

Quartic self-couplings in Inert Doublet Model and DM data

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collaboration with I.F. Ginzburg , K.A. Kanishchev and M. Krawczyk
[Phys.Rev.D82:123533,2010, arXiv:hep-ph/1009.5099, 1104.3326, 1107.1991]

Motivation

Two Higgs Doublet Model (2HDM):

- two scalar $SU(2)_W$ doublets Φ_S, Φ_D with the same hypercharge $Y = 1$
- rich phenomenology: CP violation in the scalar sector, heavy Higgs, ...
- **different types of extrema** (e.g. possible violation of $U(1)_{EM}$ or CP)
→ evolution of vacuum state in the past
- 2HDM with an exact Z_2 symmetry: Inert Doublet Model (IDM)
→ candidate for the dark matter

T.D. Lee, '73; Deshpande, Ma, '78; Barbieri, Hall, Rychkov '06; Cao, Ma, Rajasekaran '07

Testing IDM:

- collider constraints (LEP II, Tevatron, LHC)
→ properties of SM-like Higgs h_S and dark scalars $D_H, D_A, D^\pm$
- astrophysical data
→ WMAP, direct & indirect detection

Lundstrom et al. '08, Barbieri et al. '06, Lopez Honorez et al. '07, Hambye et al. '08,'09,

Agrawal et al. '09, Dolle et al. '09, Arina et al. '09

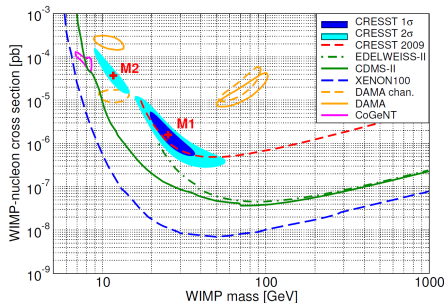
IDM constraints

LEP + S,T,U + DM relic density $\rightarrow$

- **low DM mass** $M_{D_H} \approx (3 - 8) \text{ GeV}$,
large mass splittings: $\Delta(D_A, D_H)$ and $\Delta(D^\pm, D_H)$
- **medium DM mass** $M_{D_H} \approx (40 - 160) \text{ GeV}$,
large $\Delta(D^\pm, D_H)$, small or large $\Delta(D_A, D_H)$
- **high DM mass** $M_H \approx (500 - 1000) \text{ GeV}$,
small $\Delta(D_A, D_H)$ and $\Delta(D^\pm, D_H)$

Lopez Honorez et al. '07,
Hambye et al. '08,'09,
Agrawal et al. '09,
Dolle et al. '09,
Arina et al. '09

Direct detection experiments:



detection signals (DAMA,
CoGeNT, CRESST-II) **point**
to light DM

but there is no agreement
with

exclusion limits
(XENON100, CDMS-II)

2HDM

Scalar potential V invariant under a **D -transformation** of Z_2 type:

$$D: \quad \Phi_S \rightarrow \Phi_S, \quad \Phi_D \rightarrow -\Phi_D, \quad \text{SM fields} \rightarrow \text{SM fields}$$

$$V = -\frac{1}{2} \left[m_{11}^2 \Phi_S^\dagger \Phi_S + m_{22}^2 \Phi_D^\dagger \Phi_D \right] + \frac{1}{2} \left[\lambda_1 (\Phi_S^\dagger \Phi_S)^2 + \lambda_2 (\Phi_D^\dagger \Phi_D)^2 \right] \\ + \lambda_3 (\Phi_S^\dagger \Phi_S) (\Phi_D^\dagger \Phi_D) + \lambda_4 (\Phi_S^\dagger \Phi_D) (\Phi_D^\dagger \Phi_S) + \frac{1}{2} \lambda_5 \left[(\Phi_S^\dagger \Phi_D)^2 + (\Phi_D^\dagger \Phi_S)^2 \right]$$

- all parameters $\in \mathbf{R}$ – no CP violation
- Yukawa interaction: **Model I**, only Φ_S couples to fermions
- **whole Lagrangian explicitly D -symmetric**
 $\rightarrow$ spontaneous violation by $\langle \Phi_D \rangle \neq 0$ still possible

The positivity constraints are required to have a stable vacuum:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad R + 1 > 0, \quad R_3 + 1 > 0$$

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad R = \lambda_{345} / \sqrt{\lambda_1 \lambda_2}, \quad R_3 = \lambda_3 / \sqrt{\lambda_1 \lambda_2}$$

Positivity constraints $\rightarrow$ extremum with the lowest energy is
the global minimum (vacuum).

