Simultaneous decoupling of bottom and charm quarks

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Outline

Decoupling Theorem

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[T. Appelquist, J. Carazzone, 1975]

Consider a *full theory* including a heavy field with mass m_h. A Feynmangraph containing this heavy field and external momenta $q_i \ll m_h$ will be suppressed by $(q_i/m_h)^j$, $j > 0$. Therefore physical processes at low energies may be described by a theory, arising from the full theory, by deleting all graphs that contain m_h.

- $\overline{\text{MS}}$-scheme does not satisfy this theorem.
- Decoupling procedure has to be imposed
- Construct an *effective theory* from the full theory, which does not contain the heavy field, but *redefined* light fields and parameters

Framework

- Effective Lagrangian contains only light degrees of freedom

$$\mathcal{L}'\left(\alpha_s^0, m_q^0, \xi^0; \psi_q^0, A_\mu^{0,a}, c^{0,a}; \zeta_i^0\right) = \mathcal{L}^{\text{QCD}}\left(\alpha_s^{0\prime}, m_q^{0\prime}, \xi^{0\prime}; \psi_q^{0\prime}, A_\mu^{0\prime,a}, c^{0\prime,a}\right), \qquad (1)$$

- With the definitions of the bare decoupling constants

$$\alpha_s^{0\prime} = \zeta_\alpha^0 \alpha_s^0, \; A_\mu^{0\prime,a} = \zeta_A^{0^{1/2}} A_\mu^{0,a}, \; c^{0\prime,a} = \zeta_c^{0^{1/2}} c^{0,a}, \; \text{etc.} \qquad (2)$$

- Use of renormalization condition $\alpha_s^0 = \mu^{2\epsilon} Z_\alpha \alpha_s$

$$\alpha_s'\left(\mu'\right) = \left[\left(\frac{\mu}{\mu'}\right)^{2\epsilon} \frac{Z_\alpha\left(\alpha_s\left(\mu\right)\right)}{Z_\alpha'\left(\alpha_s'\left(\mu'\right)\right)} \zeta_\alpha^0\right] \alpha_s\left(\mu\right) = \zeta_\alpha\left(\mu, \mu'\right) \alpha_s\left(\mu\right) \qquad (3)$$

Matching

Green's functions of the light fields in the effective theory and the full theory have to be equal (up to suppressed terms), e.g. for the ghost propagator

$$
\begin{aligned}
&\frac{-\delta^{ab}}{q^2\left(1+\Pi_c^0\left(q^2\right)\right)} \\
&= i\int dx\, e^{iqx}\left\langle Tc^{0,a}(x)c^{0,b}(0)\right\rangle \\
&= \frac{i}{\zeta_c^0}\int dx\, e^{iqx}\left\langle Tc^{0\prime,a}(x)c^{0\prime,b}(0)\right\rangle + \mathcal{O}\left(\frac{q^2}{m_h^2}\right) \\
&= \frac{-\delta^{ab}}{q^2\zeta_c^0\left(1+\Pi_c^{0\prime}\left(q^2\right)\right)} \\
&\to \zeta_c^0 = \frac{1+\Pi_c^0\left(q^2\right)}{1+\Pi_c^{0\prime}\left(q^2\right)}
\end{aligned}
\tag{4}
$$

Matching

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- Nullify the external momentum

