Towards measuring information in radioastronomy

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PUNCH4NFDI project

TA5 WP 1 ”Implications for discovery potential and reproducibility”

“… interplay between reproducibility of filtered and refined data and its implications for the discovery potential.”

“A large focus of TA 5 is on the actual implementation of efficient real-time filtering.”

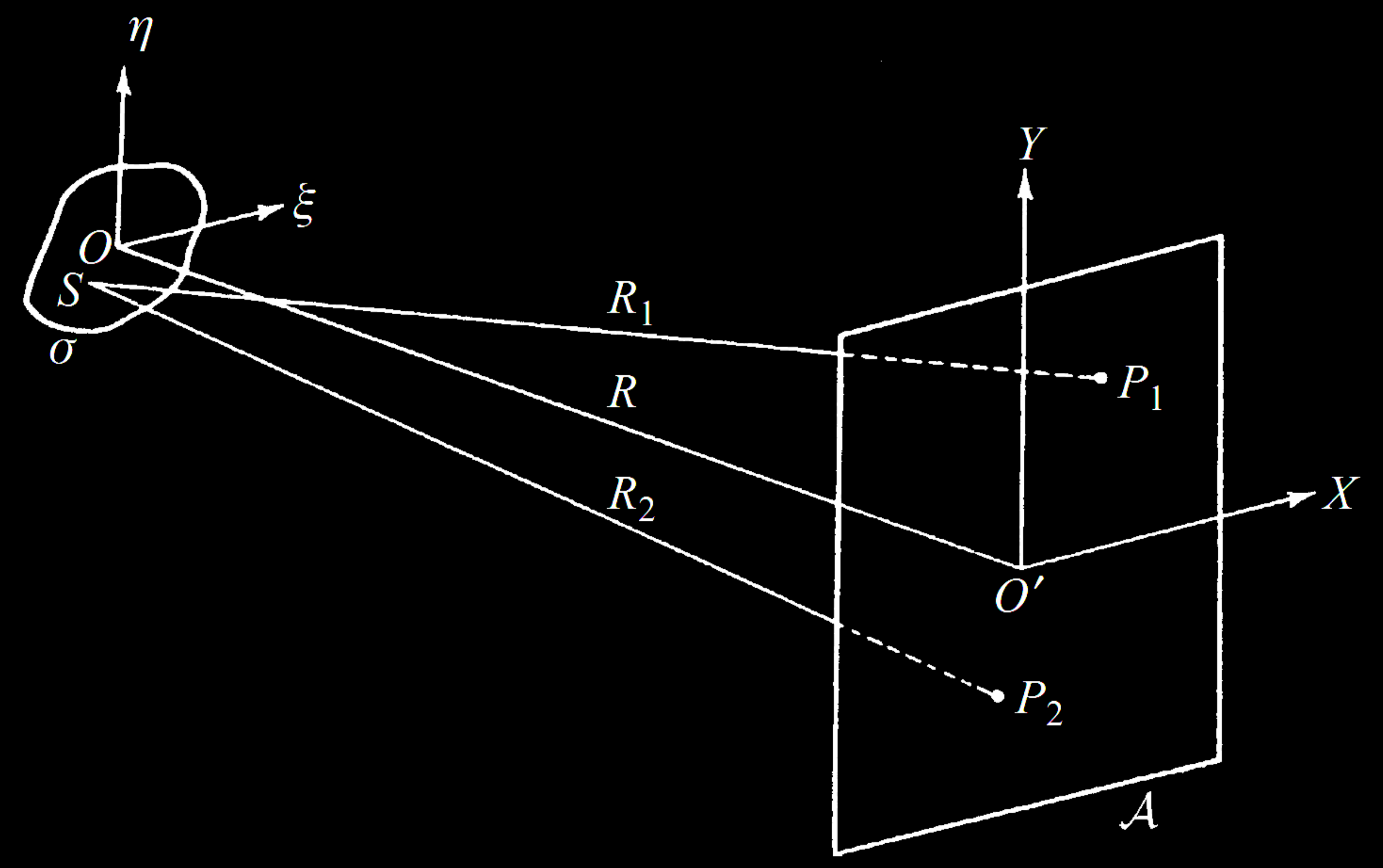
Identification of “information flow”

