A fitting formula for the effects of massive neutrinos in the nonlinear regime

Daniel Francisco Boriero
Instituto de Física "Gleb Wataghin", UNICAMP, 13083-859 Campinas, Brasil

In collaboration with Yvonne Wong, RWTH Aachen, Germany

Content

Motivation

- Theoretical tool to
  - measure neutrino mass, if not possible then
  - put better constraints on neutrino mass, if not possible then
  - make constraints on neutrino mass more reliable, if not possible then
  - decrease the degeneracy among other cosmological parameters and the neutrino mass, therefore increasing the theoretical precision.
- The nonlinear effects need to be fully described because
  - the prediction accuracy is required to get below 1% to account for the scales and the precision that future surveys will measure galaxy populations,
- A fitting formula is useful because
  - although N-Body simulations already traced the effects of massive neutrinos, its time and CPU consuming make it impracticable to work as theoretical counterpart everytime a cosmological model is tested.

- **Matter Power Spectrum** definition: two point correlation of all matter density perturbation

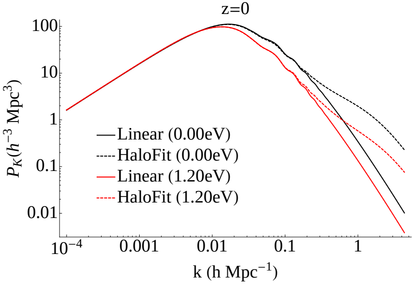
$$(2\pi)^3 P(k)\delta^3(\mathbf{k}-\mathbf{k}') \equiv \left\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}^*(\mathbf{k}') \right\rangle , \qquad (1)$$

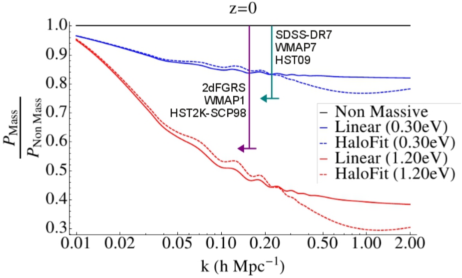
which is usually calculated with first order perturbation theory.
- Neutrino mass has a linear relation with the neutrino density which is a hot component of dark matter

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93.8eV} , \qquad (2)$$

- HDM manifests itself as a suppression of power on small scales

$$\frac{\Delta P(k)}{P(k)}\Big|_{\text{Linear}} \sim -8\frac{\Omega_\nu}{\Omega_M} , \quad \frac{\Delta P(k)}{P(k)}\Big|_{\text{Halofit}} \sim -10.3\frac{\Omega_\nu}{\Omega_M} , \qquad (3)$$





- **Halofit**[1] was created to fit the galaxy clustering by gravitational colapse

$$\frac{k^3}{2\pi^2} \times \underbrace{P(k)}_{\text{Non-Linear Spectrum}} = \underbrace{\Delta^2(k)}_{\text{Dimensionless Spectrum}} = \underbrace{\Delta^2_Q(k)}_{\text{Quasi-Linear Term}} + \underbrace{\Delta^2_H(k)}_{\text{Halo Term}} , \quad (4)$$

$$\Delta^2_Q(k) = \underbrace{F\left[k, \Delta^2_L(k); \alpha, \beta\right]}_{\text{Evolution of linear spectrum to the next orders of perturbation}} , \quad (5)$$

$$\Delta^2_H(k) = \underbrace{G\left[k, \Omega_M; a, b, c, f1, f2, f3, \gamma, \mu, \nu\right]}_{\text{Halo term}} , \quad \Omega_M \equiv \Omega_{CDM} + \Omega_b + \Omega_\nu , \quad (6)$$

- Halofit uses series expansion over the effective index and spectral curvature of the linear spectrum. There are 33 free parameters.
- Calibrated over N-Body simulations without massive neutrinos.
- The best fit is found in Smith[1] and it is still the one working in CAMB[2].

