Expected Tracking Performance from First Principles

Karri Folan DiPetrillo
University of Chicago
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Intro

- UChicago group studying tracking performance
  - Isaac: occupancy with timing & pointing requirements
  - Noah: Tracking efficiency & fake rates
  - Leo: p_T and impact parameter resolution

- p_T and impact parameter resolution results surprisingly good
  - w/o BIB so far

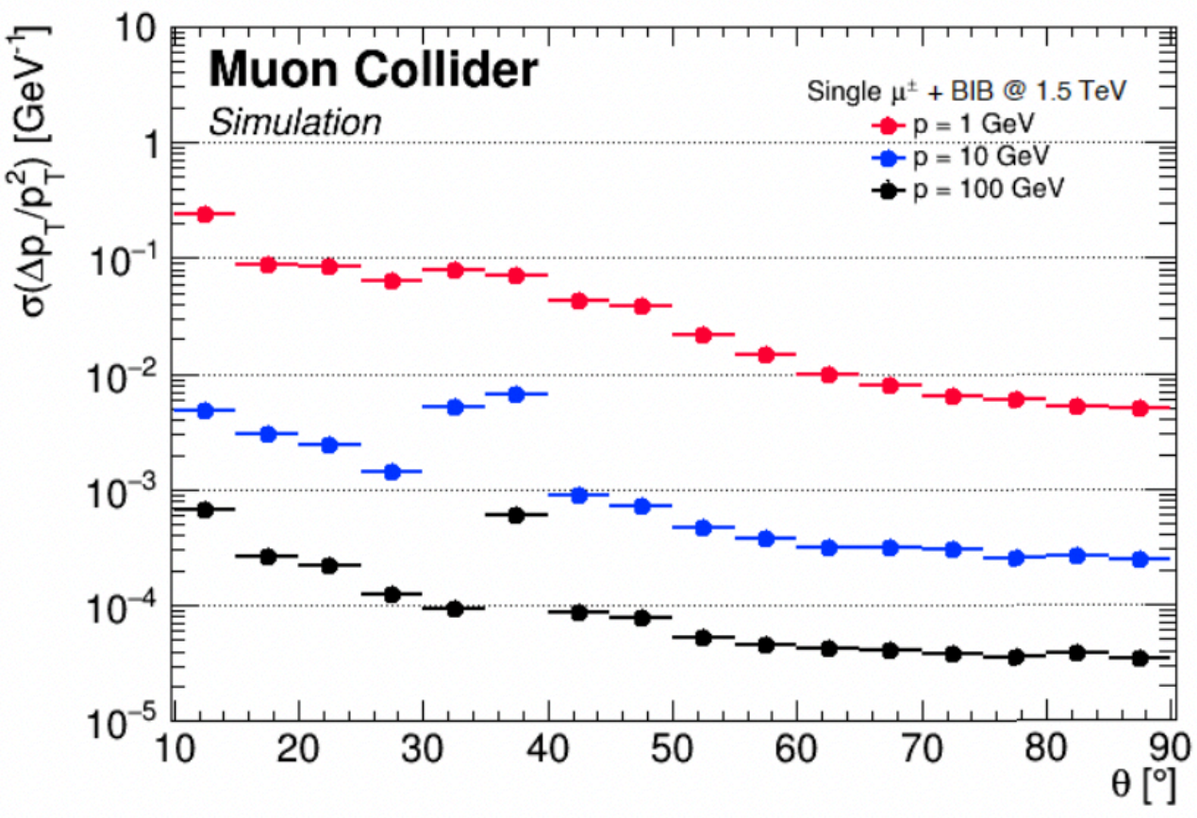
- Goal of these slides: cross-check expected & observed performance
  - Derive expected resolutions from first principles (1705.10150)
  - Compare to 3 TeV - Towards a muon collider paper

