

Reheating, Matter, Dark Matter – All You Need is Neutrino Decays. ■



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Based on arXiv:1008.2355, 1104.2750 and 1110.xxxx [hep-ph].
In collaboration with W. Buchmüller, V. Domcke and G. Vertongen.

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Cosmology meets Particle Physics | September 28, 2011

Outline

1 Idea

- Reheating through heavy neutrino decays
- Leptogenesis and gravitino dark matter

2 Scenario

- $B - L$ breaking during false vacuum decay
- Network of particle interactions

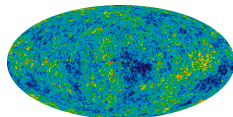
3 Analysis

- Exemplary parameter point
- Study of the parameter space
- Summary and conclusion

Origin of the hot phase of the early universe?

Epoch dominated by energy in radiation:

- ▶ Evidence: CMB ($T \simeq 0.25\text{eV}$) and BBN ($T \simeq 1\text{MeV}$).
- ▶ Paradigm: Preceded by inflation (vacuum domination).
- ▶ Transition? Reheating mechanism and T_{RH} unknown.



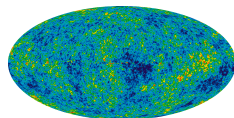
Definition T_{RH} : $H(T =: T_{\text{RH}}) = \Gamma_X \Rightarrow T_{\text{RH}} \simeq 0.2\sqrt{\Gamma_X M_P}$.

Γ_X free parameter!

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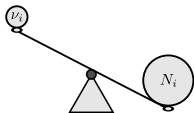
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$$m_\nu = -m_D \frac{1}{M} m_D^T$$



Type I seesaw:

Add heavy Majorana neutrinos N_i to the SM.

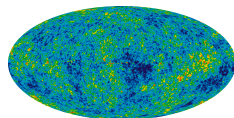
Our idea: Entropy from heavy neutrino decays

- ▶ Assume dominant $N_1 + \tilde{N}_1$ abundance after inflation.
- ▶ At tree-level $\Gamma_{N_1} = \frac{\tilde{m}_1}{4\pi} (M_1/v_{\text{EW}})^2$, $\tilde{m}_1 := (m_D^\dagger m_D)_{11}/M_1$.
- ▶ $T_{\text{RH}} = T_{\text{RH}}(\tilde{m}_1, M_1) \sim 10^{9 \div 10} \text{ GeV}$ for typical $\tilde{m}_1$ & M_1 .
- ▶ Baryon asymmetry & dark matter are natural by-products.

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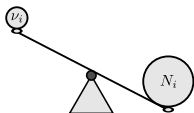
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Consistent cosmology and non-trivial parameter relations!

Baryon asymmetry and dark matter

[Fukugita & Yanagida '86]

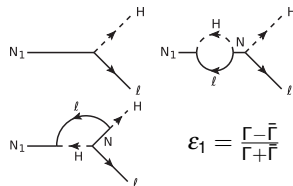
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Baryogenesis through leptogenesis:

- ▶ $\eta_B := n_B/n_\gamma \simeq 6 \times 10^{-10} \gg \eta_B^{\text{sym}} \simeq 10^{-18}$.
- ▶ SM sphalerons convert L into B asymmetry (c_{sph}).
- ▶ Seesaw & neutrino data: $M_1 \sim T_L \sim 10^{9 \div 10} \text{ GeV}$.
- ▶ Hierarchical masses M_i lead to $\eta_B = \eta_B(\tilde{m}_1, M_1)$.



CP-violating out-of-equilibrium
neutrino decays

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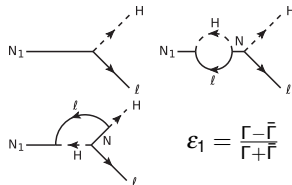
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Thermal prod. of gravitinos in SQCD:

- Do not spoil BBN or overclose the universe.
- DM for free if gravitino is heavy stable LSP.
- $\Omega_{\tilde{G}} h^2(T_{\text{RH}}, m_{\tilde{G}}, m_{\tilde{g}}) \simeq 0.11 \simeq \Omega_{\text{DM}} h^2$.



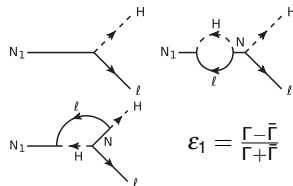
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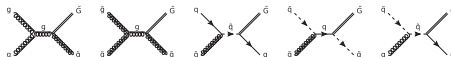
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Connect the two sectors!

