Computing x-dependent PDFs and GPDs on the lattice

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

This project is supported by the National Science Center of Poland SONATA BIS grant No 2016/22/E/ST2/00013 (2017-2022).

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 642069.

Outline of the talk

1. PDFs on the lattice
2. Quasi-PDFs
3. Other approaches
4. Selected results
5. New directions – GPDs
6. Conclusions and prospects

Collaborators:

- C. Alexandrou (Cyprus)
- M. Constantinou (Temple)
- L. Del Debbio (Edinburgh)
- T. Giani (Edinburgh)
- K. Hadjiyiannakou (Cyprus)
- K. Jansen (DESY)
- A. Scapellato (Poznań)
- F. Steffens (Bonn)

Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. D99 (2019) 114504
- K. Cichy, L. Del Debbio, T. Giani, "Parton distributions from lattice data: the nonsinglet case", JHEP 10 (2019) 137
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Quasi-PDFs with twisted mass fermions", arXiv:1910.13229, LATTICE19 proceedings
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Light-Cone Parton Distribution Functions from Lattice QCD", Phys. Rev. Lett. 121 (2018) 112001
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D98 (2018) 091503 (Rapid Communications)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)

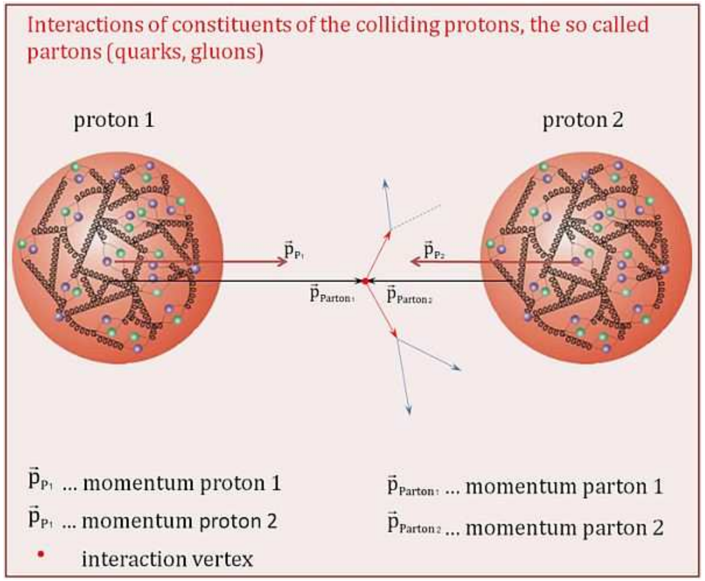
Review of the field:

- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, Adv. High Energy Phys. 2019 (2019) 3036904, arXiv: 1811.07248 [hep-lat]

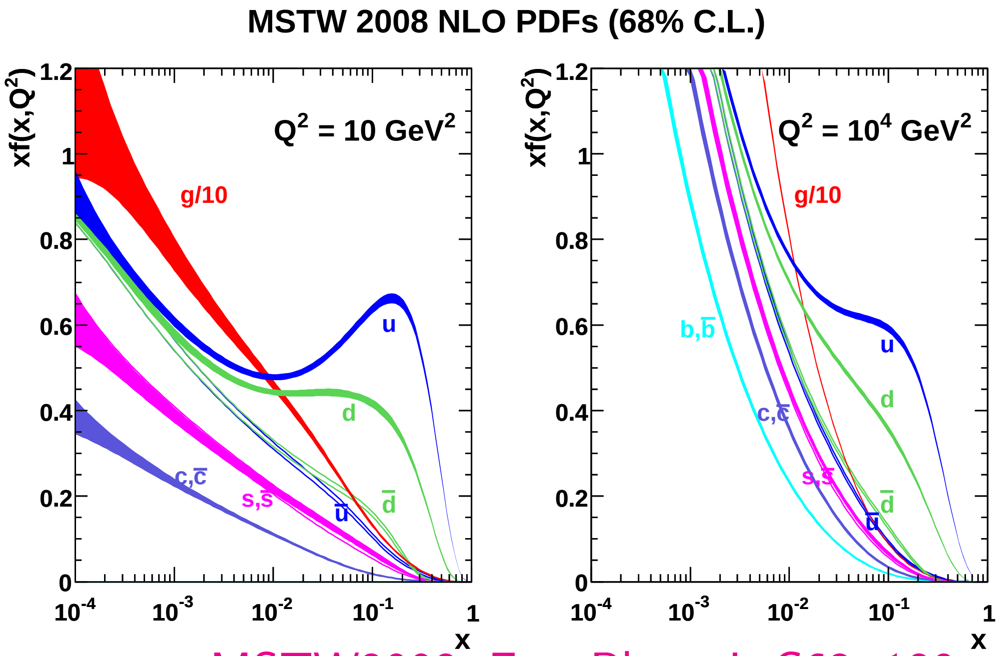
Parton distribution functions

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189

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$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

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- On the other hand, PDFs moments given in terms of local matrix elements, but only lowest 2-3 accessible.
- Recently: new **direct** approaches to get x-dependence.

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

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- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Overview of results from different approaches

results @ physical pion mass
results @ non-physical pion mass

- Quasi-distributions
  - Nucleon PDFs
    - unpolarized
      - ETMC, PRL121(2018)112001
      - ETMC, PRD99(2019)114504
      - LP3, arXiv:1803.04393
      - LP3, arXiv:1807.06566
    - helicity
      - ETMC, PRL121(2018)112001
      - ETMC, PRD99(2019)114504
      - LP3, PRL121(2018)242003
    - transversity
      - ETMC, PRD98(2018)091503(Rapid)
      - ETMC, PRD99(2019)114504
      - LP3, arXiv:1810.05043
  - Nucleon GPDs
    - ETMC, arXiv:1910.13229
  - Δ PDFs
    - ETMC/Beijing, arXiv:1907.09827
  - Pion/Kaon DA
    - LP3, PRD95(2017)094514
    - LP3, NPB939(2019)429
  - Pion PDF
    - LP3, PRD100(2019)034505
    - BNL, PRD100(2019)034516
  - Pion GPD
    - Chen et al., arXiv:1904.12376
- Pseudo-distributions
  - Nucleon PDFs
    - JLab, PRD96(2017)094503
    - JLab, arXiv:1908.09771
  - Pion PDF
    - JLab, arXiv:1909.08517
- Good LCSs
  - Pion PDF
    - JLab, PRD99(2019)074507
- Auxiliary light quark
  - Pion DA
    - Regensburg, EPJC78(2018)217
    - Regensburg, PRD98(2018)094507
- Auxiliary heavy quark
  - Pion DA
    - Taiwan/MIT, arXiv:1810.12194
- OPE without OPE
  - Nucleon F_1
    - QCDSF, PRL118(2017)242001
- Hadronic tensor
  - Nucleon W_{11}
    - χQCD, arXiv:1906.05312

Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] **and Martha Constantinou**[2]

[1]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2]*Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

Adv. High Energy Phys. 2019 (2019) 3036904, arXiv:1811.07248

Special issue *Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses*,

- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, "OPE without OPE", lattice cross sections

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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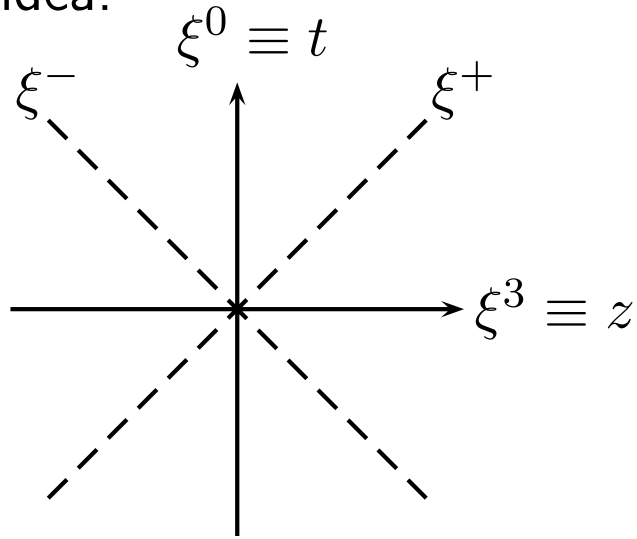
Main idea:



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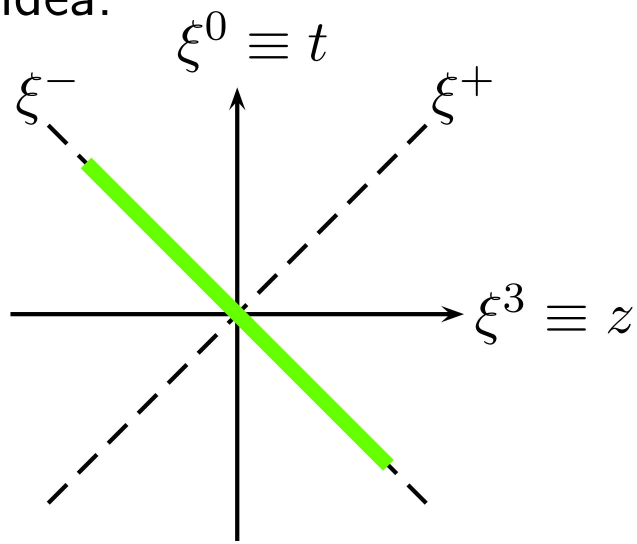

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

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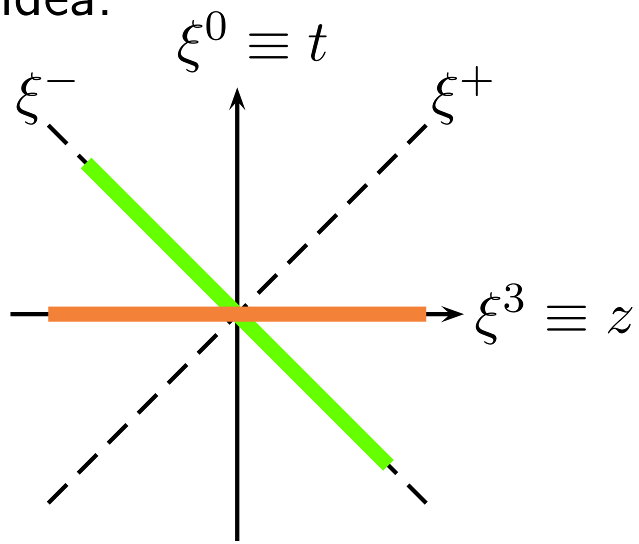

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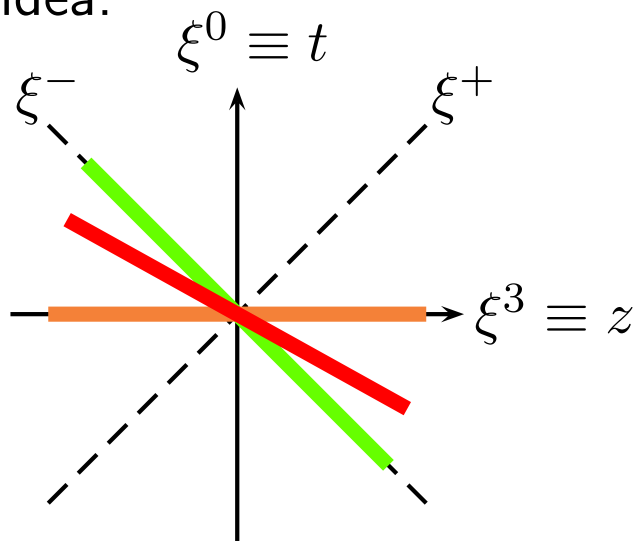

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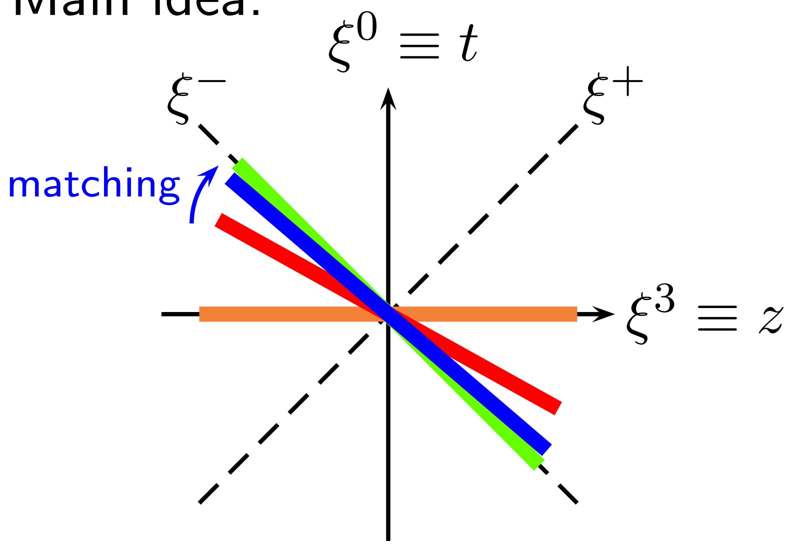

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X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

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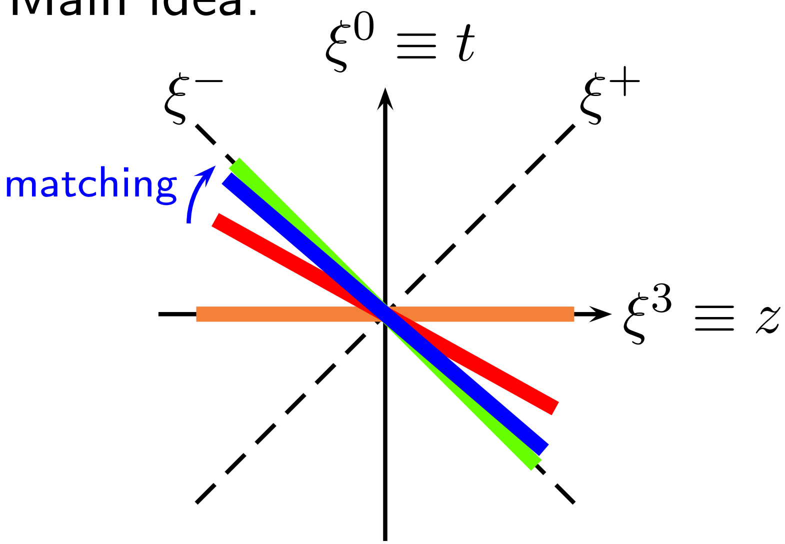

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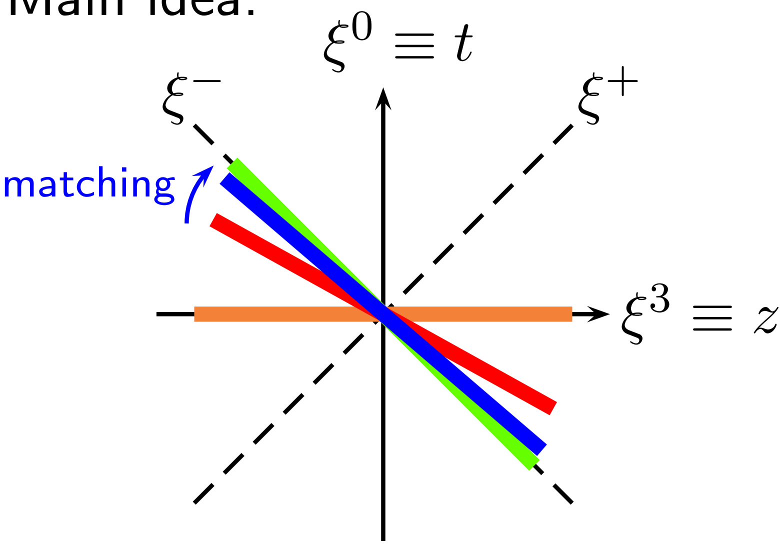

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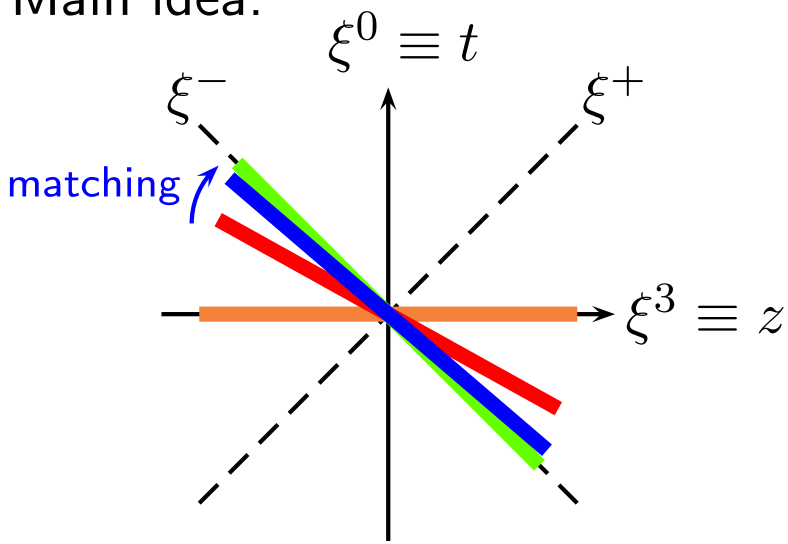

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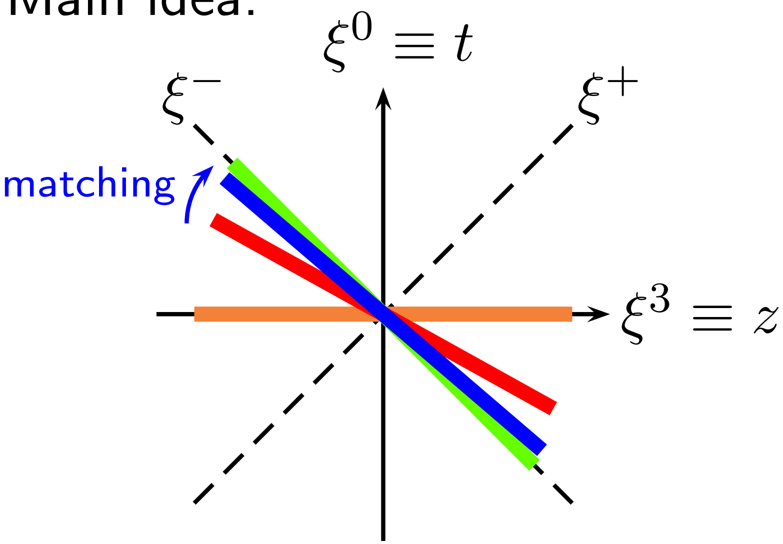

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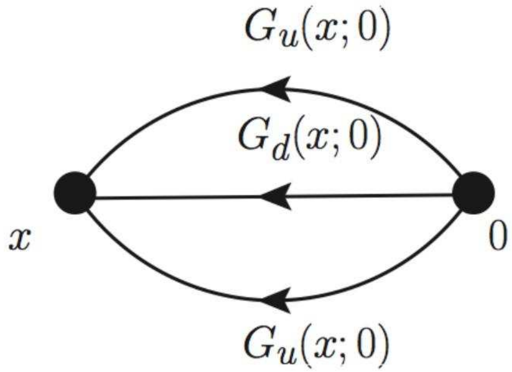

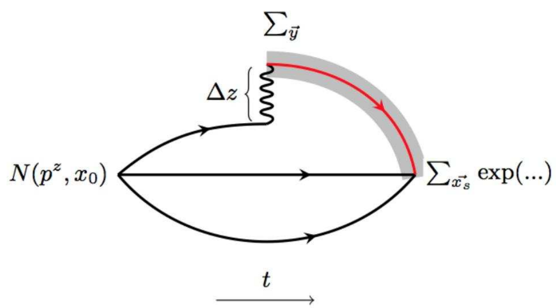

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Good lattice cross-sections

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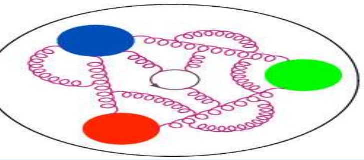

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- Another class: current-current correlators
  related idea in [V. Braun and D. Müller, EPJC 55 (2008) 349]

$$\sigma_n(\omega,\xi^2,P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle; \qquad \omega = P\cdot\xi$$

with (as one possibility):

$$\mathcal{O}_{j_1 j_2}(\xi) = \xi^{d_{j_1}+d_{j_2}-2} Z_{j_1} Z_{j_2} j_1(\xi) j_2(0).$$

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$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$

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Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

β=2.10,	$c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

	$P_3 = \frac{6\pi}{L}$		$P_3 = \frac{8\pi}{L}$		$P_3 = \frac{10\pi}{L}$	
Insertion	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$	$N_{\rm conf}$	$N_{\rm meas}$
γ^0	50	4800	425	38250	811	72990
$\gamma^5\gamma^3$	65	6240	425	38250	811	72990
σ^{3j}	50	9600	425	38250	811	72990

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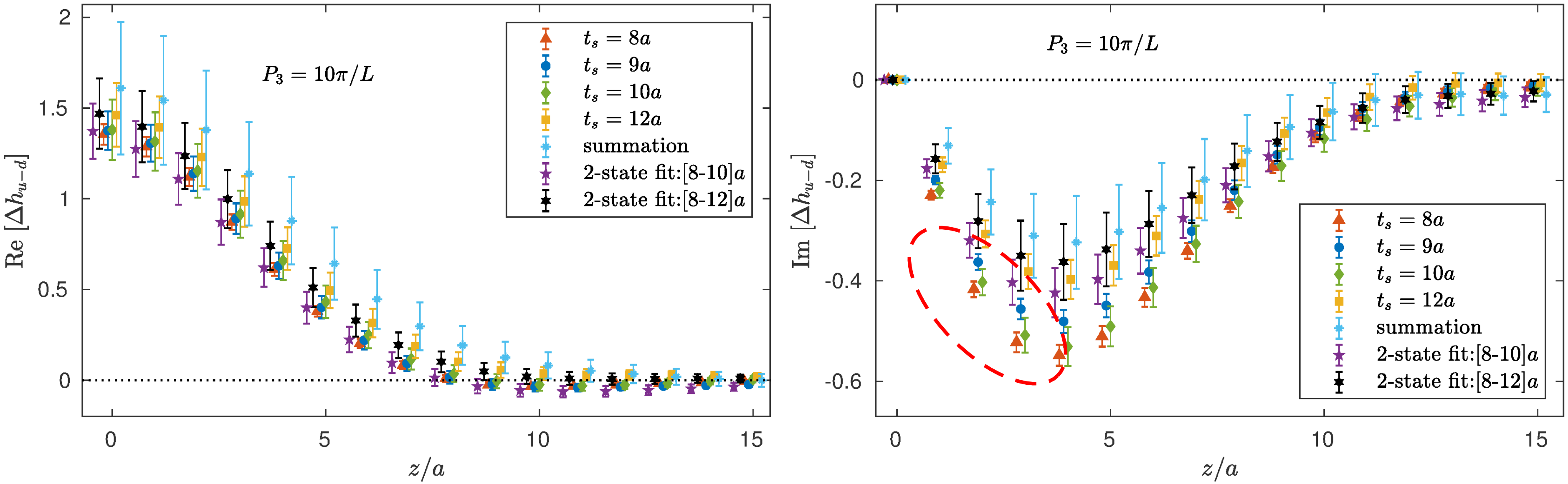

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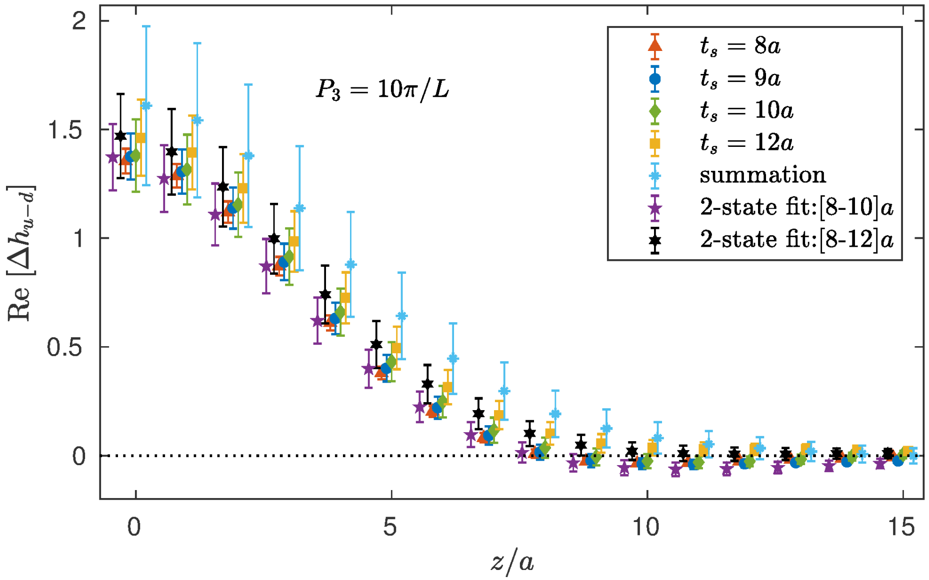

