

# Subleading- $N$ improved Parton Showers

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in collaboration with  
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# Motivation.

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Monte Carlo event generators are indispensable tools for collider phenomenology:

- account for the bulk of (soft/collinear) QCD radiation,
- include models for hadronization and multiple interactions,
- perform decays of unstable hadrons,
- finally deliver realistic, fully exclusive events.

Many improvements, particularly to include higher orders.

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This talk: Attempt to quantify the impact of subleading- $N$  contributions.

Ultimately want to answer:

- Are they important at all?
- For which observables/phasespace regions?

Clearly state: *Subleading- $N$  improved* instead of  $N = 3$  exact.

Evolution at amplitude level hard to put into current MC frameworks.

# Outline.

- Brief recaps of main ingredients.
- Colour matrix element corrections.
- Technicalities.
- Some first results.
- Conclusions and outlook.

# Dipole factorization.

[Catani & Seymour, Nucl.Phys.B485:291-419,1997]

Tree-level matrix elements in singly-unresolved limits:

$$|\mathcal{M}_{n+1}(p_1, \dots, p_{n+1})|^2 \sim \sum_{(ik)} \sum_{j \neq i, k} \langle \mathcal{M}_n(\dots, \tilde{p}_{ij}, \dots, \tilde{p}_k, \dots) | \mathbf{V}_{ij,k} | \mathcal{M}_n(\dots, \tilde{p}_{ij}, \dots, \tilde{p}_k, \dots) \rangle$$

Neglecting spin correlations,  $\mathbf{V}$  only contains colour correlations:

$$\mathbf{V}_{ij,k} = -V_{ij,k} \frac{\mathbf{T}_{ij} \cdot \mathbf{T}_k}{\mathbf{T}_{ij}^2}$$

# Showers based on dipole factorization(s).

[Krauss & Schumann; Dinsdale & Weinzierl; Krauss & Winter; Giele & Skands; Gieseke & SP, ...]

$$P_{l \rightarrow ij} \rightarrow V_{l,k \rightarrow ij,k}$$



Evolution different from just  $1 \rightarrow 2$ : All 'dipoles' competing.  
Momentum conservation locally in each branching.

# Showers based on dipole factorization.

$p_{\perp}$ -ordered CS type showers in detail:

[SP & S. Gieseke, JHEP01(2011)024]

- Check Sudakov anomalous dimensions  
→ LL and NLL coefficients ok, recoils power suppressed.
- Explicitly demonstrate coherent gluon emission  
→ guaranteed by  $p_{\perp}$  ordering, virtuality ordering *not* coherent.
- New prescription for initial state radiation  
→ proper build-up of  $p_{\perp}$  in hadronic collisions.

Implemented as add-on module to Herwig++.

# Matrix element corrections.

Want hardest emission driven by exact real emission matrix element.

[e.g. Seymour, Comput.Phys.Commun.90:95-101,1995.]

Want  $n$  hardest emissions driven by exact real emission matrix elements.

[→ merging – see work by CKKW, Lönnblad ...]

What it boils down to ...

$$P_{\text{parton shower}} \approx \frac{|\mathcal{M}_{n+1}|^2}{|\mathcal{M}_n|^2} \rightarrow \frac{|\mathcal{M}_{n+1}|^2}{|\mathcal{M}_n|^2}$$

Mind that this is an *exact* factorization at the level of cross sections.

## Colour matrix element corrections.

Treat colour correlations along the lines of matrix element corrections.  
For emission off a  $n$ -parton configuration:

$$V_{l,k \rightarrow ij,k} \rightarrow -V_{l,k \rightarrow ij,k} \times \frac{1}{|\mathcal{M}_n|^2} \langle \mathcal{M}_n | \frac{\mathbf{T}_l \cdot \mathbf{T}_k}{\mathbf{T}_j^2} | \mathcal{M}_n \rangle$$

Directly follows from the dipole factorization formula.

Algorithmic:

- Correct 'dipole' emission by colour matrix element corrections.  
→ Not chains of dipoles, but all pairs.
- Calculate next amplitude from underlying dipole factorization.