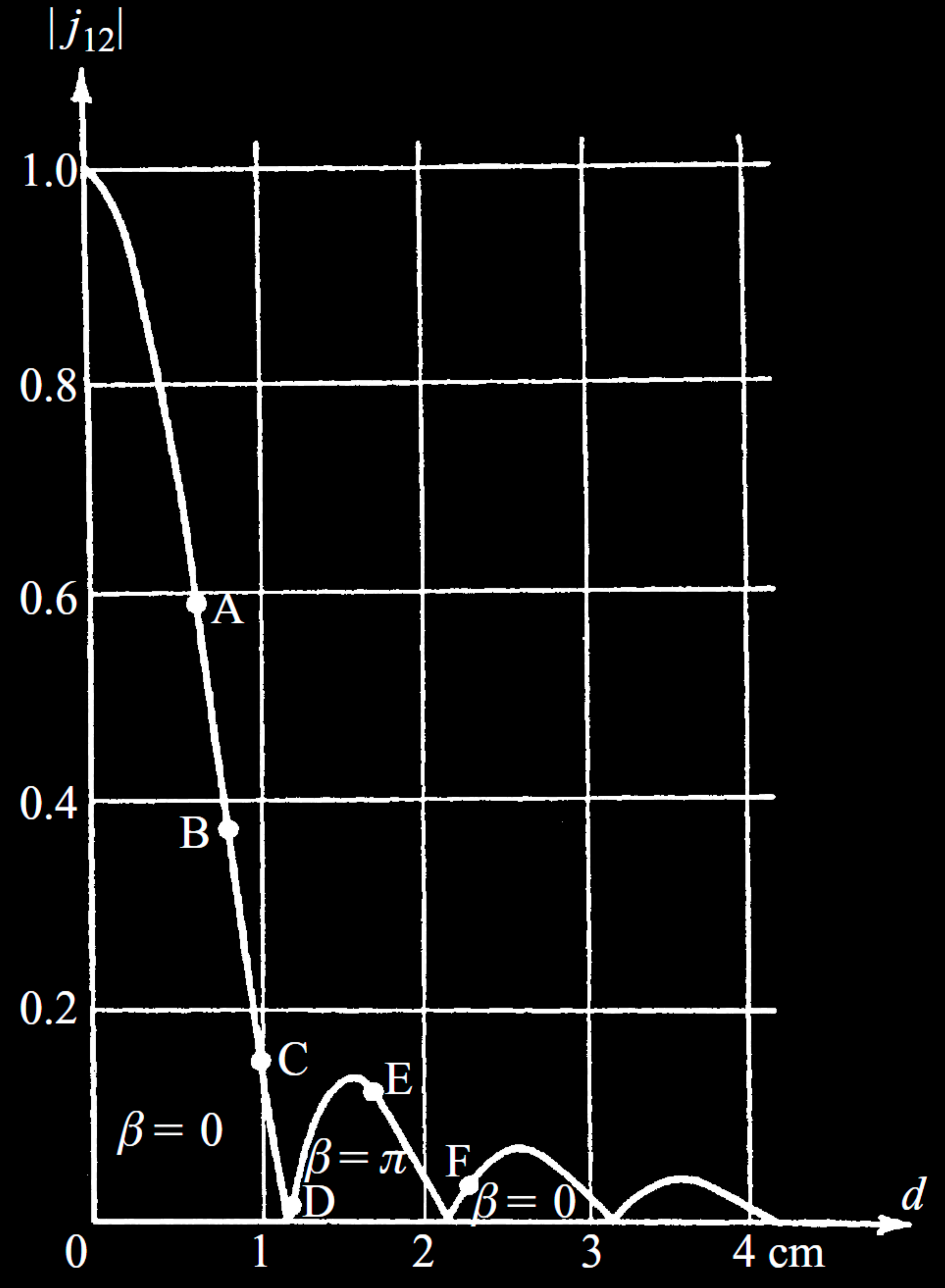
- Simulation of the coherence effect in interferometry (…)
- Level of the individual antenna
  - Identification of transients
  - Compression of the overall time series