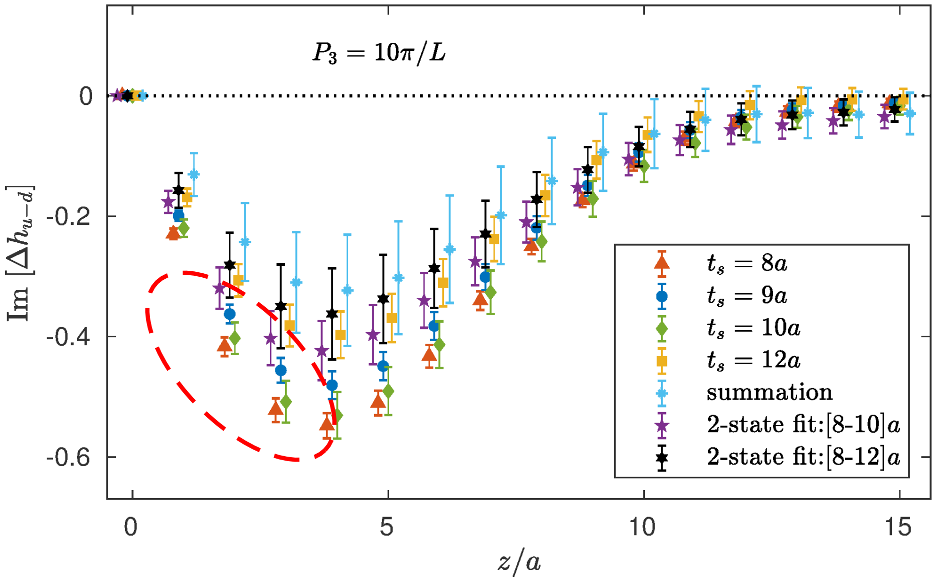

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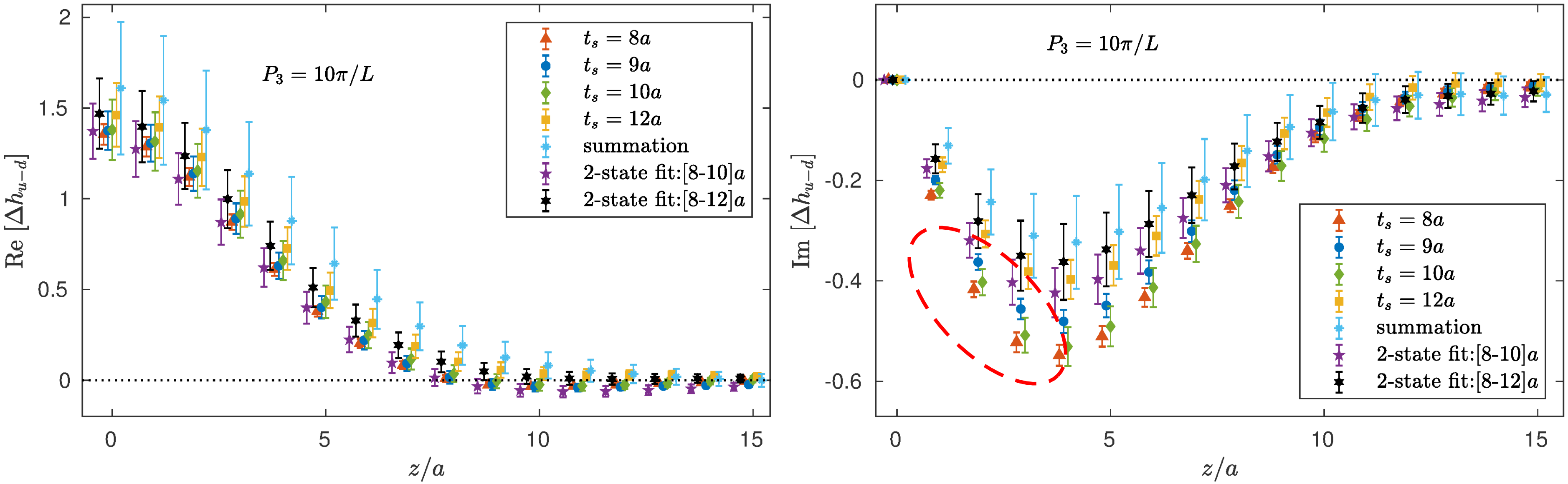

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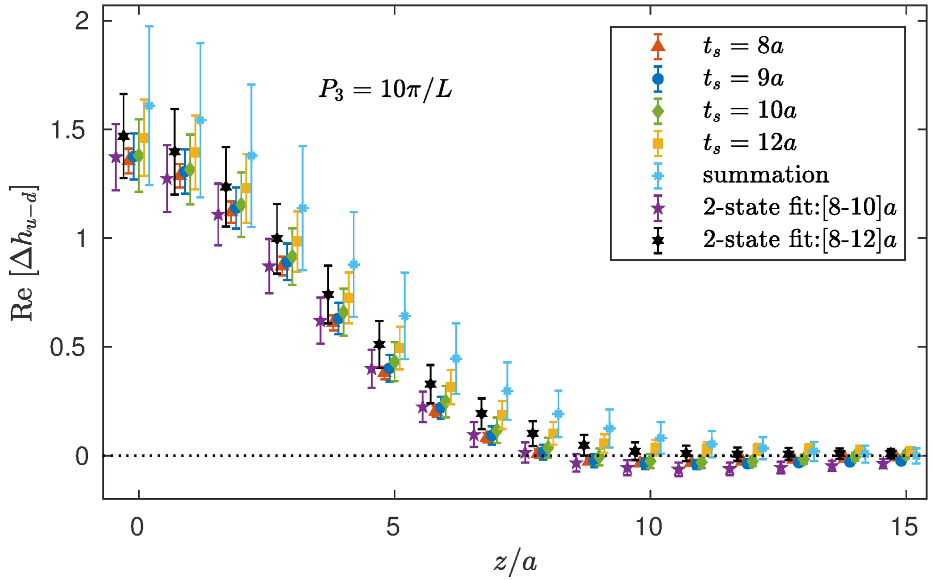

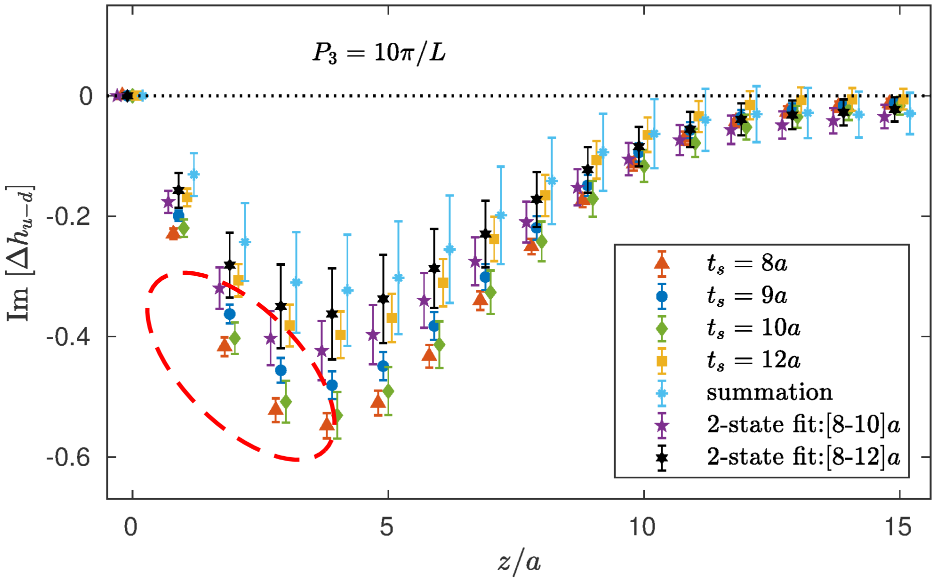

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Our largest momentum: ≈ 1.4 GeV

- safely below UV cut-off,
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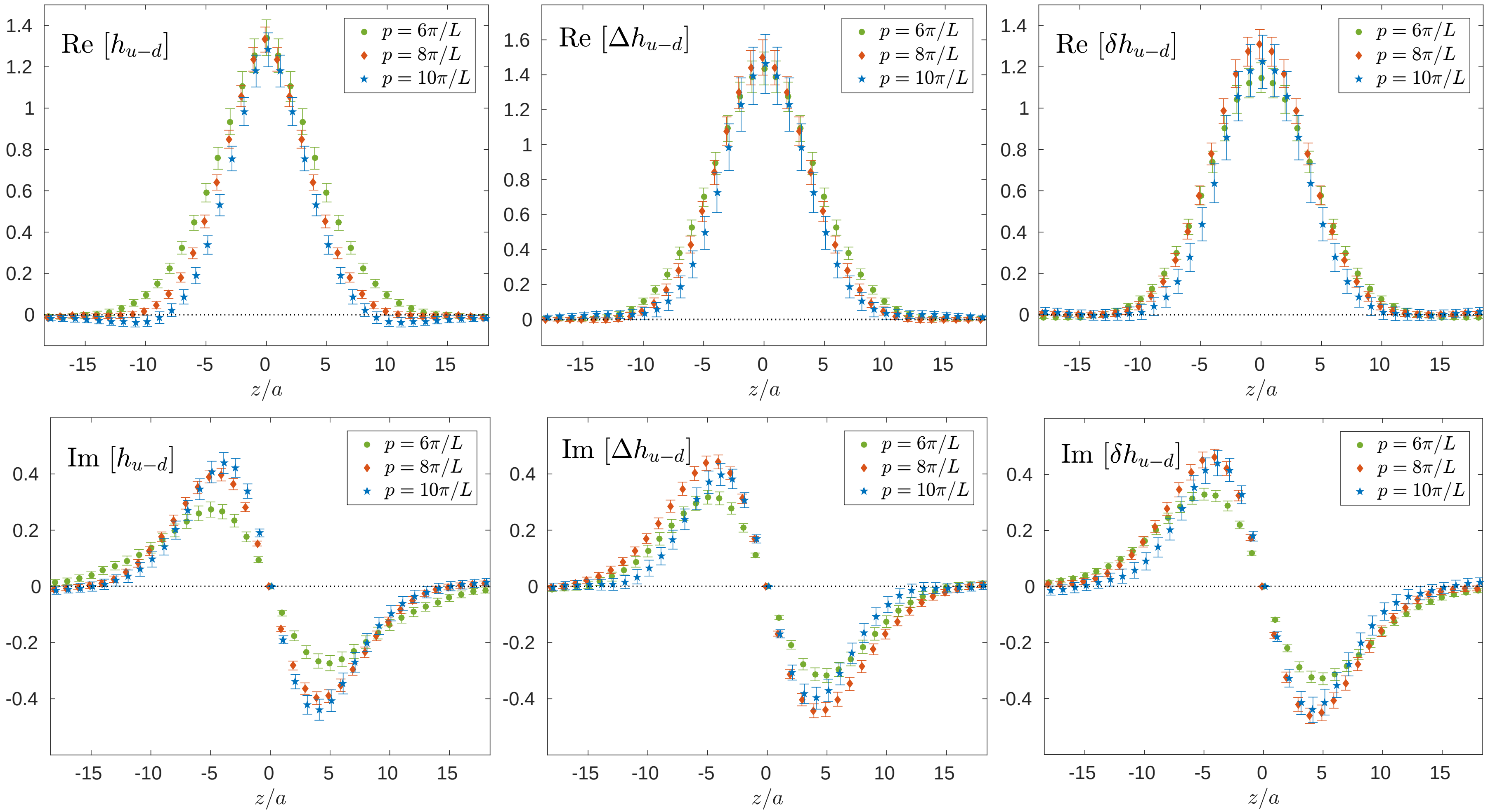
Bare matrix elements at $t_s = 12a$

unpolarized (γ_0) | helicity ($\gamma_5\gamma_3$) | transversity (σ_{3i})



STATISTICS:

unpolarized	helicity	transversity
$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
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Steps 2-4

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Historical remarks:

Renormalization

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1. it was very important to clarify the issue of renormalizability of the quasi-PDFs:

   T. Ishikawa, Y.-Q. Ma, J.-W. Qiu, S. Yoshida, Phys. Rev. D96 (2017) 094019

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Two types of divergences that need to be removed:

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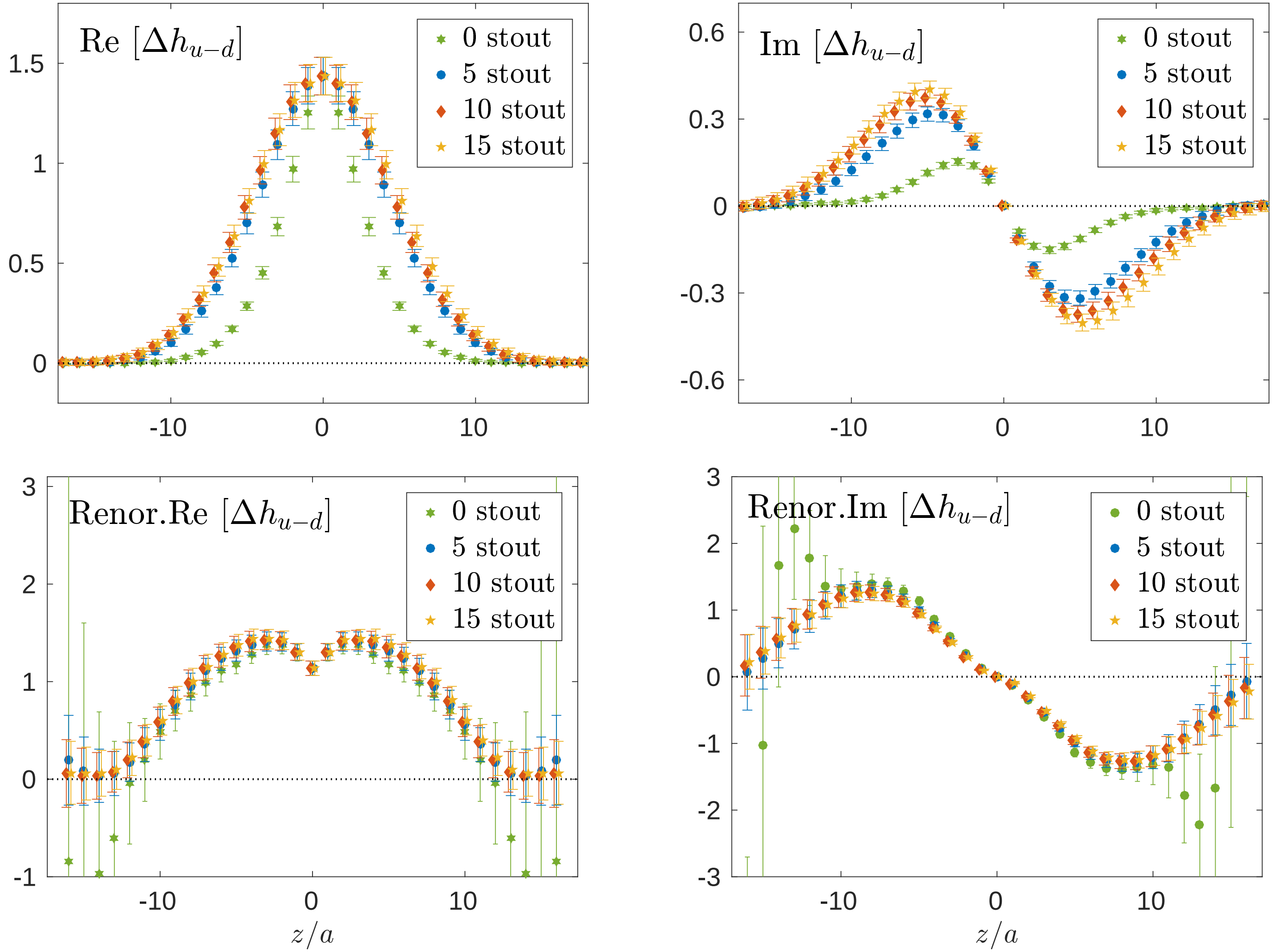
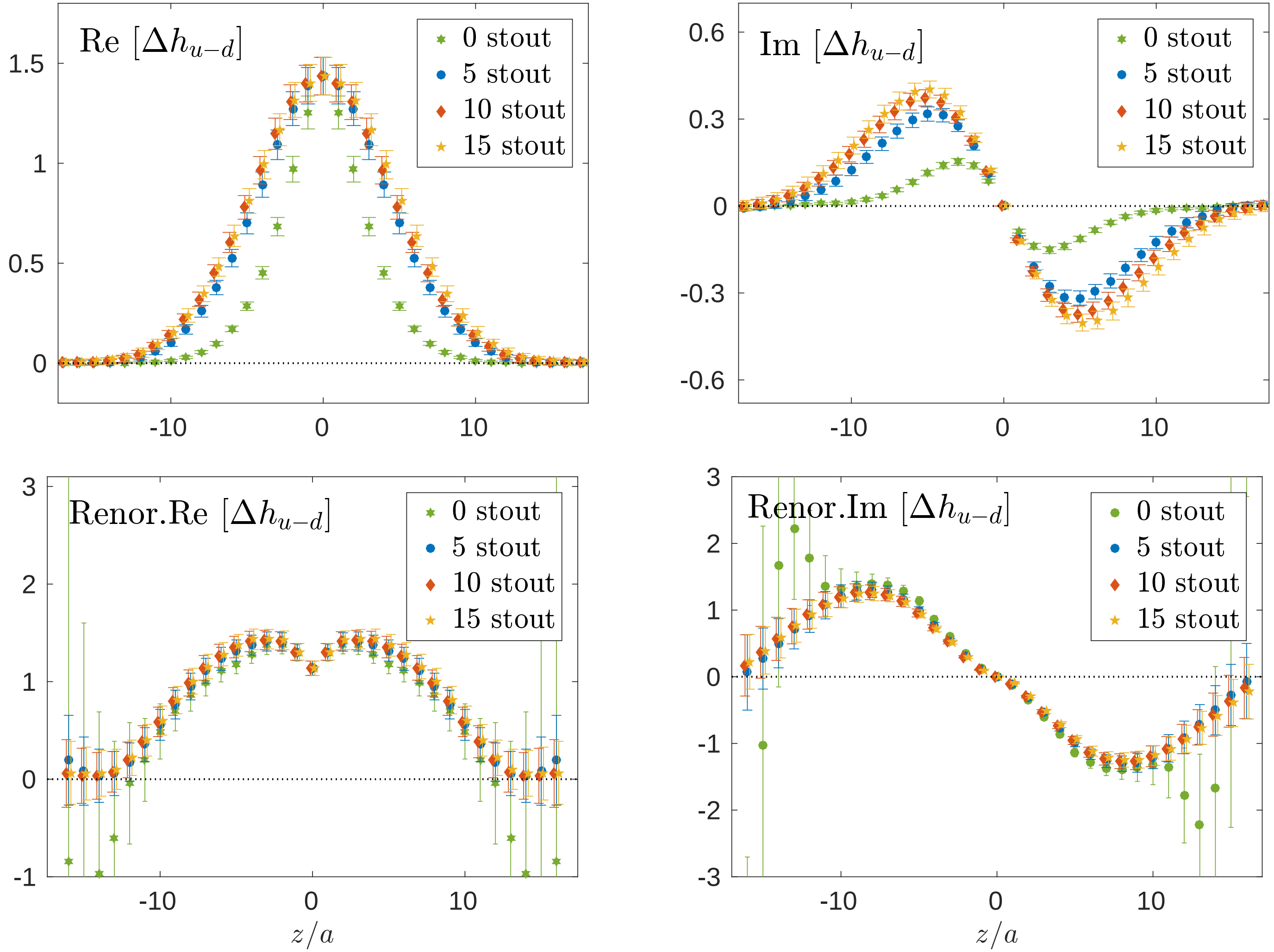
- power divergence related to the Wilson line; resums into a multiplicative exponential factor, $\exp\left(-\delta m|z|/a + c|z|\right)$
  δm – strength of the divergence, operator independent,
  c – arbitrary scale (fixed by the renormalization prescription).

Nucleon momentum $\frac{6\pi}{48}$



Renormalized matrix elements for helicity PDFs

Nucleon momentum $\frac{6\pi}{48}$



Important self-consistency check for the renormalization procedure!

