RWTH AACHEN UNIVERSITY

Institut Theoretische Teilchenphysik und Kosmologie

Bayesian analysis of current DD experiments

- based on JCAP09(2011)022 in collaboration with J. Hamann and Y. Wong
- added analysis of 2011 CoGeNT data
- model comparison with bayesian evidence

Chiara Arina

DESY theory workshop
Hamburg, 27-30 september
Cosmology meets Particle Physics: Ideas and Measurements

Bayesian inference and analysis method

$$\mathcal{P}(\theta|X)\mathrm{d}\theta \propto \mathcal{L}(X|\theta)\cdot\pi(\theta)\mathrm{d}\theta$$

Posterior probability function (PDF)

Likelihood

Prior

- Marginalization over systematic and astrophysical observables (integration over all the parameters other than WIMP mass and sigma)

Experiment	Parameter	Prior
All	$\log(m_{\mathrm{DM}}/\mathrm{GeV})$	$0 \to 3$
All	$\log(\sigma_n^{\mathrm{SI}}/\mathrm{cm}^2)$	$-44(-46) \to -38$
DAMA	q_{Na}	$0.2 \to 0.4$
DAMA	q_{I}	$0.06 \to 0.1$
CoGeNT	C	$0 \to 10$ cpd/kg/keVee
CoGeNT	$\mathcal{E}_0$	$0 \to 30$ keVee
CoGeNT	A	$0 \to 10$ cpd/kg/keVee
Xenon100	L_{eff}	$-0.01 \to 0.18$
Experiment	**Observable**	**Gaussian Prior**
All	Local standard of rest	$v_0^{\mathrm{obs}} = 230 \pm 24.4$ km s^{-1}
All	Escape velocity	$v_{\mathrm{esc}}^{\mathrm{obs}} = 544 \pm 39$ km s^{-1}
All	Local DM density	$\rho_\odot^{\mathrm{obs}} = 0.4 \pm 0.2$ GeV cm^{-3}
All	Virial mass	$M_{\mathrm{vir}}^{\mathrm{obs}} = 2.7 \pm 0.3 \times 10^{12} M_\odot$

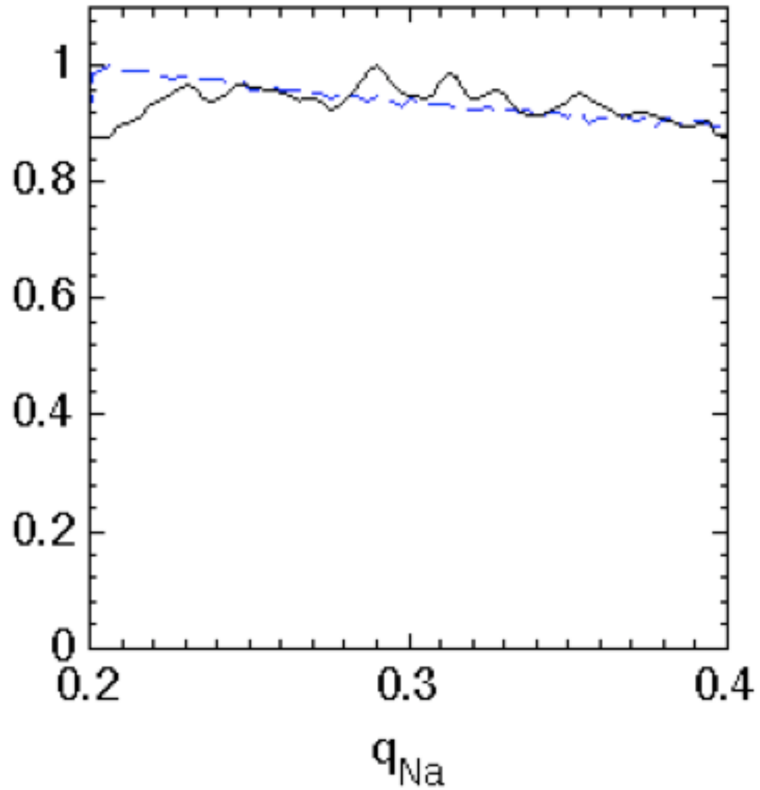
systematics

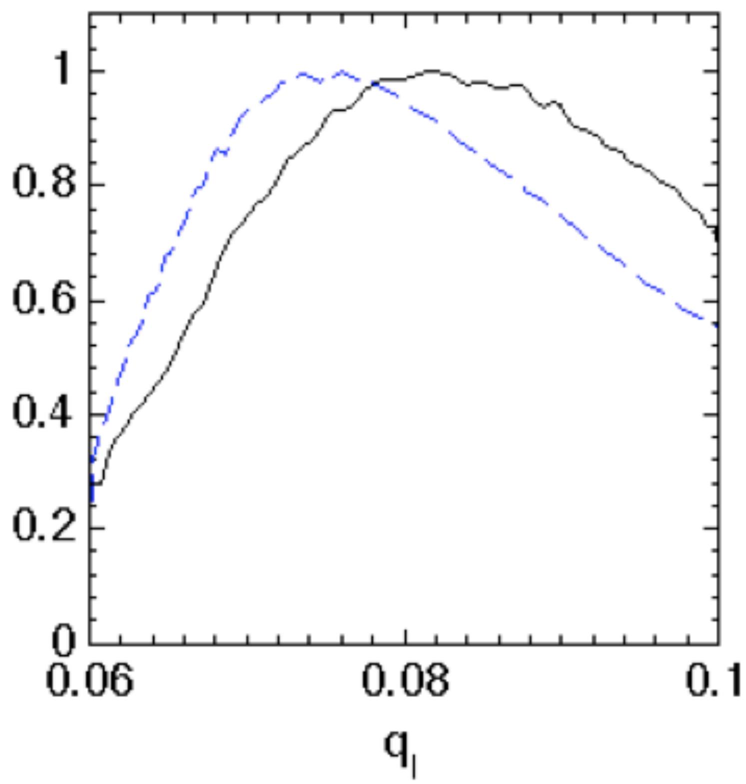
astrophysical parameters

Posterior sampled via MCMC techniques

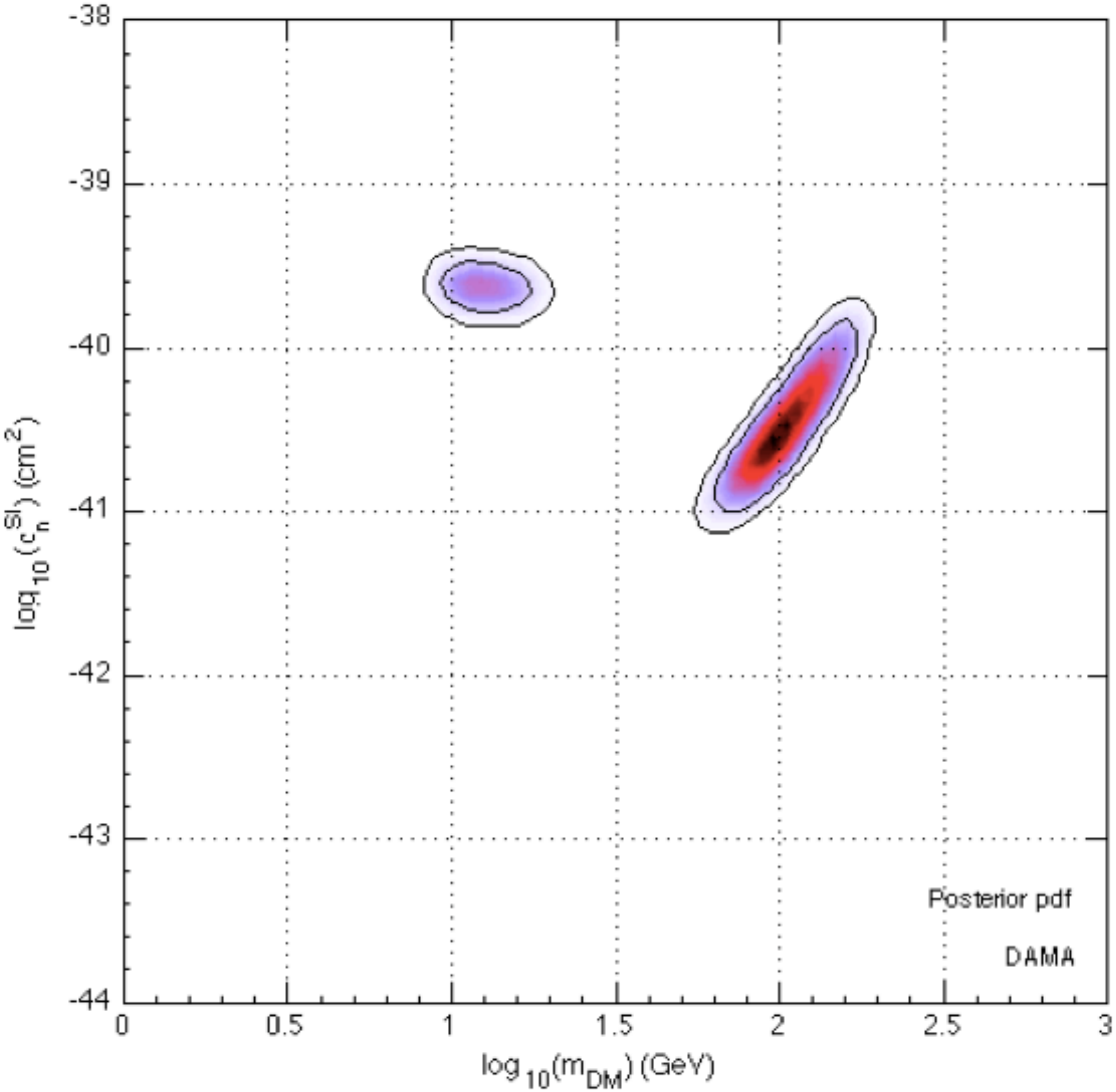
Inference: results for DAMA/LIBRA and SMH

1D marginalized posterior PDF
quenching factors (nuisance)





2D marginal credible regions at 90 and 99%



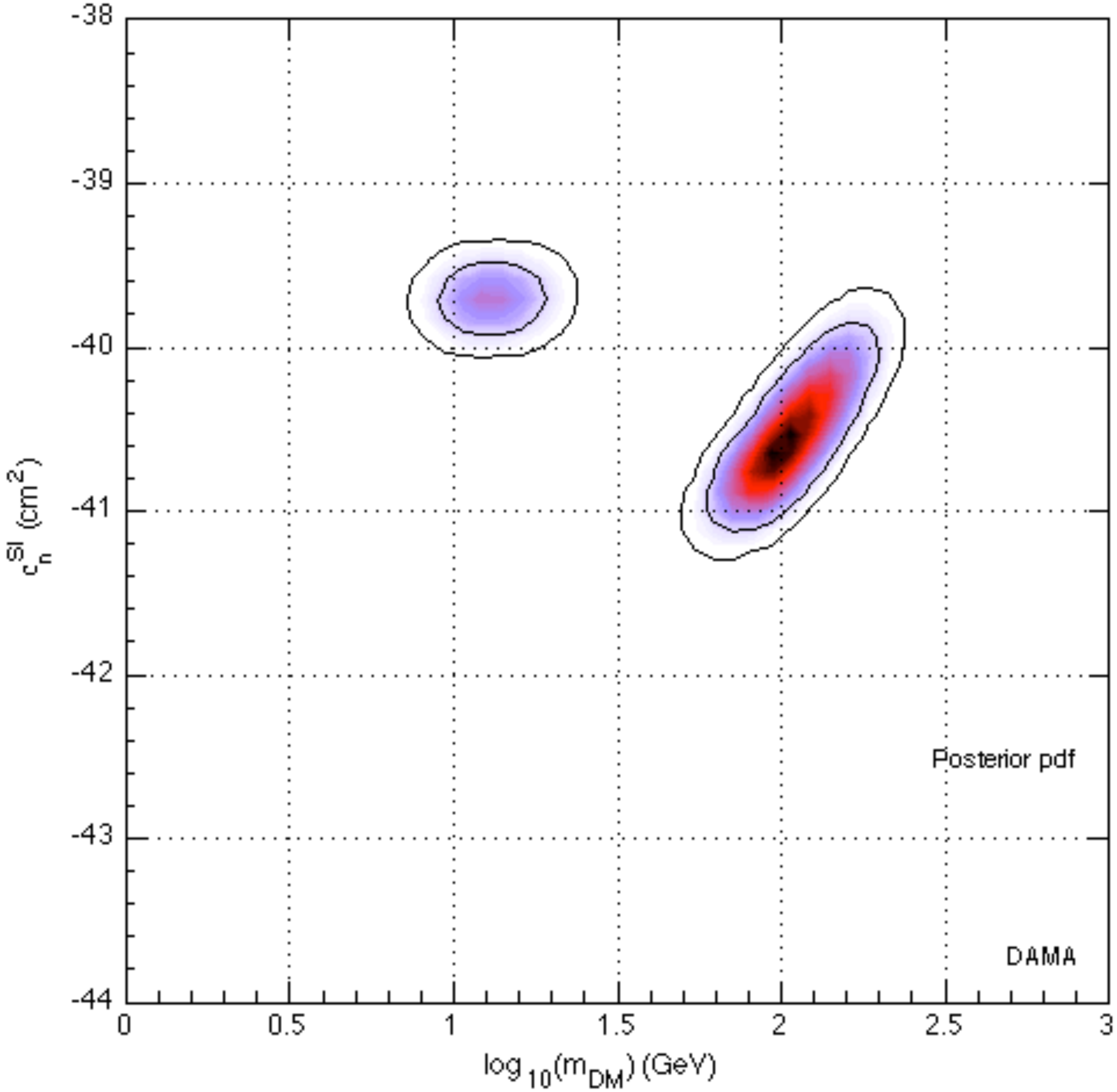
Matches with profile likelihood analysis

Varying astrophysics results for DAMA/LIBRA inference, NFW DM profile

- 1D marginalized posterior PDF quenching factors as SMH

Preferred values for astrophysics:

$$v_0^{\rm obs} = 220^{+40}_{-20}\ {\rm km\ s^{-1}}$$
$$v_{\rm esc}^{\rm obs} = 558^{+19}_{-16}\ {\rm km\ s^{-1}}$$
$$\rho_\odot^{\rm obs} = 0.38^{-0.09}_{+0.15}\ {\rm GeV\ cm^{-3}}$$



- 2D regions at 90, 99% are larger than SMH case
- volume effect due to the integration over all possible velocities and density values of the halo at the Sun position
- same behavior for Einasto, Burkert and cored isothermal profile