- Statistical signal processing/detection
- Optimal processing/detection

Correlation measurement (VCZ theorem)

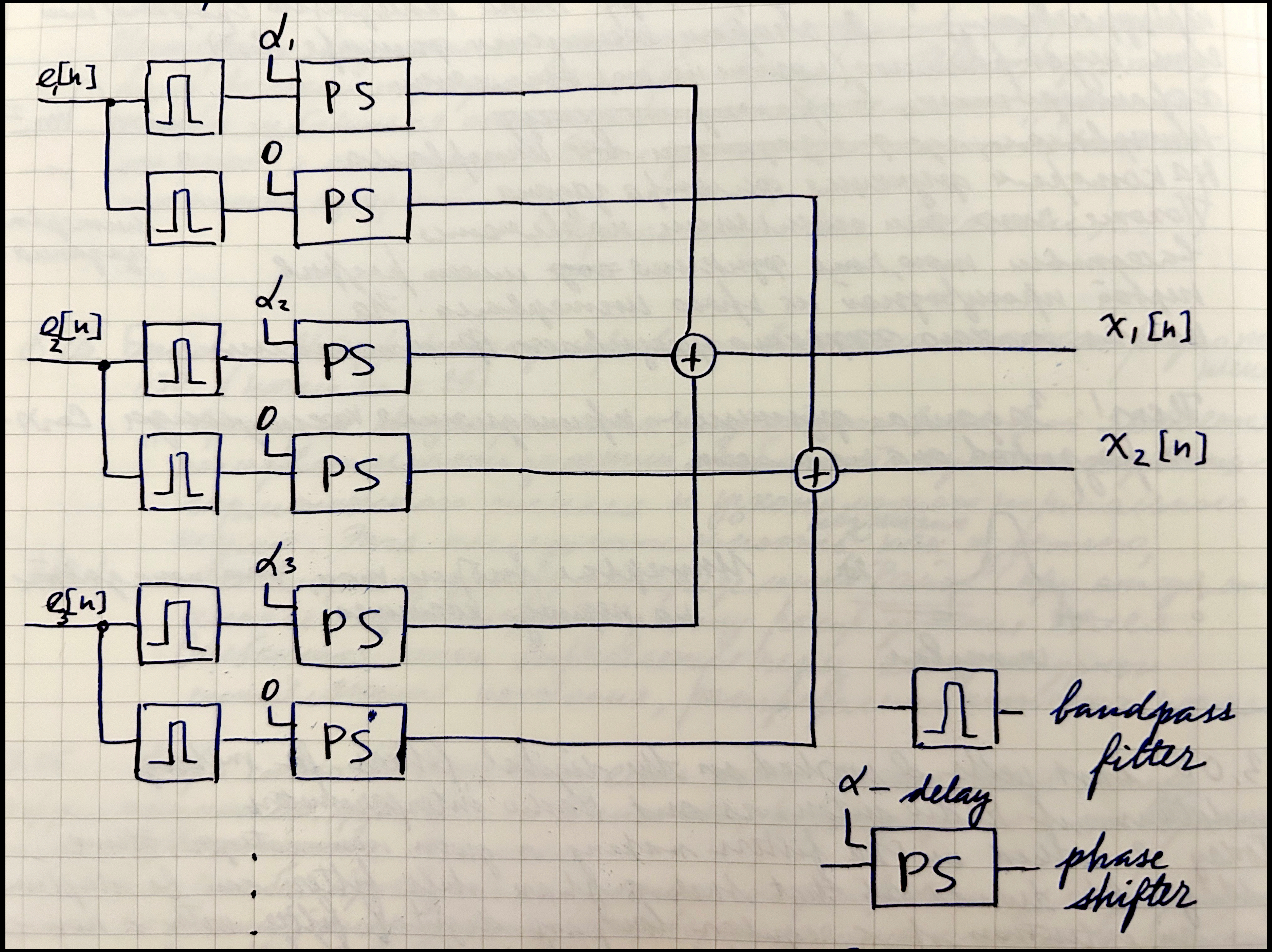
The effect: an incoherent source starts to be visible as coherent at a large distance.



