Non-Gaussianity in single field models without slow-roll

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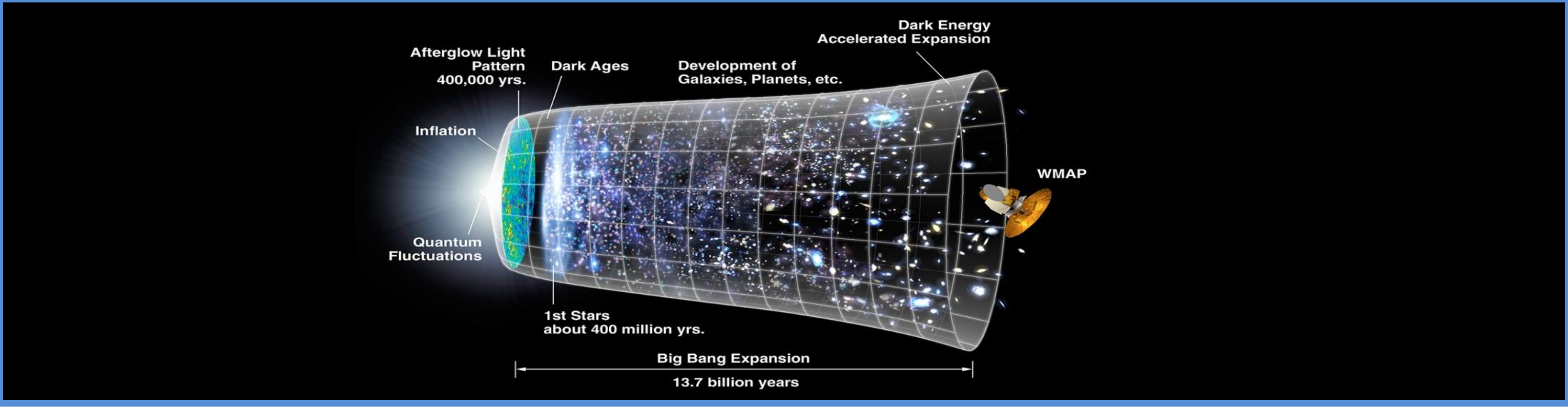
Single field inflation



Scalar field action: $$S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R - \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right)$$

Higher derivatives: $$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(\phi, \partial\phi, \partial^2\phi, ..)\right]$$

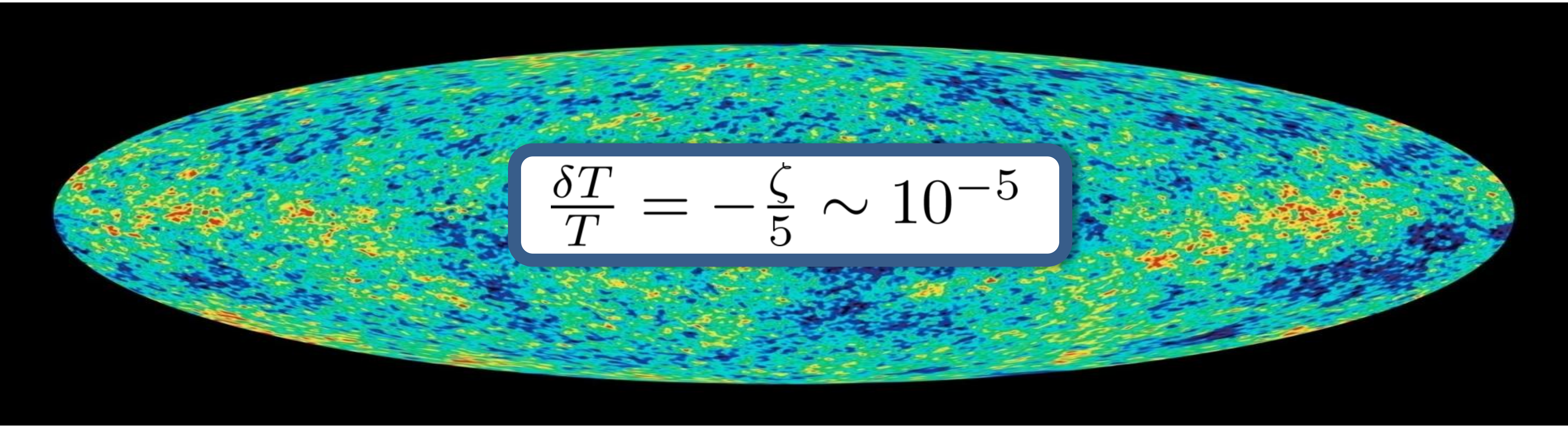
Effective theory: $$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(X, \phi)\right] \qquad X = -\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$$

