Aspects of low x physics from saturation to entanglement entropy

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NCN

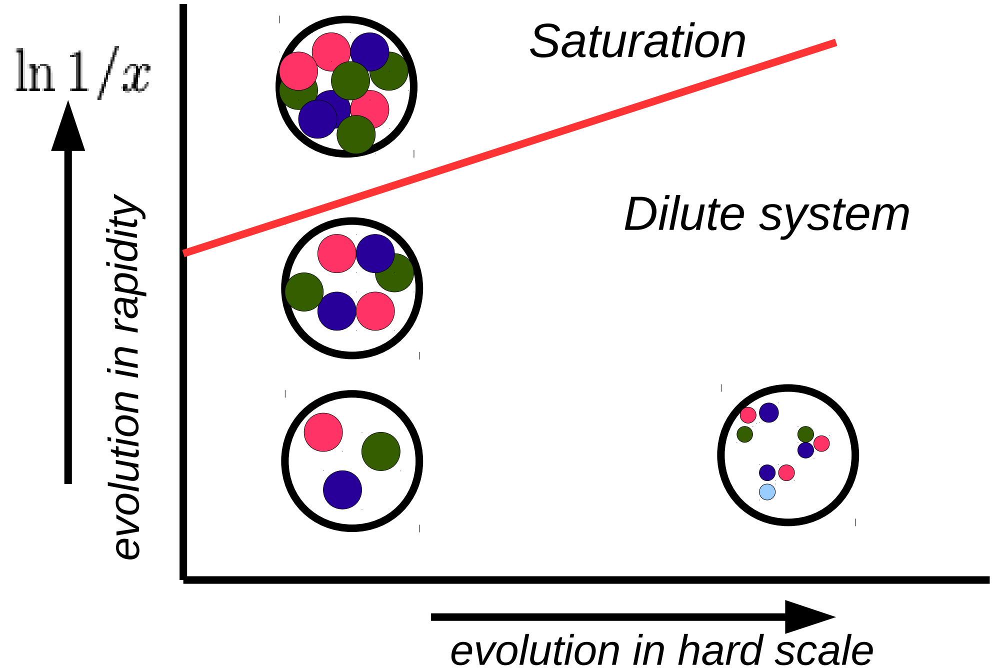
Based on
Eur. Phys. J. C 83, 947 (2023)
A. van Hameren, H. Kakkad, P. Kotko, K. Kutak, S. Sapeta

Gluons at high energies

Saturation – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

L.V. Gribov, E.M. Levin, M.G. Ryskin
Phys.Rept. 100 (1983) 1-150

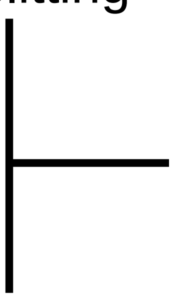
Larry D. McLerran, Raju Venugopalan
Phys.Rev. D49 (1994) 3352-3355



On microscopic level it means that
gluon apart splitting recombine

splitting

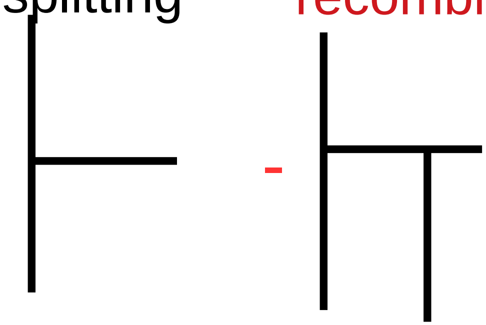
Linear evolution
Equation
BFKL



Nonlinear evolution
equations
BK, JIMWLK
Balitcky-Kovchegov,

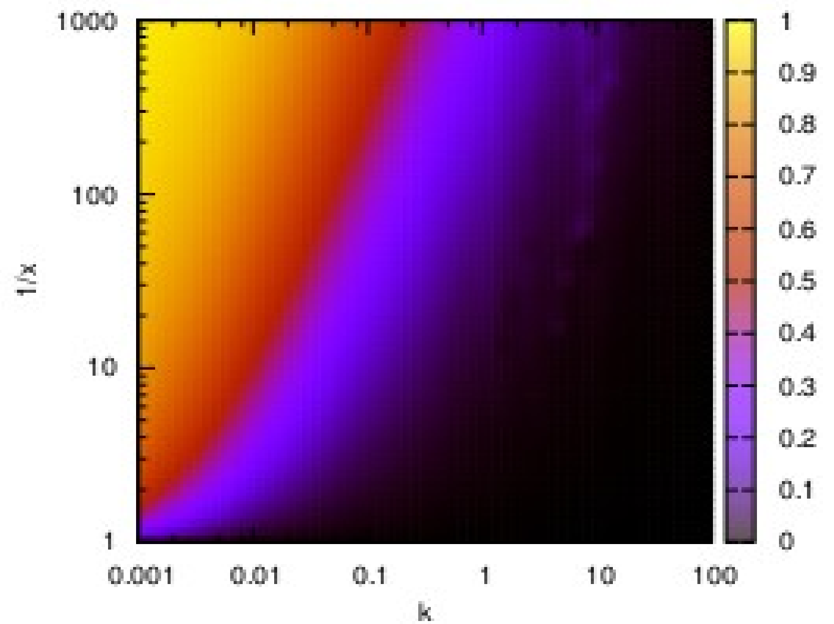
Jailian-Marian,Iancu
McLerran,Weigert,Leonidov,Kovner

splitting - recombination



Bartels,
Wusthoff ‘93

Gluons at high energies



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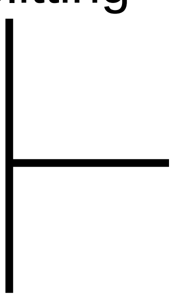

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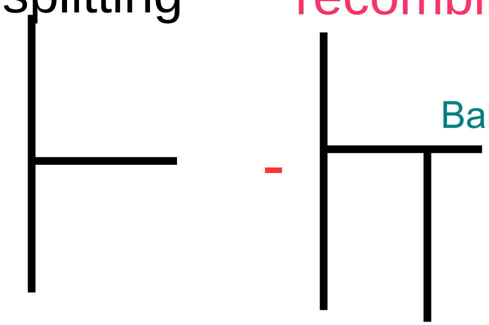


Bartels, Wusthoff'93

Proton structure function and dipole cross section

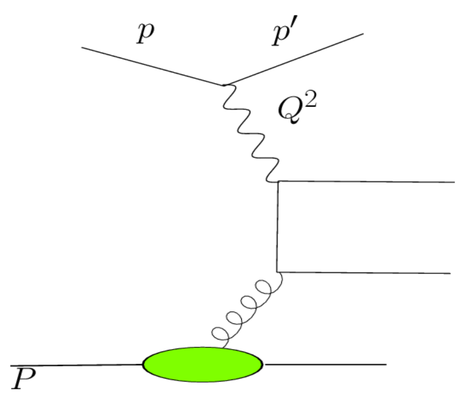
$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2}\alpha_s \sum_q e_q^2 \int d^2k\, \mathcal{F}(x,k^2)\left(S_L(k^2,Q^2,m_q^2) + S_T(k^2,Q^2,m_q^2)\right)$$

dipole gluon density

impact factors ~ hard coefficients

calculable within kT factorization
Catani, Ciafaloni, Hautmann '91
Collins, Ellis '91

In the kt factorization

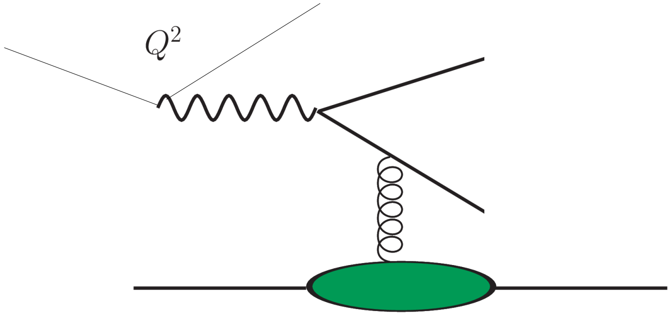


In the dipole formalism

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int d^2b \int_0^1 dz \int d^2r \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right) N(x,r,b)$$

wave function

dipole amplitude

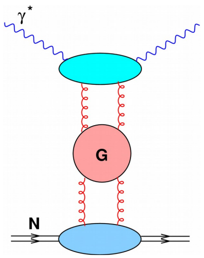


Momentum space vs coordinate space

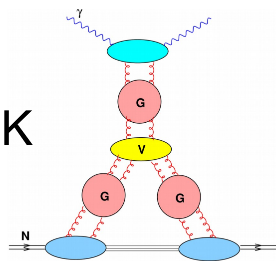
momentum space - Bjorken frame

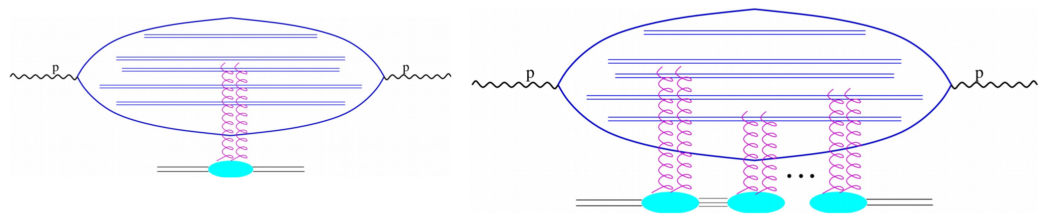
position space - Mueller frame

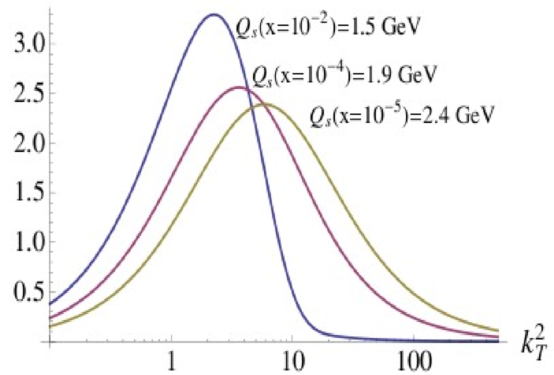
BFKL



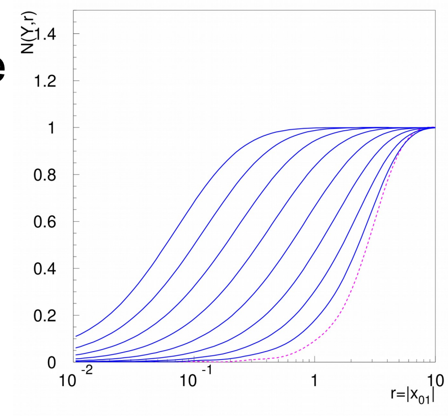
BK







gluon ~ color dipole



from *A. Stasto Acta Phys.Polon. B35 (2004) 3069-3102*

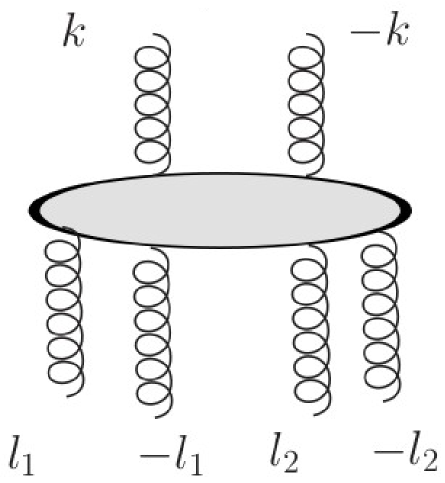
$$\mathcal{F}(x,k) = \mathcal{F} + K_{ms} \otimes \mathcal{F}(x,k) - \frac{1}{R^2} TPV \otimes \mathcal{F}(x,k)^2 \qquad N(x,r,b) = N_0 + K_{ps} \otimes \left(N(x,r,b) - N(x,r,b)^2\right)$$