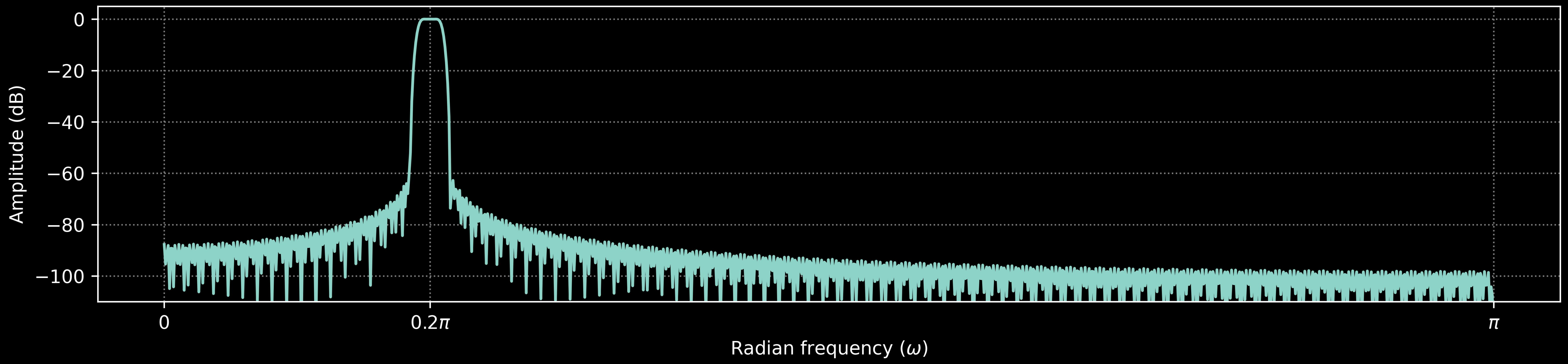


Principles of Optics by Born and Wolf

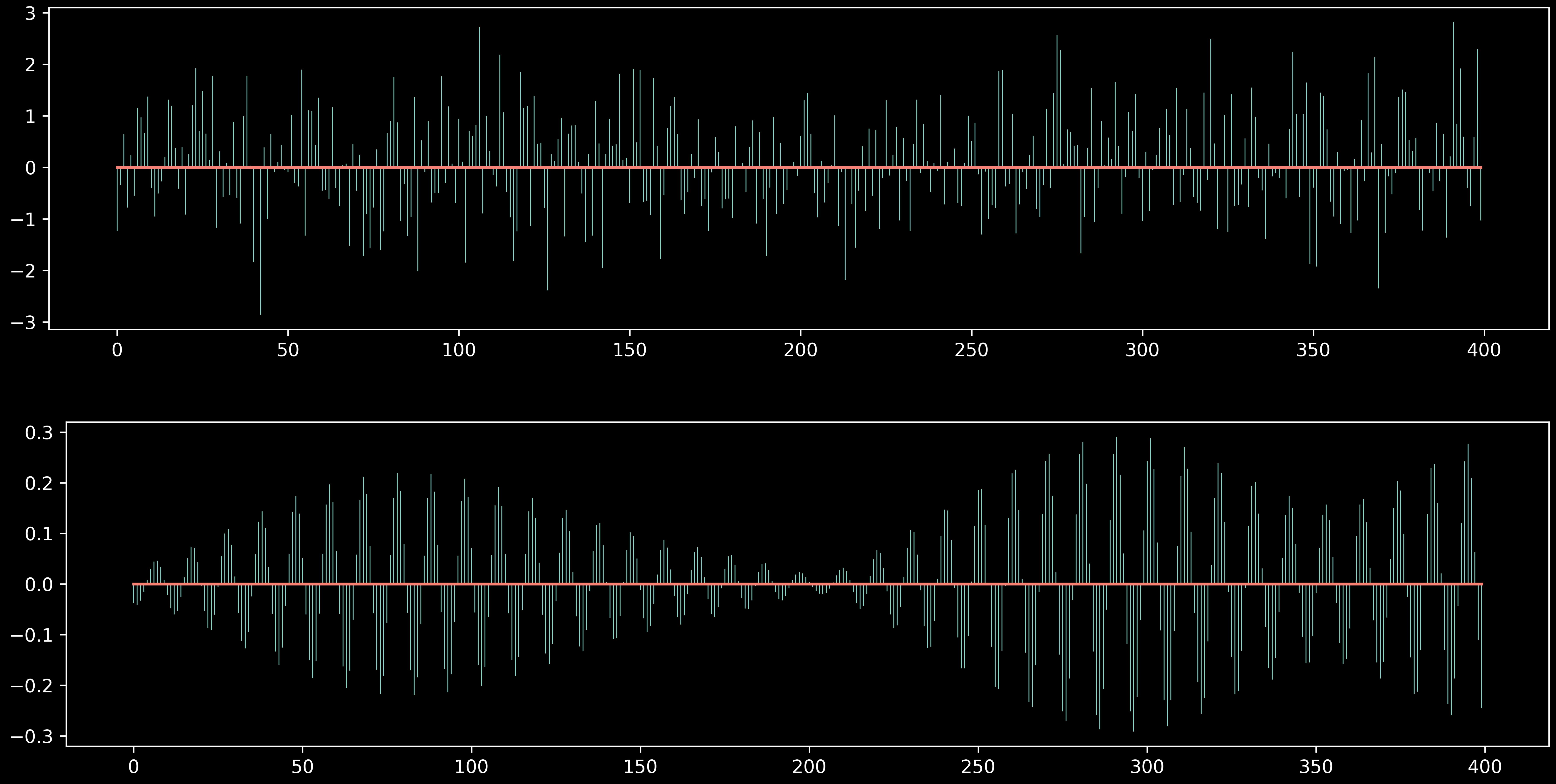
VCZ theorem — Simulation scheme



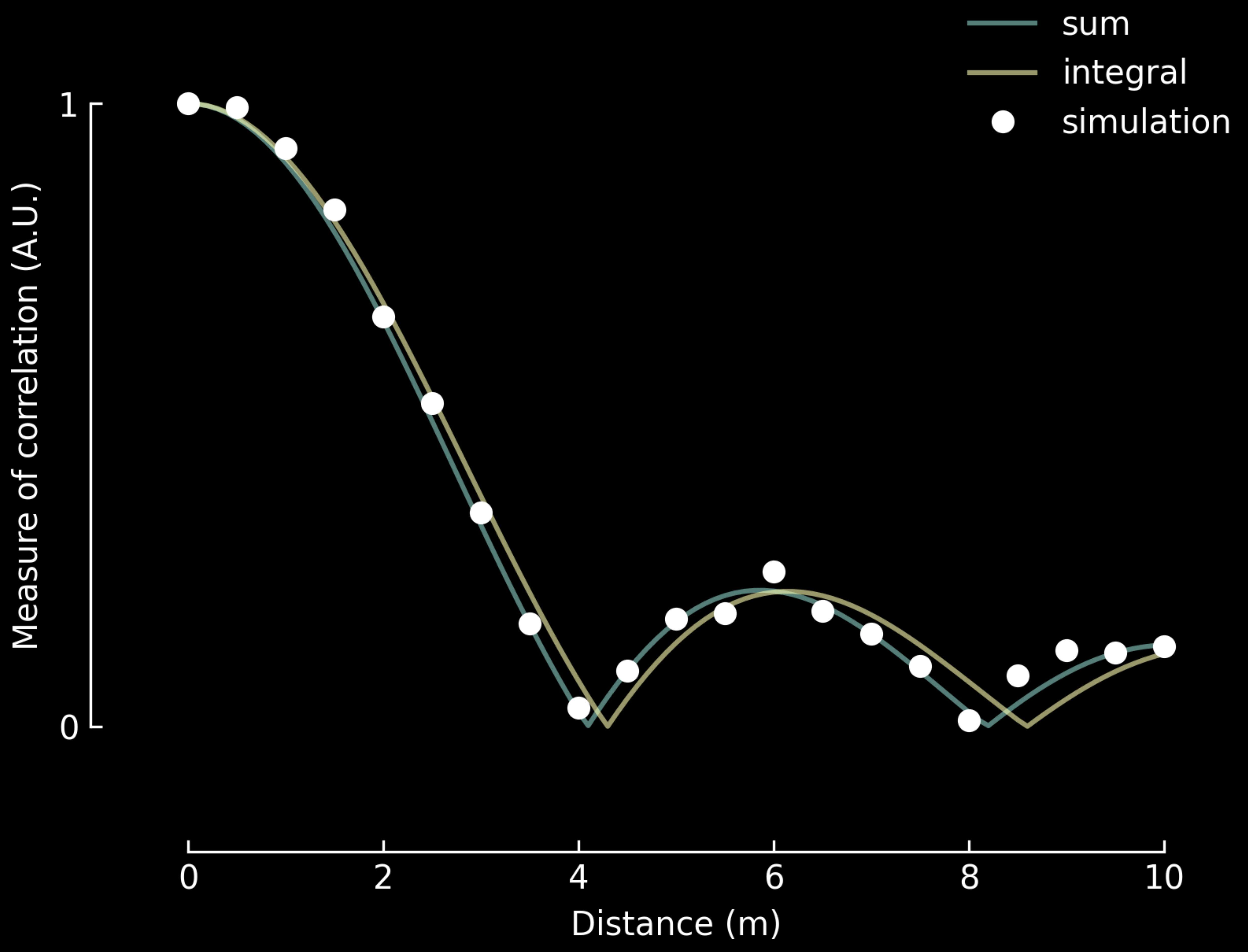
VCZ theorem — Bandpass filter



VCZ theorem — Bandpass filter



VCZ theorem — Simulation results



Level of single antenna

- Transient detection (pulsars, FRB, …)

- Statistical signal processing (detection)
- Optimal detection (matched filtering)

Statistical signal processing (detection)

