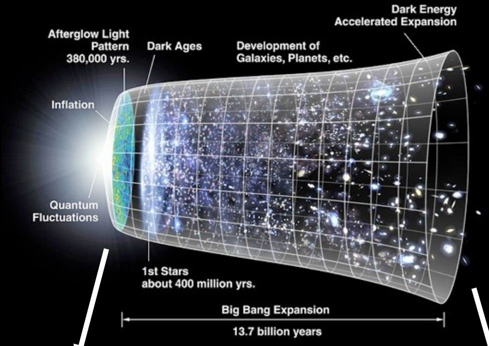
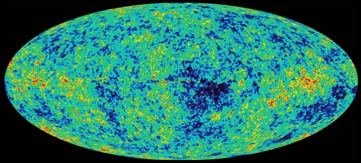
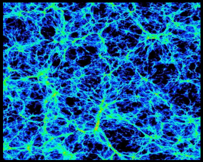
The Matter Bispectrum

Cornelius Rampf

DESY THEORY WORKSHOP
September 29, 2011







$\Delta T = \frac{T}{T_0} \leq \mathcal{O}(10^{-5})$, with $T_0 = 2.725$K

mass density

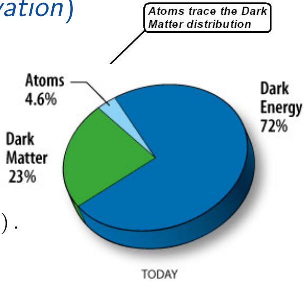
$$\rho(\boldsymbol{x}, t) \equiv \overline{\rho}(t)\,[1 + \delta(\boldsymbol{x}, t)]$$

separation of global effects from local effects: mean density   density contrast

Large Scale Structure (LSS)

The dynamics rely on solving the fluid equations for Dark Matter:

- Continuity equation (*mass conservation*)
- Euler equation (*momentum conservation*)
- coupled to the Poisson equation



$$\frac{\partial\delta(\boldsymbol{x},\tau)}{\partial\tau}+\nabla\cdot\{[1+\delta(\boldsymbol{x},\tau)]\boldsymbol{u}(\boldsymbol{x},\tau)\}=0\,,$$
$$\frac{\partial\boldsymbol{u}(\boldsymbol{x},\tau)}{\partial\tau}+\mathcal{H}(\tau)\boldsymbol{u}(\boldsymbol{x},\tau)+\boldsymbol{u}(\boldsymbol{x},\tau)\cdot\nabla\boldsymbol{u}(\boldsymbol{x},\tau)=-\nabla\Phi(\boldsymbol{x},\tau)\,.$$
$$\Delta\Phi(\boldsymbol{x},\tau)=4\pi Ga^2(\tau)\bar{\rho}(\tau)\delta(\boldsymbol{x},\tau)\,,$$

δ	density contrast	$\mathcal{H}=\mathrm{d}\ln a/\mathrm{d}\tau$	conformal Hubble parameter
$\mathrm{d}\tau=\mathrm{d}t/a$	conformal time	$\boldsymbol{u}$	peculiar velocity

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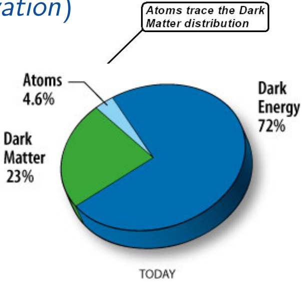


Essentially two ways to solve it:

→ **N-Body simulations**

- \+ full non-linear treatment
- – very time consuming
- – essentially a black-box

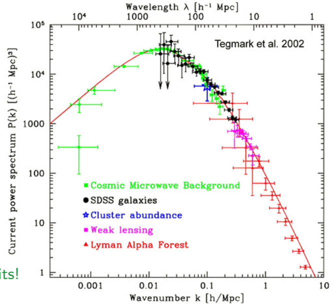
→ **Semi-analytic methods**

- \+ solve the EoMs in an approximate regime
- \+ more insight in dynamics
- – break down at length scales $< \mathcal{O}(1 - 5\text{Mpc})$

What are the observables for the LSS? → $\rho(\boldsymbol{x},t) \equiv \bar{\rho}(t)\,[1+\delta(\boldsymbol{x},t)]$

Construct *statistical* quantities out of the density contrast δ:

1.) **Power spectrum:** $\left\langle \tilde{\delta}(\boldsymbol{k})\tilde{\delta}(\boldsymbol{k}')\right\rangle_c = (2\pi)^3\delta_D^{(3)}\left(\boldsymbol{k}+\boldsymbol{k}'\right)P(k)$



... and current galaxy & weak lensing surveys will give us even tighter limits!