3 TeV performance

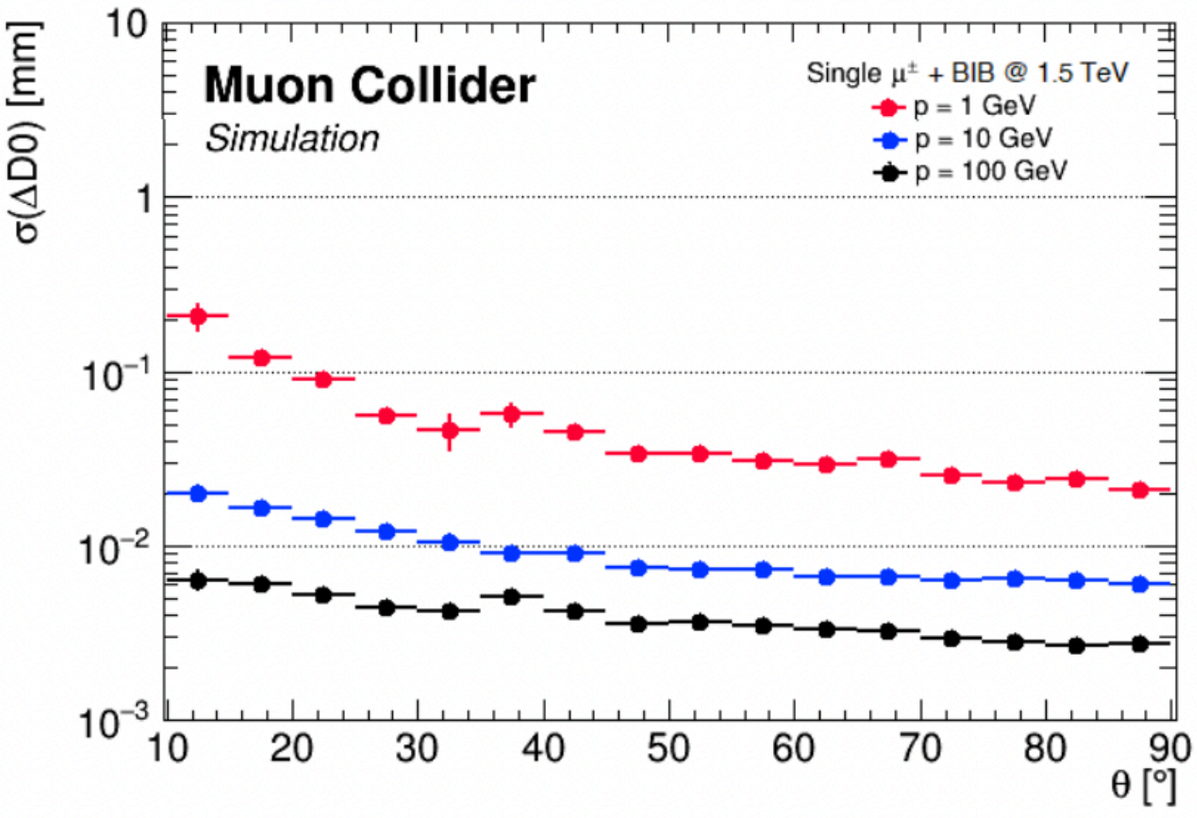
2303.08533

We're using the same detector geometry (including B-field)
so we should observe the same performance

