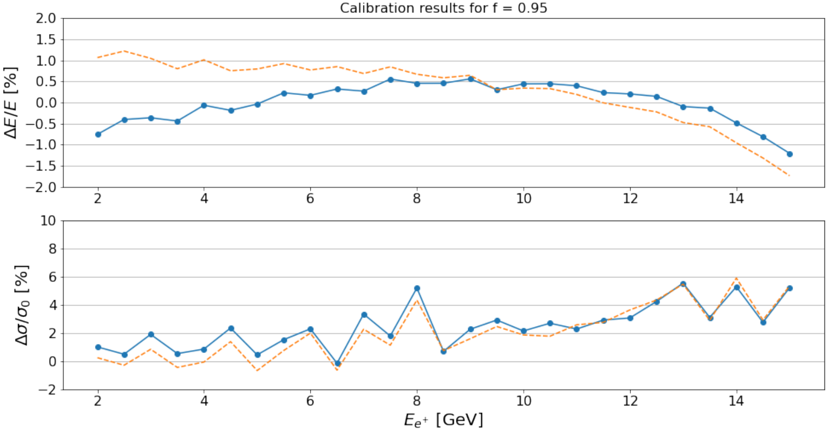


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.95$

overcorrected linearity?

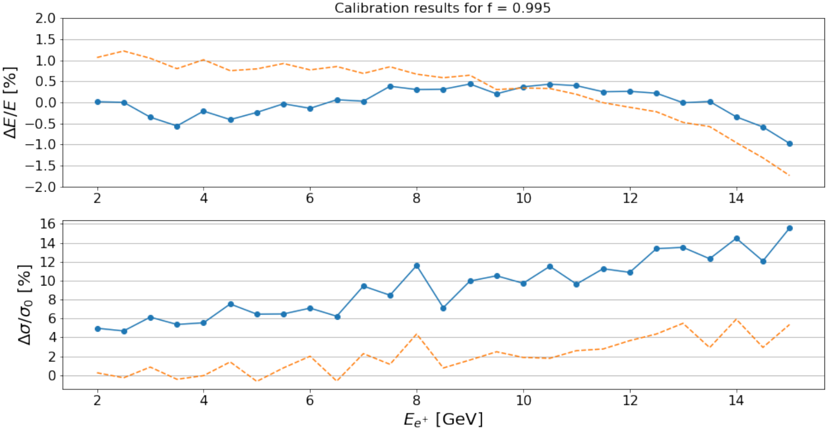


Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.995$

resolution gets worse



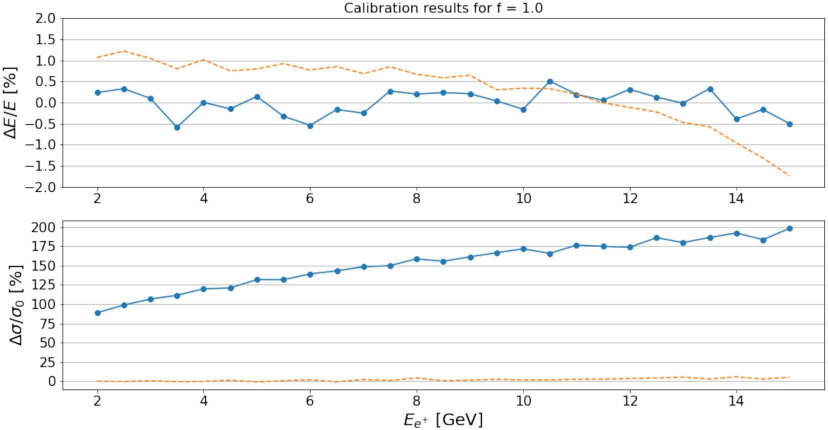
Impact of relative weight f

Expected performance based on simulation results for positron energy from 2 to 15 GeV.