Spontaneous Symmetry Breaking

The EW symmetric extremum:

$$\langle \Phi_S \rangle = \langle \Phi_D \rangle = 0$$

local minimum if $m_{11,22}^2 < 0$.

Barroso et al., '05

The general type of EWSB v.e.v:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix}$$

$u \neq 0 \implies U(1)_{EM}$ broken:

- charge breaking extremum (Ch)

$u = 0 \implies U(1)_{EM}$ conserved:

- $v_{S,D} \neq 0$ neutral mixed extremum (M)
- $v_D = 0$ neutral inert extremum (I_1)

Deshpande, Ma, '78; Barbieri, Hall, Rychkov, '06

- $v_S = 0$ neutral inertlike extremum (I_2)

Extrema: charge breaking and mixed

Charge breaking extremum Ch :

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \end{pmatrix}$$

- $U(1)_{EM}$ symmetry broken by $u \neq 0$ – **massive photon**
- not a case that is realized now, **a possible vacuum in the past if** today there is **charged DM particle** $\rightarrow$ evolution through Ch excluded
Ginzburg, Kanishchev, Krawczyk, Sokolowska '10

Mixed extremum M :

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- **CP conserving**, $\tan \beta = v_D/v_S$
 - **massive $Z^0, W^\pm$, massless photon**,
- 5 physical Higgs bosons $H^\pm, A, H, h$, **no DM candidate**

Extrema: inert and inertlike

Deshpande, Ma, '78, Barbieri et al., '06

Inert extremum I_1 :

$$\langle \Phi_S \rangle = \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Φ_S as in SM (SM-like Higgs boson h_S)
- Φ_D – "dark" or inert doublet with 4 dark scalars ($D_H, D_A, D^\pm$), no interaction with fermions
- **exact D -symmetry** – both in Lagrangian and in the extremum
- only Φ_D has odd D -parity
 - **the lightest scalar is a candidate for the dark matter**

Inertlike extremum I_2 :

$$\langle \Phi_S \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \Phi_D \rangle = \begin{pmatrix} 0 \\ v_D \end{pmatrix}$$

- Φ_S and Φ_D exchange roles
- fermions massless at tree-level (Model I, only Φ_S couples to fermions)
- **no DM candidate**

Extremum energy

I_1, I_2 and M can overlap in λ parameter space:

- $R > 1 \rightarrow I_1$ and I_2 extrema can coexist:

$$\mathcal{E}_{I_1} - \mathcal{E}_{I_2} = -\frac{m_{11}^4}{8\lambda_1} + \frac{m_{22}^4}{8\lambda_2}$$

$\rightarrow \mathcal{E}_{I_1} - \mathcal{E}_{I_2} < 0$ for I_1 vacuum

$\rightarrow$ possible coexistence of **minima** I_2 and I_1

$\rightarrow$ no M extremum

- $-1 < R < 1 \rightarrow$ also M extremum can be realized:

$$\mathcal{E}_{I_1} - \mathcal{E}_M = \frac{(m_{11}^2 \lambda_{345} - m_{22}^2 \lambda_1)^2}{8\lambda_1^2 \lambda_2 (1 - R^2)}; \quad \mathcal{E}_{I_2} - \mathcal{E}_M = \frac{(m_{22}^2 \lambda_{345} - m_{11}^2 \lambda_2)^2}{8\lambda_1 \lambda_2^2 (1 - R^2)}$$

$$|R| < 1 \Rightarrow \mathcal{E}_{I_1} - \mathcal{E}_M > 0, \quad \mathcal{E}_{I_2} - \mathcal{E}_M > 0$$