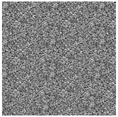
So why are we interested in higher order correlations?

2.) **Bispectrum:** $\boxed{\left\langle \tilde{\delta}(\boldsymbol{k}_1)\tilde{\delta}(\boldsymbol{k}_2)\tilde{\delta}(\boldsymbol{k}_3)\right\rangle_c = (2\pi)^3\delta_D^{(3)}\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)B(k_1,k_2,k_3)}$

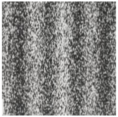
Why are we interested in the Bispectrum?

$P(k)$ is *insensitive* to any phase correlations of $\tilde{\delta}(\boldsymbol{k}) = Ae^{i\varphi_{\boldsymbol{k}}}$

($\varphi_{\boldsymbol{k}}$ is randomly distributed for Gaussian fields)



"Gaussian"



"non-Gaussian"

Bispectrum B is only non-vanishing if they are phase correlations ("non-Gaussianity")

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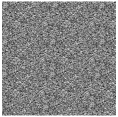

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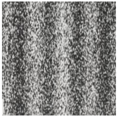

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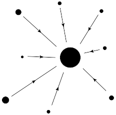
Non-Gaussianities through...



- (non-linear) gravitational evolution

  e.g. continuity equation: $\dot{\delta} + \nabla \cdot \{[1+\delta]\boldsymbol{u}\} = 0$

  → verify Newtonian approx., GR ...

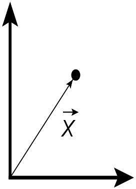
- initial conditions

  modifying the simplest models of inflation lead to $B^{\text{initial}} \neq 0$.

  (deviation from Bunch-Davies vacuum, non-canonical kinetic terms, additional fields)

Analytic Methods for the Matter Bispectrum: Perturbation Theory (PT)

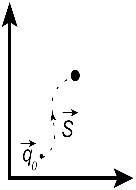
Standard PT



$$\vec{X} = \vec{q}_0 + \vec{S}(\vec{q}_0, t)$$

$\uparrow$ displacement field

Lagrangian PT



Fourier transform EoMs

Linearise EoMs, treat non-linearities as a small correction

$$\tilde{\delta} \approx \tilde{\delta}^{(1)} + \tilde{\delta}^{(2)} + \ldots$$

[Bernardeau et al. 01 and references therein]

$$\tilde{\boldsymbol{S}} \approx \tilde{\boldsymbol{S}}^{(1)} + \tilde{\boldsymbol{S}}^{(2)} + \ldots$$

[Zel'dovich 70, Buchert 87, Matsubara 08,. . .]

in Standard PT:

$$\tilde\delta^{(n)}(\boldsymbol{k}) = D^n \int \mathrm{d}^3\boldsymbol{p}_1 \cdots \int \mathrm{d}^3\boldsymbol{p}_n \delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{p}_{1\cdots n}) F_n^{(s)}(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)\,\tilde\delta^{(1)}(\boldsymbol{p}_1)\cdots\tilde\delta^{(1)}(\boldsymbol{p}_n)$$

linear growth function

Symmetrised kernels (embed the mode-coupling)

$\rightarrow$ $\boxed{\langle\tilde\delta\tilde\delta\tilde\delta\rangle_c \approx \langle\tilde\delta^{(2)}\tilde\delta^{(1)}\tilde\delta^{(1)}\rangle_c + \langle\tilde\delta^{(2)}\tilde\delta^{(2)}\tilde\delta^{(2)}\rangle_c + \langle\tilde\delta^{(3)}\tilde\delta^{(2)}\tilde\delta^{(1)}\rangle_c + \langle\tilde\delta^{(4)}\tilde\delta^{(1)}\tilde\delta^{(1)}\rangle_c + \ldots,}$

[Scoccimarro 1999, Sefussatti et al. 2010,...]

for Gaussian initial conditions.

