Threshold resummation for squark- and gluino hadroproduction

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based on

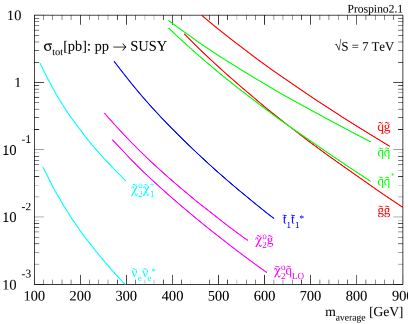
W. Beenakker, S.B., M. Krämer, A. Kulesza, E. Laenen, I. Niessen [JHEP 0912 (2009) 041]
W. Beenakker, S.B., M. Krämer, A. Kulesza, E. Laenen, L. Motyka, I. Niessen [Int.J.Mod.Phys.A26:2637-2664, 2011]
W. Beenakker, SB, M. D'Onofrio, M. Krämer, A. Kulesza, E. Laenen, I. Niessen, M. Martinez [arXiv: 1106.5647]

and work in preparation

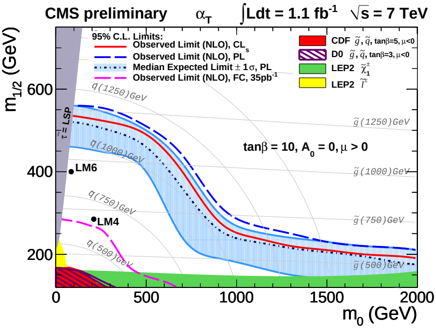
Production of SUSY particles at the LHC

Framework: MSSM with R-parity conservation

[Beenakker,Höpker,Krämer,Plehn,Spira,Zerwas'96-'98]



[arXiv: 1109.2352]

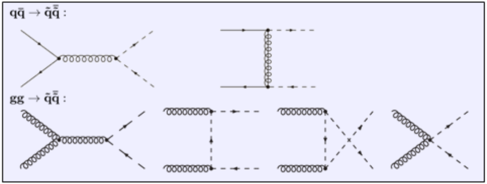


- Squarks and gluinos are produced with high production rates
  $\rightarrow$ offer strongest sensitivity for SUSY searches
- Total cross section are used to derive exclusion limits
  $\rightarrow$ precise theoretical prediction are necessary

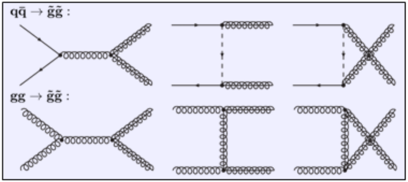
Classification of processes

Processes at LO: [Kane, Leveille '82; Harrison, Llewellyn, Smith '83; Eichten, Dawson, Quigg '85]

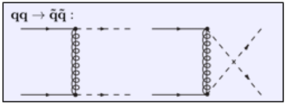
Squark-antisquark:



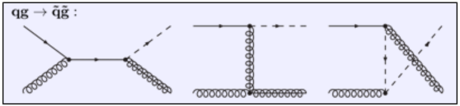
Gluino-gluino:



Squark-squark:



Squark-gluino:



Assume all squarks $\tilde{q} = (\tilde{q}_L, \tilde{q}_R)$ with $\tilde{q} \neq \tilde{t}$ mass degenerate

NLO SUSY-QCD calculation

NLO SUSY-QCD corrections [Beenakker et al. ´96]

- Large positive corrections, depending in detail on squark- and gluino mass
- Significant part can be attributed to the threshold region $\hat{s} \approx 4m^2$

NLO partonic cross section near threshold $\beta = \sqrt{1 - 4m^2/\hat{s}} \to 0$:

$$\hat{\sigma}^{(\mathrm{NLO})} = \hat{\sigma}^{(0)} \left[\alpha_s \{a \log^2(\beta^2) + b \log(\beta^2) + c \log(\beta^2) \log(\mu^2/m^2) + d\,(1/\beta)\}\right]$$

