Lattice Calculations for the anomalous magnetic moment of the Muon

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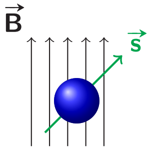
Magnetic moment of leptons $(\mathbf{e}, \boldsymbol{\mu}, \boldsymbol{\tau})$

- magnetic moment $\vec{\mu}$ of the lepton ℓ due to its spin $\vec{s}$ and electric charge $\mathbf{e}$

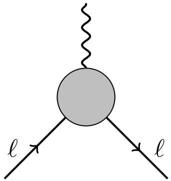
$$\vec{\mu} = g \frac{e}{2m_\ell} \vec{s}$$



torque $\quad \vec{\tau} = \vec{\mu} \times \vec{B}$

- **g**-factor: without quantum fluctuations for a lepton one finds $\mathbf{g = 2}$
- deviation from the value "2" due to quantum loops $\rightarrow$ anomalous magnetic moment of lepton ℓ

$$a_\ell = \frac{g_\ell - 2}{2}$$



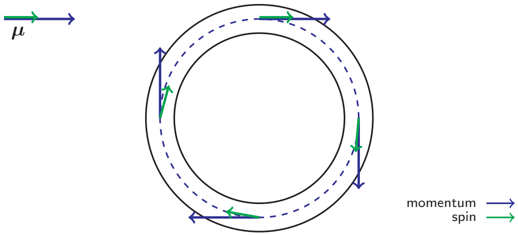
$$\langle \ell(p') | j^\gamma_\mu | \ell(p) \rangle = (-ie)\bar{u}(p') \left[\gamma_\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2) \right] u(p)$$

- $\mathbf{F_1(0) = 1}$ (electric charge) $\qquad$ $\mathbf{F_2(0) = a_\ell}$ (anomalous magnetic moment)

a_μ: Experiment vs. Theory

- measured and calculated very precisely $\longrightarrow$ test of the Standard Model
- experiment: polarized muons in a magnetic field [Bennet et al., Phys.Rev. **D73**, 072003 (2006)]

$$a_\mu = 11659209.1(5.4)(3.3) \times 10^{-10}$$



$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

- new experiments at Fermilab and JPARC $\rightarrow$ reduce error by $\approx$ **4**
  $\rightarrow$ experiment at Fermilab is running
  $\rightarrow$ first results exected end of 2019

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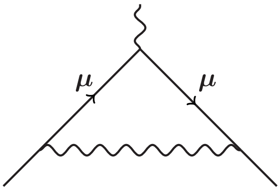
em $\quad \mathbf{(11658471.895 \pm 0.008) \times 10^{-10}}$ [Kinoshita et al., Phys.Rev.Lett. **109**, 111808 (2012)]



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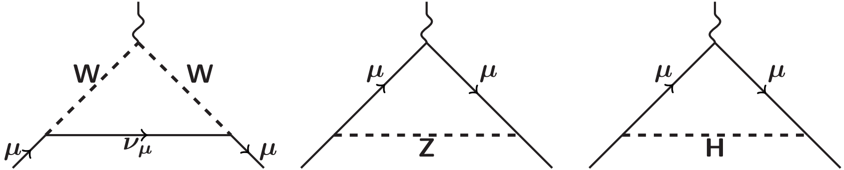

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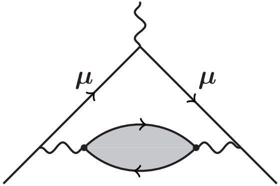

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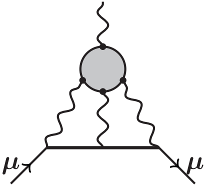

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- Comparison of theory and experiment: **3.8σ** deviation

$$\mathbf{\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 27.9(6.3)^{Exp}(3.6)^{SM} \times 10^{-10}}$$

required precision to match upcoming experiments $\quad \mathbf{\Delta a_\mu^{hvp} \lesssim 0.2\%} \quad \mathbf{\Delta a_\mu^{lbl} \lesssim 10\%}$

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	$\Delta \mathbf{a}_\mu^{\mathrm{hvp}}$
target	$\lesssim \mathbf{0.2\%}$
current **R**-ratio	$\approx \mathbf{0.5\%}$
current lattice	$\approx \mathbf{2-3\%}$

- Comparison of theory and experiment: $\mathbf{3.8\sigma}$ deviation

$$\Delta \mathbf{a}_\mu = \mathbf{a}_\mu^{\mathrm{exp}} - \mathbf{a}_\mu^{\mathrm{SM}} = \mathbf{27.9(6.3)^{Exp}(3.6)^{SM}} \times \mathbf{10^{-10}}$$

required precision to match upcoming experiments	$\Delta \mathbf{a}_\mu^{\mathrm{hvp}} \lesssim \mathbf{0.2\%}$	$\Delta \mathbf{a}_\mu^{\mathrm{lbl}} \lesssim \mathbf{10\%}$

Outline

Hadronic Vacuum Polarisation

- Introduction
- light quark contribution
- strange and charm quark contribution
- disconnected contribution
- Isospin Breaking corrections to the HVP
- Summary and Prospects

Hadronic light-by-light scattering

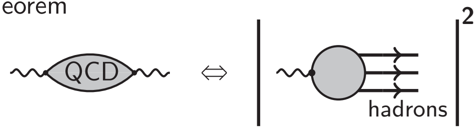
- Introduction
- Lattice Calculations
- Summary and Prospects

Final remarks

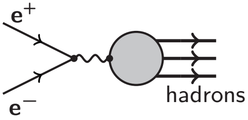
Outline

Hadronic Vacuum Polarisation (HVP) from the **R**-ratio

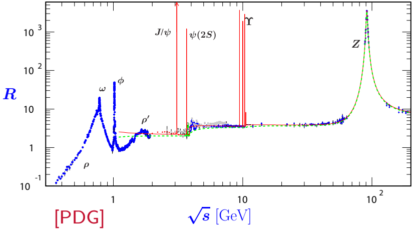
- current best theoretical estimate uses experimental data
- optical theorem



