Goldstone bosons in Higgs inflation

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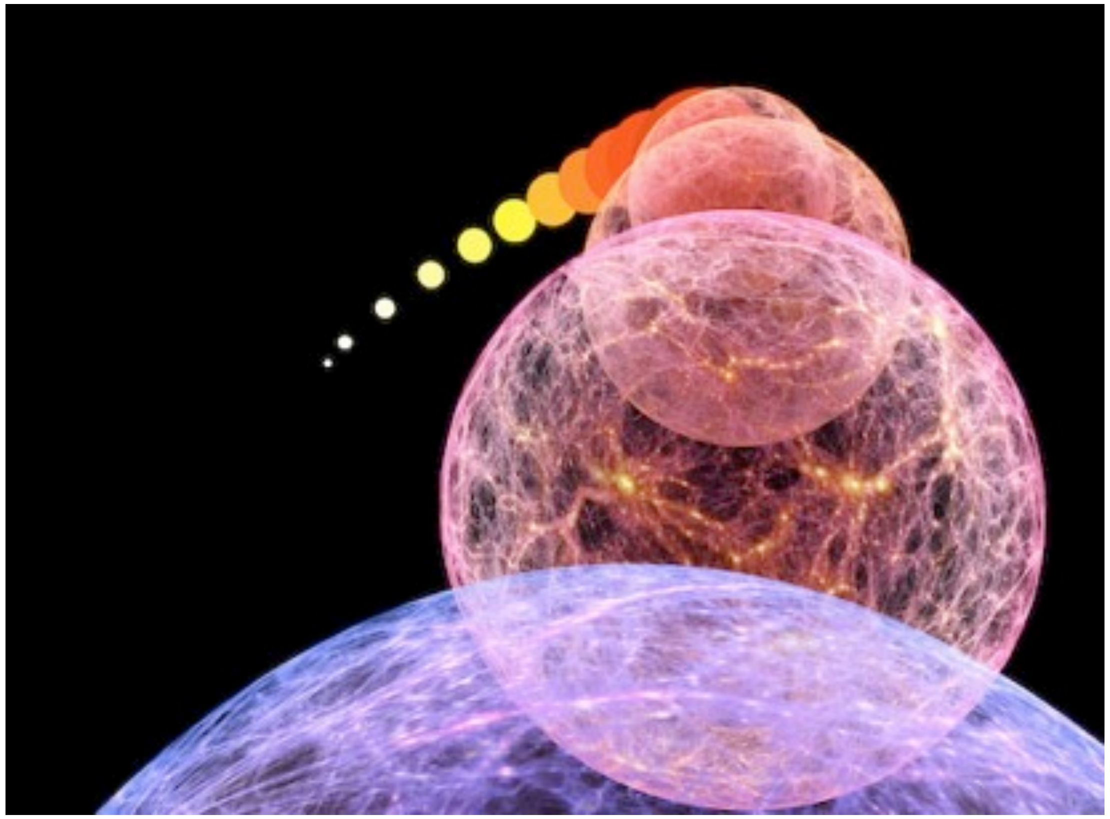
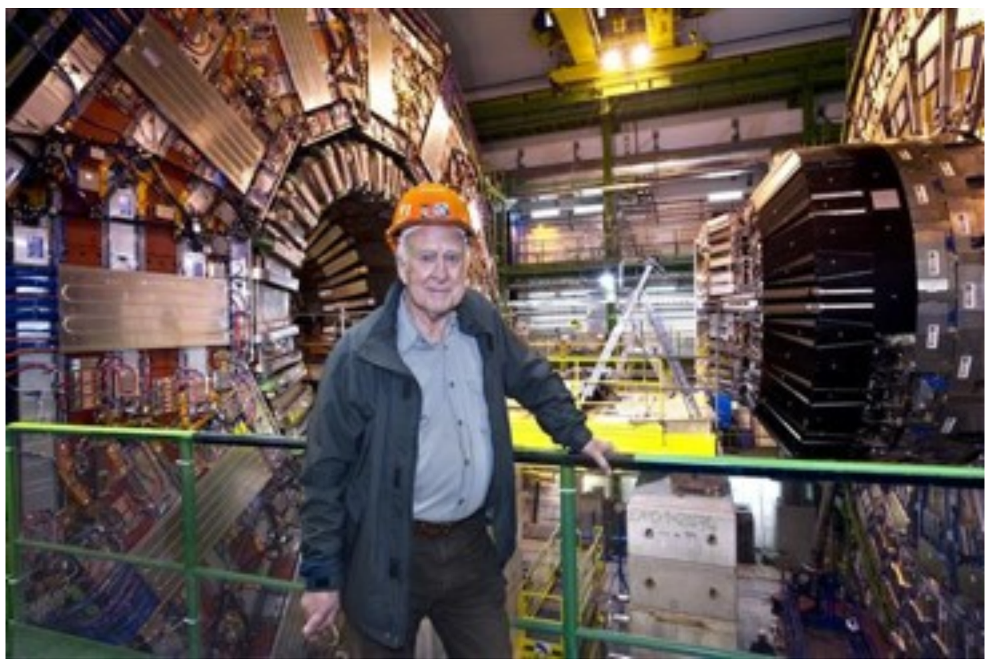
Inflation



