Scale Setting in Nf=2+1 QCD with Wilson fermions

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RQCD Collaboration:arXiv:2211.03744

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Overview

Determine the gradient flow scale at the physical point $t_{0,ph}$ using m_{Ξ}.

- ⋆ Introductory remarks on setting the scale in lattice simulations.
- ⋆ Gradient flow scale.
- ⋆ CLS ensembles.
- ⋆ Continuum, quark mass and finite volume fits to the baryon octet masses (also consider decuplet baryons). Fit forms for quark mass dependence. Estimation of systematics.
- ⋆ Extraction of the baryon sigma terms $\sigma_{\pi B}$ and σ_{sB} using the Feynman-Hellmann theorem.
- ⋆ Determination of m_ℓ, m_s, $m_u - m_d$.
- ⋆ Summary and outlook

Setting the scale

Input to lattice simulations: g, m_q.

Output: dimensionless quantities $X^{latt} = (a^n X^{phys})$ for X with dimension n.

Need to fix m_q and a using input from Nature.

Predictions for some quantities are reaching sub-percent precision:

m_s (0.6%), m_c (0.4%), m_b (0.3%),
f_π (0.6%), f_K (0.2%), f_D (0.3%), f_{D_s} (0.2%)

Precision determinations of the scale are needed.

Dimensionless combinations less sensitive to scale setting: ratios (m_c/m_s, f_π/f_K), form factors ($f_+^{K\pi}(0)$) ...

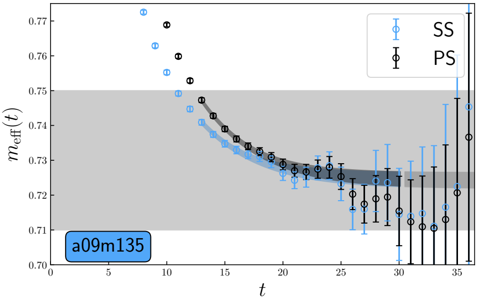
However, a_μ^{HVP} to better than 1% requires scale to a few per mille accuracy (through am_μ) [Aoyama et al.,2006.04822].

Setting the scale

Choice for expt. input should be well defined and precisely determined in expt. and also on the lattice.

For $m_{u,d,s}$: M_π and M_K, sensitive to quark mass: $M_{PS}^2 \sim B_0(m_{q1} + m_{q2})$.

Not so many choices for setting the scale $a = (X^{phys}/X^{latt})^{-n}$ (at the physical point): need a stable particle (under strong decay).



Baryon masses, m_N, m_Ξ, m_Ω

Signal to noise problem

e.g. [CalLat,2011.12166], m_Ω

Gray band, prior for the ground state.

Setting the scale

Leptonic decay constants: f_π, and f_K.

Experimental decay rates give $|V_{ud}|f_\pi$ and $|V_{us}|f_K$ plus QED.

Heavy-light or quarkonium spectrum below decay thresholds: problem of disretisation effects.

If the simulations reproduce nature then it does not matter which quantity is used to set the scale.

For some quantities lattice precision is such that deviations from Nature may be seen for ($m_u = m_d$, electrically neutral) iso-QCD simulations.

$m_u \neq m_d$ and QED effects are $O(1\%)$.

Some QCD+QED simulations with are being performed. Different approaches, QED_L [Hayakawa and Uno,0804.2044], QED_M [Endres et al.,1507.08916], QED_{C^*} [Lucini et al.,1509.01636],. . .

Dynamical $N_f = 1+1+1+1$ QCD+QED_L [BMWc,1406.4088]. Most simulations include QED in the electro-quenched approximation. Alternatively, compute isospin breaking effects perturbatively, e.g. [RM123,1303.4896].

Subracting isospin-breaking effects is ambiguous. Choose a prescription.

Setting the scale

Intermediate scales: cannot be directly related to expt..

Aim: dependence on m_q is mild and very precisely computed. Defined in terms of the gauge fields.

Sommer scale r_0 defined through the static quark force [Sommer,hep-lat/9310022]

$$r_0^2 F(r) = c = 1.65$$

A related scale: r_1 with $c = 1.0$ [Bernard et al.,hep-lat/0002028].

Also in use: **gradient flow scales t_0 and w_0.**

Consider combinations with other observables,

e.g. $(\sqrt{t_0} M_{\Xi})^{latt} = (\sqrt{t_0} M_{\Xi})^{phys}$.

Gradient flow scale t_0

Gradient flow equation [Lüscher,1006.4518]:

$$a^2 \frac{d}{dt} V_t(x,\mu) = -g_0^2 \cdot \partial_{x,\mu} S_G(V_t) \cdot V_t(x,\mu) \qquad V_t(x,\mu)|_{t=0} = U(x,\mu)$$

where S_G is a lattice gauge action.

Flow time t has dimensions a^2.

Useful properties: local gauge-invariant combinations of the smoothed fields do not require renormalization at $t > 0$ and their correlation functions have no short-distance singularities.

Define the gradient flow time t_0 in terms of the average action density $E(t)$

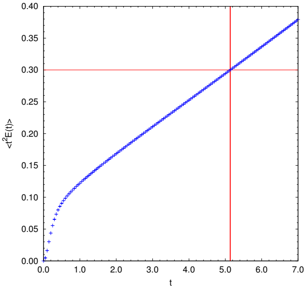
$$t^2 E(t)\Big|_{t=t_0} = c = 0.3, \quad E(t) = \frac{1}{V_4}\int_{V_4} d^4x \, \frac{1}{4} G^a_{\mu\nu}(x,t) G^a_{\mu\nu}(x,t)$$

where $G^a_{\mu\nu}(x,t)$ is the field strength tensor evaluated using V_t.

Gradient flow scale t_0

Value of $c = 0.3$ chosen so that errors are small and there is a linear dependence on t.

Discretisation effects: S_G, $E(t)$, ensemble action,. . .

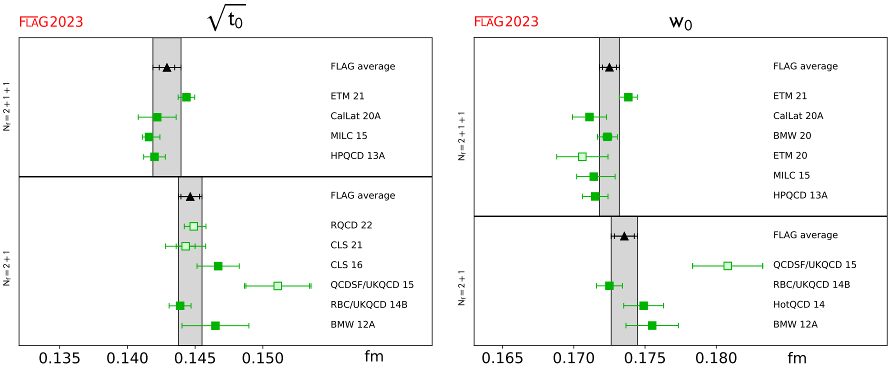


Different choices: e.g. Zeuthen flow, [Ramos and Sint,1508.05552], Symanzik flow [Fodor et al.,1203.4469], [MILC,1311.1474]. Also [Cheng et al.,1404.0984],

This work: Wilson flow and clover leaf definition of $E(t)$.

Alternative scale w_0, where $w_0^2 \partial_t (t^2 \langle E(t) \rangle)_{t=w_0^2} = 0.3$ [BMWc,1203.4469]

t_0 and w_0

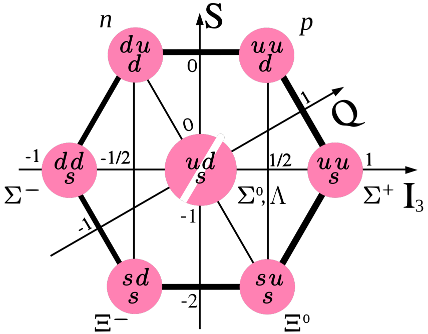


FLAG average $N_f = 2+1$: $\sqrt{t_0} = 0.14464(87)$ fm, $w_0 = 0.17355(92)$ fm.