- $\Omega_{\tilde{G}} h^2(\tilde{m}_1, M_1, m_{\tilde{G}}, m_{\tilde{g}}) = \Omega_{\text{DM}} h^2$
- ▶ Keep $m_{\tilde{g}}$ fixed & solve for M_1 .
 - ▶ $M_1, T_{\text{RH}}, \eta_B = f(\tilde{m}_1, m_{\tilde{G}})$.
 - ▶ Impose $\eta_B(\tilde{m}_1, m_{\tilde{G}}) \geq \eta_B^{\text{obs}}$.
 - ▶ Mutual bounds $\tilde{m}_1 \leftrightarrow m_{\tilde{G}}$.

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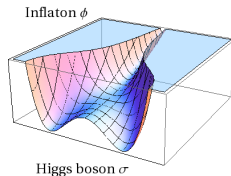
Generating a dominant nonthermal $N_1 + \tilde{N}_1$ abundance

SSB of $U(1)_{B-L}$ at v_{B-L} by gauge singlet σ :

- ▶ Seesaw: $\frac{1}{2} M_i (\overline{n_{Ri}^c} n_{Ri} + \text{h.c.})$ violates lepton number L .
- ▶ Interpretation: $\frac{1}{2} h_i^{(n)} \left(\overline{n_{Ri}^c} n_{Ri} \frac{\sigma}{\sqrt{2}} + \text{h.c.} \right), \sigma \rightarrow \sqrt{2} v_{B-L} + \sigma$.

- ▶ σ be the field coupling to the inflaton ϕ in hybrid inflation.
- ▶ False vacuum decay breaks $B - L$ and ends inflation.

Hybrid inflation =
Chaotic inflation + SSB



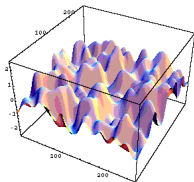
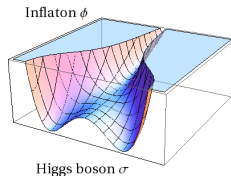
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Tachyonic preheating: [Felder et al. '01] [García-Bellido & Ruiz Morales '02]

- ▶ Instability for $\phi < \phi_{\text{crit}}$ causes explosive spinodal growth of long-wavelength Higgs modes: Waterfall transition!
- ▶ False vacuum energy density $\rightarrow$ Nonrelativistic σ bosons, DOFs coupled to σ : Gauge, inflation & neutrino multiplets.

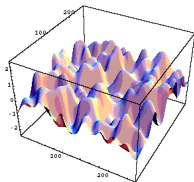
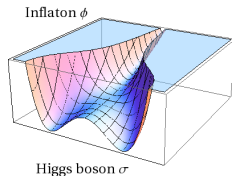
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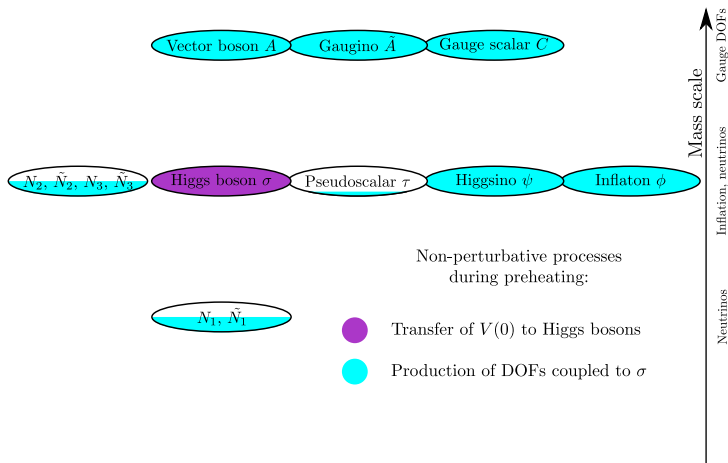


