Review of TMD studies

Alexey Vladimirov

ECFA-NuPECC-APPEC Workshop

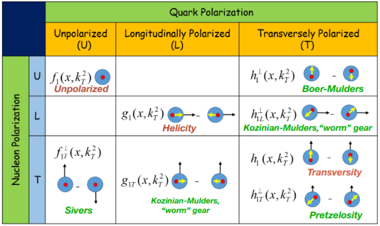
Synergies between the EIC and the LHC

IPARCOS

Comunidad de Madrid

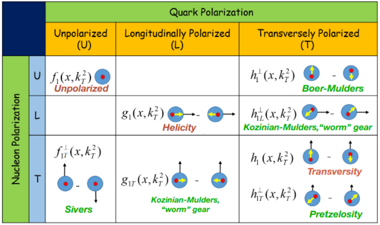
VNIVERSITAS COMPLVTENSIS MATRITENSIS

December 14, 2023



Nucleon Polarization	Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	$f_1(x,k_T^2)$ Unpolarized		$h_1^{\perp}(x,k_T^2)$ Boer-Mulders
L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^{\perp}(x,k_T^2)$ Kozinian-Mulders, "worm" gear
T	$f_{1T}^{\perp}(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Kozinian-Mulders, "worm" gear	$h_1(x,k_T^2)$ Transversity $h_{1T}^{\perp}(x,k_T^2)$ Pretzelosity

Nowadays, extractions of TMDs are **mainly** driven by LHC, despite it has not perfect low-q_T resolution and no polarization.
LHC is perturbation-theory dominated, and thus we can polish our codes and prepare them for future.
Future is for EIC, which will be perfect machine for TMDs.



Selection of topics for this review

- New fits of pion and proton unpolarized TMD (PDFs and FFs)
  - New precision N^{4}LL
  - Better estimation of uncertainties
  - More problems...
- New achievements in the theory
  - Matching TMD $\to$ small-x dipole
  - Collinear distributions from TMDs
  - Power corrections (!)
- First computation of TMD from lattice

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Collaboration

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

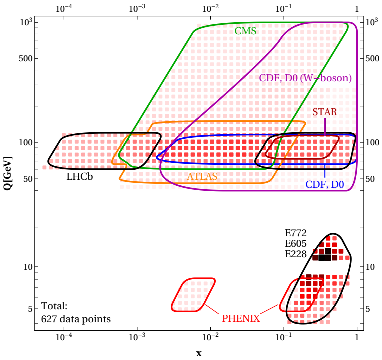
ArXiV: 2304.03302

- 350+ pages on TMD factorization and related topic
- Good introduction to the topic
- Various topics, from definitions to models, small-x, sub-leading power, etc.

ART23

[V.Moos, I.Scimemi, AV, P.Zurita, 2305.07473]

First extraction at N^{4}LL



*** data included for the first time**

- ATLAS
  - Z-boson at 8 (y-diff.)
  - **Z-boson at 13 TeV (0.1% prec.!)**
- CMS
  - Z-boson at 7 and 8 TeV
  - Z-boson at 13 TeV (y-diff.)
  - **Z/γ up to $Q = 1000$GeV**
- LHCb
  - Z-boson at 7 and 8 TeV
  - **Z-boson at 13 TeV (y-diff.)**
- **Further more:**
  - Z-boson at Tevatron
  - **W-boson at Tevatron**
  - **Z-boson at RHIC**
  - DY at PHENIX
  - DY at FERMILAB (fix target)

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627 data points

vs. 457 in SV19
vs. 484 in MAP22

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[V.Moos, I.Scimemi, AV, P.Zurita, 2305.07473]

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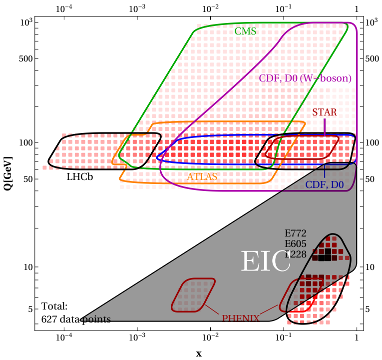