## Calculating the next amplitude.

From just soft gluon factorization:

$$|\mathcal{M}_{n+1}\rangle = \sum_l \frac{\epsilon^*(p_j) \cdot p_l}{p_j \cdot p_l} \mathbf{T}_l |\mathcal{M}_n\rangle$$

Instead use  $|\mathcal{M}_n\rangle\langle\mathcal{M}_n|$  as the fundamental object:

$$|\mathcal{M}_{n+1}\rangle\langle\mathcal{M}_{n+1}| = - \sum_{(lk)} \frac{V_{l,k \rightarrow ij,k}}{\mathbf{T}_l^2} \mathbf{T}_k |\mathcal{M}_n\rangle\langle\mathcal{M}_n| \mathbf{T}_l$$

Allows to include collinear contributions.

Don't have to deal with explicit wavefunctions.

# Technicalities.

Need colour basis for (in principle) arbitrary number of legs.

Working horse: `ColorFull`

- C++ implementation of trace basis algebra [M. Sjö Dahl, in preparation]
- Supplemented by `boost::blas` for efficiency
- Interfaced to `Matchbox` framework and dipole shower modules

[SP & S. Gieseke, arXiv:1109.6256]

The main MC algorithm can be extended to cope with the case at hand (negative splitting ‘probabilities’).

[SP & M. Sjö Dahl, arXiv:1109.6180]

## Some first results.

Proof-of-concept sandbox:  $e^+e^- \rightarrow$  jets.

Up to 6 improved gluon emissions.

Three scenarios considered to study systematics:

- $\mathbf{T}_i \cdot \mathbf{T}_j / \mathbf{T}_i^2$  exact,  $C_F$  exact: 'full correlations'
- $\mathbf{T}_i \cdot \mathbf{T}_j / \mathbf{T}_i^2$  in large  $N$ ,  $C_F$  exact: 'large- $N$  correlations'
- $\mathbf{T}_i \cdot \mathbf{T}_j / \mathbf{T}_i^2$  in large  $N$ ,  $C_F \rightarrow C_A/2$ : 'strict large- $N$  correlations'

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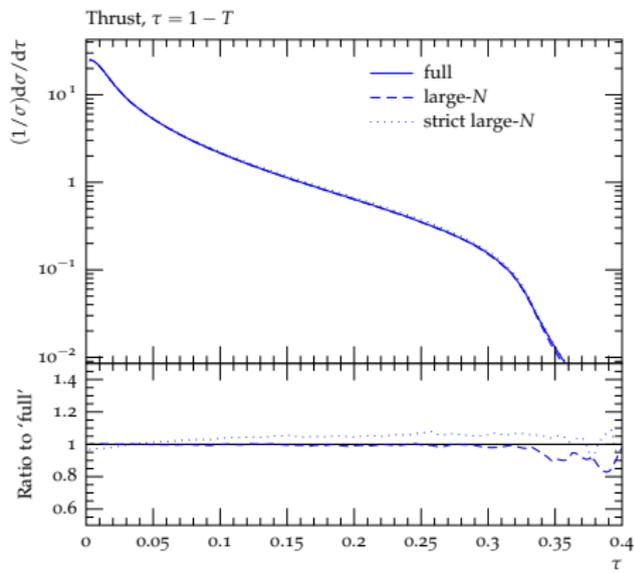
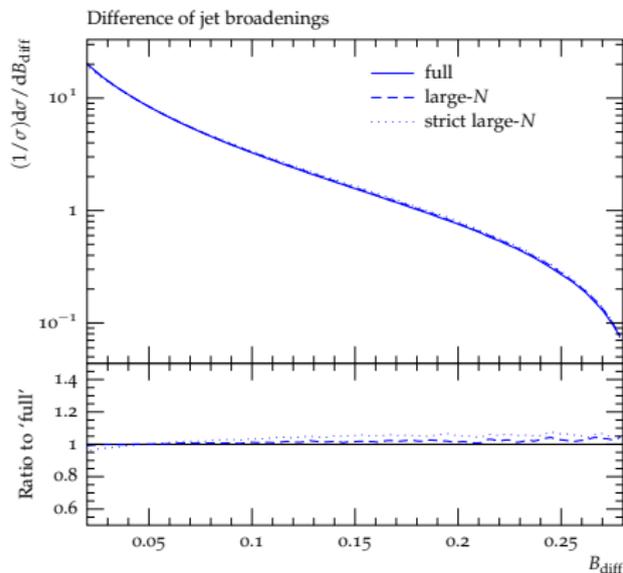
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Typical parton shower approximation.