- **R**-ratio $\quad \mathbf{R(s)} = \dfrac{\sigma(\mathbf{e^+ e^-} \rightarrow \text{hadrons}, \mathbf{s})}{\sigma(\mathbf{e^+ e^-} \rightarrow \mu^+\mu^-, \mathbf{s})}$



$$\mathbf{a}_\mu^{\text{hvp}} = \left(\frac{\alpha \mathbf{m}_\mu}{3\pi}\right)^2 \int_{\mathbf{m}_\pi^2}^{\infty} \mathbf{ds}\, \frac{\mathbf{R(s)K(s)}}{\mathbf{s}^2}$$



[PDG]

recent results:

$\mathbf{a}_\mu^{\text{hvp}} = \mathbf{689.46(3.25)}$ [Jegerlehner 18]

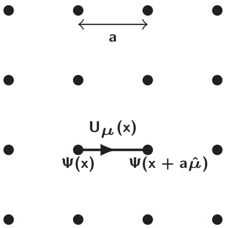
$\mathbf{a}_\mu^{\text{hvp}} = \mathbf{693.1(3.4)}$ [DHMZ 17]

$\mathbf{a}_\mu^{\text{hvp}} = \mathbf{693.37(2.46)}$ [KNT 18]

$\approx$ **0.5%** precision

- first principles calculation of HVP $\rightarrow$ lattice QCD

QCD on the lattice



- Wick rotation $(t \to -ix_0)$ to Euclidean space-time
- Discretize space-time by a hypercubic lattice Λ
- Quantize QCD using Euclidean path integrals

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi,\overline{\Psi}]\mathcal{D}[U]\, e^{-S_E[\Psi,\overline{\Psi},U]}\, A(U,\Psi,\bar{\Psi})$$

$\longrightarrow$ can be split into fermionic and gluonic part

- Calculate gluonic expectation values using Monte Carlo techniques:

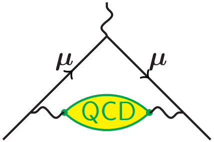
$$\langle\langle A \rangle_F \rangle_G = \int \mathcal{D}[U]\, \langle A \rangle_F\, P(U) \approx \frac{1}{N_{cfg}} \sum_{n=1}^{N_{cfg}} \langle A \rangle_F$$

average over gluonic gauge configurations U distributed according to

$$P(U) = \frac{1}{Z} (\det D)^{N_f}\, e^{-S_G[U]}$$

- extrapolate to the continuum $(a \to 0)$ and infinite volume $(V \to \infty)$

Hadronic Vacuum Polarisation (HVP) from the Lattice



- $\Pi_{\mu\nu}(Q) \equiv \int d^4x\, e^{i\,Q\cdot x} \left\langle j^\gamma_\mu(x)\; j^\gamma_\nu(0)\right\rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\, \Pi(Q^2)$
- electromagnetic current $\quad j^\gamma_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$
- hadronic contribution to the anomalous magnetic moment of the muon
[T. Blum, Phys.Rev.Lett.91, 052001 (2003)]

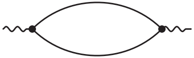
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, K(Q^2)\, \hat{\Pi}(Q^2) \qquad \text{with} \qquad \hat{\Pi}(Q^2) = 4\,\pi^2 \left[\Pi(Q^2) - \Pi(0)\right]$$

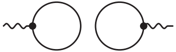
- subtracted HVP from vector correlator [Bernecker and Meyer, Eur.Phys.J. **A47**, 148 (2011)]

$$C(t) = \frac{1}{3}\sum_{k=0}^{2}\sum_{\vec{x}} \langle j^\gamma_k(\vec{x},t) j^\gamma_k(0)\rangle \qquad \hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t)\left[\frac{\cos(Qt)-1}{Q^2} + \frac{1}{2}t^2\right] \qquad a_\mu^{\rm hvp} = \int_0^\infty dt\, f(t) C(t)$$

- flavour decomposition (isospin symmetric QCD)

$$C(t) = \frac{5}{9}C^\ell(t) + \frac{1}{9}C^s(t) + \frac{4}{9}C^c(t) + C^{\rm disc}(t)$$

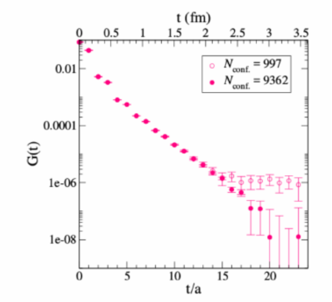




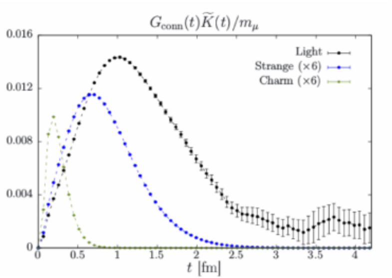
Outline

Vector correlator and long distance Signal-to-Noise problem

- examples for light-quark vector correlator at physical point



[C. Davies *et al*, arXiv:1902.04223]



[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]

- signal deteriorates for large **t**
- need noise reduction techniques to control statistical error on raw data
  - all-mode-averaging (AMA) [T. Blum *et al.*, Phys. Rev. **D88**, 094503 (2013)], [G. Bali *et al.*, Comput.Phys.Commun. 181 (2010) 1570-1583]
  - huge reduction in error when using low-mode-averaging (LMA) [T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)], [C. Aubin *et al*, arXiv:1905.09307]
- possible strategy: replace correlator by (multi-) exponential fit for $\mathbf{t} > \mathbf{t_c}$

Bounding method

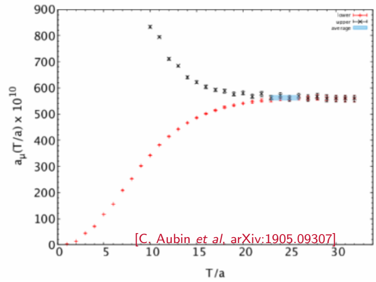
- spectral representation of the vector correlator

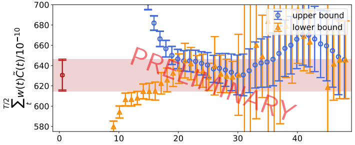
$$C(t) = \sum_n \frac{A_n^2}{2E_n} e^{-E_n t} \qquad A_n^2 > 0$$