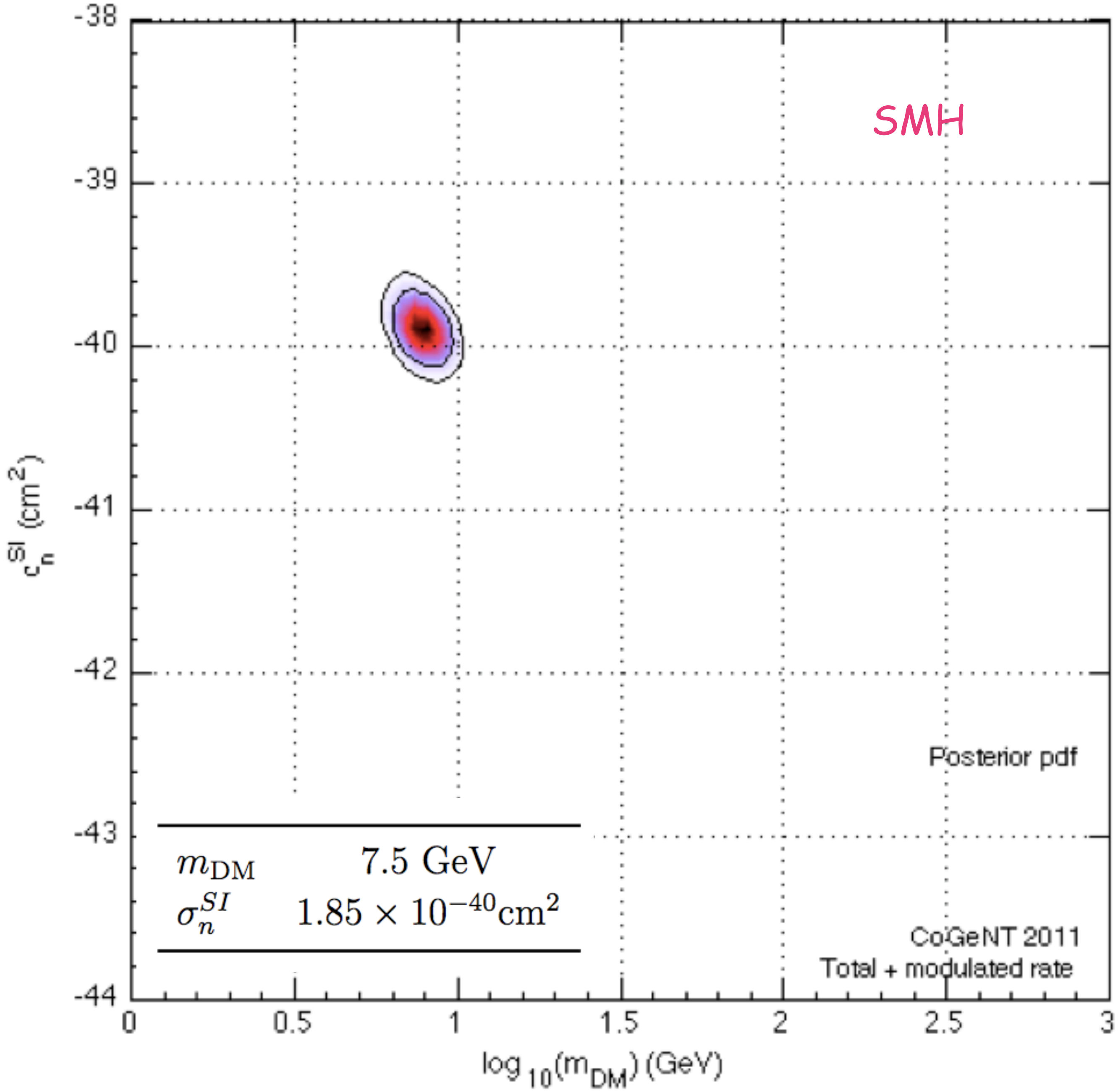
CoGeNT 2011

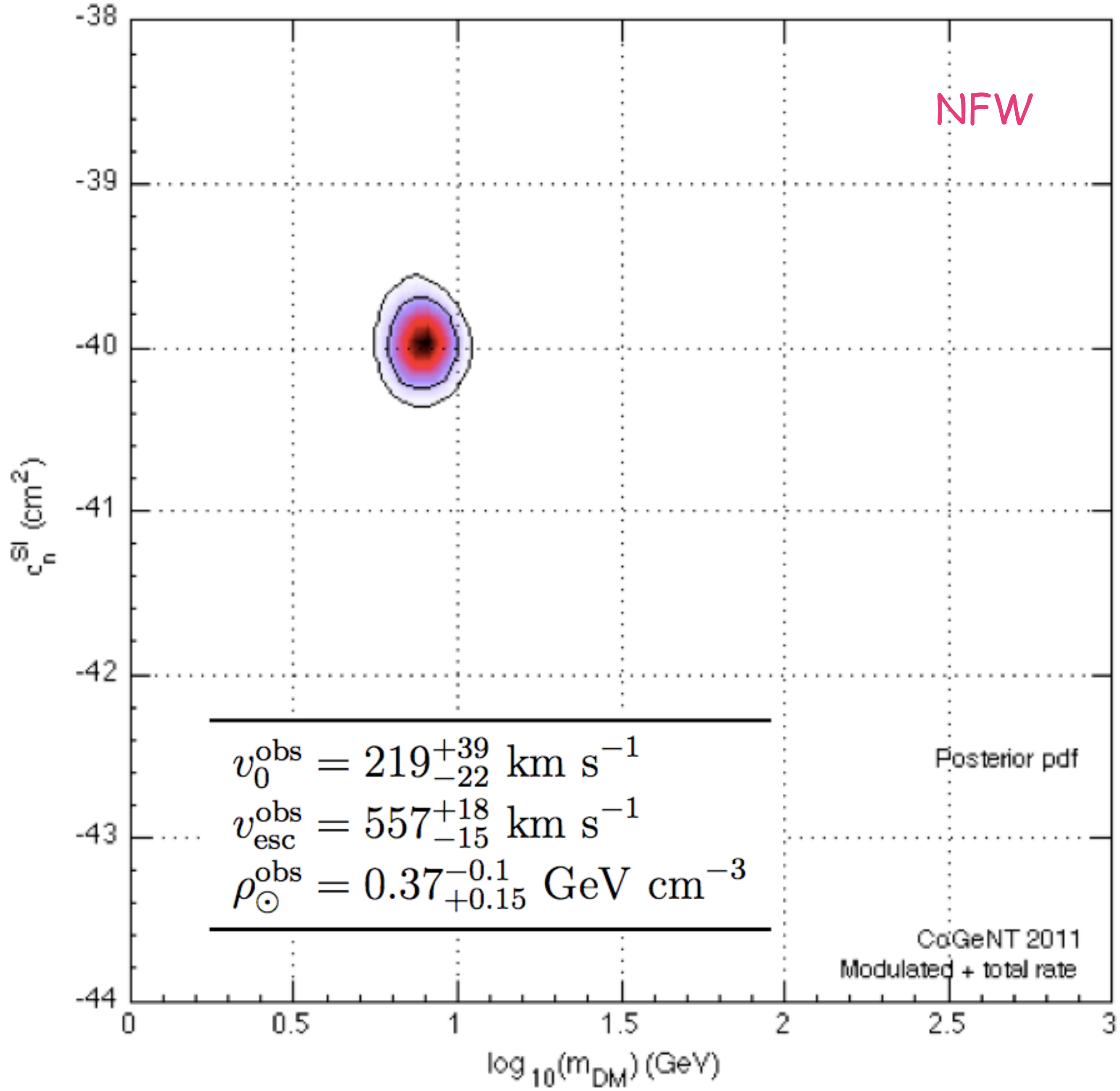
(data courtesy of CoGeNT coll.)

$$\ln \mathcal{L}_{\rm CoGeNT} = \ln \mathcal{L}_{\rm TR} + \ln \mathcal{L}_{\rm MR}$$

- 3 nuisance parameters for background modeling
- less affected by volume effects
- 2D region matches profile likelihood

2D marginal credible regions at 90 and 99%



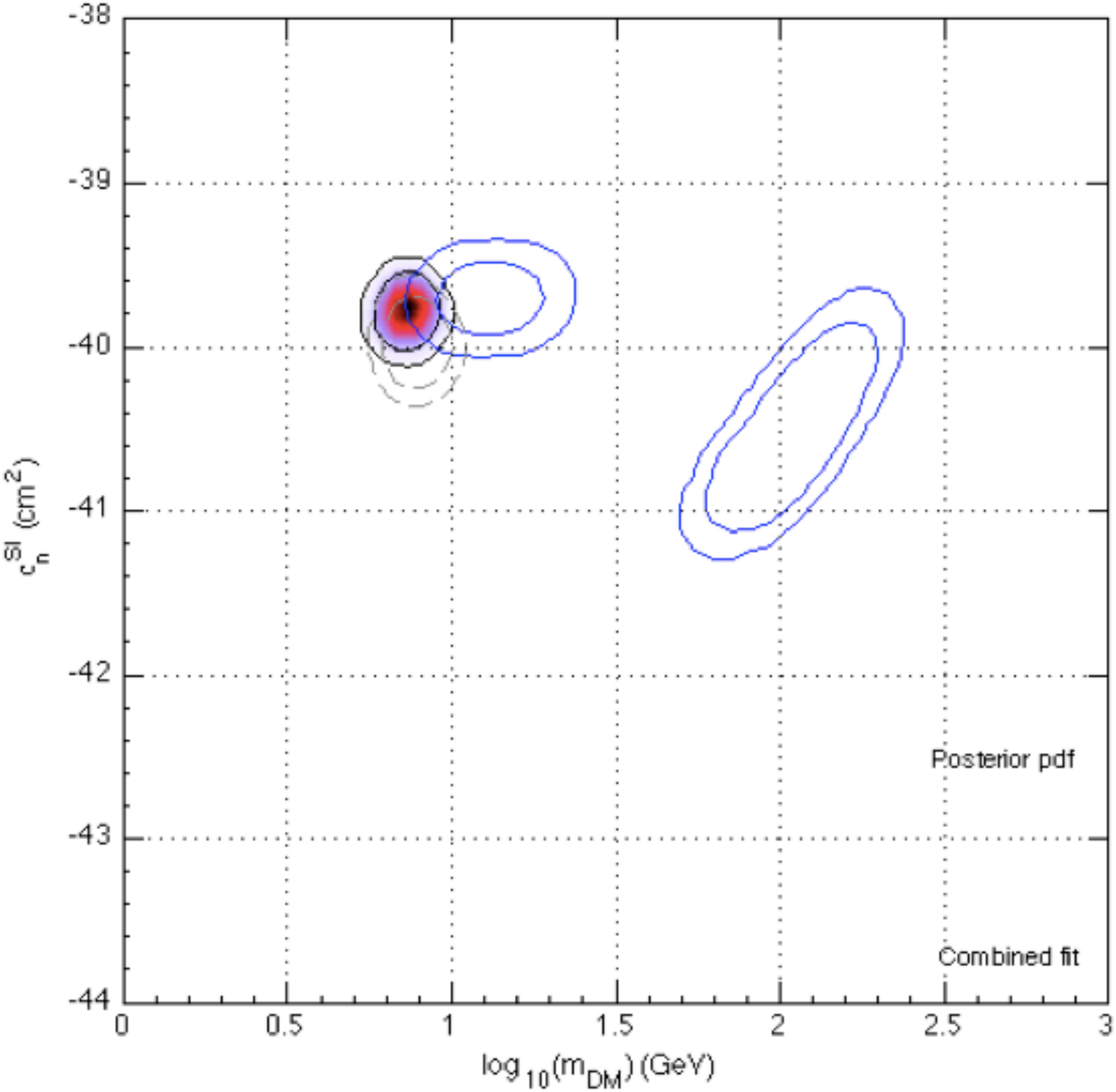


$m_{\rm DM} = 7.4$ GeV
$\sigma_n^{SI} = 9.7 \times 10^{-41}{\rm cm}^2$

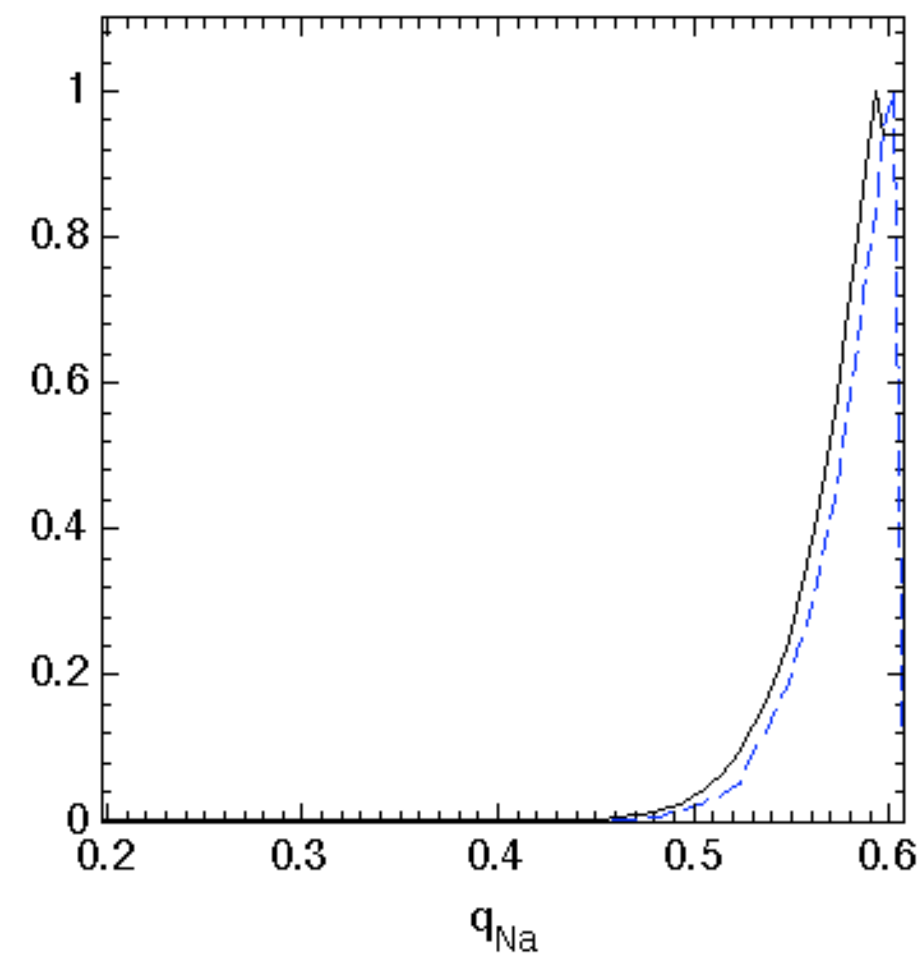
Analysis of the modulated work in progress in collaboration with J. Hamann, R. Trotta and Y. Wong

DAMA and CoGeNT, combined fit

2D marginal credible regions at 90 and 99%



- quenching factor prefers now the value 0.57 (same behavior also for SMH)



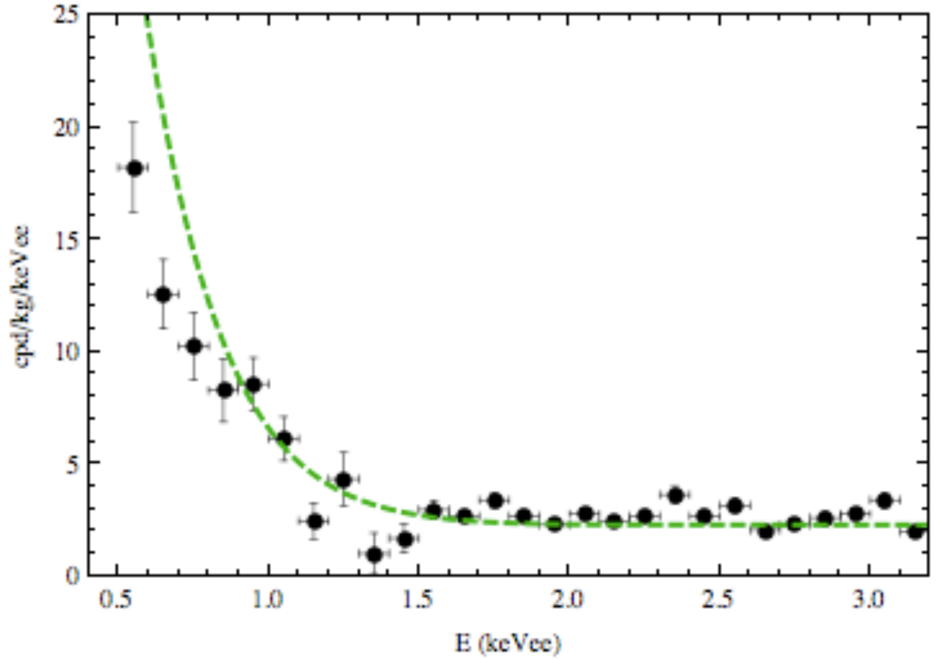
Combined fit pushes the quenching factor at corner values