Will focus on θ=90° for simplicity



σ(Δp_T/p_T) ~ 0.3-0.5%



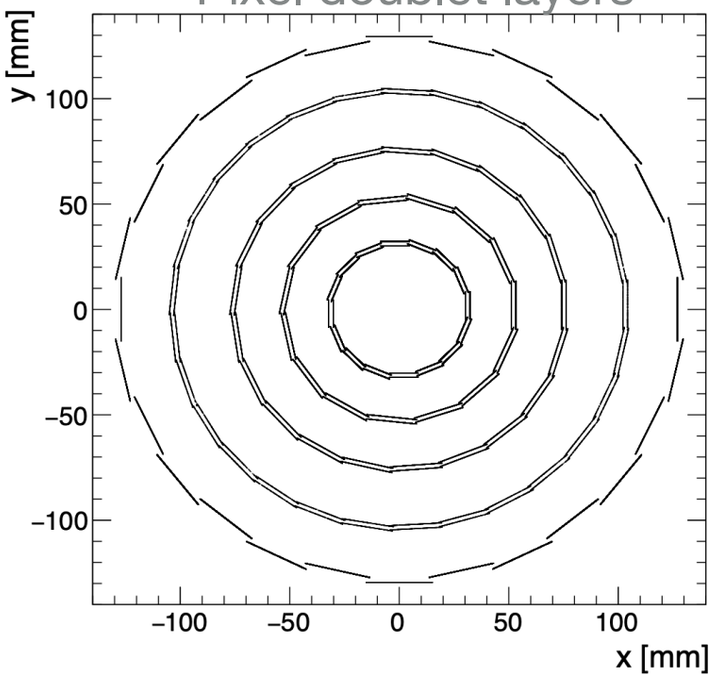
σ(d₀) ~ 3-20 μm

Tracker geometry reminder

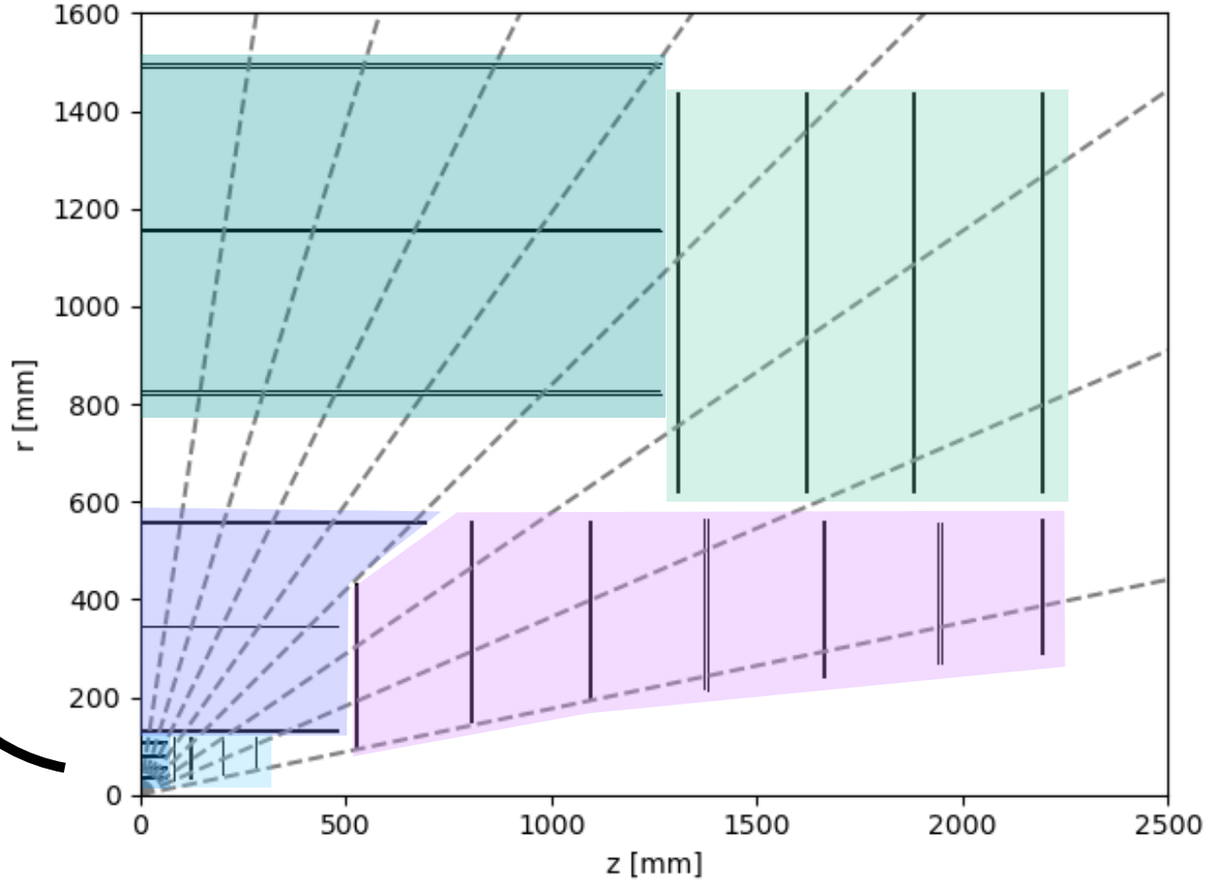
2303.08533

	Vertex Detector	Inner Tracker	Outer Tracker
cell size	25x25 μm^2	50 μm x 1 mm	50 μm x 10 mm
thickness	50 μm	100 μm	100 μm
σ_t	30 ps	60 ps	60 ps