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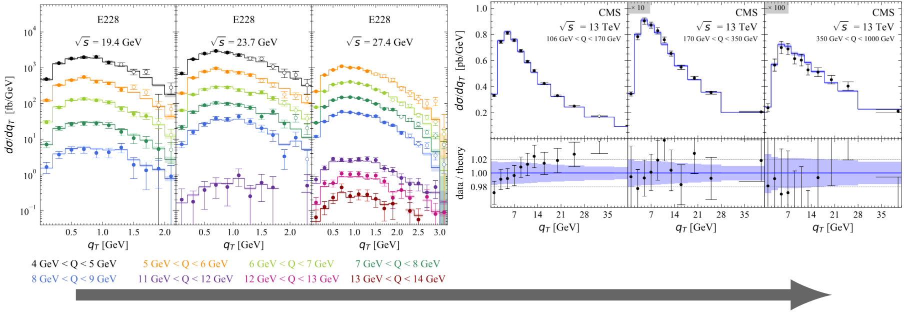
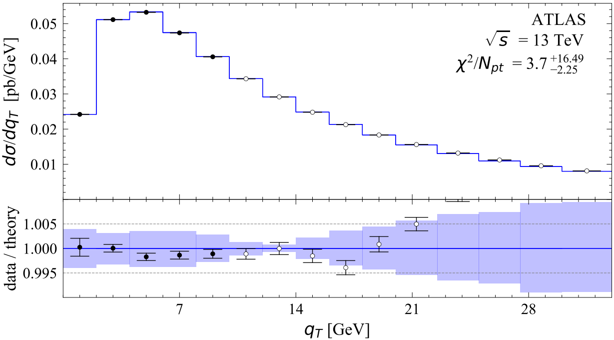


4GeV 1000GeV

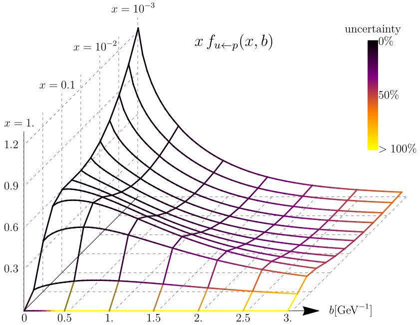
Very presice test of TMD evolution

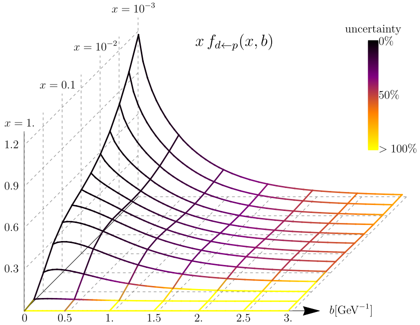


$$\text{TOTAL } (N_{\text{pt}} = 627): \quad \chi^2/N_{\text{pt}} = 0.96^{+0.09}_{-0.01}$$

ART23

[V.Moos, I.Scimemi, AV, P.Zurita, 2305.07473]

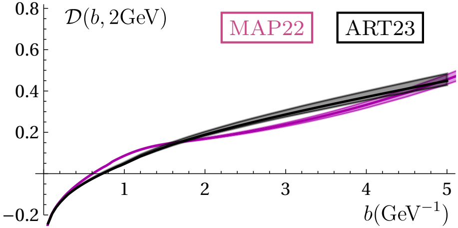




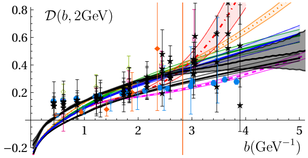
Extra features of analyses:

- Flavor dependent NP-ansatz (first time!)
  - 2 parameters per flavor
  - u, d, $\bar{u}$, $\bar{d}$, rest
- New parametrization for Collins-Soper kernel (3 parameters)
- Consistent inclusion of the PDF uncertainty (first time!)
- *artemide*

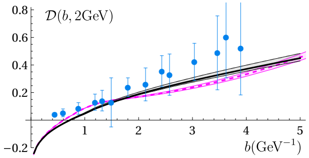
Collins-Soper kernel



Very small uncertanties
(despite huge uncertanties in TMDPDFs)



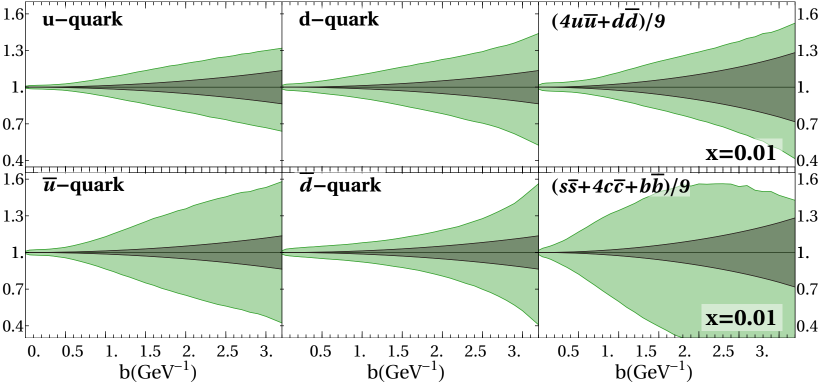
all extractions 2019-2023
pheno + lattice