dipole unintegrated gluon density

Evolved with BK dipole amplitude – expectation value of product of Wilson lines in fundamental representation

related by Fourier transform

The TPV and BK equation



$$\mathcal{V}(k,-k;l_1,-l_1,l_2,-l_2)=\frac{\pi\alpha_s^2}{N_c R^2}\left[2\theta(l_1^2-k^2)\theta(l_2^2-k^2)+k^2\ln\frac{l_1^2}{l_2^2}\delta(l_1^2-k^2)\theta(l_2^2-l_1^2)\right.$$
$$\left.+k^2\ln\frac{l_2^2}{l_1^2}\delta(l_2^2-k^2)\theta(l_1^2-l_2^2)\right]$$

Bartels,Kutak '07

Anticollinear pole dominates

TPV is 0 in DLL

$$\mathcal{F}(x,k^2)=\mathcal{F}_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2)-k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\mathcal{F}(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{2\alpha_s^2\pi}{N_c R^2}\int_{x/x_0}^1\frac{dz}{z}\left\{\left[\int_{k^2}^\infty\frac{dl^2}{l^2}\mathcal{F}(x/z,l^2)\right]^2+\mathcal{F}(x/z,k^2)\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x/z,l^2)\right\}$$

when k>>l

$$\mathcal{F}(x,k^2)=\mathcal{F}_0(x,k^2)+\bar{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_{k_0^2}^{k^2}dl^2\frac{\mathcal{F}(x/z,l^2)}{k^2}$$

For more recent studies see Motyka, Sadzikowski'14
Motyka, DIS 2022 talk

The dipole cross section and integrated gluon

$$\sigma(x,r) = \frac{4\pi^2}{N_c}\int \frac{dk^2}{k^2}(1 - J_0(kr))\mathcal{F}(x,k^2)$$

$$\sigma(x,r) \approx \frac{4\pi^2}{N_c}\int \frac{dk^2}{k^2}\left(1 - \left(1 - \frac{k^2r^2}{4}\right)\right)\mathcal{F}(x,k^2)$$

$$\sigma(x,r) \approx \frac{\pi^2}{N_c} r^2 xg(x, 1/r^2)$$

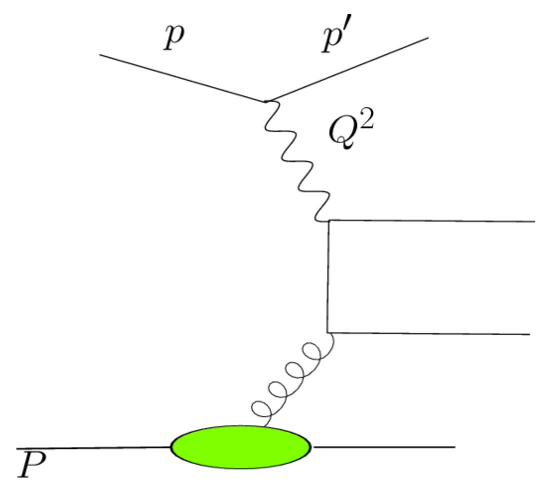
$$\sigma(x,r) = \sigma_0 N(x,r)$$

$$N(x,r) \approx xg(x, 1/r^2)$$

For fixed dipole size one has.

$$xg(x,Q^2) \equiv \int^{Q^2} dk^2 \mathcal{F}(x,k^2)$$

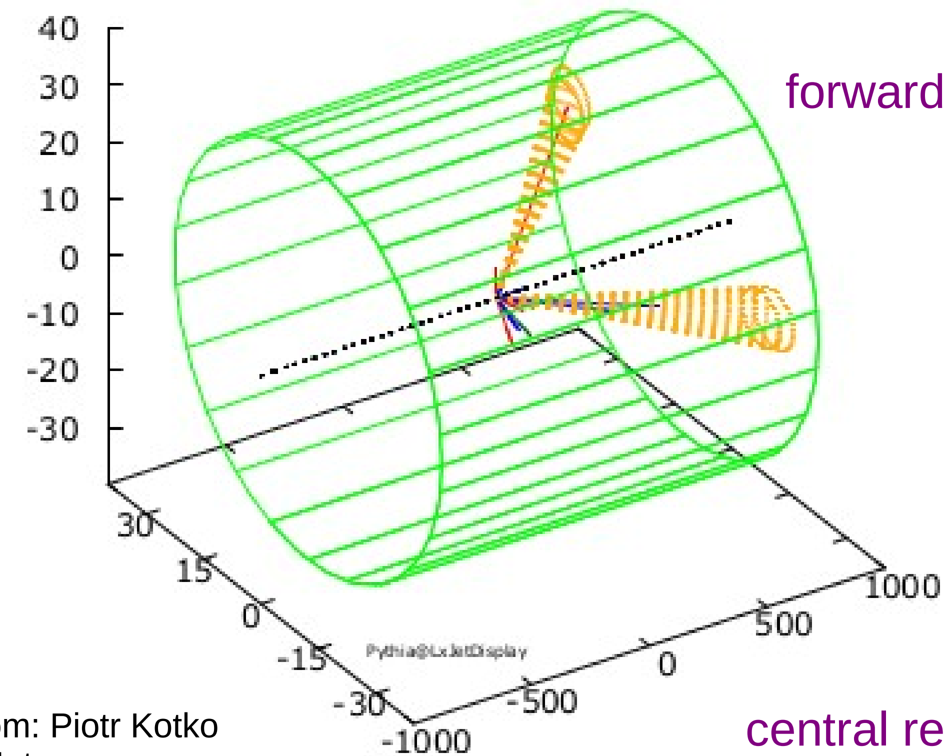
integrated gluon density



In the context of the scale dependent GBW model this approximation is viewed as linear approximation

generating function for dipoles

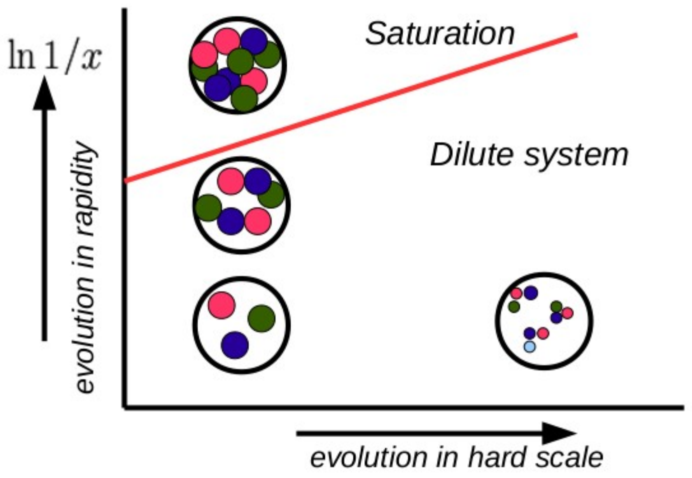
p – A (dilute-dense) forward-forward di-jets



forward region

central region

From: Piotr Kotko
LxJet



It originated from the aim to provide predictions for forward-forward jet production at the LHC

Forward-central jet see
Deak, Hautmann, Jung Kutak '09

ITMD

ITMD = small x Improved Transverse Momentum Dependent factorization

- accounts for saturation
- correct gauge structure i.e. uses gauge links to define TMD's
- takes into account kinematical effects – the whole phase space is available at LO
- is implemented in MC event generator KaTie, LxJet
- valid in region $p_T > Q_s$, k_T can by any. p_T is hard final state momentum, k_T is inbalance

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} \frac{x_1 f_{a/p}(x_1)}{1+\delta_{cd}} \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd}(P_t, k_t)\Phi^{(i)}_{ag\to cd}(x_2, k_t)$$

(one of reprezentations of the ITMD formula)

Generic structure: transverse momentum enters hard factors and gluon distributions gluon distribution depends on color flow

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren JHEP 12 (2016) 034

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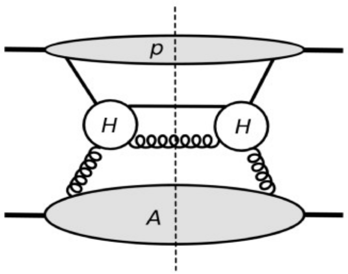
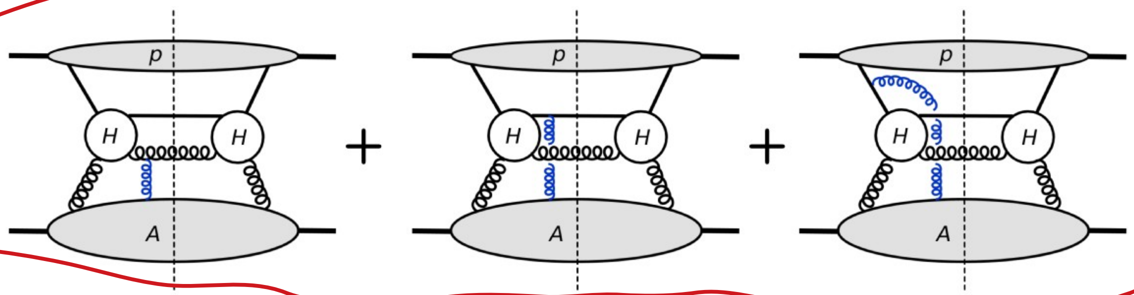
See also:
T. Altinoluk, C. Marquet, P. Taels
JHEP 06 (2021) 085
For developments for massive final states