$$\zeta_c^0 = \frac{1 + \Pi_c^0(0)}{1 + \Pi_c^{0\prime}(0)} \tag{5}$$

- Scaleless integrals vanish within dimensional regularization

$$\zeta_c^0 = 1 + \Pi_c^{0,h}(0) \tag{6}$$

- Only massive vacuum integrals have to be considered

Analogous relations

- Decoupling for gluon field

$$\zeta_A^0 = 1 + \Pi_A^{0,h}(0) \tag{7}$$

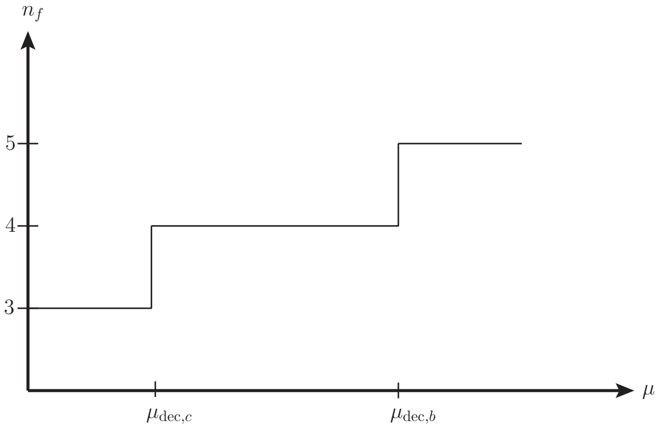
- Decoupling for ghost-gluon vertex

$$\zeta_{A\bar{c}c}^0 = 1 + \Gamma_{A\bar{c}c}^{0,h}(0,0) \tag{8}$$

- Decoupling for strong coupling constant

$$\zeta_\alpha^0 = \frac{\left(\zeta_{A\bar{c}c}^0\right)^2}{\left(\zeta_c^0\right)^2 \zeta_A^0} \tag{9}$$

Conventional decoupling of bottom and charm



Conventional 2-step approach

Conventional decoupling of bottom and charm

$$\alpha_s^{(n_l+1)}(\mu_{\text{dec},b}) = \zeta_\alpha(\mu_{\text{dec},b}, M_b)\, \alpha_s^{(n_f=n_l+2)}(\mu_{\text{dec},b}),$$
$$\alpha_s^{(n_l)}(\mu_{\text{dec},c}) = \zeta_\alpha(\mu_{\text{dec},c}, M_c)\, \alpha_s^{(n_f=n_l+1)}(\mu_{\text{dec},c}). \qquad (10)$$

- Conv. approach: decouple one quark after another
- Tower of effective theories
- Each effective theory contains only one massive quark
- well investigated (up to 4-loop order) [W. Bernreuther, K. G. Chetyrkin, B. A. Kniehl, Y. Schröder, C. Sturm, M. Steinhauser, W. Wetzel, 1982-2006]

Simultaneous decoupling of bottom and charm

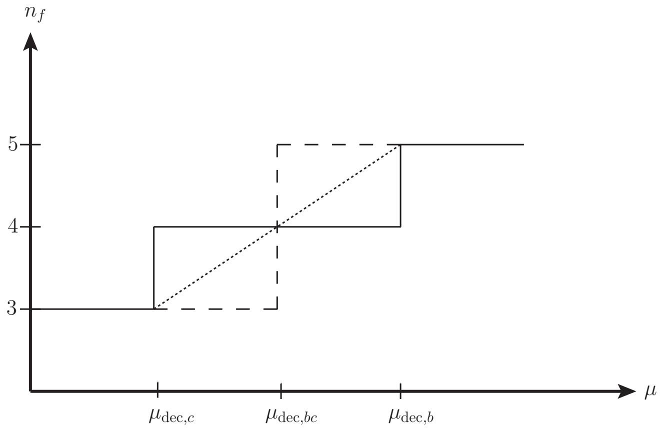
- Note that $M_c/M_b \not\ll 1$
- Take into account powers of M_c/M_b

$$\alpha_s^{(n_l)}(\mu_{\text{dec},c}) = \zeta_\alpha^{(bc)}(\mu_{\text{dec},b}, \mu_{\text{dec},c}, M_b, M_c)\, \alpha_s^{(n_f=n_l+2)}(\mu_{\text{dec},b})\,. \qquad (11)$$

- Sim. approach: decouple two quarks at once, skipping the $n_l + 1$-flavour theory
- Equalize the decoupling scales

$$\alpha_s^{(n_l)}(\mu_{\text{dec},bc}) = \zeta_\alpha^{(bc)}(\mu_{\text{dec},bc}, M_b, M_c)\, \alpha_s^{(n_f=n_l+2)}(\mu_{\text{dec},bc})\,. \qquad (12)$$