MAP22+ART23+mean lattice

$$\lim_{b\to 0} F(x,b) = C(x,b) \otimes q(x) + \mathcal{O}(b^2)$$

Propagation of PDF uncertainty into TMD uncertainty



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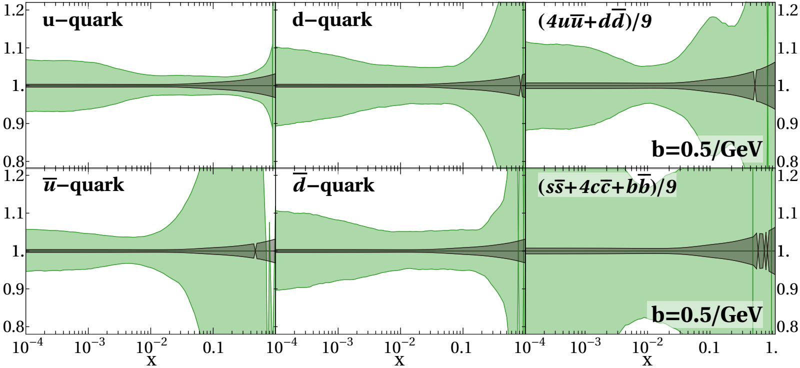
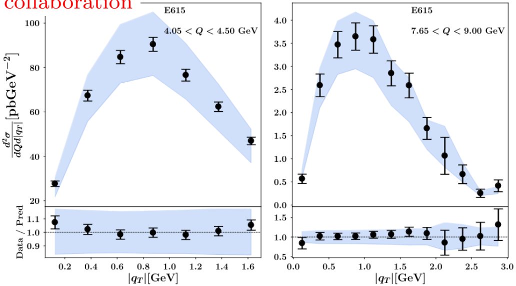


Pion TMDPDF

AV	[1907.10356]	N^2LL	Problems with normalization
MAP	[2210.01733]	N^3LL	Problems with normalization
JAM	[2302.01192]	NLO	TMD and PDF fitted simultaneously

MAP
collaboration



here curve is normalized to data

Problem with factorization or with collinear PDF?

Pion TMDPDF

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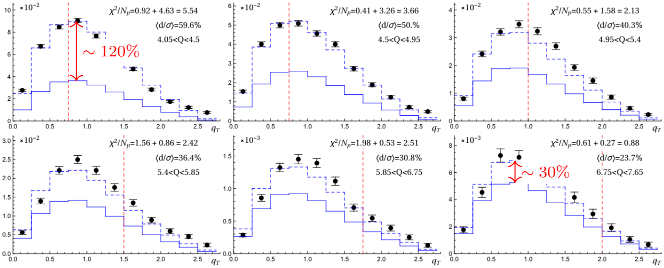
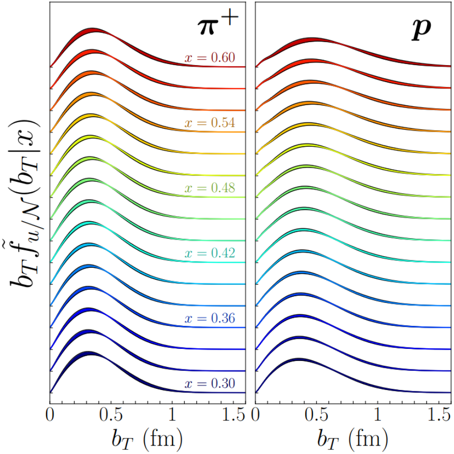
[1907.10356]

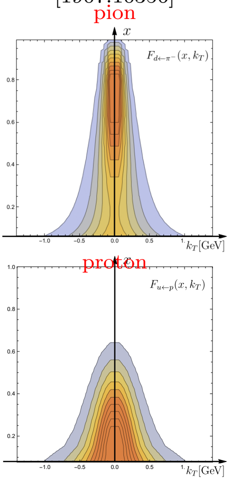


Problem with factorization or with collinear PDF?

JAM collaboration [2302.01192]

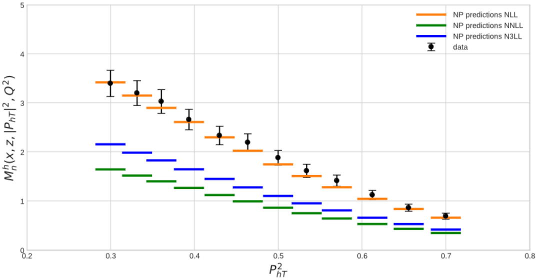


[1907.10356]



Problems with normalization at lower-energy

example of COMPASS-bin by MAP22 [2206.07598]



Multiple observations of normalization problems at $Q < 10 - 15$GeV

- $\sim 30\%$ at $Q \sim 10 - 15$GeV (πDY, DY)
- $\sim 100 - 150\%$ at $Q \sim 3 - 5$GeV (πDY, DY, SIDIS)

Possible source is **power corrections!**

Problems with normalization at lower-energy

SV19

E228-200	43	1.01	35.3%	1.12	34.6%
E228-300	53	0.91	28.8%	1.01	27.8%
E228-400	76	0.87	20.1%	0.95	18.9%
E772	35	1.86	8.9%	1.93	7.9%
E605	53	0.57	20.7%	0.60	19.5%
Low energy DY total	263	0.97		1.04	