- bound for the correlator for $t \geq t_c$ [S. Borsanyi *et al.*, Phys. Rev. D 96, 074507 (2017)], [T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)]

$$0 \leq C(t_c)\, e^{-E_{t_c}(t-t_c)} \leq C(t) \leq C(t_c)\, e^{-E_0(t-t_c)}$$

- E_{t_c}: effective mass of the correlator at t_c
- E_0: finite volume ground state energy, two pions with one unit of momentum
- use correlator data for $t < t_c$
- use upper and lower bound for $t \geq t_c$ vary t_c

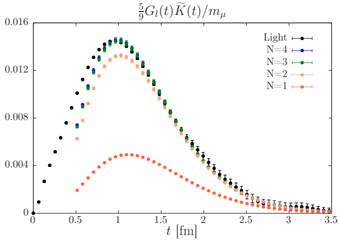




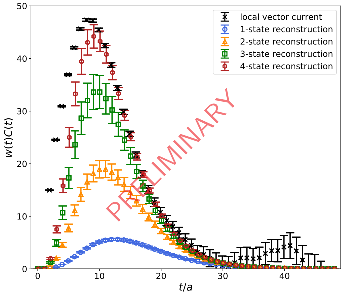
[Plot by A. Meyer (RBC/UKQCD) @ Lattice 2019]

Reconstruction of the long distance tail

- dedicated spectroscopy study, GEVP with different operators with overlap to two pions
- determine energies $\mathbf{E_n}$ and overlap factors $\mathbf{A_n}$ for lowest $\mathbf{N}$ states
- reconstruct the long distance tail of vector correlator



[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]



[Plot by A. Meyer (RBC/UKQCD) @ Lattice 2019]

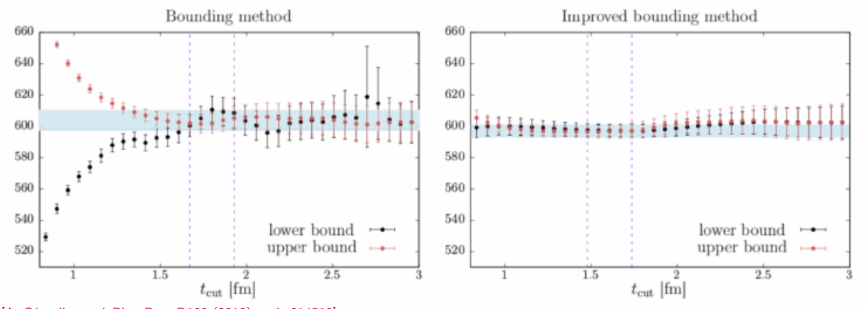
- can be used for improving the bounding method

Improved bounding method

- Improved bounding method using **N** lowest states [A. Meyer @ Lattice 2018], [A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]

$$\tilde{C}(t) = C(t) - \sum_{n=0}^{N-1} \frac{A_n^2}{2E_n} e^{-E_n t} \qquad 0 \le \tilde{C}(t_c)\, e^{-E_{t_c}(t-t_c)} \le \tilde{C}(t) \le \tilde{C}(t_c)\, e^{-E_N(t-t_c)}$$

- upper and lower bound overlap for smaller t_c
- a_μ^{hvp} can be extracted with smaller error

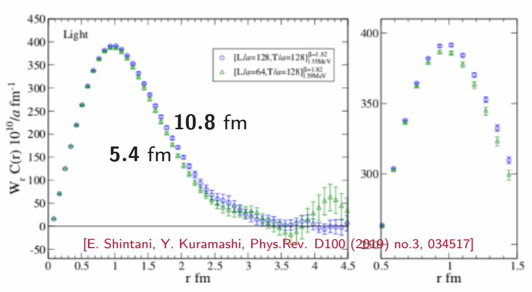


[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]

Finite volume (FV) effects

- dominated by two pion state - important at large t
- finite volume effects of $\sim \mathcal{O}(\mathbf{20-30\times 10^{-10}})$ for typical lattice sizes $\sim \mathcal{O}(\mathbf{5-6}\ \text{fm})$ at physical point, see e.g. [E. Shintani, Y. Kuramashi, Phys.Rev. D100 (2019) no.3, 034517], [A. Gérardin, Phys.Rev. D100 (2019) no.1, 014510], [C. Aubin *et al*, arXiv:1905.09307], [C. Lehner @ Lattice 2019]

- study using ensembles with different volumes



[E. Shintani, Y. Kuramashi, Phys.Rev. D100 (2019) no.3, 034517]

$\rightarrow$ FV effects about **1.7**$\times$ larger than NLO ChiPT

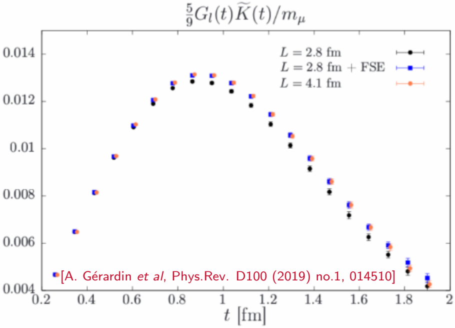
- similar observation
  - ETMC [D. Giusti *et al*, Phys. Rev. **D98**, 114504 (2018)]
    using timelike pion form factor
  - RBC/UKQCD [C. Lehner @ Lattice 2019]
    using different volumes or timelike pion form factor
- finite volume effects in NNLO ChiPT
  [C. Aubin *et al*, arXiv:1905.09307],
  [J. Bijnens, J. Relefors, JHEP 1712 (2017) 114]

  $\rightarrow$ additional FV effects from NNLO ChiPT $\approx \mathbf{0.4-0.45}$ of NLO FV effects [C. Aubin *et al*, arXiv:1905.09307]

- $\mathcal{O}(\mathbf{e^{-m_\pi L}})$ FV corrections using Hamiltonian approach (neglecting $\mathcal{O}(\mathbf{e^{-\sqrt{2}m_\pi L}})$)
  [M. Hansen, A. Patella, arXiv:1904.10010]

Finite volume effects from the timelike pion form factor

- long-distance contribution of vector correlator given in terms of the timelike pion form factor
- Gounaris-Sakurai (GS) parameterisation of the timelike pion form factor
  [H. Meyer, Phys.Rev.Lett. **107** (2011) 072002], [A. Francis *et al*, Phys.Rev. **D88** (2013) 054502]
- infinite volume long distance correlator from GS
- finite volume long distance correlator from GS & Lellouch-Lüscher formalism