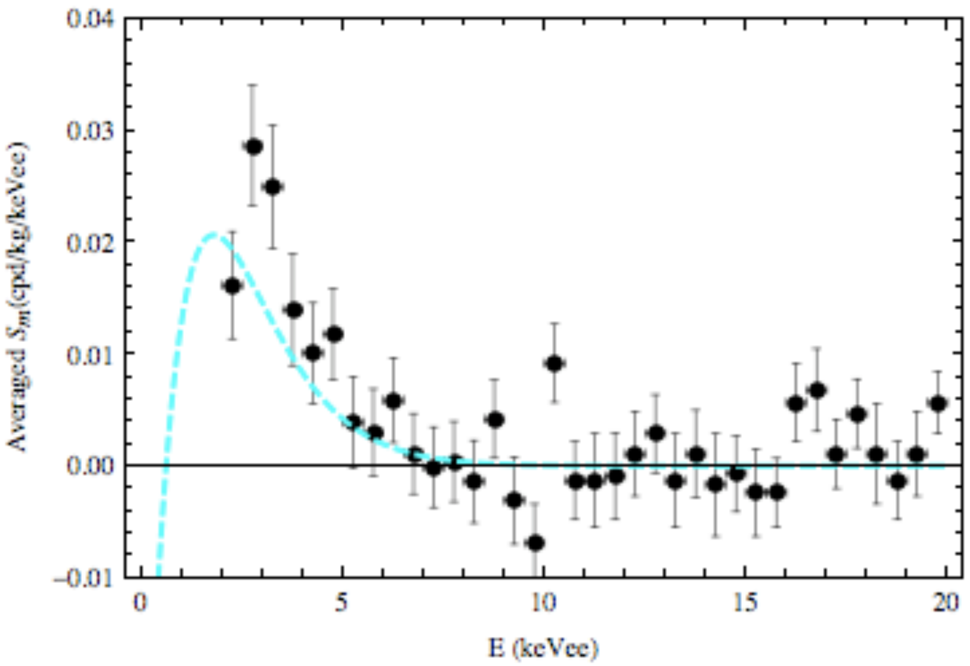
More on the combined fit

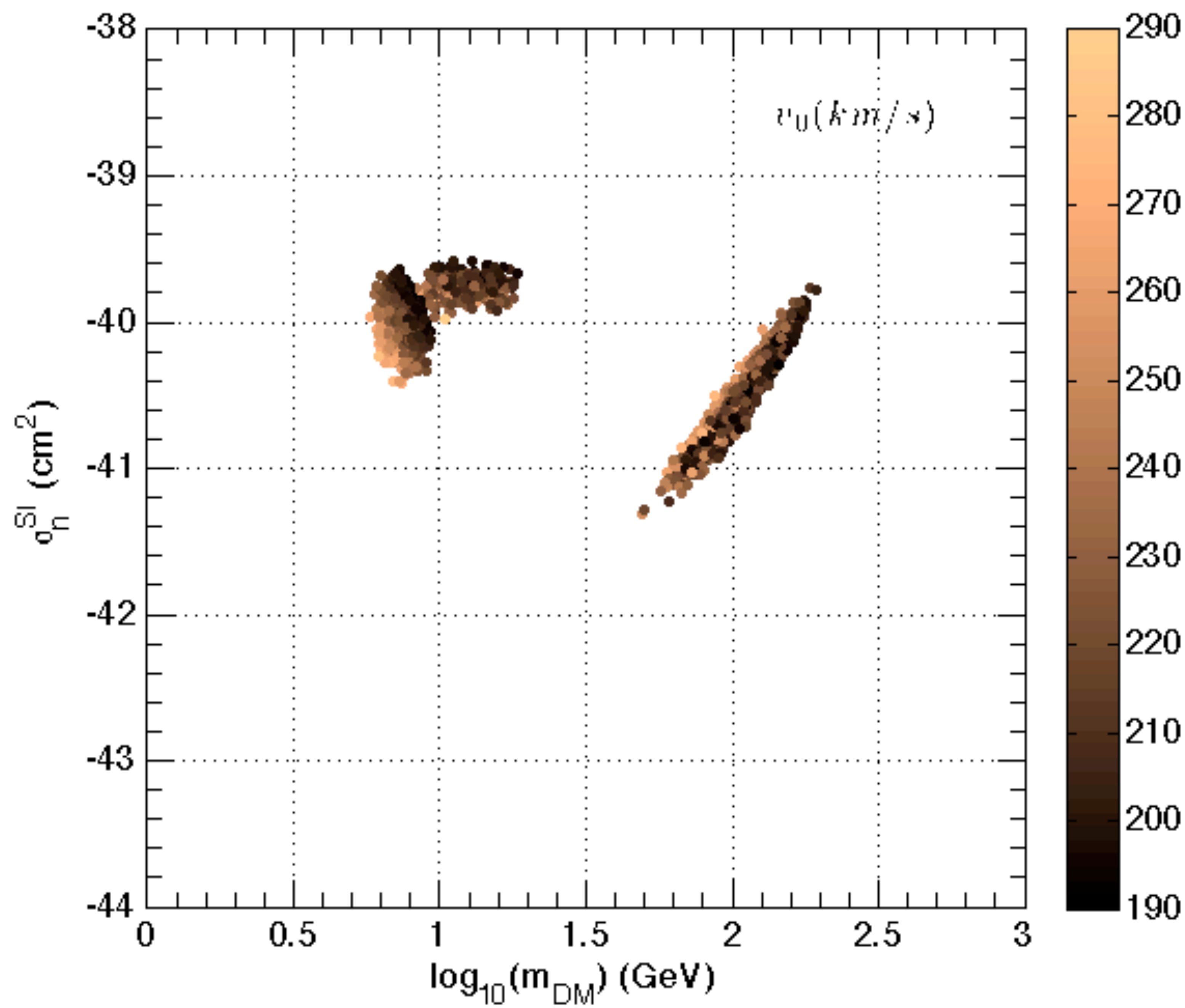
$$m_{\rm DM} = 7.\ ({\rm GeV})$$
$$\sigma_n^{SI} = 1.53 \times 10^{-40}\ ({\rm cm}^2)$$

$$q_{Na} = 0.57 \pm 0.03$$
$$A = 1.7 \pm 1.1\ ({\rm cpd/kg/keVee})$$
$$C = 1.2 \pm 0.8\ ({\rm cpd/kg/keVee})$$
$$\mathcal{E}_0 = 15.1 \pm 8.9\ ({\rm keVee})$$

$$v_0 = 214^{+32}_{-21}\ ({\rm km\ s^{-1}})$$
$$v_{\rm esc} = 556^{+14}_{-15}\ ({\rm km\ s^{-1}})$$
$$\rho_\odot = 0.35^{+0.14}_{-0.09}\ ({\rm GeV\ cm^{-3}})$$







Bayesian evidence for evaluating the support for astrophysics

$$\mathcal{Z} = \int \mathcal{L}(X|\theta)\pi(\theta)d^D\theta$$

$$\mathcal{P}(\mathcal{M}|X) \propto \mathcal{Z}\ \pi(\mathcal{M})$$

$$\frac{\mathcal{P}(\mathcal{M}_0|X)}{\mathcal{P}(\mathcal{M}_1|X)} = B_{01}\frac{\pi(\mathcal{M}_0)}{\pi(\mathcal{M}_1)}$$

Bayes factor for model comparison

Empirical Jeffreys' scale for determining the strength of evidence

$\lvert \ln B_{01} \rvert$	Odds	Probability	Strength of evidence
< 1.0	$\lesssim 3:1$	< 0.750	Inconclusive
1.0	$\sim 3:1$	0.750	Weak evidence
2.5	$\sim 12:1$	0.923	Moderate evidence
5.0	$\sim 150:1$	0.993	Strong evidence

$$p(\mathcal{M}_0) = p(\mathcal{M}_1) = 1/2 \qquad p(\mathcal{M}_0|d) + p(\mathcal{M}_1|d) = 1$$

Results from evidence calculations

CASE 1: individual experiment and SMH compared to NFW

Model 0 = DAMA + SMH
Model 1 = DAMA + NFW (nested in model 0)

$$\ln B_{01} = 5.3$$

Model 0 = CoGeNT + SMH
Model 1 = CoGeNT + NFW

$$\ln B_{01} = 3.9$$

Moderate evidence against inclusion of astrophysics

CASE 2: combined fit and SMH compared to NFW

Model 0 = DAMA + CoGeNT + SMH
Model 1 = DAMA + CoGeNT + NFW

$$\log \mathcal{Z}_{0=\mathrm{SMH}} = -99.87 \pm 0.03$$

$$\log \mathcal{Z}_{1=\mathrm{NFW}} = -68.25 \pm 0.03$$

$$\log \mathrm{B}_{01} = -32$$

Very Strong evidence for inclusion of astrophysics

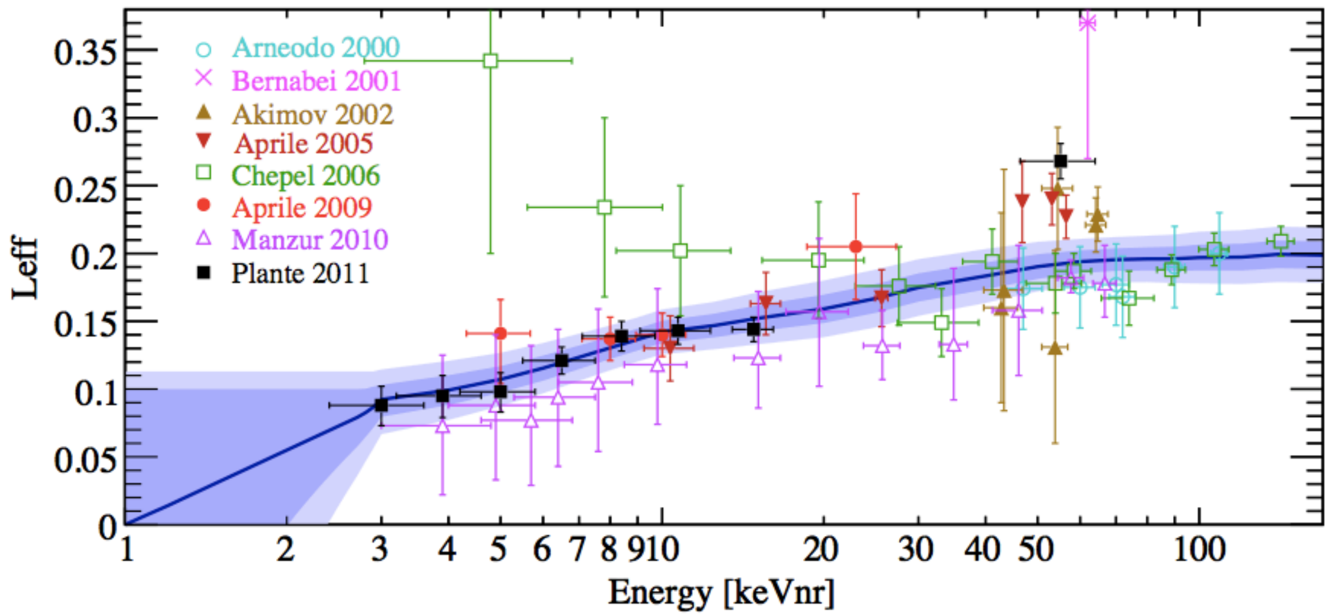
- A single direct detection experiment does not constraint astrophysical DM models
- Combined experiments need astrophysical parameters for compatibility

Xenon100: what about the compatibility with current exclusion bounds?

- 3 events seen, likelihood follows a Poisson distribution
- expected background of 1.8 +- 0.6, numerical marginalization
- considered Poisson fluctuation below threshold

$$\ln \mathcal{L}_{\rm Xenon} = \ln \mathcal{L}_{\rm Events} + \ln \mathcal{L}_{\rm L_{eff}}$$

from arXiv:1104.2549



- Scintillation efficiency is a systematic of the experimental set-up
- treated as nuisance parameter with truncated gaussian prior and marginalized over

$$S_1(E) = \mathrm{L_{eff}}(E)\ \mathrm{L}_y\ E\ \frac{S_{\rm nr}}{S_{\rm ee}}$$

$$\frac{\mathrm{d}R}{\mathrm{d}S_1} = \int_0^\infty \mathrm{d}E\ \frac{\mathrm{d}R}{\mathrm{d}E} \times P(S_1|\bar{S}_1(E))$$

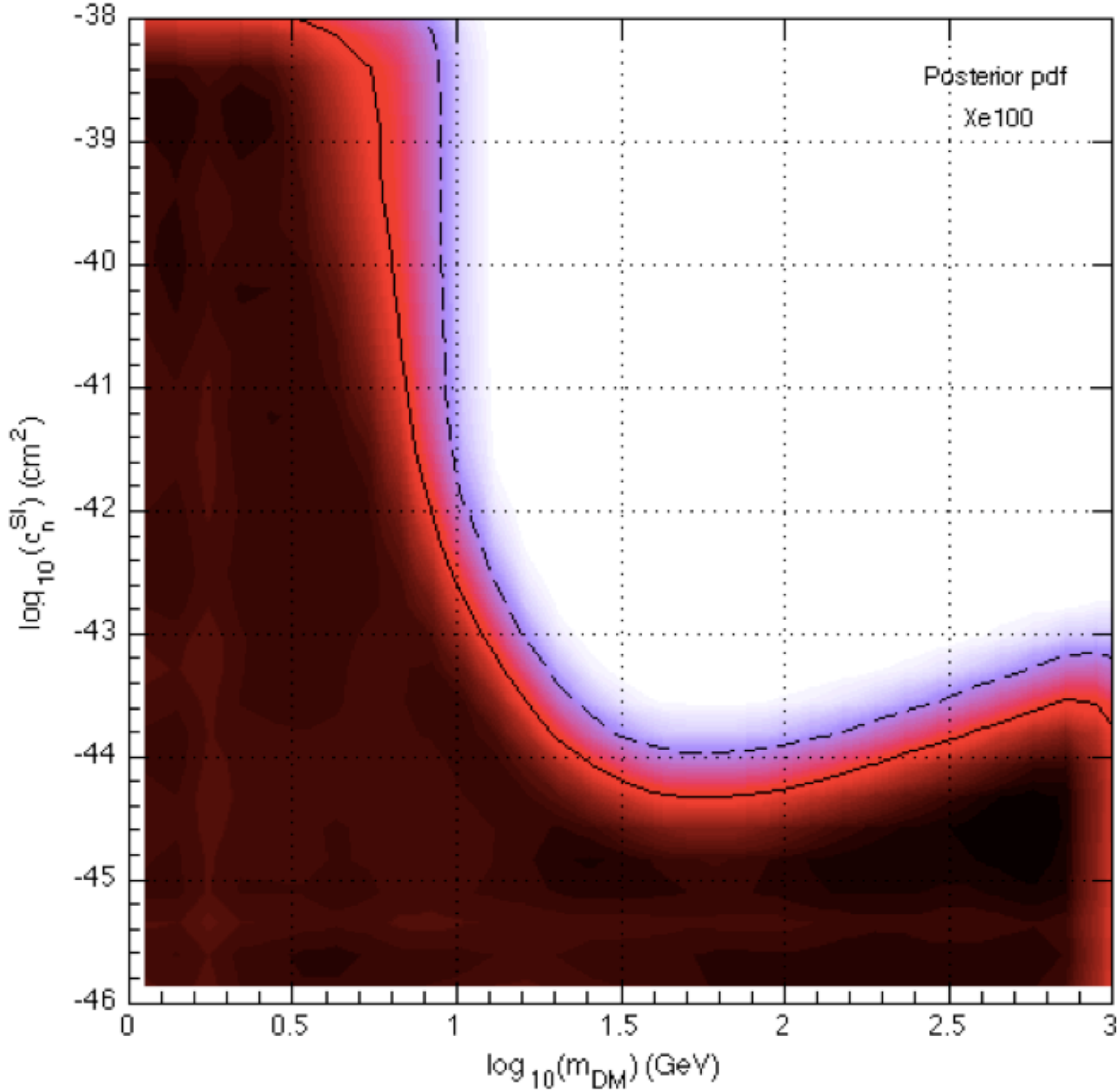
$$S = M_{\rm det} T \sum_{n=\rm PE_{min}}^{\rm PE_{max}} \frac{\mathrm{d}R}{\mathrm{d}S_1}$$

$$\ln \mathcal{L}_{\rm L_{eff}} = -\frac{(m-\bar{m})^2}{2\sigma_m^2}$$

$$\ln \mathcal{L}_{\rm Events} = -S - B + 3 + \sum_{i=1}^{3} \ln\left(\left.\frac{\mathrm{d}R}{\mathrm{d}S_1}\right|_i + \frac{B}{\bar{B}} \left.\frac{\mathrm{d}N_B}{\mathrm{d}S_1}\right|_i \right) + C_{\rm norm}$$

Xenon100: what about the compatibility with current exclusion bounds?