Step 5

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
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$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
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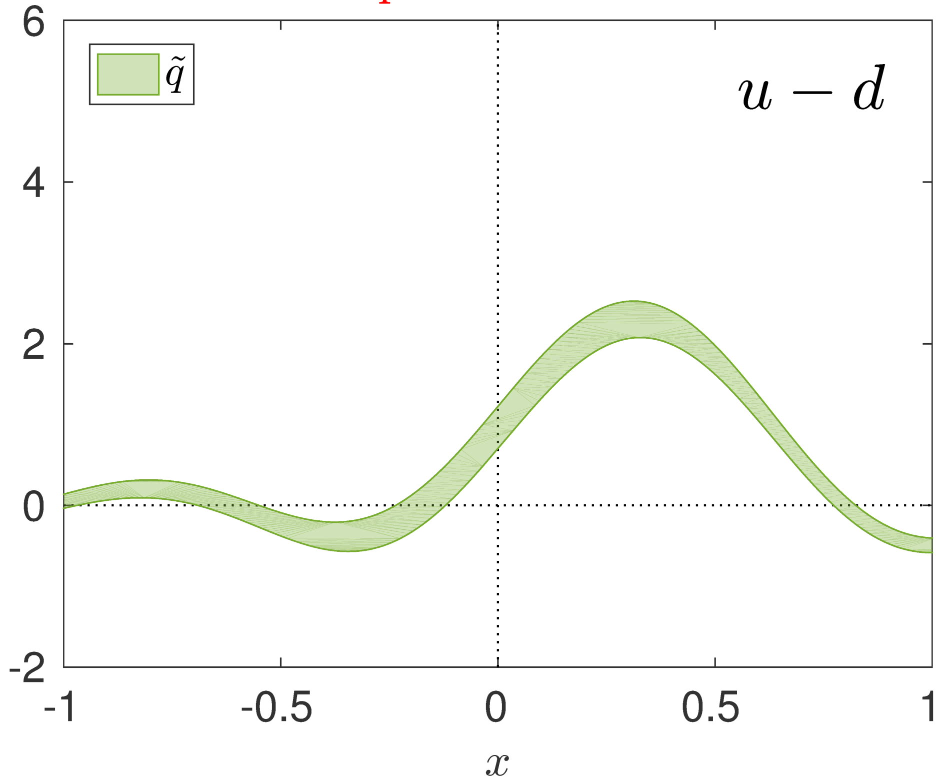
Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

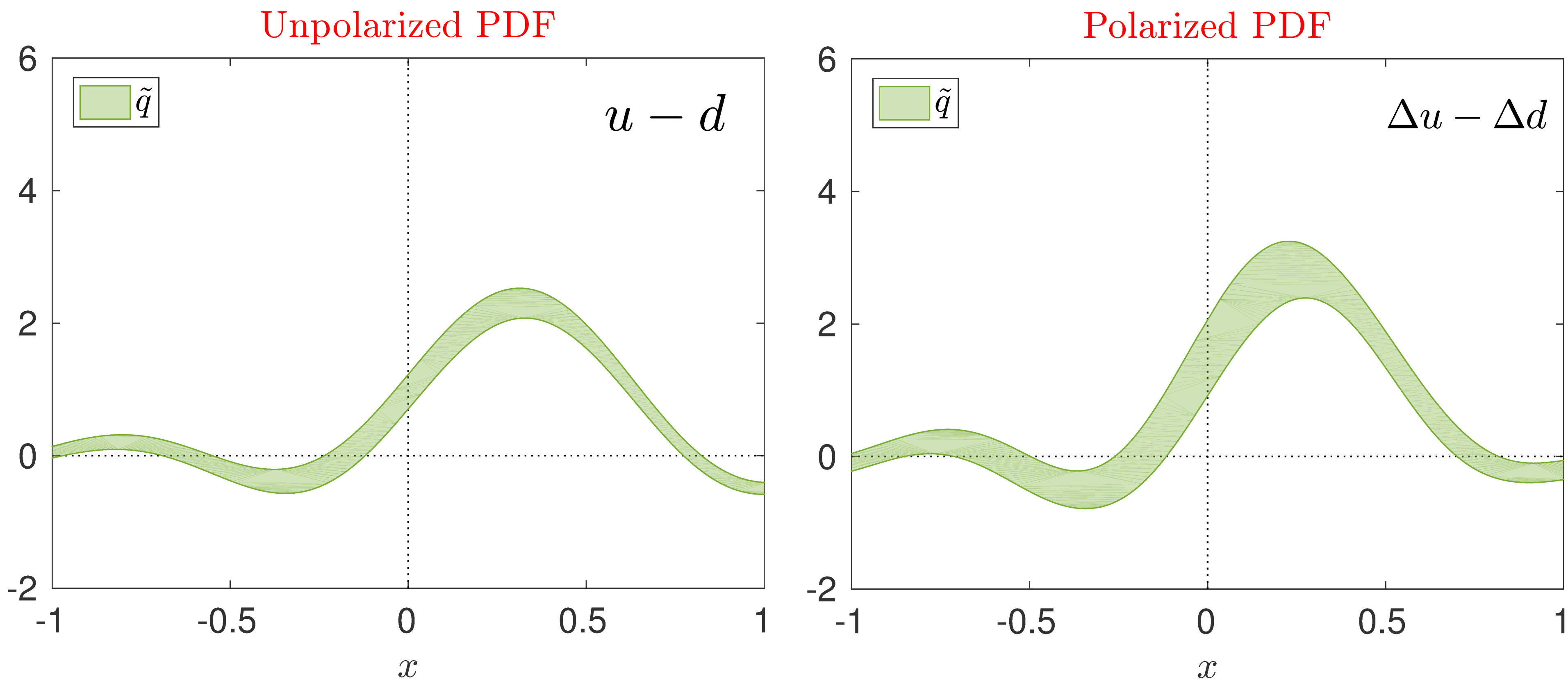
Unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Fourier transform

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Quasi-PDFs + pheno

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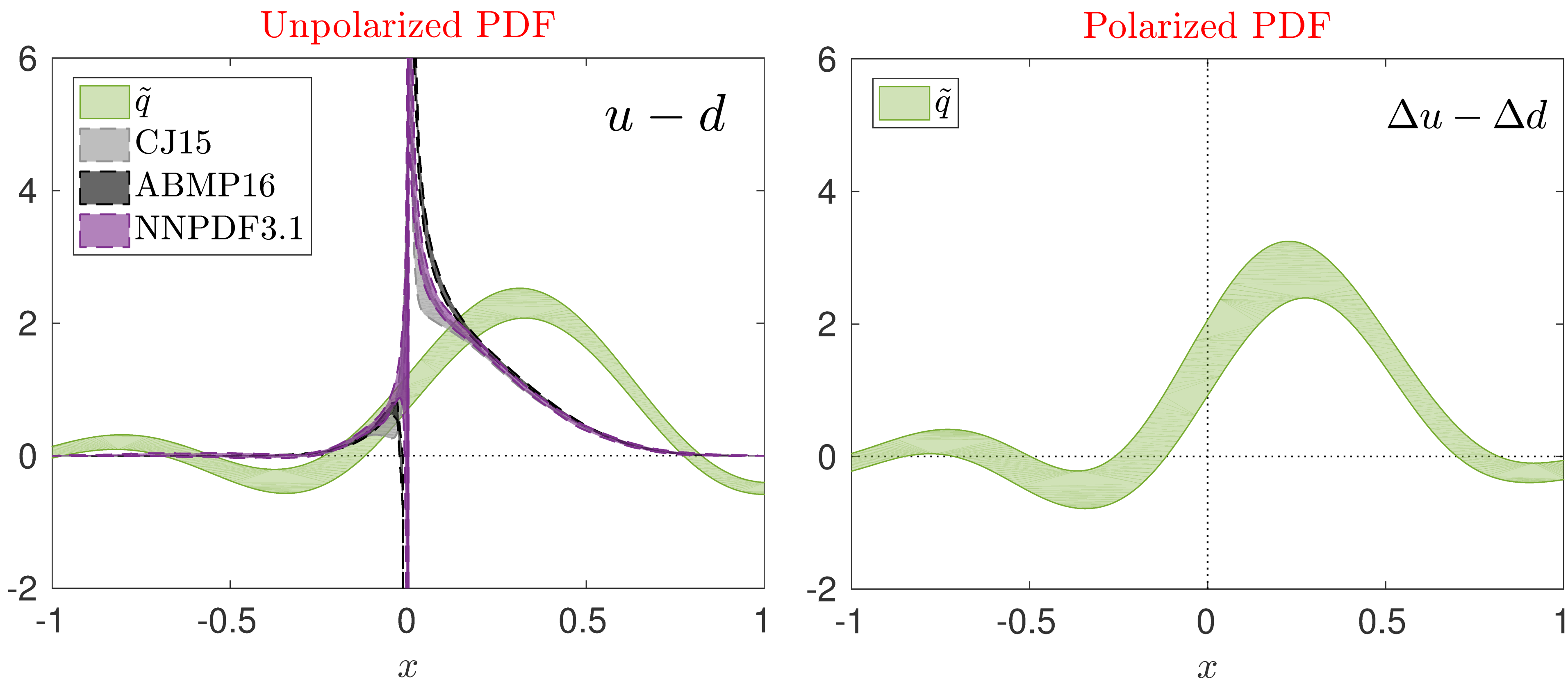

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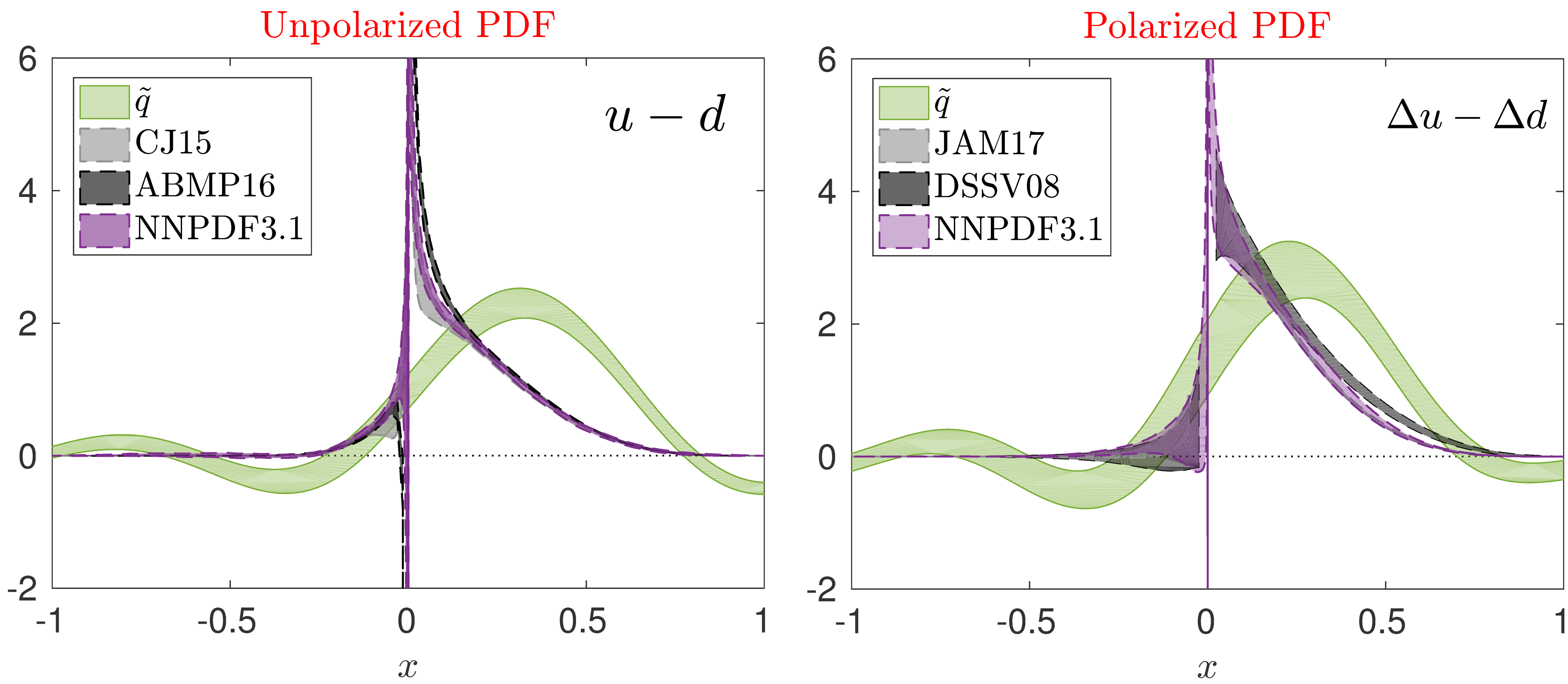

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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Strategies for matching

Matching is the essence of LaMET and was subject to important developments over the years:

- transverse momentum cut-off scheme PDFs [X. Xiong et al., Phys. Rev. D90 (2014) 014051]
- same for unpolarized and helicity GPDs [X. Ji et al., Phys. Rev. D92 (2015) 014039]
- same for transversity GPDs [X. Xiong, J. Zhang, Phys. Rev. D92 (2015) 054037]
- $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$, non-singlet and singlet PDFs [W. Wang, S. Zhao, R. Zhu, EPJC 78 (2018) 147]
- RI $\to \overline{\mathrm{MS}}$, unpolarized PDFs ($\gamma_3$) [I.W. Stewart, Y. Zhao, Phys. Rev. D97 (2018) 054512]
- $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$, treatment of UV log divergence in wave function corrections (but: violates vector current conservation) [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]
- $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$, treatment of UV log divergence in wave function corrections (preserves vector current conservation) [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]
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- RI $\to \overline{\mathrm{MS}}$, transversity PDFs [Y.-S. Liu et al., arXiv:1810.05043]
- RI $\to \overline{\mathrm{MS}}$, non-singlet GPDs [Y.-S. Liu et al., Phys. Rev. D100 (2019) 034006]
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The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

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C – matching kernel $\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2 P_3^2}{\xi^2\mu^2} \left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

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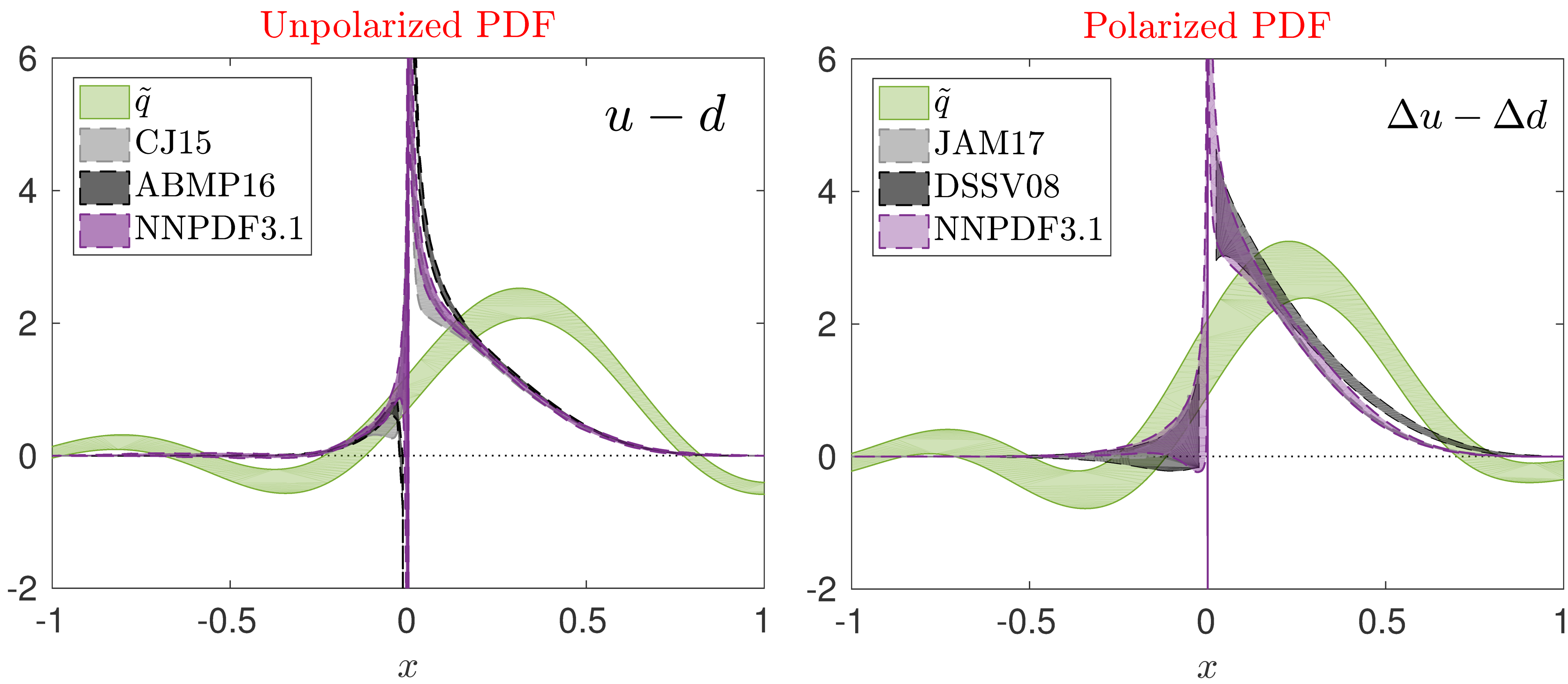
- Thus, needs modified renormalization scheme for input quasi-PDF $\to$ $\mathrm{M}\overline{\mathrm{MS}}$ scheme.
- In this procedure, vector current is **conserved**.

Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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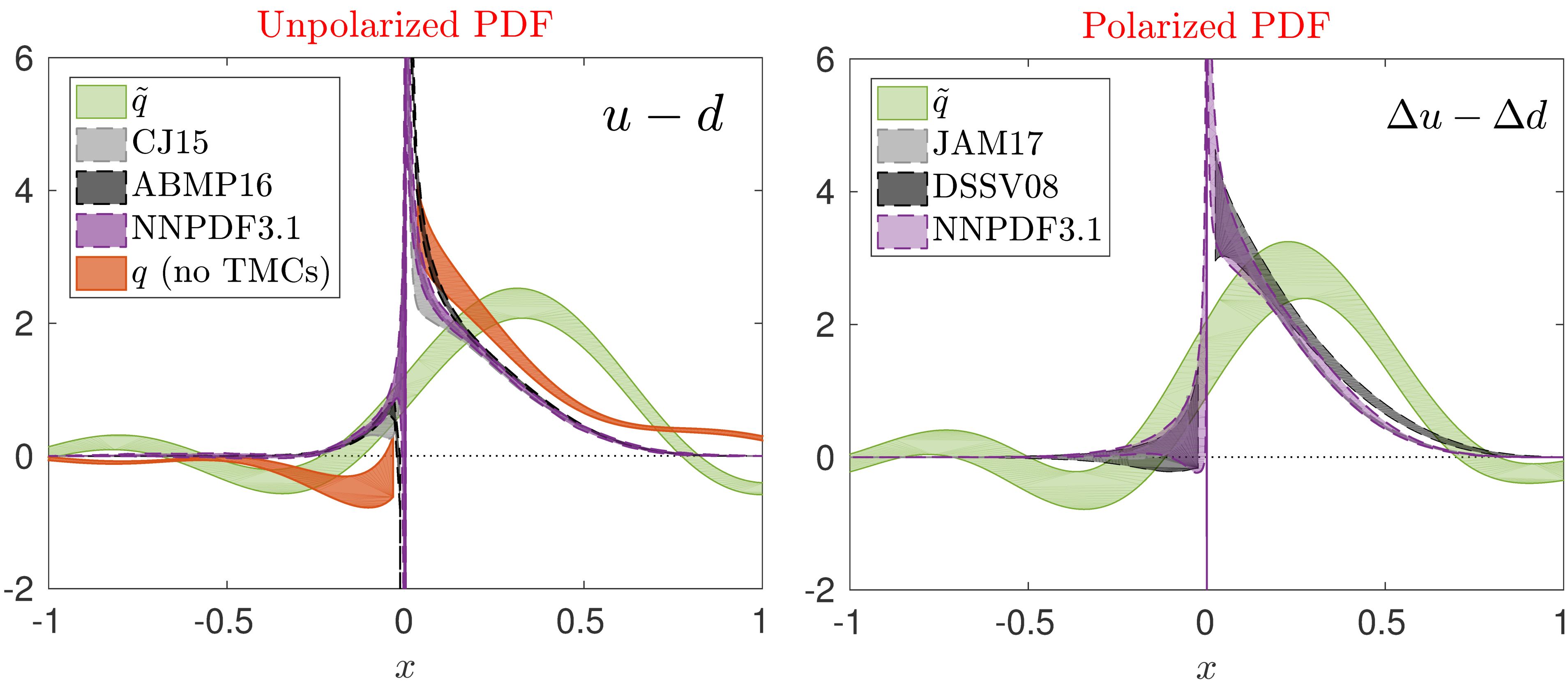

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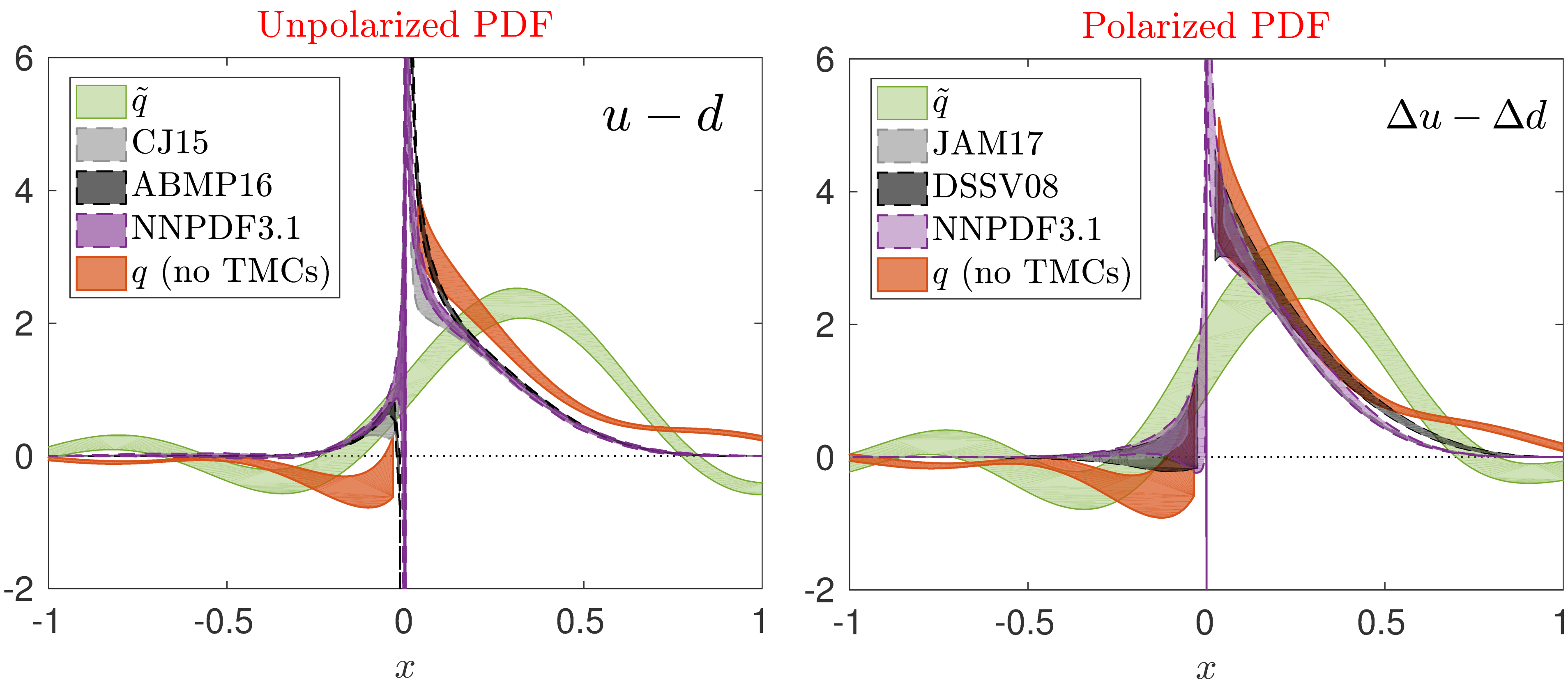

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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Nucleon mass corrections

In the infinite momentum frame, nucleon mass does not matter, i.e. $m_N/P_3 = 0$.

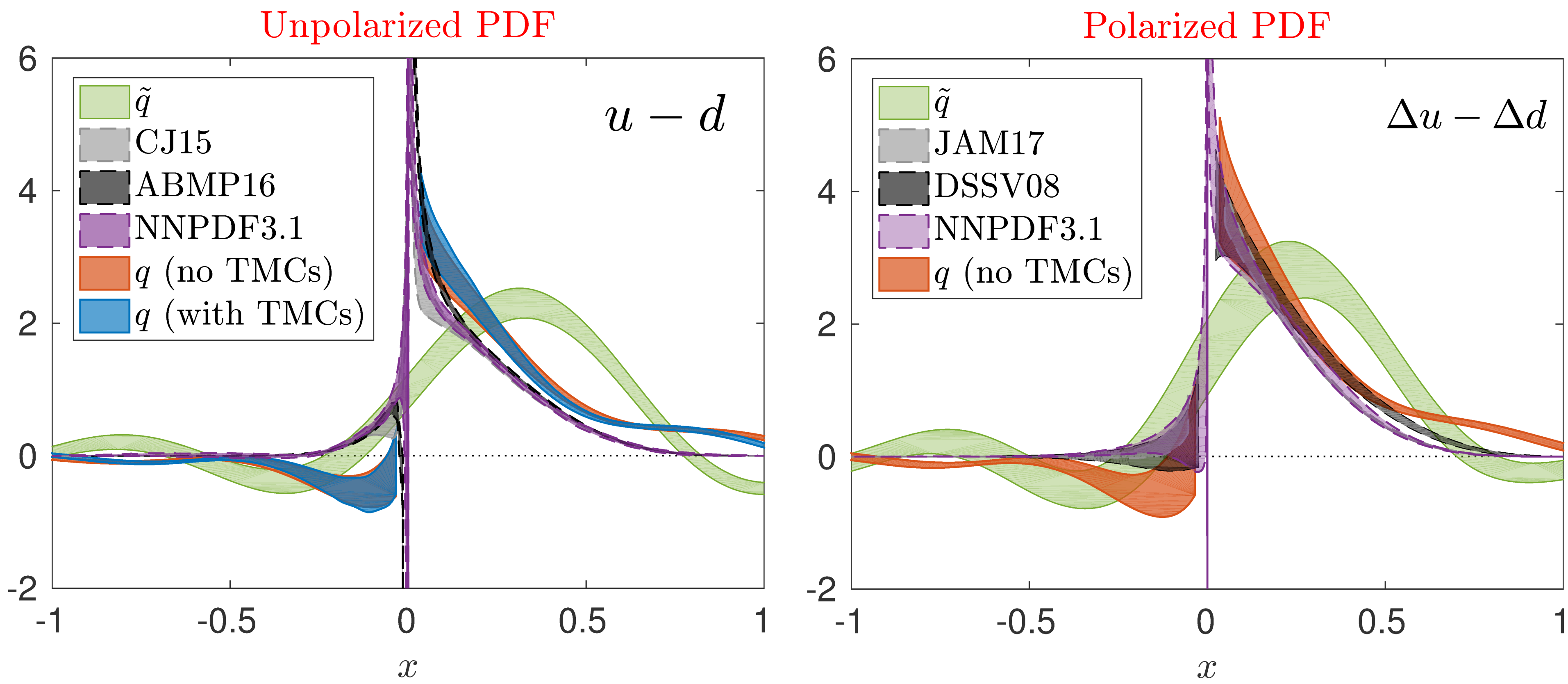
Here, we work with nucleon boosted to finite momentum P_3 and we need to correct for $m_N/P_3 \neq 0$.