- $\mathbf{m_\pi = 280}$ MeV, two different volumes
- finite size effects (FSE) corrected using timelike pion form factor

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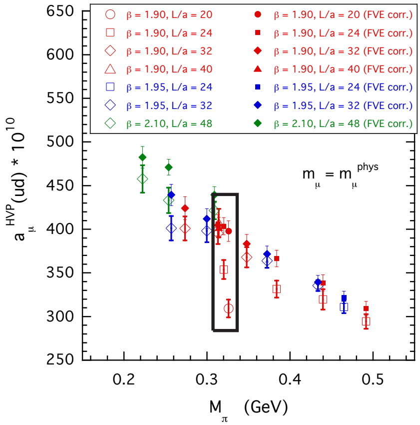
- ETMC [D. Giusti, *et al*, Phys. Rev. D 98, 114504 (2018)]
  GS and perturbative QCD for small **t**



scale setting

- $\mathbf{a}_\mu^{\mathbf{hvp}}$ depends on the scale through $\mathbf{am}_\mu$ in the kernel

- scale set by quantity $\mathbf{\Lambda}$ with error $\mathbf{\Delta\Lambda}$

$$\Delta a_\mu^{hvp} = \left| \Lambda \frac{da_\mu^{hvp}}{d\Lambda} \right| \cdot \frac{\Delta\Lambda}{\Lambda} = \left| M_\mu \frac{da_\mu^{hvp}}{dM_\mu} \right| \cdot \frac{\Delta\Lambda}{\Lambda} \qquad M_\mu = \frac{m_\mu}{\Lambda}$$

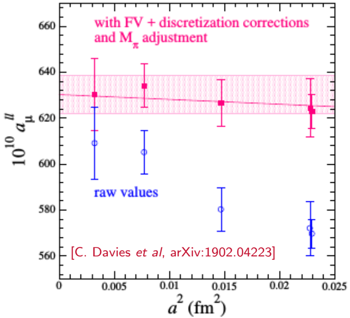
$\longrightarrow$ relative error on $\mathbf{\Lambda}$ amplified by $\approx \mathbf{1.8}$ in relative error for $\mathbf{a}_\mu$ [M. Della Morte, VG, *et al*, JHEP 1710 (2017) 020]
$\longrightarrow$ for $\mathbf{0.2\%}$ error on $\mathbf{a}_\mu^{\mathrm{hvp}}$ need $\lesssim \mathbf{0.1\%}$ on lattice spacing

- precise scale setting, e.g. RBC/UKQCD using $\mathbf{\Omega}$-Baryon [T. Blum, VG, *et al*, Phys.Rev.Lett. 121 (2018) no.2, 022003]
$\approx \mathbf{0.2 - 0.3\%}$ on lattice spacing

extrapolation to the physical point

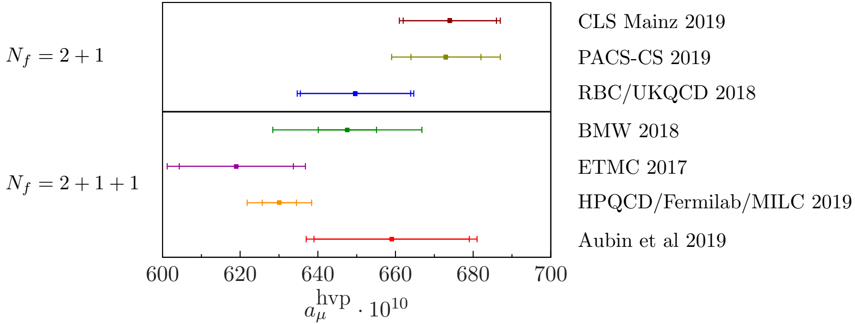
- chiral extrapolation
  - most calculations now done using (or including) ensembles at the physical point
  - chiral extrapolation if necessary

- continuum extrapolation
  - discretization effects depend on action used
  - ideally work in fully $\mathcal{O}(\mathbf{a})$ improved setup
    $\rightarrow$ actions usually $\mathcal{O}(\mathbf{a})$-improved
    $\rightarrow$ $\mathcal{O}(\mathbf{a})$-improvement of vector current, if necessary [A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]
  - ideally at least three lattice spacings



- HISQ action
- data points corrected for discretization effects from taste splitting

comparison - light quark results

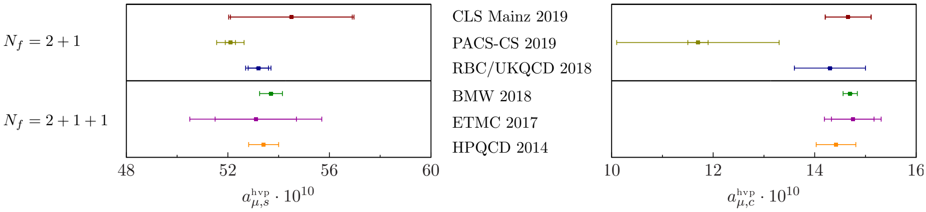


- errors from **1.3% − 3.3%**
- $\approx \mathbf{2\sigma}$ discrepancy between smallest and largest results
- compare intermediate quantities, e.g. time-moments $\mathbf{G_{2n} = \int_{-\infty}^{\infty} dt\ t^{2n} C(t)}$ or $\mathbf{a_\mu^{\text{hvp}}}$ from time window

Outline

Strange and Charm HVP

- suffers less from long-distance noise-to-signal problem and finite volume effects than light contribution
- charm usually large discretization effects



- errors on total HVP $\lesssim$ **0.4%** $\lesssim$ **0.3%**

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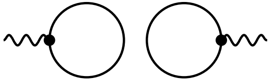
disconnected HVP



- quark-disconnected Wick contraction
- **SU(3)** suppressed
- quark loop

$$\Delta_\mu^f(t) = \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_\mu S^f(x,x)\right]$$

- all-to-all propagators, calculate stochastically
- light-strange cancellation [V.G. *et al*, PoS LATTICE2014 (2014) 128]

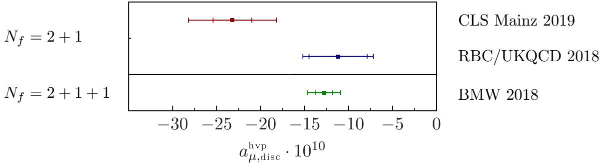
$$C^{\mathrm{disc}}(t) = \frac{1}{9}\left\langle (\Delta^\ell(t) - \Delta^s(t)) \cdot (\Delta^\ell(0) - \Delta^s(0)) \right\rangle$$