Formula for TMD gluons and gauge links

$$\mathcal{F}\left(x,k_T\right) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3\, P^+}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\, \langle P|\, \mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\hat{F}^{i+}\left(\xi^+=0,\xi^-,\vec{\xi}_T\right)\right\}|P\rangle$$





Valid for large transversal momentum and was obtained in a specific gauge

From *S. Sapeta*

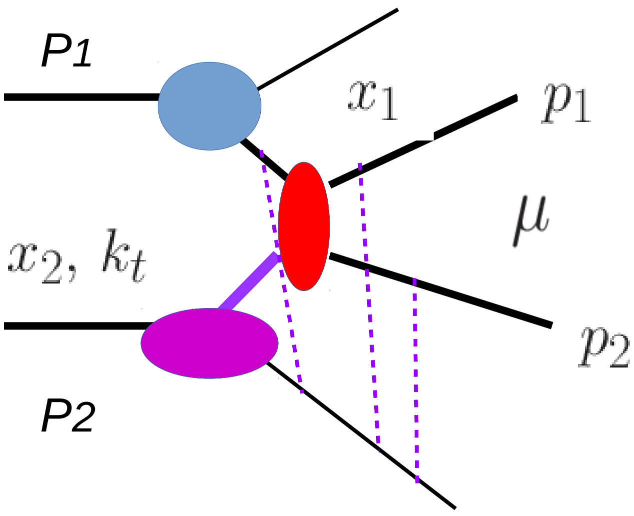
similar diagrams with 2,3,….gluon exchanges. All this need to be resummed

C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{F}\left(x,k_T\right) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3\, P^+}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\, \langle P|\, \mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\mathcal{U}_{C_1}\hat{F}^{i+}(\xi)\,\mathcal{U}_{C_2}\right\}|P\rangle$$

Hard part defines the path of the gauge link $\mathcal{U}^{[C]}(\eta;\xi) = \mathcal{P}\exp\left[-ig\int_C \mathrm{d}z\cdot A(z)\right]$

The ITMD factorization for di-jets



- The color structure is separated from kinematic part of the amplitude by means of the color decomposition.
- The TMD gluon distributions are derived for the color structures following

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren,
JHEP 1509 (2015) 106

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska
JHEP 12 (2016) 034

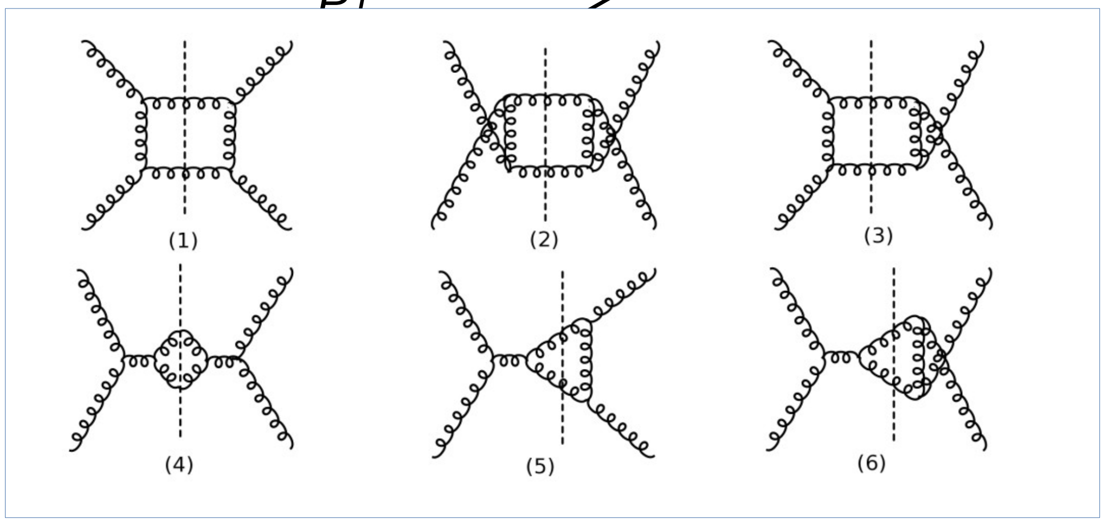
Formalism implemented in Monte Carlo programs KaTie by A. van Hameren

gauge invariant amplitudes with k_t and TMDs

Example for g* g → g g

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2}\, x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$

ITMD - hard factors



from

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan
Phys.Rev. D83 (2011) 105005

The same gauge link and as in TMD 's
Fabio Dominguez, Bo-Wen Xiao, Feng Yuan
Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan
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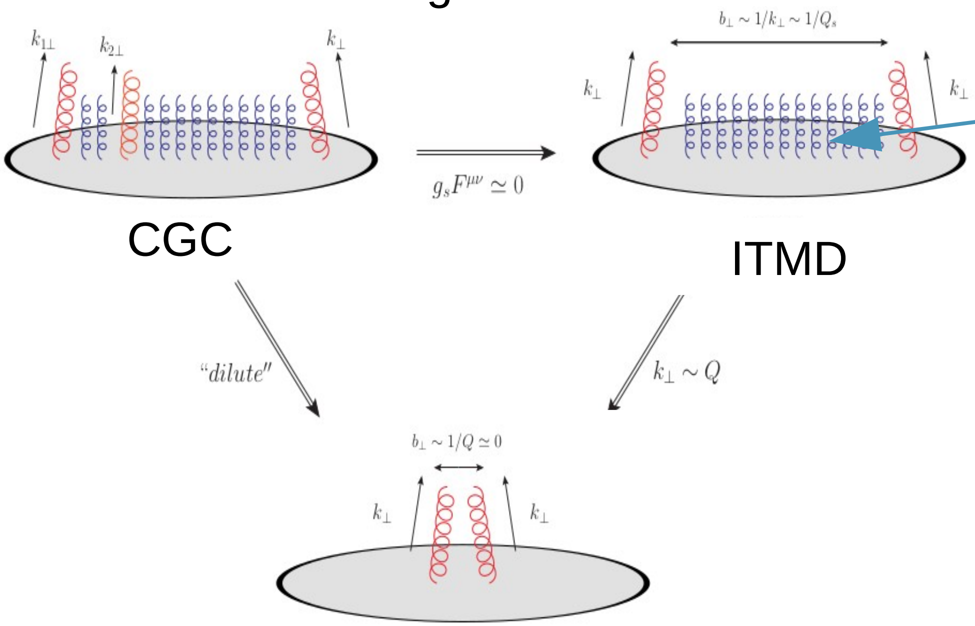
ITMD from CGC

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156
T. Altinoluk, R. Boussarie, JHEP10(2019)208

Expansion in distance - parameter entering as argument Wilson lines appearing in generic CGC amplitude i.e. amplitude for propagation in strong color field of target



kT factorization

Multiple soft emissions

from R. Boussarie
Initial Stages 2019

Wandzura-Wilczek approximmation ITMD neglects higher genuine twist contributions i.e. hard gluon exchanges between the target and the amplitude, while it resums all kinematic twist

Set of basic TMD’s for 2, 3 and 4 jets

$$\mathcal{F}_{qg}^{(1)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle$$

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$$\mathcal{F}_{gg}^{(2)}(x,k_T) = \text{F.T.}\,\frac{1}{N_c}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\right]\text{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

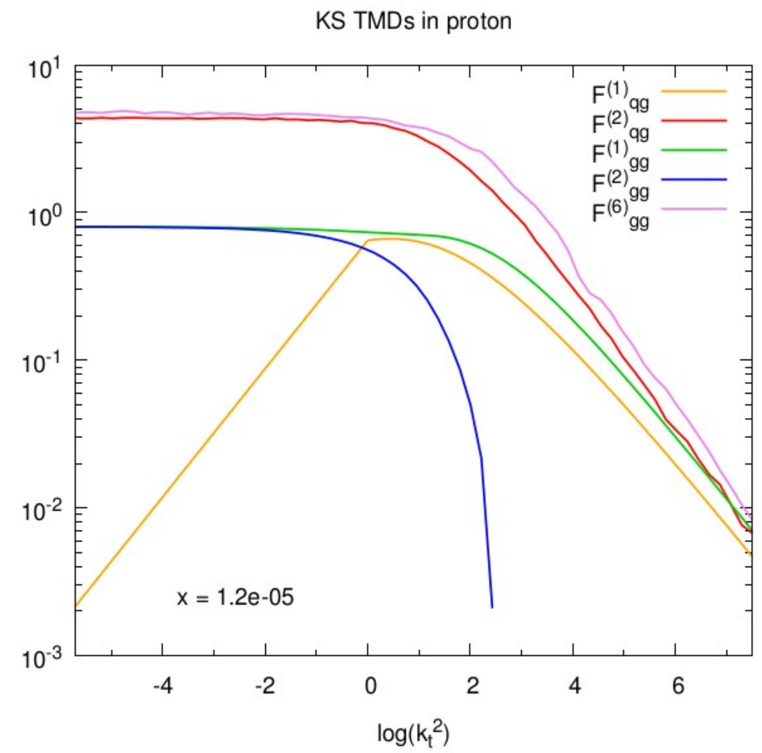
$$\mathcal{F}_{gg}^{(4)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = \text{F.T.}\left\langle \text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\frac{\text{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T) = \text{F.T.}\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c}\text{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Bury, Kotko, Kutak ’18