$f = 1.0$

best linearity but very poor resolution

calibration very sensitive to MC fluctuations, some calibration factors negative



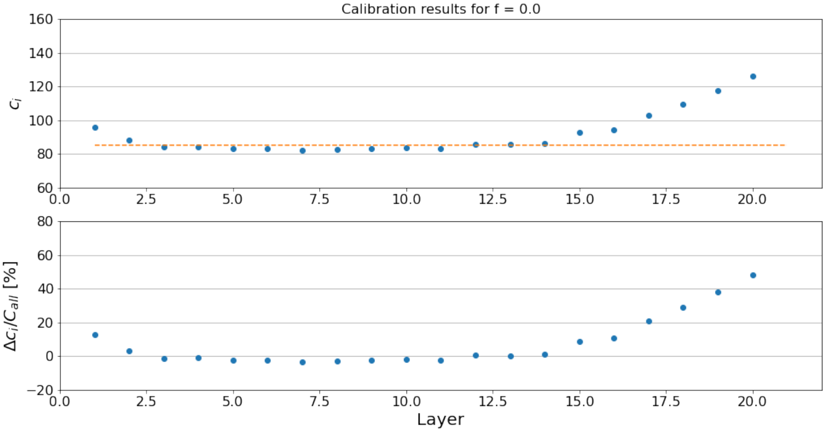
Outline

Calibration factors calorimeter with $20 \times 1\ X_0$ layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0$

resolution optimization

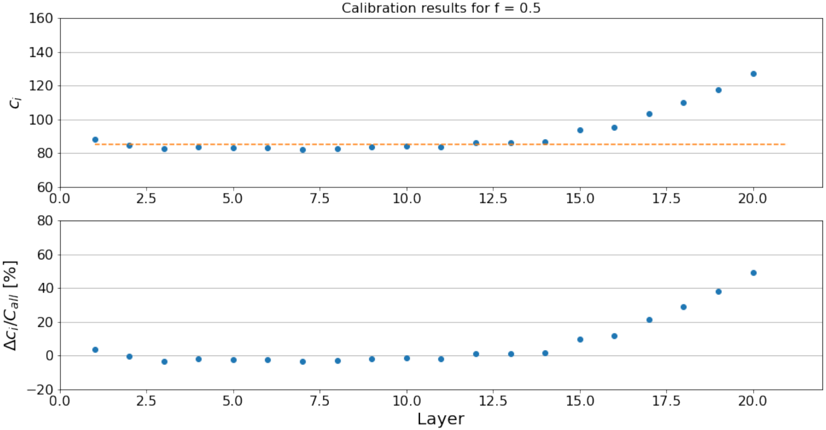


Calibration factors calorimeter with $20 \times 1\ X_0$ layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.5$

balanced optimization



Optimization tests

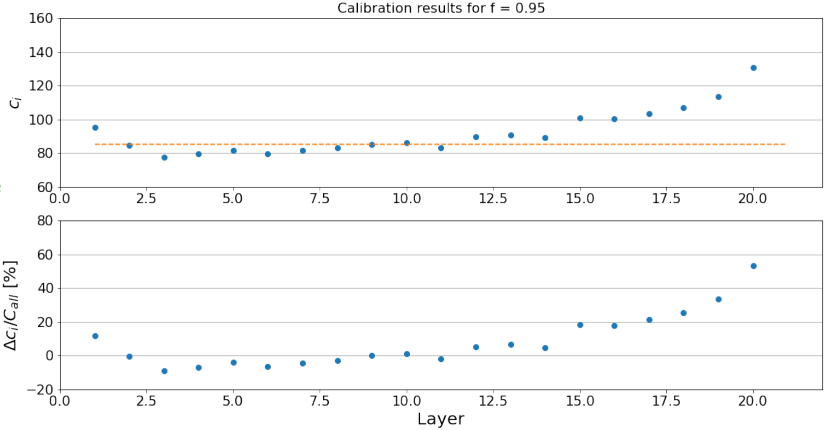
Calibration factors calorimeter with $20 \times 1\ X_0$ layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.95$

more focus on linearity

slight calibration slope visible

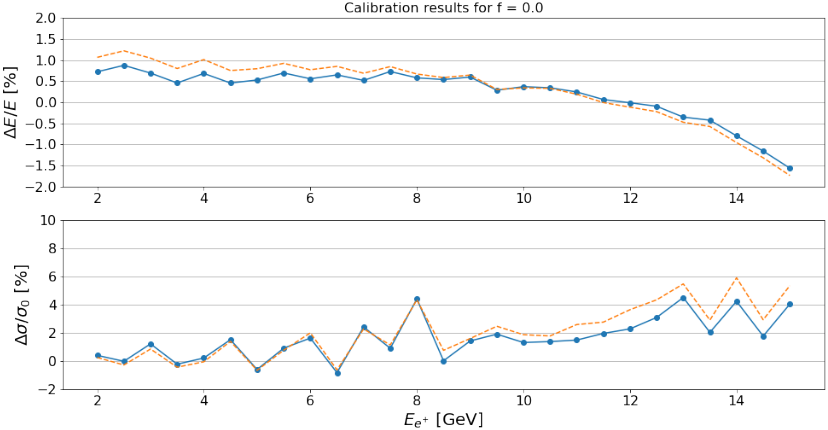


Expected performance calorimeter with $20 \times 1\ X_0$ layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0$