- further noise reduction
  - [T. Blum *et al*, Phys. Rev. Lett. 116, 232002 (2016)] low-mode averaging and sparsened noise sources for high modes
  - [A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510] hierarchical probing [A. Stathopoulos *et al*, arXiv:1302.4018]
  - frequency-splitting estimators [L. Giusti *et al*, Eur.Phys.J. C79 (2019) no.7, 586]

disconnected HVP

- quark-disconnected Wick contraction
- **SU(3)** suppressed
- quark loop

$$\Delta^f_\mu(t) = \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_\mu S^f(x,x)\right]$$



- errors on total HVP **0.3 − 0.7%**

- all-to-all propagators, calculate stochastically
- light-strange cancellation [V.G. *et al*, PoS LATTICE2014 (2014) 128]

$$C^{\mathrm{disc}}(t) = \frac{1}{9}\left\langle (\Delta^\ell(t) - \Delta^s(t)) \cdot (\Delta^\ell(0) - \Delta^s(0)) \right\rangle$$

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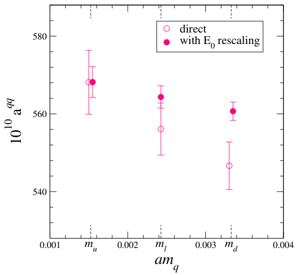
Isospin Breaking Corrections

- lattice calculations usually done in the isospin symmetric limit
- two sources of isospin breaking effects
  - different masses for up- and down quark (of $\mathcal{O}((\mathbf{m_d} - \mathbf{m_u})/\mathbf{\Lambda}_{\mathrm{QCD}}))$
  - Quarks have electrical charge (of $\mathcal{O}(\alpha))$
- lattice calculation aiming at $\lesssim \mathbf{1\%}$ precision requires to include isospin breaking

- separation of strong IB and QED effects requires renormalization scheme
- definition of "physical point" in a "QCD only world" also scheme dependent
  $\longrightarrow$ results shown above without QED and isospin breaking for $\mathbf{m}_\pi \approx \mathbf{135}$ MeV

Strong isospin corrections from the lattice

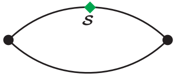
- use different up, down quark masses
- sea quark effects:
  $\longrightarrow$ configurations with different up, down masses
- results [B. Chakraborty *et al.* Phys. Rev. Lett. **120** 152001 (2018)]
  $\delta a_\mu = 7.7(3.7) \times 10^{-10}$ $\quad N_f = 2+1+1$
  $\delta a_\mu = 9.0(2.3) \times 10^{-10}$ $\quad N_f = 1+1+1+1$



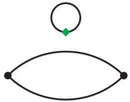
- perturbative expansion in $\Delta m = (m_u - m_d)$
  [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

$$\langle O \rangle_{m_u \neq m_d} = \langle O \rangle_{m_u = m_d} + \Delta m \frac{\partial}{\partial m} \langle O \rangle \Big|_{m_u = m_d} + \mathcal{O}\left(\Delta m^2\right)$$

- ETMC [D. Giusti *et al*, Phys.Rev. D99 (2019) no.11, 114502]
  $\delta a_\mu = 6.0(2.3) \times 10^{-10}$
- RBC/UKQCD [T. Blum, VG *et al*, Phys.Rev.Lett. 121 (2018) no.2, 022003]
  $\delta a_\mu = 10.6(4.3)_S \times 10^{-10}$



sea quark effects:

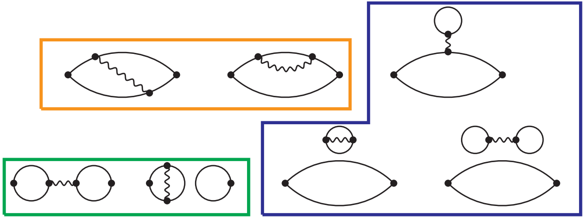


QED corrections from the lattice

- Euclidean path integral including QED

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \overline{\Psi}]\mathcal{D}[U]\mathcal{D}[A]\ O\ e^{-S_F[\Psi,\overline{\Psi},U,A]}\ e^{-S_G[U]}\ e^{-S_\gamma[A]}$$

- Finite Volume corrections for QED on the lattice
  $\rightarrow$ $1/(m_\pi L)^3$ for QED corrections to HVP in QED_L
  [J. Bijnens *et al*, Phys.Rev. D100 (2019) no.1, 014508], [D.Giusti *et al*, JHEP 1710 (2017) 157]
  $\rightarrow$ negligible for required precision
- perturbative expansion of the path integral in α [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]



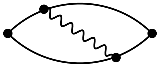
quark-connected

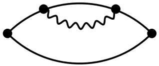
quark-disconnected

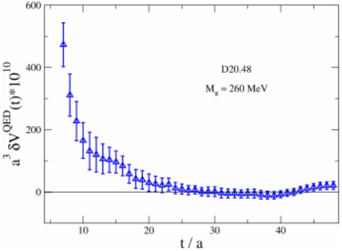
sea-quark effects

Results QED corrections

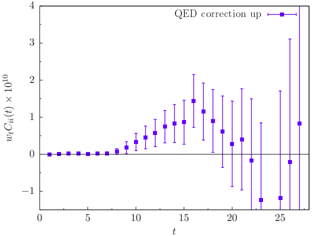
- connected contributions in electro-quenched approximation







[D. Giusti *et al*, Phys.Rev. **D99** (2019) no.11, 114502]



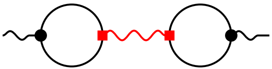
[T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)]
[VG *et al.*, PoS LATTICE2018 (2018) 134]

- several pion masses, extrapolation to physical point
  $\delta a_\mu^{hvp} = 1.1(1.0) \times 10^{-10}$

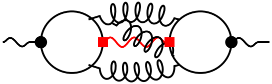
- directly at physical point, single lattice spacing
  $\delta a_\mu^{hvp} = 5.9(5.7) \times 10^{-10}$