These corrections were derived in:
[J.W. Chen et al., Nucl.Phys. B911 (2016) 246-273, arXiv:1603.06664 [hep-ph]]

Important feature: particle number is conserved in nucleon mass corrections.

Matched PDF + TMCs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



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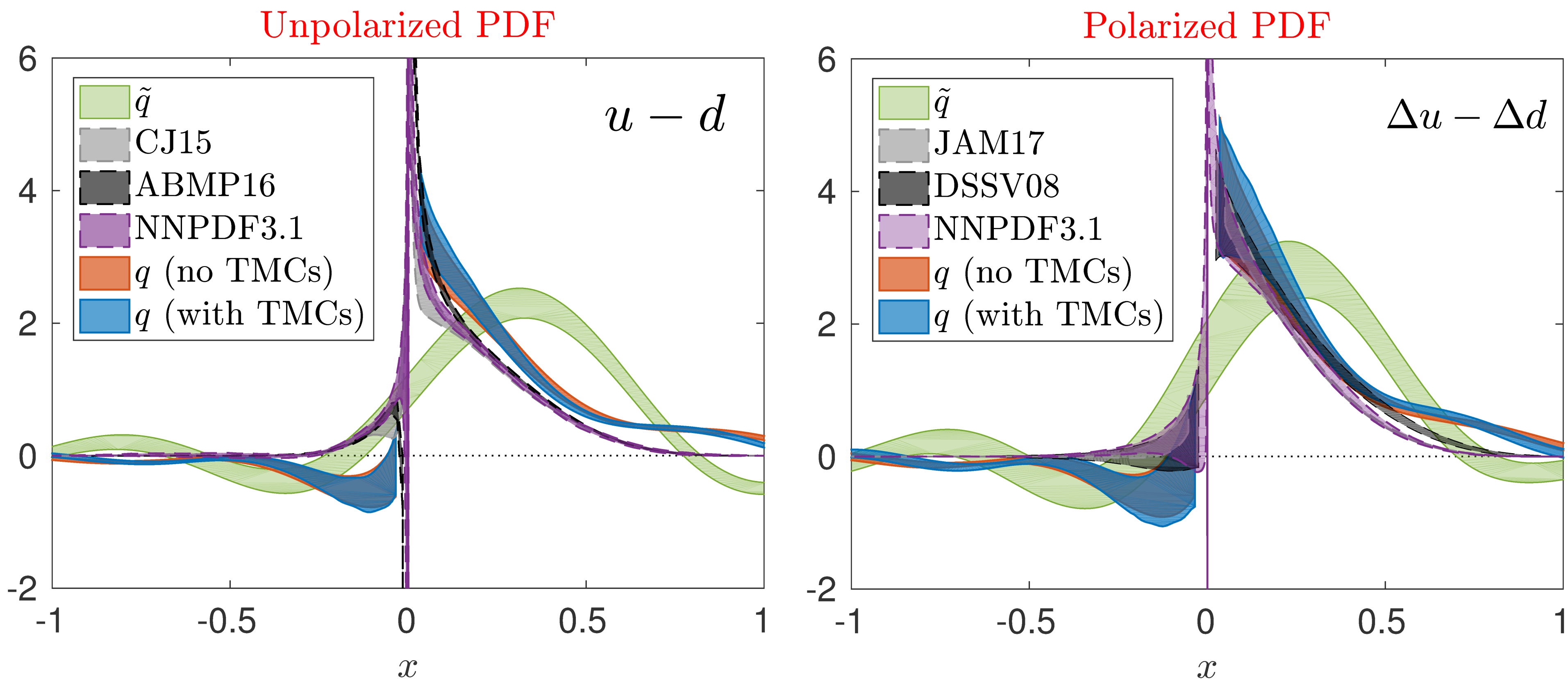
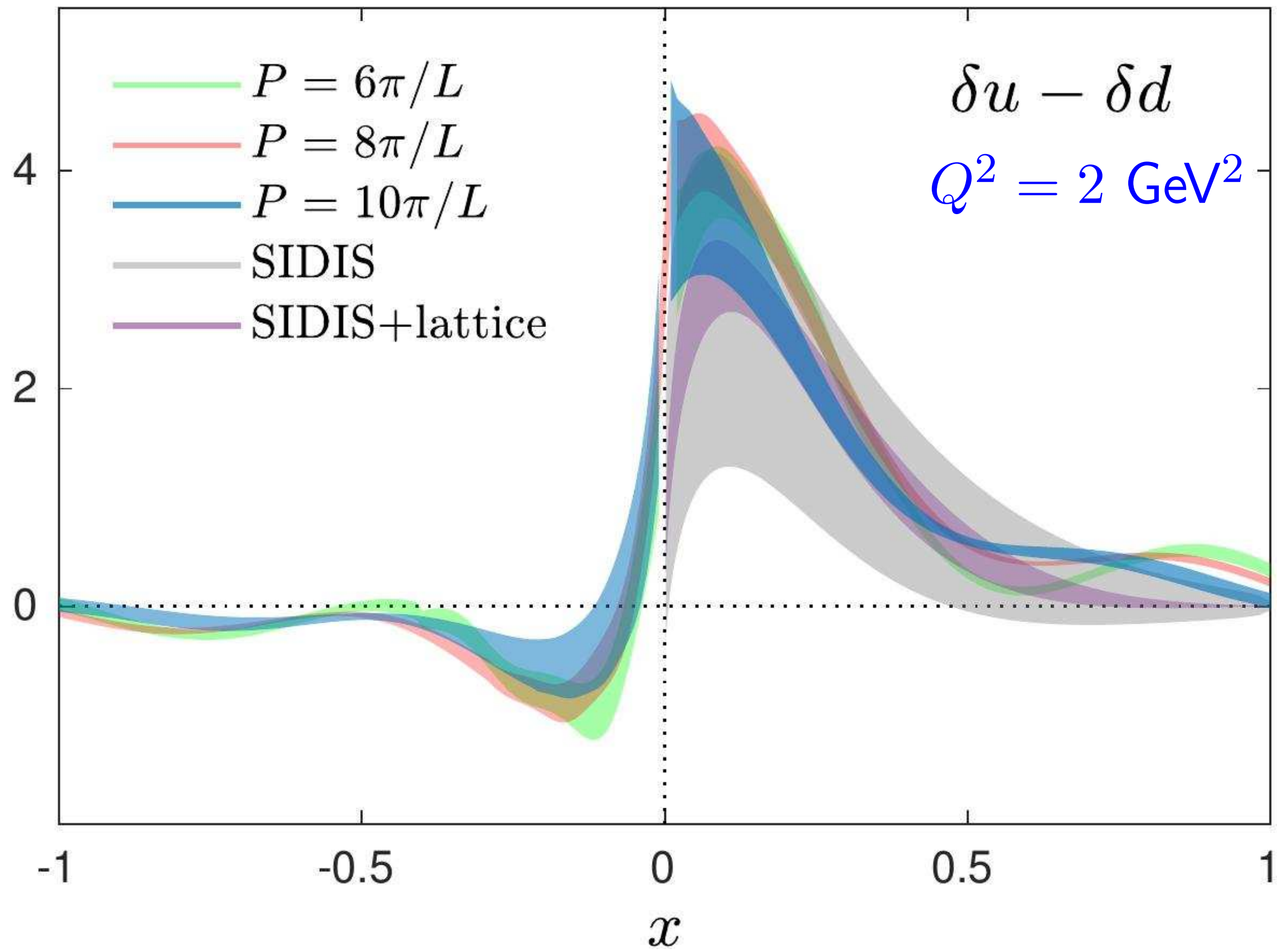


C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Transversity PDF

C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)



Statistical precision already much better than the precision of phenomenological fits from SIDIS: JAM Collaboration, Phys. Rev. Lett. 120 (2018) 152502

Different systematic effects:

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- pion mass

Systematics

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Different systematic effects:

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Investigation of several of these systematics in:
C. Alexandrou et al. [ETM Collaboration], "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point",
Phys. Rev. D99 (2019) 114504.

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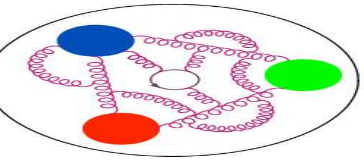

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After renormalization, the above MEs can be decomposed into MEs of quasi-GPDs:

$$\mathcal{M}(z,t,\xi;\,\mu_R;\,\Gamma,\overline{\Gamma}) = \mathcal{K}_H(\Gamma,\overline{\Gamma})H(z,t,\xi;\mu_R) + \mathcal{K}_E(\Gamma,\overline{\Gamma})E(z,t,\xi;\mu_R).$$

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For example, for unpolarized GPDs one can disentangle the H and E MEs using the projectors: $\Gamma_0 \equiv \frac{1+\gamma_0}{4}$ and $\Gamma_2 \equiv \frac{1+\gamma_0}{4}\gamma_5\gamma_2$.

Bare matrix elements

Lattice setup: ETMC, arXiv:1910.13229

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
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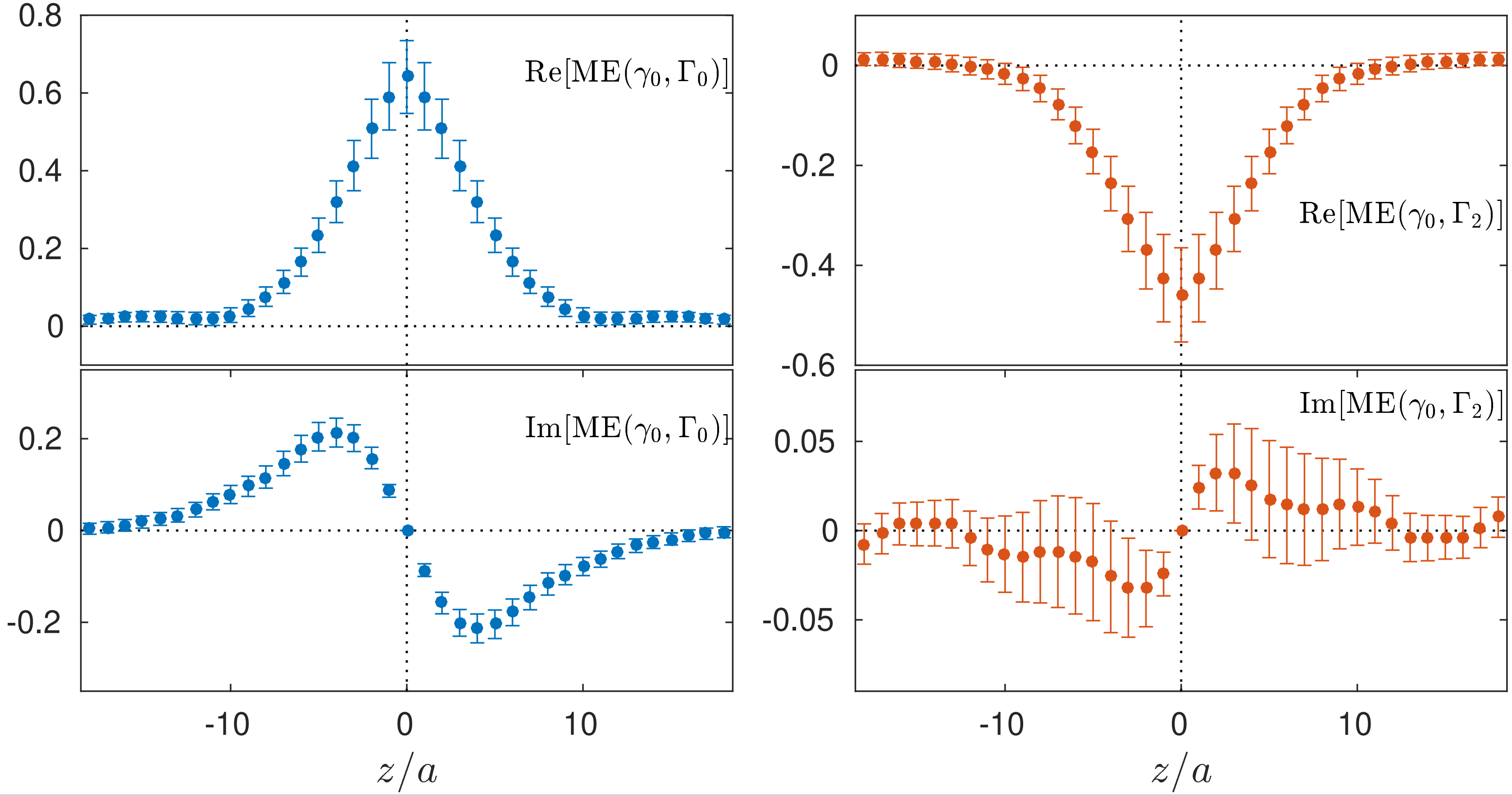

$P_3 = 0.83$ GeV
$Q^2 = 0.69$ GeV^2
$\xi = 0$

left: Γ_0 projector
right: Γ_2 projector

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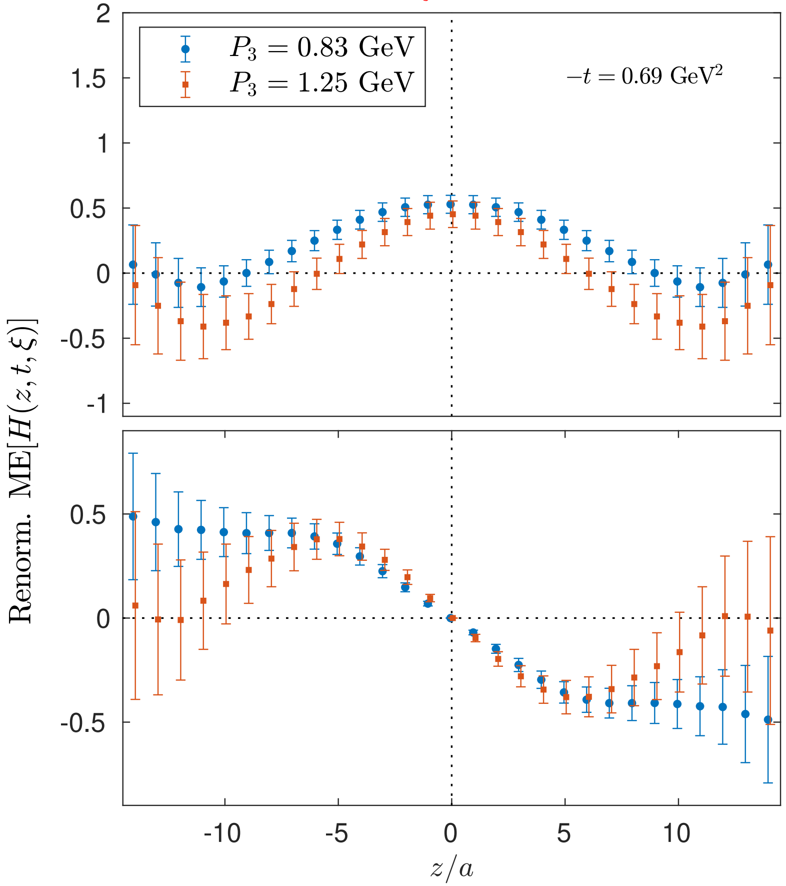

H-function

left: $\xi = 0$

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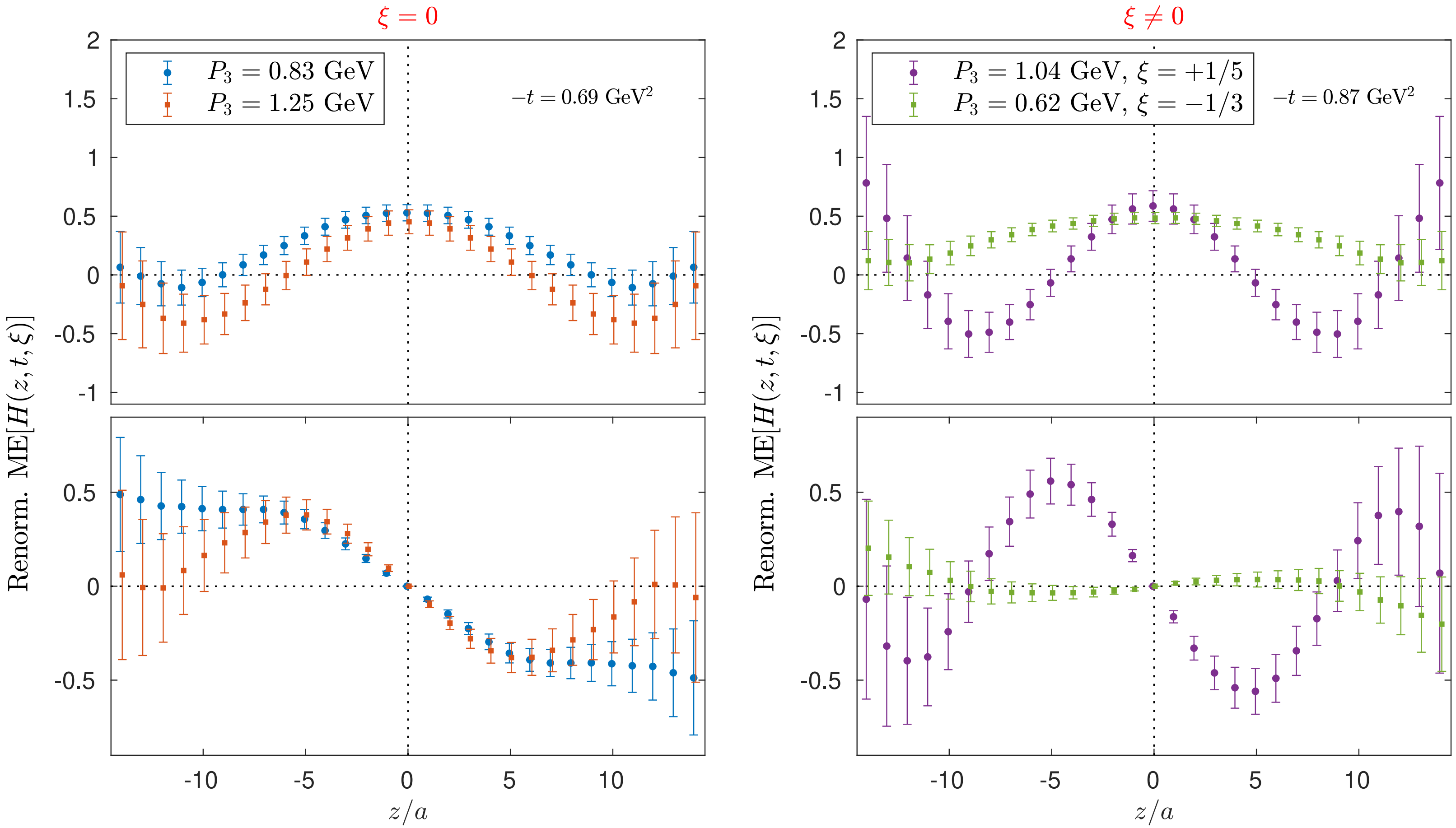

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left: $\xi = 0$

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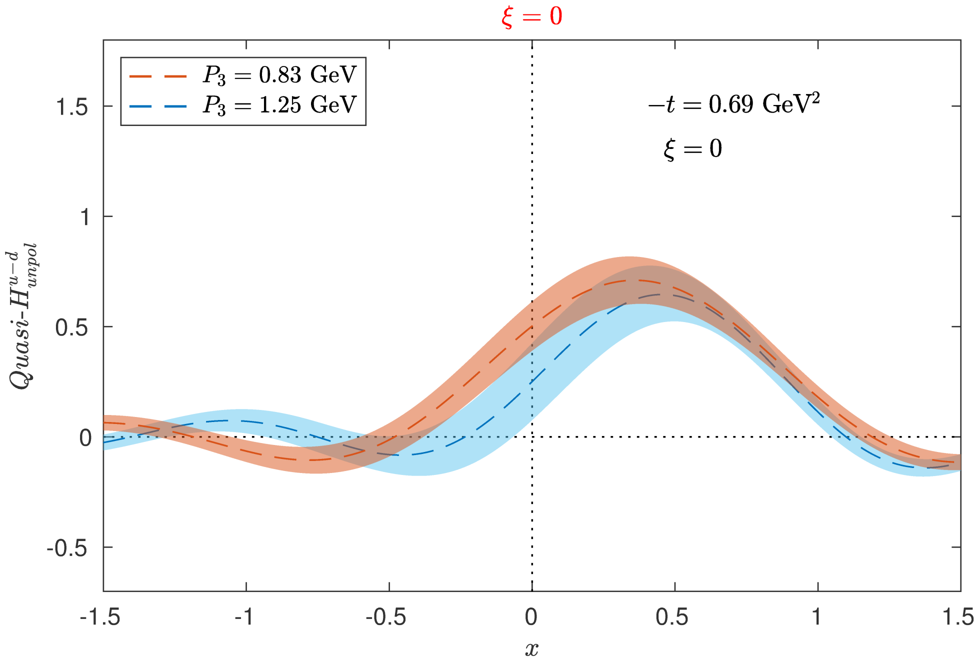

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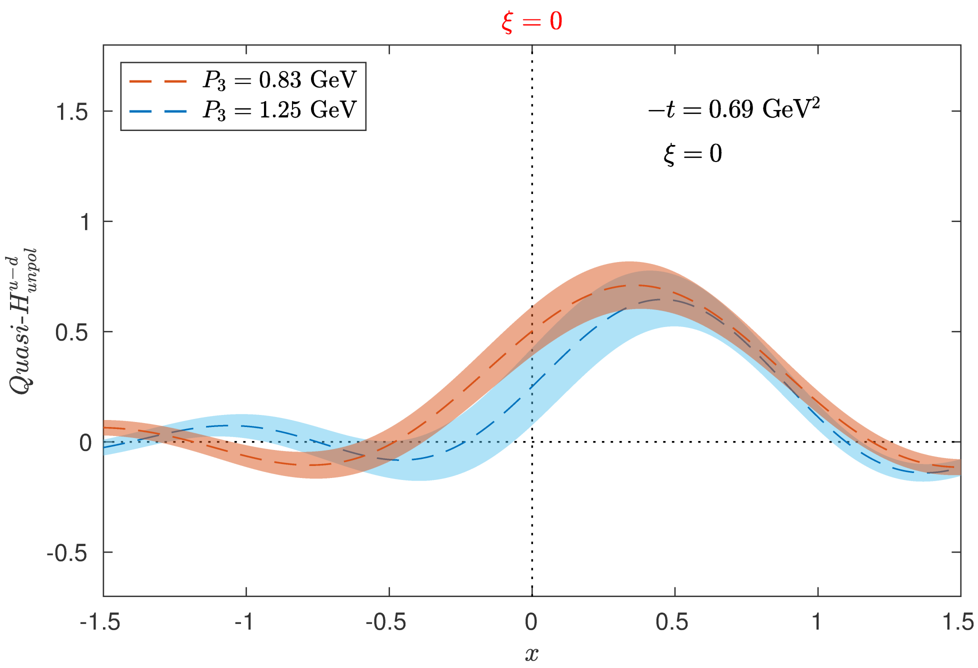