$\rightarrow$ to have I_1 vacuum we need $v_{S,D}^2|_M < 0$ (v.e.v of M extremum)

$\rightarrow$ no coexistence of minima

Evolution of the Universe

Ivanov '08; Ginzburg, Kanishchev, Ivanov '09

Ginzburg, Kanishchev, Krawczyk, Sokołowska '10

- assumption: today **Inert Model** is realized, however, in the past some other extrema could have been lower
- evolution of the Universe due to the thermal corrections to the potential

$$V_G(T) = \text{Tr}(V e^{-H/T}) / \text{Tr}(e^{-H/T}) \equiv V(T=0) + \Delta V(T)$$

- $\Delta V(T)$ – leading corrections $\propto T^2$ given by diagrams:



$\Rightarrow$ fixed quartic terms, quadratic (mass) terms change with T

$$\Delta V(T) = \frac{1}{2} c_1 T^2 \Phi_S^\dagger \Phi_S + \frac{1}{2} c_2 T^2 \Phi_D^\dagger \Phi_D$$

- **change of $V \rightarrow m_{ii}^2(T) \rightarrow v_i^2|_{I_1, I_2, M}(T) \rightarrow$ change of ground state**

Evolution of the Universe

Scalar, **bosonic** and **fermionic** contributions to $\Delta V \rightarrow m_{ii}^2(T)$:

$$m_{11}^2(T) = m_{11}^2 - c_1 T^2, \quad m_{22}^2(T) = m_{22}^2 - c_2 T^2$$

$$c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8} + \frac{g_t^2 + g_b^2}{2}, \quad c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8}$$

- **fermionic contribution** in c_1 (Model I)
- $c_1 + c_2 > 0$ from positivity constrains
- c_1 and c_2 **positive** to restore EW symmetry in the past

For a given T we determine:

- sign of $v_i^2|_{I_1, I_2, M}(T) \rightarrow$ possible existence of a given extremum
- values of λ_i (fixed) $\rightarrow$ existence of a local minimum
- value of **extremum energy** $\rightarrow$ global minimum

$\Rightarrow$ **sequences of possible phase transitions**

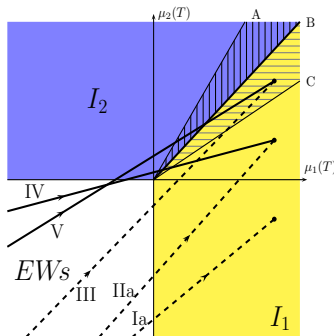
For $u = 0$ (neutral extrema) three separate cases of $EWs \rightarrow \dots \rightarrow I_1$:

$$R = \lambda_{345}/\sqrt{\lambda_1 \lambda_2} : \quad R > 1, \quad 1 > R > 0, \quad 0 > R > -1$$

Possible rays: $R > 1$

The possible sequences of phase transitions (**rays**) on $(\mu_1(T), \mu_2(T))$ plane:

$$\mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \quad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}.$$



$R > 1$

horizontal hatch – I_1 global, I_2 local min,
vertical hatch – I_2 global, I_1 local min;

$$\begin{aligned} A : \mu_2(T) &= \mu_1(T)R, \\ B : \mu_2(T) &= \mu_1(T), \\ C : \mu_2(T) &= \mu_1(T)/R \end{aligned}$$

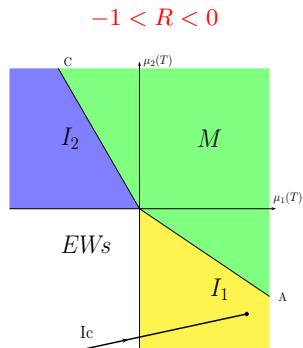
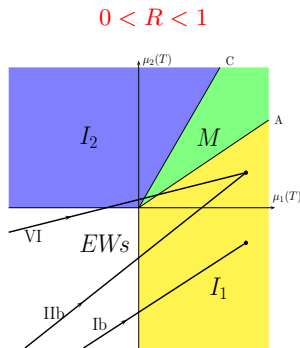
$EWs \rightarrow I_1$

