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Fakultät Physik
Theoretische Physik III

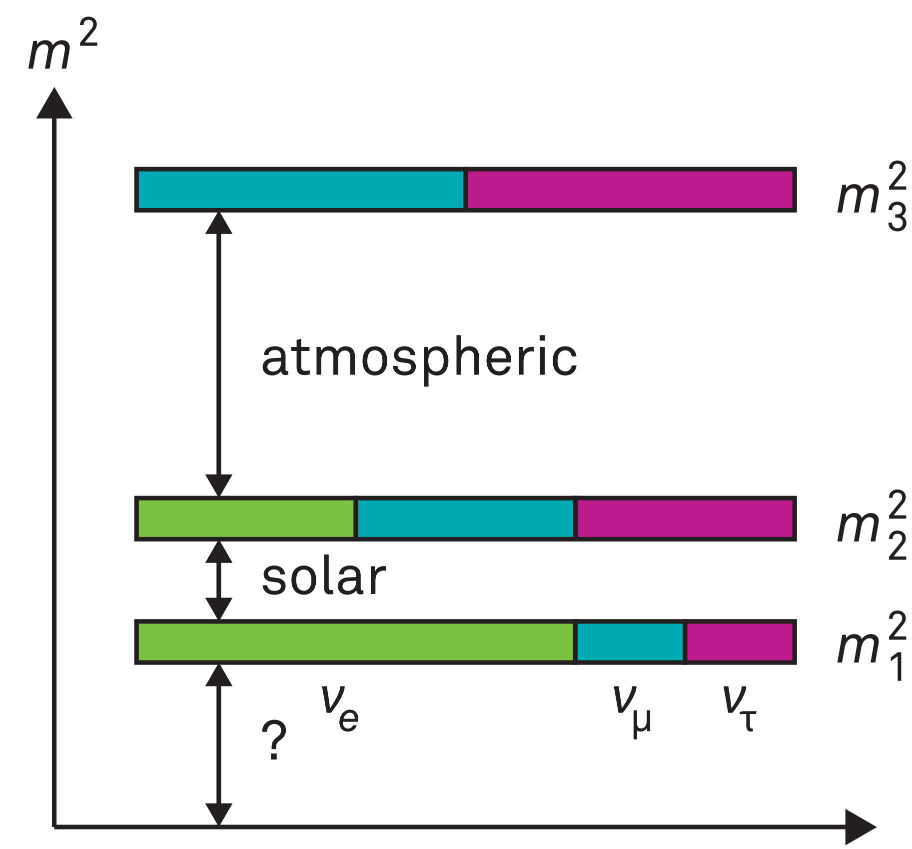
Knotted strings and leptonic flavor structure

T. W. Kephart, P. L., H. Päs, *arXiv*:1106.6201

The flavor puzzle

- Mixing in neutrino and quark sectors is non-trivially different
- CKM mixing small, mass hierarchies large
- PMNS mixing large, mass hierarchy much smaller
- Is this the result of anarchy or is some kind of structure behind it?

$$V_{\mathrm{TBM}} = \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \text{small} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



One answer: discrete symmetries

- Produce mixings via Yukawa sector
- All kinds of non-abelian discrete symmetries like S_3, A_4, S_4, … can be used to approach the flavor puzzle (e.g. Bhattacharyya, Leser, Päs, *arXiv*:1006.5597)
  - through **specific choice of representations** for particle content
  - through **vacuum alignment** of extra scalars

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A different approach

- Masses and mixings of neutrinos are very different from other particles of SM.
- Through a seesaw model, a link between low energy neutrino physics and New Physics at a high scale can be constructed.
- Idea: Leptonic flavor structure arises from topological configuration of knotted strings.