ART23

PHENIX	3	0.29	0.12	$0.42^{+0.15}_{-0.10}$	10.%
STAR	11	1.91	0.28	$2.19^{+0.51}_{-0.31}$	15.%
E288 (200)	43	0.31	0.07	$0.38^{+0.12}_{-0.05}$	44.%
(300)	53	0.36	0.07	$0.43^{+0.08}_{-0.04}$	48.%
(400)	79	0.37	0.05	$0.48^{+0.11}_{-0.03}$	48.%
E772	35	0.87	0.21	$1.08^{+0.08}_{-0.05}$	27.%
E605	53	0.18	0.21	$0.39^{+0.03}_{-0.00}$	49.%

Warning! large data uncertainties

MAP22

PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
DY collider total	251	1.86	0.2	2.06
E288 200 GeV	30	0.35	0.19	0.54
E288 300 GeV	39	0.33	0.09	0.42
E288 400 GeV	61	0.5	0.11	0.61
E772	53	1.52	1.03	2.56
E605	50	1.26	0.44	1.7

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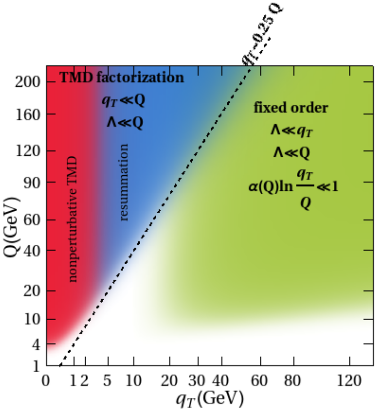


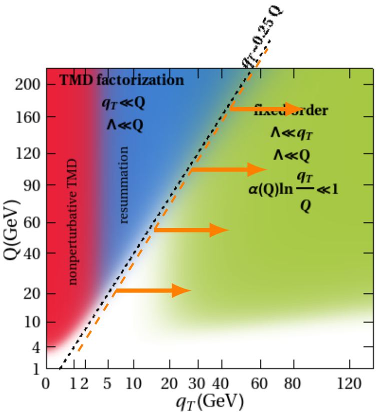
Power corrections:
(many works during last year)

- I.Stewart, A.Gao, et al,
- S.Rodini, AV, et al,
- I.Balitsky, et al,
- ...

NLP TMD factorization is done!
e.g. [2306.09495] for SIDIS
(it is much more complicated than one expected)

TMD factorization at NLP

- 4 TMDFFs, 16 TMDPDFs of twist-3
- NLP restoration of frame-invariance, gauge invariance, boost invariance
- NLO expression for coefficient functions
- LO evolution for twist-3 TMDs
- Qiu-Sterman-like terms in TMD factorization



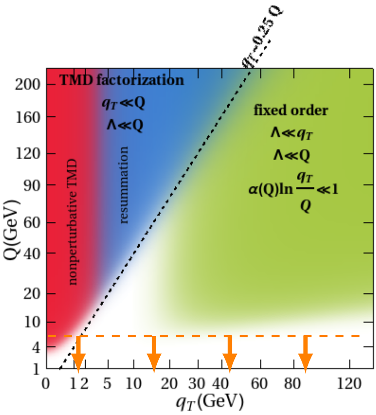
Power corrections:

1. q_T/Q-corrections
Y-term

2. Λ/Q & M/Q-corrections
higher-twist
target-mass

3. k_T/Q-corrections
kinematic

[AV,2307.13054]



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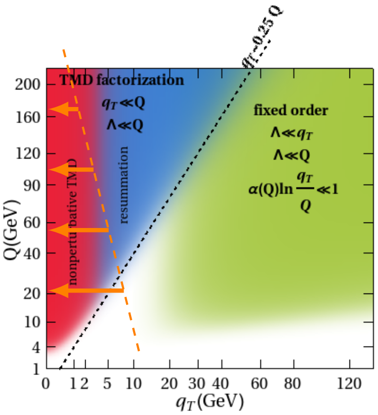

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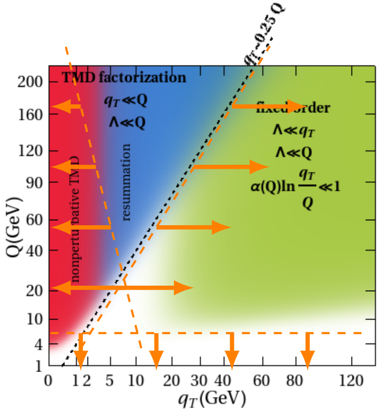

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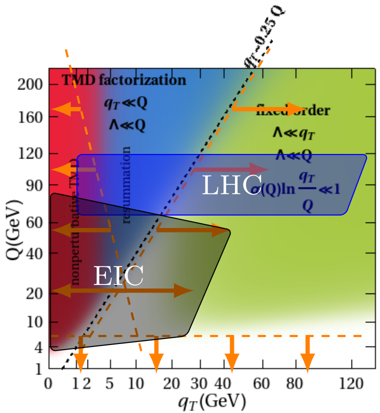

