Massive Abelian gauge symmetries and Fluxes in F-theory

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Based on T. Grimm, M.K., E. Palti and T. Weigand [arXiv: 1107.3842]

Outline

1. Motivation
2. Geometric realization of non-Abelian gauge symmetry in F-theory
3. U(1) symmetries and Stückelberg mechanism in Type IIB string theory
4. Supergravity description of geometrically massive U(1)'s and their fluxes in F-theory
5. Time permitting: Geometry of massive U(1)'s
6. Summary and Outlook

Motivation

- U(1) symmetries lead to selection rules on allowed Yukawa couplings and can impact on moduli stabilization
  ➡ direct phenomenological relevance
- U(1) symmetries easily included in Type IIB models with D7-branes
- Only the nature of non-abelian ADE groups in F-theory well-understood
- How do selection rules behave under uplift IIB $\longrightarrow$ F-theory?
- What is F-theoretic analog of U(1)'s which acquire mass through purely geometric Stückelberg mechanism?
- U(1) fluxes in IIB relevant e.g. for chiral matter
  $\longrightarrow$ corresponding mechanism in F-theory?

Non-Abelian gauge groups in F-theory

Consider F-theory on elliptically fibered 4-fold $Y_4 \longrightarrow B_3$

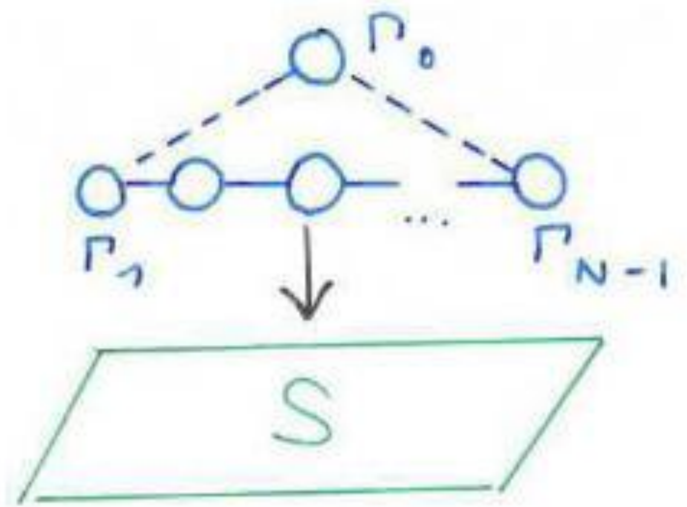
Singularity of fibration along $S \subset B_3$ ⟷ ADE group G along S

[Berschadsky et al.; Morrison,Vafa '96]

To study geometry: resolve singularities

⟶ nonsingular 4-fold $\hat{Y}_4$

- S replaced by blow-up divisors D_i , i = 1, …, rk(G)
- D_i = $\mathbb{P}^1$ fibration $\Gamma_i \hookrightarrow D_i \to S$



- Intersections of Γ_i, Γ_0

↕

extended Dynkin diagram of G

Extended node Γ_0 not homologically independent

Geometric picture of gauge bosons

- Resolution $\longleftrightarrow$ moving to Coulomb branch: $G \longrightarrow U(1)^{rk(G)}$
- Singular space Y_4 $\longleftrightarrow$ zero volume limit of $\mathbb{P}^1$'s Γ_i

- Off-diagonal generators in G $\longleftrightarrow$ membranes wrapped on Γ_i

  $\longrightarrow$ massless only in singular limit

- Cartan U(1)'s from expansion of C_3 along exceptional divisors:

  $C_3 = \sum_i A^i \wedge w_i + \ldots$ w_i = Poincare duals of D_i

Total number of U(1)'s $n_{U(1)} = h^{(1,1)}(\hat{Y}_4) - h^{(1,1)}(B_3) - 1$

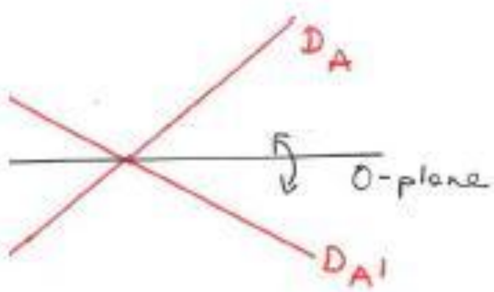
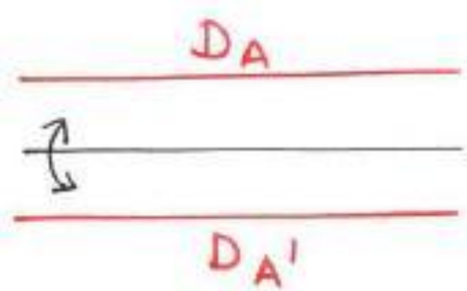
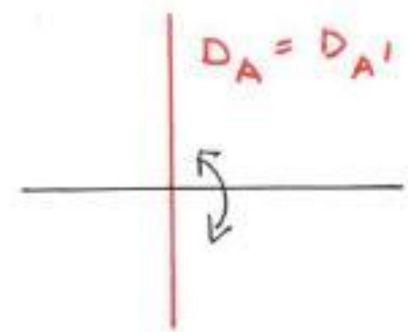
[Morrison,Vafa '96]

$\longrightarrow$ What is the nature of non-Cartan U(1)'s?