- Need a scalar field slowly rolling down an almost flat potential
- “Elegant” approach: susy, sugra, strings
- “Economical” approach: Higgs

Economical approach

- Didn't the Standard Model contain a scalar field?



$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - g^{\mu\nu} (D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) - \lambda \big(\mathcal{H}^\dagger \mathcal{H} - v^2 \big)^2 \right]$$

PROBLEM: flat potential ≠ Mexican hat

Solution: non-minimal Higgs-graviton coupling

Bezrukov & Shaposhnikov, 0710.3755

$$S_J = \int d^4x\sqrt{-\hat{g}}\left[\left(\frac{M_p^2}{2} - \xi\mathcal{H}^\dagger\mathcal{H}\right)R(\hat{g}_{\mu\nu}) - \hat{g}^{\mu\nu}(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H}) - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - v^2\right)^2\right]$$

(Jordan)

Conformal transformation

$$\hat{g}_{\mu\nu} = \omega^2 g_{\mu\nu} \to g_{\mu\nu} \qquad \omega = \left(1 + \frac{2\xi\mathcal{H}^\dagger\mathcal{H}}{M_p^2}\right)^{-1/2}$$

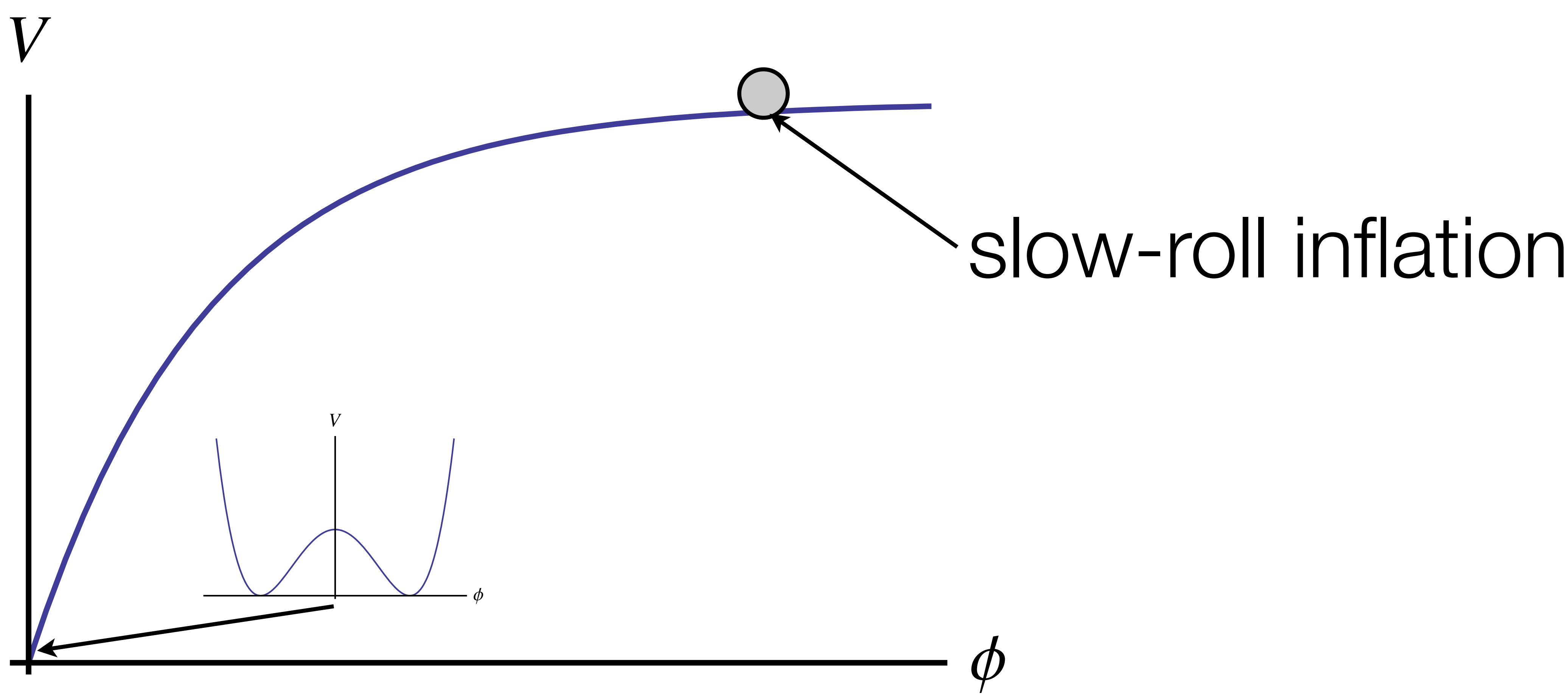
$$\begin{aligned} S_E \quad = \quad & \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R(g_{\mu\nu}) - g^{\mu\nu}\left(3\frac{\omega^4\xi^2}{M_p^2}\partial_\mu\left(\mathcal{H}^\dagger\mathcal{H}\right)\partial_\nu\left(\mathcal{H}^\dagger\mathcal{H}\right)\right.\right. \\ & \left.\left. +\omega^2(D_\mu\mathcal{H})^\dagger(D_\nu\mathcal{H})\right) - \omega^4\lambda\left(\mathcal{H}^\dagger\mathcal{H} - v^2\right)^2\right]. \end{aligned}$$

(Einstein)

- non-minimal coupling removed (back to Einstein gravity)
- non-canonical kinetic terms, deformed potential

Different field regimes

- low field: Standard Model Higgs potential
- high field: slow-roll inflation potential $$\lambda \to \lambda' \ll \lambda$$
- Higgs inflation should ultimately connect these regimes



Higgs field: gauge variant inflaton

- During inflation, the Higgs field slowly rolls down (it is the inflaton)
- It is **NOT** in the minimum of the Mexican hat potential
- A gauge variant scalar field in a symmetry-breaking configuration has Goldstone bosons associated to it