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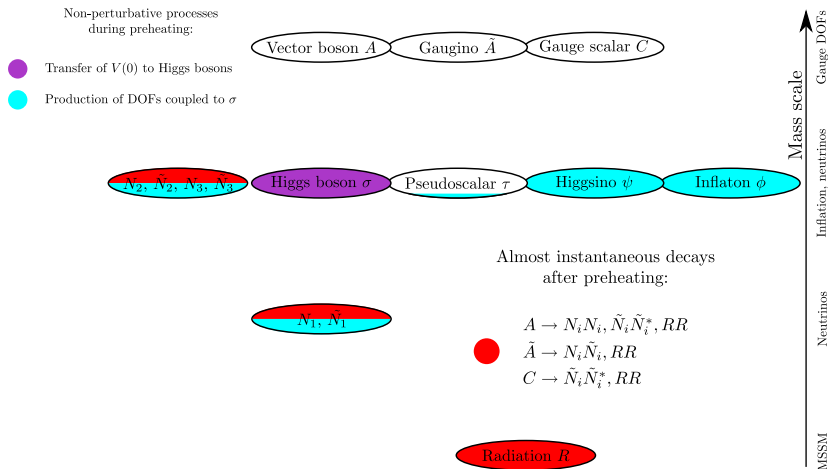
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Assume $2M_1 \ll m_\sigma \lesssim 2M_{2,3}$ s.t. σ decays exclusively into N_1 and $\tilde{N}_1$ after SSB.

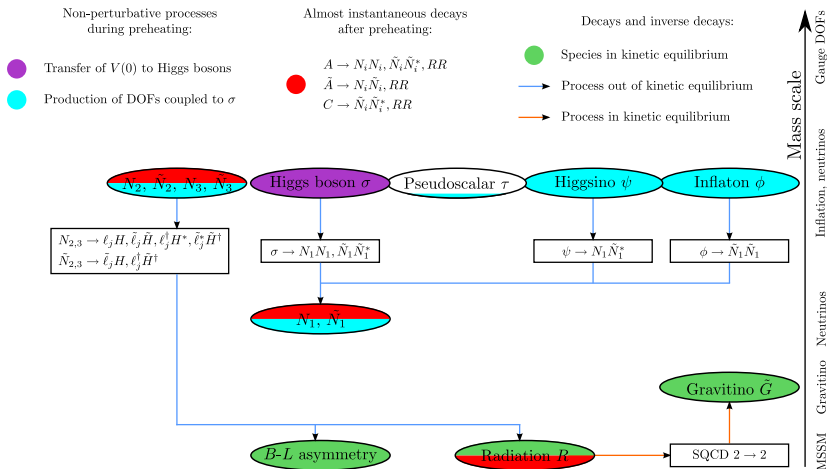
Particle production during tachyonic preheating