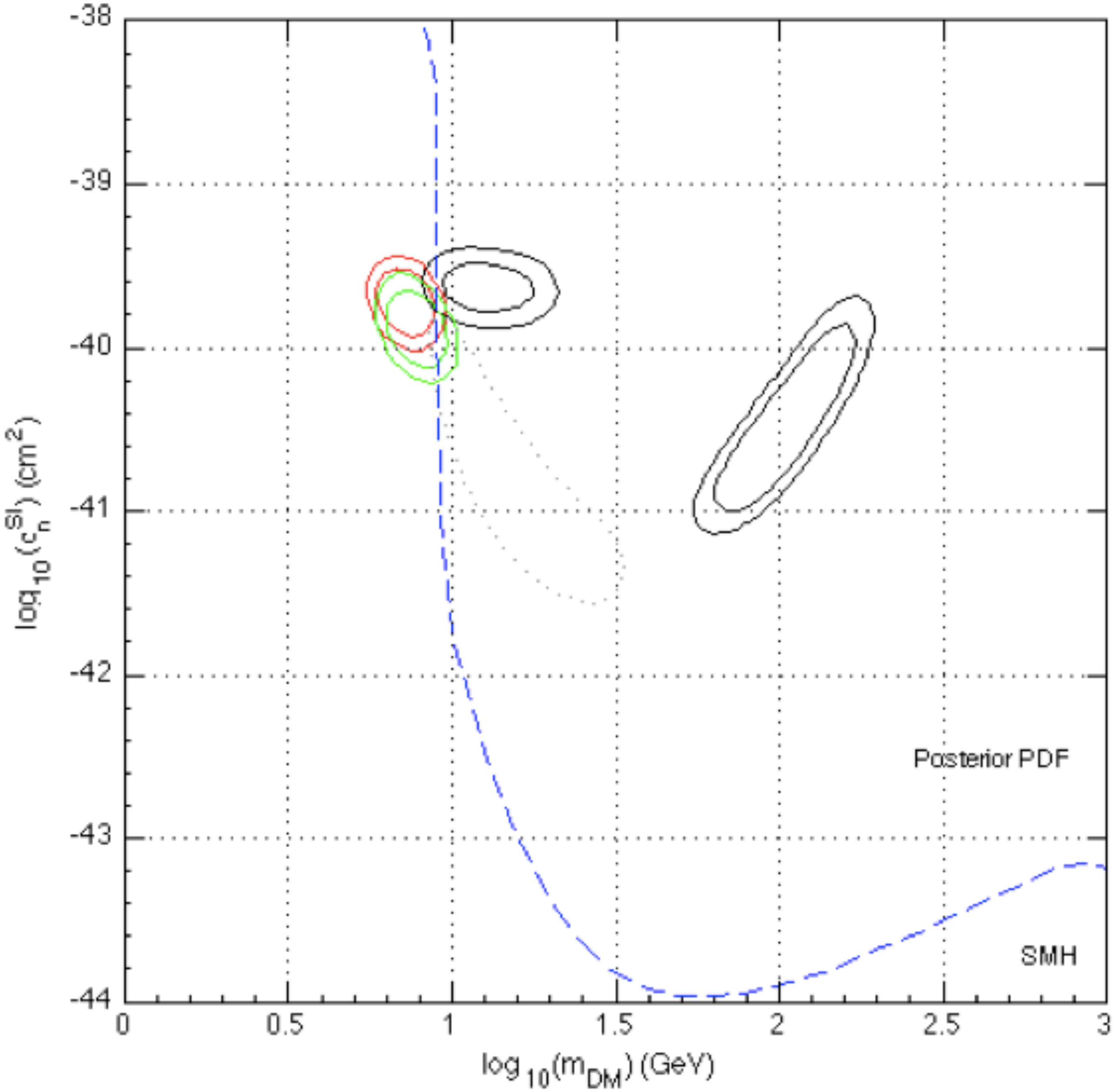
2D marginal credible regions at 90% + $90_S\%$

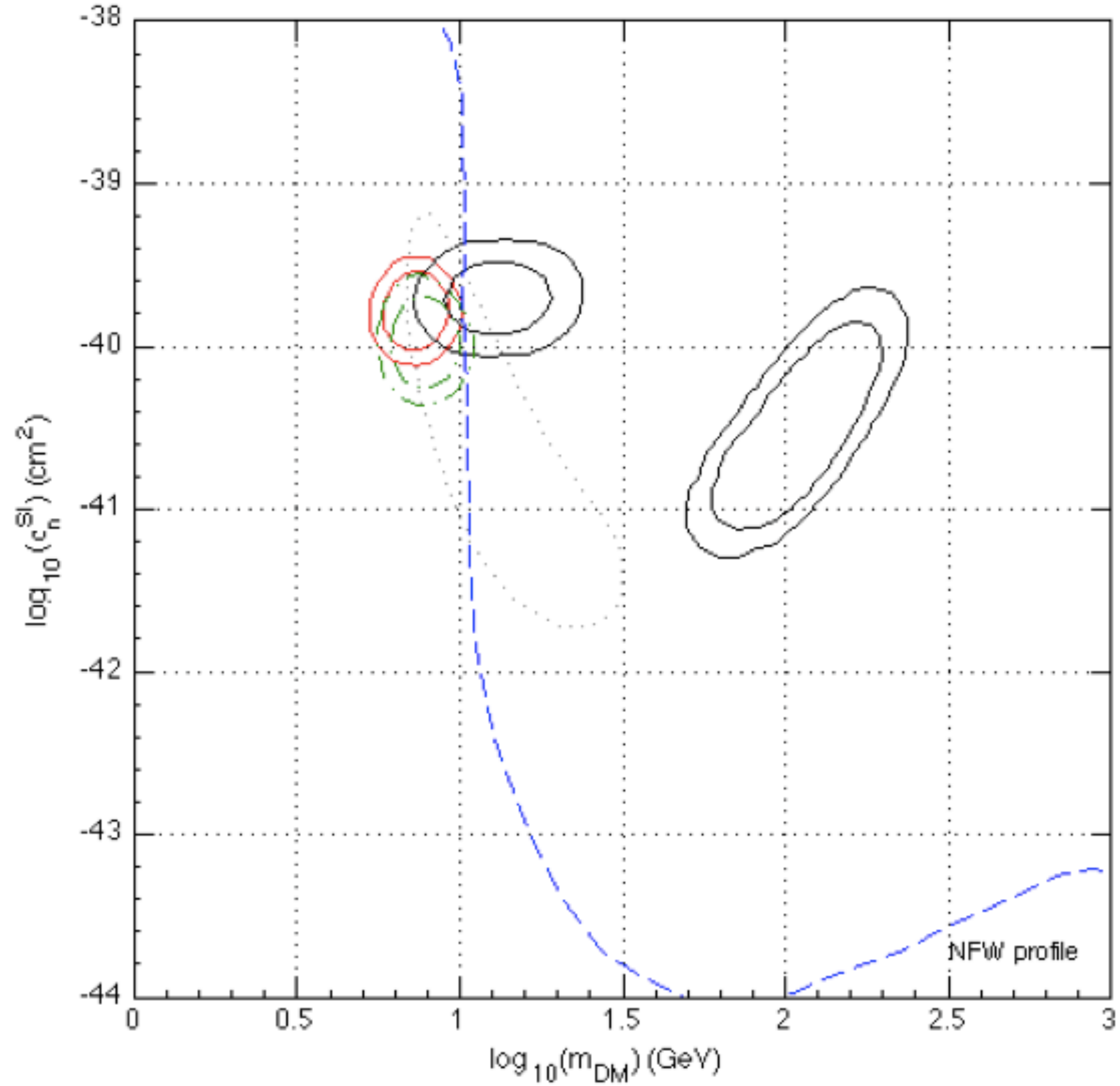


$$\Delta\chi^2_{\rm eff} \leq 2.7$$

$$\mathcal{P}_{\rm mar}(m_{\rm DM}, \sigma_n^{\rm SI}|X) = \mathcal{P}_{\rm mar}(S_x|X)$$

Experimental excess + Xenon100 exclusion bound





- for SMH the experiments are marginally compatible at 90% C.L.
- Varying astrophysics augments the compatibility between the exclusion bound and DAMA, CoGeNT
- the combined fit is compatible at 90% C.L. with Xe100 bound

Summary

- Bayesian inference of Xenon100, DAMA and CoGeNT data for elastic scattering off nucleus, with particular attention to the light WIMP region

- Marginalization over the systematics and nuisance parameters characteristic of each experiment

- Inclusion of velocity distributions arising from DM density profile and marginalization over astrophysical variables

- Astrophysical uncertainties can not be yet constrained by direct detection experiment alone

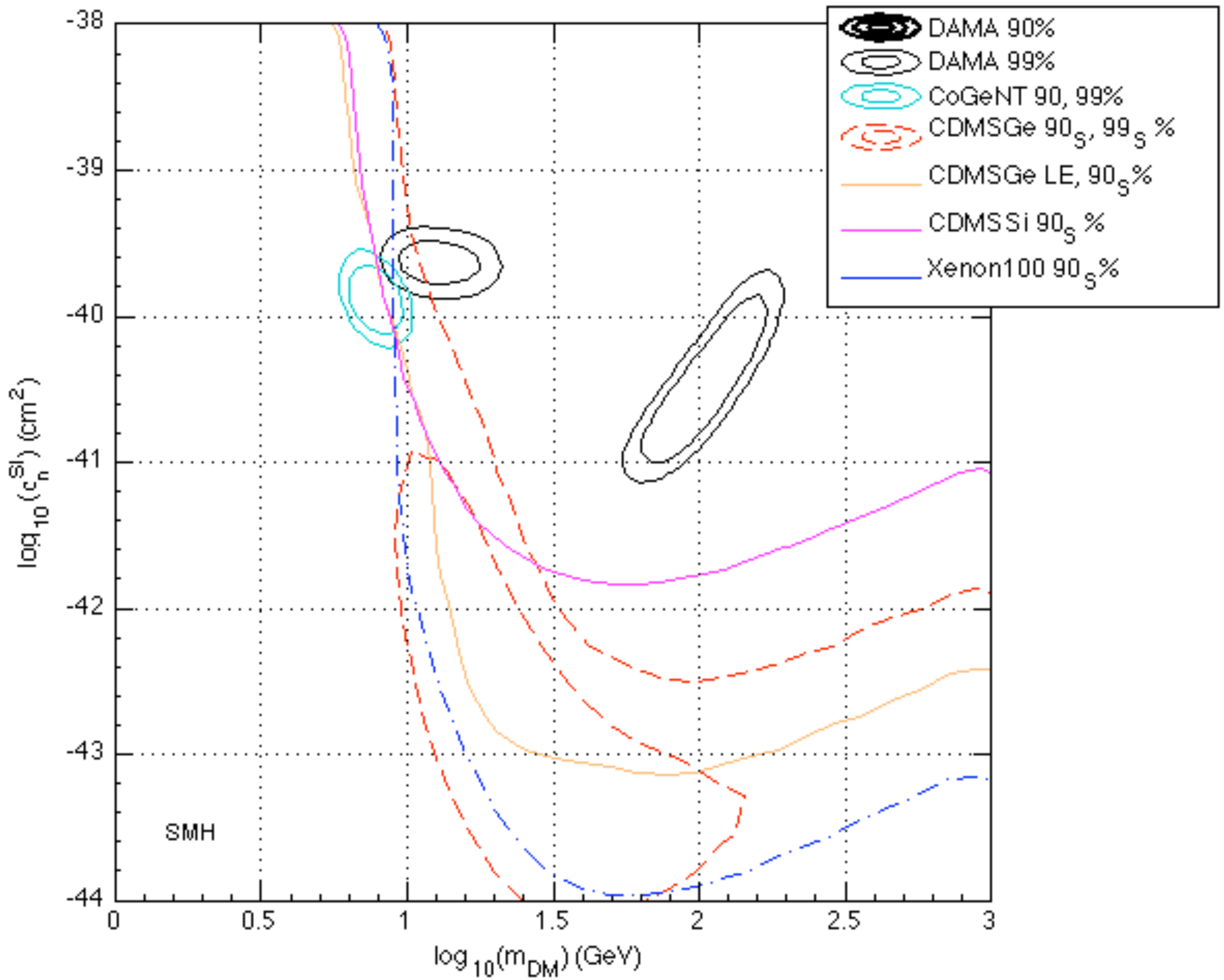
- Combining the data of CoGeNT 2011 and DAMA can constrain astrophysics

- Combined fit implies large value of the quenching factor on Sodium for DAMA and small local standard of rest velocity

- Combined fit as well as single experiment are compatible at 90% with Xenon100

Back-up slides

2D region for SMH, all experiments



Varying the velocity distribution

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho_\odot}{m_{\mathrm{DM}}} \int_{v'>v'_{\mathrm{min}}} \mathrm{d}^3 v' \, \frac{\mathrm{d}\sigma}{\mathrm{d}E} \, v' \, f(\vec{v}'(t))$$

Velocity distribution at the Sun position

$$\rho_{\mathrm{DM}}(\vec{r}) = \int \mathrm{d}^3 v \; F(\vec{v}, \vec{r})$$

$$f(\vec{v}'(t)) \equiv F(\vec{v}, \vec{R}_\odot)/\rho_\odot$$

$$\rho_\odot \equiv \rho_{\mathrm{DM}}(R_\odot)$$

- Spherically symmetric halos produce isotropic velocity distribution
- Considered as example the case of the NFW density profile

$$\ln \mathcal{L}_{\mathrm{Astro}} = -\frac{(v_0 - \bar{v}_0^{\mathrm{obs}})^2}{2\sigma_{v_0}^2} - \frac{(v_{\mathrm{esc}} - \bar{v}_{\mathrm{esc}}^{\mathrm{obs}})^2}{2\sigma_{v_{\mathrm{esc}}}^2} - \frac{(\rho_\odot - \bar{\rho}_\odot^{\mathrm{obs}})^2}{2\sigma_{\rho_\odot}^2} - \frac{(M_{\mathrm{vir}} - \bar{M}_{\mathrm{vir}}^{\mathrm{obs}})^2}{2\sigma_{M_{\mathrm{vir}}}^2}$$

$$v_0^{\mathrm{obs}} = 230 \pm 24.4 \ \mathrm{km\ s^{-1}}$$

$$v_{\mathrm{esc}}^{\mathrm{obs}} = 544 \pm 39 \ \mathrm{km\ s^{-1}}$$

$$\rho_\odot^{\mathrm{obs}} = 0.4 \pm 0.2 \ \mathrm{GeV\ cm^{-3}}$$

$$M_{\mathrm{vir}}^{\mathrm{obs}} = 2.7 \pm 0.3 \times 10^{12} M_\odot$$

- Astrophysical variables are nuisance parameters with a gaussian prior centered on the measured value
- Marginalization over astrophysical uncertainties

Xenon100

- 3 events seen, likelihood follows a Poisson distribution
- expected background of 1.8 +- 0.6, numerical marginalization
- exposure of 1481 kg days
- energy range from 4 -> 30 PE

$$\ln \mathcal{L}_{\text{Events}} = -S - B + 3 + \sum_{i=1}^{3} \ln\left(\left.\frac{\mathrm{d}R}{\mathrm{d}S_1}\right|_i + \frac{B}{\bar{B}} \left.\frac{\mathrm{d}N_B}{\mathrm{d}S_1}\right|_i \right) + C_{\text{norm}}$$

$$\ln \mathcal{L}_{\mathrm{L_{eff}}} = -\frac{(m-\bar{m})^2}{2\sigma_m^2}$$

$$\mathrm{L_{eff}}(E) = \begin{cases} \bar{\mathrm{L}}_{\mathrm{eff}}(E), & E \geq 3 \text{ keVnr}, \\ \max\{m[\ln(E/\text{keVnr}) - \ln 3] + 0.09,\ 0\}, & 1 < E/\text{keVnr} < 3 \end{cases}$$

$$S_1(E) = \mathrm{L_{eff}}(E)\ \mathrm{L}_y\ E\ \frac{S_{\text{nr}}}{S_{\text{ee}}}$$

conversion between keVnr and PE

$$S = M_{\text{det}} T \sum_{n=\mathrm{PE_{min}}}^{\mathrm{PE_{max}}} \frac{\mathrm{d}R}{\mathrm{d}S_1}$$

$$\frac{\mathrm{d}R}{\mathrm{d}S_1} = \int_0^\infty \mathrm{d}E\, \frac{\mathrm{d}R}{\mathrm{d}E} \times P(S_1|\bar{S}_1(E))$$