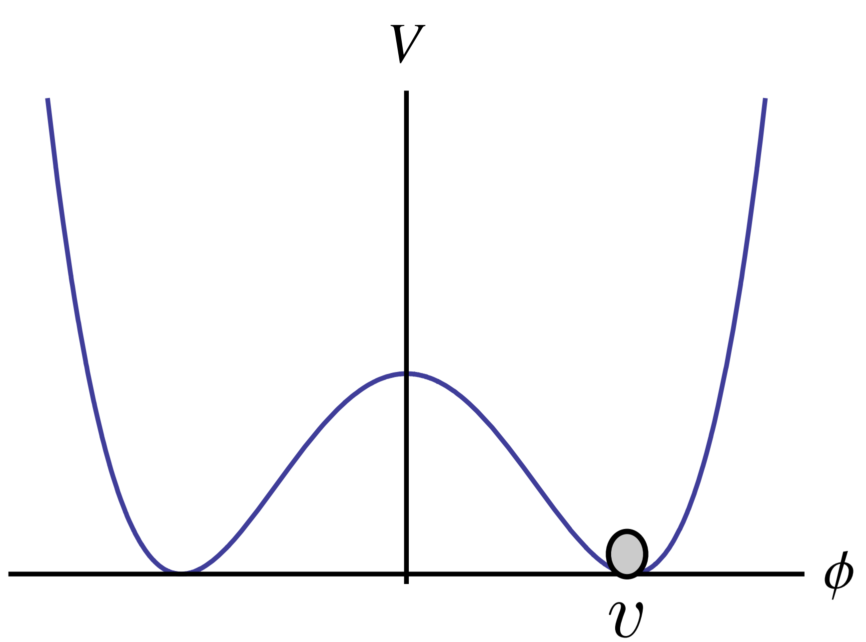
Goldstone Mechanism in Standard Model

[U(1) toy model]

$$\Phi(t,\vec{x}) = \frac{1}{\sqrt{2}}\left[v + h(t,\vec{x}) + i\theta(t,\vec{x})\right]$$

- v: time-independent VEV (246 GeV)
- h: quantum Higgs field
- θ: massless Goldstone boson

$$m_\theta^2 \equiv \left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\text{cl}} = \left.\frac{1}{\phi}\frac{\partial V}{\partial \phi}\right|_{\text{cl}} = 0$$



- Goldstone boson is unphysical: disappears from theory in unitary gauge. Its associated degree of freedom renders the U(1) gauge boson massive.

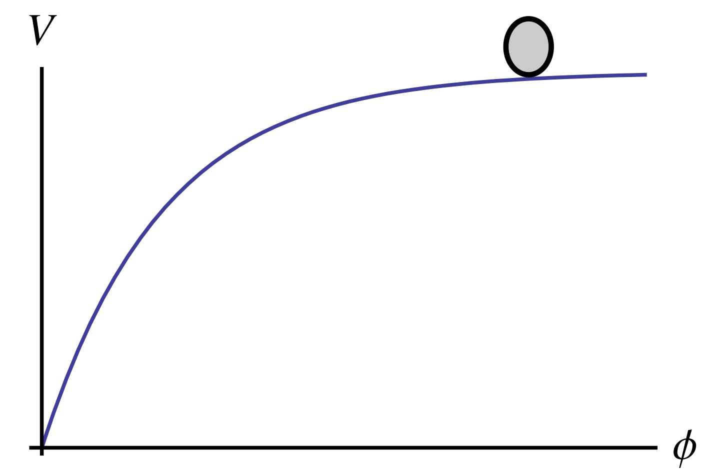
Goldstone Mechanism in Higgs Inflation

[U(1) toy model]

$$\Phi(t,\vec{x}) = \frac{1}{\sqrt{2}}\left[\phi(t) + h(t,\vec{x}) + i\theta(t,\vec{x})\right]$$

- φ: time-dependent classical background field
- h: quantum Higgs field
- θ: massive Goldstone boson

$$m_\theta^2 \equiv \left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\rm cl} = \left.\frac{1}{\phi}\frac{\partial V}{\partial\phi}\right|_{\rm cl} = -\left.\frac{\ddot{\phi}}{\phi}\right|_{\rm cl} \neq 0$$



- Goldstone boson still disappears from theory in unitary gauge. Its associated degree of freedom still renders the U(1) gauge boson massive. Is it still unphysical?