Soft-gluon corrections

Coulomb corrections

Generic form of higher-order corrections near threshold:

$$\hat{\sigma} = \hat{\sigma}^{(0)} \times \left[1 + \alpha_s(L^2 + L + \ldots) + \ldots + \alpha_s^n(L^{2n} + L^{2n-1} + \ldots)\right] \qquad L = \log(\beta^2)$$

- Logarithmic terms become large near threshold
- Spoil convergent behaviour of perturbative series in α_s

→ Requires all-order summation

→ Soft-gluon resummation

Theoretical Status

- NLO SUSY-QCD corrections [Beenakker et al.'96]
- NLL-resummed corrections ($\tilde{q}\bar{\tilde{q}}, \tilde{g}\tilde{g}$) [Kulesza, Motyka '08,'09]
- Resummation of leading Coulomb-corrections [Kulesza, Motyka '09]
- combined (soft-gluon & Coulomb) NLL-resummed corrections ($\tilde{q}\bar{\tilde{q}}$)

  [Beneke, Falgari, Schwinn '09,'10]
- approximate NNLO contributions ($\tilde{q}\bar{\tilde{q}}$) [Langenfeld, Moch '09]
- bound state effects in $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{g}$ [Hagiwara, Yokoya '09]

  [Kauth et al. '09,'11][Kauth, Kress, Kühn '11]

- NLO EW corrections [Beccaria et al. '07,'08][Hollik, Kollar, Trenkel '07][Hollik, Mirabella '08] [Hollik, Mirabella, Trenkel '08][Mirabella '09][Germer et al. '09, '11]
- LO EW and QCD-EW interference [Bozzi, Fuks, Klasen '05][Alan, Cankocak, Demir '07] [Bornhauser et al '07][Hollik, Kollar, Trenkel '07][Hollik, Mirabella '08][Hollik, Mirabella, Trenkel '08][Germer et al. '09,'11]

Soft-gluon resummation

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman, Bonciani, Catani, Mangano, Nason '96 – '98]

- Perform resummation of soft-gluon contributions using approach in Mellin-space

$$\sigma_{h_A h_B \to kl}\big(N, \{m^2\}\big) \;\equiv\; \int_0^1 d\rho\, \rho^{N-1}\, \sigma_{h_A h_B \to kl}\big(\rho, \{m^2\}\big)$$

- Hadronic cross section for the production of two massive coloured sparticles *k,l*

$$\sigma_{h_A h_B \to kl}\big(\rho, \{m^2\}\big) \;=\; \sum_{i,j} \int dx_1\, dx_2\, dz\; \delta\left(z - \frac{\rho}{x_1 x_2}\right)\; f_{i/h_A}(x_1, \mu^2)\, f_{j/h_B}(x_2, \mu^2)\, \hat{\sigma}_{ij \to kl}\big(z, \{m^2\}, \mu^2\big)$$

with $\rho = \dfrac{(m_k + m_l)^2}{S}$ and $z = \dfrac{(m_k + m_l)^2}{\hat{s}}$

$$\rightarrow \quad \sigma_{h_A h_B \to kl}\big(N, \{m^2\}\big) = \sum_{i,j} f_{i/h_A}(N{+}1, \mu^2)\, f_{j/h_B}(N{+}1, \mu^2)\, \hat{\sigma}_{ij \to kl}\big(N, \{m^2\}, \mu^2\big)$$

- Form of soft-gluon corrections $\alpha_s^n \log^m(N)$ $\quad m \leq 2n \qquad N \to \infty$

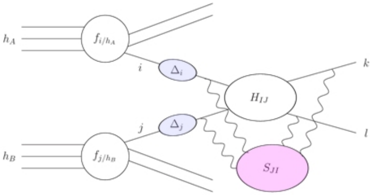
- Resummation: $\hat{\sigma}^{(\mathrm{res})}_{ij \to kl}(N) \;=\; \exp\Big[\underbrace{L g_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots\Big] \times P(\alpha_s) \qquad L = \log(N)$

Soft-gluon resummation for coloured heavy (s)particles

[Contopanagos, Kidonakis, Laenen, Oderda, Sterman '96-'98; Bonciani, Catani, Mangano, Nason '98]