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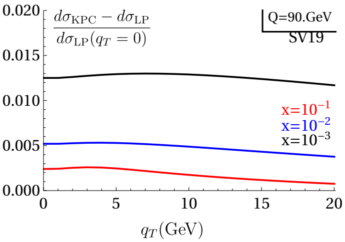
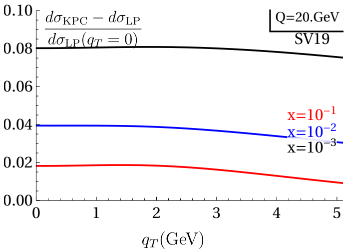
[AV,2307.13054]

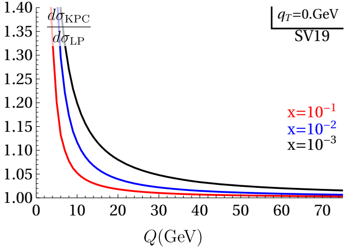
This explains why there are problems with low-k_T at $Q \sim 10$GeV
LHC is "pure" perturbation theory
EIC will be "more interesting"

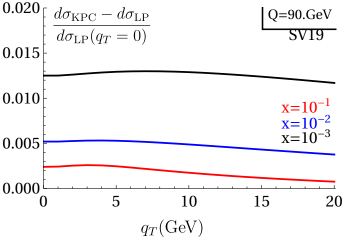


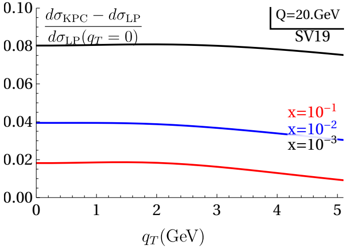


Kinematic power corrections

- Correction for the non-collinear parton momentum
- Restore EM-gauge-invariance (charge conservation)
- Restore frame-invariance
- Can be summed up at all powers [AV,2307.13054]
- Non-zero at $q_T = 0$







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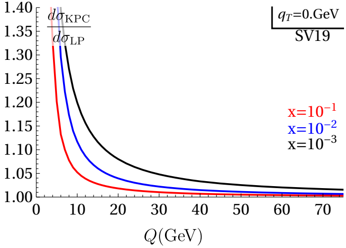
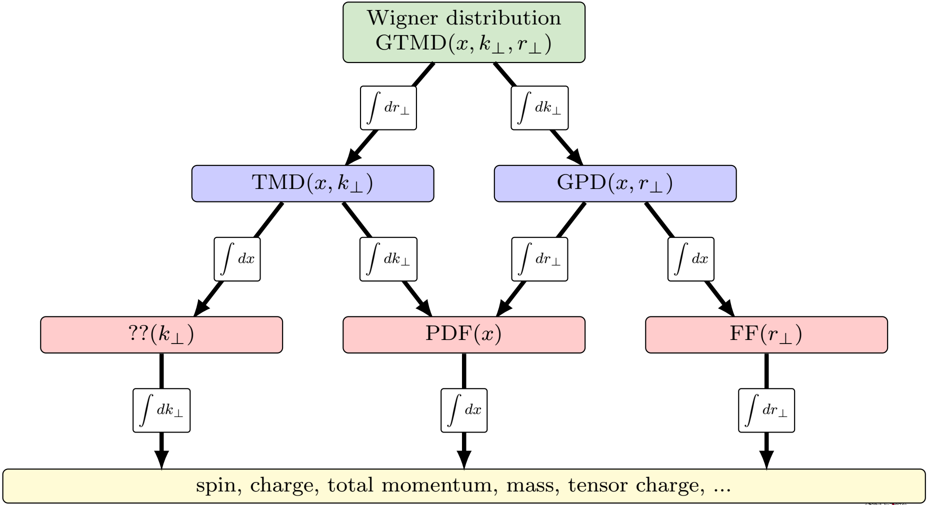
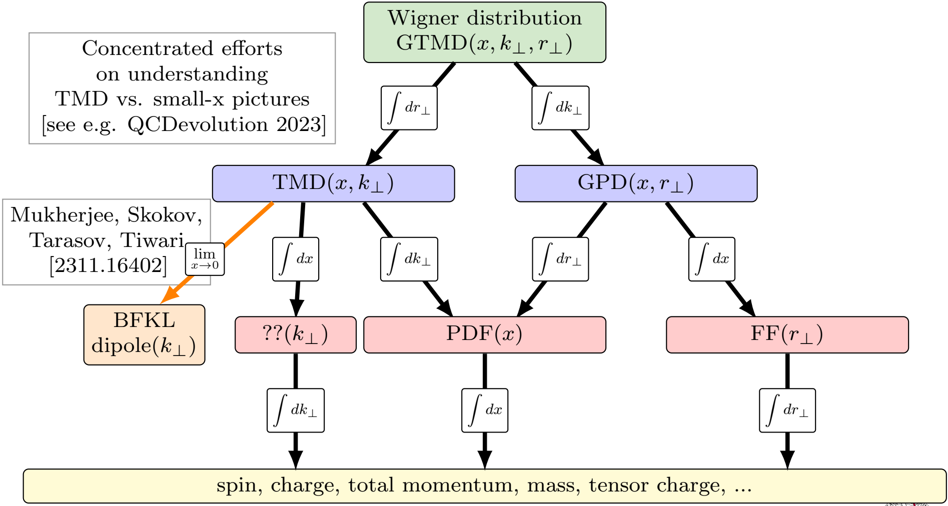


Requires further investigation

If true, all earlier phenomenology of TMDs is concerned.