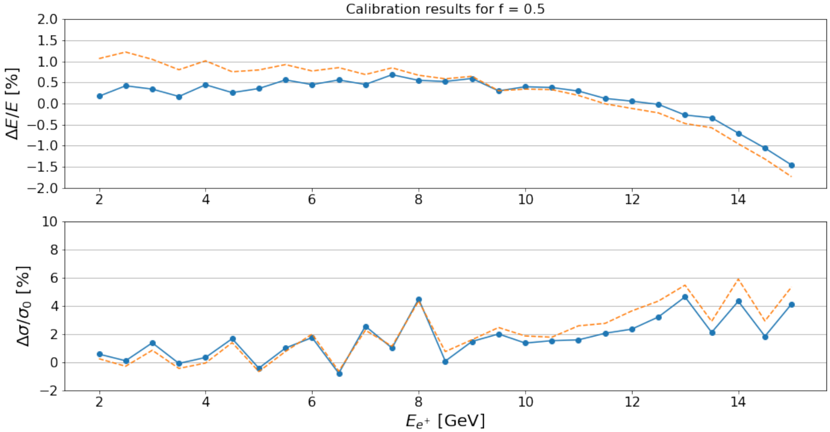
resolution optimization



Expected performance calorimeter with $20 \times 1\ X_0$ layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.5$
balanced optimization



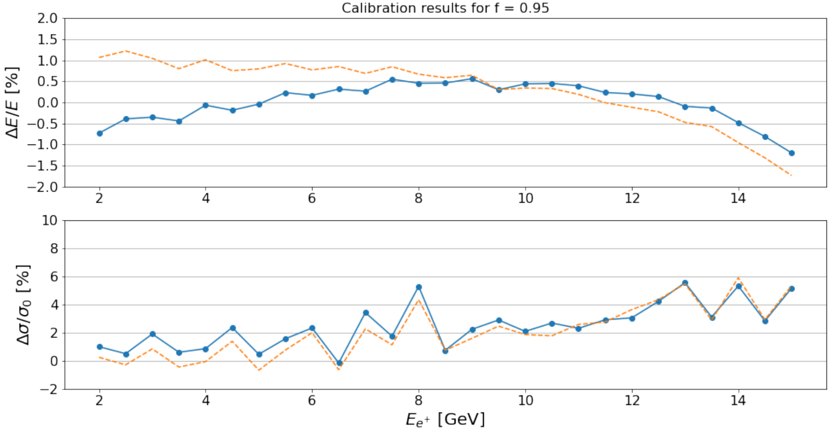
Optimization tests

Expected performance calorimeter with 20 $\times$ 1 X_0 layers

Based on simulation results for positron energy from 2 to 15 GeV.

$f = 0.95$

more focus on linearity



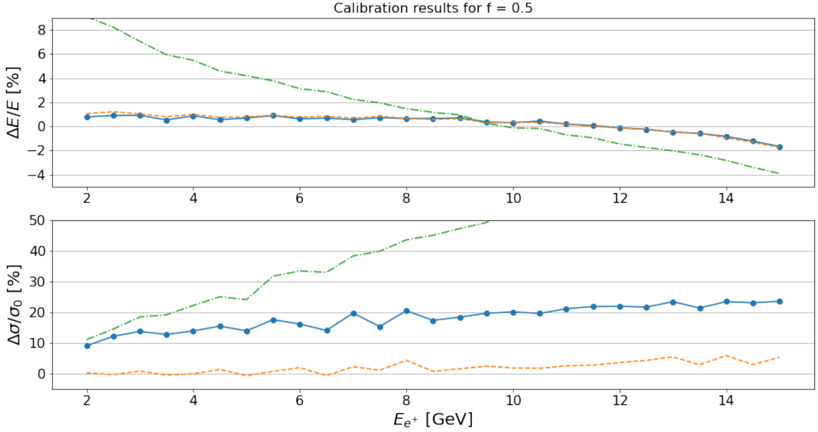
Expected performance calorimeter with 15 sensor layers