U(1) symmetries in Type IIB Orientifolds

Consider N_A D7-branes along divisor D_A and image D_A'

- Possible brane configurations and gauge groups:

$[D_A] \neq [D_A']$	$D_A \neq D_A'$ but $[D_A] = [O7] = [D_A']$	$D_A = D_A'$ or $[D_A] = [D_A'] \neq [O7]$
$U(N_A)=U(1) \times SU(N_A)$	$U(N_A)=U(1) \times SU(N_A)$	Symplectic or orthogonal
		

Focus on first two cases in the following

Stückelberg masses in IIB

Dimensional reduction of CS action

$$\int_{D_A} \sum_{2p} C_{2p} e^{2\pi\alpha' F^A - B} + \int_{D_{A'}} \sum_{2p} C_{2p} e^{2\pi\alpha' F^{A'} - B}$$

[Jockers,Louis '04, …]

➡ gauging of scalars in RR forms, e.g.

$$C_2 = c^a \omega_a + \ldots, \qquad \omega_a \in H_-^{(1,1)}(X_3) \quad \Longrightarrow \quad \nabla c^a = dc^a - Q_A^a A^A$$

- $[D_A] = [D_{A'}]$: gauging only in the presence of nontrivial gauge flux
- $[D_A] \neq [D_{A'}]$: gauging independent of flux
  "geometric gauging"

➡ U(1) becomes massive by "eating" axion

U(1) fluxes in IIB

- Non-trivial U(1) gauge flux controls chirality of matter at intersection of brane stacks A, B

$$I_{AB} = -\int_{X_3} [D_A] \wedge [D_B] \wedge (\tilde{\mathcal{F}}_0^A - \tilde{\mathcal{F}}_0^B) \, , \qquad \tilde{\mathcal{F}}_0^A = 2\pi\alpha' \mathcal{F}_0^A - \iota^* B_+$$

- Flux-induced gauging $\nabla T_\alpha = dT_\alpha - iQ_{A\alpha} A^A$,

$$Q_{A\alpha} \propto \int \omega_\alpha \wedge \left[\tilde{\mathcal{F}}_0^A \wedge [D_A] + \tilde{\mathcal{F}}_0'^A \wedge [D_A'] \right]$$

- Fluxes contribute to D3- and D5-tadpoles, e.g. the D3-tadpole cancellation condition

$$N_{D3} + N_{\rm gauge} = \frac{N_{03}}{4} + \sum_A N_A \frac{\chi_0(D_A)}{24} + \frac{\chi(D_{07})}{6},$$

$$N_{\rm gauge} = -\tfrac{1}{4} \sum_A N_A \left(\int_{D_A} \tilde{\mathcal{F}}_0^A \wedge \tilde{\mathcal{F}}_0^A + \int_{D_A'} \tilde{\mathcal{F}'}_0^A \wedge \tilde{\mathcal{F}'}_0^A \right)$$

Gauged axions in F-theory

For $[D_A] \neq [D_A']$: Stückelberg mechanism is independent of flux

➡ expect this to be realized geometrically in F-theory

➡ M-theory supergravity reduction should reproduce gauging of the axions

Recall massless axions from C_3 in F-theory effective action

[Grimm '10]

$$C_3 = c^a \mathfrak{a}_a + b_a \mathfrak{b}^a + \ldots, \qquad \mathfrak{a}_a, \mathfrak{b}^a \in H^3(Y) - H^3(B)$$

Take local $SL(2,\mathbb{Z})$ basis dx, dy of 1-forms on non-singular torus fiber . Relation to IIB quantities:

$$\mathfrak{a}_a = \omega_a \wedge dx\,; \qquad \mathfrak{b}_a = \omega_a \wedge dy, \qquad \omega_a \in H_-^{(1,1)}(X_3)$$

$$C_3 = C_2 \wedge dx + B_2 \wedge dy + \ldots$$

Gauged axions in F-theory

Proposal: geometric gauging from non-harmonic forms in M-theory dimensional reduction [Grimm,Weigand '10; Grimm,Palti,MK,Weigand '11]

➡ Include non-harmonic forms satisfying

$$d\mathrm{w}_{0A} = Q^a_A\,\alpha_a\,, \qquad d\beta^a = \tilde{Q}^a_A\,\tilde{\mathrm{w}}^{bA}$$

$$\rightarrow C_3 = A^{iA} \wedge \mathrm{w}_{iA} + A^{0A} \wedge \mathrm{w}_{0A} + c^a\alpha_a + b_a\beta^a + \ldots$$

$$\rightarrow G_4 = F^{iA} \wedge \omega_{iA} + F^{0A} \wedge \mathrm{w}_{0A} + \nabla \mathbf{c^a} \wedge \alpha_{\mathbf{a}} + db_a \wedge \beta^a + b_a\, d\beta^a + \ldots$$

$$\nabla c^a = dc^a - Q^a_A A^A$$

Ansatz precisely reproduces gauging and D-terms found in the Type IIB reduction!