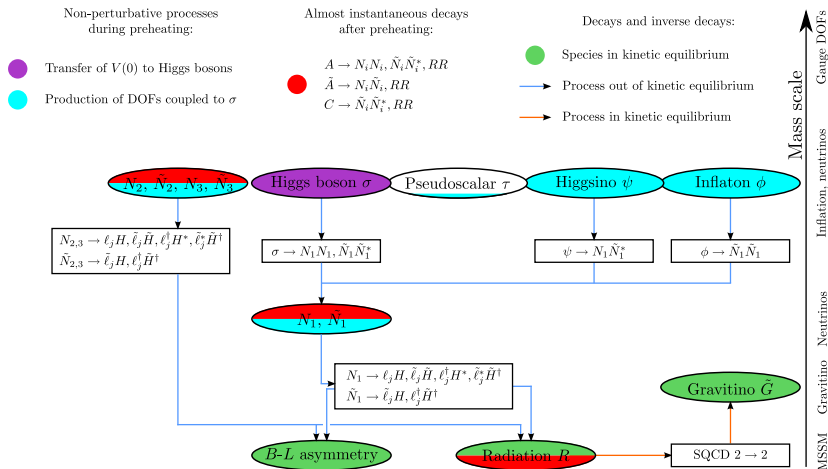
Decay of the gauge degrees of freedom



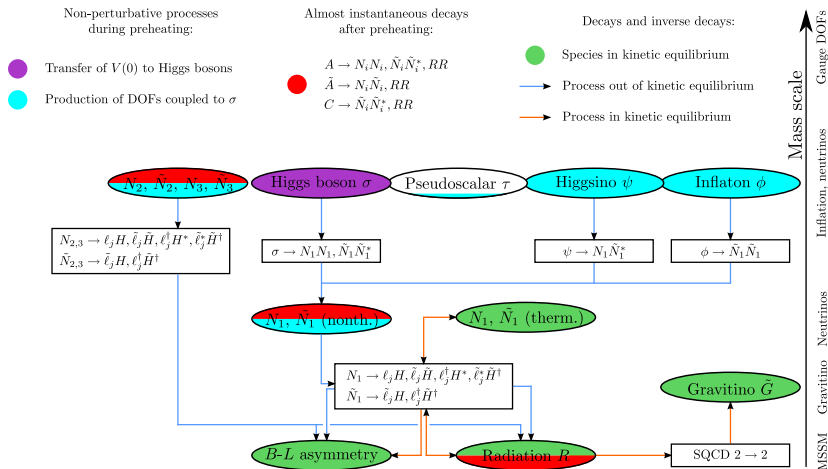
Decay of $N_2, \tilde{N}_2, N_3, \tilde{N}_3, \sigma, \tau, \psi$ and ϕ



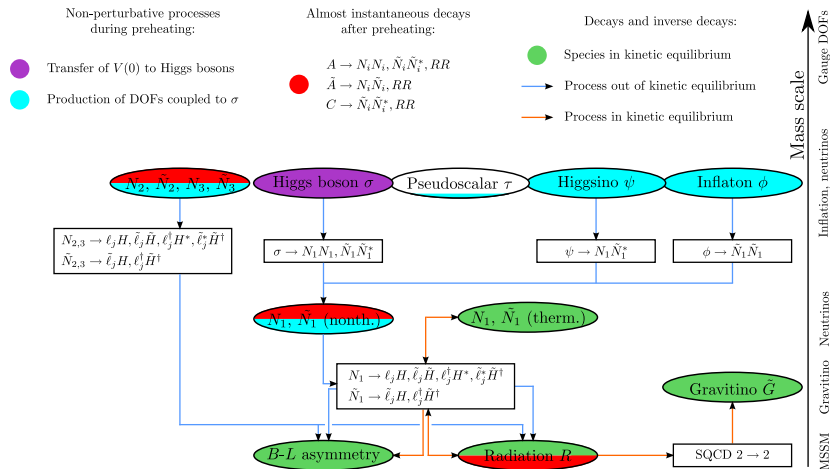
Decay of nonthermal neutrinos



Production and decay of thermal neutrinos



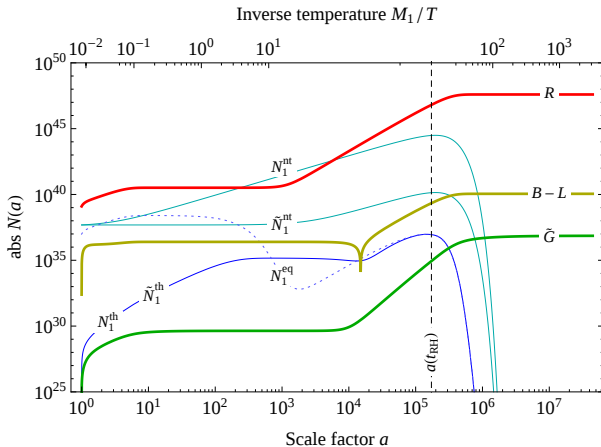
Production and decay of thermal neutrinos



Math: Boltzmann equations, coupled system of non-linear first-order PDEs.

Solve for phase space distr. funcs. & number densities in an expanding FLRW background.

Comoving number densities



$$N(a) = a^3 n(a)$$

$$v_{B-L} = 5 \times 10^{15} \text{ GeV}$$

$$M_1 = 2 \times 10^{11} \text{ GeV}$$

$$\tilde{m}_1 = 3 \times 10^{-4} \text{ eV}$$

$$m_{\tilde{G}} = 160 \text{ GeV}$$

$$m_{\tilde{g}} = 1000 \text{ GeV}$$

$$\epsilon_1^{\max} = +2 \times 10^{-5}$$

$$\epsilon_{2,3}^{\max} = -5 \times 10^{-3}$$

Large enough baryon asymmetry:

$$\eta_B^{\max} = 9 \times 10^{-9} > \eta_B^{\text{obs}} \simeq 6 \times 10^{-10}$$