Pixel doublet layers



B=3.57 T



Radius of Curvature & Sagitta

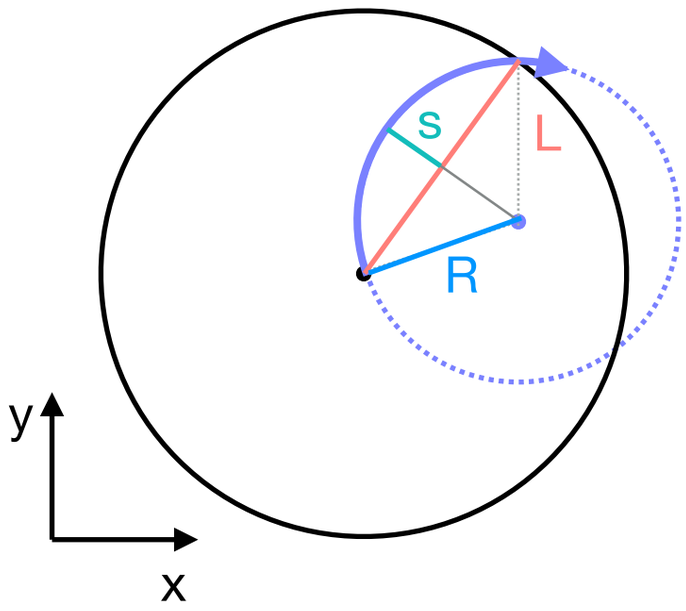
Particles follow a helical trajectory in solenoidal field
First sanity check: getting a feel for radius of curvature and sagitta