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

$10 \times 1\ X_0 + 5 \times 2\ X_0$

orange: 20 layers
green: uniform calibration

linearity preserved
resolution 10-20% worse



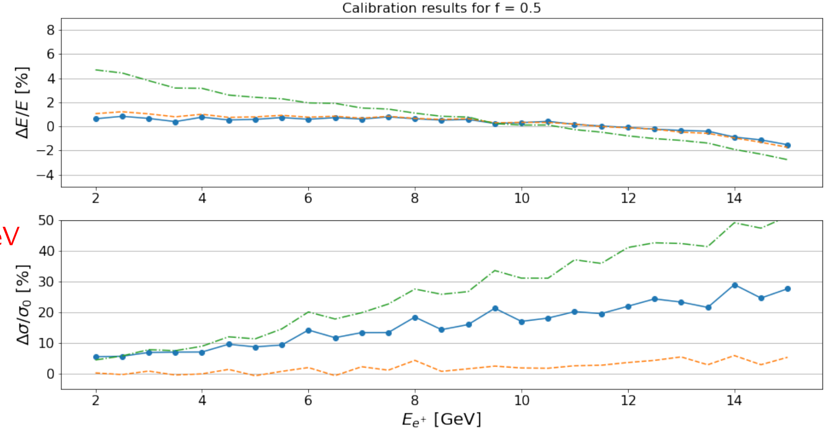
Expected performance calorimeter with 15 sensor layers