- Based on near threshold factorisation of the cross section

$$\hat{\sigma}_{ij\to kl}(N) = \underbrace{\Delta_i\ \Delta_j}_{\text{soft-collinear}} \sum_{IJ} H_{ij\to kl,JI}\ \underbrace{S_{ij\to kl,IJ}}_{\text{wide-angle soft}}$$



- Evolution equations

  e.g.: $$\mu\frac{d}{d\mu}S_{JI} = -\Gamma^{\dagger}_{JK}\,S_{KI} - S_{JK}\,\Gamma_{KI}$$

- Solving evolution equations $\longrightarrow$ resummed expressions

$$\tilde{\sigma}^{(\mathrm{res})}_{ij\to\tilde{q}\bar{\tilde{q}}}(N,\{m^2\},\mu^2) = \sum_I \tilde{\sigma}^{(0)}_{ij\to kl,I}(N,\{m^2\},\mu^2)\ C_{ij\to kl,I}(N,\{m^2\},\mu^2)$$
$$\times\ \Delta_i(N+1,Q^2,\mu^2)\,\Delta_j(N+1,Q^2,\mu^2)\,\Delta^{(\mathrm{s})}_{ij\to kl,I}(N+1,Q^2,\mu^2)$$

- $\Delta_i = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{\mu^2}^{Q^2(1-z)^2}\frac{dq^2}{q^2}A_i(\alpha_s(q^2))$: resums soft and collinear gluon radiation
- $\Delta^{(\mathrm{s})}_{ij\to kl,I} = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z}D_{ij\to kl,I}(\alpha_s((1-z)^2Q^2))$: resums wide-angle soft gluon radiation

Matching with fixed-order calculation

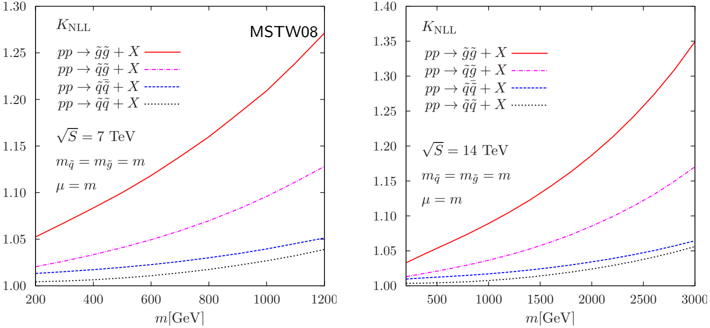
NLO and NLL-resummed are combined through a matching procedure:

$$
\begin{aligned}
\sigma^{\mathrm{NLO+NLL}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2) \quad &= \quad \sum_{i,j} \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f_{i/h_A}(N+1, \mu^2)\, f_{j/h_B}(N+1, \mu^2) \\
&\times \quad \left[\hat{\sigma}^{\mathrm{res,NLL}}_{ij \to kl}(N, \{m^2\}, \mu^2) \;-\; \hat{\sigma}^{\mathrm{res,NLL}}_{ij \to kl}(N, \{m^2\}, \mu^2)\Big|_{(\mathrm{NLO})} \right] \\
&+ \quad \sigma^{\mathrm{NLO}}_{h_A h_B \to kl}(\rho, \{m^2\}, \mu^2)
\end{aligned}
$$

- Avoids double counting of logarithmic terms
- Using "minimal prescription" for the contour of the inverse Mellin transform [Catani et al., '96]
- NLO cross section calculated with PROSPINO [Beenakker, Höpker, Krämer, Plehn, Spira, Zerwas, '96-'98]

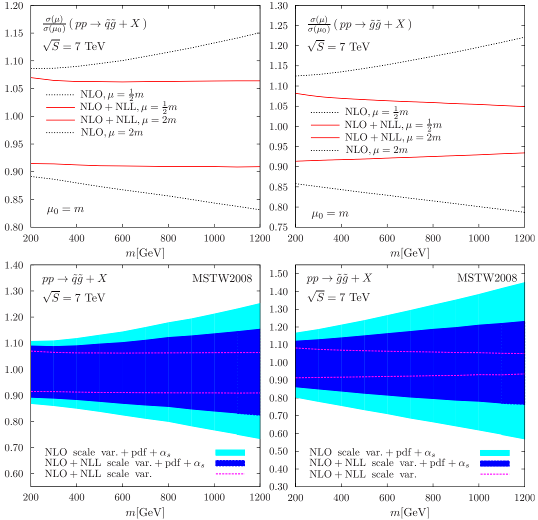
$K_{\mathrm{NLL}} = \sigma_{\mathrm{NLO+NLL}}/\sigma_{\mathrm{NLO}}$ at the LHC