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[Scoccimarro 1999, Sefussatti et al. 2010,...] for Gaussian initial conditions.

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in Lagrangian PT:

$$\tilde{\boldsymbol{S}}^{(n)}(\boldsymbol{k}) = -iD^n \int \mathrm{d}^3\boldsymbol{p}_1 \cdots \int \mathrm{d}^3\boldsymbol{p}_n \delta_D^{(3)}(\boldsymbol{k}-\boldsymbol{p}_{1\cdots n}) \boldsymbol{L}_n^{(s)}(\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n)\,\tilde\delta^{(1)}(\boldsymbol{p}_1)\cdots\tilde\delta^{(1)}(\boldsymbol{p}_n)$$

The Jacobian of the non-Galilean transformation can be written as $\tilde\delta(\boldsymbol{k}) = \int \mathrm{d}^3q\, e^{i\boldsymbol{k}\cdot\boldsymbol{q}}\left(e^{i\boldsymbol{k}\cdot\boldsymbol{S}(\boldsymbol{q})}-1\right)$.

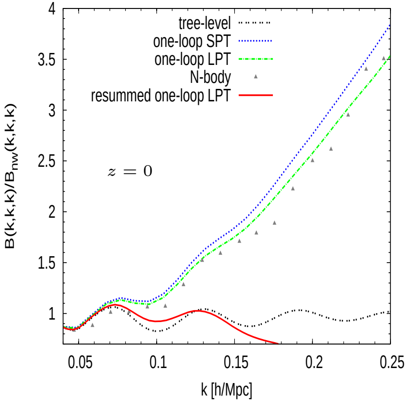
$$\boxed{B(k_1,\,k_2,\,k_3) = \int \mathrm{d}^3\Delta_{21}\mathrm{d}^3\Delta_{31}\,e^{i\boldsymbol{k}_2\cdot\boldsymbol{\Delta}_{21}+i\boldsymbol{k}_3\cdot\boldsymbol{\Delta}_{31}}\left(\underbrace{\left\langle e^{i\boldsymbol{k}_2\cdot[\boldsymbol{S}(\boldsymbol{q}_2)-\boldsymbol{S}(\boldsymbol{q}_1)]+i\boldsymbol{k}_3\cdot[\boldsymbol{S}(\boldsymbol{q}_3)-\boldsymbol{S}(\boldsymbol{q}_1)]}\right\rangle}_{}-1\right)}$$

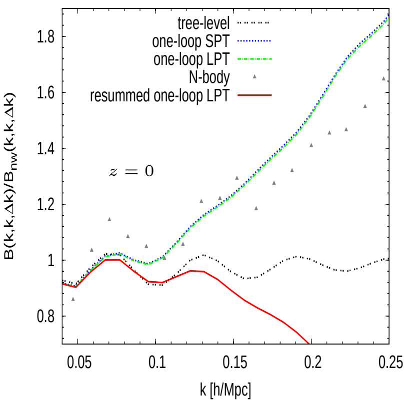
[Y. Y. Y. Wong and CR, in preparation] $\Delta_{ij} \equiv \boldsymbol{q}_i - \boldsymbol{q}_j$.

Leads to a similar relation like above $\longleftarrow$ Taylor expand exponential $\longleftarrow$ use cumulant expansion theorem

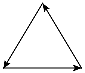
Results *for Gaussian initial conditions and at* $z = 0$

[Y. Y. Y. Wong and CR in preparation, N-body results are from Desjacques et al. 2010]

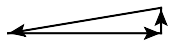




equilateral case



squeezed case, $\Delta k = 0.012h/\text{Mpc}$



Conclusions/Future Work

- the Matter Bispectrum is an important tool (primordial non-Gaussianity?)
- for the first time we can build the Bispectrum in LPT as well
- extending to non-Gaussian initial conditions is fairly easy

There are some neat features and advantages for the LPT Bispectrum:

- calculation of the Bispectrum directly in redshift space
- resummation techniques
- one-loop correction to the (real-space) 3-point correlation function

pictures are from:

- `http://crd.lbl.gov/~borrill/cmb/planck/`
- `http://map.gsfc.nasa.gov/media/080998/`
- `http://www.cfa.harvard.edu/ta/images/`
- `http://www.antapex.org/`