The most general Lorentz invariant action for a single scalar field minimally coupled to gravity that contains at most first derivatives of the field

Primordial perturbations

Scalar perturbations in metric: $ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a^2(t)\gamma_{ij}dx^i dx^j$

Gauge invariant curvature perturbation: $\zeta = \Phi\frac{5+3w}{3(1+w)} + \frac{2}{3(1+w)}\frac{\Phi'}{\mathcal{H}}$



$$\frac{\delta T}{T} = -\frac{\zeta}{5} \sim 10^{-5}$$

Observables $|\mathbf{P}_\zeta|, \mathbf{n_s}, \mathcal{A}, \mathbf{r}, \mathbf{n}_{NG}\ldots$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5\delta^3(\mathbf{k}_1+\mathbf{k}_2)P_\zeta\frac{1}{2k_1^3}$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta^2\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

A slow-rolling single field

Scalar field action: $$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(X,\phi)\right]$$

Slow-roll parameters: $$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}, \dots \quad \text{and} \quad \epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \quad \eta_s \equiv \frac{\dot{\epsilon}_s}{\epsilon_s H}, \dots$$

Slow-roll condition: $$\epsilon_i \ll 1 \quad \text{and} \quad \eta_i \ll 1$$

Canonical n_s: $$n_s - 1 \equiv \frac{\mathrm{d}\ln P_\zeta}{\mathrm{d}\ln k} = -6\epsilon + 2\eta$$

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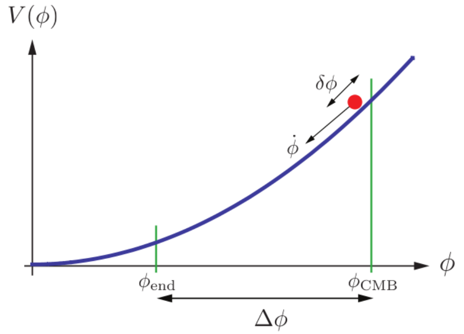
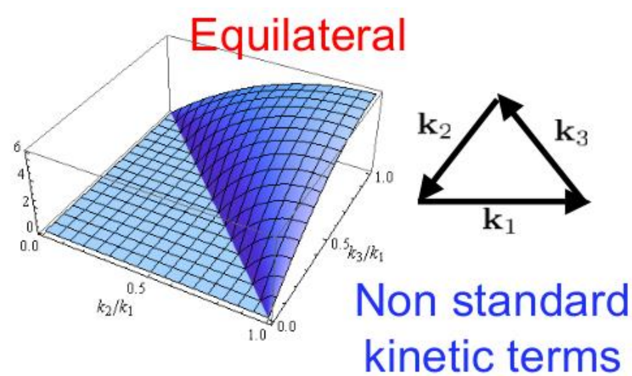
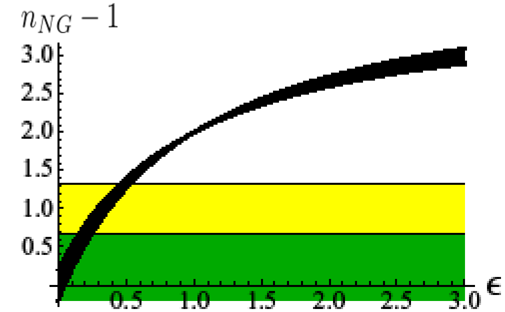
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta{}^2\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

Typically equilateral non-Gaussianity, negligible running n_{NG}

cf. Chen '10 , Khoury, Piazza '09 , Senatore et al. '09 , Seery, Lidsey '05







Plots by: Baumann; Renaux-Petel, Chen; JN

Slow-roll constraints

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \sim 0, \quad \eta_s \equiv \frac{\dot{\epsilon}_s}{\epsilon_s H} \sim 0$$

Spectral index:

$$n_s - 1 \equiv \frac{d\ln P_\zeta}{d\ln k} = \frac{2\epsilon + \epsilon_s}{\epsilon + \epsilon_s - 1}$$