- ray **Ia** – I_2 is not an extremum
- ray **IIa** – I_2 is an extremum, but never was a (local) minimum
- ray **III** – I_2 is a **local** minimum, but never was a global minimum

$EWs \rightarrow I_2 \rightarrow I_1$

- ray **IV** – I_2 is not a local minimum, but was a global minimum in the past
- ray **V** – I_2 is a **local** minimum, it was a global minimum in the past
- **unique possibility: 1st order phase transition $I_2 \rightarrow I_1$**

Possible rays: $|R| < 1$



$EWs \rightarrow I_1$

- rays **Ib**, **Ic** – I_2 is not an extremum
- rays **IIb** – I_2 is an extremum, but never was a (local) minimum

$EWs \rightarrow I_2 \rightarrow M \rightarrow I_1$

- ray **VI** – I_2, M were global minima in the past

Physical parameters

$$(\mu_1, \mu_2, R, c_1, c_2) \rightarrow (M_{h_S}, M_{D^\pm}, M_{D_A}, M_{D_H}, \lambda_{345}, \lambda_2)$$

λ_{345} – **triple** and **quartic** coupling: $D_H D_H h_S$ and $D_H D_H h_S h_S$

- main annihilation channel $D_H D_H \rightarrow h_S \rightarrow f \bar{f}$:

$$\sigma \propto \lambda_{345}^2 / (4M_{D_H}^2 - M_{h_S}^2)^2 \Rightarrow \text{constraints from relic density data}$$

- DM-nucleon elastic scattering via h_S :

$$\sigma_{DM,N} \propto \lambda_{345}^2 / (M_{D_H} + M_N)^2 \Rightarrow \text{direct detection experiments}$$

λ_2 – **quartic** self-coupling $D_H D_H D_H D_H$

- no influence on DM relic density
- non-accessible in the colliders
- limits λ_{345} through positivity constraints
- important for the type of evolution

$(\lambda_{345}, \lambda_2)$ phase space

We introduce the $(\lambda_{345}, \lambda_2)$ plane:

- **fixed** scalar masses, λ_{345} and λ_2 **vary**
- conditions for $(\mu_1, \mu_2) \rightarrow$ conditions for $(\lambda_{345}, \lambda_2)$
- each ray in separate region of $(\lambda_{345}, \lambda_2)$
- low, medium and high DM mass
→ different rays possible in each region

We check the agreement with:

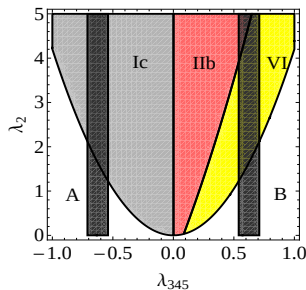
- collider constraints & EWPT
- direct detection
- estimation of $\Omega_{DM} h^2$ (micrOMEGAs)

Dark Matter data and constraints on quartic couplings in IDM, arXiv:1107.1991

All examples here for SM-like Higgs mass $M_{h_S} = 120$ GeV.

Low DM mass region

- low DM mass $M_{D_H} \approx (3 - 8)$ GeV
- large mass splittings: $M_{D_A} \approx M_{D^\pm} \approx 100$ GeV
→ no coannihilation, mimics singlet DM
- direct detection measurements: excluded by XENON100, allowed by CRESST-II, DAMA/Libra and CoGent
- Fermi-LAT data: strong gamma-ray flux limits



- limited number of rays
- large λ_2 needed, rather large λ_{345}
- T of final transition lower for large λ_{345}
→ $T_{M,1} \approx M_{D_H}$
→ next order of correction to V needed

A – excluded by positivity constraints

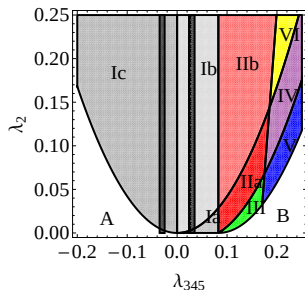
B – I_2 is a global minimum

vertical bounds – WMAP-allowed region

$$M_{D_H} = 5 \text{ GeV}, \quad M_{D_A} = 105 \text{ GeV}, \quad M_{D^\pm} = 110 \text{ GeV}$$