Wigner distribution
GTMD$(x, k_\perp, r_\perp)$
$\int dr_\perp$
$\int dk_\perp$
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
??$(k_\perp)$
PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...



Concentrated efforts on understanding TMD vs. small-x pictures [see e.g. QCDevolution 2023]
Wigner distribution
GTMD($x, k_\perp, r_\perp$)
$\int dr_\perp$
$\int dk_\perp$
TMD($x, k_\perp$)
GPD($x, r_\perp$)
Mukherjee, Skokov, Tarasov, Tiwari [2311.16402]
$\lim_{x\to 0}$
$\int dx$
$\int dk_\perp$
$\int dr_\perp$
$\int dx$
BFKL dipole($k_\perp$)
??($k_\perp$)
PDF(x)
FF($r_\perp$)
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$\int dx$
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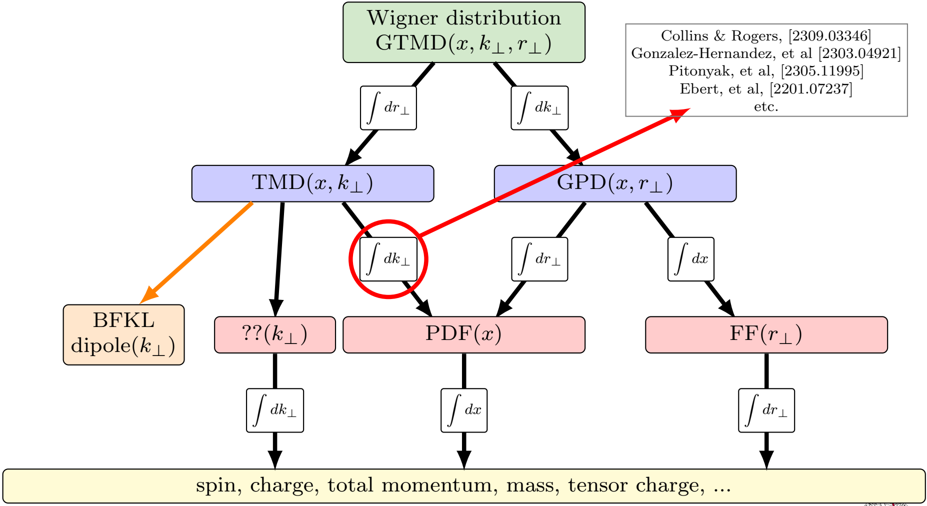
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Collins & Rogers, [2309.03346]
Gonzalez-Hernandez, et al [2303.04921]
Pitonyak, et al, [2305.11995]
Ebert, et al, [2201.07237]
etc.
TMD$(x, k_\perp)$
GPD$(x, r_\perp)$
$\int dk_\perp$
$\int dr_\perp$
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PDF(x)
FF$(r_\perp)$
$\int dk_\perp$
$\int dx$
$\int dr_\perp$
spin, charge, total momentum, mass, tensor charge, ...

Collinear distribution from TMDs

Naively

$f_1(x,\mathbf{k}_T)$ unpol.	$\int dk_T$ →	$q(x)$ unpol.
$h_1(x,\mathbf{k}_T)$ transver.	$\int dk_T$ →	$\Delta q(x)$ transver.
$f_1^{\perp}(x,\mathbf{k}_T)$ Sivers	$\int dk_T$ →	$T(-x,0,x)$ Qiu-Sterman
$h_1^{\perp}(x,\mathbf{k}_T)$ Boer-Mulders	$\int dk_T$ →	$E(-x,0,x)$ e(x)
$\mathrm{TMD}(x,\mathbf{k}_T)$	$\int dk_T$ →	$\mathrm{PDF}(x)$

Collinear distribution from TMDs
Properly

TMD		Coefficient	Collinear
$f_1(x,\mathbf{k}_T,\mu,\zeta)$ unpol.	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ N^{3}LO	$q(x,\mu)$ unpol.
$h_1(x,\mathbf{k}_T,\mu,\zeta)$ transver.	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ N^{2}LO	$\Delta q(x,\mu)$ transver.
$f_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Sivers	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ NLO	$T(-x,0,x,\mu)$ Qiu-Sterman
$h_1^\perp(x,\mathbf{k}_T,\mu,\zeta)$ Boer-Mulders	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$ NLO	$E(-x,0,x,\mu)$ e(x)
$\text{TMD}(x,\mathbf{k}_T,\mu,\zeta)$	OPE →	$C(x,k_\perp;\mu,\zeta)\otimes$	$\text{PDF}(x,\mu)$