Conditions: A_1 - signal is present, A_0 - no signal

Decisions: A_1^* - signal is present, A_0^* - no signal

$A_0^* A_0$ — correct non-detection ($\hat{F} = P(A_0^* \,|\, A_0)$)

$A_1^* A_0$ — “false alarm” or false positive detection ($F = P(A_1^* \,|\, A_0)$)

$A_0^* A_1$ — missing the signal ($\hat{D} = P(A_0^* \,|\, A_1)$)

$A_1^* A_1$ — correct detection of the signal ($D = P(A_1^* \,|\, A_1)$)

Statistical signal processing (detection)

Conditions: A_1 - signal is present, A_0 - no signal

Decisions: A_1^* - signal is present, A_0^* - no signal

$\hat{F} = P(A_0^* | A_0)$ $\quad$ $\hat{D} = P(A_0^* | A_1)$ $\quad$ $\hat{F} + F = 1$

$F = P(A_1^* | A_0)$ $\quad$ $D = P(A_1^* | A_1)$ $\quad$ $\hat{D} + D = 1$

$$P(A_0^*, A_0) + P(A_1^*, A_0) + P(A_0^*, A_1) + P(A_1^*, A_1) = 1$$

Mean risk:

$$\bar{r} = \sum_i r_i P_i = r_1 P(A_0^*, A_0) + r_2 P(A_1^*, A_0) + r_3 P(A_0^*, A_1) + r_4 P(A_1^*, A_1)$$
$$= r_F P(A_1^*, A_0) + r_{\hat{D}} P(A_0^*, A_1) = r_F F P(A_0) + r_{\hat{D}} \hat{D} P(A_1)$$