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

$15 \times 1\ X_0$

slightly better linearity

better resolution below 12 GeV



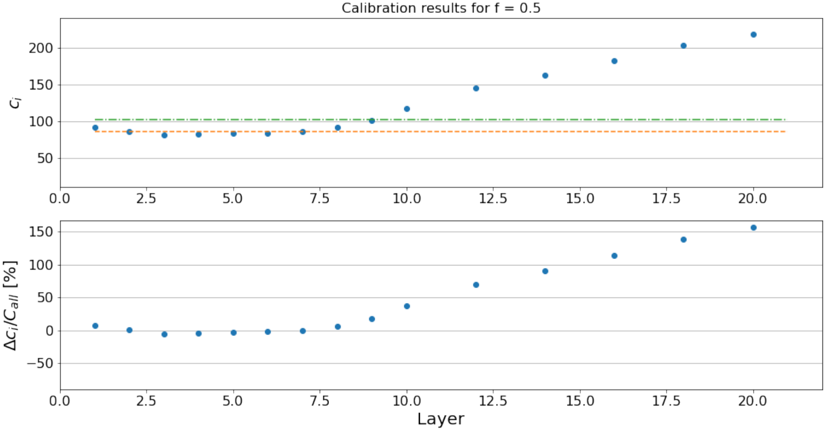
Optimization tests

Calibration factors calorimeter with 15 sensor layers

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

$10 \times 1\ X_0 + 5 \times 2\ X_0$

clearly more complicated than expected

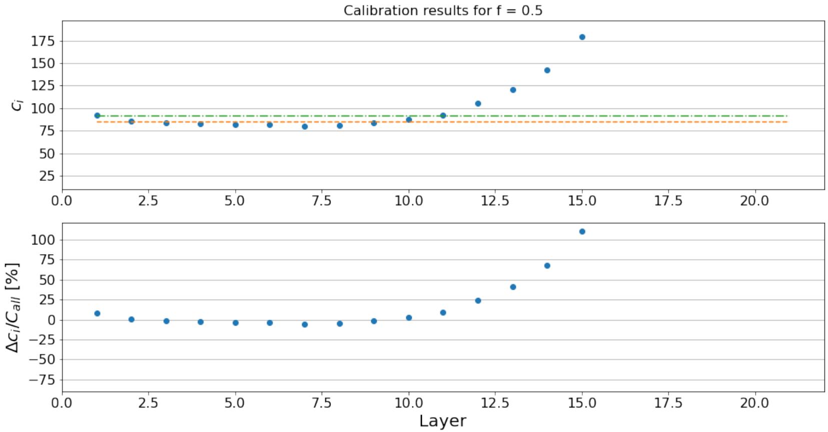


Calibration factors calorimeter with 15 sensor layers

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

$15 \times 1\ X_0$

effective leakage compensation visible

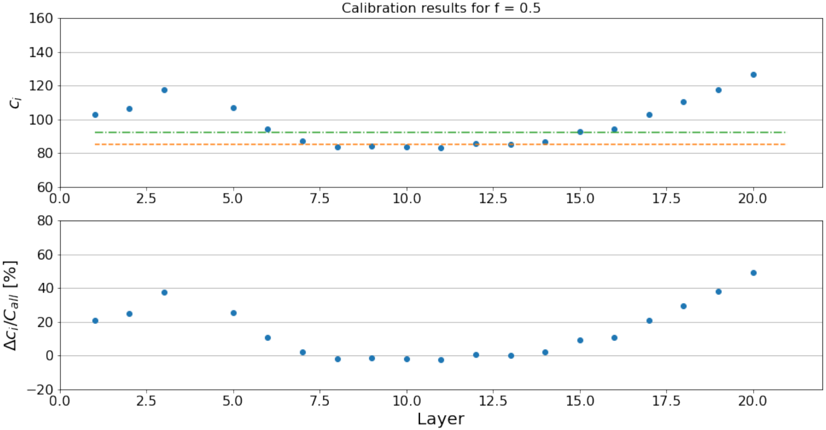


Calibration factors calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 4 damaged

effective loss compensation



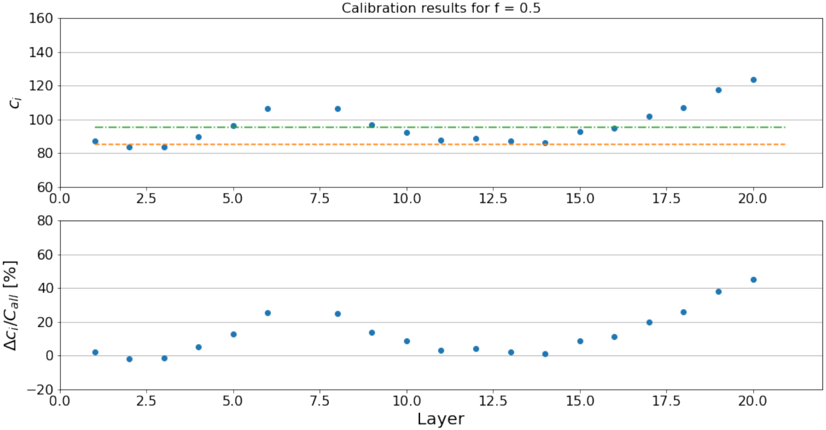
Optimization tests

Calibration factors calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 7 damaged

effective loss compensation

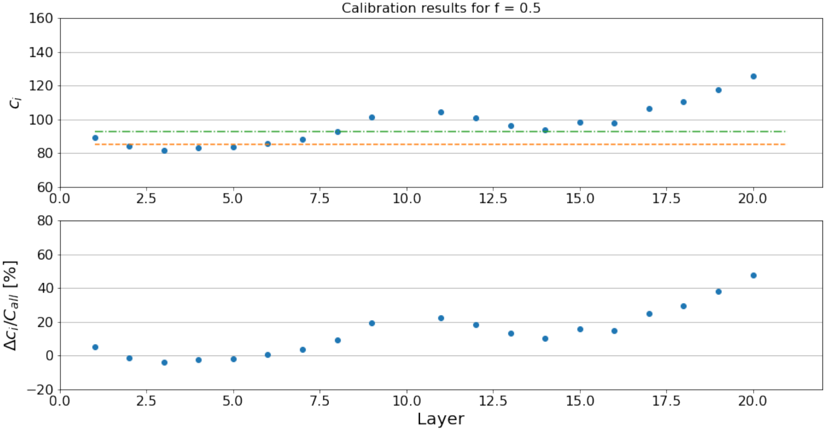


Calibration factors calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 10 damaged