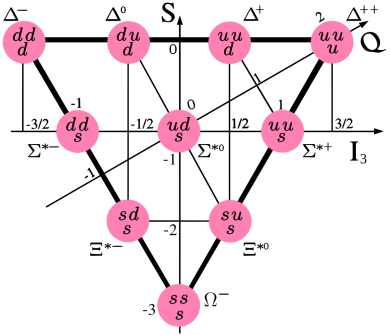
ETM 21 (f_π), CalLat 20 (m_Ω), MILC 15 (f_π, f_K), HPQCD 13 (f_π), CLS 16, CLS 21 (f_π, f_K), QCDSF/UKQCD 15 (m_O, m_V), RBC/UKQCD 14 (m_Ω), BMWc 12 (m_Ω).

Baryon spectrum

Octet: $J^P = \frac{1}{2}^+$



decuplet $J^P = \frac{3}{2}^+$



Simulating QCD (isospin-symmetric, electrically neutral): N, Σ, Λ, Ξ and Δ, Σ^*, Ξ^*, Ω.

Unstable under strong decay: $\Delta \to N\pi$, $\Sigma^* \to \Lambda\pi, \Sigma\pi$ and $\Xi^* \to \Xi\pi$.

Correcting expt. masses for electrical and quark mass isospin breaking effects

QCD values for M_π and M_K from [FLAG 16,1607.00299].

mass	value/MeV	Expt./MeV	mass	value/MeV	Expt./MeV
M_π	**134.8(3)**	134.98 (π^0)			
M_K	**494.2(3)**	497.61 (K^0)			
m_N	937.53(6)	938.27 (p)	m_Δ	1231(60)	1231(56) (Δ^0)
m_Λ	1115.68(1)	1115.68	m_{Σ^*}	1383(20)	1383(18) (Σ^{*+})
m_Σ	1190.67(12)	1192.64 (Σ^0)	m_{Ξ^*}	1532(5)	1532(5) (Ξ^{*0})
m_Ξ	**1316.9(3)**	1314.86 (Ξ^0)	m_Ω	1669.5(3.0)	1672.45(29)

$\overline{M}_\pi = \hat{M}_{\pi^+}$, $\overline{M}_K = \sqrt{\frac{1}{2}(\hat{M}^2_{K^+} + \hat{M}^2_{K^0})}$.
cf. [FLAG 21,2111.09849], $\overline{M}_\pi = M_{\pi^0}$, $\overline{M}_K = M_{K^0}$.

For the unstable decuplet baryons, the Breit-Wigner mass is given with an error of $\Gamma/2$ (with rounding).

Correcting expt. masses for electrical and quark mass isospin breaking effects

Assume $\Delta m_B = \Delta Q^2 \delta m^{\mathrm{QED}} - \Delta I_3 \delta m^{\mathrm{QCD}}$

Consistent with convention that neutral particles do not receive QED corrections.

Also with $\Delta m_N - \Delta m_\Sigma - \Delta m_\Xi = 0$, Coleman-Glashow theorem (verified to within 0.13 MeV by [BMWc,1406.4088]), while expt gives 0.06(23) MeV.

Wrong, e.g. single δm^{QCD} means (CVC relation) all $g_{S,B}$ are equal.

$$\Delta m_N = m_p - m_n \approx -\delta m^{\mathrm{QCD}} + \delta m^{\mathrm{QED}}, \qquad \Delta m_\Sigma = m_{\Sigma^+} - m_{\Sigma^-} \approx -2\delta m^{\mathrm{QCD}},$$
$$\Delta m_\Xi = m_{\Xi^0} - m_{\Xi^-} \approx -\delta m^{\mathrm{QCD}} - \delta m^{\mathrm{QED}}, \qquad m_\Xi = \frac{1}{2}\left(m_{\Xi^0} + m_{\Xi^-} - \delta m^{\mathrm{QED}}\right)$$

$\delta m^{\mathrm{QCD}} \approx 4$ MeV and $\delta m^{\mathrm{QED}} \approx 2.7$ MeV. Assume same δm^{QED} for m_Ω.

Note: 1 MeV is a 0.75 ‰ effect on m_Ξ.

CLS ensembles

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen.

$N_f = 2 + 1$ flavours of non-perturbatively $O(a)$ improved Wilson fermions on tree level Symanzik improved glue. Leading $O(a^2)$ errors in hadron masses.

$\star$ **High statistics**: typically 6000-8000 MDUs, 1000-2000 configurations.

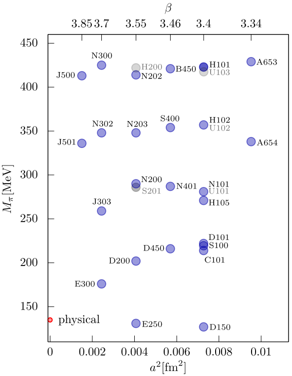
Aim to control all main sources of systematics (a, m_q and V).

$\star$ **Discretisation**: Six lattice spacings: $a = 0.1 - 0.04$ fm.

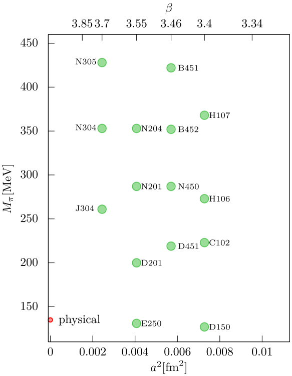
$\star$ **Finite volume**: $Lm_\pi \gtrsim 4$ with additional smaller volumes.

$\star$ **Quark mass**: $m_\pi = 410$ MeV down to m_π^{phys}.

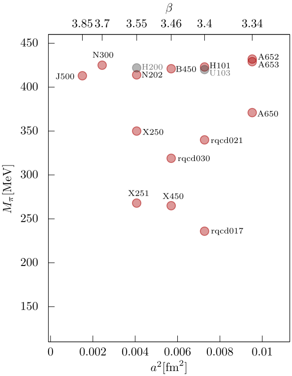
CLS ensembles: m_π vs a^2



$2m_\ell + m_s = \text{const.}$



$m_s = \text{const.}$

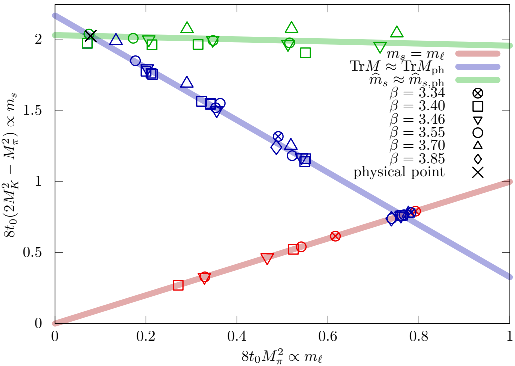


$m_\ell = m_s$

$a < 0.06$ fm open-boundary conditions in time.

$a > 0.06$ fm mixture of ensembles with periodic and open-boundary conditions.

CLS ensembles: m_ℓ-m_s plane



Three trajectories: good control over the quark mass dependence. Can correct for mis-tuning of the trajectories. Observables sensitive to m_s are tightly constrained.

$2m_\ell + m_s = \mathrm{const.}$: investigate SU(3) flavour breaking (flavour average quantities roughly constant), approach to physical point involves $m_\pi \downarrow$ and $m_K \uparrow$.

$m_\ell = m_s$: important for determination of SU(3) ChPT low energy constants (and renormalisation factors).

$t_{0,ph}$, t_0^* and $t_{0,ch}$

Determine t_0 at three points in the quark mass plane.