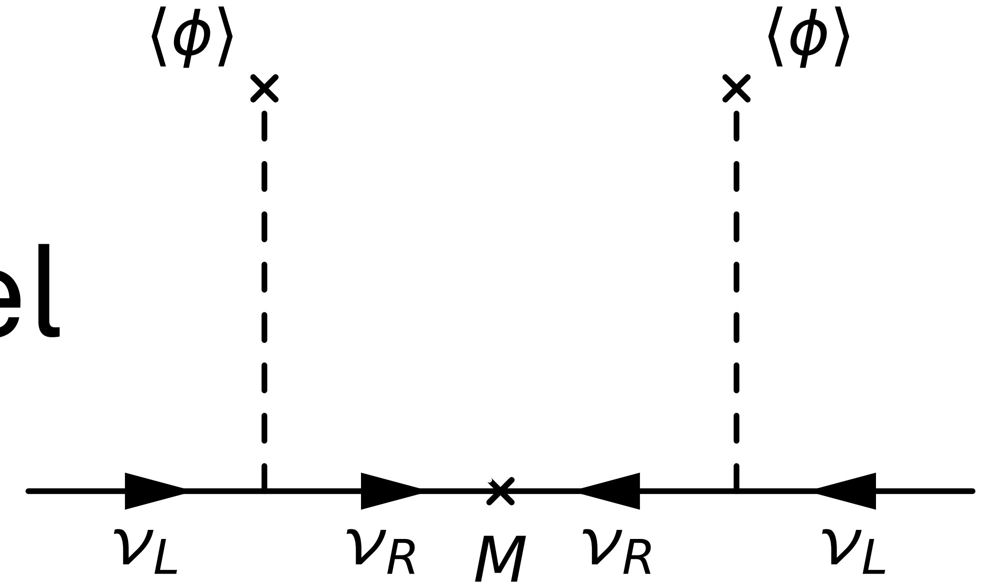
Knots and links

- Motivation:
- Closed strings can have the correct quantum numbers of a right-handed neutrino
  (e.g., Arkani-Hamed, Dimopoulos, Dvali, *arXiv*:hep-ph/9811448)
- String configurations such as knots and links can contribute to the mass of closed string states.
- As string tension tends to minimize string length, knot and link lengths can be assumed to be proportional to string spectrum.

Knots and links

- Previous applications:
- This idea using QCD flux tubes has been used to explain the spectrum of glueball candidates by Buniy, Kephart, Holmes. (*arXiv*: 0209339, 0408027, 1010.5832)
- Mapping of Standard Model particle spectrum to knotted flux tubes. (Finkelstein, *hep-th*/0408218)

Description of the model



- generic type I seesaw
- flavor structure originates from right-handed sector

$$M = \begin{pmatrix} m_1^{\mathrm{K}} & m_1^{\mathrm{L}} & m_2^{\mathrm{L}} \\ m_1^{\mathrm{L}} & m_2^{\mathrm{K}} & m_3^{\mathrm{L}} \\ m_2^{\mathrm{L}} & m_3^{\mathrm{L}} & m_3^{\mathrm{K}} \end{pmatrix} \qquad m_{\mathrm{D}} = \begin{pmatrix} m_1^{\mathrm{D}} & 0 & 0 \\ 0 & m_2^{\mathrm{D}} & 0 \\ 0 & 0 & m_3^{\mathrm{D}} \end{pmatrix}$$

$$M_\nu^{\mathrm{flv}} = m_{\mathrm{D}}^{\mathsf{T}} M^{-1} m_{\mathrm{D}}$$

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Mass spectrum

- Majorana masses $m^{K/L}$ are proportional to the spectrum of knots and links lengths:

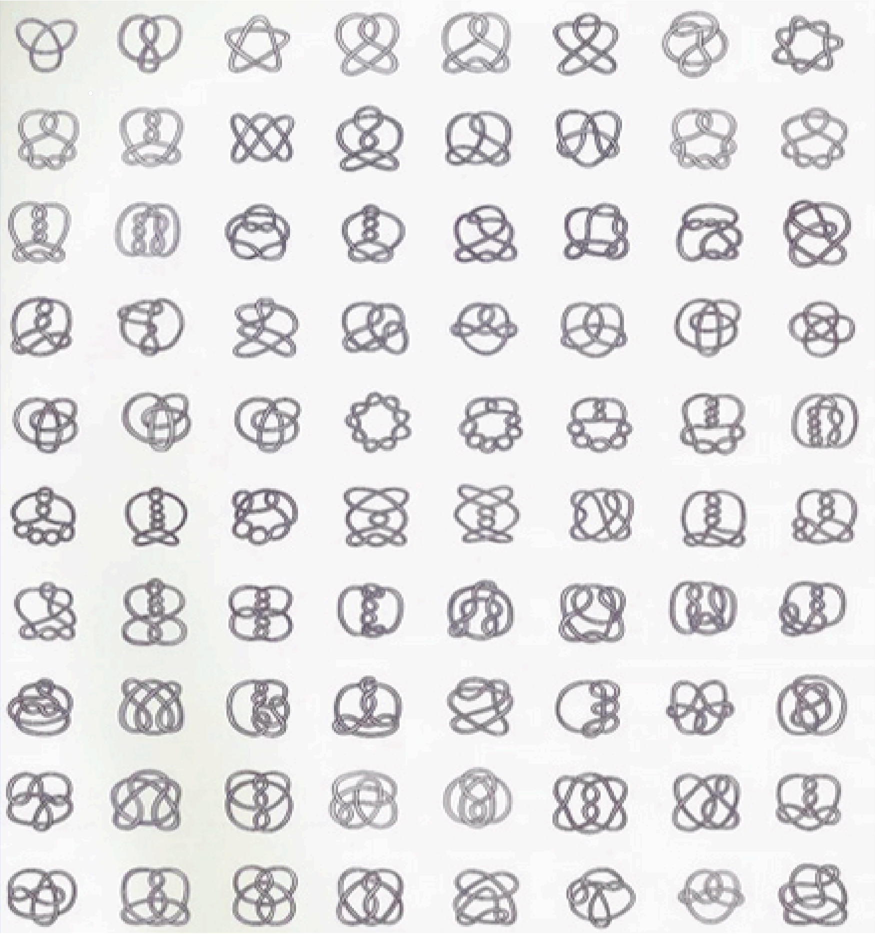
$$m_i^{K/L} = \ell_i^{K/L} \cdot m_{\mathrm{GUT}}$$