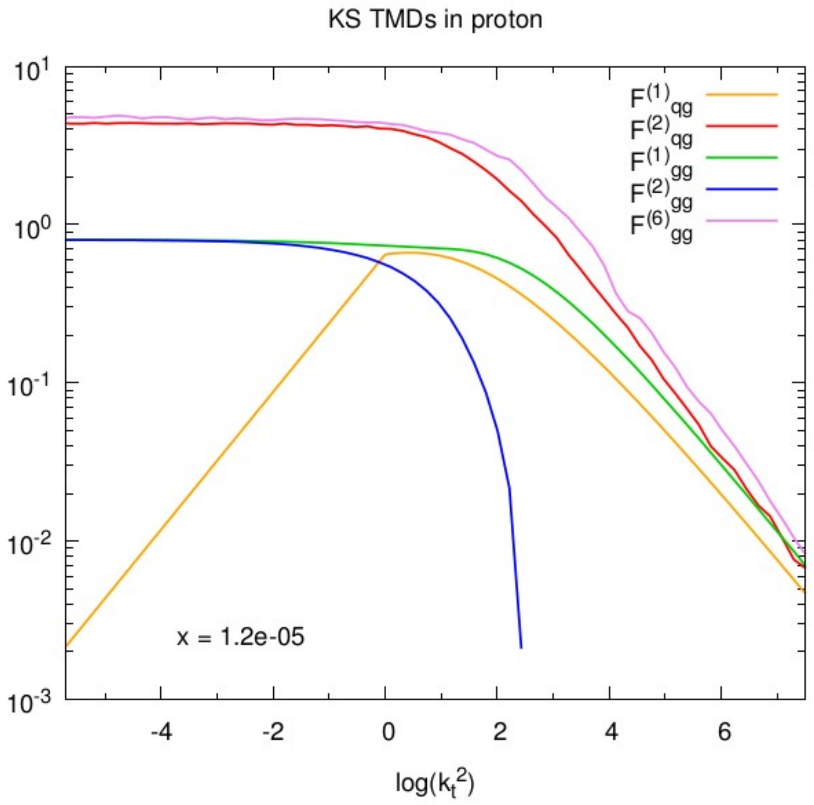


For dijets see
Dominguez,Marquet,Xiao,Yuan ‘11

Set of basic TMD's for 2, 3 and 4 jets

Bury, Kotko, Kutak '18

$$\mathcal{F}_{qg}^{(1)}(x_2,k_t) = x_2 G^{(2)}(x_2,q_t)\,,$$
$$\mathcal{F}_{qg}^{(2)}(x_2,k_t) = \int d^2q_t\, x_2 G^{(1)}(x_2,q_t) F(x_2,k_t-q_t)\,,$$
$$\mathcal{F}_{gg}^{(1)}(x_2,k_t) = \int d^2q_t\, x_2 G^{(2)}(x_2,q_t) F(x_2,k_t-q_t)\,,$$
$$\mathcal{F}_{gg}^{(2)}(x_2,k_t) = -\int d^2q_t \frac{(k_t-q_t)\cdot q_t}{q_t^2} x_2 G^{(2)}(x_2,q_t) F(x_2,k_t-q_t)\,,$$
$$\mathcal{F}_{gg}^{(6)}(x_2,k_t) = \int d^2q_t d^2q_t'\, x_2 G^{(1)}(x_2,q_t) F(x_2,q_t') F(x_2,k_t-q_t-q_t')$$



For dijets see

Dominguez,Marquet,Xiao,Yuan '11

ITMD +Sudakov

$$\frac{d\sigma^{\mathrm{pA}\to j_1 j_2+X}}{d^2P_T d^2k_T dy_1 dy_2} = \sum_{a,c,d} x_{\mathrm{p}} f_{a/\mathrm{p}}\left(x_{\mathrm{p}},\mu\right) \sum_{i=1}^{2} \mathcal{K}^{(i)}_{ag^*\to cd}\left(P_T,k_T;\mu\right) \Phi^{(i)}_{ag\to cd}\left(x_{\mathrm{A}},k_T\right)$$

Sudakov + low x

A. Mueller, B-W. Xiao, F. Yuan, 2013

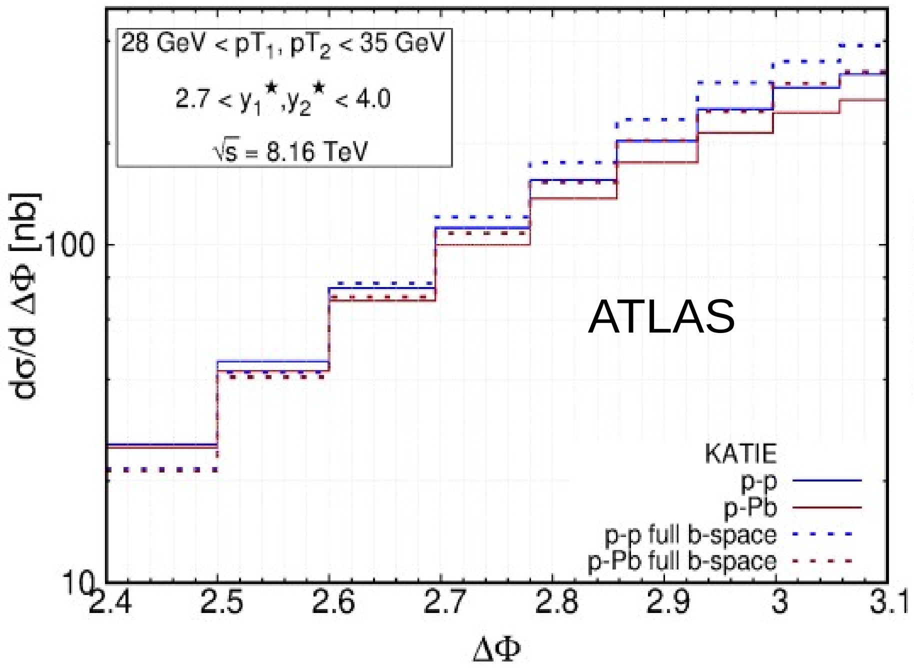
$$\frac{d\sigma^{\mathrm{pA}\to j_1 j_2+X}}{d^2P_T d^2k_T dy_1 dy_2} = \sum_{a,c,d} x_{\mathrm{p}} \sum_{i=1}^{2} \mathcal{K}^{(i)}_{ag^*\to cd}\left(P_T,k_T;\mu\right)$$

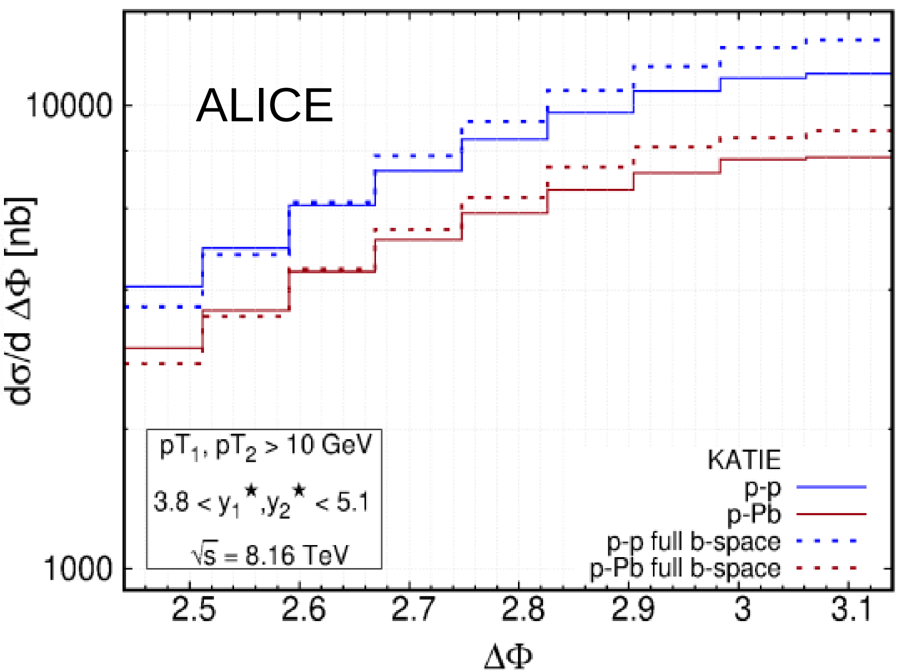
$$\times \int db_T b_T J_0(b_T k_T) f_{a/\mathrm{p}}\left(x_{\mathrm{p}},\mu_b\right) \widetilde{\Phi}^{(i)}_{ag\to cd}\left(x_{\mathrm{A}},b_T\right) e^{-S^{ag\to cd}(\mu,b_\perp)}$$

Depending on the choice of scale the collinear pdf can be put in front of the integral or kept under the integral. We consider both options.
For the unfactorized case we use

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* = b_T/\sqrt{1+b_T^2/b_{\max}^2}$$

Dijets – azimuthal angle dependence – parton level



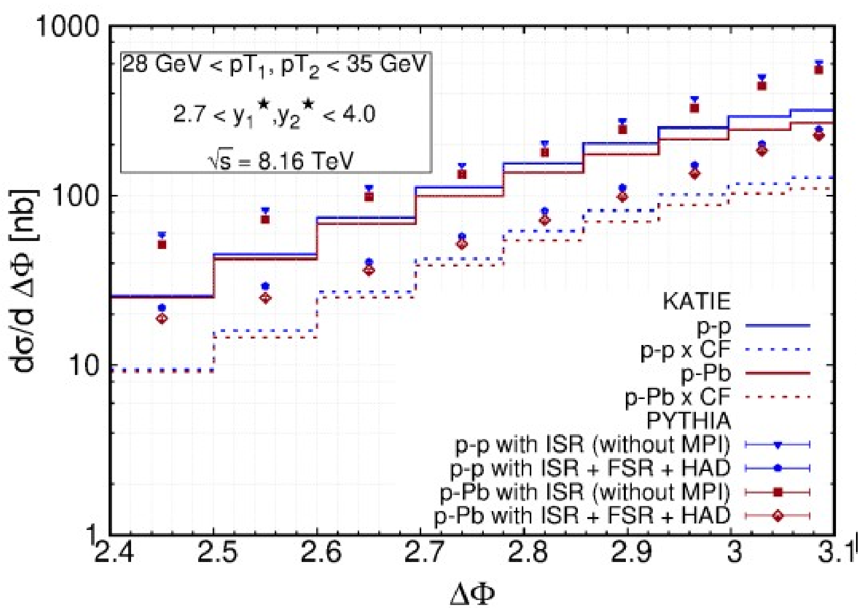


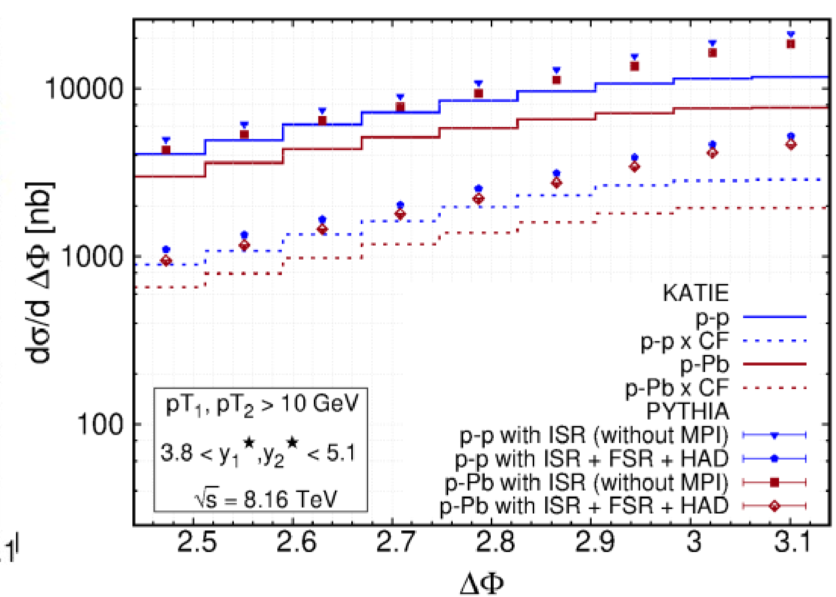
Visible differences especially for ALICE FoCal between p-p and p-Pb results.
Lower pt cut and more forward rapidities.
Not large differences between to approaches to account for Sudakov form factor.