- Physical point: $t_{0,ph}$ using $(\sqrt{t_0} m_\Xi)^{latt} = \sqrt{t_{0,ph}} m_\Xi^{ph}$.
- Chiral limit: $t_{0,ch}$ at $m_\ell = m_s = 0$, needed for the extraction of LECs.
- Reference point: t_0^* [Bruno et al.,1608.08900] Defined at the point along the symmetric line $m_s = m_\ell$ where

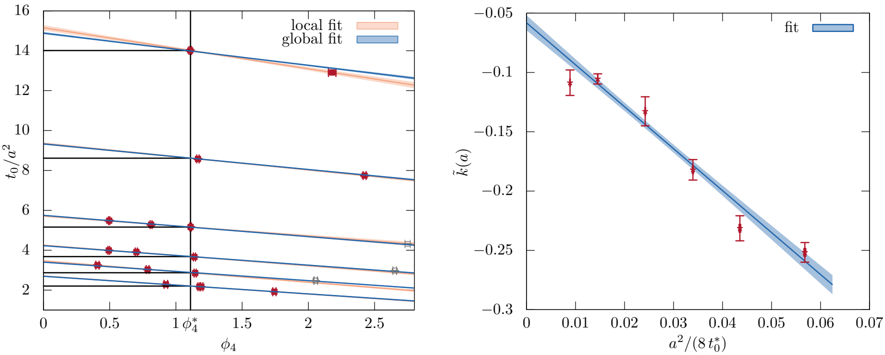
$$\phi_4^* = 8t_0^* \left(M_K^2 + \frac{M_\pi^2}{2} \right) = 12 t_0^* M_\pi^2 = 1.110.$$

Only requires an interpolation. Useful for representing relative change in the lattice spacing.

t_0 depends (mildly) on the quark mass. Known to NNLO SU(3) ChPT [Bär and Golterman,1312.4999].

Only able to resolve NLO behaviour.

$$t_0(\overline{M}, \delta M) = t_{0,\mathrm{ch}} \left(1 + k_1 \frac{3\overline{M}^2}{(4\pi F_0)^2}\right) \approx t_{0,\mathrm{ch}} \left(1 + \tilde{k}_1 8 t_0 \overline{M}^2\right),$$



From a global fit to t_0: in the continuum,

$$\tilde{k}_1 = -0.0466(62), \qquad t_0^* = 0.9655(46)\, t_{0,\mathrm{ch}}, \qquad t_0^* = 0.99947(7)\, t_{0,\mathrm{ph}}.$$

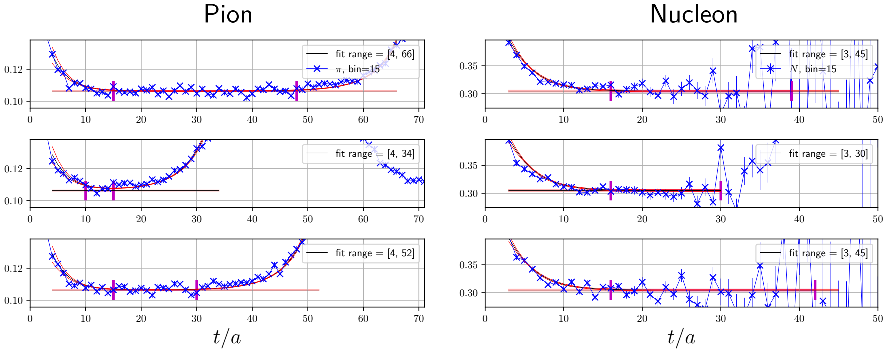
Extracting meson and baryon masses

Multiple measurements of the two-point functions on every configuration.

Smeared sources optimized to reduce excited state contamination.

For ensembles with open boundary conditions in time, hadron masses can only be extracted in the bulk.

N300 $m_s = m_\ell$, $a = 0.05$ fm, $M_\pi = 425$ MeV



Autocorrelations taken into account: errors estimated via a binsize analysis.

Extrapolation of baryon multiplets

Determine $t_{0,ph}$ using m_Ξ

Perform a continuum, quark mass and finite volume extrapolation of the baryon octet (and decuplet) masses.

$$\overline{M}^2 = \frac{1}{3}(2M_K^2 + M_\pi^2) \propto \overline{m} = \frac{1}{3}(2m_\ell + m_s), \quad \delta M^2 = 2(M_K^2 - M_\pi^2) \propto \delta m = m_s - m_\ell$$

Rescale all masses by $\sqrt{8t_0}$: $B \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$

$$\mathbb{m}_B = \sqrt{8t_0}\, m_B, \quad \overline{\mathbb{M}} = \sqrt{8t_0}\, \overline{M}, \quad \delta\mathbb{M} = \sqrt{8t_0}\, \delta M, \quad \mathbb{a} = \frac{a}{\sqrt{8t_0^*}},$$

Extrapolation of baryon multiplets

Extrapolation performed using the fit form

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, L, \mathrm{a}) = \left[\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0) + \delta m_B^{FV}(\mathbb{M}_\pi, \mathbb{M}_K, L)\right] \\ \times \left[1 + \mathrm{a}^2 \left(c + \bar{c}\,\overline{\mathbb{M}}^2 + \delta c_B\, \delta\mathbb{M}^2\right)\right].$$

Simultaneous fit to baryon multiplets with **all correlations taken into account.**

Discretisation coefficients: 6 parameters for the octet and decuplets baryons separately.

[Husung et al.,1912.08498] $a^2 \sum_i [\bar{g}(1/a)]^{\hat{\Gamma}_i} c_i$.
[Husung et al.,2111.02347]: dominant contribution expected to be $g^{1.52} a^2$,
This work: g^2 varies by a factor of 1.15, while a changes by a factor of 2.5.

Natural choice for $\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$ is to use SU(3) baryon ChPT
(and in a finite volume for δm_B^{FV}).

NNLO BChPT

BChPT: $O(p^3)$ baryon ChPT with EOMS regularisation [Ellis et al.,nucl-th/9904017]

$$m_O(M_\pi, M_K, \infty, 0) = m_0 + \bar{b}\,\overline{M}^2 + \delta b_O\,\delta M^2 + \frac{m_0^3}{(4\pi F_0)^2}\left[g_{O,\pi} f_O\left(\frac{M_\pi}{m_0}\right) + g_{O,K} f_O\left(\frac{M_K}{m_0}\right) + g_{O,\eta_8} f_O\left(\frac{M_{\eta_8}}{m_0}\right)\right],$$

where

$$f_O(x) = -2x^3\left[\sqrt{1-\frac{x^2}{4}}\,\arccos\left(\frac{x}{2}\right) + \frac{x}{2}\ln(x)\right].$$

and $M_{\eta_8}^2 = (4M_K^2 - M_\pi^2)/3$.

Depends on 6 low energy constants (LECs): m_0,
$\bar{b}$ and δb_O (depending only on b_0**,** b_F **and** b_D due to SU(3) constraints),
g_{O,π,K,η_8} (depending only on F **and** D, also appearing in ChPT expressions for g_A^O), appear in combination with F_0.

NNLO BChPT

Heavy limit (HBChPT) [Jenkins and Manohar,Phys. Lett. B 255 (1991) 558.]:
$f(x) = -x^3 + \mathcal{O}(x^4)$.

Analogous expression for the decuplet baryons ([Martin Camalich et al.,1003.1929]) involves 4 LECs.

BChPT in a finite volume (FV) gives the finite volume dependence of m_B (see e.g. [Ren et al.,1209.3641]). **No additional LECs!**

Joint baryon octet and decuplet fits: small scale expansion

If M_π not much smaller than the gap $m_D - m_O$ then at NNLO in BChPT need to consider both baryon loops in addition to meson loops.