$$M = \begin{pmatrix} m_1^{\mathrm{K}} & m_1^{\mathrm{L}} & m_2^{\mathrm{L}} \\ m_1^{\mathrm{L}} & m_2^{\mathrm{K}} & m_3^{\mathrm{L}} \\ m_2^{\mathrm{L}} & m_3^{\mathrm{L}} & m_3^{\mathrm{K}} \end{pmatrix}$$

- Caveat: We assume model, there is no detailed UV completion at the moment.

Link	Rop_p	Rop
2_1^2	25.1415	25.1334
3_1	32.7437	32.7436
4_1	42.0971	42.0887
4_1^2	40.0203	40.0122
5_1	47.2149	47.2016
5_2	49.4820	49.4701
5_1^2	49.7864	49.7716
6_1	56.7178	56.7058
6_2	57.0381	57.0235
6_3	57.8531	57.8392
6_1^2	54.3919	54.3768
6_2^2	56.7087	56.7000
6_3^2	58.1142	58.1013
6_1^3	57.8286	57.8141
6_2^3	58.0112	58.0070
6_3^3	50.5602	50.5539
7_1	61.4234	61.4067
7_2	63.8684	63.8556
7_3	63.9430	63.9285
7_4	64.2836	64.2687
7_5	65.2705	65.2560
7_6	65.7068	65.6924
7_7	65.6235	65.6086
7_1^2	64.2484	64.2345
7_2^2	65.0363	65.0204
7_3^2	65.3414	65.3257
7_4^2	65.0759	65.0602
7_5^2	66.2068	66.1915
7_6^2	66.3281	66.3147
7_7^2	55.5177	55.5095
7_8^2	57.7714	57.7631
7_1^3	65.8157	65.8062
8_1	70.9833	70.9669
8_2	71.4141	71.3985

$\ell_i^{K/L}$



Diagrams by Ali Roth/Cabinet, from Knots and Links by Dale Rolfsen

Determining the hierarchy

- After applying the seesaw formula, it is possible to diagonalize the mass matrix approximately for three scenarios:
  - normal, inverted, degenerate hierarchies.
- Each scenario leads to some conditions on the mass parameters:

Inverted / Degenerate

- Both are *disfavored* by the model:
- Inverted hierarchy leads to conditions that require all the neutrino masses to be zero.
- Degenerate spectrum would force off-diagonal entries of M to be zero.

$$m_1^D m_3^D \neq 0$$

$$K_2 = K_1 \left(m_2^D\right)^2 / \left(m_1^D\right)^2$$

$$K_3 = K_1 \left(m_3^D\right)^2 / \left(m_1^D\right)^2$$

$$K_3 \neq 0$$

$$K_1 K_2 \neq 0$$

$$K_1 \neq 0$$

$$\boxed{L_1 = L_2 = L_3 = 0}$$

$$K_1 K_2 \left(m_3^D\right)^2 + \tilde{m}^2 = 0$$

Ashton, Cantarella, Piatek, Rawdon, 1002.1723

Link	Rop_p	Rop
2_1^2	25.1415	25.1334
3_1	32.7437	32.7436
4_1	42.0971	42.0887
4_1^2	40.0203	40.0122
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8_1	70.9833	70.9669
8_2	71.4141	71.3985

Expectations

- The model predicts a normal neutrino mass hierarchy.
- It should provide a better fit than a random spectrum for $M^{K/L}$.
- Almost degenerate spectrum of knots and links lengths should fulfill conditions for normal hierarchy easily.