Medium DM mass region

- medium DM mass $M_{D_H} \approx (45 - 160)$ GeV
- large $(D_H, D^\pm)$ mass splittings: $M_{D^\pm} - M_{D_H} \approx (50 - 90)$ GeV
 - **large (D_H, D_A) mass splitting:** $M_{D_A} - M_{D_H} \approx (50 - 90)$ GeV
 $\Rightarrow$ **no coannihilation**
 - **small (D_H, D_A) mass splitting:** $M_{D_A} - M_{D_H} < 8$ GeV
 $\Rightarrow$ **coannihilation**



- all rays possible for all masses
- $\Omega_{DM} h^2$: strong dependence on M_{D_H} and $(M_{D_A} - M_{D_H})$
- for rays VI, IV, V – low T of final phase transition possible
- rays IV and V – 1st order phase transition

A – excluded by positivity constraints

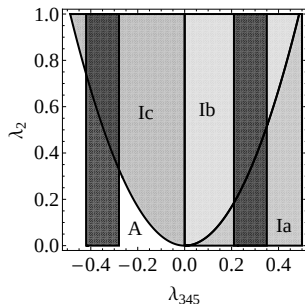
$B - I_2$ is a global minimum

vertical bounds – WMAP-allowed region

$$M_{D_H} = 50 \text{ GeV}, \quad M_{D_A} = 120 \text{ GeV}, \quad M_{D^\pm} = 120 \text{ GeV}$$

High DM mass region

- high DM mass $M_{D_H} \approx (500 - 1000)$ GeV
- **small mass splittings:** $M_{D_H} \approx M_{D_A} \approx M_{D^\pm}$
- **coannihilation between all dark particles**
- only light Higgs h_S possible (EWPT)



- **only rays Ia, Ib, Ic**
- other rays require $\lambda \approx O(20)$

A – excluded by positivity constraints
 vertical bounds – WMAP-allowed region

$$M_{D_H} = 800 \text{ GeV}, \quad M_{D_A} = 801 \text{ GeV}, \quad M_{D^\pm} = 801 \text{ GeV}$$

Medium DM mass: example

- fixed values of scalars' masses $\rightarrow (\lambda_{345}, \lambda_2)$ phase space:

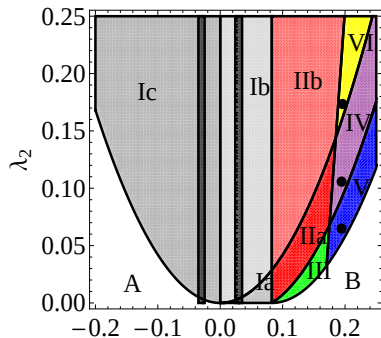
$$M_{D_H} = 50 \text{ GeV}, M_{D_A} = 120 \text{ GeV}, M_{D^\pm} = 120 \text{ GeV}, M_{h_S} = 120 \text{ GeV}$$

- fixed value of λ_{345} :

$$\lambda_{345} = 0.1945$$

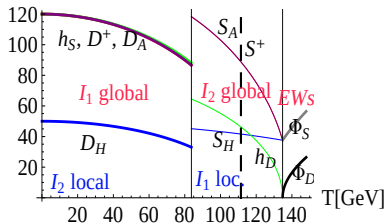
- rays may differ only by value of λ_2

Ray no.	λ_2
$EWs \rightarrow I_2 \rightarrow I_1$	
IV	$\lambda_2 = 0.1031$
V	$\lambda_2 = 0.0684$
$EWs \rightarrow I_2 \rightarrow M \rightarrow I_1$	
VI	$\lambda_2 = 0.1672$