Radius of curvature

$$R = \frac{p_{\mathrm{T}}}{0.3 \cdot B}$$

Sagitta Track Length

$$s \approx \frac{L^2}{8R} = \frac{0.3\ BL^2}{8\ p_{\mathrm{T}}}$$



L = 1.5 m
B = 3.5 T

p_T (GeV)	radius (m)	sagitta (mm)
1	0.9	295
10	9.3	30
100	93	3
1000	934	0.3

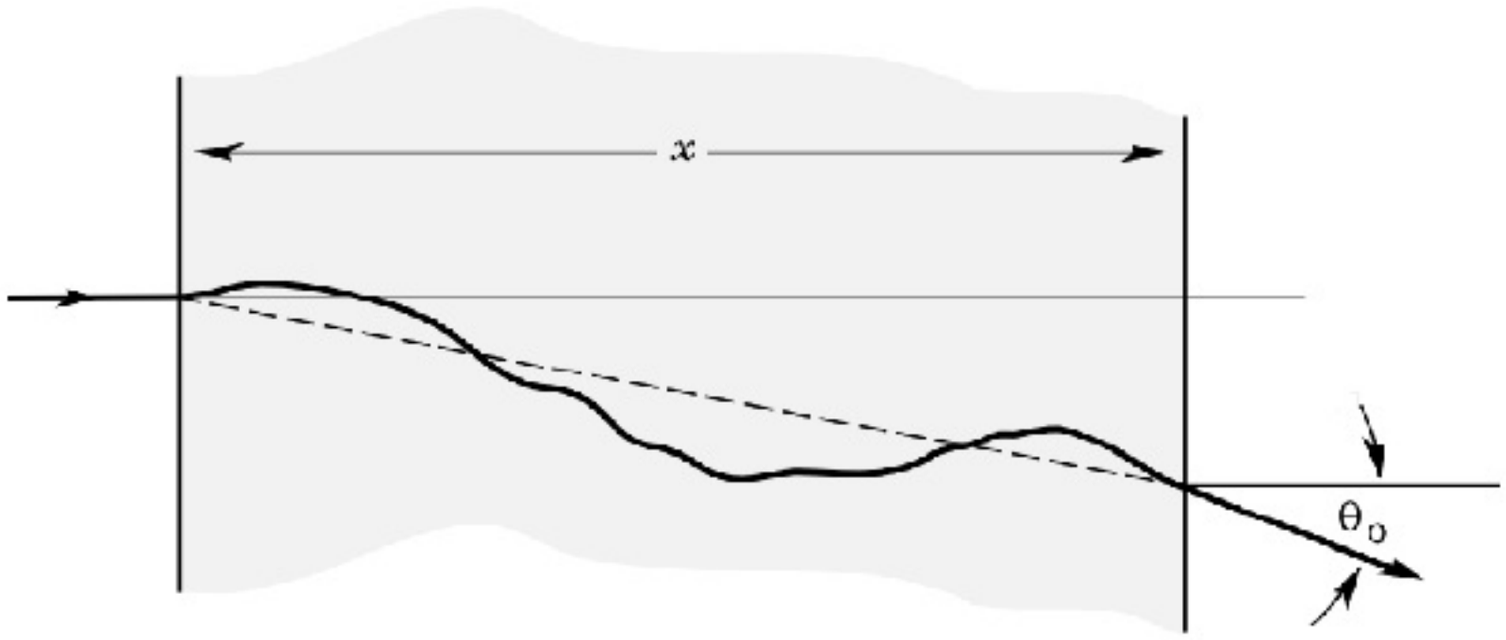
Looper if $p_T < 0.8$ GeV

Multiple scattering

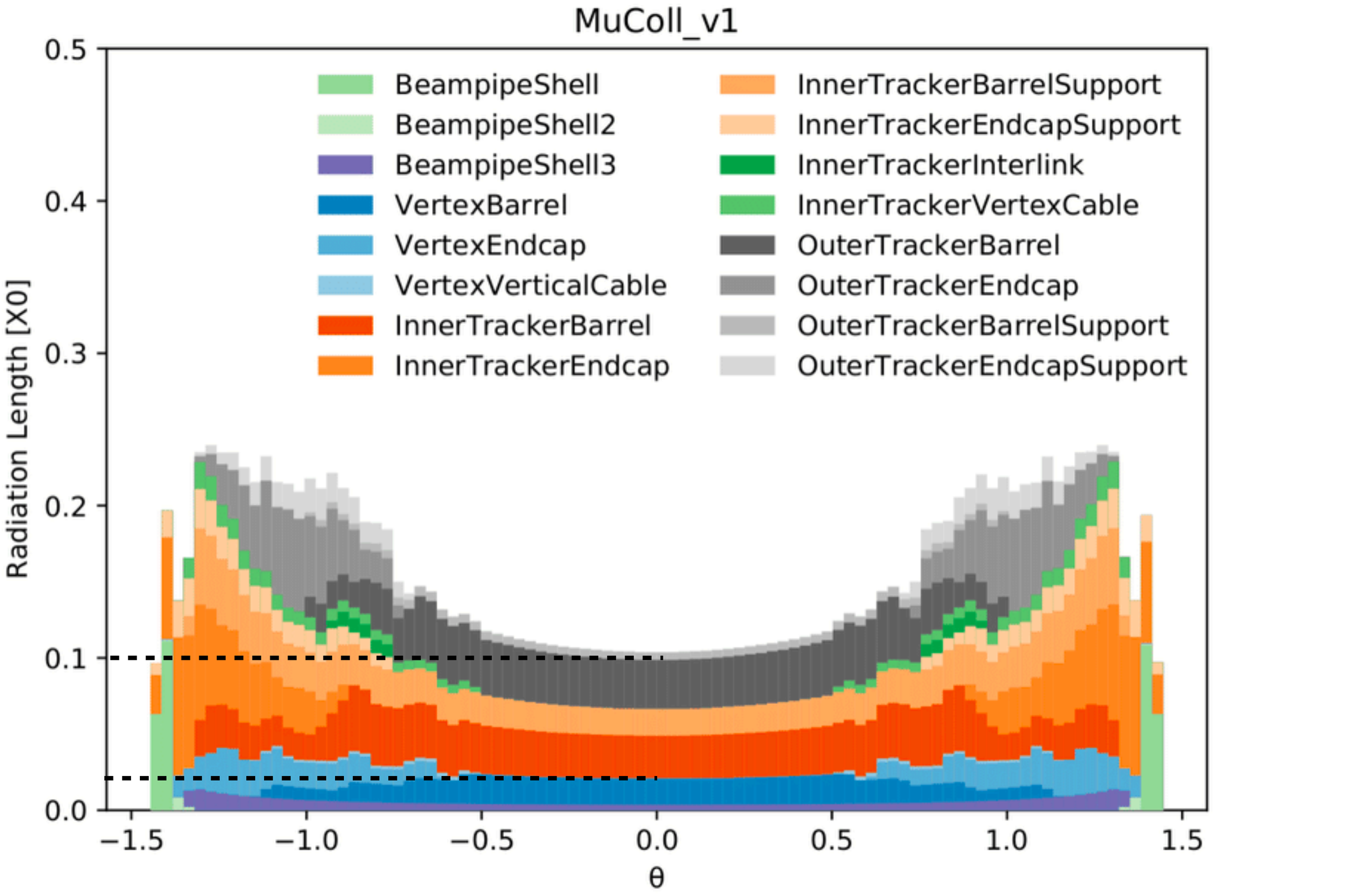
Incident particles scatter in detector material
Scattering adds a random component to particle's path
Angular dispersion modeled by Gaussian

$$\theta_0 = \frac{13.4\text{MeV}/c}{\beta p} q\sqrt{\frac{d}{X_0}}.$$

d/X_0 =#radiation lengths



Material Budget



Momentum Resolution

Depends on two factors

Error on sagitta measurement

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{sagitta}} = \frac{p_T}{0.3}\frac{\sigma_{\text{point}}}{BL^2}\sqrt{\frac{720}{N+4}}$$