[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen '09]
[Beenakker, SB, Krämer, Kulesza, Laenen, Motyka ,Niessen '11]



- K_{NLL}-factor grows with increasing sparticle mass due to importance of threshold region
- large effects for processes involving large colour charge, i.e. initial state gluons and final state gluinos

[Beenakker, SB, Krämer, Kulesza, Laenen, Motyka, Niessen '11]

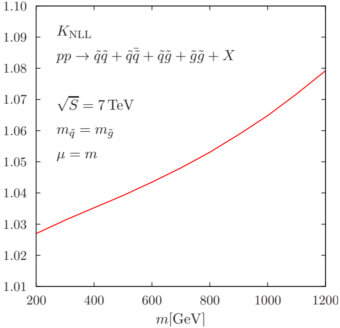


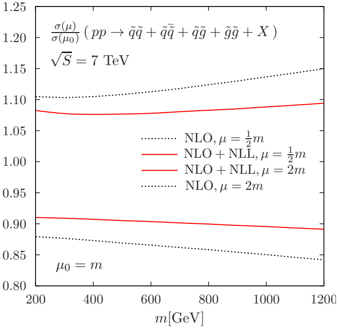
- Scale uncertainty at NLO+NLL$\lesssim$ 10%, also for $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$
- Total theory error at NLO+NLL$\lesssim$ 20%, also for $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$

Total incl. K-factor at the LHC@7TeV

Sum of all four processes: $pp \to \tilde{q}\bar{\tilde{q}} + \tilde{q}\tilde{q} + \tilde{q}\tilde{g} + \tilde{g}\tilde{g}$

[Beenakker, SB, Krämer, Kulesza, Laenen, Motyka ,Niessen '11]



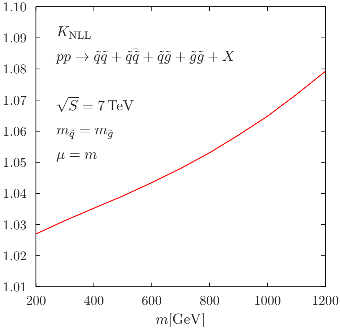


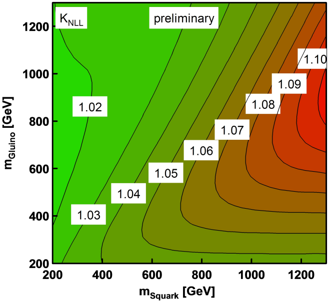
- Each individual $K_{\rm NLL}$-factor contributes with its individual weight
- Reduced scale sensitivity

Total incl. K-factor at the LHC@7TeV, Mass dependence

Sum of all four processes: $pp \to \tilde{q}\bar{\tilde{q}} + \tilde{q}\tilde{q} + \tilde{q}\tilde{g} + \tilde{g}\tilde{g}$

[Beenakker, SB, Krämer, Kulesza, Laenen, Motyka, Niessen '11]





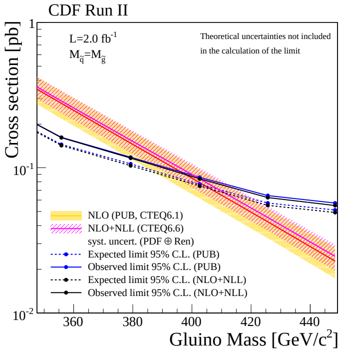
- Each individual K_{NLL}-factor contributes with its individual weight
- Reduced scale sensitivity
- Total inclusive K_{NLL}-factor up to 10%