Small scale expansion (SSE), see e.g. [Martin Camalich et al.,1003.1929]

$$m_B \mapsto m_B + \mathcal{C}^2 \frac{\delta^3}{(4\pi F_0)^2} \left[\xi_{B,\pi} h_B\left(\frac{M_\pi}{\delta}\right) + \xi_{B,K} h_B\left(\frac{M_K}{\delta}\right) + \xi_{B,\eta_8} h_B\left(\frac{M_{\eta_8}}{\delta}\right) \right],$$

for $B \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$ and

$$h_O(x) = -\left(2 - 3x^2\right)\ln\left(\frac{x}{2}\right) - \frac{1}{2}x^2 - 2w(x),$$

$$h_D(x) = \left(2 - 3x^2\right)\ln\left(\frac{x}{2}\right) + \frac{1}{2}x^2 - 2w(-x),$$

$$w(x) = \begin{cases} \left(x^2 - 1\right)^{3/2} \arccos\left(x^{-1}\right) & , \quad |x| \geq 1 \\ \left(1 - x^2\right)^{3/2} \ln\left(\left|x^{-1} + \sqrt{x^{-2} - 1}\right|\right) & , \quad |x| < 1 \end{cases} .$$

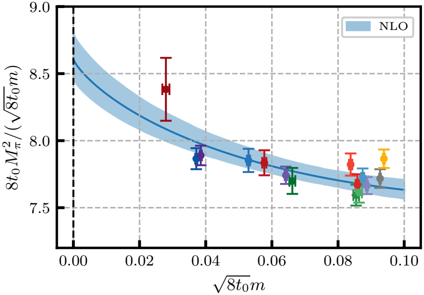
ξ_{B,π,K,η_8} are known group theoretical factors.

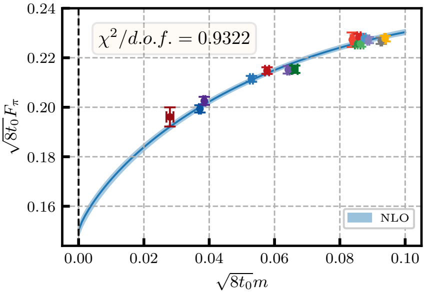
Two new LECs: $\mathcal{C}$ and $\delta = m_{D0} - m_0$.

Future: simultaneous fit to several observables

[RQCD,2201.05591]: CLS $N_f = 3$ ($m_s = m_\ell$) ensembles, $a = 0.04 - 0.1$ fm, $M_\pi = 430 - 240$ MeV.

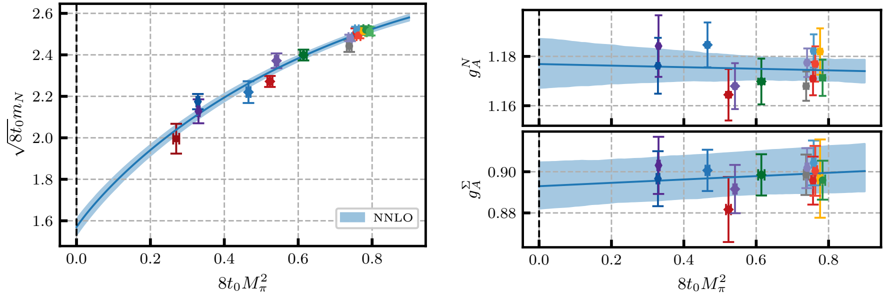
Fit M_π^2 and F_π as a function of the renormalised quark mass $m = m_\ell$ to extract the ChPT low energy constants B_0 and F_0.





Future: simultaneous fit to several observables

Fit m_N and $g_A^{N,\Sigma}$ as a function of M_π^2 to extract the ChPT low energy constants m_0, F and D.



Taylor expansion a la Gell-Mann-Okubo (GMO)

Not clear how well SU(3) NNLO BChPT or HBChPT expressions describe the baryon masses for our range of M_π and M_K (also at the physical point!).

Also consider a **Taylor expansion** about **the symmetric point** ($m_s = m_\ell$) [QCDSF,1102.5300].

$$m_O(M_\pi, M_K, \infty, 0) = m_0 + \overline{b}, \overline{M}^2 + \delta b_O\, \delta M^2 + \overline{d}\, \overline{M}^4 + \delta d_O\, \delta M^2 \overline{M}^2 + \delta e_O\, \delta M^4$$

Equivalent to form obtained expanding about $\overline{M}^{*,2}$ at the symmetric point, with different coefficients.

At NLO the expression (and LECs) is the same as in BChPT (1+3 parameters).

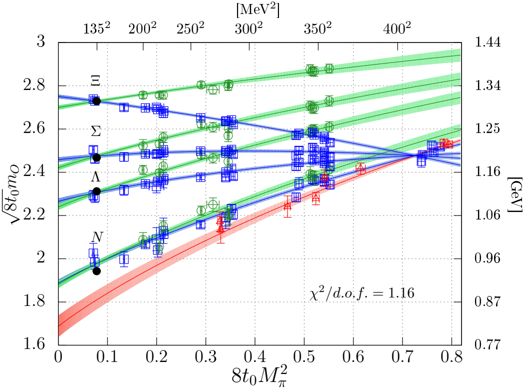
SU(3) constraints on δd_O mean NNLO terms introduce $1(\overline{d})+2(\delta d_O)+4(\delta e_O)$ parameters.

Less constrained than BChPT fit (11 compared to 6 parameters).

Expression for the decuplet baryons involves 8 coefficients.

NNLO BChPT fit to the baryon octet: m_q dependence

FV terms included in the fit. 12 parameters to fit the 4 octet baryon masses.

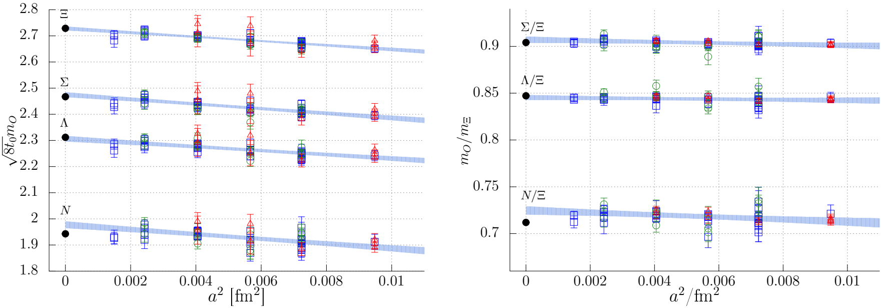


The data points are shifted to correct for finite a, finite V and mis-tuning of the trajectory.

$t_{0,ph}$ determined via an iterative procedure with corrected expt. values for $M_{\pi,ph}$, $M_{K,ph}$ and $m_{\Xi,ph}$. $\sqrt{8t_{0,ph}} \sim 0.409$ fm.

NNLO BChPT fit to the baryon octet: discretisation effects

The data points are shifted to the physical point ($M_{\pi,ph}$, $M_{K,ph}$).

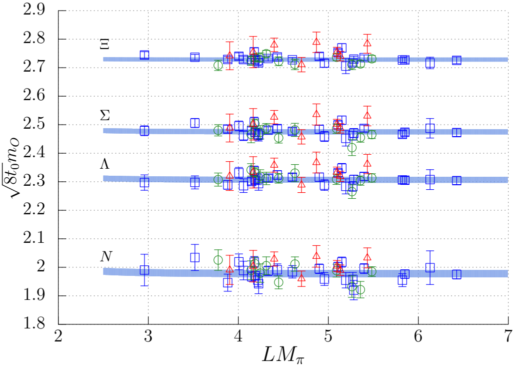


Discretisation effects are mild: $1 + \mathbb{a}^2\left(c + \bar{c}\,\overline{\mathbb{M}}^2 + \delta c_O\,\delta\mathbb{M}^2\right)$. Around 3% from $a = 0.1$ fm to $a = 0$.

$\mathbb{a}^2\delta c_O\,\delta\mathbb{M}^2$ terms are small. Higher order terms not significant.

NNLO BChPT fit to the baryon octet: finite volume effects

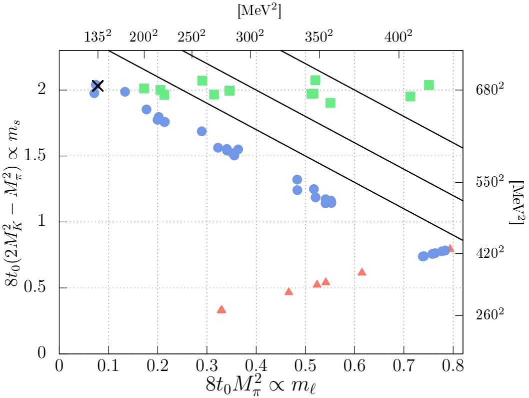
The data points are shifted to the physical point in the continuum limit.



Finite volume effects are small, however, including FV terms in the fit (which involves no extra coefficients) improves the fit quality.