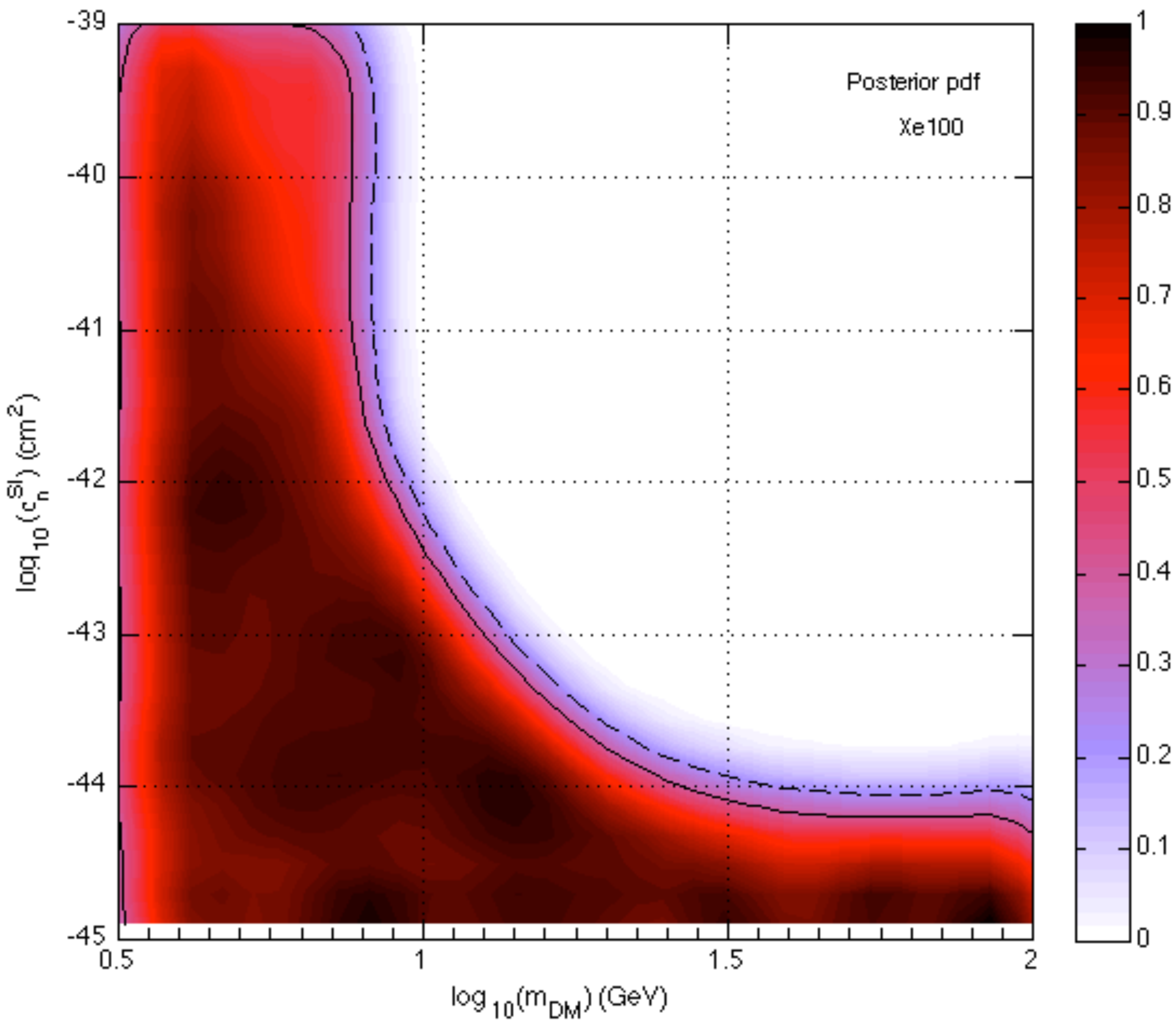
$$\mathrm{d}N_B/\mathrm{d}S_1 = 0.069/(1481 \text{ kg days})$$

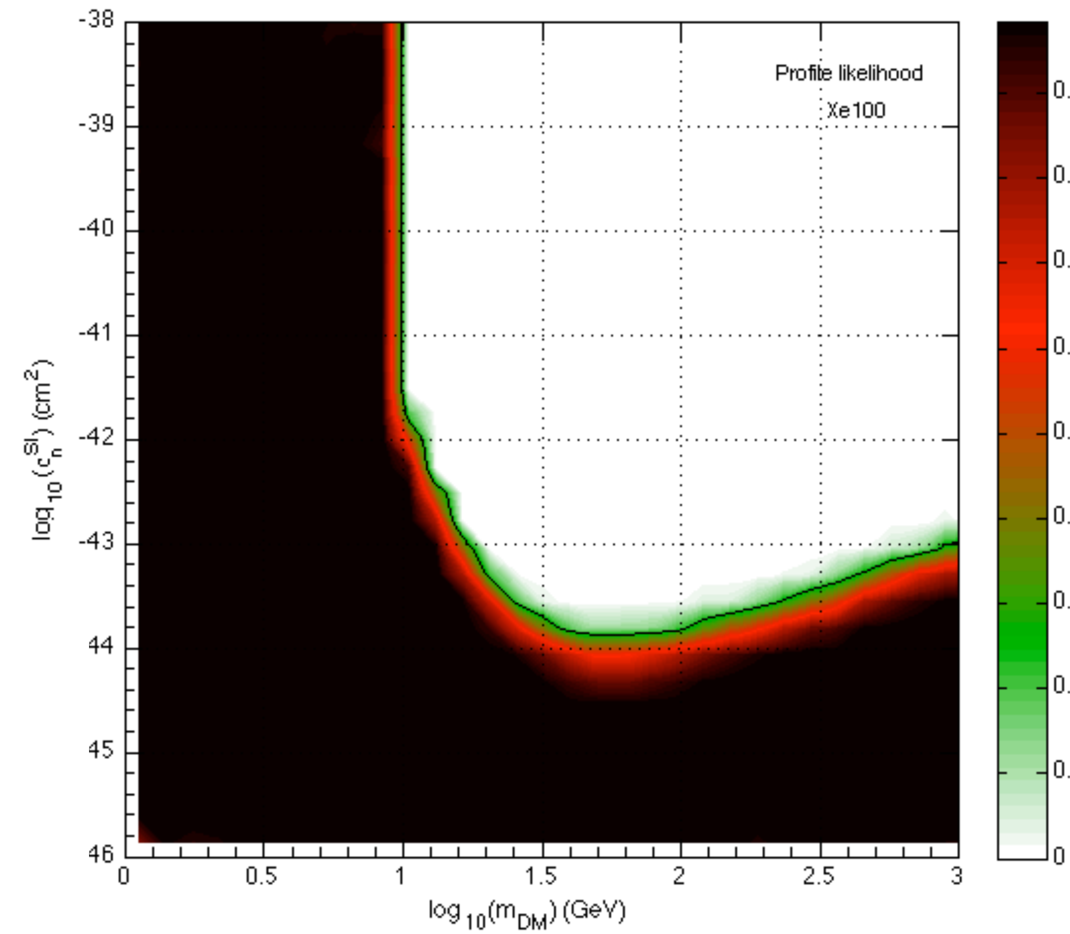
$$C_{\text{norm}} = \textstyle\sum_{i=1,2,3} \ln(M_{\text{det}} T)$$

All the likelihoods are normalized such that $\ln \mathcal{L} = 0$ if the background matches exactly the number of observed events

Xenon100: results of bayesian inference for the 2D marginalized credible region at 90% C.L.

Data are not constraining therefore the upper bound depends on the prior choice





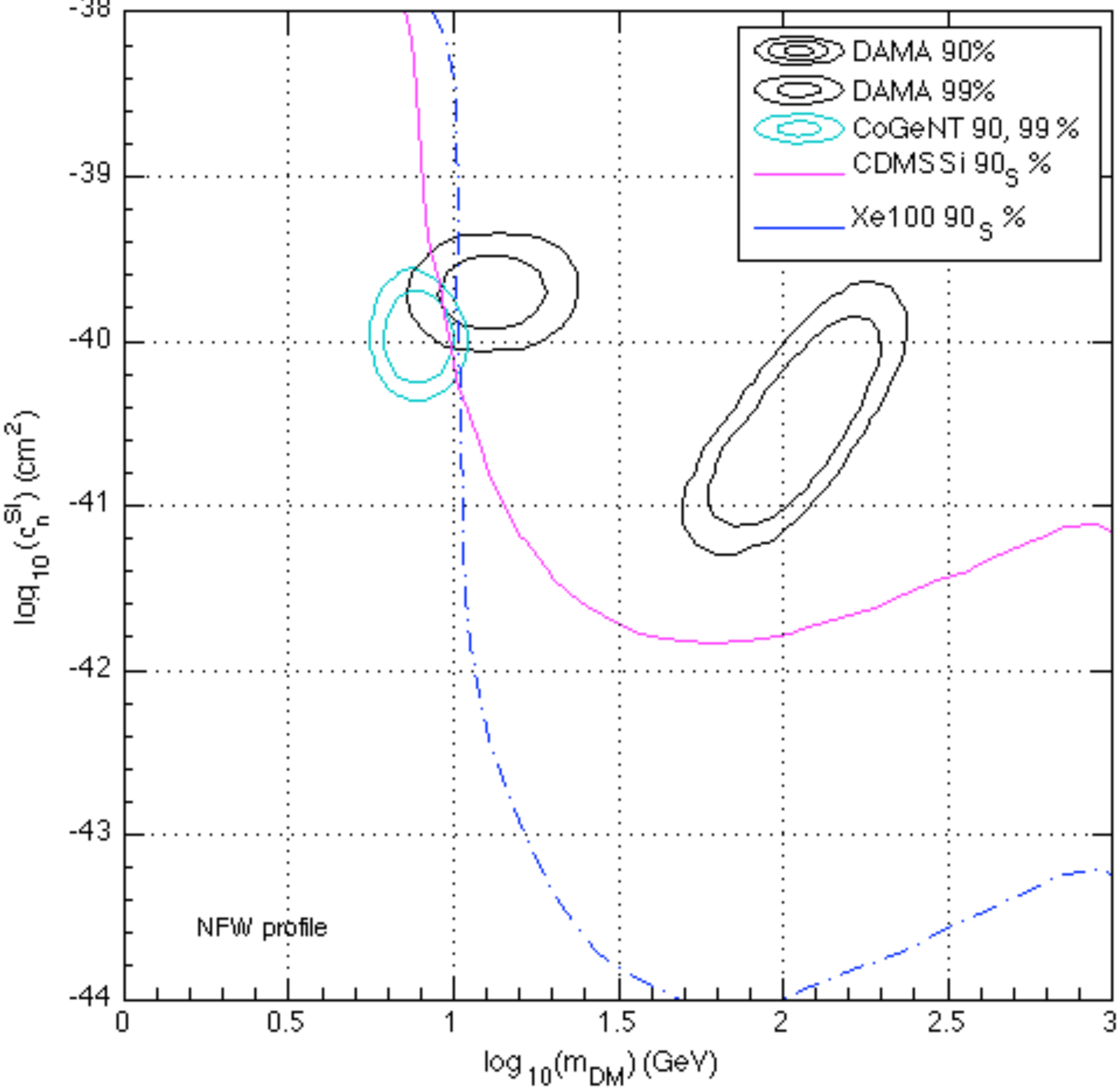
- - - $90_S\%$ = Invariant exclusion bound based on the S signal with bayesian interpretation

S_x @ $x\%$

$$\mathcal{P}_{\rm mar}(m_{\rm DM}, \sigma_n^{\rm SI}|X) = \mathcal{P}_{\rm mar}(S_x|X)$$

$S \leq 5.2$ $\qquad$ $\Delta\chi^2_{\rm eff} \leq 2.7$

2D credible regions for NFW density profile case



Preferred values for the astrophysical observables

	v_0 (km s^{-1})	v_{esc} (km s^{-1})	$\rho_\odot$ (GeV cm^{-3})
DAMA	220^{+40}_{-20}	558^{+19}_{-16}	$0.37^{+0.15}_{-0.09}$
CoGeNT	219^{+38}_{-18}	559 ± 17	$0.37^{+0.20}_{-0.08}$
CDMSSi	218^{+44}_{-19}	560^{+19}_{-18}	$0.36^{+0.18}_{-0.09}$
Xenon100	219^{+43}_{-20}	559 ± 18	$0.37^{+0.16}_{-0.08}$

- Volume effect due to marginalization over astrophysical variables

CoGeNT 2011 (SMH)

Germanium cryogenic detector
detector mass 0.33 kg
live time 442 days
total exposure 148.56 kg days

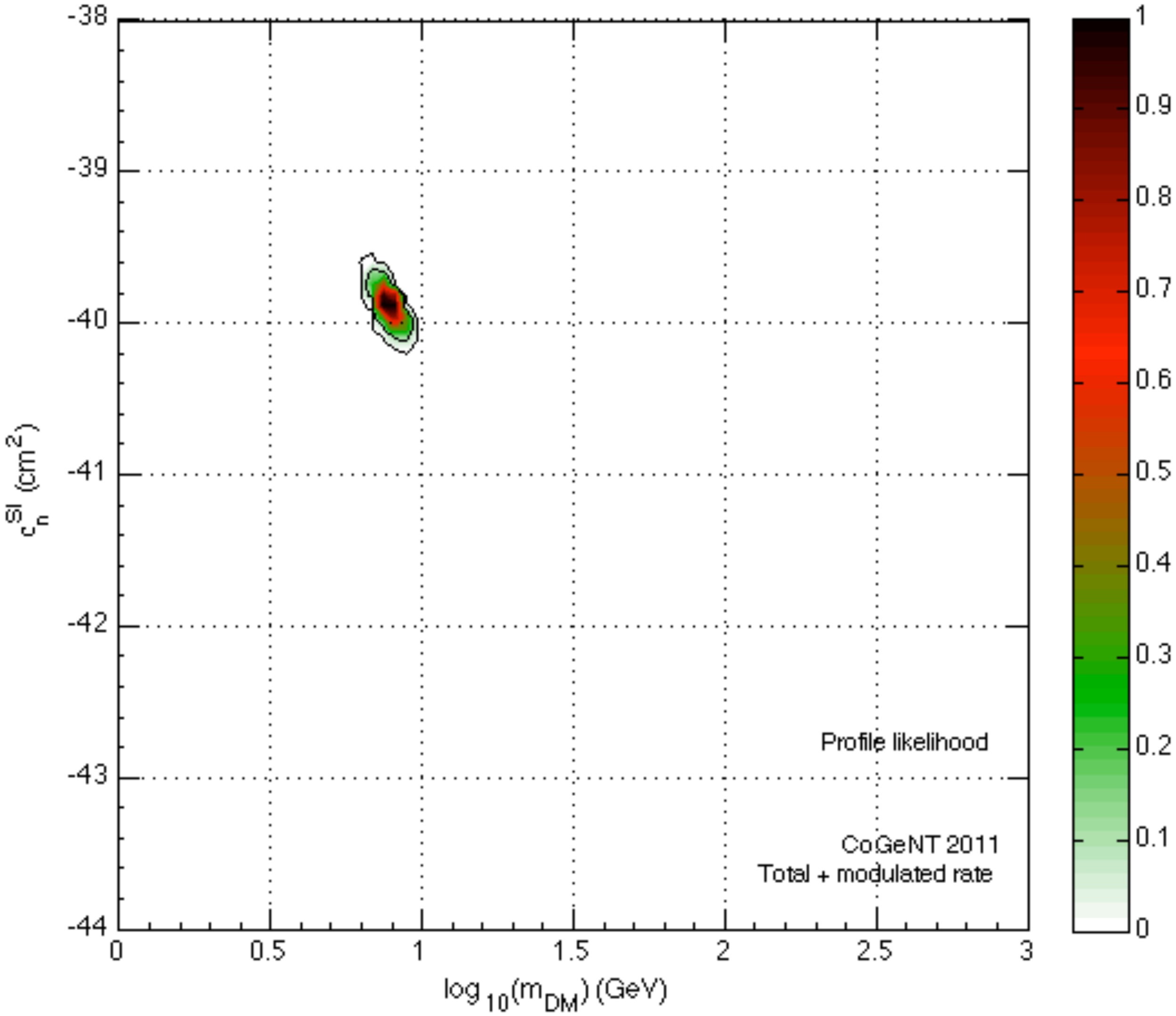
- Data analysis and binning follow arXiv:1106.0650 [astro-ph.CO]
- Radioactive peaks subtracted as prescribed by the collaboration
- Analysis of the total rate with a background (27 bins)
- Analysis of the modulated rate without background in 3 energy bins

Total rate : 27 bins of width 0.1 keVee
energy range 0.5- 3.2 keVee

$$\ln \mathcal{L}_{\rm TR} = -\frac{\chi^2}{2} = -\sum_{i=1}^{27} \frac{((S_i + b_i) - C_i)^2}{2\sigma_i^2}$$

$$\ln \mathcal{L}_{MR} = -\frac{\chi^2}{2} = -\sum_{j=1}^{3} \frac{(S^i_{\rm theo} - S^i_{\rm m})^2}{2\sigma_i^2}$$