Effect on mass limits from squark- and gluino searches at the Tevatron

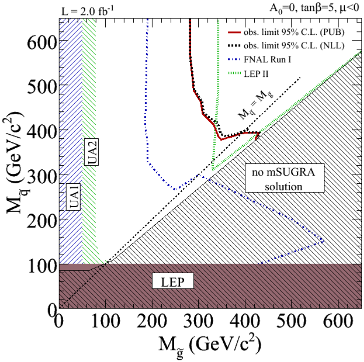
[Beenakker, SB, D'Onofrio, Krämer, Kulesza, Laenen, Martinez, Niessen '11]

Re-analysis of limits from CDF [CDF, PRL 102, 121801, '09]:

Limit on cross section:

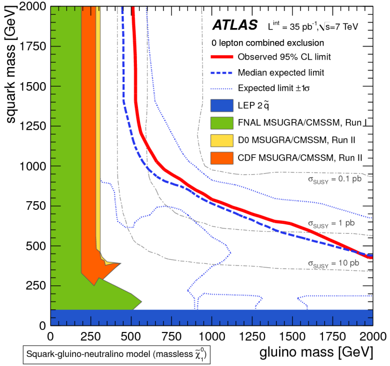


Limits on squark- and gluino mass:

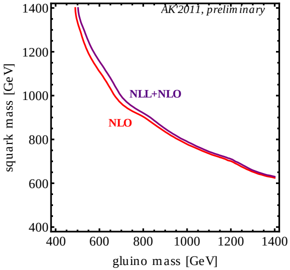


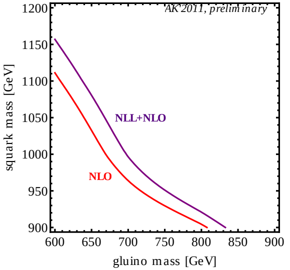
- Upper limit on cross section: Improvement of ∼ 10% compared to published analysis in [CDF, PRL 102, 121801, '09]
- $m_{\tilde{q}} \sim m_{\tilde{g}}$: Improvement of 5-10GeV compared to published analysis in [CDF, PRL 102, 121801, '09]

[PBL 701 (2011) 186]



Plot courtesy of A. Kulesza





- mass shift up to 50 GeV
- for higher masses larger effects expected

Public Code: NLL-fast

Available at: (LPCC BSM Cross Section Working Group)

http://web.physik.rwth-aachen.de/service/wiki/bin/view/Main/SUSYCrossSections

Files for NLL-fast

- main program: interpolation_susy.f
- grids: grids.tar

```
c Parameters to be given by user:
c  - process:
c      'sb'  squark-antisquark production,
c      'ss'  squark-squark production,
c      'gg'  gluino-gluino production,
c      'sg'  squark-gluino production,
c      'tot' sum of all subprocesses
c  - masses for which the NLL+NLO cross section can be calculated:
c       MSQ  squark mass        500 GeV < MSQ < 2000 GeV,
c       MGL  gluino mass        500 GeV < MGL < 2000 GeV,
c  - order of interpolation polynomials:
c       NITP       recommended NITP=4 or NITP=5.
```

Sample output

```
# LHC @ 7 TeV, MSTW 2008 NLO
# process: tot
# ms[GeV]  mg[GeV]  LO[pb]     NLO[pb]    NLL+NLO[pb]  d_mu+[pb]  d_mu-[pb]   d_pdf+[%]  d_pdf-[%]  d_as+[%] d_as-[%]  K_NLO   K_NLL
-----------------------------------------------------------------------------------------------------------------------------------
   500.     500.    0.102E+02  0.127E+02  0.132E+02    0.101E+01  -0.124E+01    3.2        -3.0       1.9      -2.3     1.25    1.04
   600.     600.    0.310E+01  0.386E+01  0.403E+01    0.314E+00  -0.388E+00    3.7        -3.2       1.9      -2.2     1.25    1.04
   700.     700.    0.106E+01  0.132E+01  0.138E+01    0.110E+00  -0.135E+00    4.1        -3.3       1.8      -2.0     1.25    1.05
```