Results QED corrections

- leading QED correction to the disconnected HVP

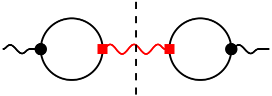


gluons between the quarks



$\rightarrow$ QED correction to LO HVP

no gluons between the quarks

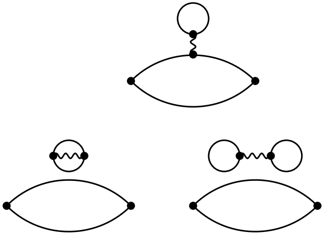


$\rightarrow$ included in NLO HVP

- QED correction to disconnected HVP [T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)]

$$\mathbf{a}_{\mu}^{\text{QED, disc}} = \mathbf{-6.9(2.1)(1.4) \times 10^{-10}}$$

- QED corrections from sea-quark effects
- diagrams at least $\mathbf{1/N_c}$ suppressed
  $\rightarrow$ could be **33%** of connected
  $\rightarrow$ need to be studied for sub-percent precision on total HVP



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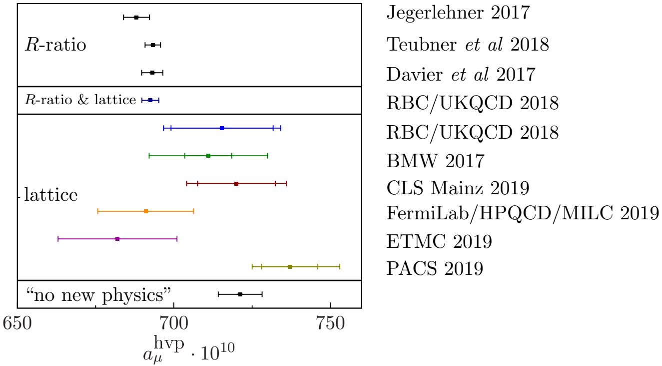
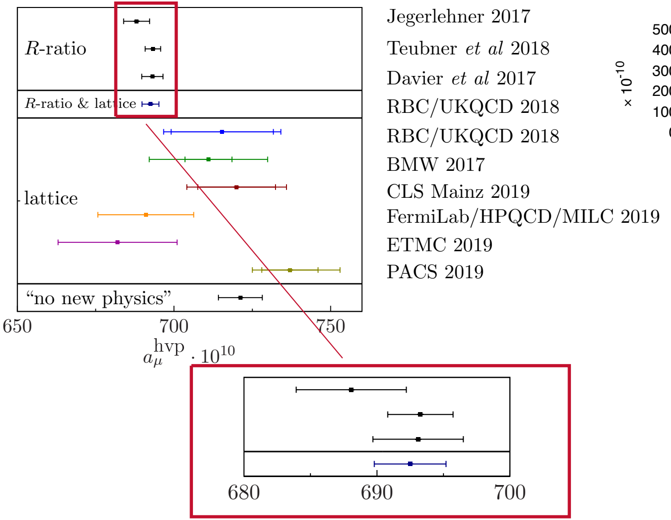
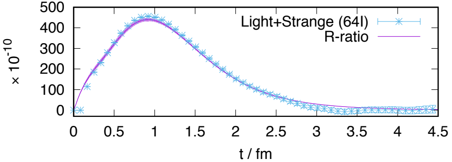
Full HVP comparison



Full HVP comparison

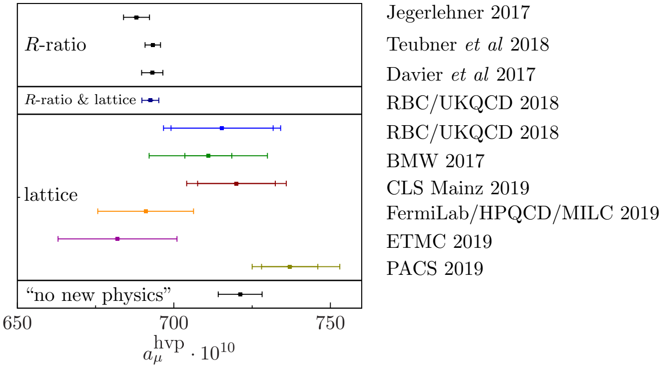


- combining lattice with **R**-ratio
  [RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 022003]

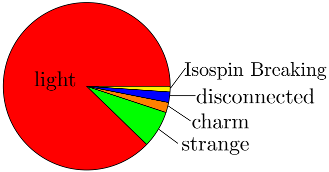


- short and long distance from **R**-ratio
- intermediate distance from lattice

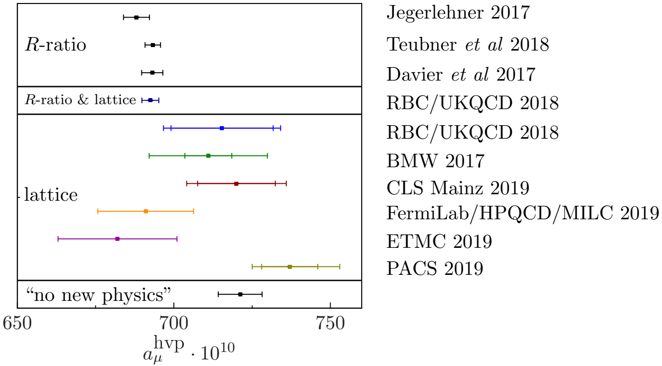
Full HVP comparison

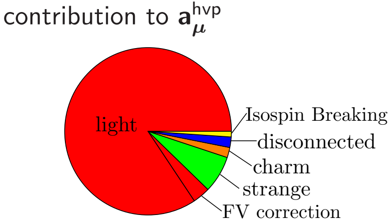


contribution to $\mathbf{a}_{\boldsymbol{\mu}}^{\text{hvp}}$



Full HVP comparison





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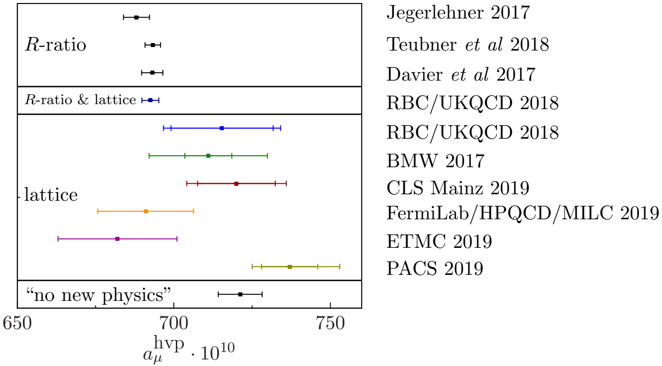
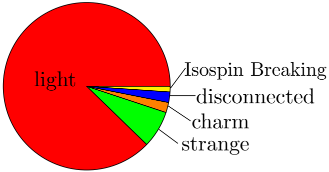
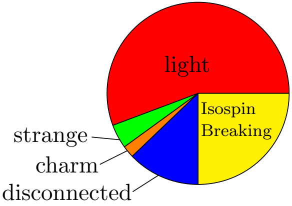


contribution to $\mathbf{a}_{\boldsymbol{\mu}}^{\mathsf{hvp}}$



contribution to $\boldsymbol{\Delta}\mathbf{a}_{\boldsymbol{\mu}}^{\mathsf{hvp}} \approx \mathbf{2.5\%}$