All data are corrected by the efficiency factor, ranging from 0.7 to 0.82



ΔE_i (keVee)	S_m (cpd/kg/keVee)
$0.5 - 0.9$	1.10 ± 0.39
$0.9 - 3.0$	0.60 ± 0.12
$3.0 - 4.5$	0.07 ± 0.9

Best fit point

$m_{\rm DM}$	7.5 ± 1.1 (GeV)
σ_n^{SI}	$(1.85 \pm 0.12) \times 10^{-40}({\rm cm}^2)$
$\mathcal{E}_0$	15.51 ± 8.6 (keVee)
C	1.41 ± 0.76(cpd/kg/keVee)
A	1.62 ± 1.30(cpd/kg/keVee)

$$b_i = \frac{1}{\Delta_b} \int_{\mathcal{E}_i}^{\mathcal{E}_{i+1}} \frac{{\rm d}B}{{\rm d}\mathcal{E}} {\rm d}\mathcal{E} \qquad \frac{{\rm d}B}{{\rm d}\mathcal{E}} = C + A \exp(-\mathcal{E}/\mathcal{E}_0)$$

CoGeNT 2011

radioactive peaks

Element	$\mathcal{E}_p$ (keVee)	σ_p (keVee)	$\tau_{1/2}$ (days)	N_0
^{73}As	1.414	0.077	80.	12.7
^{68}Ge	1.298	0.077	271.	638.9
^{68}Ga	1.194	0.076	271.	52.8
^{65}Zn	1.096	0.075	244.	211.2
^{56}Ni	0.926	0.075	5.9	1.53
^{56,58}Co	0.846	0.074	71.	9.44
^{57}Co	0.846	0.074	271.	2.59
^{55}Fe	0.769	0.074	996.	44.9
^{55}Mn	0.695	0.073	312.	21.1
^{51}Cr	0.628	0.073	28.	2.93
^{49}V	0.564	0.073	330.	14.9

$$P^A_{\rm rad}(\mathcal{E}_{\rm min}, \mathcal{E}_{\rm max}) = \int_{\mathcal{E}_{\rm min}}^{\mathcal{E}_{\rm max}} {\rm Gaussian}(\mathcal{E}, \mathcal{E}_p, \sigma_p) d\mathcal{E}$$

$$D^A(t_1, t_2) = \left(\exp(-\frac{\ln 2}{\tau_{1/2}} t_1) - \exp(-\frac{\ln 2}{\tau_{1/2}} t_2) \right)$$

$$N^A_{\rm tot}(\mathcal{E}_{\rm min}, \mathcal{E}_{\rm max}, t_1, t_2) = N_0 P^A_{\rm rad}(\mathcal{E}_{\rm min}, \mathcal{E}_{\rm max}) D^A(t_1, t_2)$$

Velocity distribution from DM density profile

Assuming equilibrium between gravitational force and pressure:

$$F(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\left[\int_0^{\varepsilon} \frac{\mathrm{d}^2\rho_{\rm DM}}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\varepsilon-\Psi}} + \frac{1}{\sqrt{\varepsilon}}\left(\frac{\mathrm{d}\rho_{\rm DM}}{\mathrm{d}\Psi}\right)\Big|_{\Psi=0}\right]$$

Eddigton formula for isotropic and spherically symmetric DM density profiles

Poisson equation for the gravitational potential including contribution from the bulge and disk:

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\Psi}{\mathrm{d}r} = -4\pi G[\rho_{\rm DM} + \rho_{\rm disk} + \rho_{\rm bulge}]$$

$$\rho_{\rm DM}(r) = \rho_s\left(\frac{r}{r_s}\right)^{-1}\left(1+\left(\frac{r}{r_s}\right)\right)^{-2}$$ NFW

$$\rho_{\rm disk}(r) = \frac{M_{\rm disk}}{4\pi r_{\rm disk}^2}\frac{e^{-r/r_{\rm disk}}}{r}$$

$$\rho_{\rm bulge}(r) = M_{\rm bulge}\delta_D^{(3)}(\vec{r})$$

The velocity distribution is translated to the reference frame of the Earth:

$$\int_{v'>v'_{\rm min}} \mathrm{d}^3v' \, \frac{f(\vec{v'}(t))}{v'} \to 2\pi\rho_\odot^{-1}\int_{v'>v'_{\rm min}} \mathrm{d}v' \; v' \int_{-1}^{1} \mathrm{d}\alpha \; F\Big(\Psi_\odot - \frac{1}{2}v^2\Big)$$

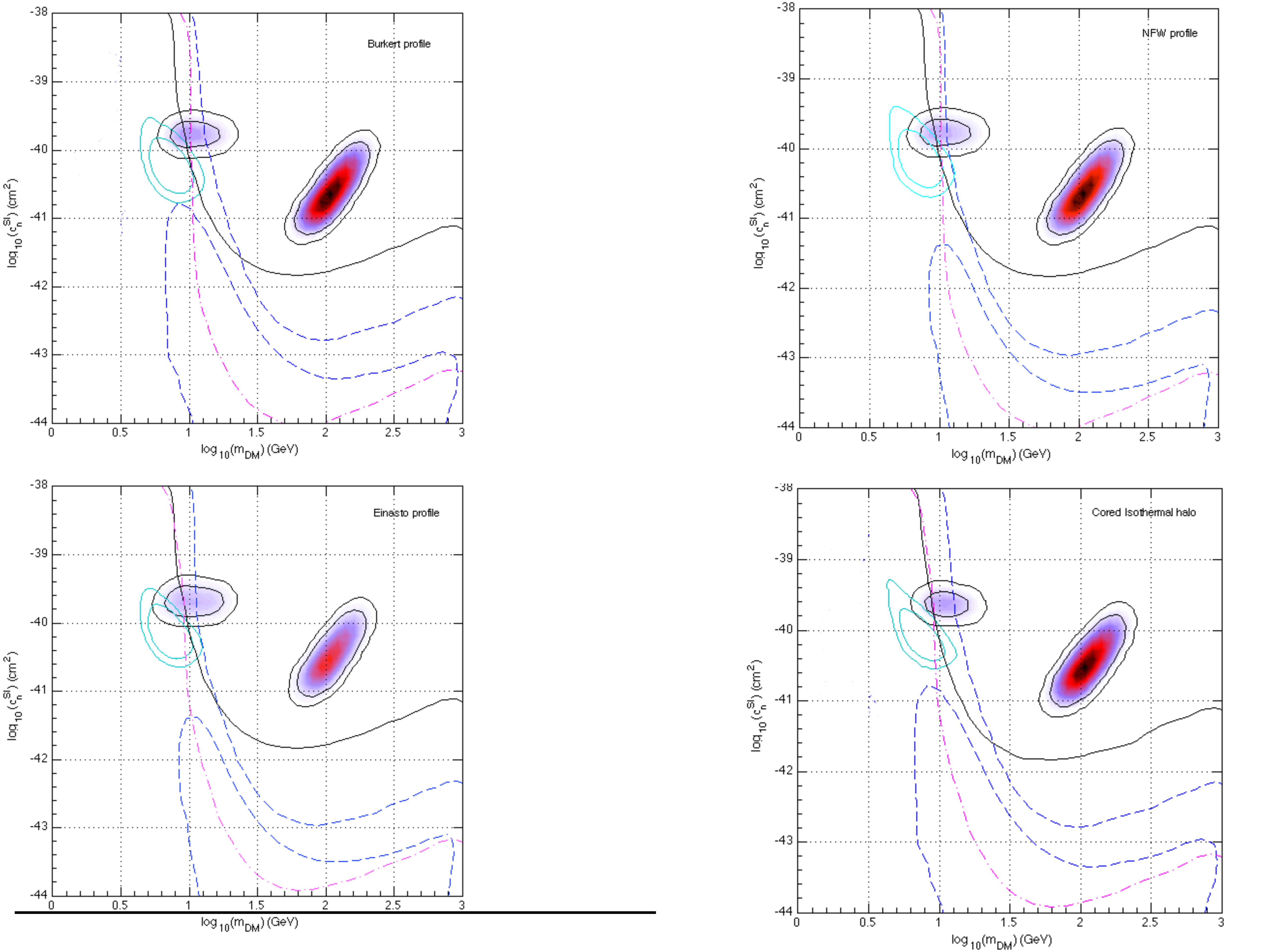
$$v_0 \equiv \sqrt{-r\frac{\mathrm{d}\Psi}{\mathrm{d}r}}\Big|_{r=R_\odot}$$

$$v^2 = |\vec{v'} + \vec{v}_\oplus|^2 = v'^2 + v_\oplus^2 + 2v'v_\oplus\alpha\,,$$

$$v_\oplus = |\vec{v}_\odot + \vec{v''}_{\oplus,\rm rot}| = v_\odot + v''_{\oplus,\rm rot}\cos\gamma\cos[2\pi(t-t_0)/T]$$

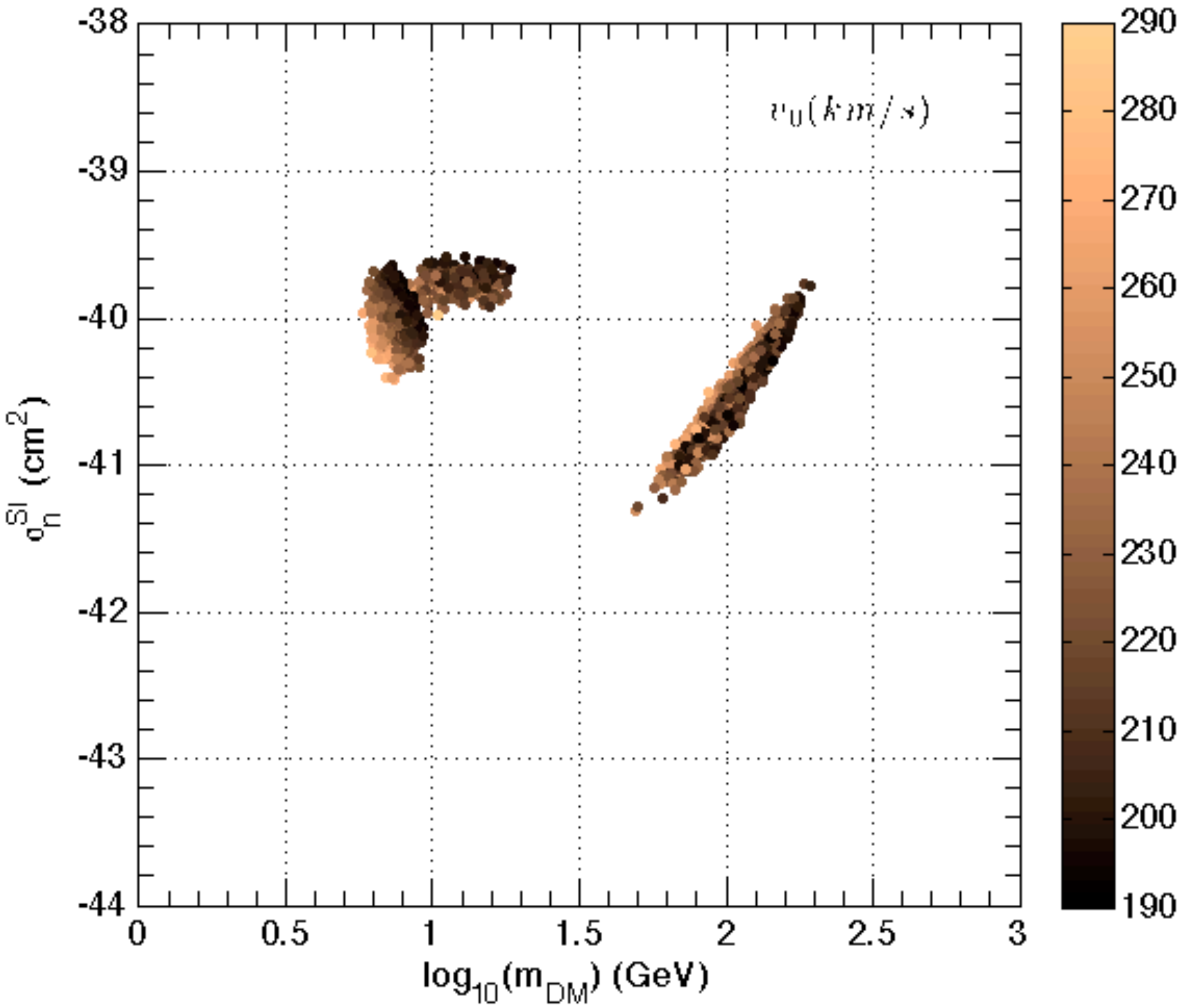
$$v_{\rm esc} = \sqrt{2\Psi}\Big|_{r=R_\odot}$$