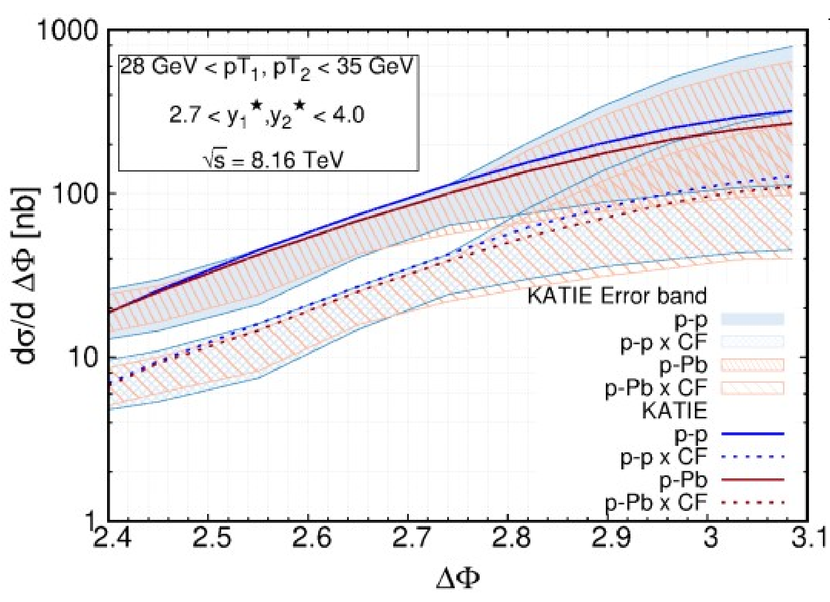
For earlier results see

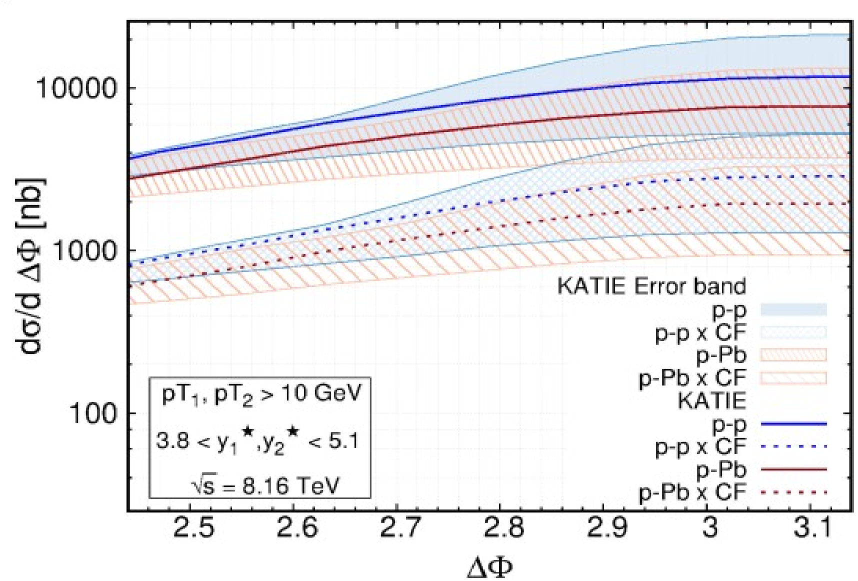
A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

Azimuthal angle dependence – adding correction factor

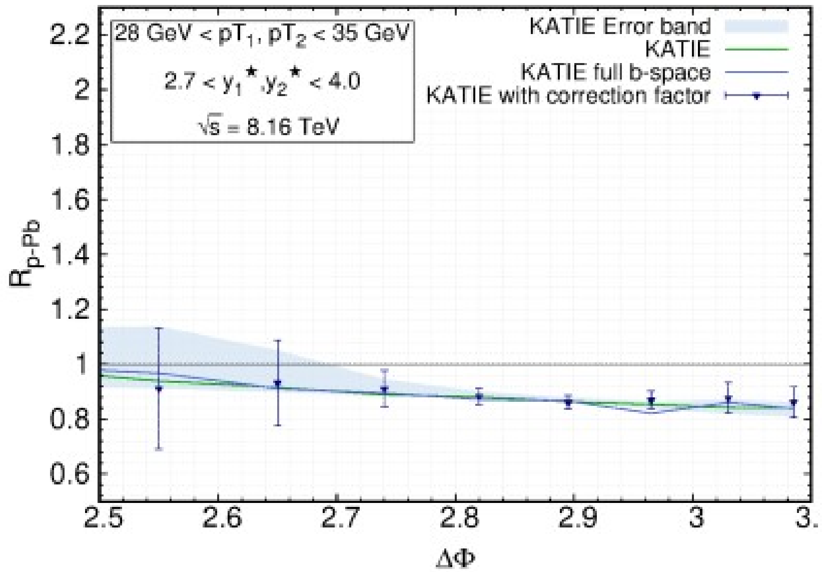


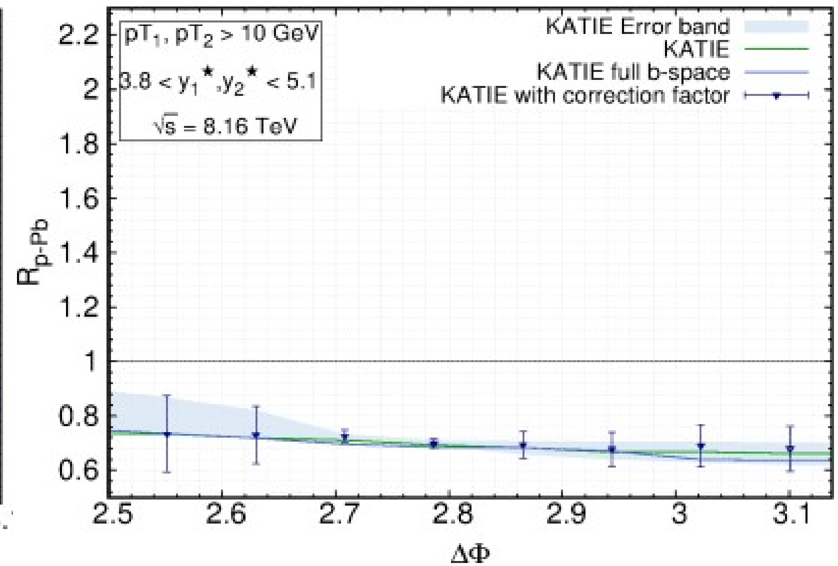






Nuclear modification ratio

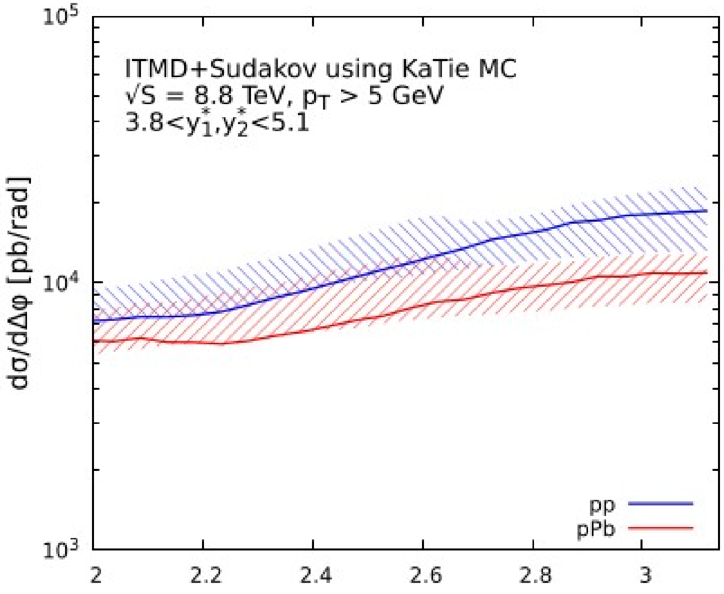


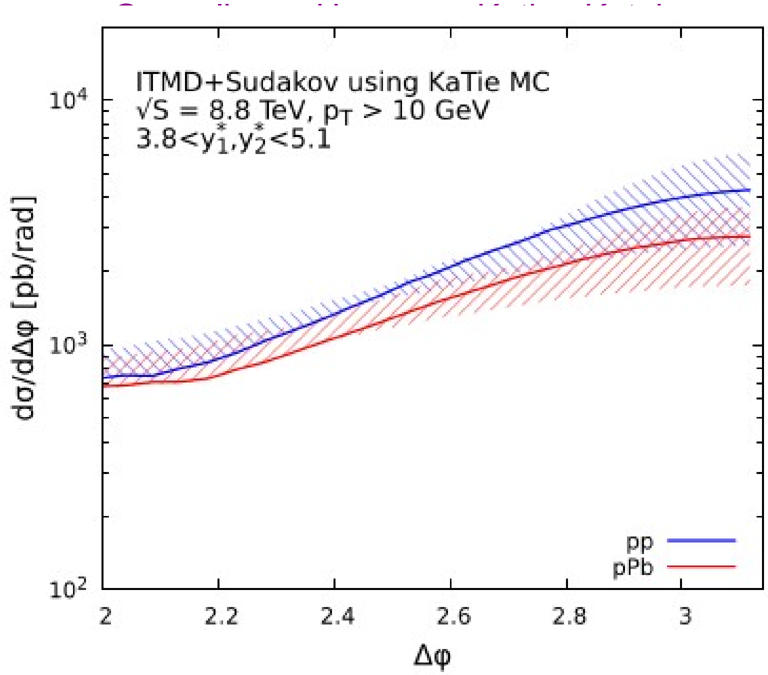


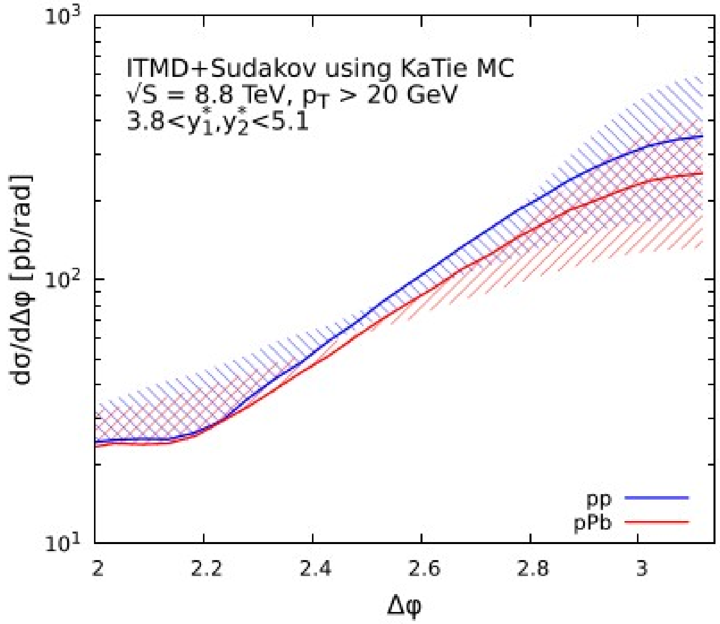
Visible suppression in both ATLAS and ALICE kinematical setup.
Correction factor effectively cancels. Strong saturation signal.