$$K_3/K_2 = \left(m_2^D\right)^2 / \left(m_3^D\right)^2$$

$$L_2/L_1 = m_3^D/m_2^D$$

$$K_2 L_2 = L_1 L_3$$

$$K_1 K_2 \neq L_1^2$$

$$\tilde{m}^2 = 2\left(m_3^D\right)^2 \left(L_1^2 - K_1 K_2\right)$$

Numerical analysis

- Dirac masses are set to electroweak order, but are allowed to vary in fit.
- Squared mass differences are mostly determined by Dirac sector.
- Mixing parameters are calculated for each possible combination of links and knots.
- Double beta decay parameter $m_{\beta\beta}$, lightest neutrino mass m_0 and angle θ_{13} are calculated.
- Comparison with random numbers.

Numerical results

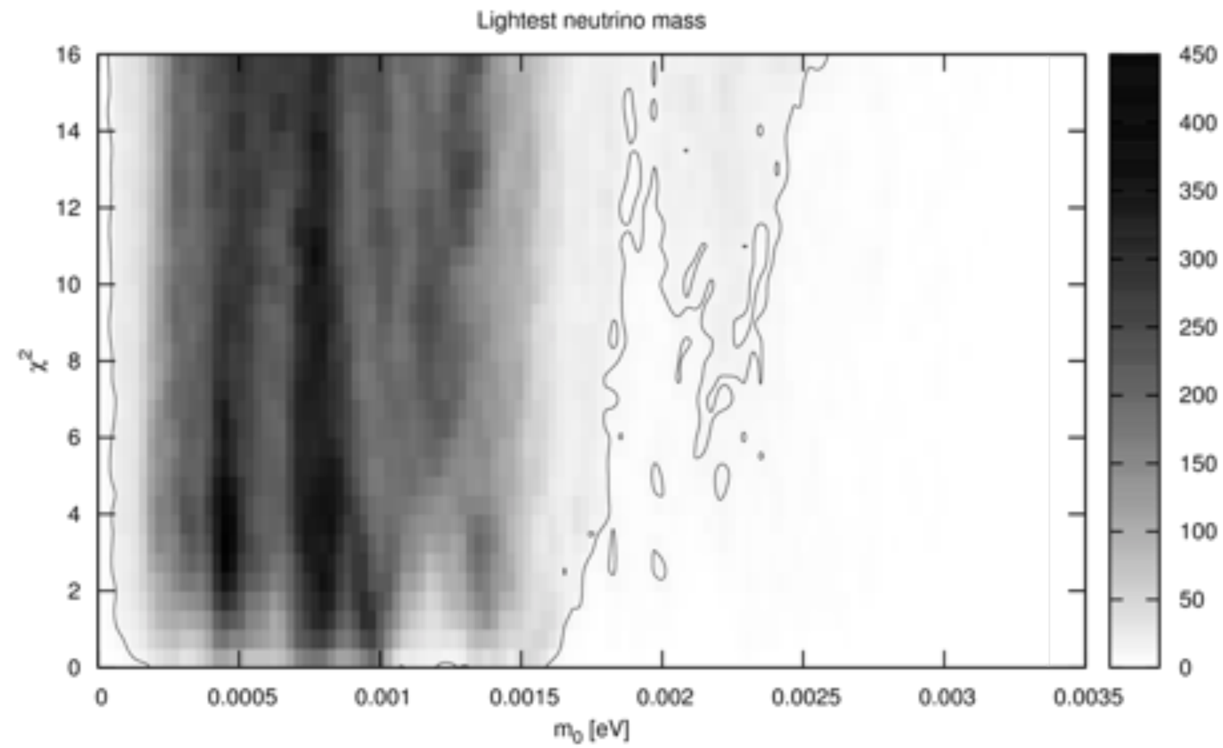
- Of all possible 10^7 combinations, 3% yield an acceptable fit.
- About 300000 models with normal hierarchy, 400 with inverted.
- The best fit lies in the normal hierarchy regime.
- Consistently higher number of acceptable models than runs with random numbers.