- Tracker length, L = 1.5 m
- B-field = 3.5 T
- Hit resolution σ_{point} (m) ~ 8 μm
  - single hit resolution = pitch/√12
  - with multiple hits per cluster ~0.8 pitch/√12
- Number of measurements, N
  - Doublets as separate layers = 14
  - Doublets as single layers = 10

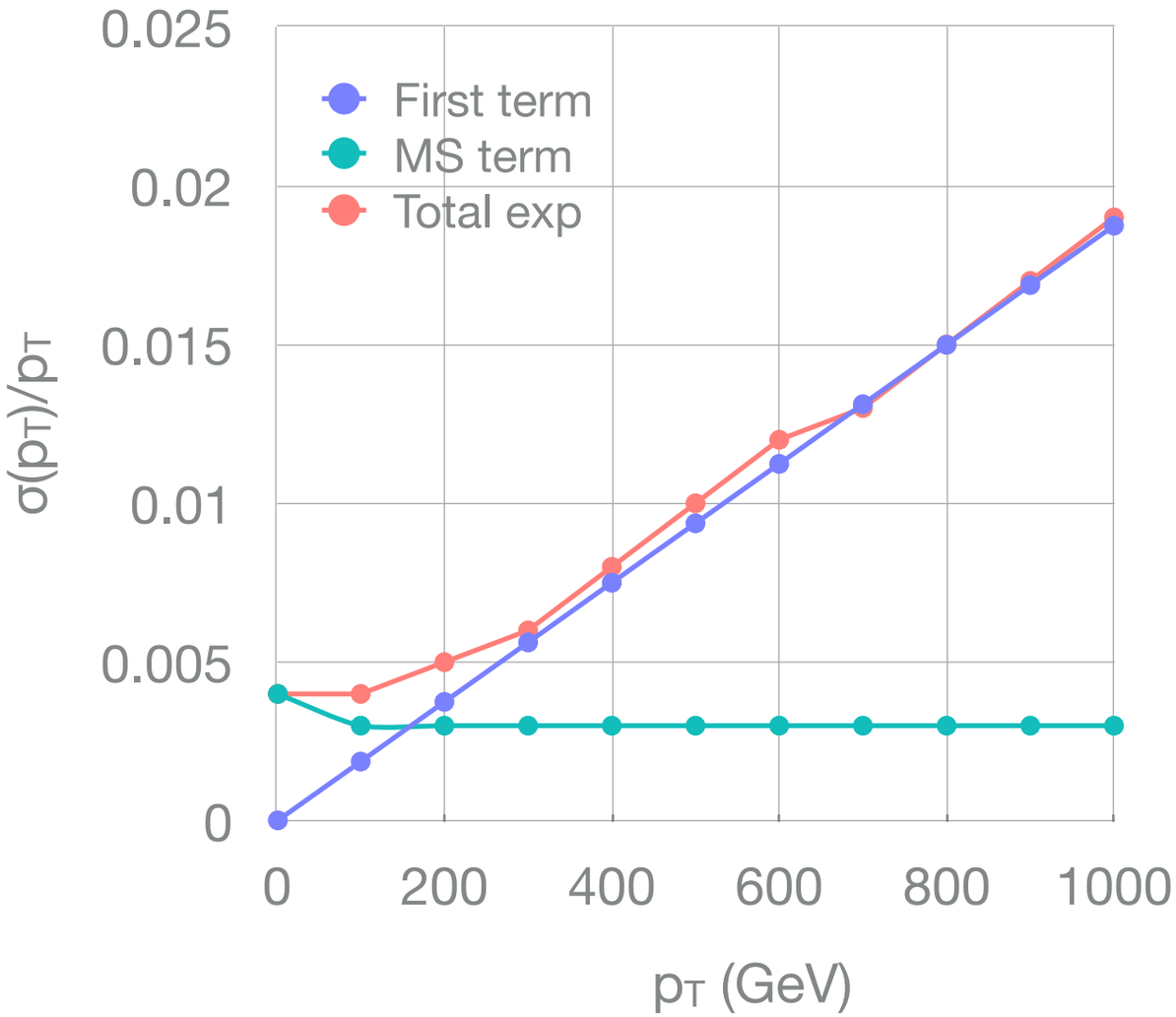
Multiple Scattering

$$\left(\frac{\sigma_{p_T}}{p_T}\right)_{\text{MS}} = \frac{0.0136}{0.3\ \beta\ BL}\sqrt{\frac{x/\sin\theta}{X_0}}\sqrt{C_N}$$