$$R_{\mathrm{p-Pb}} = \frac{\frac{d\sigma^{p+Pb}}{d\mathcal{O}}}{A\frac{d\sigma^{p+p}}{d\mathcal{O}}}$$

Photon and jet at LHC







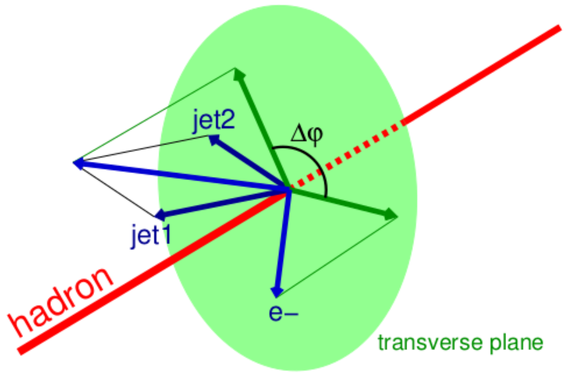
Only dipole gluon density + Sudakov form factor
strong saturation effects

For hadrons see

J. Jalilian-Marian, A. H. Rezaeian '12,….
Benic, Garcia-Montero, Perkov' 22,….

Dijets in DIS

P. Kotko, KK. S. Sapeta, A. van Hameren, E. Zarow, EPJC



$$d\sigma_{eh\to e'+2j+X} = \int \frac{dx}{x}\frac{d^2k_T}{\pi}\mathcal{F}^{(3)}_{gg}(x,k_T,\mu)\,\frac{1}{4xP_e\cdot P_h}\,d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|^2$$

$$S^{g\to q\bar{q}}_{\rm Sud}(\mu,b_T) = \frac{\alpha_s N_c}{4\pi}\ln^2\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

A. Mueller, B-W. Xiao, F. Yuan, 2013

We do not take into account linearly polarised gluons

Related studies for dijet/dihadron at EIC

Back-to-back regime using MV model + Sudakov
L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014

Full CGC calculations
A. Dumitru, V. Skokov, 2018

H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019
F. Salazar, B. Schenke, 2020

R. Boussarie, H. Mäntysaari, F. Salazar, B. Schenke
JHEP 09 (2021) 178

Y. Zhao et . Al 2105.08818

Marquet, Taels, Altinoluk, JHEP 06 (2021) 085

P.Taels, T.Altinoluk, G. Beuf, C.Marquet ‘22

P. Caucal, F. Salazar, B. Schenke, T. Stebel, R. Venugopalan '23

F_2 structure function – kt factorization

$$F_2(x,Q^2) = S(Q^2,k^2,\beta) \otimes \mathcal{F}(x/z,k^2)\Theta\left(1-\frac{x}{z}\right)$$

derived in the linear regime

Catani, Ciafaloni,Hautman '90

longitudinal momentum fraction of the photon carried by the quark

$$z = \left[1 + \frac{\kappa'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k^2}{Q^2}\right]^{-1}$$

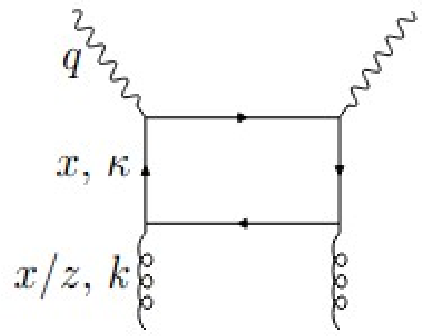
shifted quark momentum

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \int_0^1 dz \int d^2r (|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2)\sigma(x,r)$$

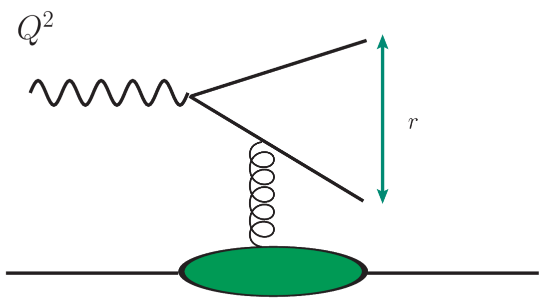
wave function

dipole cross section

In the kt factorization

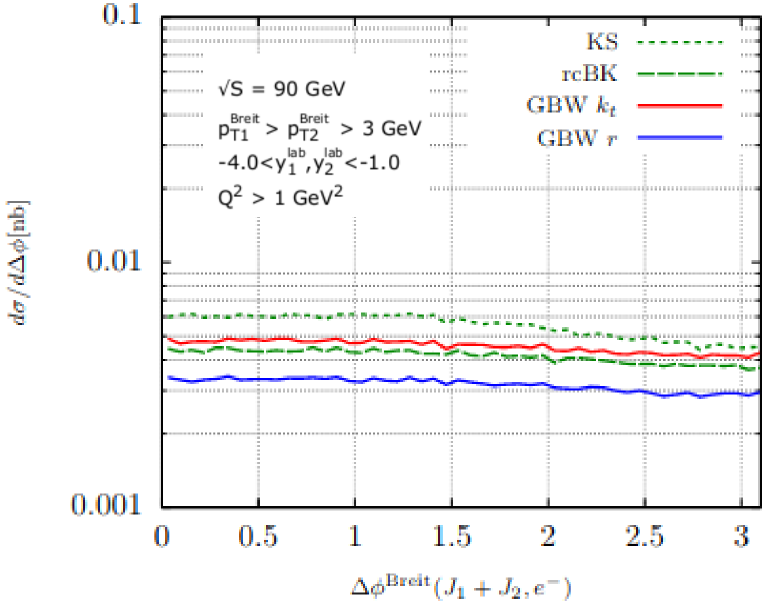


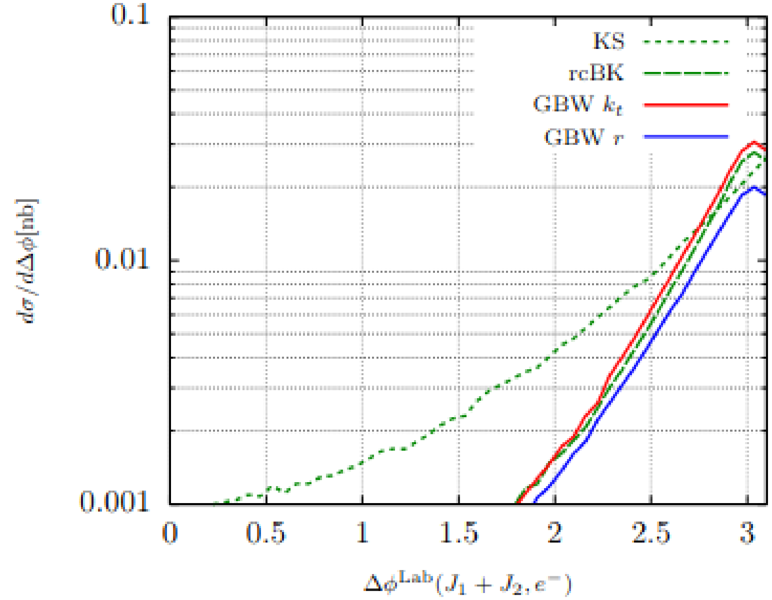
In the dipole formalism

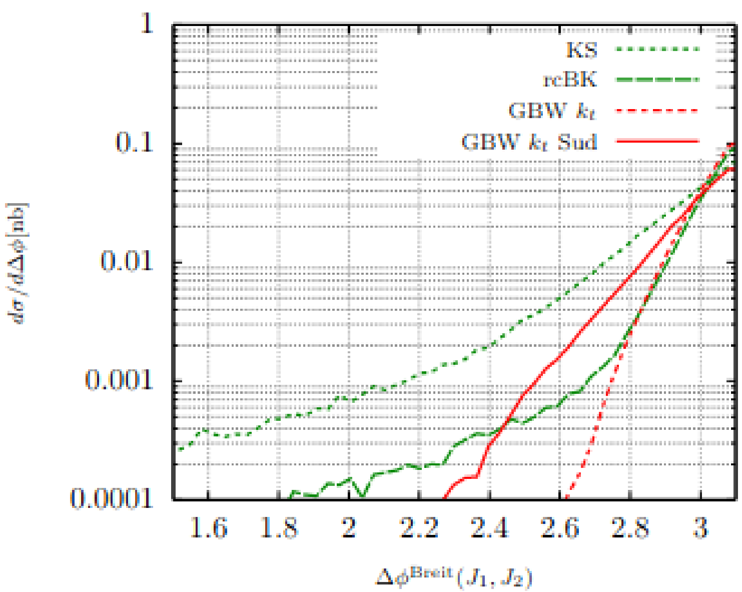


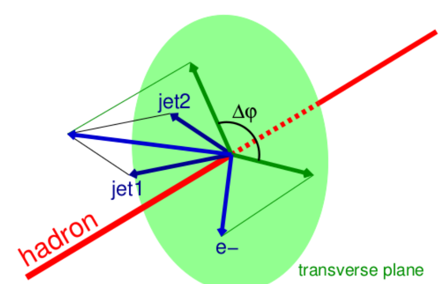
Dijets in DIS – detailed kinematics vs approximate kinematics