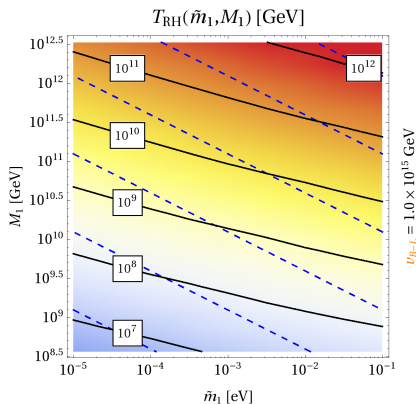
Right DM abundance:

$$\Omega_{\tilde{G}} h^2 = 0.11 = \Omega_{\text{DM}}^{\text{obs}} h^2$$

Reheating temperature and baryon asymmetry

$$M_1(\nu_{B-L}, \tilde{m}_1, m_{\tilde{G}}, m_{\tilde{g}}) \text{ s. t. } \Omega_{\tilde{G}} h^2 = 0.11$$

$$\eta_B(\nu_{B-L}, \tilde{m}_1, M_1) = \eta_B^S + \eta_B^T > \eta_B^{\text{obs}}$$

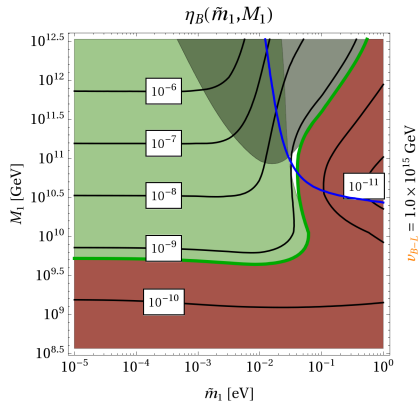
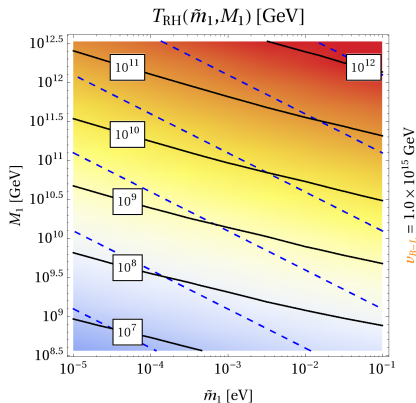


- ▶ Results of nonsupersymmetric study (arXiv:1104.2750 [hep-ph]).
- ▶ Inclusion of full particle spectrum is work in progress.
- ▶ Publication soon, qualitative findings remain unchanged.

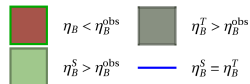
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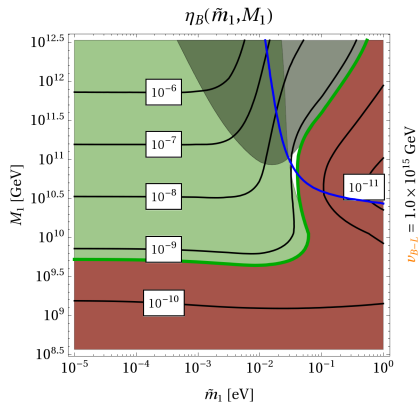
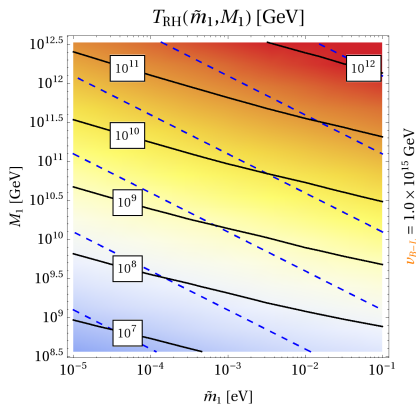
Appreciate: Thermal bounds significantly alleviated!



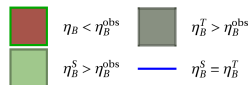
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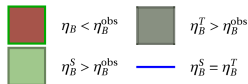
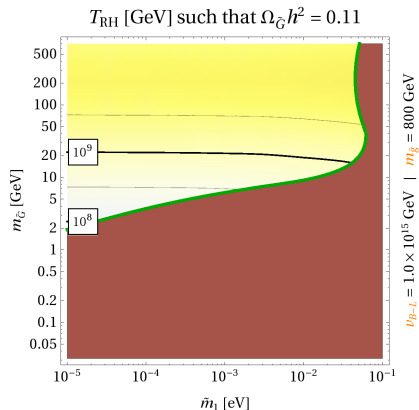
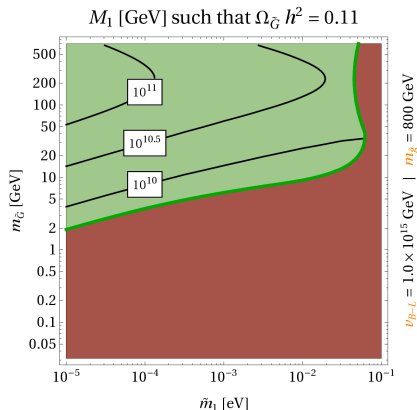
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Connection between SUGRA and neutrino parameters



Scenario works in large region of parameter space!