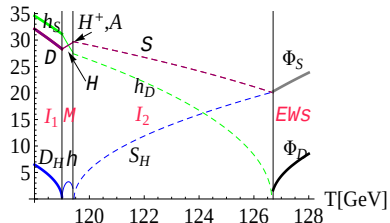
Medium DM mass: example

IV	$T_{EWSB} = 131.7 \text{ GeV}, T_{2,1} = 107.5 \text{ GeV}$
V	$T_{EWSB} = 134.8 \text{ GeV}, T_{2,1} = 83.7 \text{ GeV}$
VI	$T_{EWSB} = 126.7 \text{ GeV}, T_{2,M} = 119.4 \text{ GeV}, T_{M,1} = 119.0 \text{ GeV}$



scalar masses for ray V

1st order transition – discontinuity,
dashed line for $T = 111 \text{ GeV}$:
 I_1 local minimum appears
coexistence of I_1 and I_2 for $T = 0$



scalar masses for ray VI

2nd order transition – continuity:
 $(S_H, h, D_H), (S_A, A, D_A),$
 $(S^\pm, H^\pm, D^\pm), (h_D, H, h_S)$
notation: $D = D_A, D^\pm, S = S_A, S^\pm$

Different behaviour for different λ_2 .

Conclusions

- Today – DM from Inert Model (collider data, EWPT, WMAP).
- Different types of extrema can be realized in the past.
- Possible sequences of phase transitions:

$$EWs \rightarrow I_2 \rightarrow M \rightarrow I_1$$

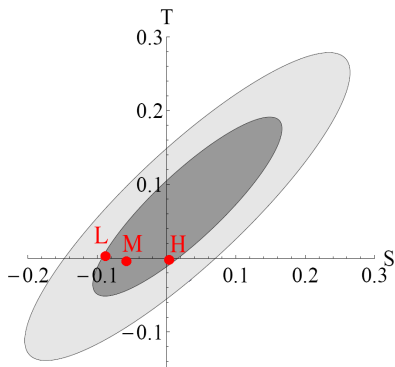
$$EWs \rightarrow I_2 \rightarrow I_1$$

$$EWs \rightarrow I_1$$

- Inert phase may appear for lower T than in one-step EWSB.
- λ_2 important for the evolution and constraints for λ_{345} .
- Different properties of low, medium and high DM mass cases.
- Need for the further corrections to V .

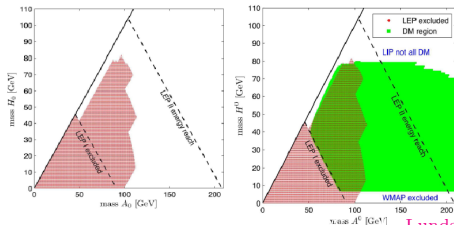
Backup slides: S,T,U oblique parameters

low DM mass	$S = -0.96 \cdot 10^{-2}, \quad T = 5.6 \cdot 10^{-3}$
medium DM mass	$S = -0.57 \cdot 10^{-2}, \quad T = -1.76 \cdot 10^{-3}$
high DM mass	$S = 7.12 \cdot 10^{-3}, \quad T = -1.76 \cdot 10^{-3}$



calculations done with 2HDMC-1.0.3; S,T ellipses taken from arXiv:1011.6188

Backup slides: LEP II data



Lundstrom et al. '08

LEP II data analysis: exclusion of

$$M_{D_H} < 80 \text{ GeV}, M_{D_A} < 100 \text{ GeV} \text{ and } \delta_A > 8 \text{ GeV}$$

For $\delta_A < 8$ GeV the LEP I limit $M_{D_H} + M_{D_A} > M_Z$ applies.

Backup slides: Indirect detection

Gamma rays observations:

- low DM mass: Fermi-LAT gamma-ray flux limits in tension with the expected DM signal
- medium DM mass: possible clear monochromatic γ line (one-loop)
 - so far no observation from Fermi-LAT
 - IDM gamma lines allowed but not with strong boost factor
- high DM mass: signals fall below observational limits unless large boost $\gtrsim 100$ is present

Neutrinos:

- IDM not very constrained by neutrino data
- strongest limits: Super-Kamiokande results challenge low DM mass

Positrons and Antiprotons:

- absence of direct coupling to fermions: no strong positron signals expected
- IDM within the observational limits for positron signal
- low DM mass: weak positron signal, fairly strong antiproton signal

M. Gustafsson, *The Inert Doublet Model and its Phenomenology and references within*