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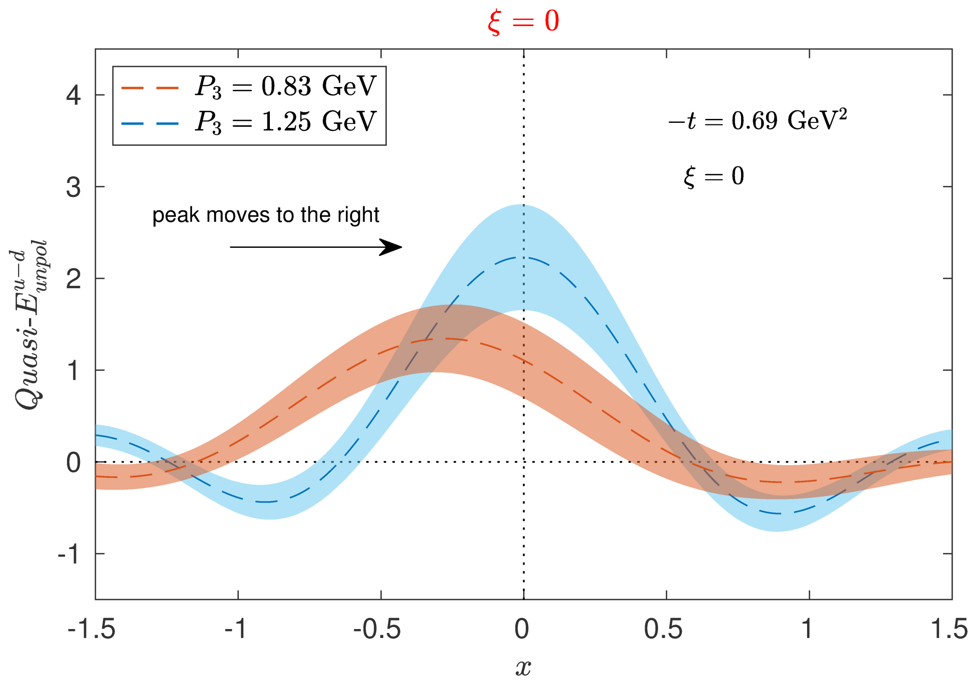

quasi-E function

After matching: H and E functions

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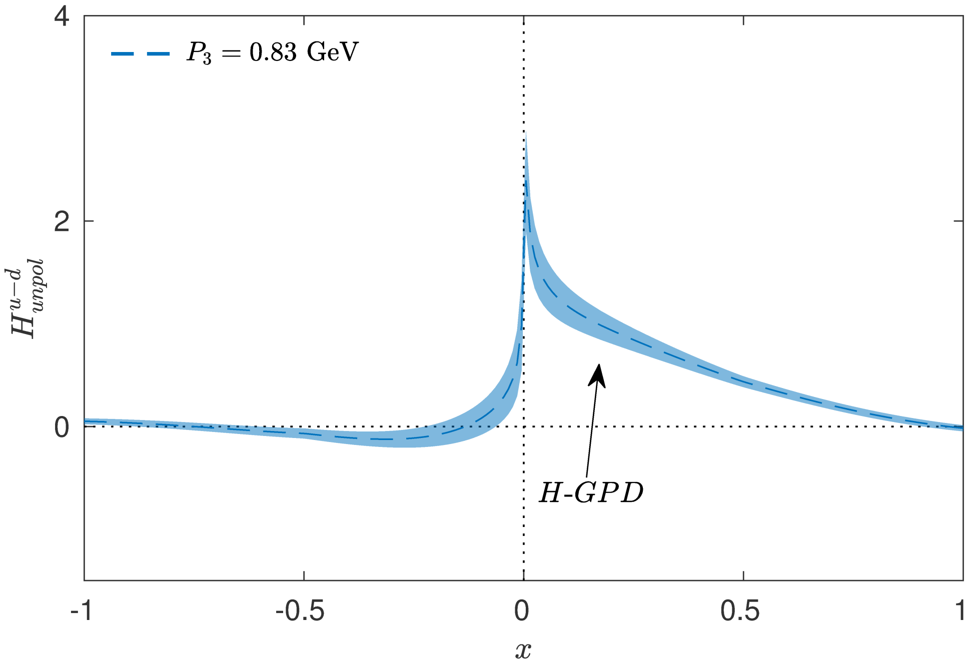

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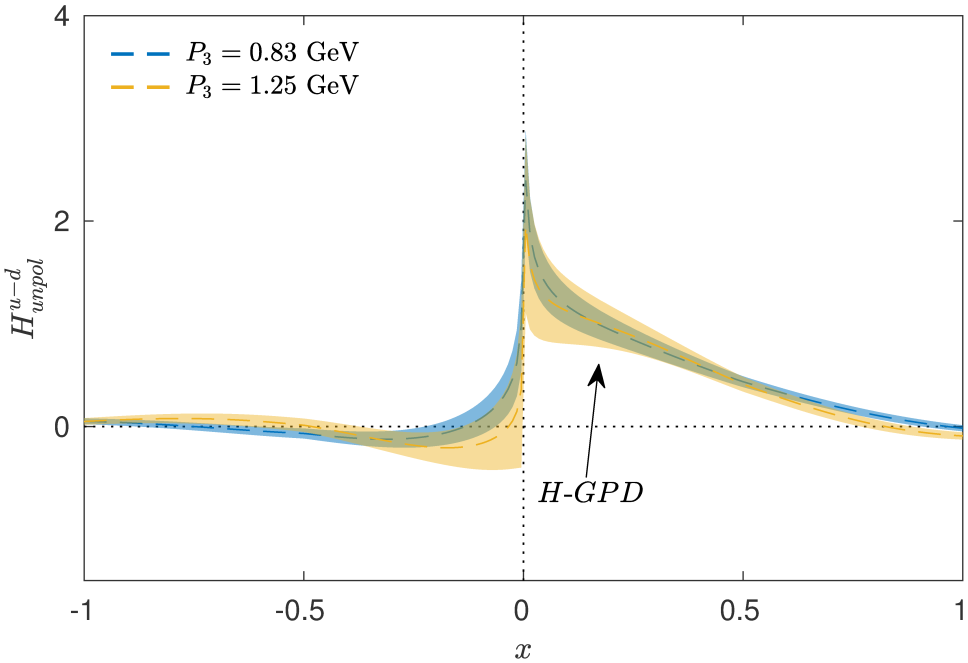

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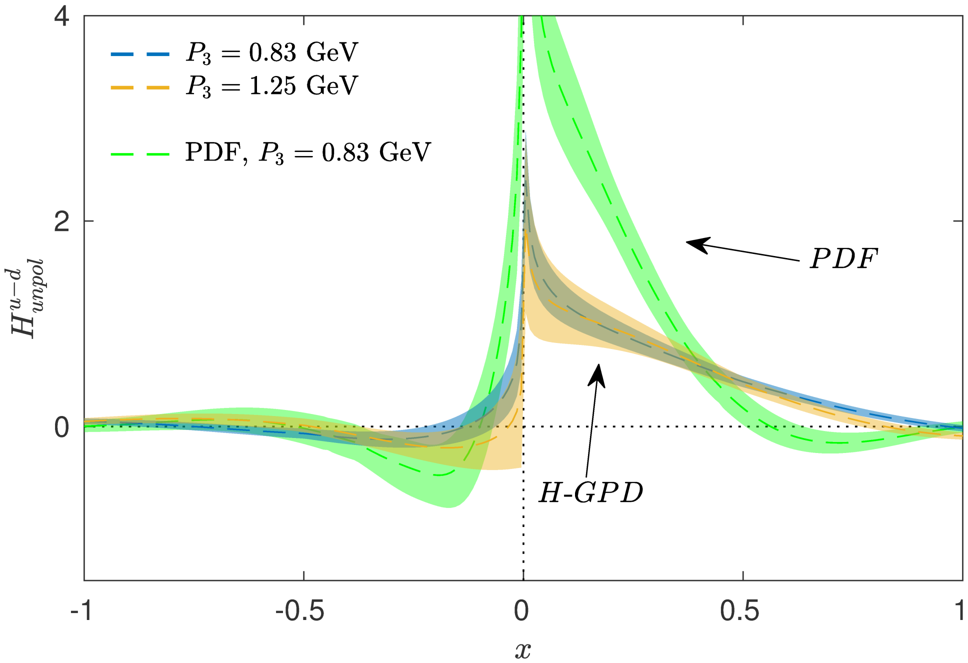

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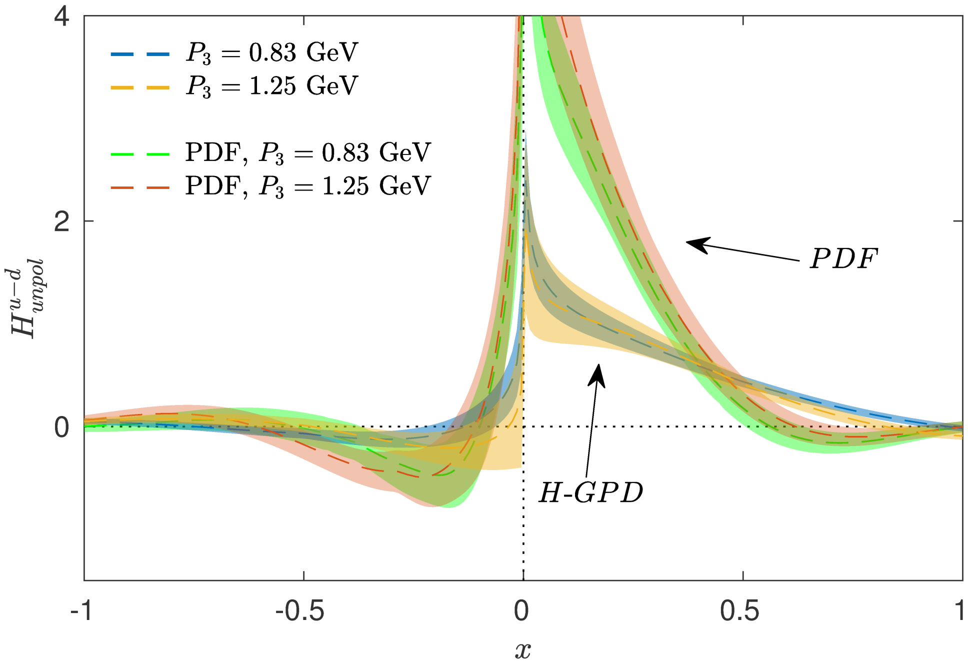

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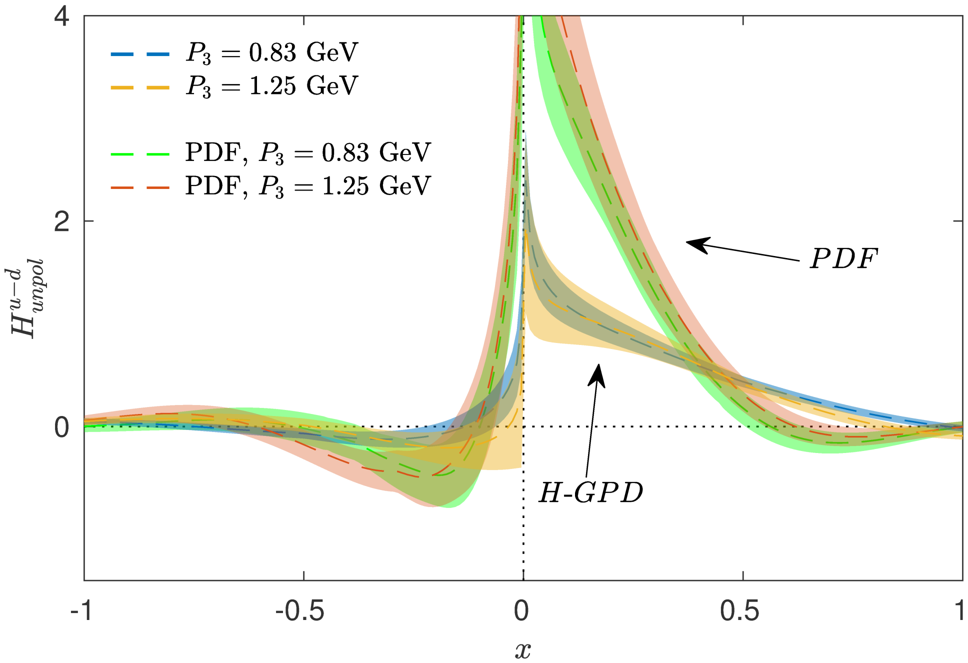

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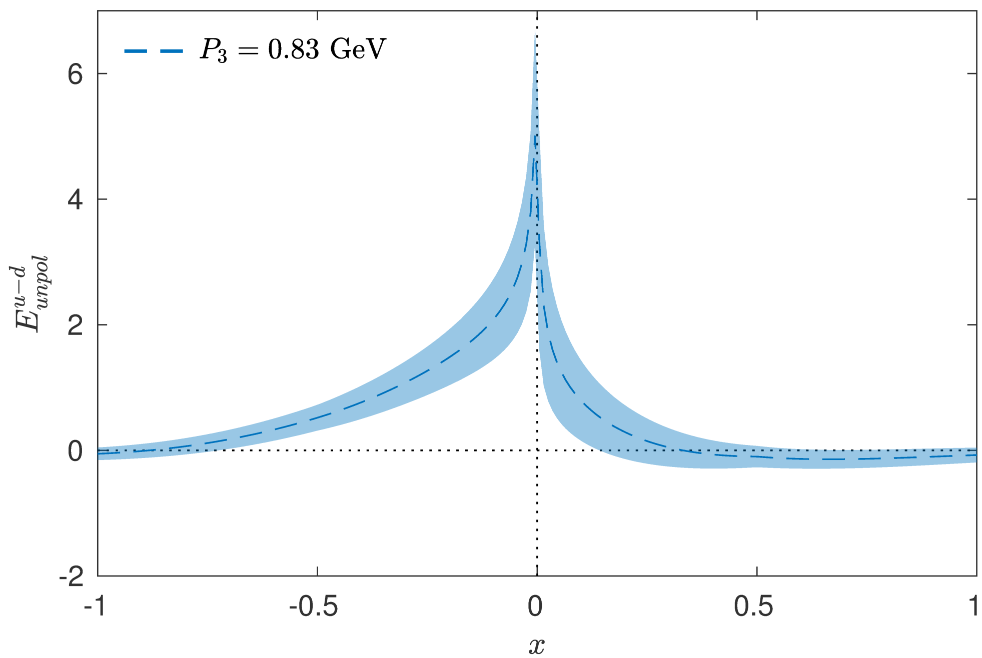

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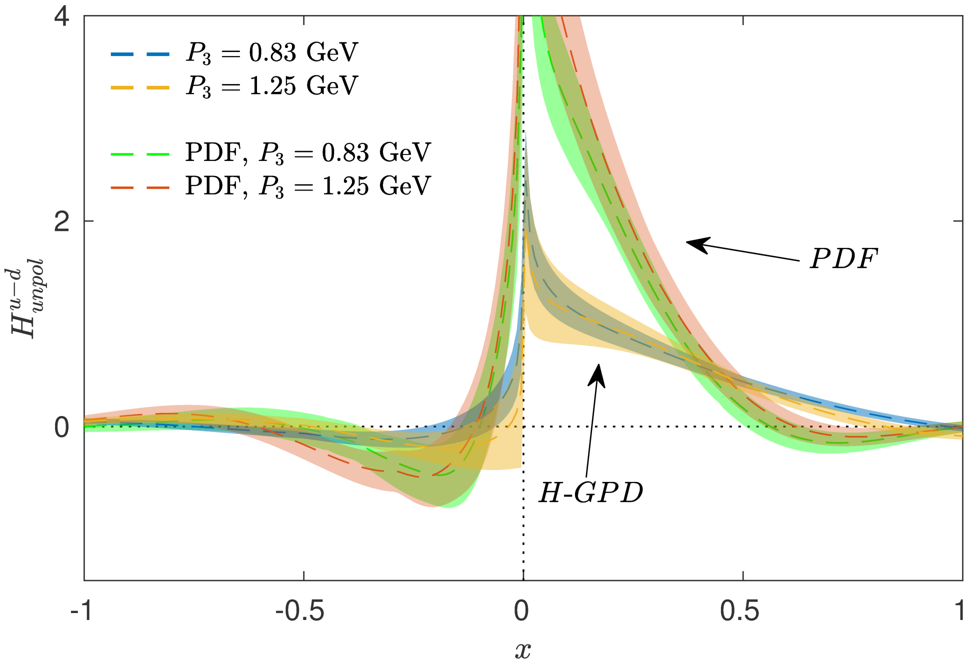

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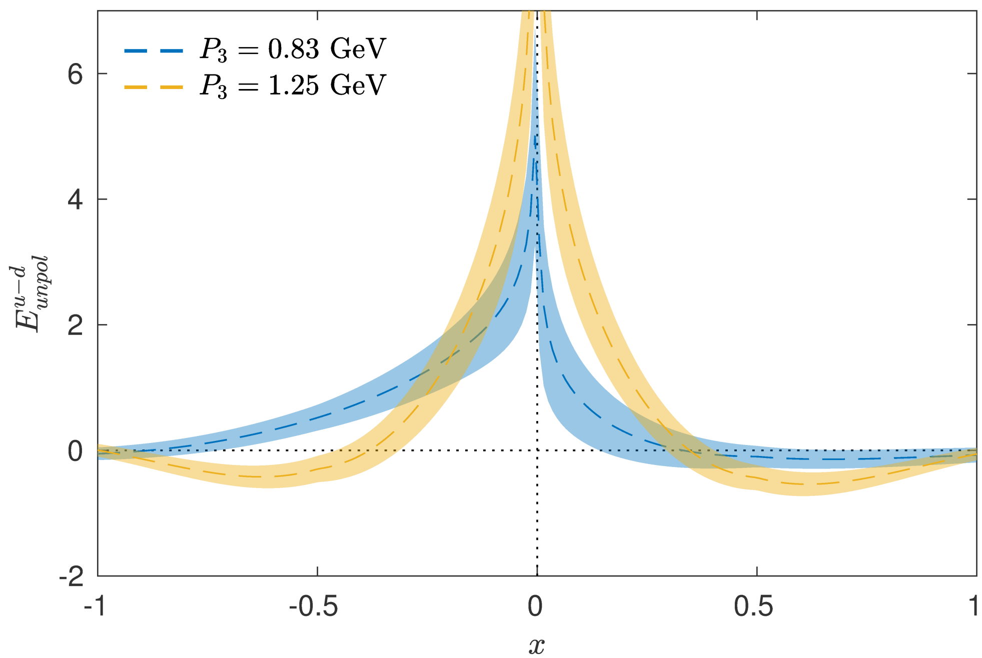

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- NN parametrization: $V_3/T_3\,(x,\mu) \propto x^{\alpha_{V/T}}\,(1-x)^{\beta_{V/T}}\,\mathrm{NN}_{V/T}\,(x)$.

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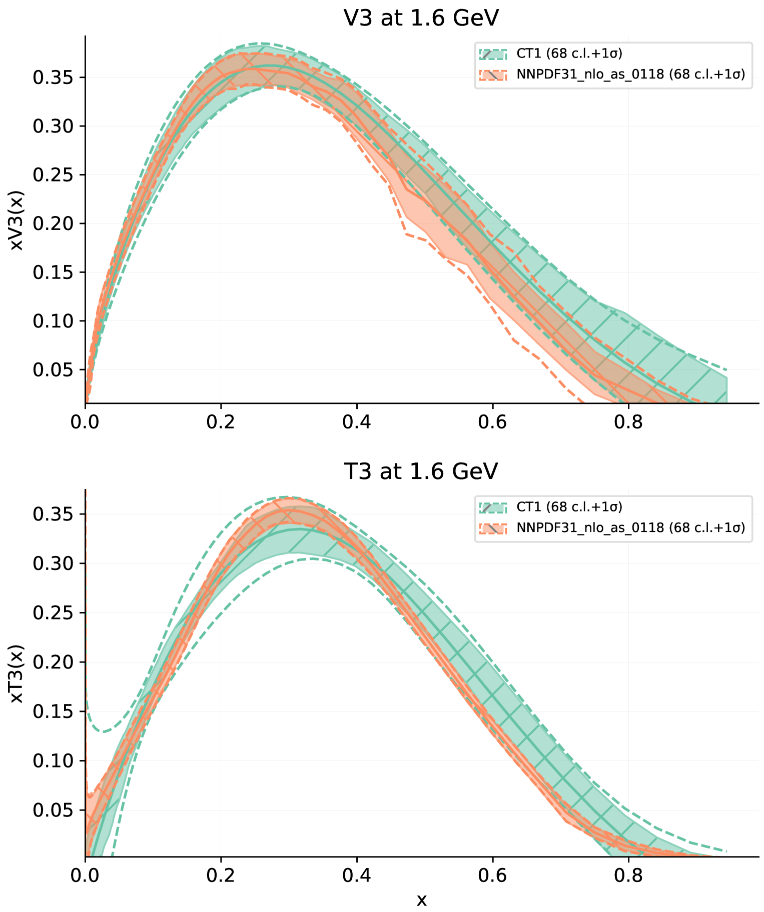

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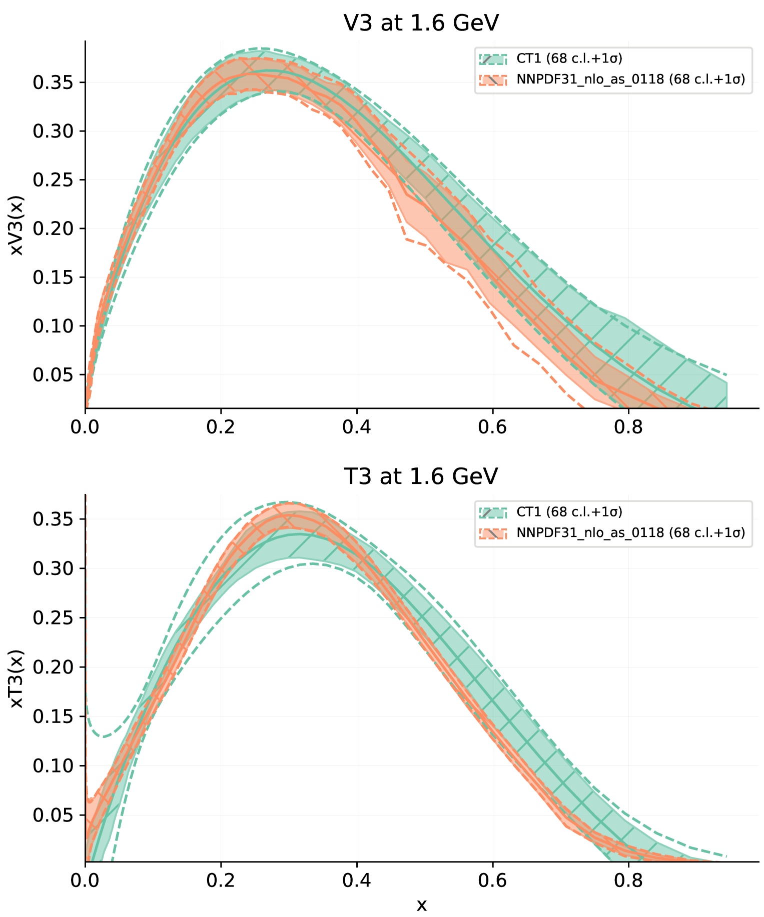

Very robust result!

pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

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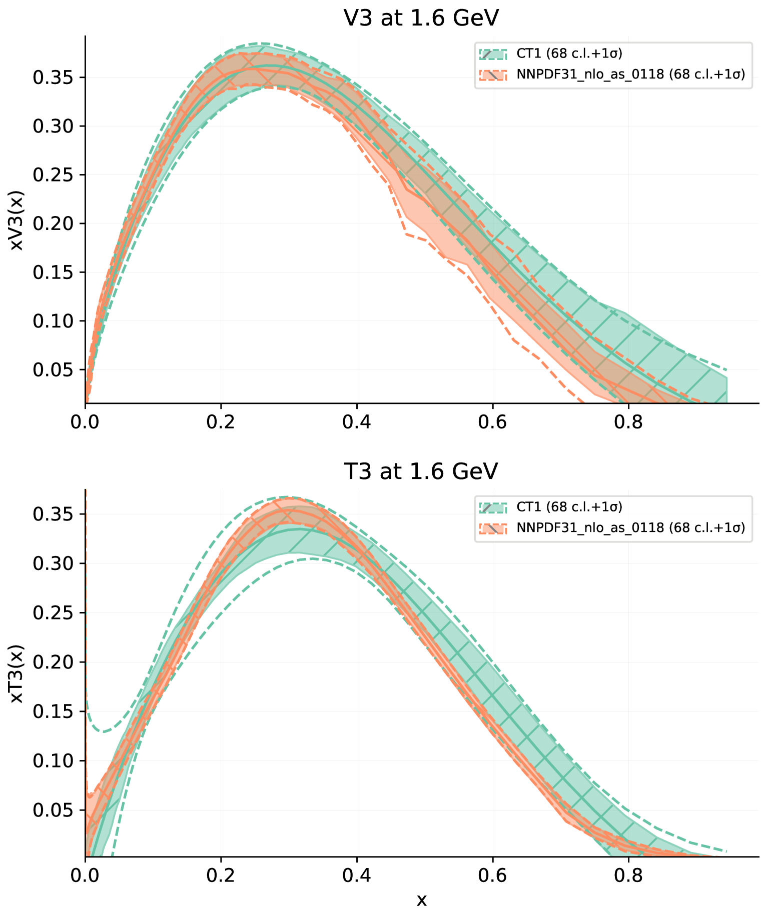


only error of NNPDF

Very robust result!

pseudo data:
1. DGLAP evolution 1.65→2 GeV
2. inverse matching
3. inverse Fourier

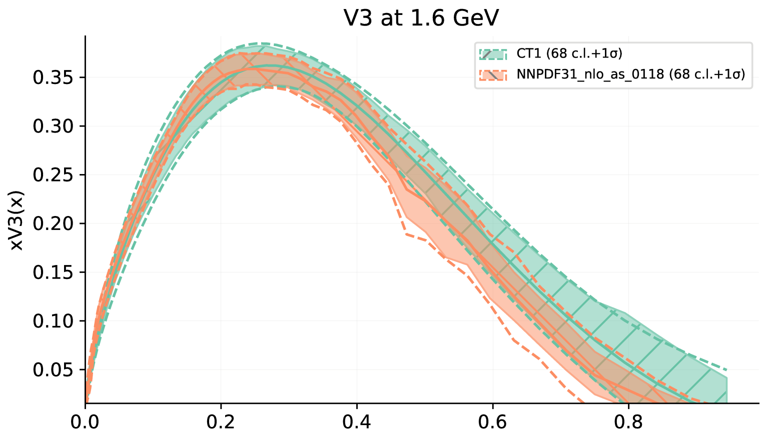
reconstruction:
1. NN fit
2. matching
3. DGLAP evolution 2→1.65 GeV

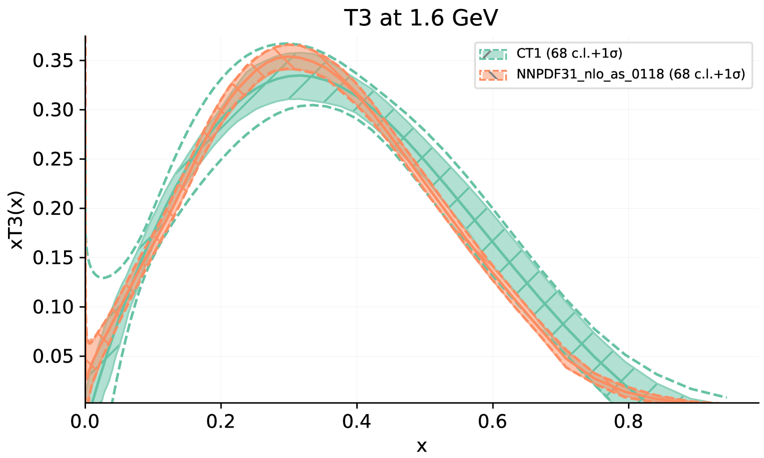
Shows the power of the convolution ⊛ in constraining PDFs! (only 16 lat. points!)