Tensor modes:

$$r \approx 2^{2\mu - 3} \frac{(1-\varepsilon)^{2\mu-1}}{(\epsilon_s + \varepsilon - 1)^2} \left| \frac{\Gamma(\mu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 16 c_s(k_\zeta) \varepsilon \left(\frac{H_h}{H_\zeta} \right)^2$$

$$\varepsilon c_s < 0.023 \quad \text{at } 2\sigma \text{ confidence}$$

Agarwal & Bean '09, Lorenz et al. '08, Lidsey & Huston '07, Stewart & Lyth '93

$$\varepsilon \lesssim 0.4 \, ^*$$

$^* \bar{c}_s \gtrsim 0.05$

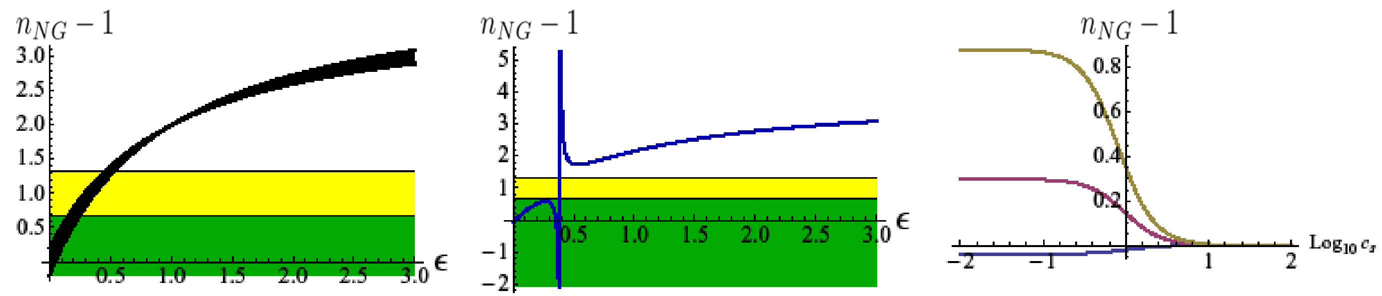
$$n_s = 0.973 \pm 0.028, \quad r < 0.24 \qquad \text{(WMAP7, } 2\sigma \text{ confidence)}$$

Running non-Gaussianities

$$f_{\rm NL} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3} \qquad n_{\rm NG} - 1 \equiv \frac{{\rm d} \ln |f_{\rm NL}|}{{\rm d} \ln K}$$

Fast-roll generically gives rise to a large blue running n_{NG}.

Running Non-Gaussianity



cf. JN & Magueijo '11 , Byrnes et al. '10, Khoury & Piazza '09, LoVerde et al. '08

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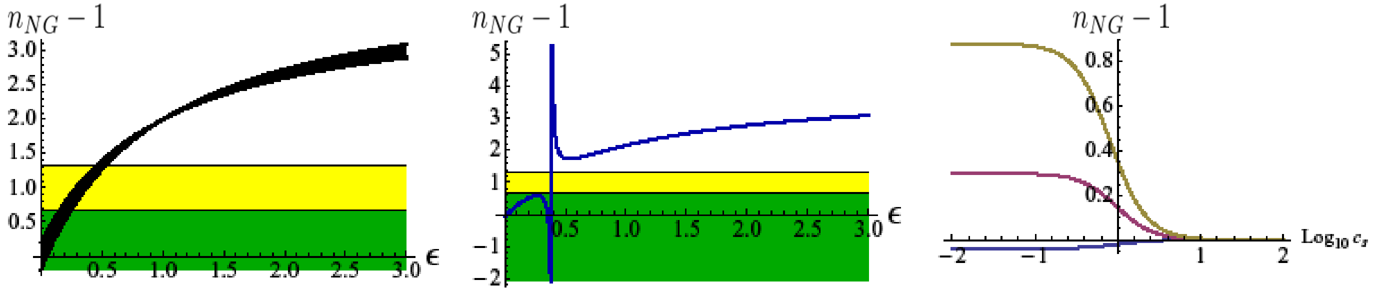
Running Non-Gaussianity