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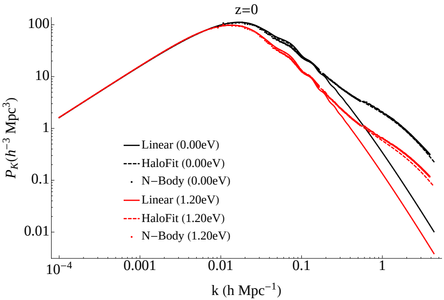
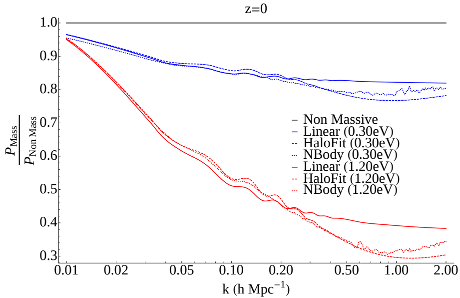
[1] R. E. Smith et. al., Monthly Notices of the Royal Astronomical Society 341, 1311 (2003).
[2] A. Lewis, A. Challinor, and A. Lasenby, Astrophys. J. 538, 473 (2000).

- **N-Body simulations** tracked the effect of thermal neutrino motion[3]
- Simulations based upon GADGET-2[4]

$$\frac{\Delta P(k)}{P(k)}\Big|_{\text{Halofit}} \sim -10.3\frac{\Omega_\nu}{\Omega_M}\;, \quad \frac{\Delta P(k)}{P(k)}\Big|_{\text{N-Body}} \sim -9.8\frac{\Omega_\nu}{\Omega_M}\;, \tag{7}$$





[3] J. Brandbyge, S. Hannestad, T. Haugblle, and B. Thomsen, JCAP 08, 020 (2008).
[4] V. Springel, Mon.Not.Roy.Astron.Soc. 364, 1105 (2005).

- **A Halofit correction** is required because the conventional Halofit doesn't account properly for massive neutrinos

$$\underbrace{P(k,z,m_\nu)}_{\text{Corrected Halofit Spectrum}} \equiv \underbrace{P(k,z)}_{\text{Original Halofit Spectrum}} \times \underbrace{r(k,z,m_\nu)}_{\text{Our fitting formula calibratred over N-Body simulation}} , \tag{8}$$

- Calibration procedure

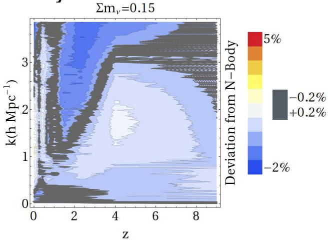
$$r(k,z,m_\nu) = Fit\{ \underbrace{\left(\frac{\text{N-Body}(k,z,m_\nu)}{\text{N-Body}(k,z,0)}\right)}_{\text{Suppression calculated by N-Body}} \div \underbrace{\left(\frac{P(k,z)}{P_{m_\nu=0}(k,z)}\right)}_{\text{Suppression obtained by original Halofit}} \} , \tag{9}$$

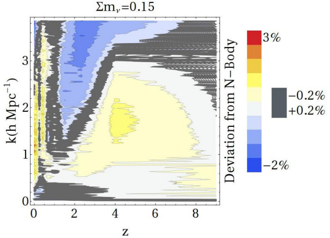
$$r(k,z,m_\nu) \times \left(\frac{P(k,z)}{P_{m_\nu=0}(k,z)}\right) \longrightarrow \text{Mimic N-Body suppression as good as possible.}$$

- The calibration was made using N-Body simulations provided by S. Hannestad[5].
- (m_ν:0 $\rightarrow$ 1.2eV), (V:256^3, 1024^3, 4096^3), (z:0 $\rightarrow$ 49), (k:0.0077 $\rightarrow$ 3.9/hMpc).

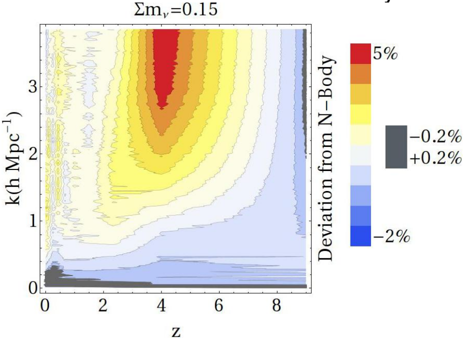
[5] J. Brandbyge, S. Hannestad, T. Haugblle, and B. Thomsen, JCAP 08, 020 (2008).