See also:
J.Karpie et al., JHEP04(2019)057

Closure tests

- Exercise: generate pseudo data from a selected NNPDF and run fitting code over them.
- 16 "lattice points" generated (16 real, 15 imaginary)
- Test different scenarios: K.C., L. Del Debbio, T. Giani, JHEP 10 (2019) 137





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3. inverse Fourier

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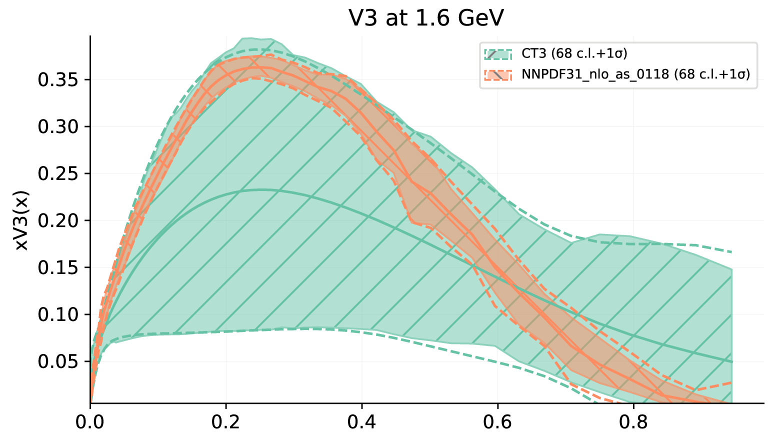

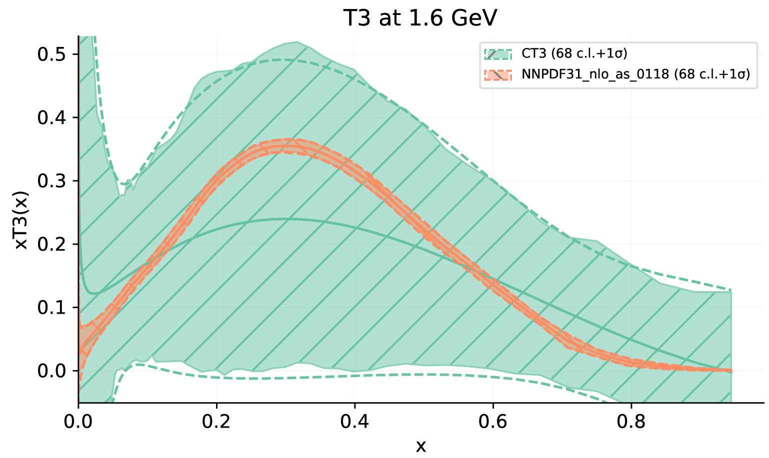

stat.error of ETMC lattice data + a scenario for systematics

- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.

Fitting actual lattice data

- We also took actual ETMC lattice data for the unpolarized case and used the NNPDF framework to calculate the resulting V_3 and T_3 distributions.
- We took actual statistical errors and considered different scenarios for systematics:

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
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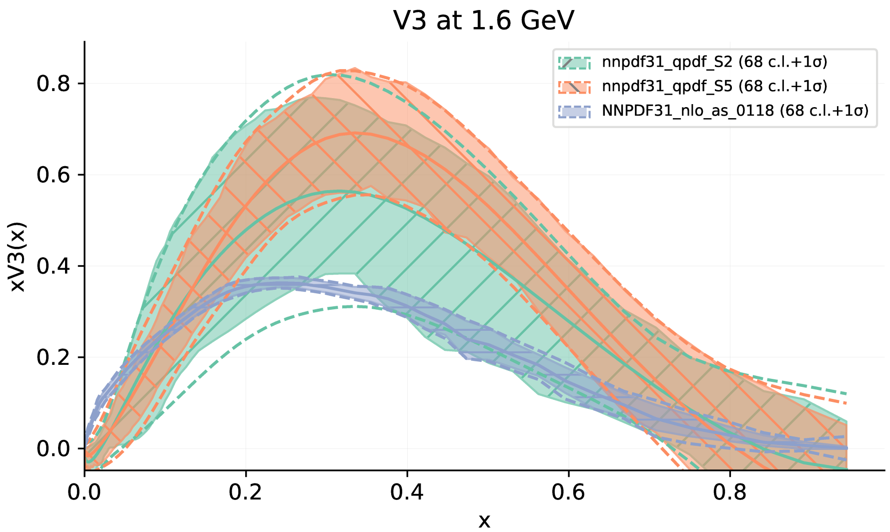
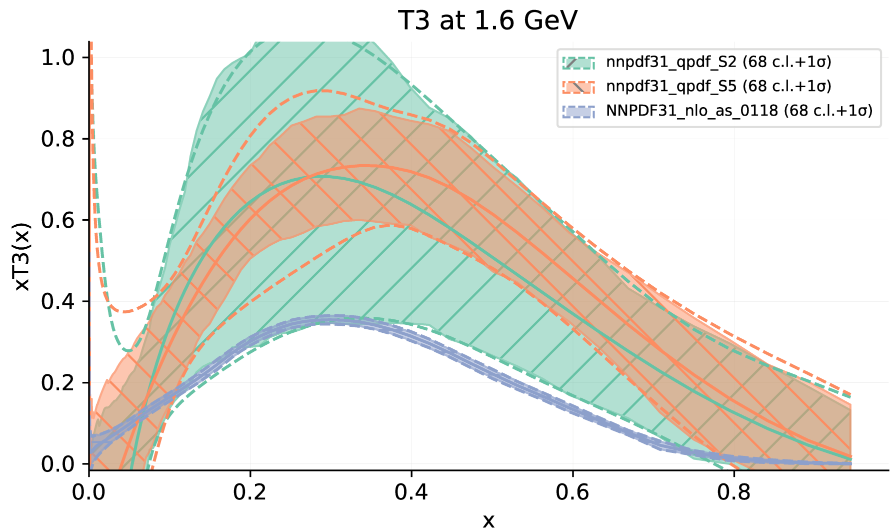
Results from scenarios S2 and S5 ("realistic"):

K.C., L. Del Debbio, T. Giani
JHEP 10 (2019) 137





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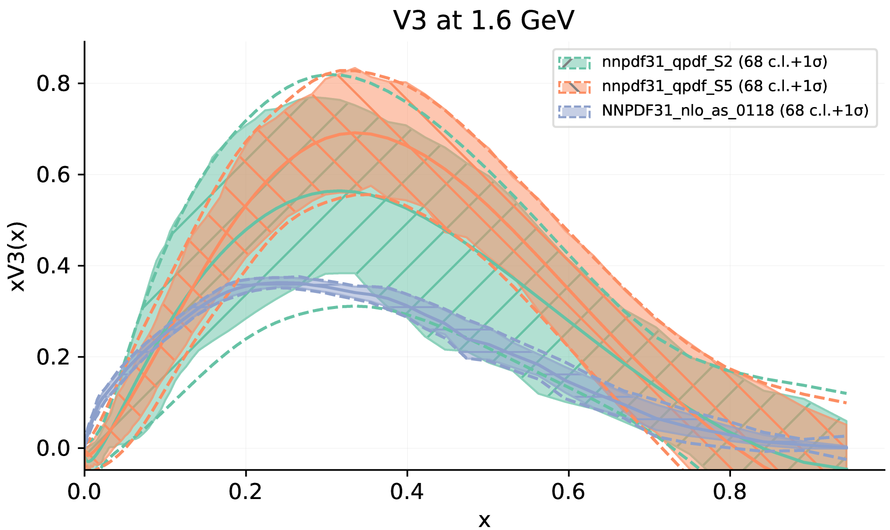

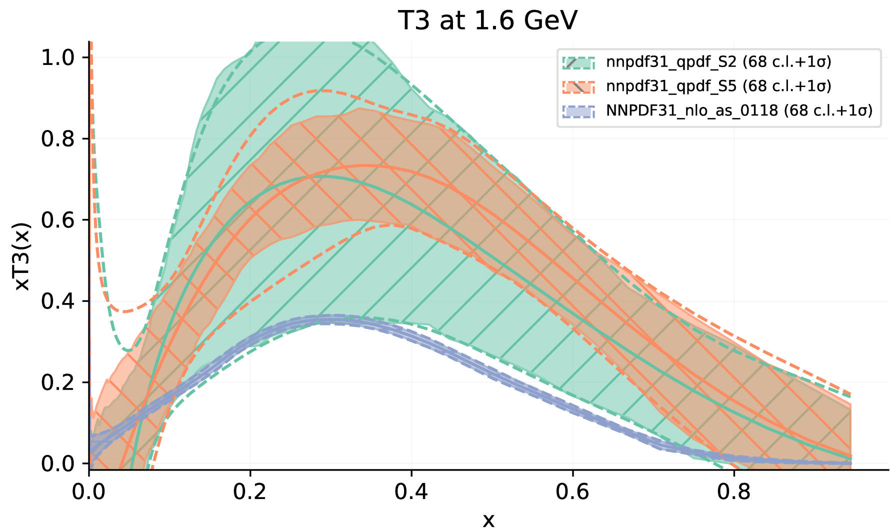

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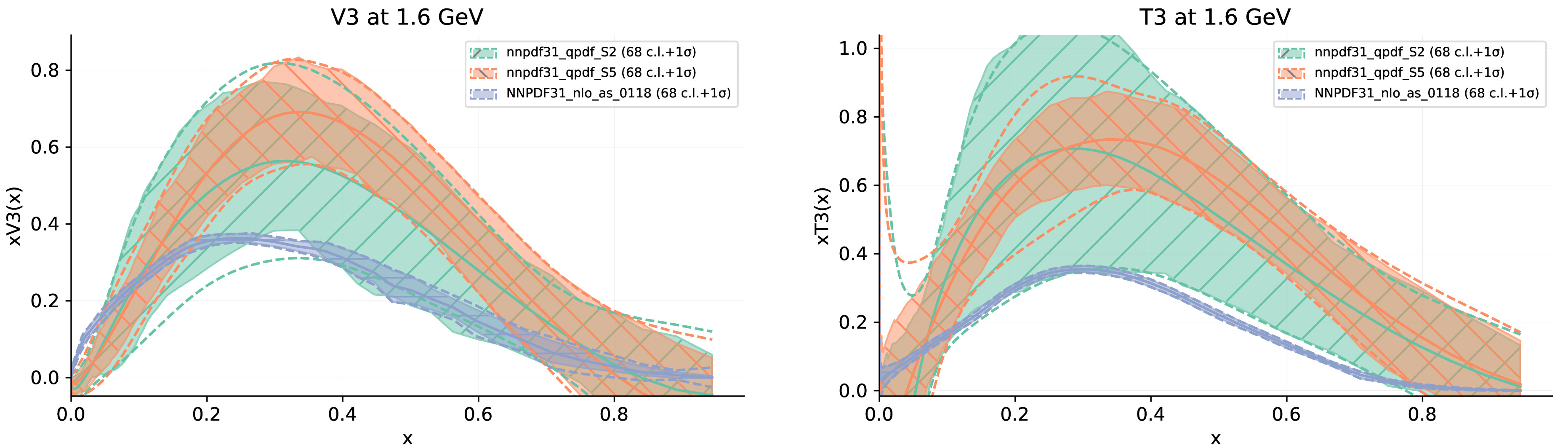

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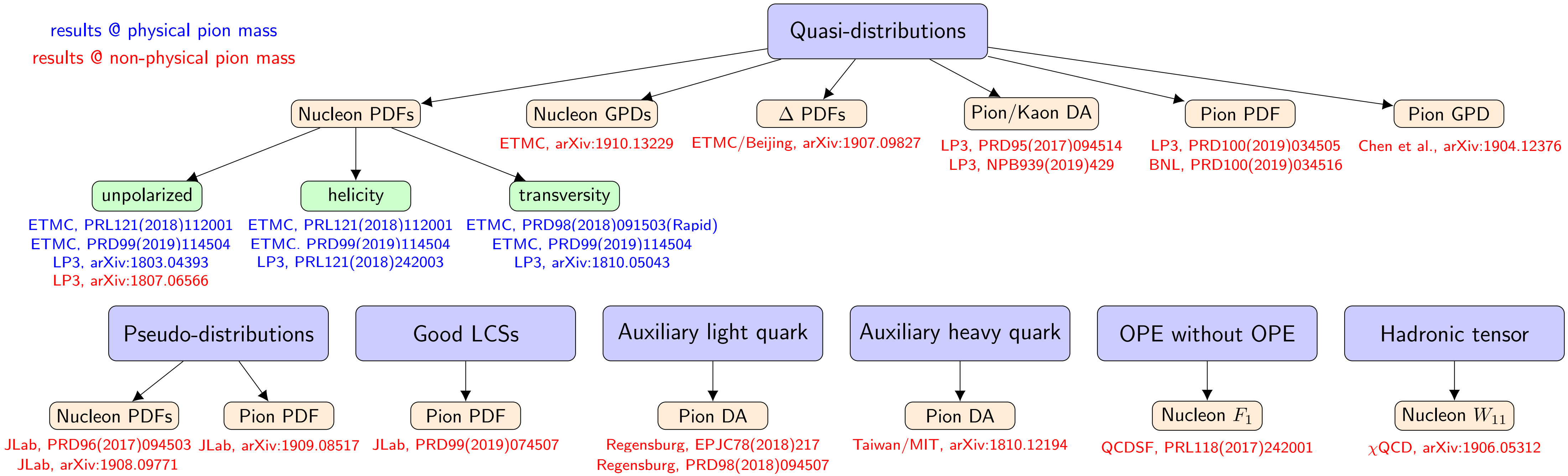
Overview of results from different approaches

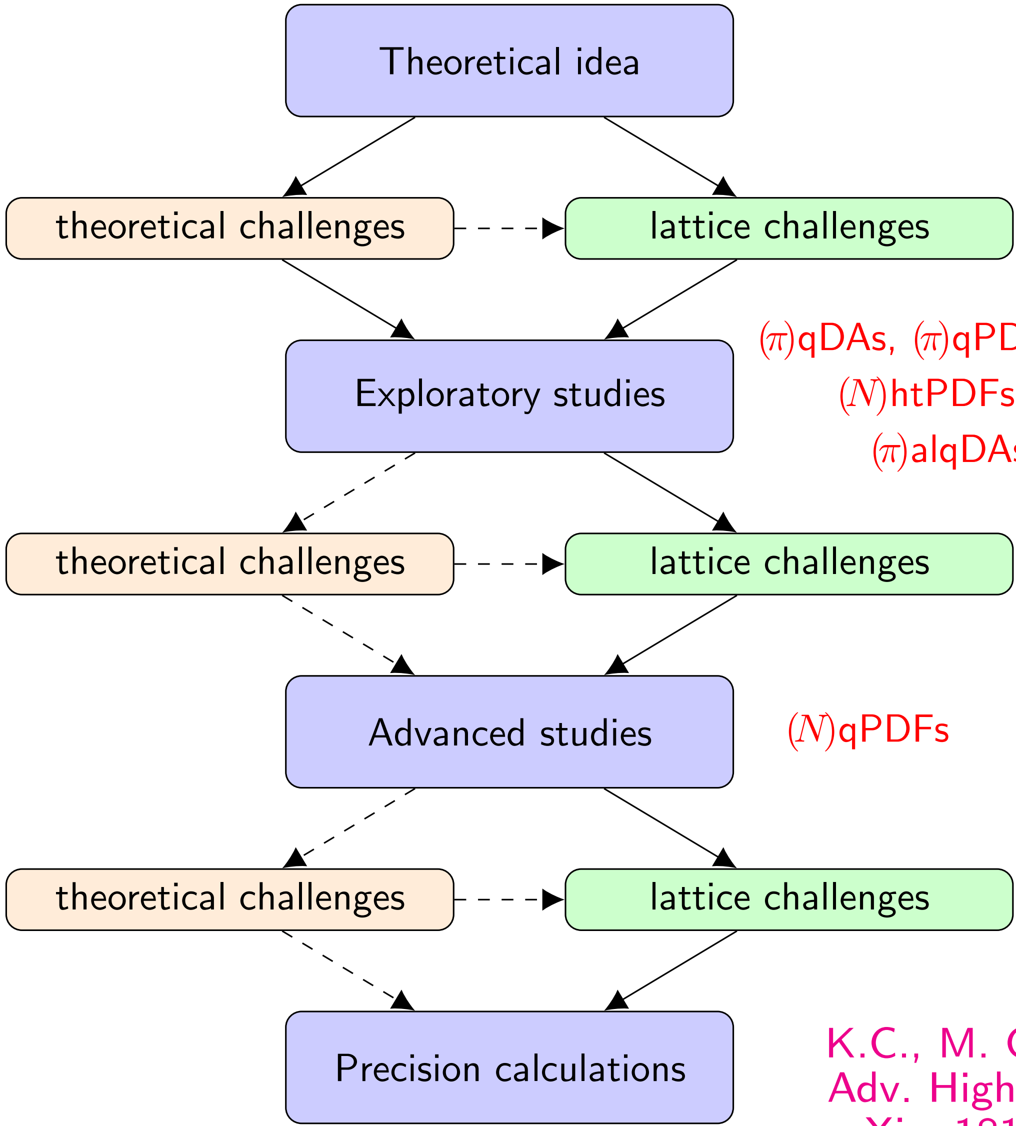


Approaches to light-cone PDFs

2018

(N)qTMDs

(N)qGPDs



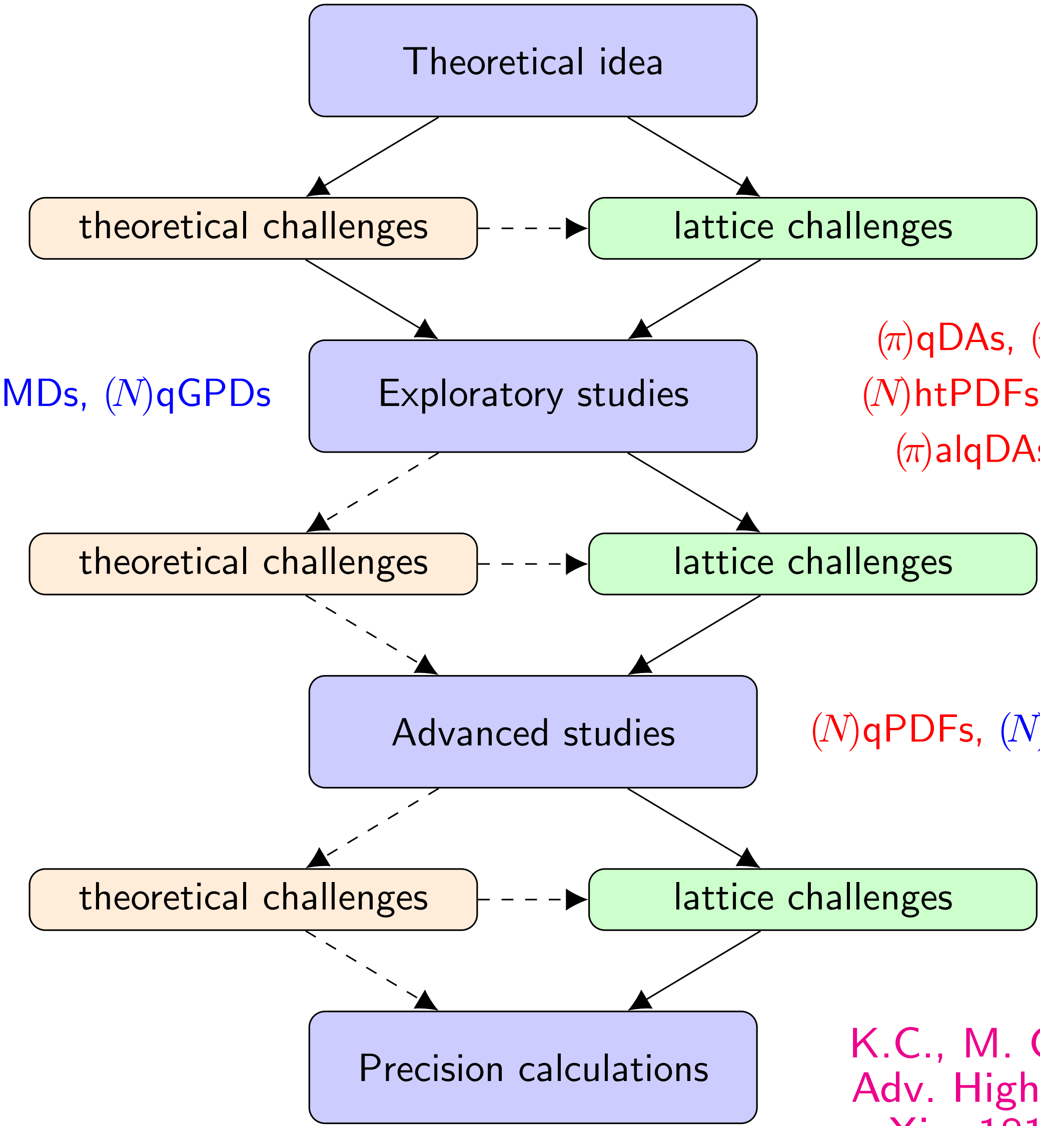
(π)qDAs, (π)qPDFs, (N,π)qPDFs(g), (N)pPDFs
(N)htPDFs, (N)opePDFs, (π)ahqDAs
(π)alqDAs, (π)lcsDAs, (π)lcsPDFs

(N)qPDFs

K.C., M. Constantinou
Adv. High Energy Phys. 2019, 3036904
arXiv: 1811.07248 [hep-lat]

Approaches to light-cone PDFs

2019
(update)



(N)qTMDs, (N)qGPDs

(π)qDAs, (π)qPDFs, (N,π)qPDFs(g)
(N)htPDFs, (N)opePDFs, (π)ahqDAs
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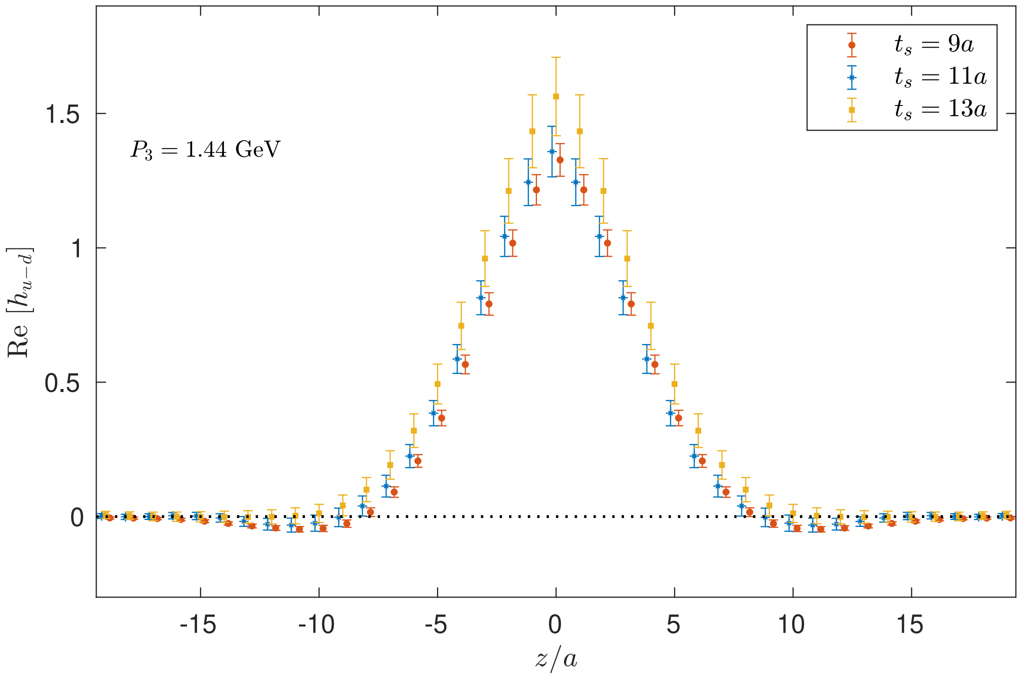
Thank you for your attention!