Goda, Kutak, Sapeta'23





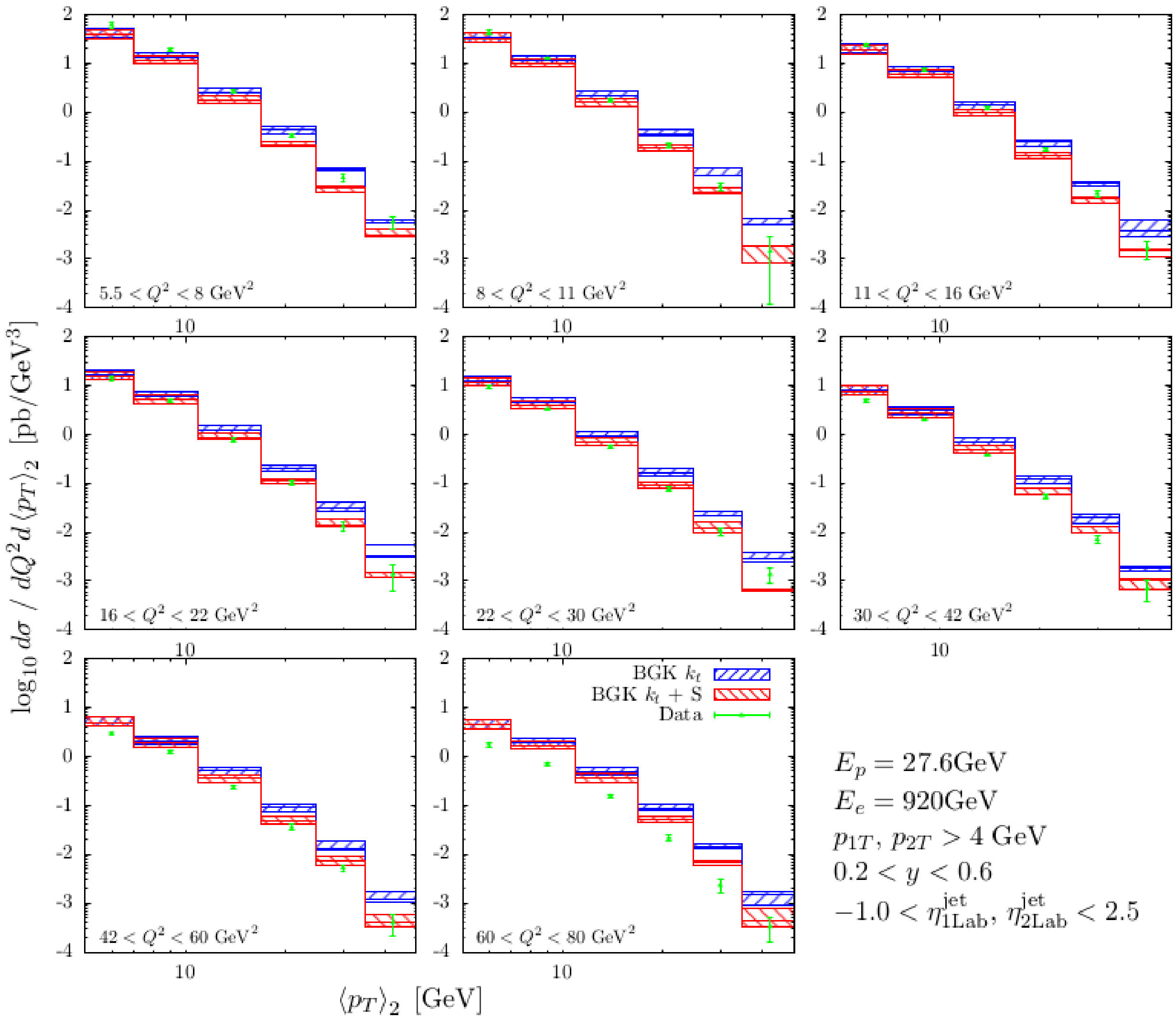




-	σ_0 [mb]	x_0 (10^{-4})	λ	χ^2/dof
r-GBW	1.907e+01	2.582e+00	3.219e-01	4.438e+00
r-GBW-massive	2.384e+01	1.117e+00	3.082e-01	5.274e+00
k_t-GBW	3.344e+01	1.333e+00	3.258e-01	4.396e+00
rc-k_t-GBW	1.520e+01	2.648e+00	3.211e-01	2.447e+00

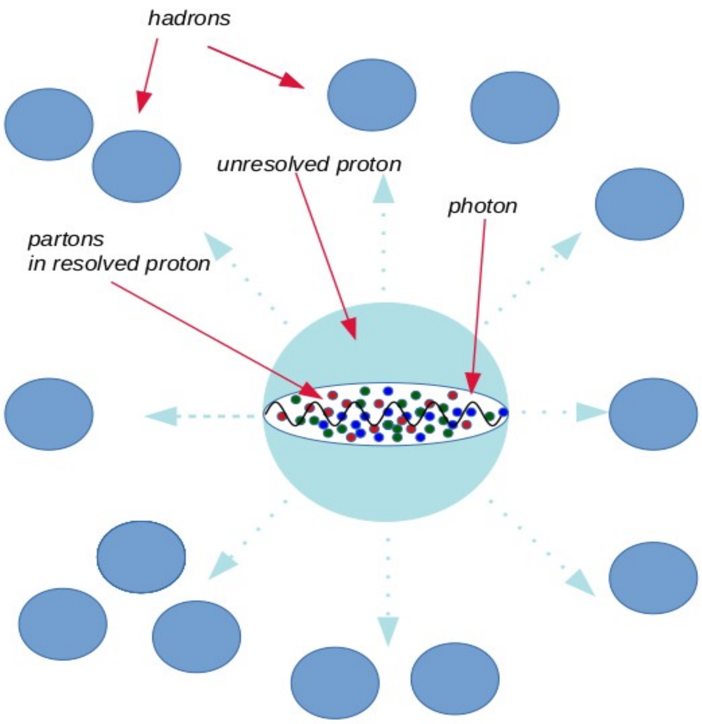
Dijets in HERA

Goda, Kutak, Sapeta'23



BGK+kt+S →saturation + evolution in hard scale + Sudakov + exact kinemmatics

Entanglement entropy in DIS



$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

The density matrix of the mixed state probed in region A

$\alpha_n^2 \equiv p_n$ probability of state with n partons

Kharzeev, Levin ‘17

$$S = -\sum_n p_n \ln p_n$$

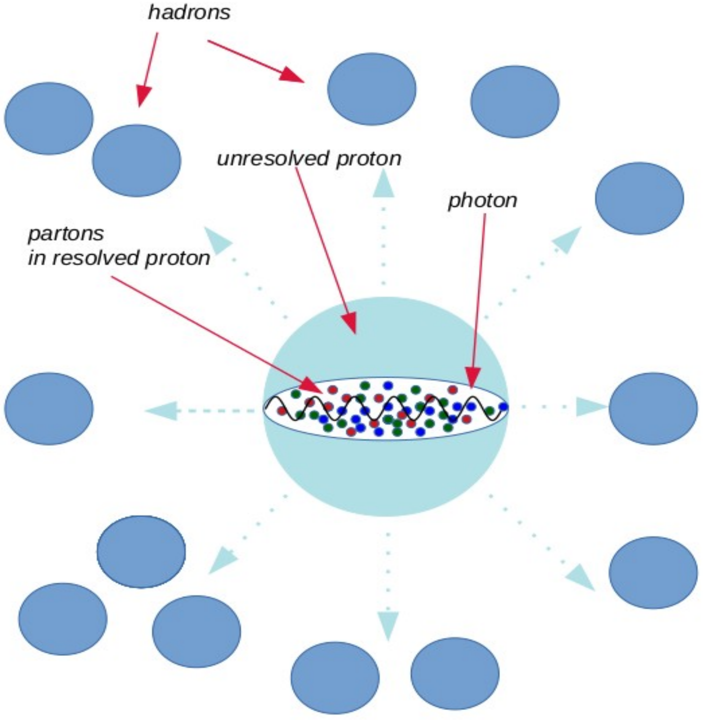
entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy. Entropy of region A is the same as entropy in region B.

Entanglement entropy – calculation and measurement

For DIS at high energies, this entanglement entropy can be calculated using

$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$

$$S_{hadron} = \sum P(N) \ln P(N)$$



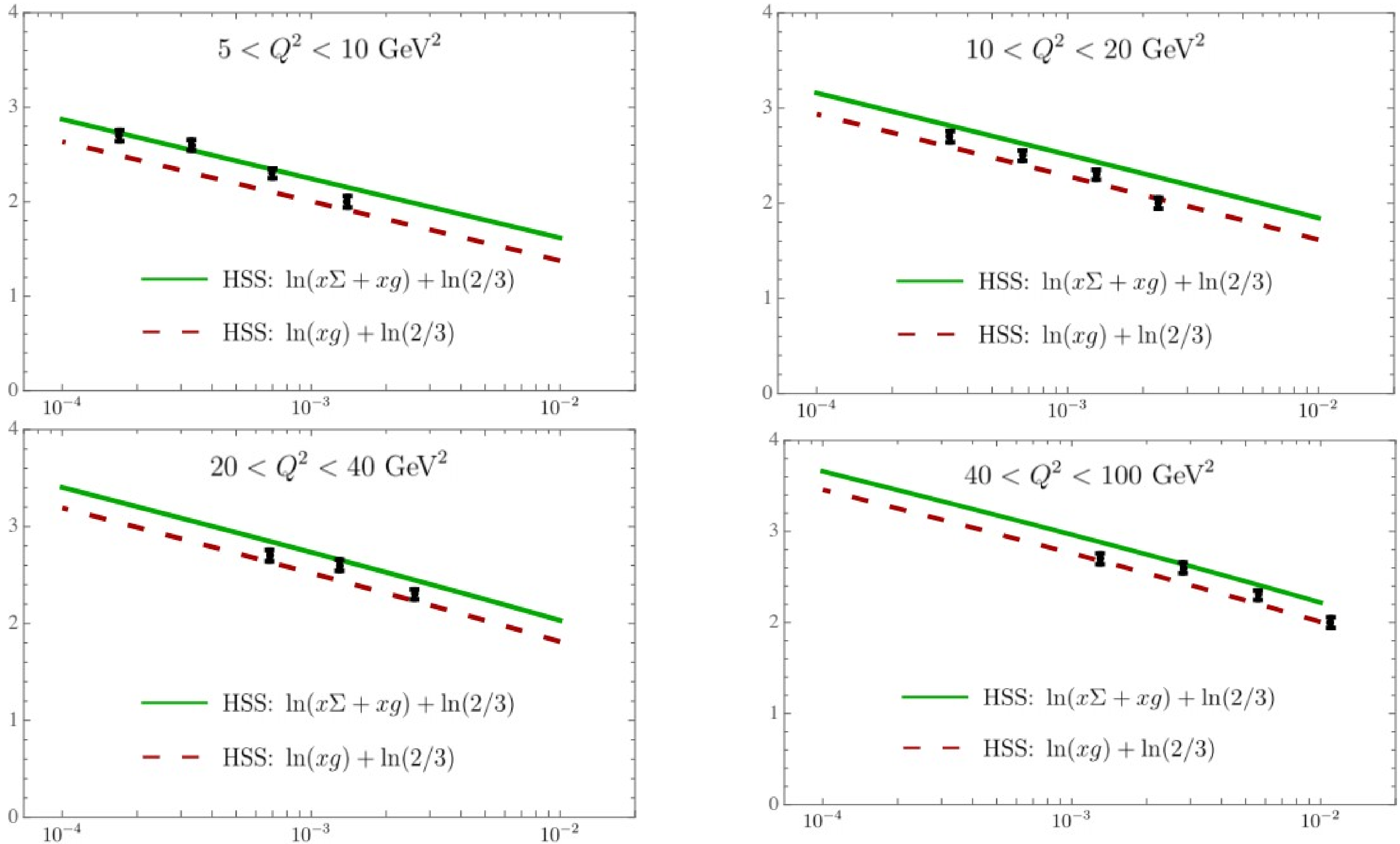
N number of measured hadrons

The charged particle multiplicity distribution measured in either the current fragmentation region or the target fragmentation region.