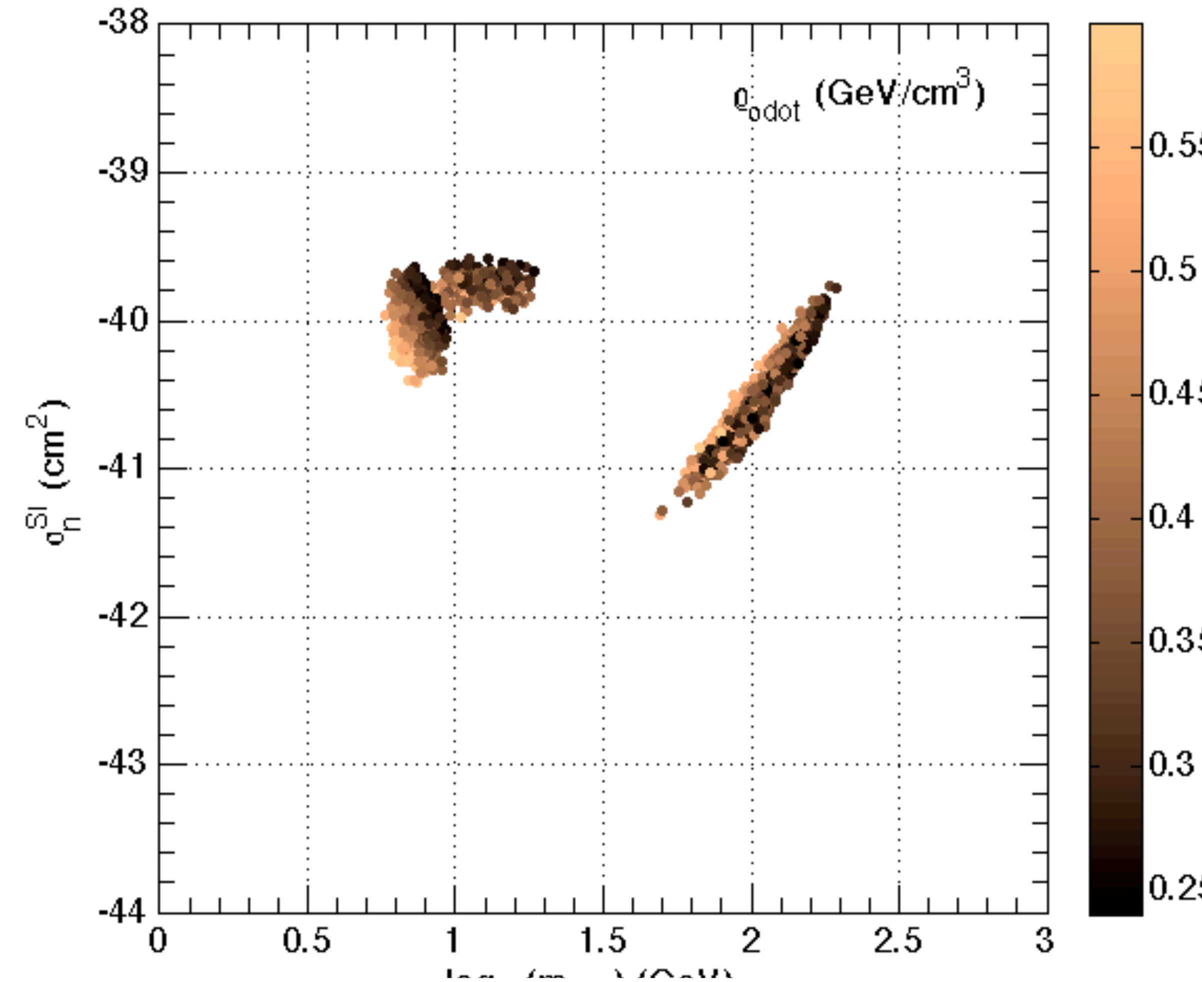
Results for various DM halos

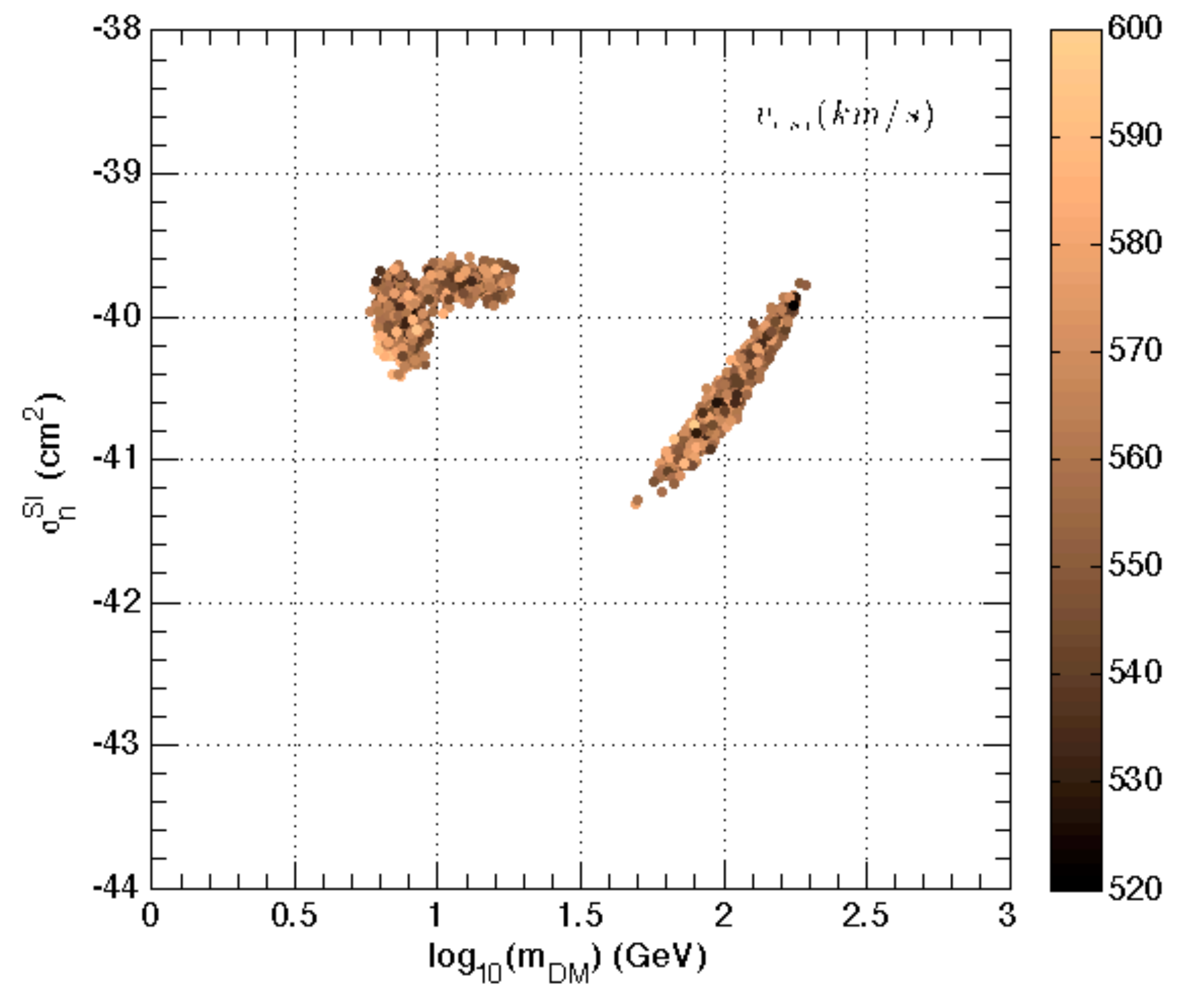


3D marginal credible regions in the plane

$v_{\rm esc}$ and v_0 , $\rho_\odot$ and $v_{\rm esc}$





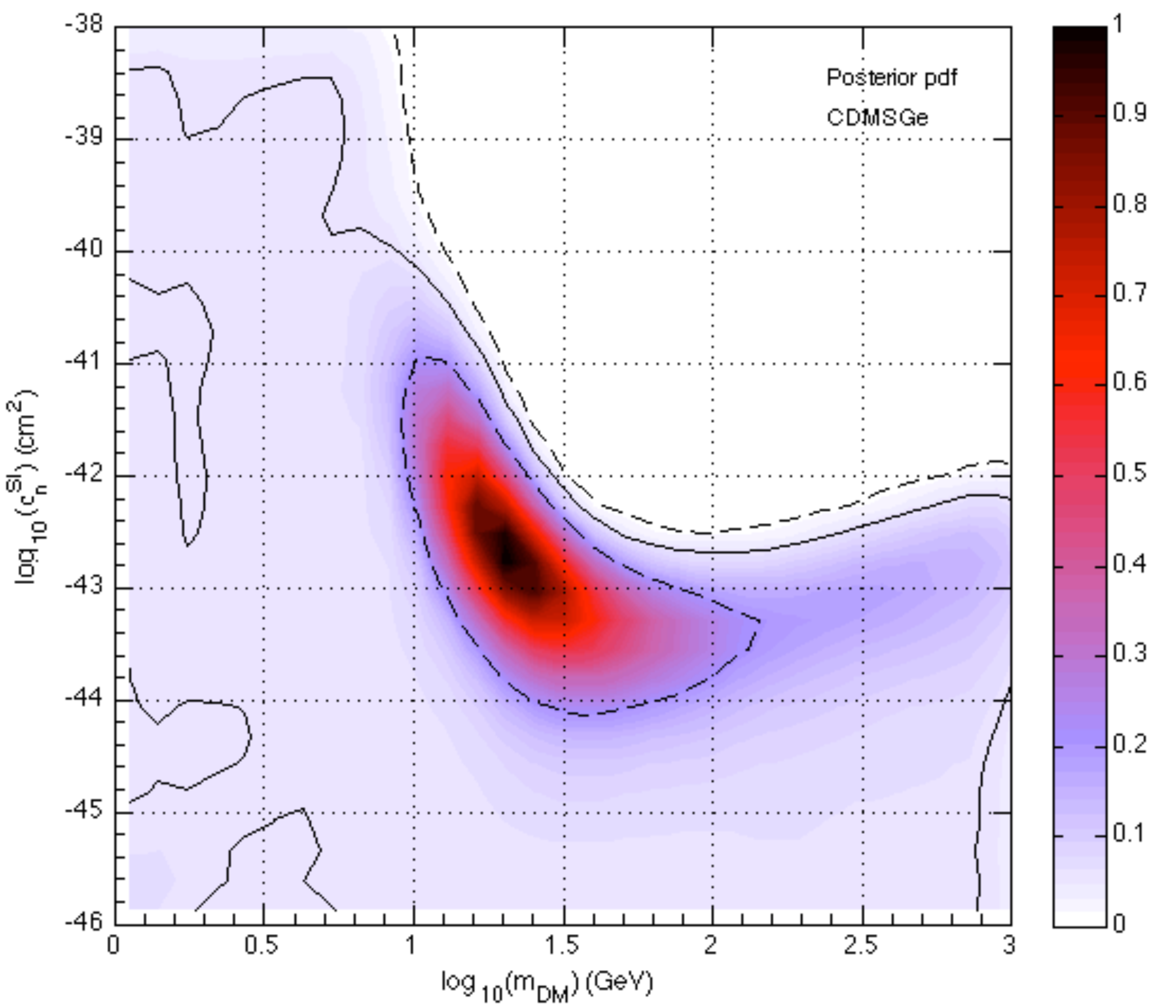


CDMSGe analysis

- 2 events seen, likelihood follows a Poisson distribution
- expected background of 1.38 +- 0.38, analytical marginalization
- exposure of 1063.2 kg days (all runs combined)
- energy range from 10 -> 100 keVnr
- used spectral information

$$\ln \mathcal{L}_{\rm CDMSGe} = -S - B + 2 + \sum_{i=1,2} \ln \left(\frac{\mathrm{dR}}{\mathrm{dE_i}} + \frac{B}{\bar{B}} \frac{\mathrm{d}N_B}{\mathrm{d}E_i} \right) + C_{\rm norm}$$

$$\ln \mathcal{L}^{\rm eff}_{\rm CDMSGe} = -S - \bar{B} + \frac{\sigma_B^2}{2} + 2 + C_{\rm norm} + \ln \left[\prod_{i=1,2} \left(\frac{\mathrm{d}R}{\mathrm{d}E_i} + \frac{\bar{B} - \sigma_B^2}{\bar{B}} \frac{\mathrm{d}N_B}{\mathrm{d}E_i} \right) + \sigma_B^2 \prod_{i=1,2} \frac{1}{\bar{B}} \frac{\mathrm{d}N_B}{\mathrm{d}E_i} \right]$$



$90_S\%$ $\quad \Delta\chi^2_{\rm eff} \leq 3.0$

$99_S\%$ $\quad \Delta\chi^2_{\rm eff} = 7.4$

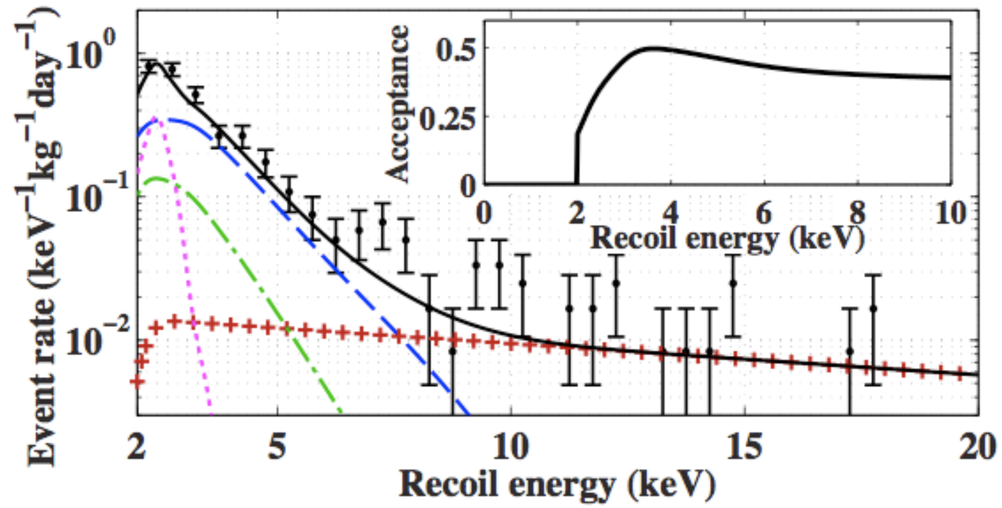
$$C_{\rm norm} = \sum_{i=1,2} \ln[M_{\rm det} T \epsilon(qE_i)]$$

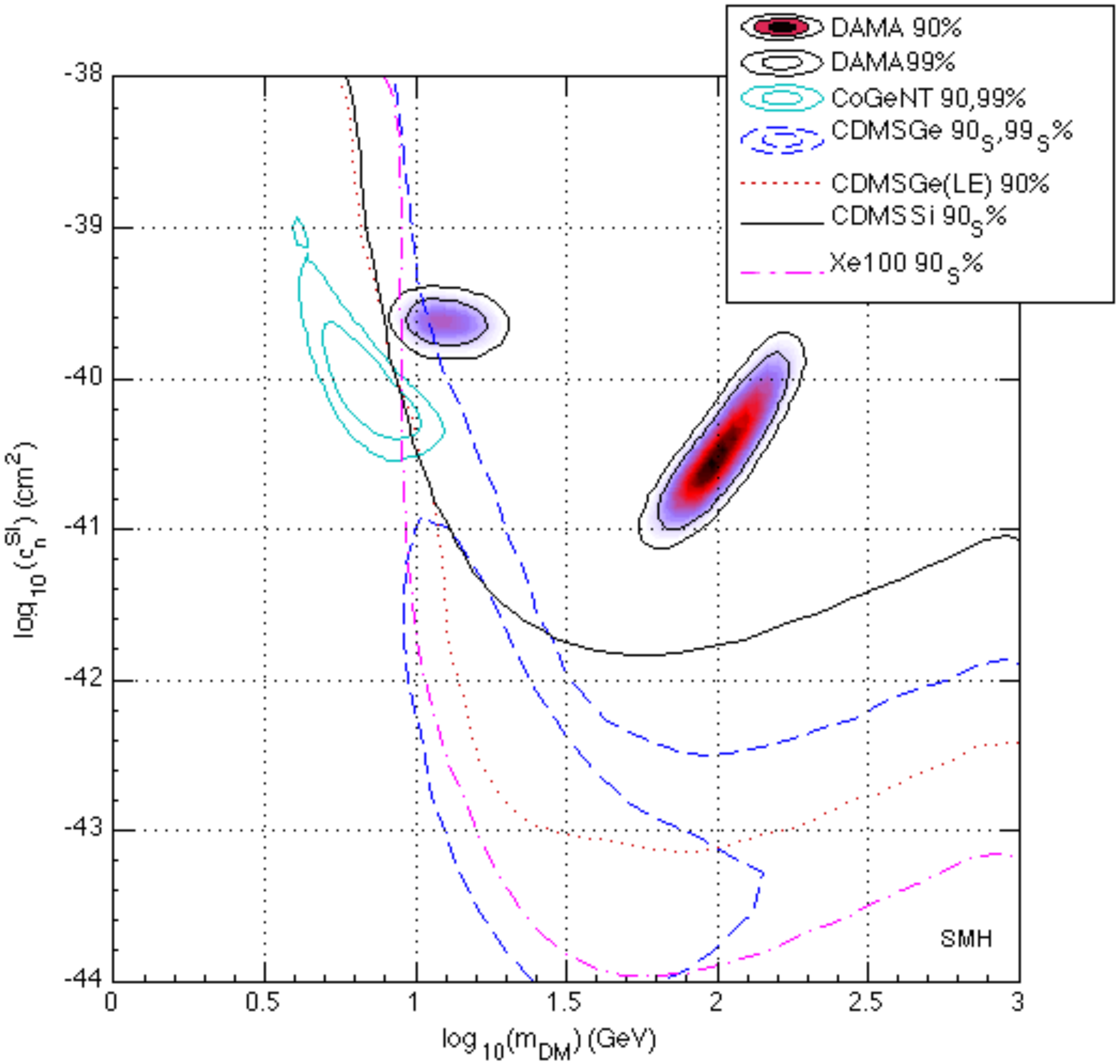
$$E_{1,2} = 12.3, 15.5 \ \mathrm{keVnr}$$