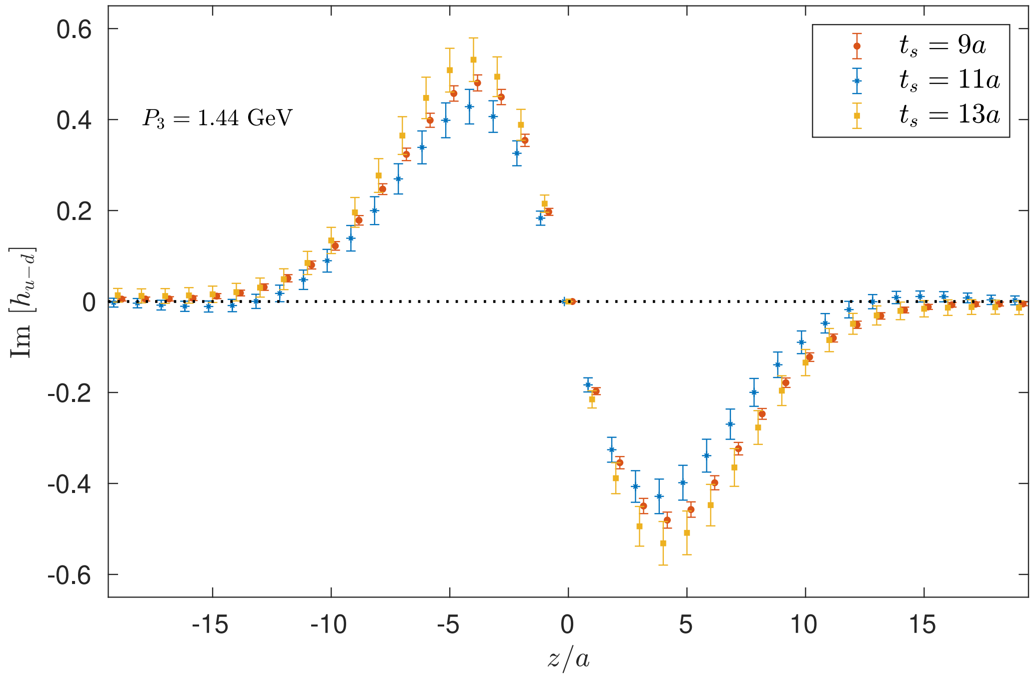
Backup slides

Preliminary new results – qPDFs $N_f = 2+1+1$

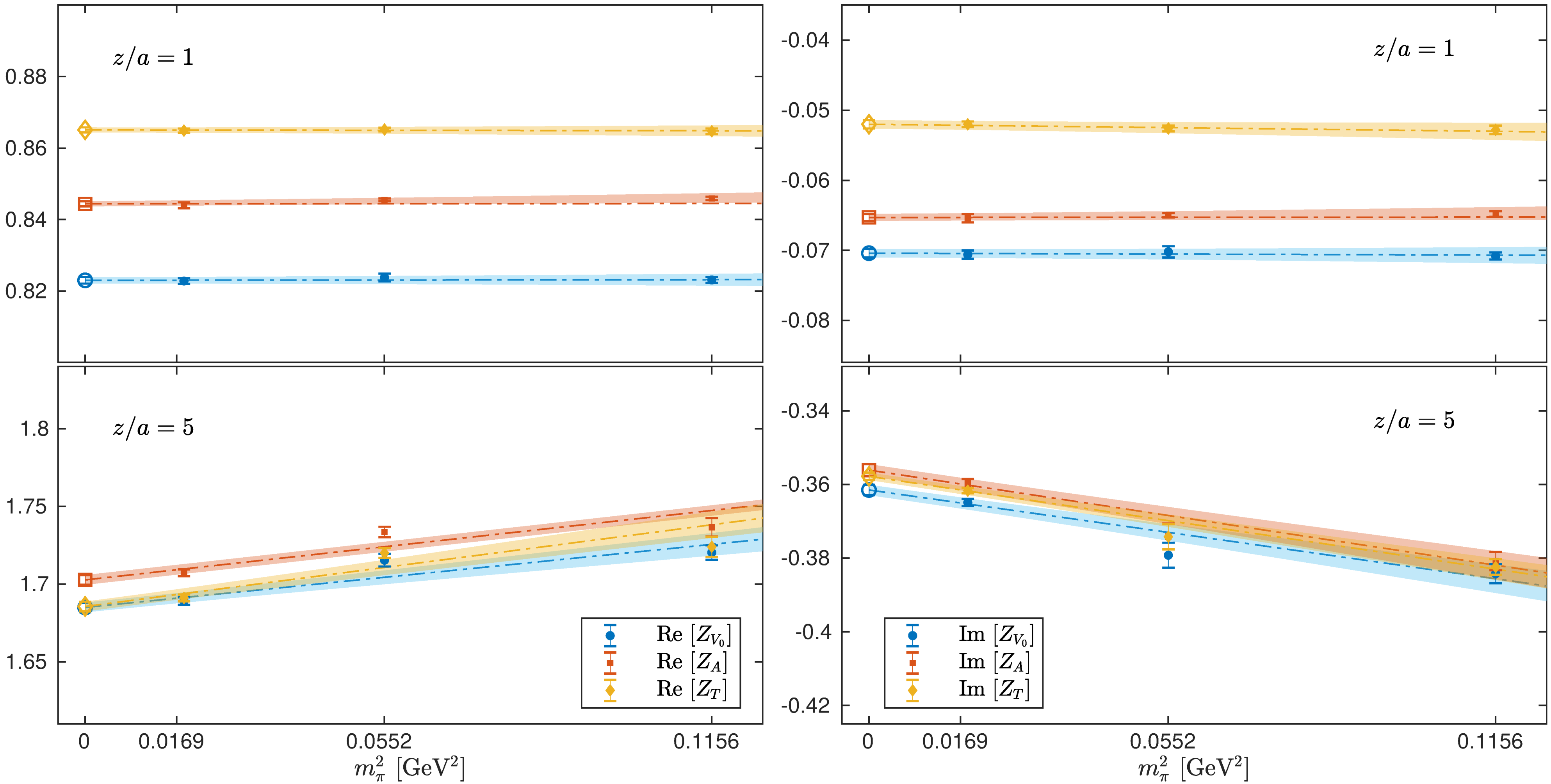
- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- $64^3 \times 128$, $L = 5.2$ fm, $m_\pi L = 3.55$,
- a=0.081 fm
- physical pion mass,
- around 30000 measurements and increasing.

ETMC, arXiv:1910.13229





Pion mass dependence of Z-factors

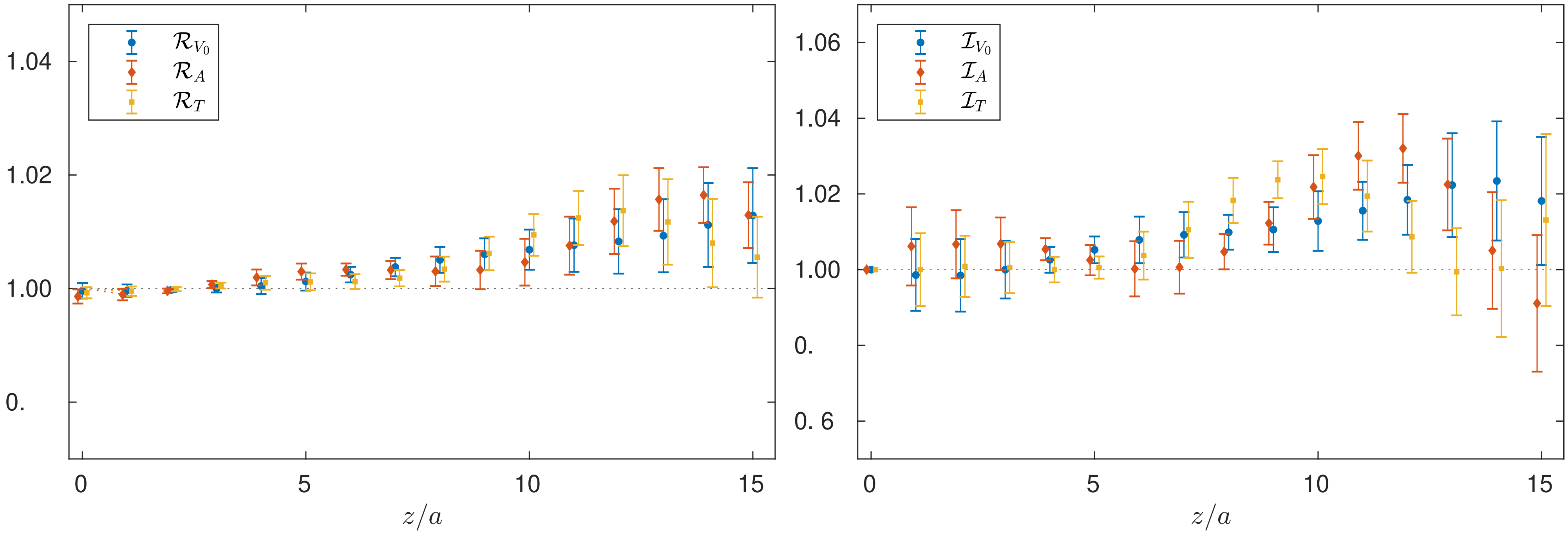


C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Possibly enhanced FVE in non-local operators suggested in:

R. Briceño, J. Guerrero, M. Hansen, C. Monahan, Phys. Rev. D98 (2018) 014511

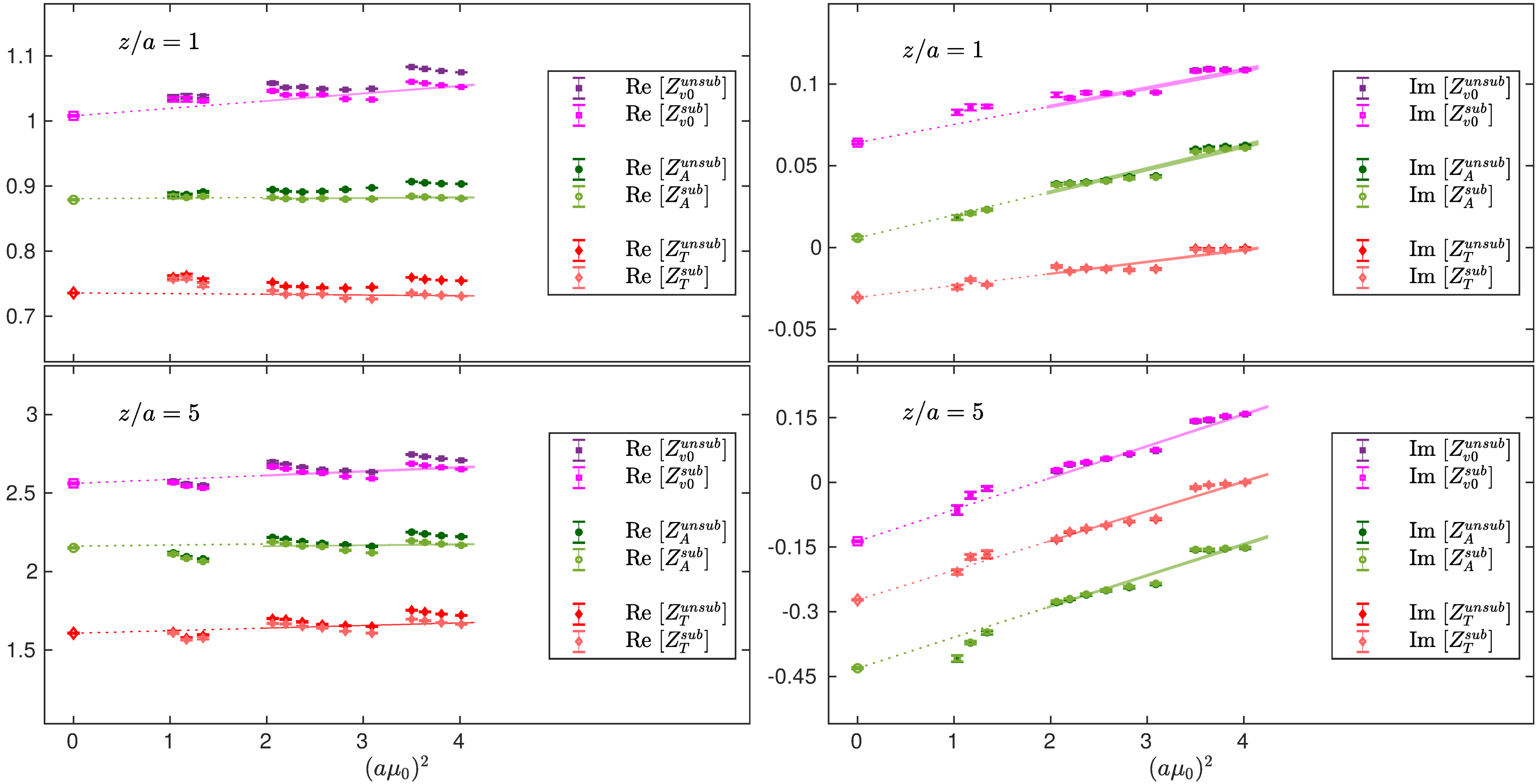
$$\mathcal{R}_{\mathcal{O}}(z) \equiv \frac{\mathrm{Re}[Z^{\mathrm{RI}'}_{\mathcal{O},64}(z,\mu_0,m_\pi)]}{\mathrm{Re}[Z^{\mathrm{RI}'}_{\mathcal{O},48}(z,\mu_0,m_\pi)]}, \qquad \mathcal{I}_{\mathcal{O}}(z) \equiv \frac{\mathrm{Im}[Z^{\mathrm{RI}'}_{\mathcal{O},64}(z,\mu_0,m_\pi)]}{\mathrm{Im}[Z^{\mathrm{RI}'}_{\mathcal{O},48}\mathcal{O}(z,\mu_0,m_\pi)]}$$



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Z-factors can have $\mathcal{O}(g^2 a^\infty)$ artefacts perturbatively subtracted

By: M. Constantinou, H. Panagopoulos, e.g. Phys. Rev. D95 (2017) 034505



C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_{+(1)}^{[1,\infty]} - \frac{3}{2\xi} & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2 P_3^2}{\xi^2\mu^2}\left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_{+(1)}^{[0,1]} & 0 < \xi < 1 \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_{+(1)}^{[-\infty,0]} - \frac{3}{2(1-\xi)} & \xi < 0, \end{cases}$$

$$+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{\mu^2}{4y^2P_3^2} + \frac{5}{2}\right), \quad \iota{=}0 \text{ for } \gamma_0 \text{ and } \iota{=}1 \text{ for } \gamma_3/\gamma_5\gamma_3.$$

Problem: violates vector current conservation:

$$\int_{-\infty}^{\infty} dx\, q(x,\mu) \neq \int_{-\infty}^{\infty} dx\, \tilde{q}(x,\mu,P_3) \qquad \text{and} \qquad \int_{-\infty}^{\infty} d\xi\, C(\xi, \xi\mu/xP_3) \neq 1,$$

which **increases** with growing P_3 (around 8% at $P_3 = 10\pi/48$).

Alternative matching: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2\mu^2} (4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- In this procedure, vector current is **conserved**.
- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF.
- However, modification **decreases** with growing P_3.

We introduce a modified $\overline{\mathrm{MS}}$ scheme ($\mathrm{M}\overline{\mathrm{MS}}$) with an extra subtraction made outside the physical region of the unintegrated vertex corrections. [C. Alexandrou et al., Phys. Rev. D99 (2019) 114504]
This renormalizes the ξ-dependence for $\xi > 1$ and $\xi < 0$.

$$\tilde{Z}^{M\overline{MS}}_{\Gamma_{\gamma^0}}(\xi) = 1 - \frac{\alpha_s}{2\pi} C_F \frac{3}{2}\left(-\frac{1}{\xi}\theta(\xi-1) - \frac{1}{1-\xi}\theta(-\xi)\right) - \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\frac{1}{4} + \frac{5}{2}\right)$$

In z-space:

$$\begin{aligned} Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\mu) &= 1 - \frac{\alpha_s}{2\pi} C_F \left(\frac{3}{2}\ln\left(\frac{1}{4}\right) + \frac{5}{2}\right) \\ &+ \frac{3}{2}\frac{\alpha_s}{2\pi} C_F \left(i\pi\frac{|z\mu|}{2z\mu} - Ci(z\mu) + \ln(z\mu) - \ln(|z\mu|) - iSi(z\mu)\right) \\ &- \frac{3}{2}\frac{\alpha_s}{2\pi} C_F e^{iz\mu}\left(\frac{2Ei(-iz\mu) - \ln(-iz\mu) + \ln(iz\mu) + i\pi Sign(z\mu)}{2}\right). \end{aligned}$$

The above has to modify the conversion factor, i.e. the conversion will be RI$\to \overline{\mathrm{MS}} \to \mathrm{M}\overline{\mathrm{MS}}$.
Consistency check: $z \to 0$ limit:

$$Z^{M\overline{MS}}_{\Gamma_{\gamma^0}}(z\to 0) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{\mu^2 z^2 e^{2\gamma_E}}{4}\right) + \frac{5}{2}\right) = Z^{Ratio}_{\Gamma_{\gamma^0}}(z\mu)$$

Exactly cancels the divergence in $\ln(z)$ present in $\overline{\mathrm{MS}}$!
(consistency with: M. Constantinou, H. Panagopoulos, Phys. Rev. D96 (2017) 054506
and with the "Ratio" scheme of T. Izubuchi et al., Phys. Rev. D98 (2018) 056004)

Another alternative matching ("ratio" scheme):

[T. Izubuchi et al., Phys. Rev. D98 (2018) 056004]

$$C\left(\xi, \frac{\mu}{|y|P_3}\right) = \delta\left(1-\xi\right) + \frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1}+1-\frac{3}{2(1-\xi)}\right)_{+(1)} & \xi>1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\frac{y^2P_3^2}{\mu^2}(4\xi(1-\xi))-1\right]+1+2\iota(1-\xi)+\frac{3}{2(1-\xi)}\right)_{+(1)} & 0<\xi<1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi}-1+\frac{3}{2(1-\xi)}\right)_{+(1)} & \xi<0 \end{cases}$$

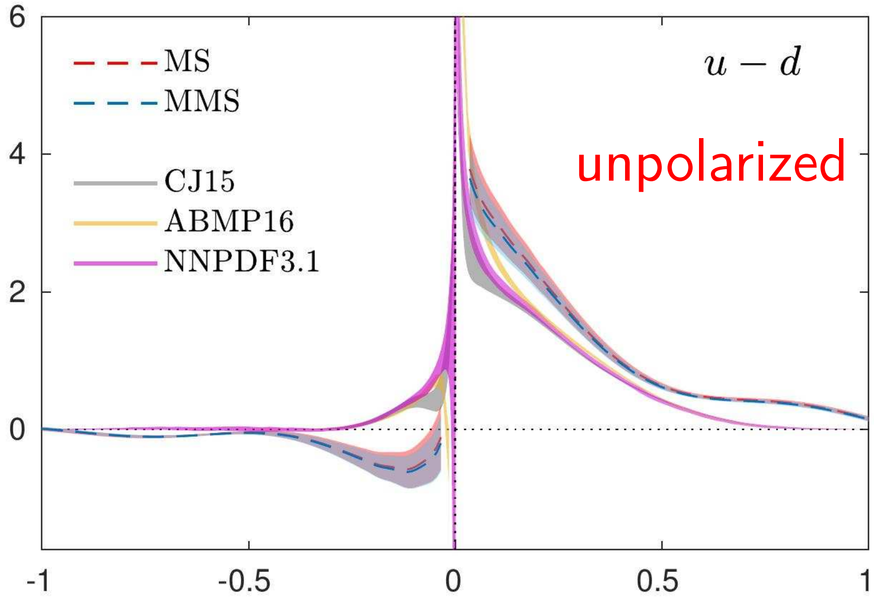
In this scheme, all regions in the ξ-integration of the plus functions (including the "physical" one) contain the same $3/2(1-\xi)$ term and no additional term appears.