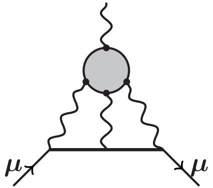
Conclusions and Prospects

- most important issues:
  - noise reduction and control of long-distance tail of the light quark correlator
  - careful estimate of finite volume effects
  - first lattice calculations of isospin breaking and QED corrections
    $\longrightarrow$ study also sea quark effects
  - achieve consensus between lattice results

Outline

Introduction

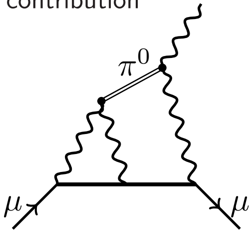
- hadronic light-by-light scattering enters at α^3



- Glasgow consensus [J. Prades, E. de Rafael, A. Vainshtein, Adv.Ser.Direct.High Energy Phys. **20** (2009) 303-317]

π^0, η, η'	$(11.4 \pm 1.3) \times 10^{-10}$
charged π loop	$(-1.9 \pm 1.9) \times 10^{-10}$
axialvector	$(1.5 \pm 1.0) \times 10^{-10}$
scalar	$(-0.7 \pm 0.7) \times 10^{-10}$
charm loops	0.2×10^{-10}
total	$(10.5 \pm 2.6) \times 10^{-10}$

e.g. π^0 contribution

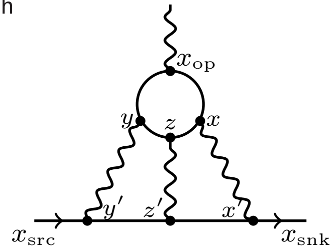


- work in progress using dispersion relations, see e.g. [G. Colangelo *et al*, JHEP 1902 (2019) 006], [G. Colangelo *et al*, Phys.Rev.Lett. 118 (2017) no.23, 232001], [G. Colangelo *et al*, JHEP 1704 (2017) 161], [M. Hoferichter *et al*, JHEP 1810 (2018) 141],[M. Hoferichter *et al*, Phys.Rev.Lett. 121 (2018) no.11, 112002], [V. Pauk and M. Vanderhaeghen, Phys.Rev. D90 (2014) no.11, 113012], . . .

Outline

light-by-light from the lattice

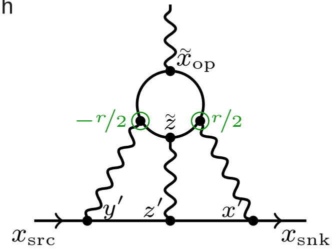
- two collaborations working on this: RBC/UKQCD and Mainz, both using position space approaches



+ 5 other permutations of $\mathbf{x'}$, $\mathbf{y'}$, $\mathbf{z'}$

light-by-light from the lattice

- two collaborations working on this: RBC/UKQCD and Mainz, both using position space approaches

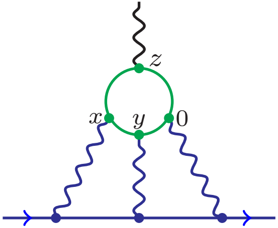


- approach proposed in [T. Blum *et al*, Phys.Rev. **D93** (2016) no.1, 014503]
  - position space sampling, i.e. stochastic evaluation of sum over **r**
  - exact photon propagators
    $\longrightarrow$ photons in $\mathrm{QED_L}$: power-law finite volume corrections
    $\longrightarrow$ infinite volume photons [T. Blum *et al*, Phys. Rev. **D96**, 034515 (2017)]

- approach proposed by Mainz [J. Green *et al*, arXiv:1510.08384],[N. Asmussen *et al*,1609.08454]

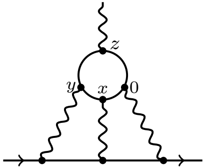
$$a_\mu^{\mathrm{lbl}} = \frac{me^6}{3}\int dx^4 dy^4\, \overline{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\; i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)$$

  - calculate $\overline{\mathcal{L}}$ (semi-) analytical in the continuum and infinite volume

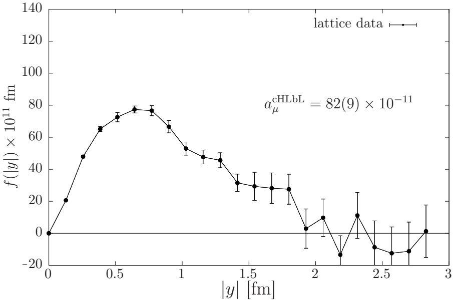


Results Mainz connected light-by-light

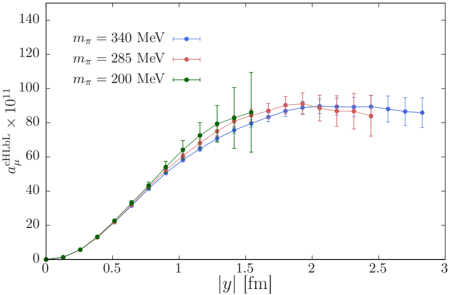
- preliminary results, see [N. Asmussen @ Lattice 2019], [N. Asmussen, g-2 workshop Mainz]
- connected diagram



$\mathbf{a_\mu^{lbl}}$ integrand ($\mathbf{m_\pi = 340}$ MeV):