- Velocity, β = 1
- Number of radiation lengths
  - $(x/\sin\theta)/X_0 = 0.1$ for particle with $p_T = \infty$ at $\eta=0$
- C_N = combinatoric factor
  - = 2.5 for N = 3
  - = 1.3 for infinite
  - Lets call it 1.5

Momentum Resolution

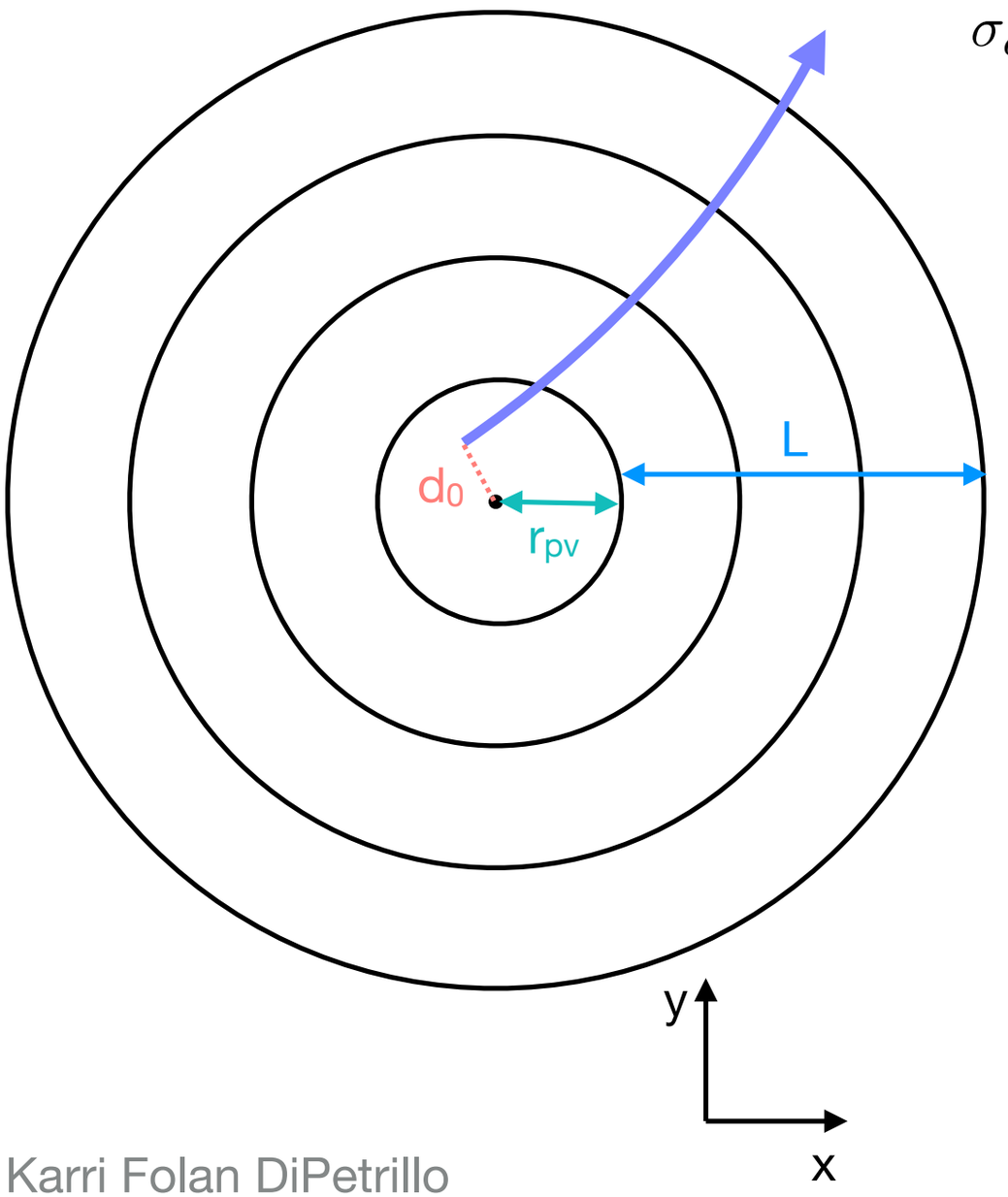
	σ(pT) / pT			
pT (GeV)	Sagitta Term	MS Term	Total Exp	Total Obs
1	0.00002	0.004	0.004	0.005
10	0.00019	0.003	0.003	0.003
100	0.00187	0.003	0.004	0.003
1000	0.01874	0.003	0.019	-