- Deviation of our fitting formula from N-Body data





- Deviation of Halofit from N-Body data



- **The fitting formula**

$$r(k,z,m_\nu) = 1 + p_1 m_\nu k \left[p_2 k^{p_3} (1 + Tanh[1-z]) + p_4 Exp\left(-\frac{(z - p_5 k^{p_6})^2}{p_7} \right) \right] ,$$

$$(p_1, p_2, p_3, p_4, p_5, p_6, p_7) \to (0.139, 0.054, 0.648, 0.727, 2.039, 0.561, 5) , \quad (10)$$

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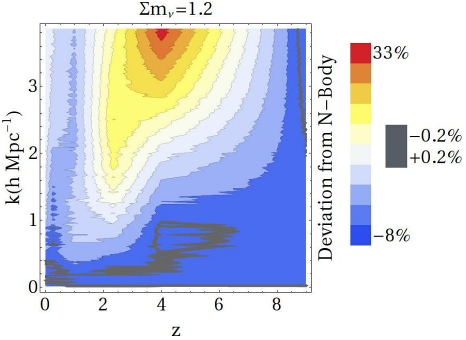

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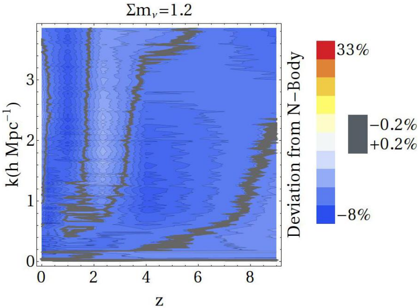
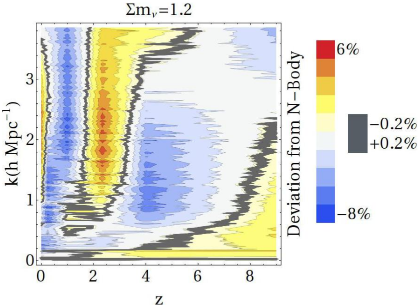
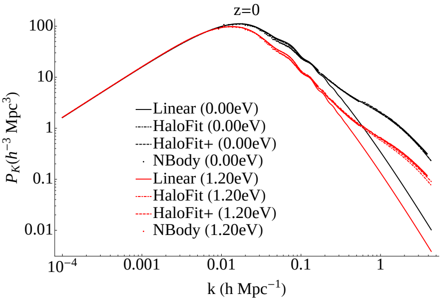
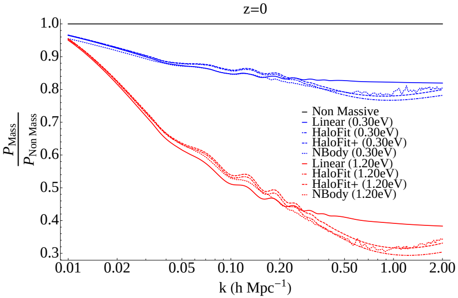




- The corrected Halofit tracks the N-Body data better than the original





- Simeon Bird, Matteo Viel and Martin G. Haehnelt,arXiv:1109.4416v1[astro-ph.CO]

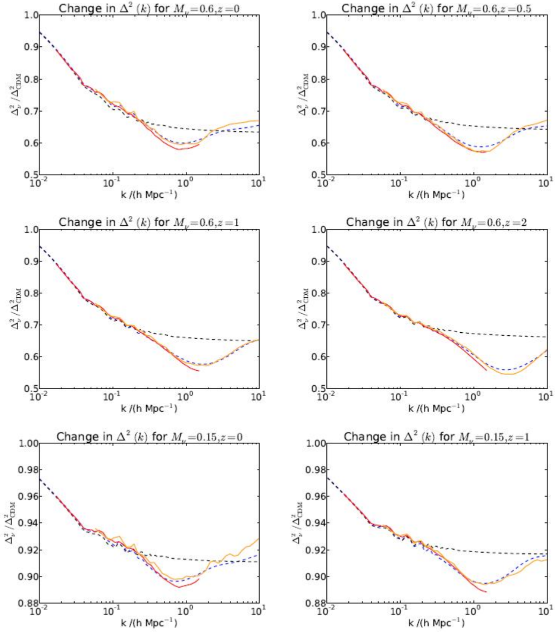


Figure 6. The effect of massive neutrinos on the matter power spectrum for a variety of neutrino masses. Solid lines show the ratio between simulations with and without massive neutrinos, for both 512 Mpc h^{-1} (red) and 150 Mpc h^{-1} (orange) boxes. The blue dashed line shows the estimated ratio using our improved fitting formula, while the black dashed line shows the prediction from linear theory.

Outlook

The next steps will be

1. test the conventional and the corrected Halofit against the own N-Body data.
2. test the corrected Halofit against the real large scale structure data.
3. evaluate the sensitivity to detect the neutrino mass in cosmological surveys in the future.