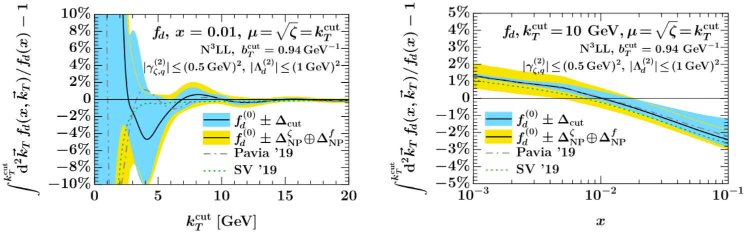
1. Coefficient function $\sim \ln^n(\mathbf{k}^2)/\mathbf{k}^2$
2. Three scales: (μ,ζ) in TMD, μ in collinear PDF

How determine collinear PDF from given TMDPDF?

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) \simeq q(x,\mu)$$

[Ebert, et al 2201.07237]
[Conzalez-Hernandez, et al, 2205.05750]

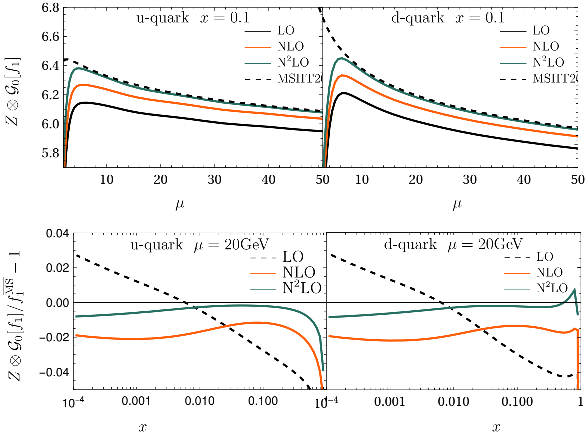


One can restore (tw2) collinear PDF up to few %. **Can we do better?**

Collinear distribution from TMDs

$$\int^{\mu} d^2\mathbf{k}_T f_1(x,\mathbf{k}_T;\mu,\mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu)\otimes q(x,\mu)$$

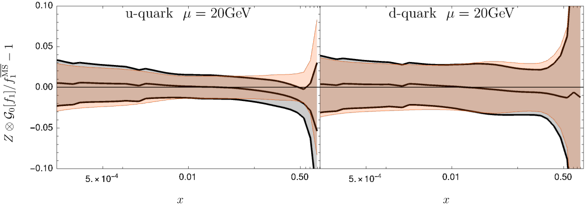
[O. del Rio, et al, 2312.????



Collinear distribution from TMDs

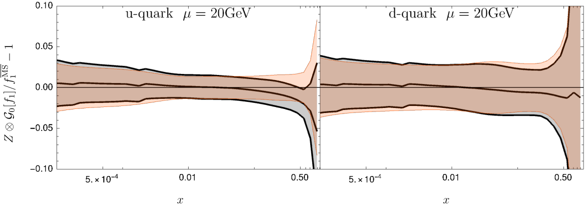
$$\int^{\mu} d^2\mathbf{k}_T f_1(x, \mathbf{k}_T; \mu, \mu^2) = Z^{\mathrm{TMD}/\overline{\mathrm{MS}}}(\mu) \otimes q(x, \mu)$$

[O. del Rio, et al, 2312.????]

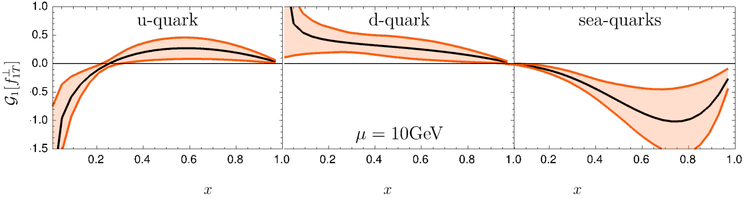


Collinear distribution from TMDs

[O. del Rio, et al, 2312.????