Assess the systematics of the fits: cuts on the data

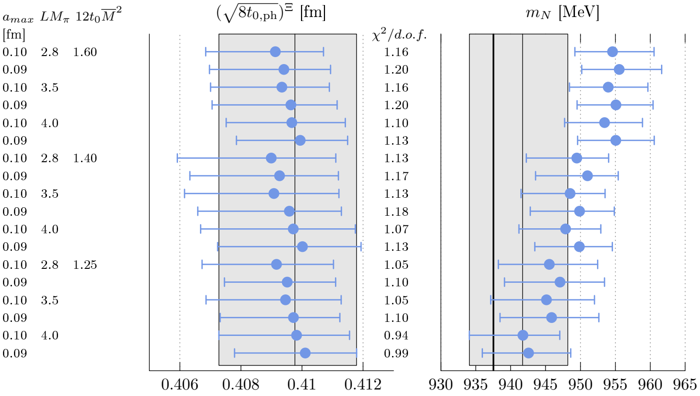
m_q dependence: (SU(3) BChPT) include $\overline{M}^2 < (498)^2$ [MeV^2] and impose further cuts $\overline{M}^2 < (466)^2$ [MeV^2], $\overline{M}^2 < (440)^2$ [MeV^2].



Finite V: start with $L > 2.3$ fm and impose cuts $LM_\pi > 3.5$ and $LM_\pi > 4.0$.

Finite a: only consider cut $a < 0.09$ fm.

Variation with cuts on the data



$\chi^2/d.o.f$ improves with cuts on $\overline{M}^2$.

Values of $\sqrt{8t_{0,ph}}$ obtained are consistent. Agreement of m_N with corrected expt. value improves with cuts on $\overline{M}^2$.

Grey bands indicate the weighted average of the results.

Weighted average of the results

Use a modified Akaike Information Criterion (AIC) [Akaike,(1998)] (following [BMWc,2002.12347])

Perform a weighted average of fit the results by weighting each fit j with

$$O_{weighted} = \sum_{j}^{N_M} w_j O_j$$

$$w_j = A \exp\left[-\frac{1}{2}\left(\chi_j^2 - N_{\mathrm{DF},j} + p_j\right)\right], \qquad \sum_{i}^{N_M} w_i = 1$$

Standard AIC: fixed data, exponent $\chi_j^2 + 2p_j$, favours fits with low χ^2 and a small number of parameters.

Modification (subtract $n_{data,j} = N_{\mathrm{DF},j} + p_j$): fix the parametrization and vary the data, favours larger $N_{\mathrm{DF},j}$.

Alternatives: [Jay and Neil,2008.01069], equivalent to subtracting $2N_{\mathrm{DF},j}$.

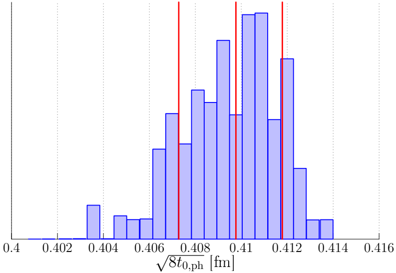
Weighted average of the results

Final result: $\sqrt{8t_{0,ph}} = 0.4098^{(20)}_{(25)}$ fm

Fits to 18 differents cuts on the data.

18×100 bootstraps weighted with w_j.

(0.5% total uncertainty)

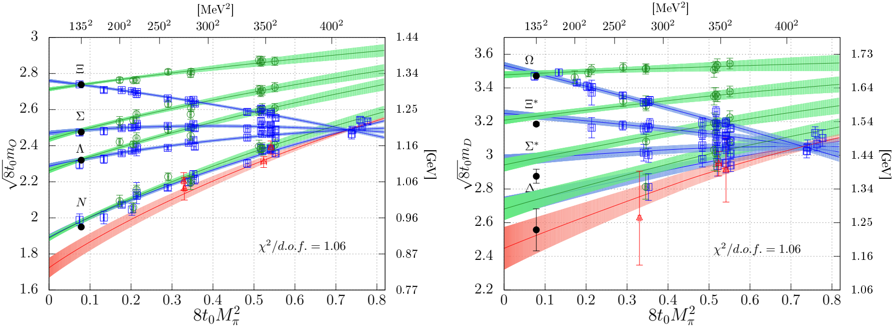


Dominated by fits with $\chi^2/d.o.f.$ of 0.94 ($N_{\mathrm{DF}} = 80$) and 0.99 ($N_{\mathrm{DF}} = 73$).

Variation with the continuum fit form

Fit the octet and decuplet masses simultaneously using NNLO BChPT + SSE + FV terms (23 parameters).

Δ, Σ^*, Ξ^* baryons are unstable: cut out data where $D \to O\pi$ in the infinite volume limit.



$\sqrt{8t_{0,ph}}$ set using m_Ξ. Best fit shown (fit form gives reasonable description of data). m_Ω consistent with corrected expt. value.

Unstable baryons: expt. masses not reproduced.

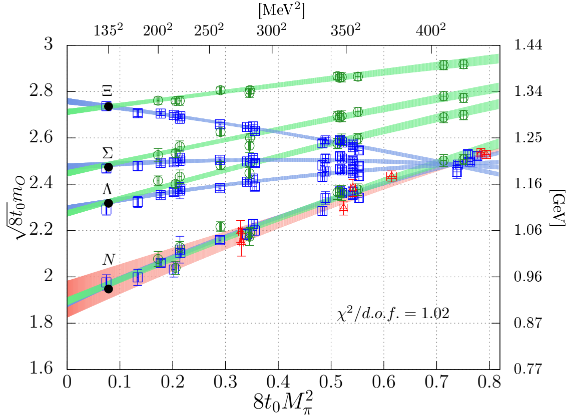
Variation with the continuum fit form

Fit the octet masses using GMO fit form (17 parameters).

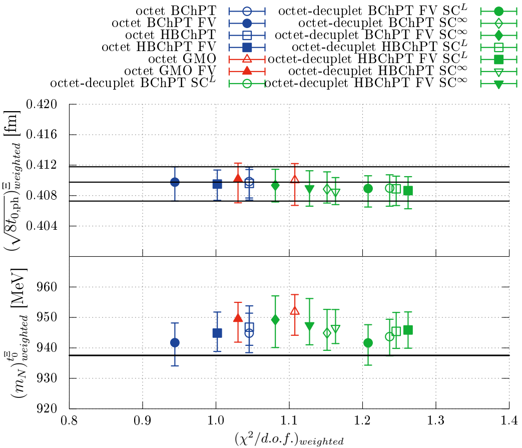
Fit is less constrained compared to BChPT (11 parameters compared to 6). Best fit shown (fit form gives reasonable description of data).

Convergence determined by $\delta M^2/\overline{M}^2$.
Want to include data from ensembles at or close to the physical point
$\rightarrow$ no m_q cut imposed.

$\sqrt{8t_{0,ph}}$ set using m_Ξ.



Variation with the continuum fit form



Results for $\sqrt{8t_{0,ph}}$ are very stable (m_Ξ tightly constrained).

Including FV terms in the fit improves the fit quality.

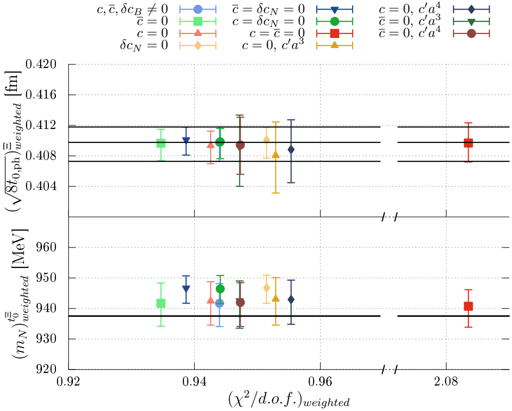
Poorer fit quality: HBChPT and GMO fit forms compared to EOMS BChPT, also when including the decuplet masses.

Variation with the fit form: discretisation effects

Discretisation effects are mild, however, only varying a-effects by cutting out the coarsest lattice spacing (only 4 ensembles).