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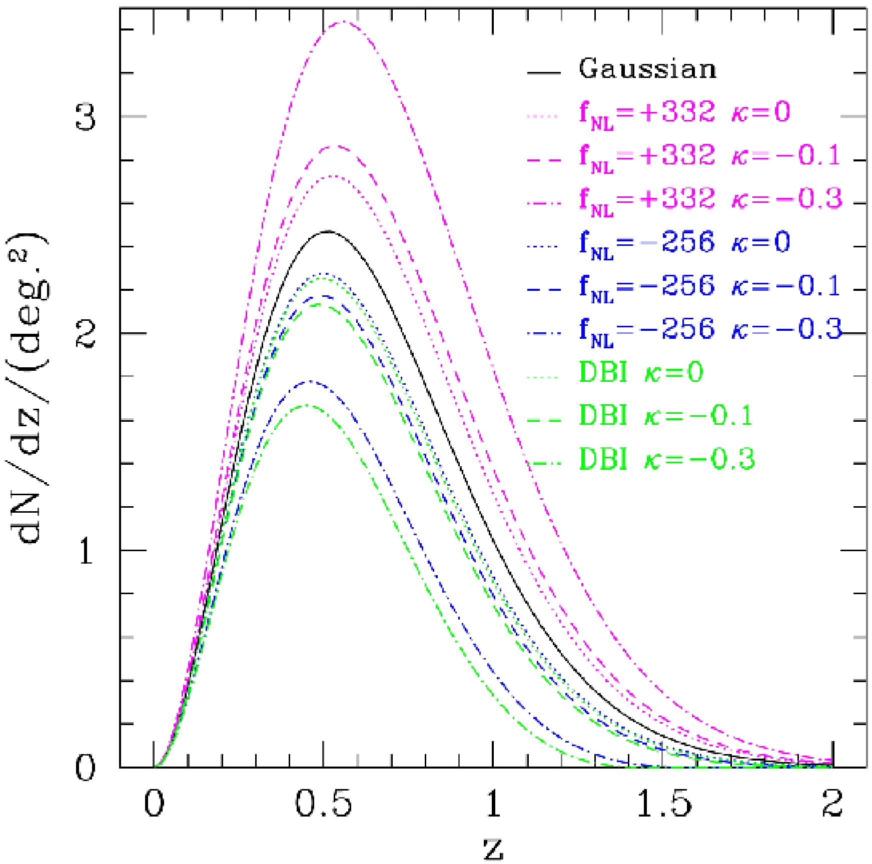
Strong coupling constraints

$$\frac{\mathcal{L}_{\rm int}}{\mathcal{L}_2} \ll 1 \qquad \Lambda^4_{\dot\zeta(\partial\zeta)^2} \sim 16\pi^2 M_{Pl}^2 \dot H \frac{c_s^5}{1-c_s^2}$$

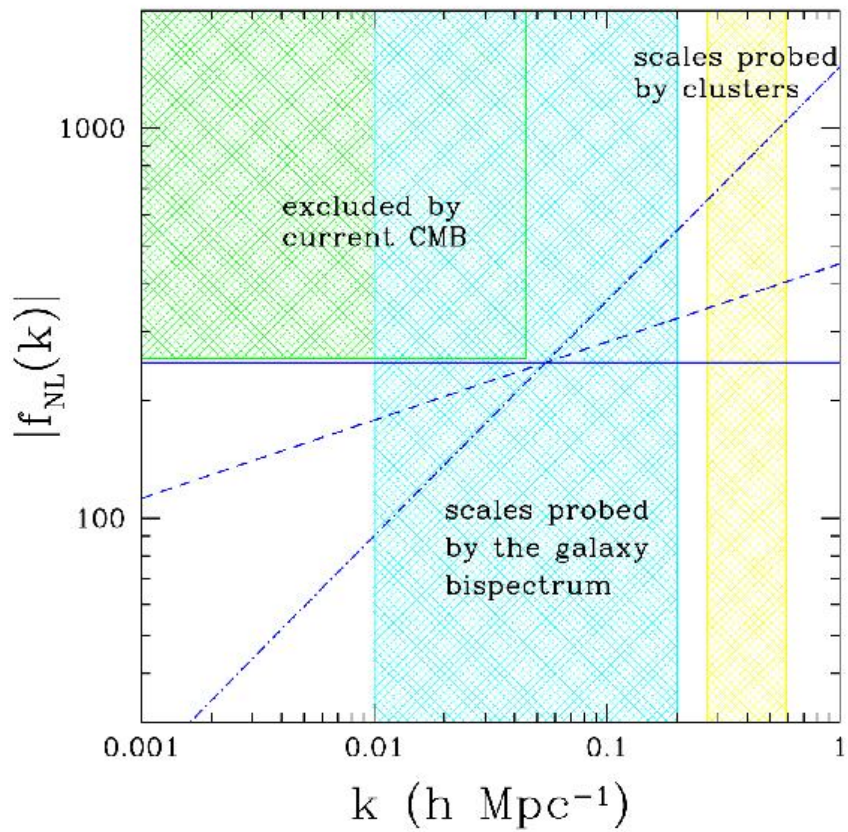
cf. Baumann et al. ‘11, Senatore et al. ‘09, Cheung et al. ‘07