$T(-x, 0, x)$ (from Sivers function [2103.03270])



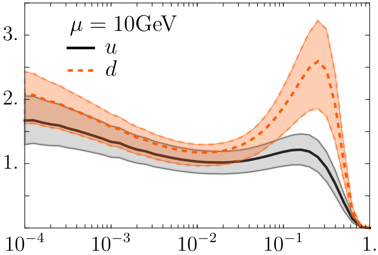
TMD-scheme (different from $\overline{\rm MS}$ at NLO)

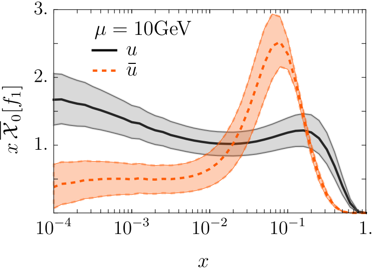
Second moment of TMD

$$\int^{\mu} d^2\mathbf{k}_T \mathbf{k}_T^2 f_1(x,\mathbf{k}_T;\mu,\mu^2) - \text{subtraction} \simeq \langle \mathbf{k}_T^2\rangle(x,\mu)$$

Equals to $\langle \bar{q}D^2q\rangle$ in TMD-scheme ($\overline{\text{MS}}$-scheme up to NLO)

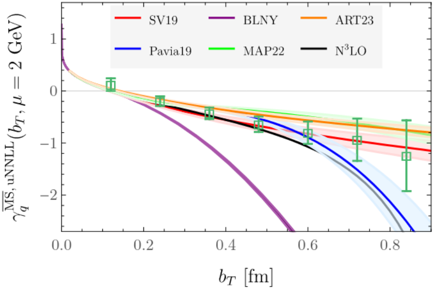
[O. del Rio, et al, 2312.????





Determination of Collins-Soper kernel from the lattice

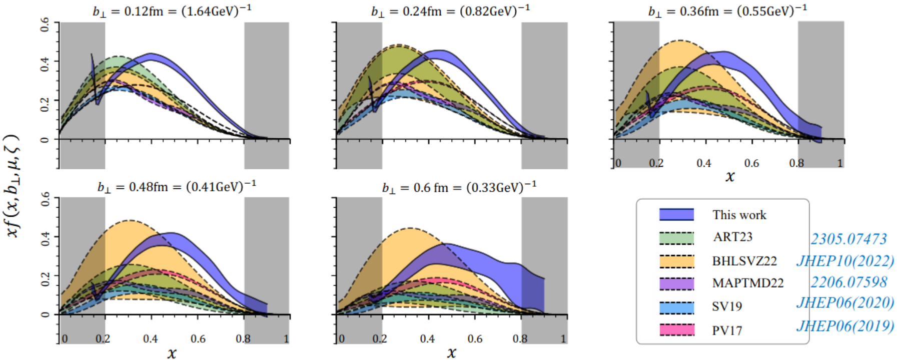
[Avkhadiev, Shanahan, Wagman, Zhao, hep-lat/2307.12359



- Physical pion mass
- NNLO accuracy
- Attention to various sources of uncertainties

First determination of the TMDPDF from the lattice

[LPC, hep-lat/2211.02340



Systematic uncertainty still unknown...

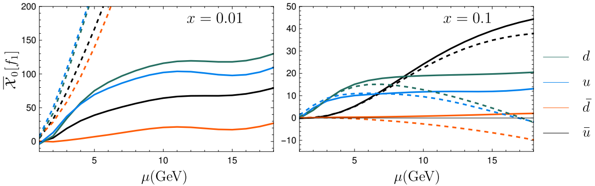
Conclusion

TMD studies are progressing, especially in the theory side (expect new wave of updated extractions next years)

- Phenomenology
  - New extractions of unpolarized TMDs
  - Systematic problems with normalization of low-energy data
- Theory
  - Power corrections [complete NLP, kinematic power corrections]
  - Better understanding of relation with small-x
  - Integral relations
- Lattice
  - Trustful Collins-Soper results
  - First attempts to determine TMDPDFs

Nowadays, extractions of TMDs are **mainly** driven by LHC, despite it has not perfect low-q_T resolution and no polarization.
LHC is perturbation-theory dominated, and thus we can polish our codes and prepare them for future.
Future is for EIC, which will be perfect machine for TMDs.

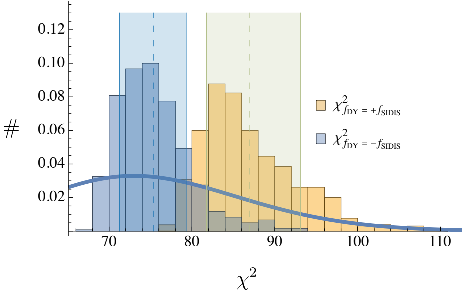
backup 1



200
150
100
50
0
$\overline{\mathcal{X}}_0[f_1]$
$x = 0.01$
5
10
15
μ(GeV)
50
40
30
20
10
0
−10
$x = 0.1$
5
10
15
μ(GeV)
d
u
$\bar{d}$
$\bar{u}$

Check sign-change

$$f_{1T}^{\perp}(SIDIS) = -f_{1T}^{\perp}(DY)$$



$$f_{1T}^{\perp}(sea) \to -f_{1T}^{\perp}(sea)$$

$\chi^2/N_{pt} = 0.88^{+0.16}_{-0.06}$ **vs.** $\chi^2/N_{pt} = 1.00^{+0.22}_{-0.08}$

Current data does not check sign-change!

Naive picture