Geometric intuition: local uplift $\omega_a \rightarrow \omega_a \wedge dx$ fails to give harmonic form if $Q^a_A \cong \omega_a|_{D_A} \neq 0$ as fibration is singular over brane.

Open: explicit construction of the α_a directly in $\hat{Y}_4$

Massive U(1)´s in F-theory

Geometrically massive gauge boson A^{0A} is at KK-scale

➡ can be consistently omitted in low energy theory

But: expect that as in IIB perturbative selection rules remain as accidental symmetries!

➡ observe same pattern of allowed Yukawa couplings in F-theory upon uplifting IIB models

[Collinucci ’08/’09; Blumenhagen,Grimm,Jurke,Weigand ’09]

Note: in F-theory models without a IIB limit, e.g. models with E8 symmetry, different selection rules are possible

In addition, selection rules may be broken non-perturbatively or by Higgsing of U(1)´s [Grimm,Weigand ’10]

Uplift of fluxes along diagonal U(1)

- Gauge fluxes of all branes encoded in single object G_4
- Dynamics of diagonal U(1) encoded in non-harmonic structure
  $\longrightarrow$ non-harmonic contributions to G_4

$$G_4 = -\tilde{\mathcal{F}}_0^{A,\alpha} \omega_\alpha \wedge \mathrm{w}_{0A} - \tilde{\mathcal{F}}_0^{A,a} \tilde{\mathrm{w}}_{aA}.$$

- Ansatz correctly reproduces flux contributions to tadpoles as well as flux-induced gauging $\nabla T_\alpha \;=\; dT_\alpha - iQ_{A\alpha} A^A$
- Chirality index from IIB is reproduced via an ansatz of the form

$$I_{AB} \propto \int \mathrm{w}_{AB} \wedge G_4$$

where w_{AB} is a 4-form describing the intersection locus of brane stacks A, B including non-harmonic contributions.

Geometry of massless diagonal U(1)'s

In IIB: D-brane and O-plane homologous

The nature of the divisor associated to the U(1) in F-theory varies depending on the intersection properties of the brane.

- Case 1: D, D' intersect
  - ➡ Corresponding D7 locus has self-intersection in F-theory
  - ➡ Singularity type worsens over intersection locus
  - ➡ additional blow-up divisor necessary to resolve, U(1) given by expanding C_3 along corresponding 2-form
- Case 2: D, D' do not intersect
  - ➡ In F-theory: D and O-plane homologous but don't intersect
  - ➡ connecting 5-chain Γ
  - ➡ A-cycle of elliptic fiber shrinks over D and O-plane
  - ➡ Fibering this cycle over Γ gives the desired divisor

Geometry of massive U(1)'s (brief)

- Recall: Tree of $\mathbb{P}^1$'s over singularity along divisor D_A obey

  homological relation $\sum_i \Gamma_{iA} - e = \partial(Q^a_A \mathcal{C}_a)$

- e corresponds to the class of the total elliptic fiber
- Proposal:
  3-chains $\mathcal{C}_a$ dual to the α_a in the sense that $\int_{\mathcal{C}_a} \alpha_b = \delta^a_b$

$\Rightarrow \int_{\mathcal{C}_a} d\mathrm{w}_{0A} = Q^a_A$

- Explicit realization of these curves best studied in the dual picture of deforming rather than resolving the singularities.

Summary

- Uplift of massless diagonal U(1) generically described in F-theory by resolution divisor over self-intersection curve
- Geometrically massive U(1)’s can be described by non-harmonic forms and live at the KK-scale
- The M-theory reduction involving these non-harmonic forms precisely reproduces the structure of the axion gaugings and D-terms found in Type IIB
- The corresponding perturbative selection rules survive in the low energy theory in the form of accidental symmetries
- Flux along diagonal U(1) can be described in F-theory by including non-harmonic forms in expansion of G_4 flux
- This correctly reproduces flux-contributions to tadpoles and chirality indices

Outlook & further work

- Precise geometric understanding of the origin of the non-harmonic forms and the uplift of cohomology groups from X_3 to Y_4 desirable

- Study possible relation between this non-harmonic structure and the similar structure found in torsional cohomology

[see e.g. Camara, Ibanez, Marchesano '11]

- Study effect of massive U(1)'s on D3/M5 instantons and their selection rules

- Compare this description of (the harmonic part of the) G_4 flux to recent explicit constructions

[e.g. Braun, Collinucci, Valandro; Marsano, Saulina, Schäfer-Nameki; Krause, Mayrhofer, Weigand '11]