$$\epsilon \lesssim 0.3 \qquad f_{NL}(\mathrm{CMB}) \lesssim 100$$

Running non-Gaussianities



$$M > M_{lim} = 1.75 \cdot 10^{14} h^{-1} M_{sun}$$



LoVerde et al. '08, Creminelli et al. '06

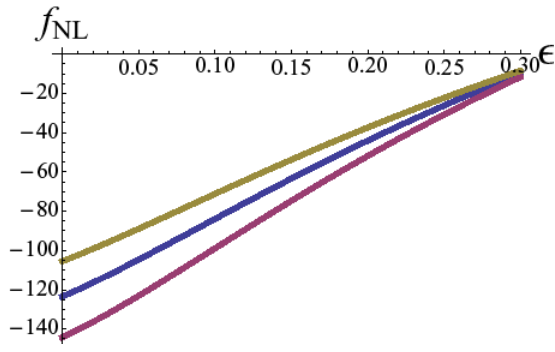
Plots from LoVerde et al. '08

Fast-roll phenomenology

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \sim 0, \quad \eta_s \equiv \frac{\dot{\epsilon_s}}{\epsilon_s H} \sim 0$$

$$f_{NL}^{\frac{\lambda}{\Sigma} \gg 1} \sim 3\frac{1+\epsilon}{n_s - 2}\frac{\lambda}{\Sigma}\cos\left(\alpha_2 \frac{\pi}{2}\right)\Gamma(3+\alpha_2)$$

Fast-roll generically suppresses the amplitude $\mathcal{A}$



JN & Magueijo '11, Magueijo, JN, Piazza '10, Khoury & Piazza '09

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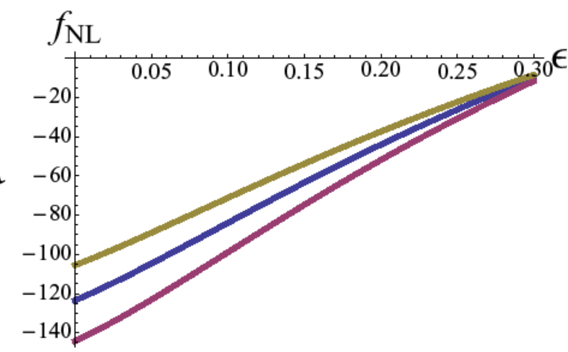
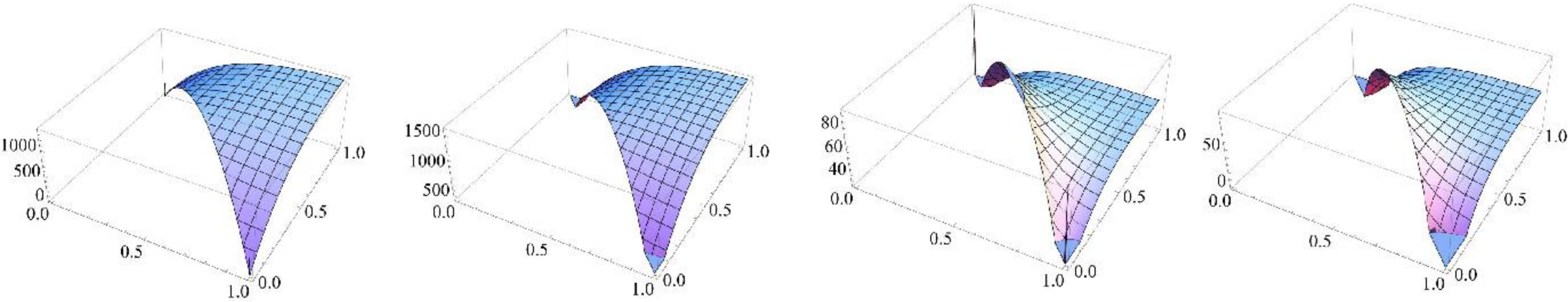
Dominant folded shapes as a signature



JN & Magueijo '11, Magueijo, JN, Piazza '10, Khoury & Piazza '09



$(\epsilon, n_s) = (0.001, 1)$, $(0.001, 0.96)$, $(0.3, 1)$, $(0.3, 0.96)$

$c_s = \mathcal{O}(0.05)$, $\frac{\lambda}{\Sigma} = \mathcal{O}(-10^4)$

Summary

- **Slow-roll violations observationally allowed**
  Bounds compatible with $\epsilon \lesssim 0.3$
- **A variety of observational signatures:**
  Suppressed f_{NL}, "folded" amplitudes, large running n_{NG}

Thank you!

JN & Magueijo, (PRD 83, 103511), arXiv:1102.0275