Investigate importance of $O(a^2)$ terms: $1 + \mathbb{a}^2 \left(c + \bar{c}\, \overline{\mathbb{M}}^2 + \delta c_O \, \delta \mathbb{M}^2 \right)$ and higher order terms using NNLO BChPT continuum fit form + FV terms:

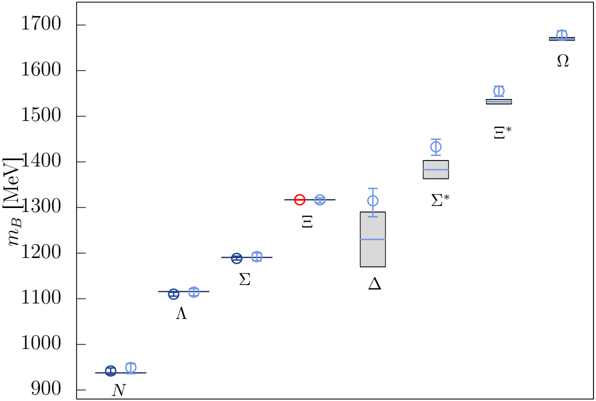
Results are very stable, even though the fit quality varies. m_N sensitive to δc_N.



Low lying baryon spectrum

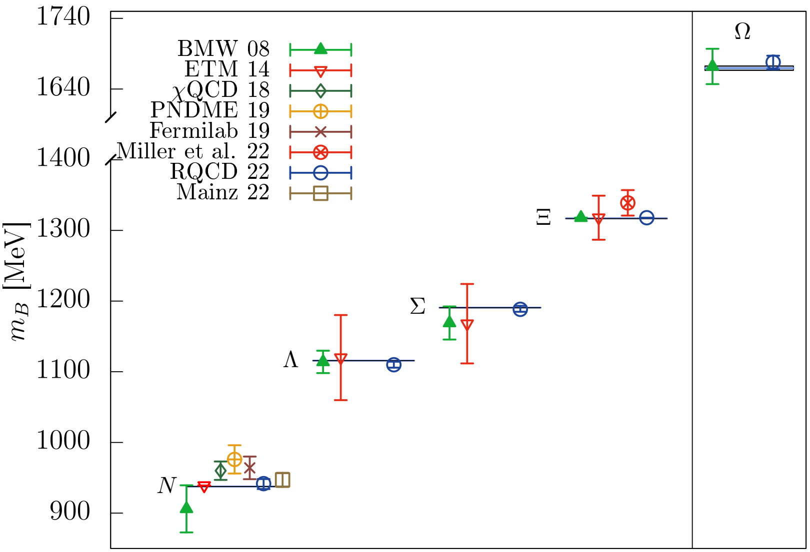
Octet baryon spectrum from BChPT fits + FV terms (determine $\sqrt{8t_{0,ph}}$)
Agreement with corrected expt. masses within 1% overall uncertainty.

Octet and decuplet masses from BChPT + SSE + FV terms.



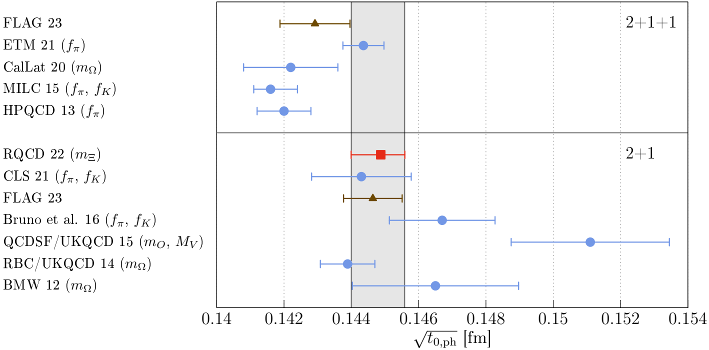
Proper treatment of the unstable decuplet baryons via the Lüscher formalism required.

Low lying baryon spectrum



Comparison with other determinations of $\sqrt{t_{0,ph}}$

Final result: $\sqrt{t_{0,ph}} = 0.1449^{(7)}_{(9)}$ fm



CLS ensembles: [Bruno et al.,1608.08900] and [CLS 21,2112.06696] (proceedings).

Also: Saez-Gonzalvo et al. (Lattice 2023, preliminary) 15 ensembles, twisted mass and improved Wilson valence quarks:

$$\sqrt{t_{0,ph}} = 0.1446(5)(3) \text{ fm from } f_\pi \text{ and } f_K.$$

Baryon sigma terms: $\sigma_{\pi B}$ and σ_{sB}

The sigma terms, can be obtained via the Feynman-Hellmann theorem.

$$\sigma_{q,B} = m_q \left[\frac{\langle B \,| \bar{q} \mathbb{1} q |\, B \rangle}{\langle B|B \rangle} - \langle \Omega \,| \bar{q} \mathbb{1} q |\, \Omega \rangle \right] = m_q \frac{\partial m_B}{\partial m_q},$$

where $|\Omega\rangle$ denotes the vacuum. Consider $\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$ and σ_{sB}.

Using the Gell-Mann-Oakes-Renner (GMOR) relation: $M_{PS}^2 \approx B_0(m_{q1} + m_{q2})$

$$\sigma_{\pi B} \approx \tilde{\sigma}_{\pi B} = M_\pi^2 \frac{\partial m_B}{\partial M_\pi^2} \qquad \sigma_{sB} \approx \tilde{\sigma}_{sB} = M_{s\bar{s}}^2 \frac{\partial m_B}{\partial M_{s\bar{s}}^2}$$

where $M_{s\bar{s}}^2 = 2M_K^2 - M_\pi^2$.

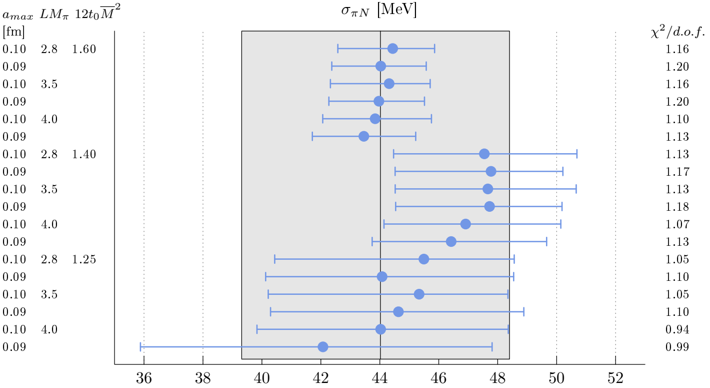
Nucleon sigma terms are relevant for computing WIMP-nucleon spin-independent cross-sections for direct dark matter detection experiments. $q \in \{c, b, t\}$ are also interesting.

Difficult to determine $\sigma_{s,N}$ via indirect (FH) approach (in particular if m_s is kept roughly constant).

Direct determinations of σ_{qB} $(q \in \{u, d, s, c\})$ have been performed.

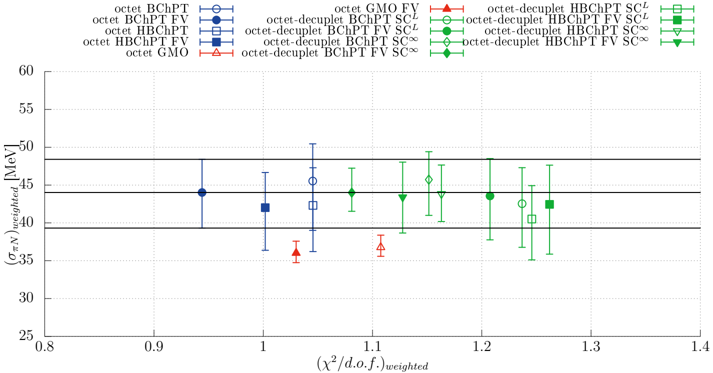
Variation of $\tilde{\sigma}_{\pi N}$ with cuts on the data

Using NNLO BChPT for $\mathrm{m}_O(\mathbb{M}_\pi, \mathbb{M}_K, \infty, 0)$.