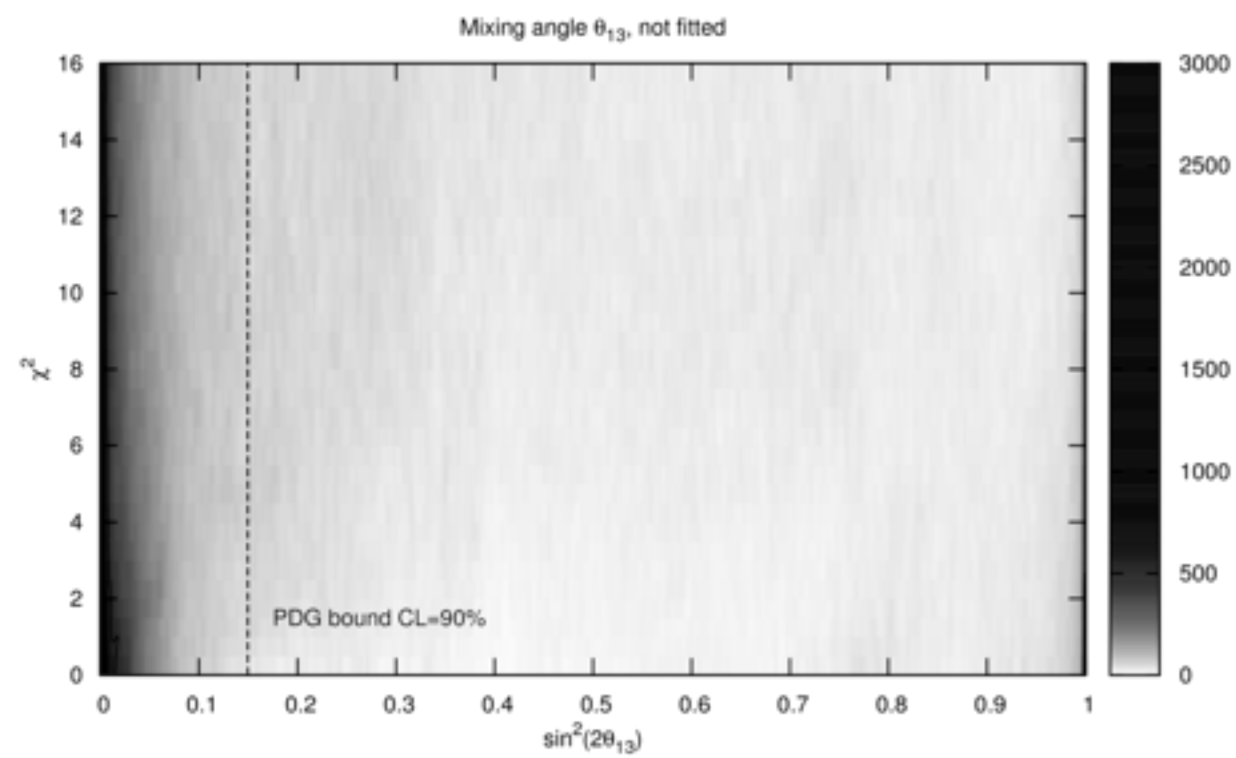
index	knot length [a.u.]	link length [a.u.]
00	32.7436	25.1334
01	42.0887	40.0122
02	47.2016	49.7716
03	49.4701	54.3768
04	56.7058	56.7000
05	57.0235	58.1013
06	57.8392	57.8141
07	61.4067	58.0070
08	63.8556	50.5539
09	63.9285	64.2345
10	64.2687	65.0204
11	65.2560	65.3257
12	65.6924	65.0602
13	65.6086	66.1915
14		66.3147
15		55.5095
16		57.7631
17		65.8062

hierarchy	χ^2	K_1	K_2	K_3	L_1	L_2	L_3	m_1^D [GeV]	m_2^D [GeV]	m_3^D [GeV]	Scale factor [GeV]
normal	0.002	01	05	11	11	12	06	100.391	104.155	101.971	$5.9 \cdot 10^{13}$
inverted	7.262	01	11	06	12	00	13	202.484	126.717	80.652	$1.6 \cdot 10^{13}$

Numerical results

- $m_{\beta\beta}$ is smaller than 0.01 eV for all normal hierarchy models.
- The lightest neutrino mass is below 0.003 eV which gives a sum of all masses well below the current bound of 0.5 eV.
- θ_{13} close to zero is preferred, but the results are still compatible with the recent results from T2K:
  $(0.03 < \sin^2 2\theta_{13} < 0.28)$





Conclusions

- The discrete spectrum of knots and links can be used to derive the neutrino mixing parameters.
- It produces a larger number of viable models than a spectrum of random numbers.
- The model favors a normal neutrino mass hierarchy and predicts a small mixing angle θ_{13}.

set	number of accepted samples normal hierarchy	 inverted hierarchy
knots and links	321952	400
random 0	120619	3685
random 1	231748	2957
random 2	266868	1941
random 3	142333	3598
random 4	145388	3427
random 5	167163	3376
random 6	244459	1283
random 7	189467	5200
random 8	256586	4078
random 9	229931	1649