- ▶ T_{RH} bound lowered: $\gtrsim 10^9$ GeV $\rightarrow \gtrsim 10^8$ GeV.
- ▶ $\tilde{m}_1$ and $m_{\tilde{G}}$ mutually constrain each other.

A common origin of entropy, matter and dark matter

Idea: Neutrino decays produce all entropy of the hot early universe.

- ▶ Reheating temperature T_{RH} determined by neutrino lifetime $\Gamma_{N_1}(\tilde{m}_1, M_1)$.
- ▶ BAU through leptogenesis and gravitino DM follow naturally + interrelated.

Scenario: Dominant nouth. N_1 abundance after false vacuum decay.

- ▶ Hybrid inflation ends in SSB of local $U(1)_{B-L}$ and tachyonic preheating.
- ▶ Vacuum energy density $\rightarrow$ Higgs bosons of $B-L$ breaking $\rightarrow N_1$ neutrinos.

Analysis: Quantitative results after solving the Boltzmann equations.

- ▶ T_{RH} , η_B and $\Omega_{\tilde{G}} h^2$ from Lagrangian parameters v_{B-L} , $\tilde{m}_1$, M_1 , $m_{\tilde{G}}$ and $m_{\tilde{g}}$.
- ▶ $\tilde{m}_1$ constrains $m_{\tilde{G}}$ and vice versa. Falsification: Measure $m_i \gtrsim 0.1 \text{ eV}$.
- ▶ Connection b/t collider searches, laboratory exps. and cosmological obs.

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Thank you for your attention!

Yukawa couplings

Superpotential for matter fields in $SU(5)$ notation:

- ▶ $W_M = h_{ij}^{(u)} \mathbf{10}_i \mathbf{10}_j H_u + h_{ij}^{(d)} \mathbf{5}_i^* \mathbf{10}_j H_d + h_{ij}^{(v)} \mathbf{5}_i^* \mathbf{1}_j H_u + h_i^{(n)} \mathbf{1}_i \mathbf{1}_i S.$
- ▶ Fermion irreps: $\mathbf{10}_i = (q_i, u_i^c, e_i^c)$; $\mathbf{5}_i^* = (d_i^c, \ell_i)$; $\mathbf{1}_i = (n_i)$; $N_i := n_i + n_i^c.$

Froggatt-Nielson $U(1)_{\text{FN}}$ flavor symmetry: [Buchmüller & Yanagida '99]

- ▶ Yukawa terms are generated from non-renorm. $U(1)_{\text{FN}}$ -inv. higher-dim. operators:
 $h_{ij} \sim \eta^{Q_i + Q_j}$; $\eta := v_{\text{FN}}/\Lambda \simeq 1/\sqrt{300}$; $\Lambda > \Lambda_{\text{GUT}}.$
- ▶ FN parameter η and charges Q_i determined from quark and lepton mass hierarchies.

ψ_i	$\mathbf{10}_3$	$\mathbf{10}_2$	$\mathbf{10}_1$	$\mathbf{5}_3^*$	$\mathbf{5}_2^*$	$\mathbf{5}_1^*$	$\mathbf{1}_3$	$\mathbf{1}_2$	$\mathbf{1}_1$
Q_i	0	1	2	a	a	$a+1$	b	c	d

Specific example: $a = 0.5$, $d = 1.5$.