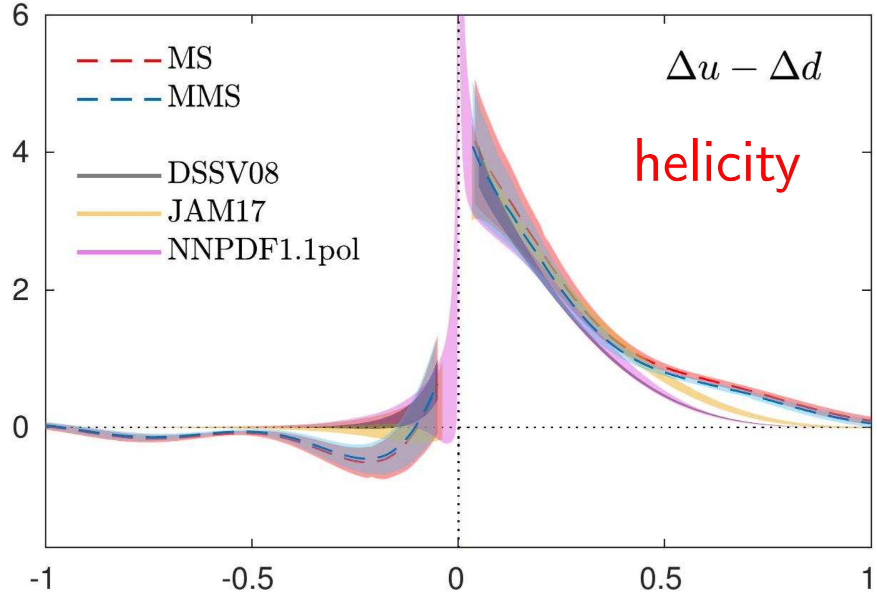
Modification of the perturbative conversion from the intermediate renormalization scheme to $\overline{\mathrm{MS}}$:

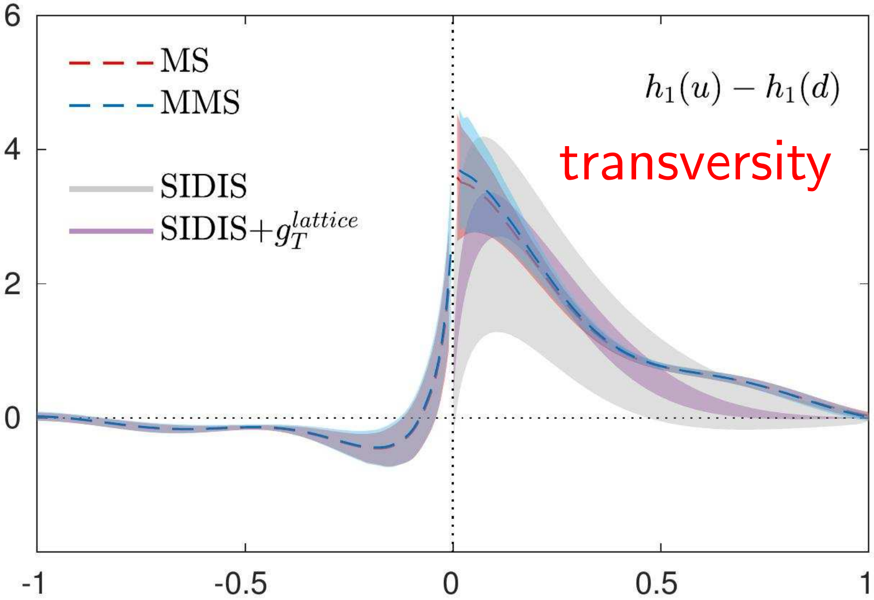
$$C_0(\mu^2z^2) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\ln(\mu^2z^2e^{2\gamma_E}/4)+\frac{5}{2}\right]$$

Caveat: modification of the *physical* ξ-region – potentially large numerical effect.

Effect from $\mathrm{M}\overline{\mathrm{MS}}$







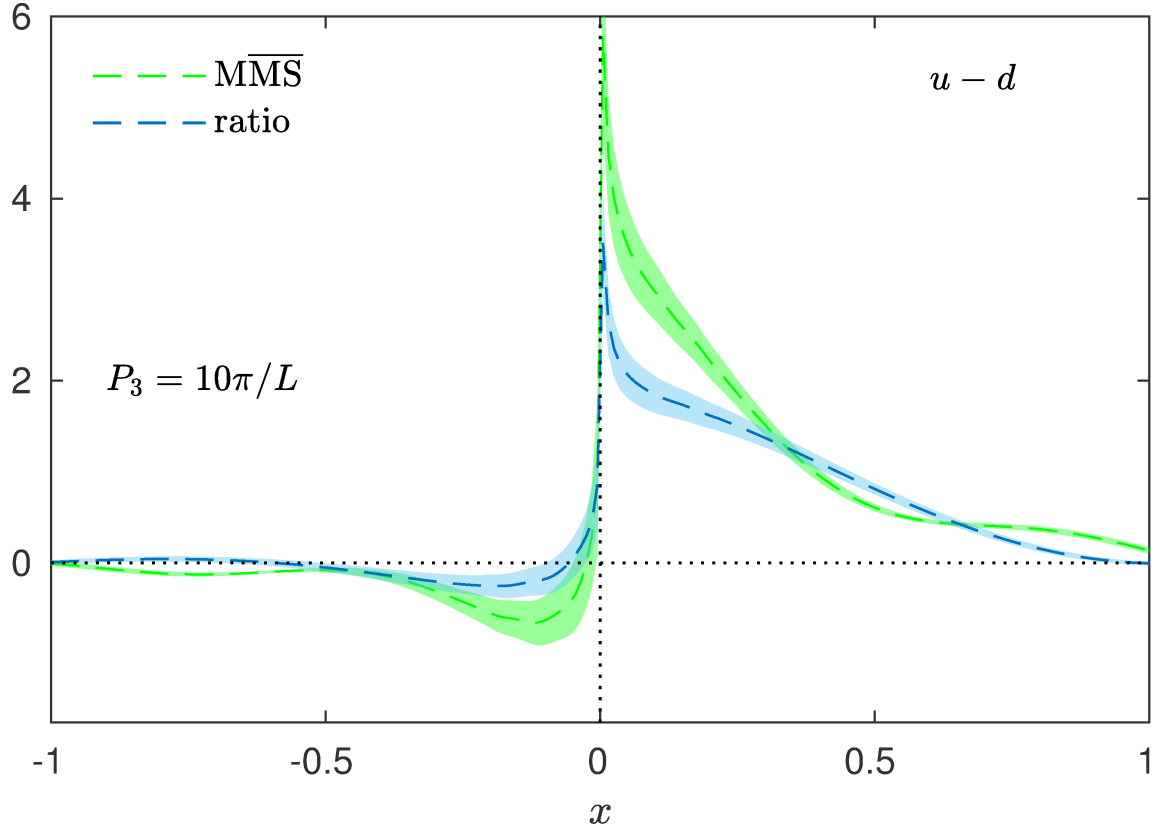
Nucleon momentum $\frac{10\pi}{48}$

As expected, the effect is very small (modification of $\overline{\mathrm{MS}}$ only in unphysical regions)

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

$\mathrm{M\overline{MS}}$ vs. "ratio" scheme

$\mathrm{M\overline{MS}}$ – modification only of the "non-physical" regions $\xi < 0$, $\xi > 1$.
"ratio" – modification also of the "physical" region $0 < \xi < 1$.

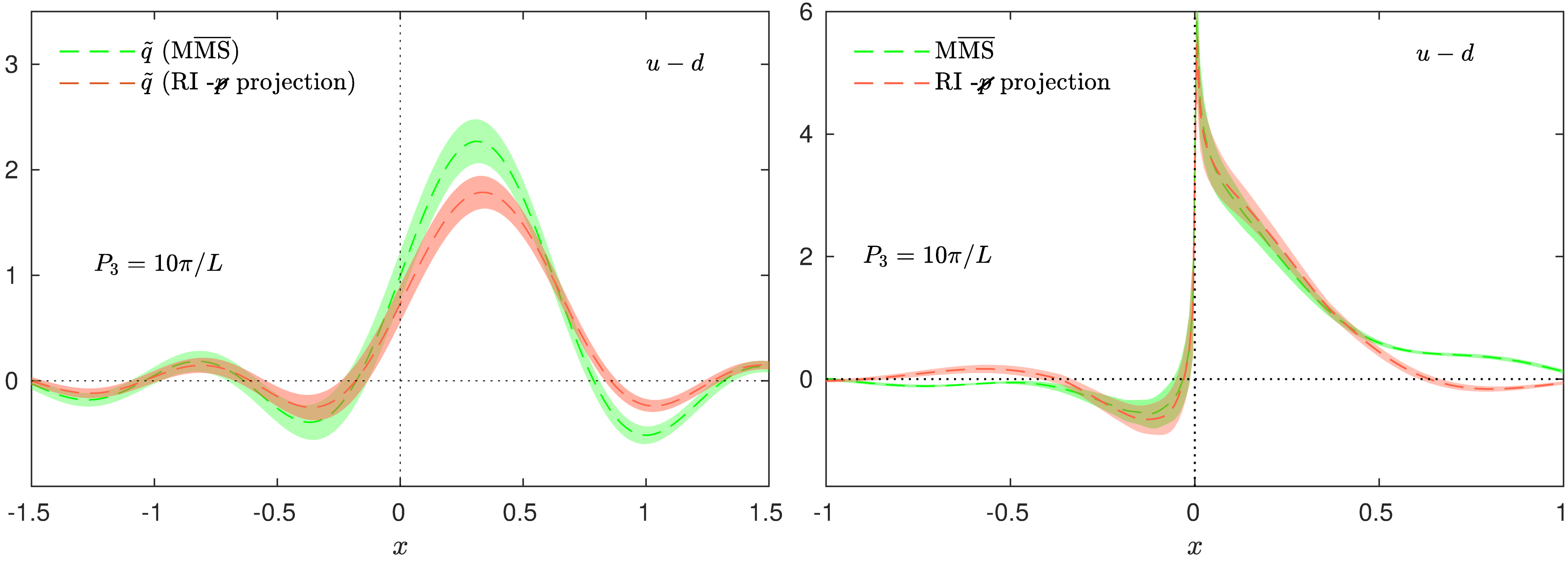


C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

$\mathrm{M\overline{MS}} \to \overline{\mathrm{MS}}$ vs. RI$\to \overline{\mathrm{MS}}$ matching

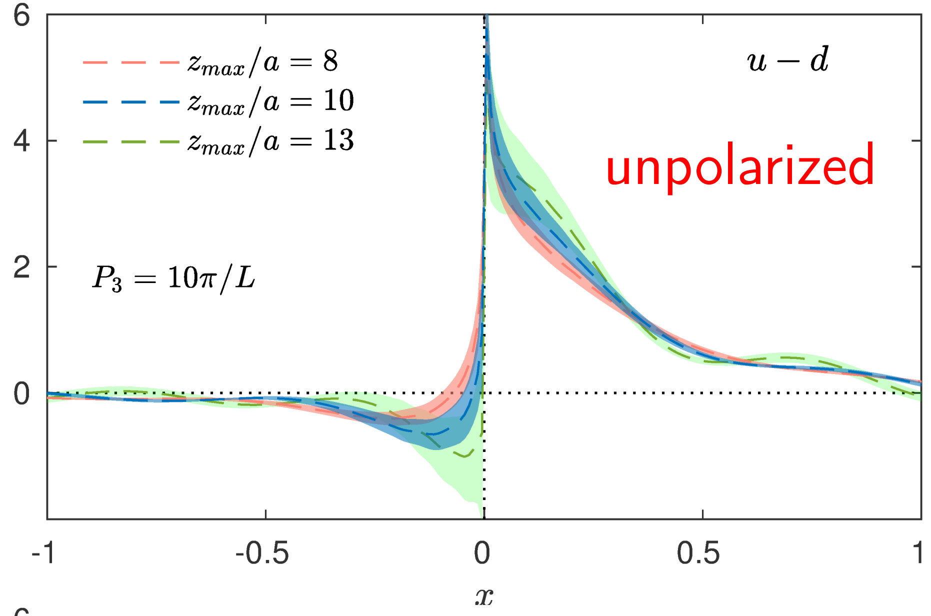
Matching can also be performed directly from the RI scheme to $\overline{\mathrm{MS}}$

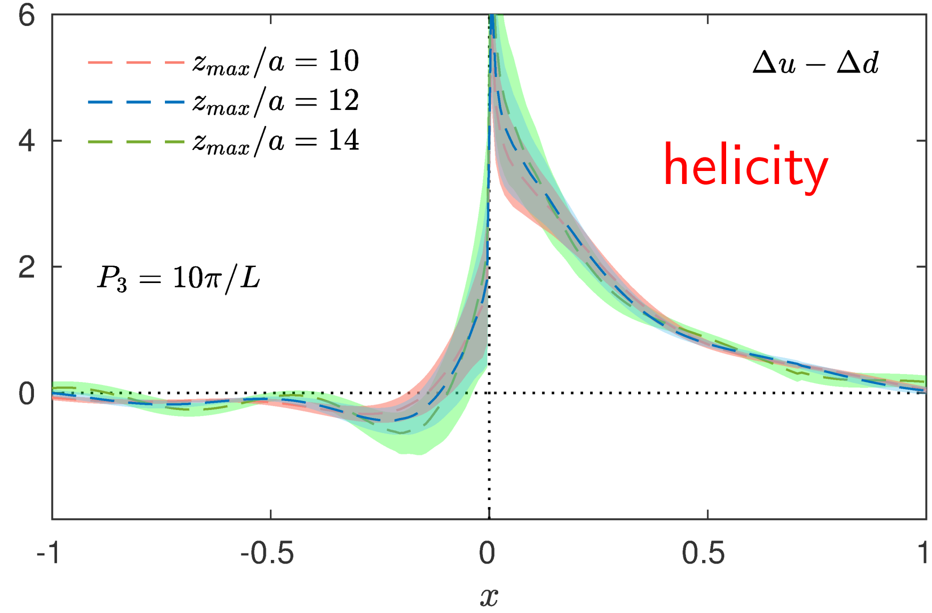
I.W. Stewart, Y. Zhao, Phys. Rev. D97 (2018) 054512

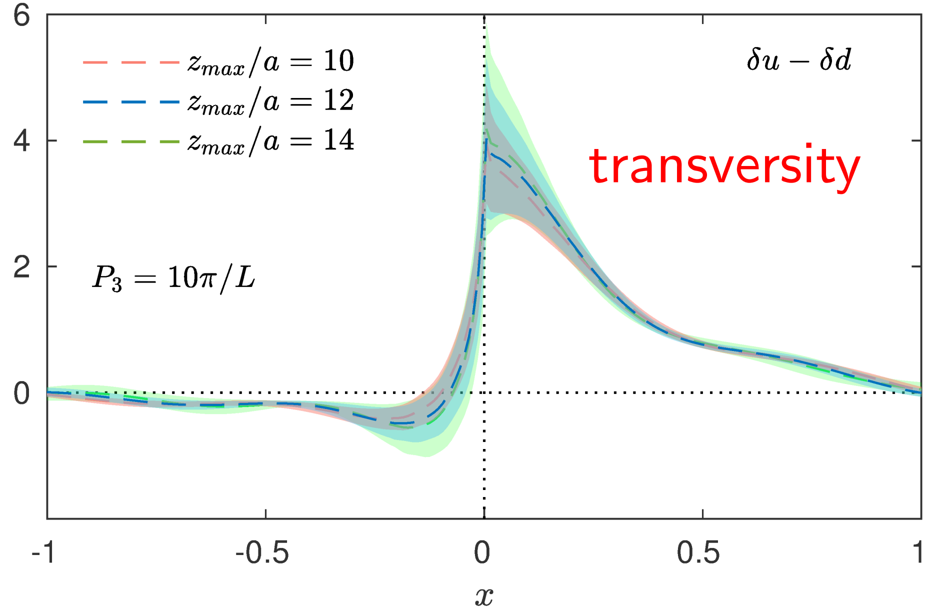


C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Truncation of Fourier transform







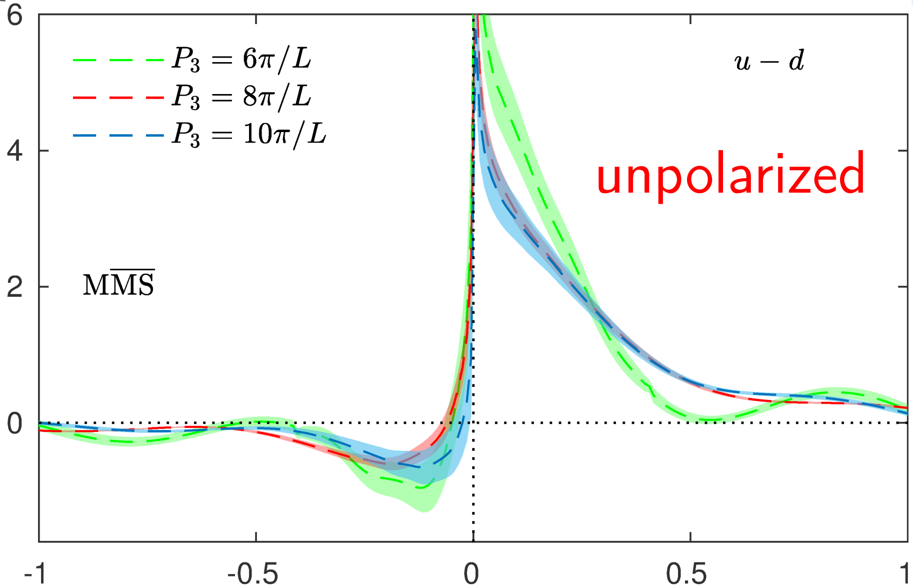
Nucleon momentum $\frac{10\pi}{48}$

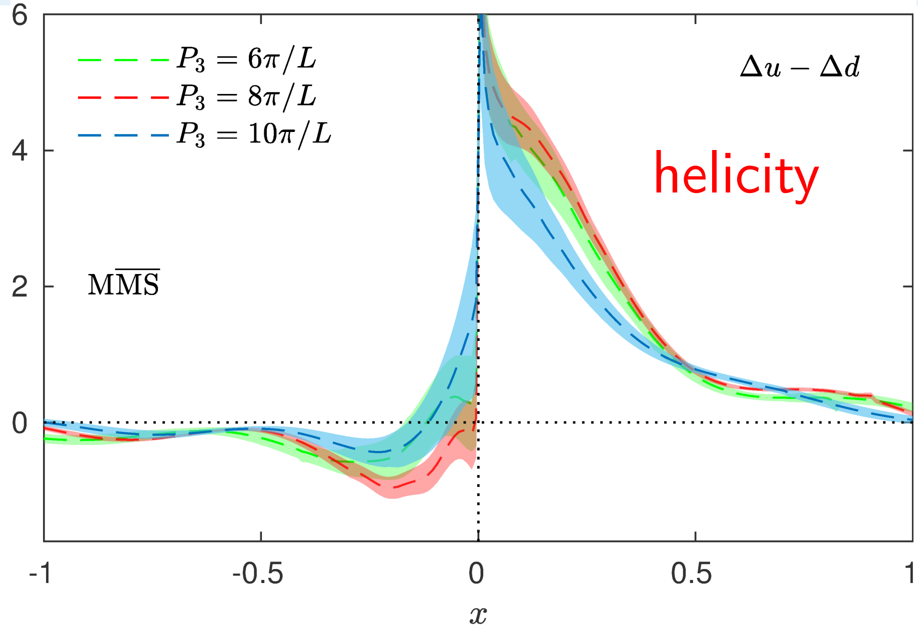
Needs the use of advanced reconstruction techniques

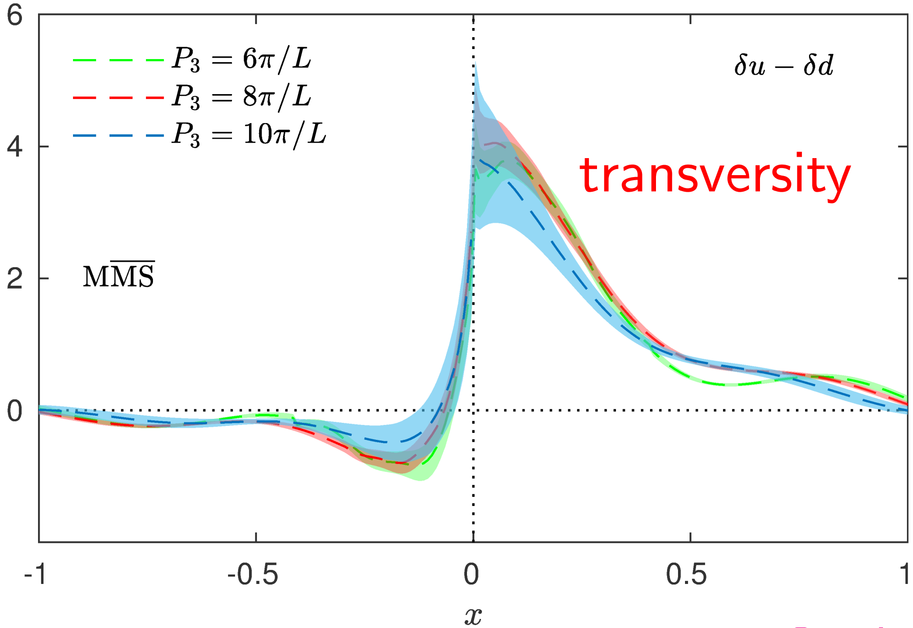
J. Karpie et al., JHEP 1904 (2019) 057

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Momentum dependence of final PDFs







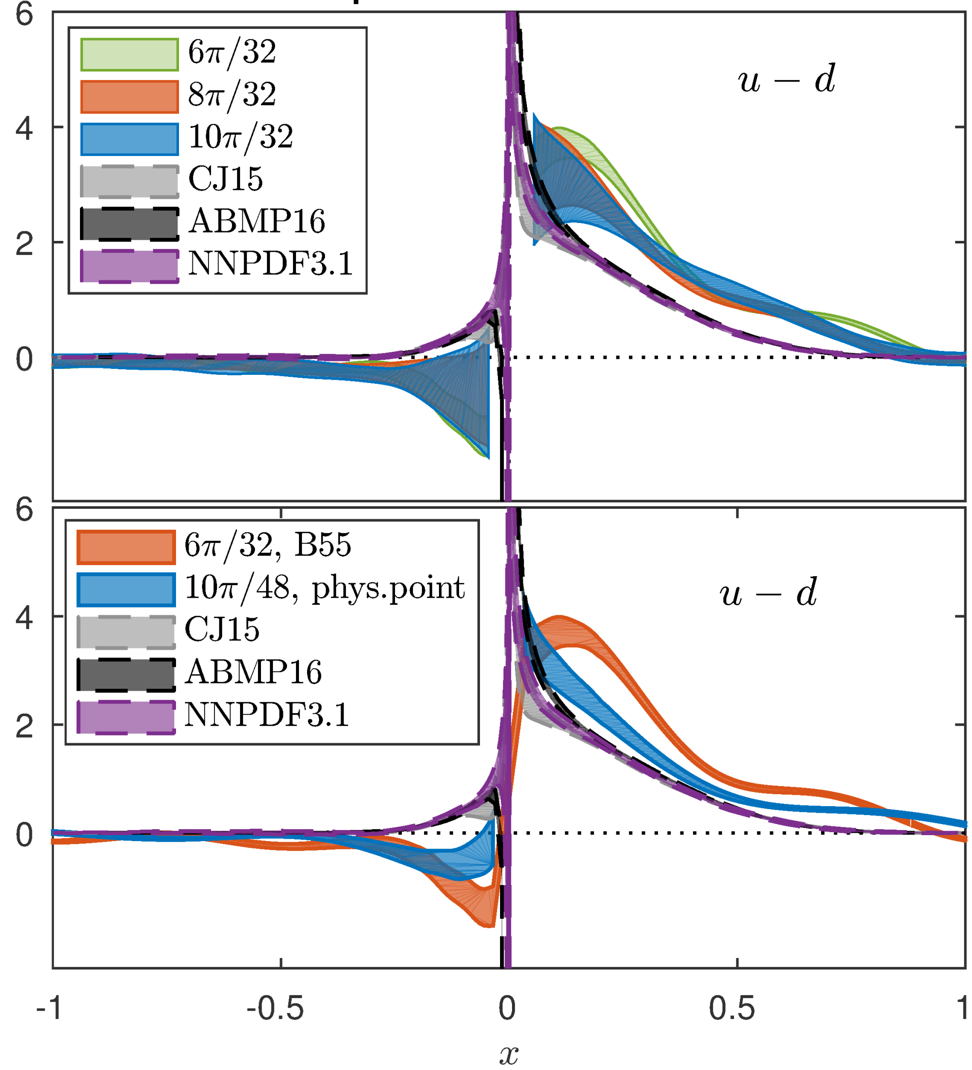
Nucleon momenta $\frac{6\pi}{48}$, $\frac{8\pi}{48}$, $\frac{10\pi}{48}$

Results seem to indicate convergence in nucleon boost

Expected HTE:
$\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001