Fraction of events with charged hadron

Results

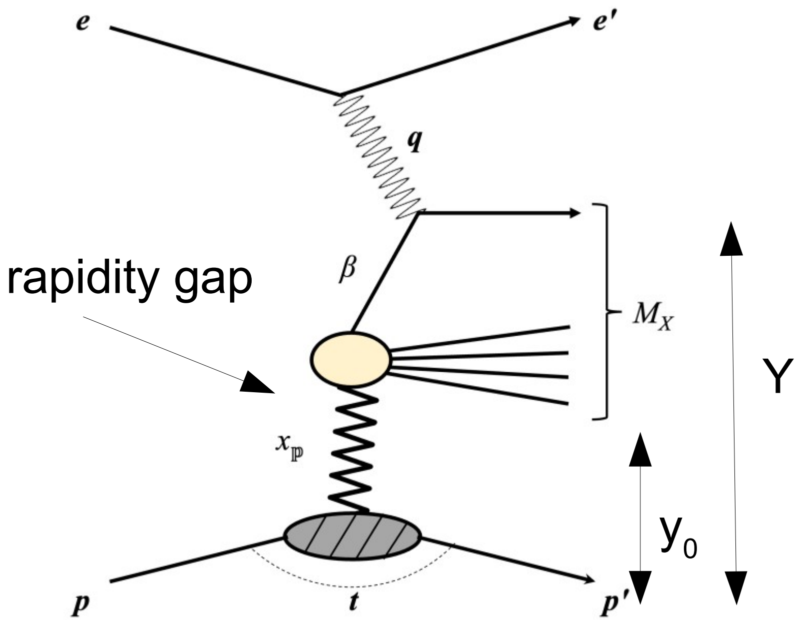
Eur.Phys.J.C 82 (2022) 2, 111 Hentschinski, Kutak



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in Diffractive Deep Inelastic Scattering



$x_{\mathbb{P}}$ proton’s momentum fraction carried by the Pomeron

β denotes the Pomeron’s momentum fraction carried by the quark interacting with the virtual photon

$x = \beta \cdot x_{\mathbb{P}}$ Bjorken x

$y_0 \simeq \ln 1/x_{\mathbb{P}}$ size of rpidity gap

$Y = \ln 1/x$

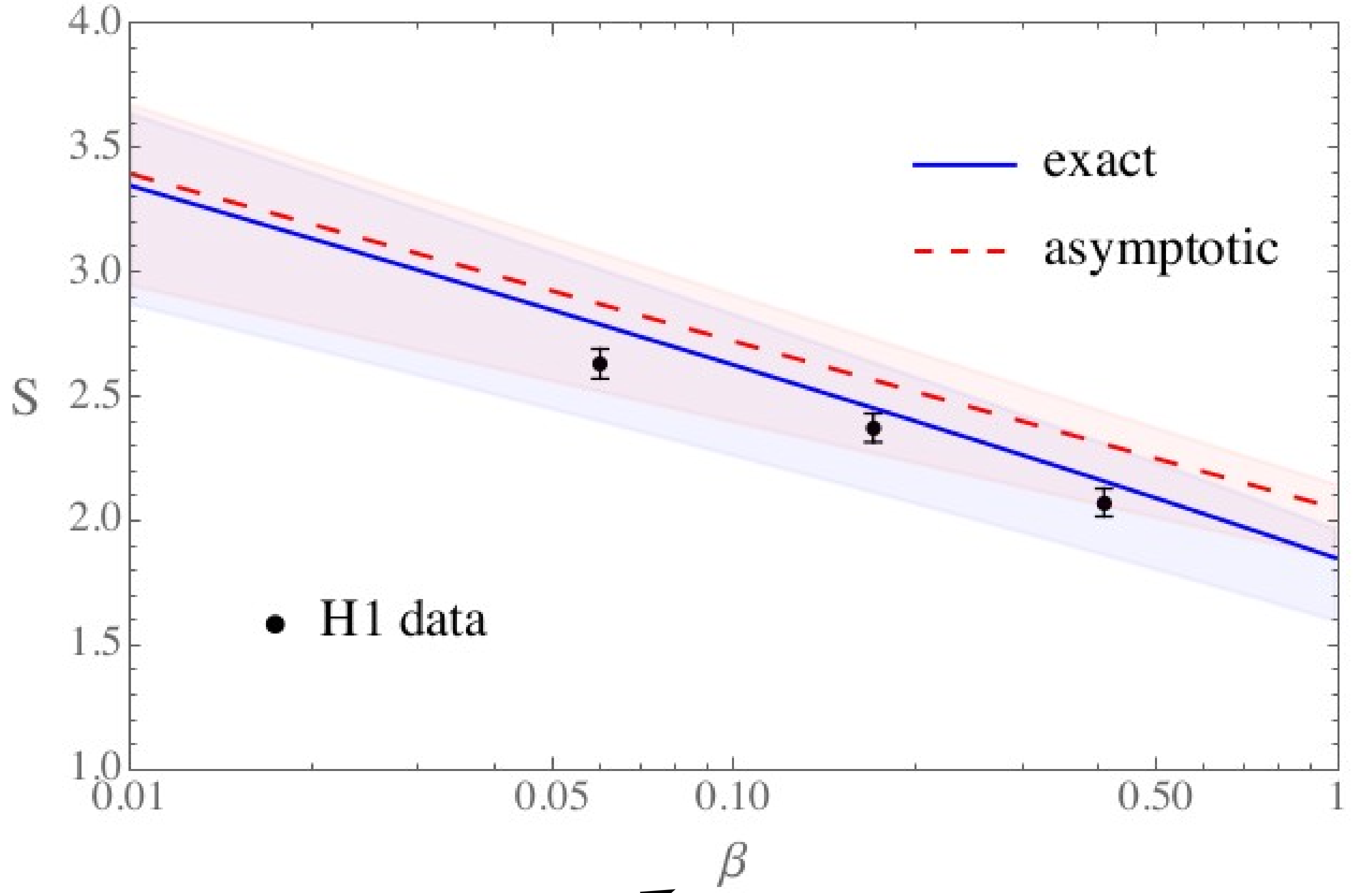
$y_X = Y - y_0 \simeq \ln 1/\beta$

Analogous evolution equation as for non-diffractive case but Initial conditions are different and there is delay because of rapidity gap.

Munier, Mueller Phys. Rev. D 98, 034021 (2018)

EE in DDIS

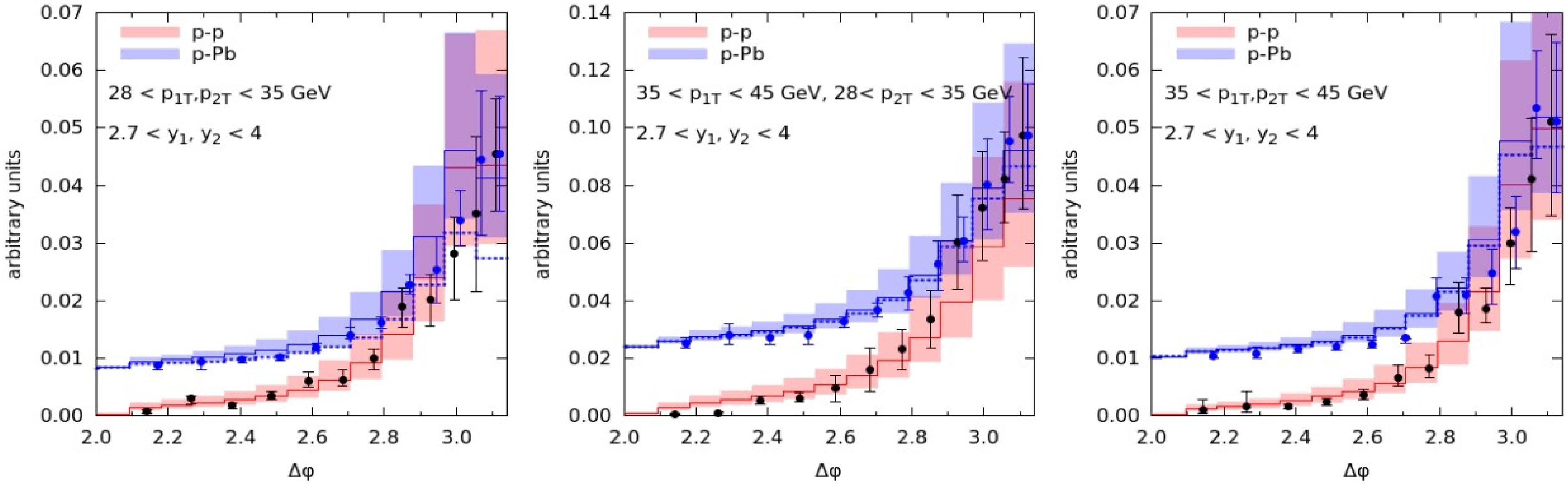
H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu
Phys. Rev. Lett. 131, 241901



momentum fraction carried by the
quark interacting with the virtual photon

BACKUP

The ITMD factorization for jets



A. Hameren, P. Kotko, K. Kutak, S. Sapeta
Phys.Lett. B795 (2019) 511-515

Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

gauge invariant amplitudes with k_t and TMDs

Example for g g → g g*

$$\frac{d\sigma^{pA\to ggX}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2}\, x_1 f_{g/p}(x_1,\mu^2) \sum_{i=1}^{6} \mathcal{F}_{gg}^{(i)} H_{gg\to gg}^{(i)}$$