- ▶ Require $M_1 \ll M_{2,3} = m_s \Rightarrow b = c = d - 1 = 0.5.$
- ▶ $v_{B-L} \sim 6 \times 10^{13} \text{ GeV}$; $M_1 \sim 10^{10} \text{ GeV}$; $M_{2,3} = m_s \sim v_{B-L}.$

Boltzmann equations

$$(S) \quad \hat{\mathcal{L}} f_S(t, p) = -\frac{m_S}{E_S} \Gamma_S^0 f_S(t, p)$$

$$(N_1^S) \quad \hat{\mathcal{L}} f_{N_1}^S(t, p) = -\frac{M_1}{E_{N_1}} \Gamma_{N_1}^0 f_{N_1}^S(t, p) + 2 \times \frac{\pi^2 \Gamma_S^0 N_S}{a^3 E_{N_1}^2} \frac{\delta(E_{N_1} - m_S/2)}{\sqrt{1 - (2M_1/m_S)^2}}$$

$$(N_1^T) \quad aH \frac{d}{da} N_{N_1}^T = -\Gamma_{N_1}^T (N_{N_1}^T - N_{N_1}^{\text{eq}})$$

$$(B-L) \quad aH \frac{d}{da} N_{B-L} = \varepsilon_1 \Gamma_{N_1}^S N_{N_1}^S + \varepsilon_1 \Gamma_{N_1}^T (N_{N_1}^T - N_{N_1}^{\text{eq}}) - \frac{N_{N_1}^{\text{eq}}}{2N_\ell^{\text{eq}}} \Gamma_{N_1}^T N_{B-L}$$

$$(\tilde{G}) \quad aH \frac{d}{da} N_{\tilde{G}} = \kappa_{\tilde{G}} \times a^3 \left(1 + \frac{m_{\tilde{g}}^2(T)}{3m_{\tilde{G}}^2}\right) \frac{54\zeta(3)g_s^2(T)}{\pi^2 M_p} T^6 \left[\ln\left(\frac{T^2}{m_{\tilde{g}}^2(T)}\right) + 0.8846 \right]$$

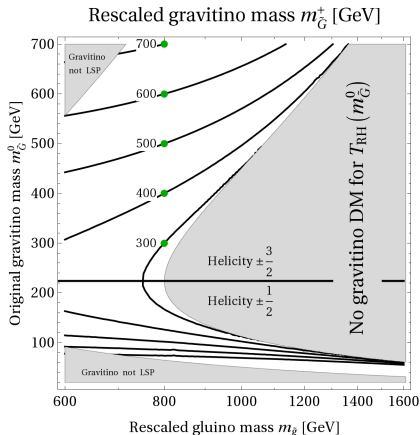
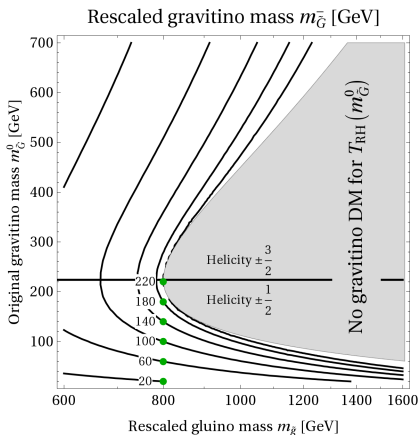
$$(R) \quad aH \frac{d}{da} N_R = r_R^S \Gamma_{N_1}^S N_{N_1}^S + r_R^T \Gamma_{N_1}^T (N_{N_1}^T - N_{N_1}^{\text{eq}})$$

with $\hat{\mathcal{L}} = \frac{\partial}{\partial t} - H p \frac{\partial}{\partial p}$, $\Gamma_{N_1}^{S/T} = \left\langle \frac{M_1}{E_{N_1}} \right\rangle_{S/T} \Gamma_{N_1}^0$, $\kappa_{\tilde{G}} \sim \mathcal{O}(1)$ and $r_R^{S/T} = \frac{3\rho_{N_1}^{S/T} n_R}{4n_{N_1}^{S/T} \rho_R}$.

Coupled system of non-linear first-order partial differential eqns.

Solve for phase space distr. funcs. & number densities in an expanding FLRW background.