$r_F = r(A_1^*, A_0)$ $\quad$ $r_{\hat{D}} = r(A_0^*, A_1)$

Criterion of the ideal observer: $\bar{r} = FP(A_0) + \hat{D}P(A_1)$

$$P(A_1^*, A_0) = P(A_0)P(A_1^* | A_0) = P(A_0)F$$

$$P(A_0^*, A_1) = P(A_1)P(A_0^* | A_1) = P(A_1)\hat{D}$$

$$\bar{r} = r_{\hat{D}} P(A_1) - [D - l_0 F] r_{\hat{D}} P(A_1)$$

Weighting criterion: $\operatorname{argmax}(D - l_0 F)$

$$l_0 = \frac{r_F P(A_0)}{r_{\hat{D}} P(A_1)}$$

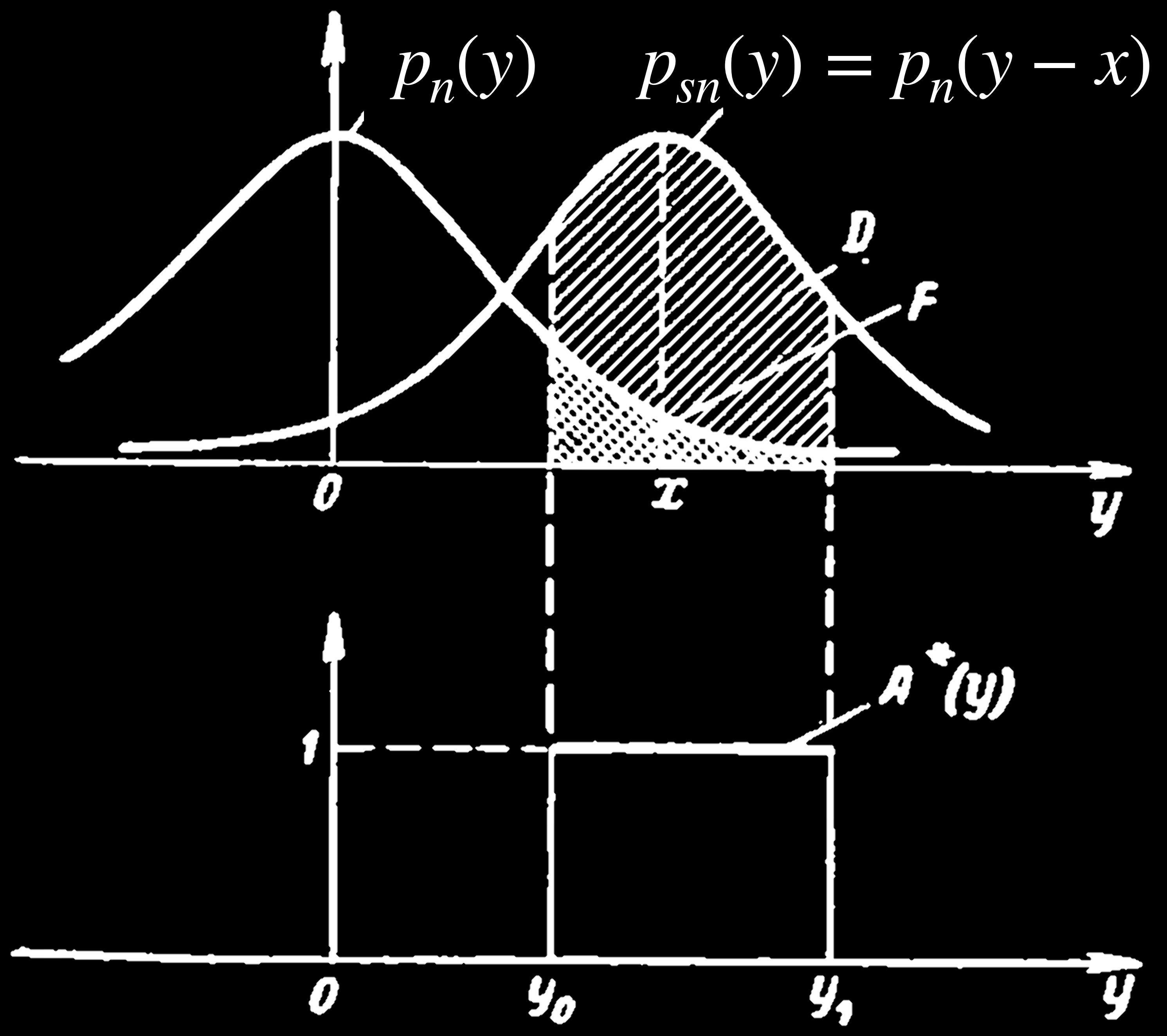
Statistical signal processing (detection)