Expected results agree with measured resolution @3 TeV
Need to test for p_T > 200 GeV to validate first term

Impact parameter resolution

Here just focus on the pixel detector



$$\sigma_{d_0} \approx \frac{\sigma_{\text{point}}}{\sqrt{N}} \sqrt{1 + \frac{12(N-1)}{(N+1)} \left(\frac{r}{L}\right)^2} \oplus \theta_0 \, r_{pv} \sqrt{\frac{N(2N-1)}{6(N-1)^2}}.$$

Error due to extrapolation

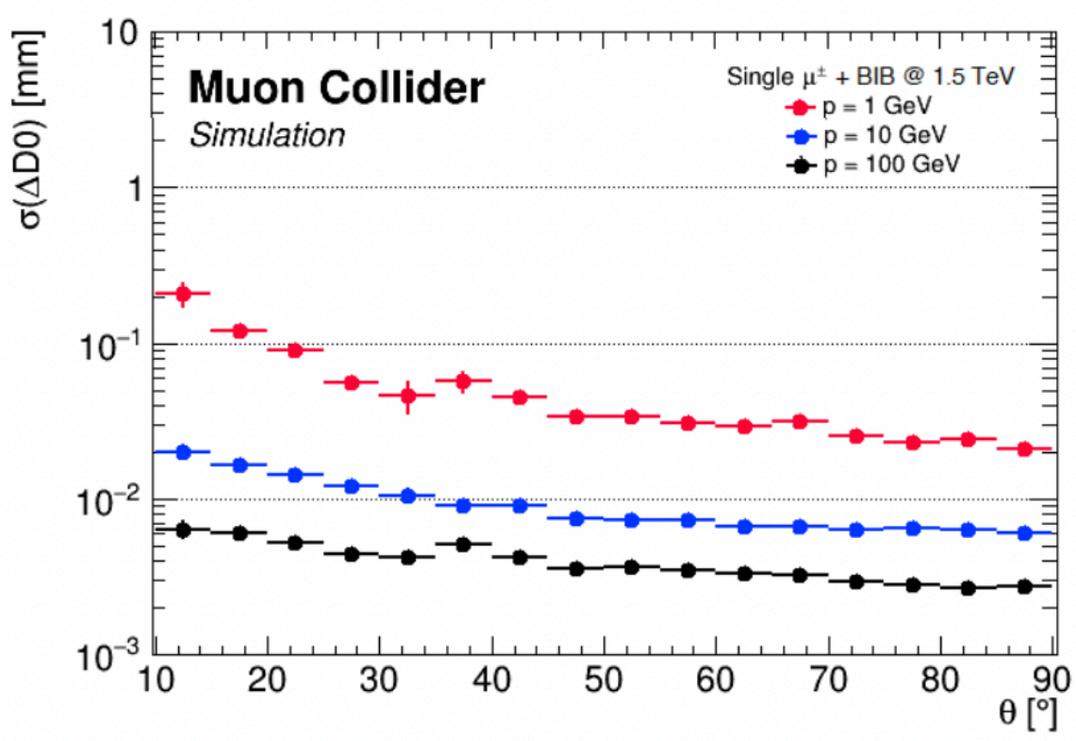
Multiple Scattering

- First term
  - σ_{point} (m) ~6 μm
  - r = 30 mm
  - L = 70 mm
  - N = 8
- Radiation lengths
  - x/X0=0.02

Impact parameter resolution

Agrees fairly well with observed resolution @3 TeV

	σ(d_0) (μm)			
p_T (GeV)	First term	MS term	Total exp	Total Obs
1	3.9	37	37	20
10	3.9	3.6	5.4	6
100	3.9	0.36	4.0	3
1000	3.9	0.036	3.9	-



Conclusions

- Cross-checked expected tracking performance with measured performance @3 TeV
- First principles calculations in good agreement with observation for 1-100 GeV muons
- Need to test p_T > 100 GeV up to 1 TeV