Weighted average of the fits: $\tilde{\sigma}_{\pi N} = 44.0^{(4.4)}_{(4.7)}$ MeV.

Variation of $\tilde{\sigma}_{\pi N}$ with continuum fit form for m_O



Additional considerations

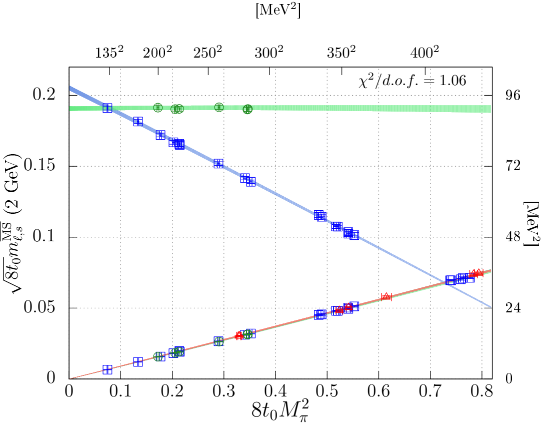
Fit to $\sqrt{8t_0}m_B$, however, t_0 depends on $\overline{M}^2$: $\tilde{\sigma}_{\pi B}$ and $\tilde{\sigma}_{sB}$ shown are corrected for this.

Want $\sigma_{\pi B}$ not $\tilde{\sigma}_{\pi B}$ etc. $\longrightarrow$ Need $\partial M_\pi/\partial m_\ell$, $\partial M_K/\partial m_\ell$, $\partial M_K/\partial m_s$.

Consistent to use GMOR relation with NNLO BChPT.

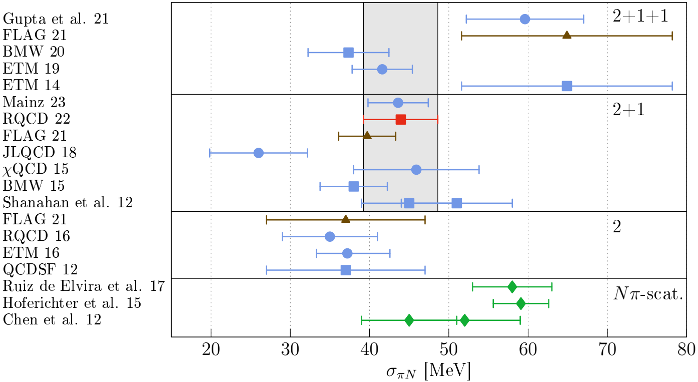
Note: all m_ℓ trajectories go to zero as $M_\pi^2 \to 0$.

To $O(p^4)$ in SU(3) ChPT [Gasser and Leutwyler,Nucl. Phys. B250 (1985)]. PCAC quark masses renormalised to $\overline{MS}$ at 2 GeV using Z_A/Z_P from [RQCD,2012.06284] determined using RI-SMOM (used $O(a)$ improvement coefficients from [ALPHA,1906.03445]).



Comparison with other determinations of $\sigma_{\pi N}$

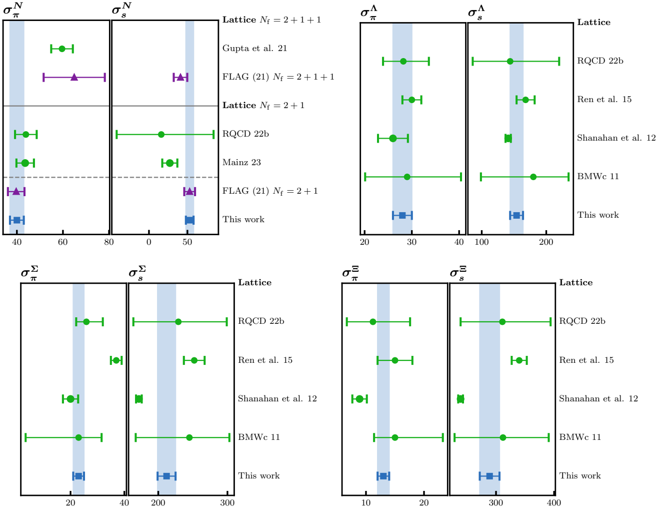
$\tilde{\sigma}_{\pi N} = 44.4^{(4.4)}_{(4.7)}$ MeV $\rightarrow \sigma_{\pi N} = 43.9^{(4.7)}_{(4.7)}$ MeV,



Some tension with the phenomenological results of [Hoferichter et al.,1506.04142] obtained using $N\pi$ scattering data.
[Hoferichter et al.,2305.07045] ($m_{\pi^\pm}$ vs m_{π^0}) $\sigma_{\pi N} = 55.9 \pm 3.5$ MeV.

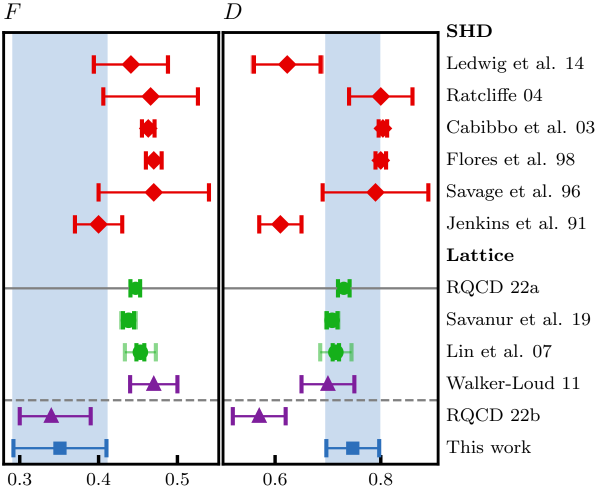
Direct determination by [Gupta et al.,2105.12095] is also larger.

Sigma terms of the octet baryons



"This work": RQCD, Pia Petrak and J. Heitger, direct determination of the sigma terms on a sub-set of the CLS ensembles ($M_\pi \gtrsim 200$ MeV). **Not all systematics are estimated.**

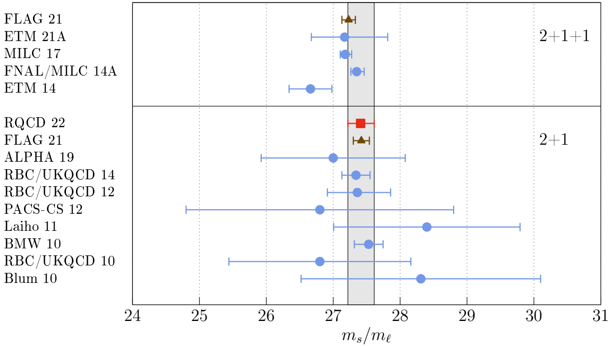
Low energy constants F and D



SHD: phenomenological analyses of semi-leptonic hyperon decays.
Lattice: Axial charges at the physical point (Savanur et al., Lin et al.), baryon masses (Walker-Loud)

Light and strange quark masses

$N_f = 3$, $m_\ell^{\overline{\rm MS}}(2\,{\rm GeV}) = 3.37(^{+4}_{-5})$ MeV, $m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 92.2(^{+8}_{-10})$ MeV



[RQCD,2305.04717]: CVC relation $\delta_m = m_u - m_d = \frac{g_V^\Sigma}{g_S^\Sigma}\Delta m_\Sigma^{QCD}$.

Estimate $\Delta m_\Sigma^{QCD} \approx (m_{\Sigma^+} - m_{\Sigma^-})/2 = -4.04(4)$ MeV (PDG) and using $g_S^\Sigma = 3.98^{(22)}_{(24)}$ gives

$$\delta_m = m_u - m_d = -2.03(12)\,\text{MeV}.$$

Leads to, with $m_\ell = (m_u + m_d)/2$, $\quad m_u/m_d = 0.54(2)$,

compared to FLAG ($N_f = 2 + 1$) $\quad m_u/m_d = 0.485(19)$ [BMWc,1604.07112].

Summary and outlook

Precision determinations of the lattice scale are important as lattice predictions for key quantities relevant for expt. and phenomenology become more precise.