Measurement of one value only: $y = Ax + n$

$$p(y\,|\,A_0) = p_n(y) \qquad p(y\,|\,A_1) = p_{sn}(y)$$

$$D = \int_{-\infty}^{\infty} A^*(y) p_{sn}(y)\, dy \qquad F = \int_{-\infty}^{\infty} A^*(y) p_n(y)\, dy$$

$$D - l_0 F = \int_{-\infty}^{\infty} p_n(y) A^*(y) [l(y) - l_0]\, dy \qquad l(y) = \frac{p_{sn}(y)}{p_n(y)}$$ — likelihood ratio



$$l(y) = \exp\left(-\frac{x^2}{2n_0^2}\right) \exp\left(\frac{xy}{n_0^2}\right)$$

xy — correlation sum/integral

$$l(y) = \exp\left(-\frac{1}{N_0}\sum_i x_i^2 \Delta t\right) \exp\left(\frac{2}{N_0}\sum_i x_i y_i \Delta t\right)$$

Optimal detection/Optimal filter in white noise

$$\mathrm{SNR} = \frac{|s_{out}|}{\sigma} = \frac{1}{\sigma}\left|\frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\,\mathrm{e}^{i\theta_s}\,K(\omega)\,\mathrm{e}^{i\phi_k + i\omega t_0}\,d\omega\right| \le \frac{1}{\sqrt{2\pi N_0}}\left|\int_{-\infty}^{\infty} S^2(\omega)\,d\omega\right| = \sqrt{\frac{E}{N_0}}$$

$K(\omega) = AS(\omega)$ $\quad\phi_k + \omega t_0 = -\,\theta_s$ — matched filter $\quad g(t) = A\,s(t_0 - t)$ — impulse response

- filter output is the correlation measurement with the expected signal (as in the likelihood ratio (!))
- matched filtration can be treated statistically (likelihoods, statistics, etc.)
- no dependence on the signal shape
- SRN depends only on the energy of signal
- linear time-invariant systems
- filter can be split (dedispersion, spectral filtration, detection on intermediate frequency, etc.)
- correlation measurements is the basis for any signal decomposition

Statistical properties of the correlations

$$z = \int_{-\infty}^{\infty} x(t)y(t)\,dt \qquad\qquad z = \sum_i x_i y_i$$

- $x(t)$ — gaussian process (zero mean and σ^2 variance)
- $y(t)$ — filter/template signal/…

$$E\{z\} = \int_{-\infty}^{\infty} x^2(t)\,dt = E \qquad\qquad D\{z\} = \sigma E$$

- All decomposition components are affected according to their energy

Level of single antenna

- Transient detection (pulsars, FRB, …)

- Signal shape it unknown in general case. Desire of generalization or signal decomposition
- Unavoidable drop of the detection threshold (payment for generalization)
- Local: Daubechies wavelets (orthogonal family of wavelets, filter bank with decimation)
- Global: Walsh-Hadamard transform (similar to discrete Fourier transform)

No showable results yet

Summary and plans

- Understanding digital signal processing is crucially important (nothing to do with electronics or hardware)

Summary
- Simple simulation of the correlation effect described by the VCZ theorem (totally from scratch).
- Identification of the internal connection of the Bayesian detection approach, likelihood ratio test, optimal filter.

Close plans
- Studying the Walsh-Hadamard and wavelet and signal decompositions (i.e. local and global) and corresponding statistical signal identification, etc.
- Measuring information in time series based on likelihood values of the individual signal components.