effective loss compensation



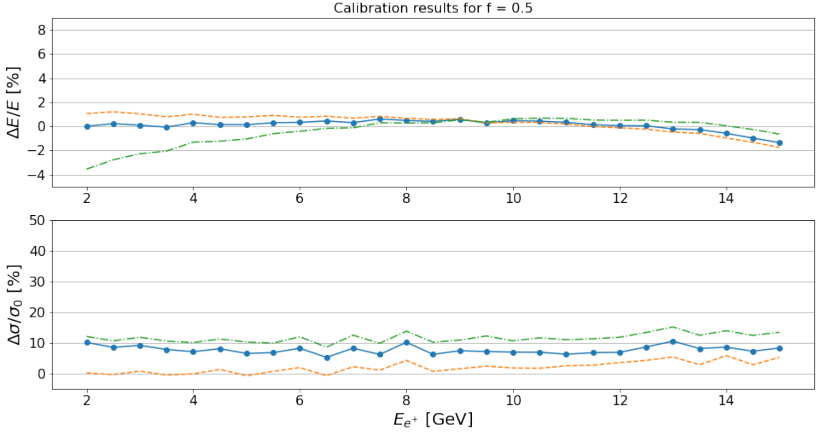
Expected performance calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 4 damaged

linearity preserved

resolution ~10% worse



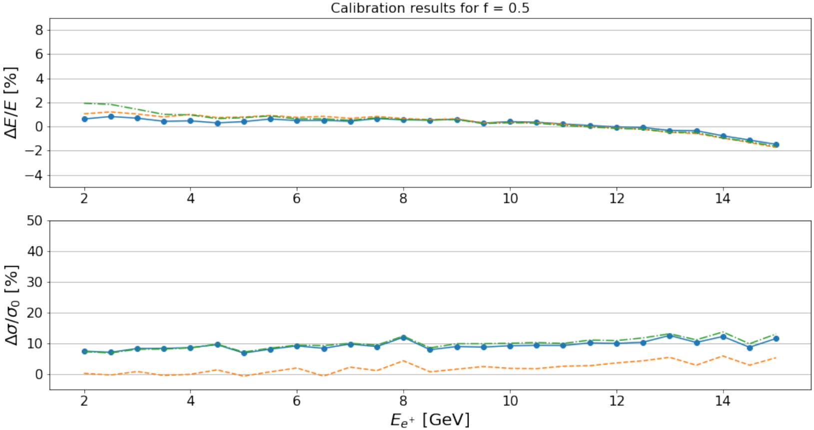
Expected performance calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 7 damaged

linearity preserved

resolution ∼10% worse



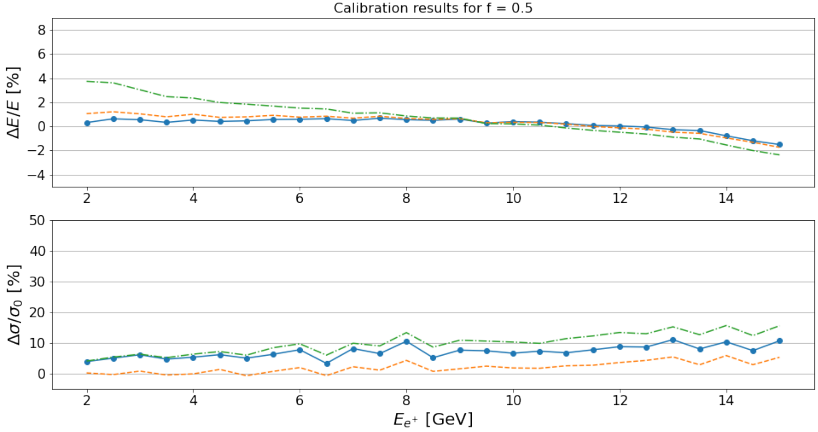
Expected performance calorimeter with 20 layers, readout for one layer broken

Based on simulation results for positron energy from 2 to 15 GeV, $f = 0.5$.

layer 10 damaged

linearity preserved

resolution $\sim$10% worse



Outline

Conclusions

General framework for calorimeter response optimization introduced.

Optimization target not uniquely defined:
linearity and resolution optimization goals differ (due to shower leakages?)

Flexible analytical method for calorimeter calibration optimization implemented.

Different calorimeter configurations can be very efficiently compared.

The framework can be used to propose the optimal ECAL-P readout configuration for running with the reduced number of sensitive layer.

Final results should still be cross-checked for consistency with with MC events