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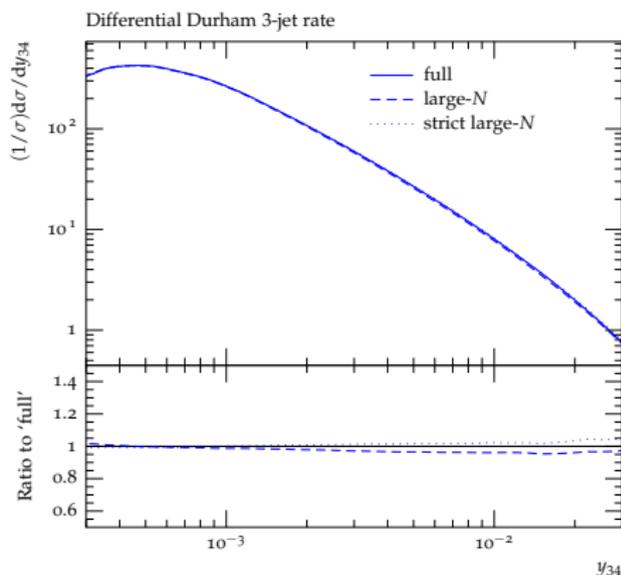
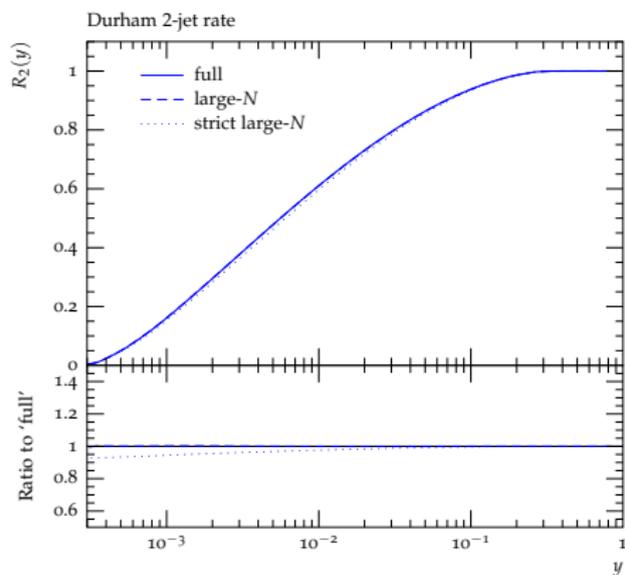
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[preliminary]

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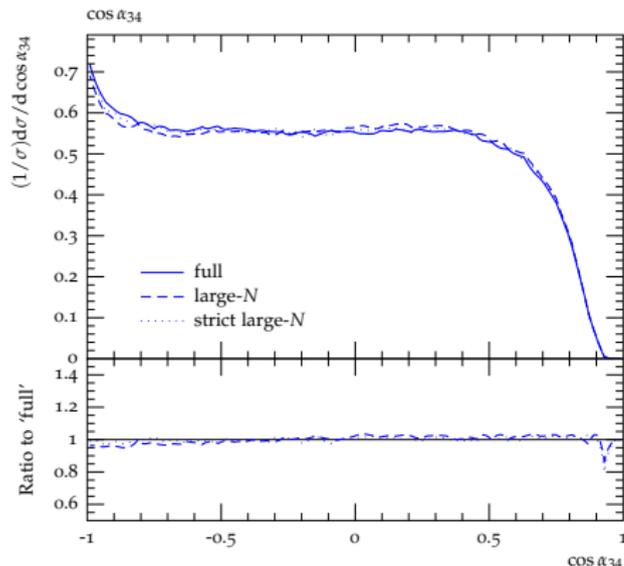
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Still no large effects.

Construct dedicated  
5, 6-jet observables  
(work in progress).

[preliminary]

# Conclusions and outlook.

Algorithm to improve parton showers by subleading- $N$  contributions.

Proof of concept implementation for  $e^+e^- \rightarrow$  jets:

- Differences at the level of or below 5%
- Larger deviations for strict large- $N$  limit with  $C_F \rightarrow C_A/2$

Outlook:

- More significant effects expected in QCD  $2 \rightarrow 2$ .
- Combine with any number of large- $N$  emissions downstream.
- Computationally less exhaustive algorithm possible?
- How should hadronization models be formulated?

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