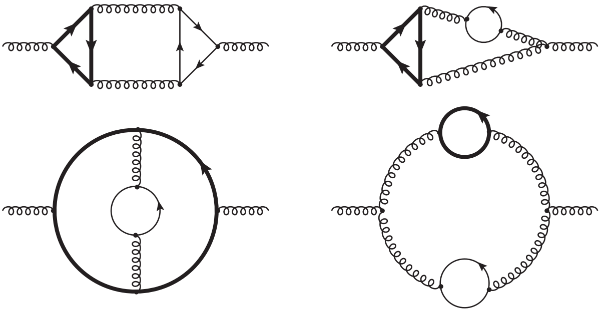
Decoupling of bottom and charm



Continuous line: conventional 2-step approach, dotted line: simultaneous 2-scale approach, dashed line: simultaneous 1-scale approach

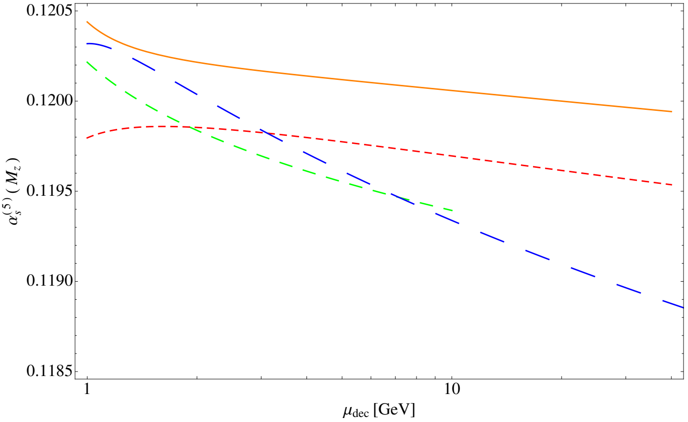
Calculation

- Simultaneous decoupling constants (at 3-loop level) require the calculation of vacuum integrals with *two* mass scales



- Automated calculation using `QGRAF` [P. Nogueira, 1993], `q2e`, `exp` [R. Harlander, T. Seidensticker, M. Steinhauser, 1998-1999], `MATAD` [M. Steinhauser, 2001] and `FIRE` [A. V. Smirnov, 2008]

Conv. vs sim. approach for $M_\tau \to M_Z$



conv. (3L dec.) $\mu_{\text{dec},c} = 3$ GeV, $\mu_{\text{dec},b} = \mu_{\text{dec}}$; conv. (3L dec.) $\mu_{\text{dec},c} = \mu_{\text{dec}}$, $\mu_{\text{dec},b} = 10$ GeV;

sim. (3L dec.) $\mu_{\text{dec},bc} = \mu_{\text{dec}}$; conv. (4L dec.) $\mu_{\text{dec},c} = 3$ GeV, $\mu_{\text{dec},b} = \mu_{\text{dec}}$.

Contribution of power-suppressed terms to the 2-step approach

$$\left(\zeta_\alpha^{(bc)}\left(\mu_{\text{dec},b},\mu_{\text{dec},c},M_b,M_c\right)\right)^{-1} =$$

$$\left(\zeta_\alpha(\mu_{\text{dec},b},M_b)\right)^{-1}\frac{Z^{(n_l+1)}_{\alpha(\mu_{\text{dec},c})}}{Z^{(n_l+1)}_{\alpha(\mu_{\text{dec},b})}}\left(\zeta_\alpha(\mu_{\text{dec},c},M_c)\right)^{-1}+\delta\left(\zeta_\alpha^{(bc)}\right)^{-1} \tag{13}$$