Coleman-Weinberg corrections

$$V_{CW} = \frac{1}{2}\sum_i (-1)^{2J_i}(2J_i+1)\int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m_i^2}$$
$$= \frac{1}{16\pi^2}\sum_i (-1)^{2J_i}(2J_i+1)(m_i^2\Lambda^2 - \frac{1}{4}m_i^2 \ln\left(\frac{\Lambda^2}{m^2}) + \dots\right).$$

(Λ: cut-off on spatial momentum)

Sum is over all fields... what about the Goldstone boson mass? Cross it out?

- YES because it is unphysical

Linde et al, 1008.2942

- "YES" would have dramatic consequences for SUSY Higgs inflation

- NO because that produces a discontinuous CW potential

Solution: compute CW-corrections for time-dependent background

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathrm{gaugekinetic}} + \mathcal{L}_{\mathrm{higgskinetic}} + \mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{gaugefixing}} + \mathcal{L}_{\mathrm{faddeev-popov}} \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\Phi(D^\mu\Phi)^\dagger - V(\Phi\Phi^\dagger) - \frac{1}{2\xi}G^2 + \bar{\eta}g\frac{\delta G}{\delta\alpha}\eta\end{aligned}$$

$$\left(G = \partial_\mu A^\mu - \xi g(\phi + h)\theta\right)$$

- Use Schwinger-Keldysh formalism (closed time path, in-in)
- Similar computations have been done before in other contexts (Boyanovsky, Heitmann & Baacke), we find other coefficients

Closed Time Path Formalism

- Double fields and sources to move forward and backward in time

- No specification of out-states needed

$$
\begin{aligned}
Z[J_-, J_+] &= \sum_\alpha < \Omega, t_{\rm in}|\alpha, t_{\rm out} >_{J_-} < \alpha, t_{\rm out}|\Omega, t_{\rm in} >_{J_+} \\
&= \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \exp\left[iS[\phi_+] - iS[\phi_-] + i\int d^4x J_+\phi_+ - i\int d^4x J_-\phi_- \right]
\end{aligned}
$$

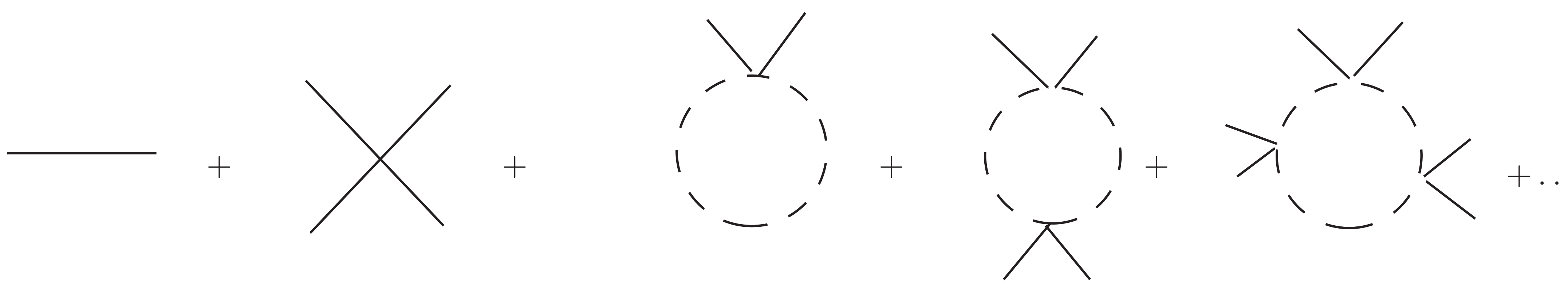
- Very usual for time-dependent problems

Calculation (I): effective action Γ

$$\phi = \phi_{\rm cl}(t) + h(\vec{x}, t)$$

(here only one real scalar field)

- Check how h changes dynamics of φ_cl
- Γ: sum of all 1PI diagrams involving at least one factor φ_cl