Determination of Wilson flow scale at the physical point with 0.5% uncertainty via m_{Ξ}:

$$\sqrt{8t_{0,ph}} = 0.4098^{(20)}_{(25)}\ [\mathrm{fm}]$$

- ⋆ Performed continuum, finite volume, quark mass simultaneous fits to the baryon octet with all correlations taken into account. Systematics on the fits were evaluated via cuts to the data and utilising the modified AIC criterion.
- ⋆ Utilising data along three trajectories in the quark mass plane means mistuning of the trajectories can easily be corrected. Provided tight constraints on m_{Ξ} at the physical point.
- ⋆ Discretisation and finite volume effects are mild for our ensembles.
- ⋆ Covariant BChPT provided a reasonable description of the data for our range of M_π and M_K ($\overline{M}^2 < 440^2$ [MeV^2]). (Large N_{DF} for the fits).
- ⋆ Corrected expt. octet baryon spectrum reproduced to within 1% uncertainties.
- ⋆ As a by product extract the sigma terms $\sigma_{\pi B}$ and σ_{sB}.
- ⋆ Also extracted the LECs which are not well known for SU(3) BChPT.

- ⋆ HBChPT and GMO expansions gave consistent results for $t_{0,ph}$ and the baryon spectrum. Including SSE terms enabled a reasonable fit of both the octet and decuplet baryons.

In the future:

Improve the $t_{0,ph}$ determination:

- ⋆ Already have additional measurements on many ensembles, which are not included in the current analysis.
- ⋆ Perform a simultaneous fit of m_B and direct determinations of $\sigma_{\pi B}$ and σ_{sB}.

In addition:

- ⋆ Test ChPT further through determination of LECs via simultaneous fits to related observables, e.g. $M_{\pi,K}$, $m_{\ell,s}$ and $F_{\pi,K}$ and m_B and g_A^B.
- ⋆ ...

Back-up slides

Global interpolation of t_0/a^2

$$\frac{t_0}{a^2} = \frac{t_0^*}{a^2}(g^2)\left[1 + \tilde{k}_1 A - 2b_a\left(a\overline{m} - am^*\right)\right] + \bar{c}A + \delta c\, 8t_0\delta M^2$$
$$= \left[f(g^2) + c_{t_0} + d_{t_0} f^{-1/2}(g^2)\right]\left(1 + \tilde{k}_1 A - 2b_a a\overline{m}\right) + \bar{c}A + \delta c\, 8t_0\delta M^2,$$

where

$$A = 8t_0\left(\overline{M}^2 - M^{*2}\right).$$

and

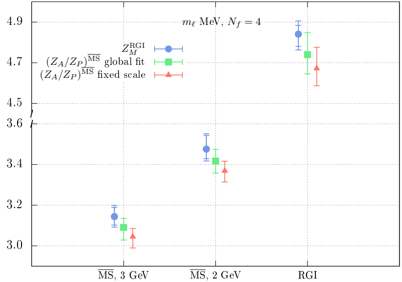
$$\frac{t_0^*}{a^2(g^2)} = \frac{t_0^*}{a_0^2(g^2)}\left[1 - 2b_a(g^2)am^*\right] = \left[f(g^2) + c_{t_0} + d_{t_0} f^{-1/2}(g^2)\right] \times \left[1 - 2b_a(g^2)am^*\right],$$

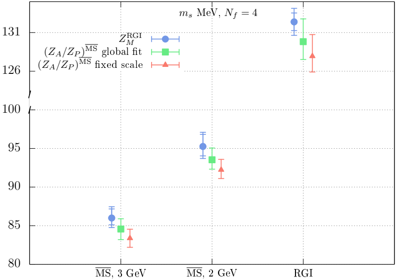
where

$$f(g^2) = \frac{t_0^* \Lambda_L^2}{h^2(g^2)} = 0.00528(36)\exp\left(\frac{16\pi^2}{\beta_0 g^2} + \frac{\beta_1}{\beta_0^2}\ln\frac{\beta_0 g^2}{16\pi^2} - b_{t_0} g^2 + \cdots\right).$$

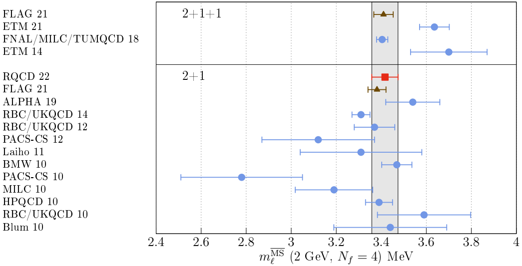
Quark masses m_ℓ and m_s

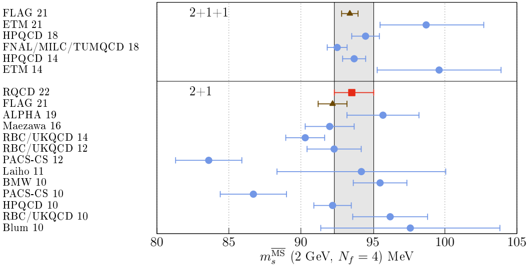
Renormalisation: RGI [ALPHA,1906.03445], RI′-SMOM [RQCD,2012.06284]



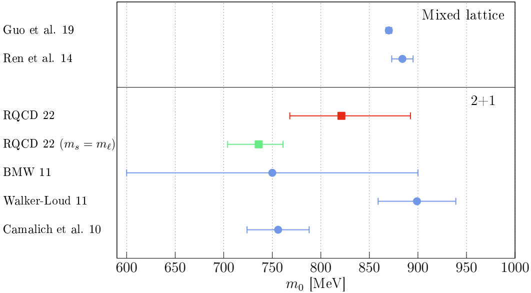


Quark masses m_ℓ and m_s

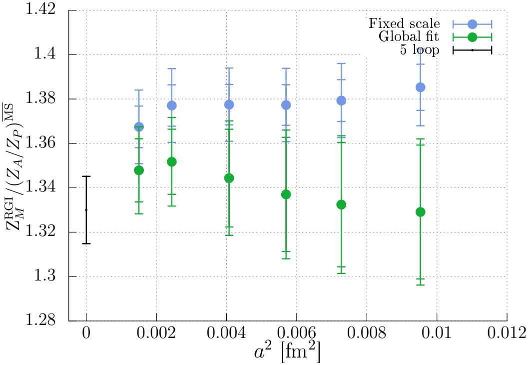




Octet baryon mass in the chiral limit



Quark masses m_ℓ and m_s



Estimates of QED and QCD isospin breaking effects

[RQCD,2305.04717]: CVC relation $\Delta m_B^{\mathrm{QCD}} = g_{S,B}(m_u - m_d)$

For the baryon octet

$$\begin{aligned}\Delta m_N^{\mathrm{QED}} &= \Delta m_N - g_S^N(m_u - m_d)\\ &= \Delta m_N - \frac{2g_S^N}{g_S^\Sigma}\Delta m_\Sigma^{\mathrm{QCD}},\\ \Delta m_\Xi^{\mathrm{QED}} &= \Delta m_\Xi - \frac{2g_S^\Xi}{g_S^\Sigma}\Delta m_\Sigma^{\mathrm{QCD}}.\end{aligned}$$

This gives

$$\begin{aligned}\Delta m_N^{\mathrm{QED}} &= 0.97^{(31)}_{(36)}\,\mathrm{MeV}, & \Delta m_N^{\mathrm{QCD}} &= -2.26^{(31)}_{(36)}\,\mathrm{MeV},\\ \Delta m_\Sigma^{\mathrm{QED}} &= 0.77(05)\,\mathrm{MeV}, & \Delta m_\Sigma^{\mathrm{QCD}} &= -4.04(04)\,\mathrm{MeV},\ (\mathrm{Expt})\\ \Delta m_\Xi^{\mathrm{QED}} &= -1.65^{(37)}_{(39)}\,\mathrm{MeV}, & \Delta m_\Xi^{\mathrm{QCD}} &= -5.20^{(42)}_{(44)}\,\mathrm{MeV}.\end{aligned}$$