CDMSGe low energy analysis

- 2-100 keVnr energy range
- 462 events combined into 16 bins
 from 2 -> 10 KeVnr and 9 from 10 to 100 keVnr
- 214 kg days

from arXiv:1011.2482





$$\ln \mathcal{L}_{\mathrm{CDMSGe(LE)}} = -\sum_{i=1}^{N_{\mathrm{bin}}} \frac{(s_i - \bar{s}_i^{\mathrm{obs}})^2}{2\sigma_i^2} + \ln \mathcal{L}_{m_{\mathrm{B}}}$$

$$s_i = \frac{1}{\Delta E_i} \int_{E_i - \Delta E_i/2}^{E_i + \Delta E_i/2} \mathrm{d}E \left[\frac{\mathrm{d}R}{\mathrm{d}E} + \mathrm{m_B}(E) \right]$$

$$\mathrm{m_B}(E) = \begin{cases} \bar{\mathrm{m}}_{\mathrm{B}}(E), & E \geq 5 \text{ keVnr}, \\ 0.1 \times 10^{a[(E/\mathrm{keVnr})-5]}, & 2 < E/\mathrm{keVnr} < 5. \end{cases}$$

$$\ln \mathcal{L}_{m_{\mathrm{B}}} = -\frac{(a - \bar{a})^2}{2\sigma_a^2}$$

prior range flat over: $-0.60 \rightarrow -0.18$

$$\Delta\chi^2_{\mathrm{eff}} < 4.6$$

$$90_S\%$$

CDMSSi analysis

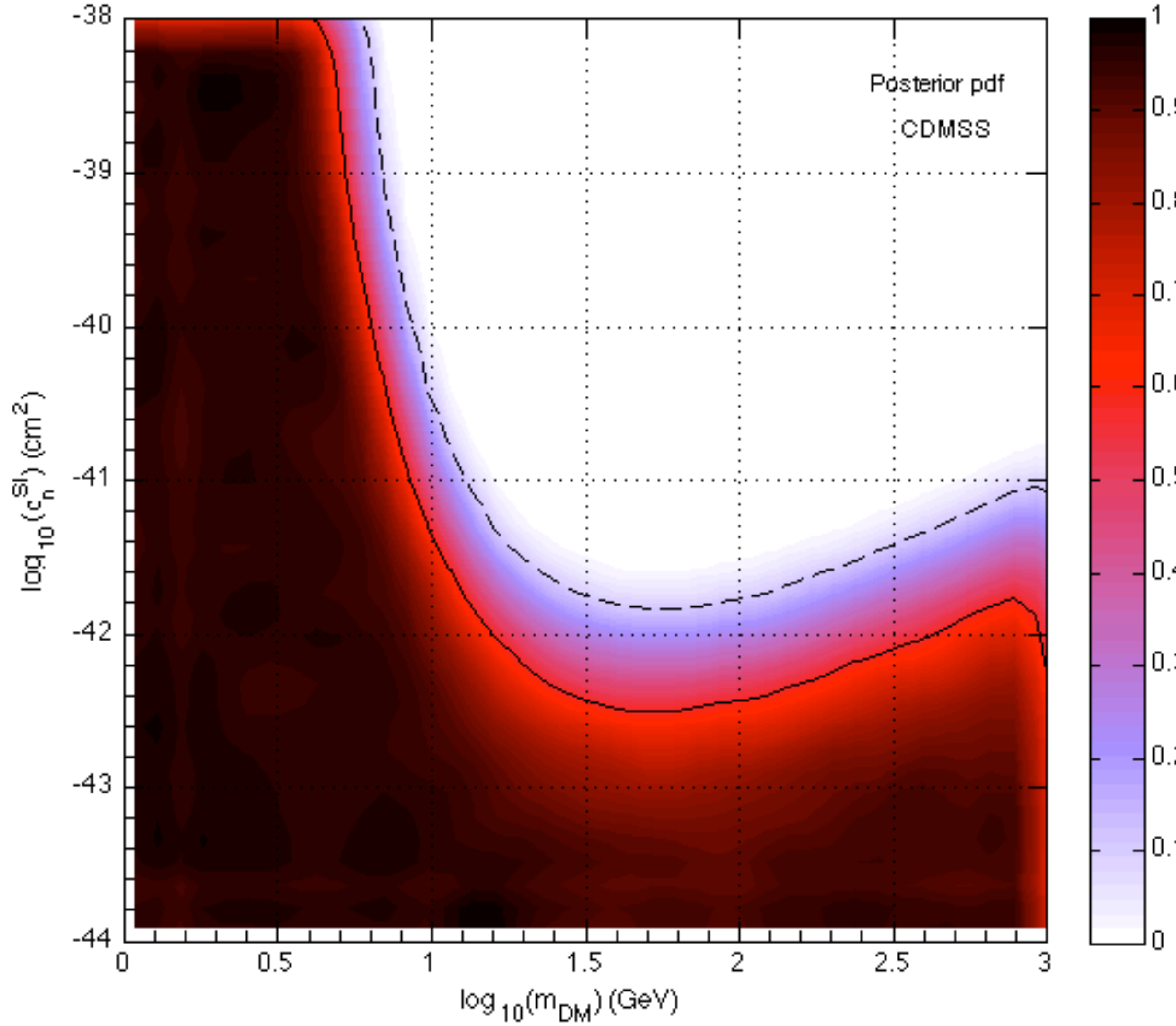
- 2 events seen, likelihood follows a Poisson distribution
- expected background of 4.4 (Be = 0.8, Bn = 3.6)
- exposure of 65.8 kg days
- energy range from 10 -> 100 keVnr

$$\ln \mathcal{L}_{\rm CDMSSi}(2|S,B) = -S - B + 2 + 2\ln\left(\frac{S+B}{2}\right)$$

$$\mathcal{L}^{\rm eff}_{\rm CDMSSi}(2|S) = \int_0^\infty {\rm d}B\ \mathcal{L}_{\rm CDMSSi}(2|S,B)\ p(B)$$

$$\Delta\chi^2_{\rm eff} \leq 4.2$$

$$S \leq 3.3$$



Analytical marginalization over the background:

$$p(B) = \frac{1}{\sqrt{2\pi\sigma_B^2}} \exp\left[-\frac{(B-\bar{B})^2}{2\sigma_B^2}\right]$$

$$\bar{B} \pm \sigma_B = 4.4 \pm 0.6,$$

$$\ln \mathcal{L}^{\rm eff}_{\rm CDMSSi} = -S - \bar{B} + \frac{\sigma_B^2}{2} + 2 + \ln\left[\frac{\sigma_B^2 + (S + \bar{B} - \sigma_B^2)^2}{4}\right]$$

Statistics

$$\mathcal{P}_{\text{mar}}(\theta_1, ..., \theta_n|X) \propto \int \mathrm{d}\psi_1...\mathrm{d}\psi_m\ \mathcal{P}(\theta_1, ..., \theta_n, \psi_1..., \psi_m|X)$$

$$\mathcal{L}_{\text{prof}}(X|\theta_1, ..., \theta_n) \propto \max_{\psi_1...\psi_m}\ \mathcal{L}(X|\theta_1, ..., \theta_n, \psi_1..., \psi_m)$$

$$\Delta\chi^2_{\text{eff}}(m_{\text{DM}}, \sigma_n^{\text{SI}}) \equiv -2\ln\mathcal{L}_{\text{prof}}(m_{\text{DM}}, \sigma_n^{\text{SI}})$$

$$\mathcal{P}(\mathcal{M}|X) \propto \mathcal{Z}\ \pi(\mathcal{M})$$

- Occam's razor effect for nested models is controlled by the volume of parameter space enclosed by the prior distribution for the new parameters
- The evidence for the more complicated model will be larger only if the quality-of-fit increases enough to offset the penalizing effect of the Occam's factor

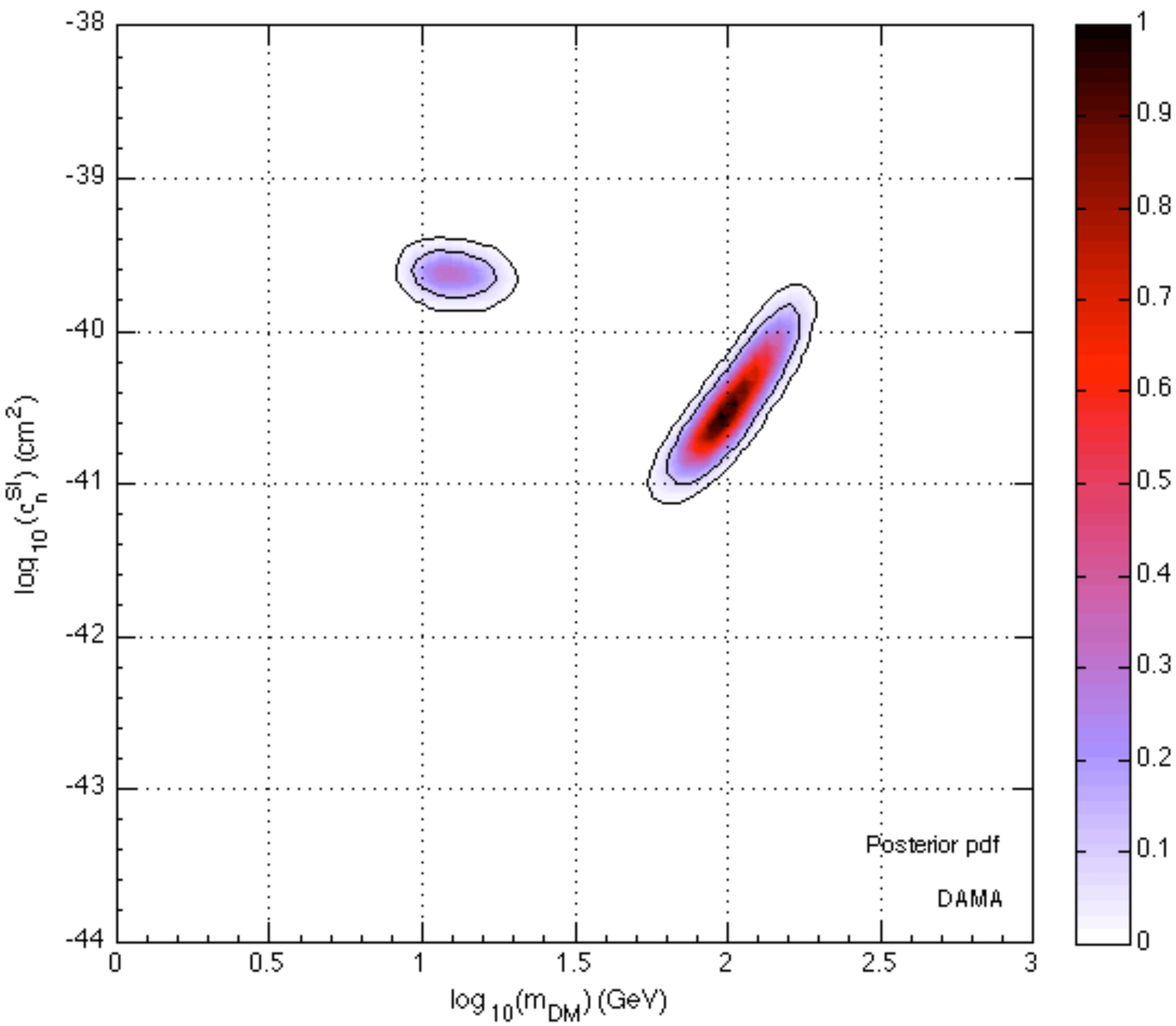
p–value	$\overline{B}_{10}$	$\ln \overline{B}_{10}$	sigma	category
0.05	2.5	0.9	2.0	
0.04	2.9	1.0	2.1	'weak' at best
0.01	8.0	2.1	2.6	
0.006	12	2.5	2.7	'moderate' at best
0.003	21	3.0	3.0	
0.001	53	4.0	3.3	
0.0003	150	5.0	3.6	'strong' at best
6×10^{-7}	43000	11	5.0	

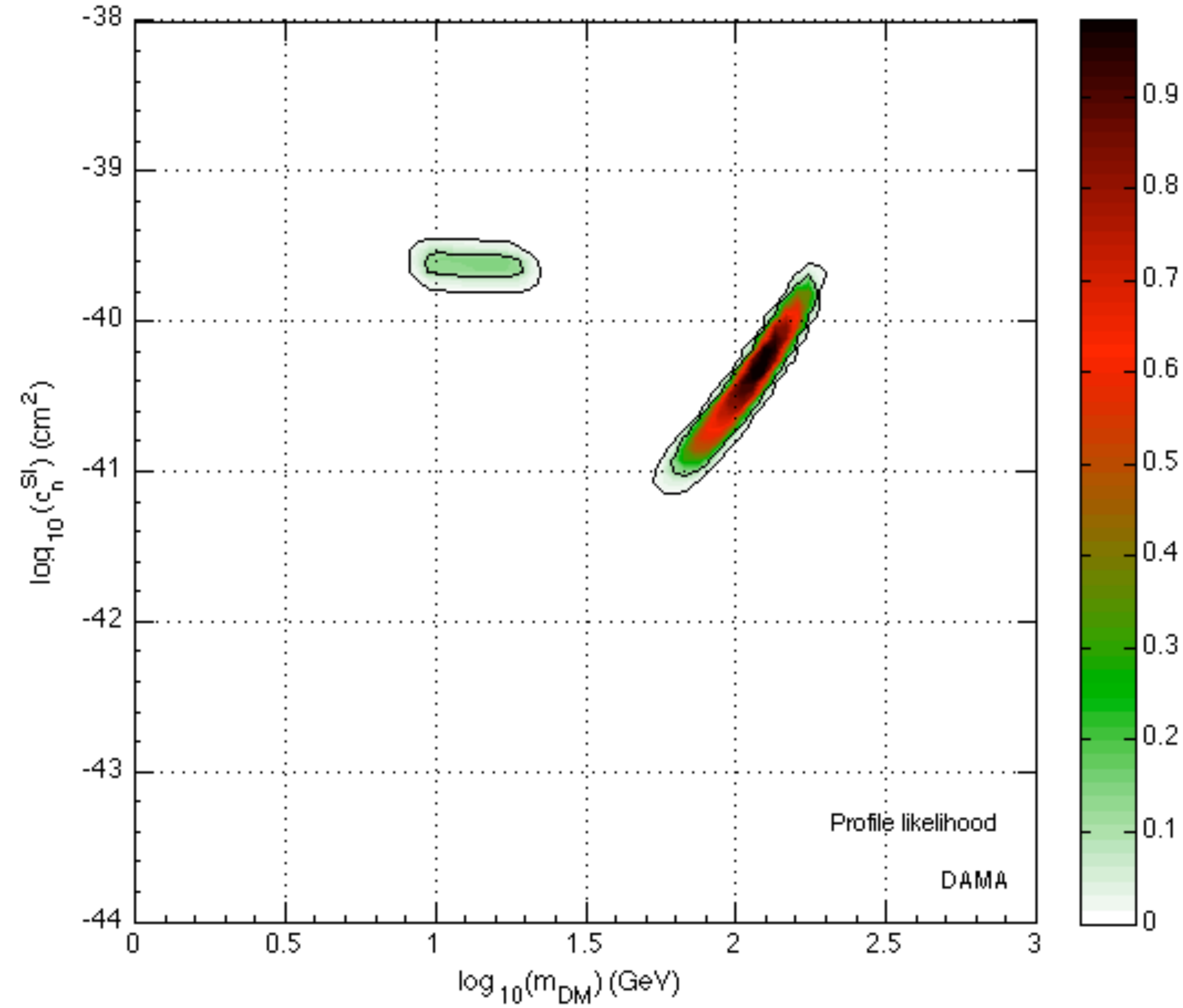
DAMA

$$\ln \mathcal{L}_{\mathrm{DAMA}} = -\sum_{i=1}^{N_{\mathrm{bin}}} \frac{(s_i - \bar{s}_i^{\mathrm{obs}})^2}{2\sigma_i^2}$$

$$\log \mathcal{Z}_{0=\mathrm{SMH}} = -9.47 \pm 0.02$$
$$\log \mathcal{Z}_{1=\mathrm{NFW}} = -14.74 \pm 0.03$$
$$\log \mathcal{Z}_{2=\mathrm{ISO}} = -18.00 \pm 0.03$$

$$\ln B_{01} = 5.3$$
$$\ln B_{02} = 8.5$$





Theoretical predictions for elastic spin-independent scattering off nucleus

Differential rate

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho_\odot}{m_{\mathrm{DM}}} \int_{v' > v'_{\mathrm{min}}} \mathrm{d}^3 v' \, \frac{\mathrm{d}\sigma}{\mathrm{d}E} \, v' \, f(\vec{v'}(t))$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{M_{\mathcal{N}} \sigma_n^{\mathrm{SI}}}{2\mu_n^2 v'^2} \, \frac{\left(f_p Z + (A-Z) f_n\right)^2}{f_n^2} \mathcal{F}^2(E)$$

$$\mathcal{E} = qE \qquad S(t) = M_{\mathrm{det}} T \int_{\mathcal{E}_1/q}^{\mathcal{E}_2/q} \mathrm{d}E \; \epsilon(qE) \, \frac{\mathrm{d}R}{\mathrm{d}E}$$

Modulated rate

$$s = \frac{1}{\mathcal{E}_2 - \mathcal{E}_1} \sum_{X=\mathrm{Na,I}} w_X \int_{\mathcal{E}_1/q_X}^{\mathcal{E}_2/q_X} \mathrm{d}E \; \frac{1}{2} \left[\frac{\mathrm{d}R_X}{\mathrm{d}E}(\text{June } 2) - \frac{\mathrm{d}R_X}{\mathrm{d}E}(\text{Dec } 2) \right]$$

$$S_{\mathrm{m\%}} = \frac{R(\mathrm{June2}) - R(\mathrm{Dec2})}{R(\mathrm{June2}) + R(\mathrm{Dec2})}$$