Rescaling the gravitino mass for $m_{\tilde{g}} \neq 800 \text{ GeV}$

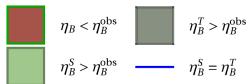
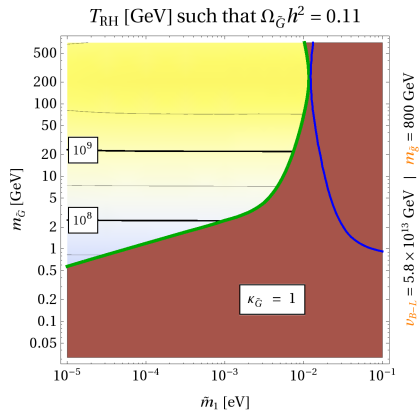
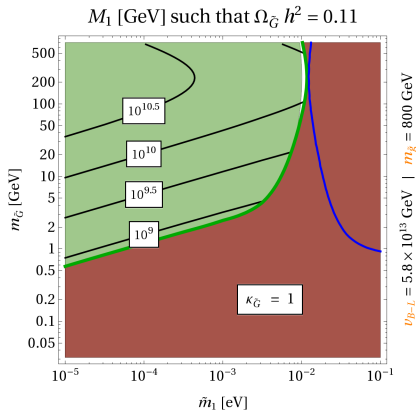


$$\text{Solve } \Omega_{\tilde{G}} h^2 (T_{\text{RH}}, m_{\tilde{G}}^0, 800 \text{ GeV}) = \Omega_{\tilde{G}} h^2 (T_{\text{RH}}, m_{\tilde{G}}, m_{\tilde{g}}).$$

- Quadratic eq. for $m_{\tilde{G}} (m_{\tilde{g}}, m_{\tilde{G}}^0)$ w/ two solutions $m_{\tilde{G}}^{\pm}$.
- For $m_{\tilde{G}}^0 \ll m_{\tilde{g}}$: $m_{\tilde{G}} = m_{\tilde{G}}^0 (m_{\tilde{g}}/800 \text{ GeV})^2$.

T_{RH} and η_B unaffected
as long as v_{B-L} , $\tilde{m}_1$ and
 M_1 are kept constant.

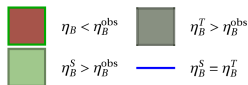
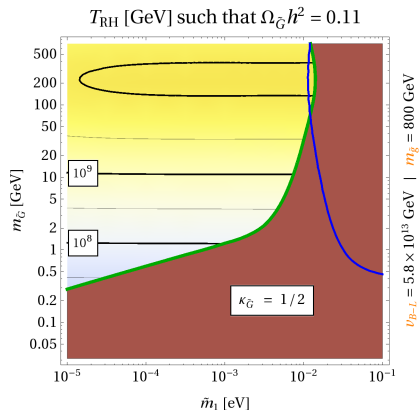
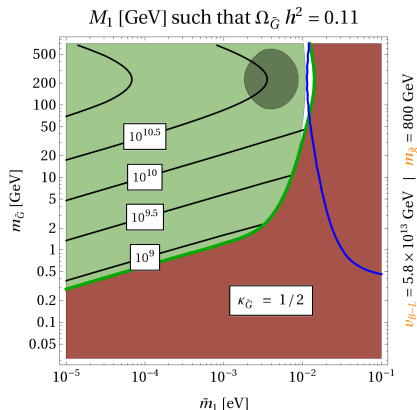
Connection between SUGRA and neutrino parameters



Scenario works in large region of parameter space!

- ▶ T_{RH} bound lowered: $\gtrsim 10^9$ GeV $\rightarrow \gtrsim 10^7$ GeV.
- ▶ $\tilde{m}_1$ and $m_{\tilde{G}}$ mutually constrain each other.

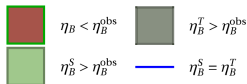
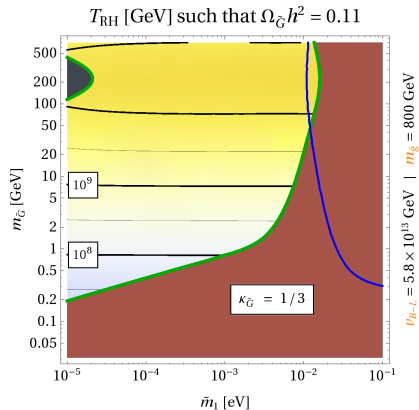
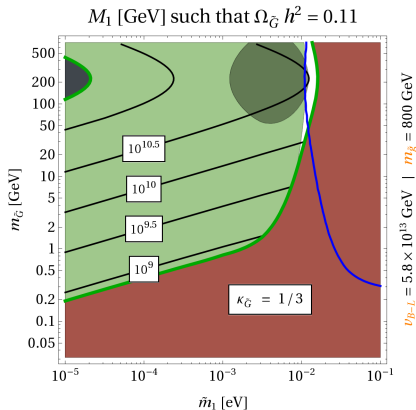
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