Threshold resummation at NNLL for $\tilde{q}\bar{\tilde{q}}$-production

$$\tilde{\sigma}^{(\mathrm{res})}_{ij\to\tilde{q}\bar{\tilde{q}}}(N,\{m^2\},\mu^2) = \sum_I \tilde{\sigma}^{(0)}_{ij\to kl,I}(N,\{m^2\},\mu^2)\, C_{ij\to kl,I}(N,\{m^2\},\mu^2) \times \Delta_i(N+1,Q^2,\mu^2)\,\Delta_j(N+1,Q^2,\mu^2)\,\Delta^{(\mathrm{s})}_{ij\to kl,I}(N+1,Q^2,\mu^2)$$

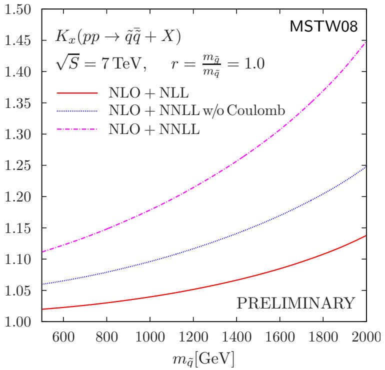
- Soft-radiative factors $\Delta_i \Delta_j \Delta^{(\mathrm{s})}_{ij\to\tilde{q}\bar{\tilde{q}},I}$: $A^{(1)}, A^{(2)}, A^{(3)}$, $D^{(1)}_I, D^{(2)}_I$ required

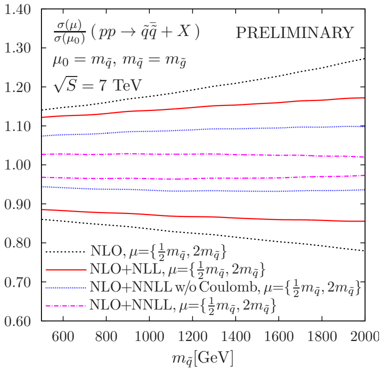
[Kodaira, Trentadue '82, Catani, D'Emilio, Trentadue '88, Moch, Vermaseren, Vogt '05][Vogt '01, Catani et. al. '03]
[Catani et. al '96, Kidonakis, Sterman '96, Czakon, Mitov, Sterman '09, Beneke, Falgari, Schwinn '09]

- $C_I^{\mathrm{NNLL}} = \left(1 + \frac{\alpha_{\mathrm{s}}}{\pi}\,\mathcal{C}_I^{\mathrm{Coul},(1)}(N,\{m^2\},\mu^2)\right)\,\left(1 + \frac{\alpha_{\mathrm{s}}}{\pi}\,\mathcal{C}_I^{(1)}(\{m^2\},\mu^2)\right)$
  - Factorisation into Coulomb correction and hard matching coefficient
    [Beneke, Falgari, Schwinn '10]
  - $\mathcal{C}_I^{\mathrm{Coul},(1)}(N,\{m^2\},\mu^2)$: Leading Coulomb correction (in N-space)
  - $\mathcal{C}_I^{(1)}(\{m^2\},\mu^2)$: Hard matching coefficients,
    Calculated analytic expressions at one-loop for $q\bar{q}/gg \to \tilde{q}\bar{\tilde{q}}$
    [Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]

$K_{\mathrm{NNLL}} = \sigma_{\mathrm{NLO+NNLL}}/\sigma_{\mathrm{NLO}}$ and scale variation

[Beenakker, SB, Krämer, Kulesza, Laenen, Niessen, in prep.]





- significant corrections beyond NLL
  - can be mostly attributed to incorporating hard matching coefficients and leading Coulomb corrections
- significant reduction of theoretical error due to scale variation

Summary

- NLO+NLL matched predictions for total cross sections for all possible squark and gluino pair-production processes
  - significant enhancement of NLO cross section predictions ($\sim$ 5%-40%)
  - most pronounced for processes involving large colour charge
  - significant reduction of theoretical error due to scale variation
  - improvement of mass limits possible

- Public code available: **NLL-fast**

  http://web.physik.rwth-aachen.de/service/wiki/bin/view/Main/SUSYCrossSections

- NNLL resummed predictions for squark-antisquark production