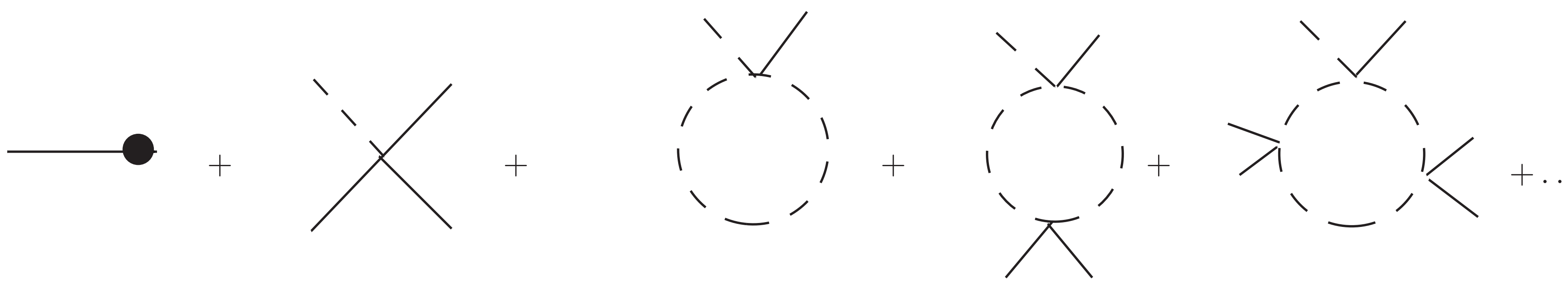
- Non-perturbatively: sum over loop diagrams gives harmonic oscillator all over spacetime

$$\Gamma = \int d^4x \frac{1}{2}\partial_\mu \phi_{\rm cl} \partial^\mu \phi_{\rm cl} - V(\phi_{\rm cl}) - \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} E_{\vec{k}}$$

$$E_{\vec{k}} = \sqrt{k^2 + V''(\phi_{\rm cl})}$$

Calculation (II): dΓ/dΦ

- One external quantum field



- Non-perturbatively: summation over loop diagrams gives resummed equal-time Feynman propagator

$$\frac{d\Gamma}{d\phi} = \partial_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} - \frac{1}{2} V'''(t) \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2E_{\vec{k}}}$$

Calculation (III)

- work in terms of $d\Gamma/d\Phi$, in the end integrate back to Γ
- calculate contributions from all fields in the problem
- complications from terms mixing Goldstone and vector bosons

Result

$$\Gamma = \int d^4x \left\{ \mathcal{L}_{\rm cl} + \frac{1}{16\pi^2}\left[\Lambda^2\left(V_{hh} + V_{\theta\theta} + 3m_A^2\right) - \frac{1}{4}\ln\Lambda\left(V_{hh}^2 + V_{\theta\theta}^2 + 3m_A^4 - 6V_{\theta\theta}m_A^2\right)\right]\right\} + \mathcal{O}(\hbar^2).$$

- Continuous effective action and potential
- Potential reduces to ordinary Coleman-Weinberg in static limit
- **Goldstone bosons do contribute to effective potential** (induced by massive gauge boson?)
- Gauge invariant (ξ-independent)
- unitary gauge ill-defined (limit ξ→∞ does not commute with limit k→∞ in momentum integrals)
- all details in arXiv[1104.4897]

Conclusion & remark

- From a U(1) toy model it already follows that in Higgs inflation Goldstone bosons will contribute to the effective potential

- Effects will be small during inflation, but significant at its end

To do...

- Generalize from U(1) to SU(2) x U(1) (trivial)
- Generalize from Minkowski to FLRW (complicated: gauge field components couple in their equations of motion)
- include non-canonical kinetic terms (difficult for fast field evolution after inflation)
- apply renormalisation group analysis, check for corrections from time-dependent masses