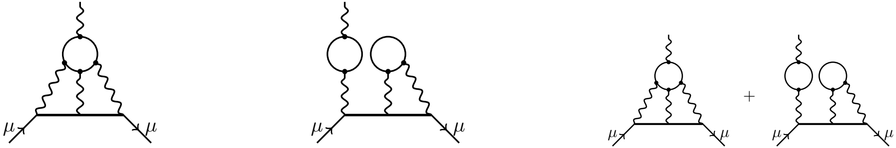


$\mathbf{a_\mu^{lbl}}$ partial integration up to $|\mathbf{y}|$:

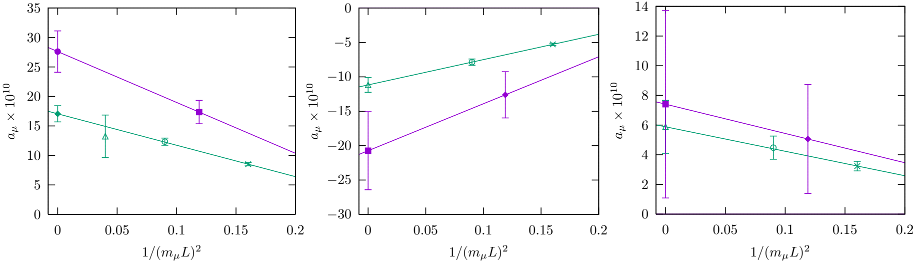


Results RBC/UKQCD connected + leading disconnected light-by-light

- preliminary results, see [T. Blum @ Lattice 2019]



- continuum and infinite volume extrapolation $\mathrm{QED_L}$



$\mathbf{a_\mu^{clbl} = 27.61(3.51)(0.32)}$ $\mathbf{a_\mu^{dlbl} = -20.20(5.65)}$ $\mathbf{a_\mu^{lbl} = 7.41(6.32)(0.32)}$ $(\times 10^{-10})$

Results RBC/UKQCD connected + leading disconnected light-by-light

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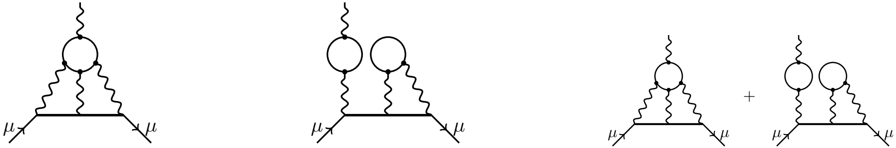
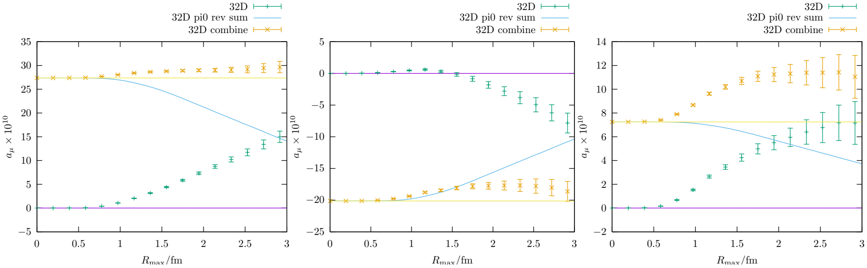


- QED$_\infty$, combined with π^0-pole contribution from model for long distances $\geq$ R_{max}



- work in progress: replace model by lattice calculation of $\pi^0 \to \gamma\gamma$, see [L. Jin @ Lattice 2019]

Outline

Conclusions - light-by-light

- two collaborations working on lattice calculations
  - RBC/UKQCD: first result (connected+leading disconnected) extrapolated to physical point
  - Mainz: connected contribution
- important check: consistency with Glasgow Consensus?

  $\rightarrow$ would need $\approx \mathbf{3}\times$ larger $\mathbf{a}_\mu^{\mathrm{lbl}}$ than Glasgow Consensus to explain $\mathbf{a}_\mu$ discrepancy

  $\rightarrow$ preliminary lattice results suggest this is unlikely

- lattice calculations of the pion transition form factor $\pi^0 \rightarrow \gamma\gamma$
  [A. Gérardin *et al*, Phys.Rev. D94 (2016) no.7, 074507],[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.3, 034520]

  $\rightarrow$ pion pole contribution to $\mathbf{a}_\mu^{\mathrm{lbl}}$

  $\rightarrow$ constrain long-distance tail to $\mathbf{a}_\mu^{\mathrm{lbl}}$ lattice calculation

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  $\rightarrow$ would need $\approx$ **3**$\times$ larger $\mathbf{a}_\mu^{\mathbf{lbl}}$ than Glasgow Consensus to explain $\mathbf{a}_\mu$ discrepancy

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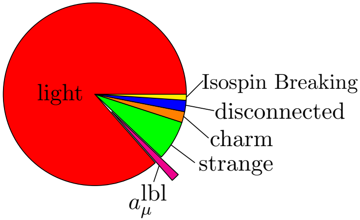
  $\rightarrow$ pion pole contribution to $\mathbf{a}_\mu^{\mathbf{lbl}}$

  $\rightarrow$ constrain long-distance tail to $\mathbf{a}_\mu^{\mathbf{lbl}}$ lattice calculation



size of light-by-light vs HVP

Outline

Hadronic Vacuum Polarisation

- Introduction
- light quark contribution
- strange and charm quark contribution
- disconnected contribution
- Isospin Breaking corrections to the HVP
- Summary and Prospects

Hadronic light-by-light scattering

- Introduction
- Lattice Calculations
- Summary and Prospects

Final remarks

Final remarks

- $\mathbf{a}_\mu$ measured and calculated very precisely
  $\rightarrow$ test of the Standard Model
  $\rightarrow$ new experiment running at Fermilab
  $\rightarrow$ largest uncertainty in Standard Model prediction from hadronic contributions

- huge effort in the lattice community to calculate hadronic contributions from first principles

- work in progress on g-2 Theory Whitepaper from the Muon g-2 Theory Initiative, several workshops since 2017, last workshop: September 9 - 13, 2019 at INT

- hadronic vacuum polarisation contribution to $\mathbf{a}_\mu$
  - first lattice calculations of $\mathbf{a}_\mu^{\mathrm{hvp}}$ with $\lesssim \mathbf{1\%}$ precision available within $\mathcal{O}(\text{year})$
  - $\lesssim \mathbf{0.2\%}$ within a few years

Thank you

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