- $\delta(\zeta_\alpha^{(bc)})^{-1}$ is of $\mathcal{O}(M_c/M_b)$ and independent of $\mu_{\text{dec},c},\mu_{\text{dec},b}$:

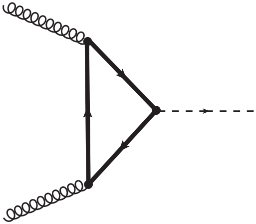
$$\delta\left(\zeta_\alpha^{(bc)}\right)^{-1}\approx 0.170\left(\frac{\alpha_s^{(3)}\left(\mu_{\text{dec},c}\right)}{\pi}\right)^3, \tag{14}$$

$$\delta\alpha_s^{(5)}\left(\mu_{\text{dec},b}\right)=\left(\delta\zeta_\alpha^{(bc)}\right)^{-1}\alpha_s^{(3)}\left(\mu_{\text{dec},c}\right)\sim 10^{-5}. \tag{15}$$

- Effect is $\sim 10^{-1}\times$ 4-loop contribution

BSM theories containing several heavy quarks

- in the SM $\sigma(gg \to H)$ largest of all higgs-production cross-sections
- dominant term from top-loop



- BSM theories with several heavy quarks (e.g. 4th generation) with top-like Higgs couplings: $\sigma(gg \to H)^{(n_h)}/\sigma(gg \to H)^{(SM)} \approx n_h^2$
- tests and constraints from LHC

Coefficient function C_1

- Production and decay of intermediate-mass Higgs boson can be described by an effective Lagrangian, where the top quark t and further generations of heavy quarks are integrated out

$$\mathcal{L}_{eff} = -\frac{\phi}{v} C_1 \mathcal{O}_1, \text{with } \mathcal{O}_1 = G_{\mu\nu} G^{\mu\nu} \tag{16}$$

- Low-energy theorem [K. G. Chetyrkin, B. A. Kniehl, M. Steinhauser, 1998]

$$-2C_1 = M_t^2 \frac{\partial}{\partial M_t^2} \ln\left(\zeta_\alpha^{(t't)}\right) + M_{t'}^2 \frac{\partial}{\partial M_{t'}^2} \ln\left(\zeta_\alpha^{(t't)}\right), \tag{17}$$

- At 3-loop level max. two quarks in one diagram
- Generalization of the result for an *arbitrary number* of heavy quarks
- Confirmation of our result with that obtained in [A. Anastasiou, R. Boughezal, E. Furlan, 2010]

Summary

- Computation of simultaneous decoupling constant for α_s at 3 loops
  - exact evaluation of 2-mass Feynmandiagrams
- numerical effect of $(\frac{M_c}{M_b})^i$ terms is small
- C_1 for extansions of the Standard Model with several heavy quarks

$$
\begin{aligned}
C_1 = &\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)\left\{-\frac{1}{6}T_F\,(n_t+n_{t'})\right\}\\
&+\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2\left\{\left(\frac{-5C_AT_F}{24}+\frac{C_FT_F}{8}\right)(n_t+n_{t'})\right\}\\
&+\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3\left\{\left(\frac{5C_AT_F^2}{576}+\frac{C_FT_F^2}{72}\right)(n_t+n_{t'})^2\right.\\
&+L_{tt'}\left(\frac{-7C_A^2T_F}{96}+\frac{11C_AC_FT_F}{96}-\frac{C_FT_F^2n_l}{12}\right)\\
&\left.+(n_t+n_{t'})\left(\frac{-1063C_A^2T_F}{3456}+\frac{25C_AC_FT_F}{72}-\frac{9C_F^2T_F}{64}+\frac{47C_AT_F^2n_l}{864}+\frac{5C_FT_F^2n_l}{96}\right)\right\}
\end{aligned}
\tag{18}
$$

with

$$
L_{tt'} = n_t\ln\left(\frac{\mu^2}{M_t^2}\right)+n_{t'}\ln\left(\frac{\mu^2}{M_{t'}^2}\right). \tag{19}
$$