Correlations in multiparton interactions

Tomas Kasemets[1]

[1]DESY, Notketraße 85, 22607 Hamburg, Germany

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Many features of multi-parton interactions are present already in double parton scattering. The two hard scatterings are connected via the double parton distributions, producing effects not present in the case of a single hard interaction. We examine the impact of correlations between initial state partons on the differential cross section for the double Drell-Yan process. The polarizations of the interacting quarks are found to induce correlations between the decay planes of the vector bosons.

1 Introduction

Interactions where more than one of the partons take part in hard collisions can provide new insights about the structure of the proton and also constitute important backgrounds to other processes, such as Higgs production [1].

In studies of multi-parton interactions an often neglected feature are the correlations between the hard collisions. Although such approximations can be partially motivated, under certain conditions, it is still necessary to examine the correlation effects in greater detail. Double parton scattering possess many properties of multi-parton interactions. As a first examination we study the double Drell-Yan process ($\gamma^*, Z, W^{\pm}$) [2, 3, 4]. It has the advantage of being theoretically clean and well understood in the single parton scattering case. We assume that the cross section can be factorized into hard parts, calculable by perturbation theory, and soft parts described by parton densities. The first steps towards a proof of this assumption in the context of the double Drell-Yan process has been taken in [5] and [6]. We calculate the differential cross section taking many of the correlation effects into account, in order to examine how the correlations in the distributions of the two partons propagate into measurable quantities.

2 Double parton interactions

Double parton interactions exhibit many features not present in single parton scattering. The normal parton distribution functions are replaced by double parton distributions, DPDs, and there can be interferences between the two hard collisions.

When more than one parton in the proton interacts, it is only the sum of the momenta and quantum numbers which have to match between the amplitude and the conjugate amplitude [7, 8, 9]. This is illustrated for the double Drell-Yan process in figure 1. Therefore, a parton in the amplitude can have a different momentum than its partner in the conjugate. The momentum difference r in one interaction has to be balanced by the other. Colors of the quarks can be matched in the canonical way inside each hard interaction. But there is also the possibility to

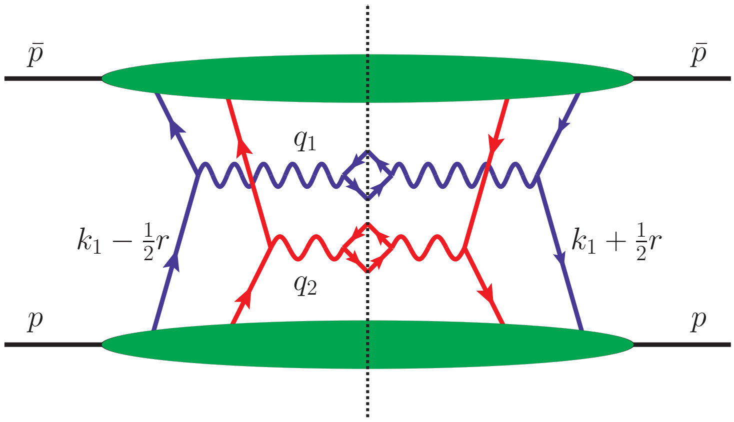

Figure 1: The double Drell-Yan process where two quarks in the right moving proton, with momentum p, interact with two anti-quarks from the left moving proton, with momentum $\bar{p}$. q_1 and q_2 are the momenta of the vector bosons from interaction one and two, while k_1 is the averaged momentum carried by the parton, in interaction 1, from the right moving proton. r is the momentum miss-match and has to be balanced by collision 2. Barred labels indicate that the quantities are from the left moving proton and the two interactions are labeled by the subscripts 1 and 2 respectively.

match the colors between the two collisions, producing color interference terms. Similarly there can be interference in flavor when the two colliding partons from a proton are different. There can even be fermion number interference between quarks/anti-quarks, but we will not consider this in the following. For the cross section differential in the transverse momenta of the vector bosons, $\boldsymbol{q}_i$, the double interactions are not power suppressed compared to single interactions producing the same final state [8] and are important in certain regions of phase space [10]. The power suppression arises when integrating over transverse momenta, due to the larger phase space in the case of a single interaction. k_1 and k_2 are average momenta of the partons taking part in hard interaction 1 and 2 respectively, where the average is taken over the amplitude and its conjugate. Fourier transforming the transverse momentum difference $\boldsymbol{r}$ into position space, we obtain the transverse distance $\boldsymbol{y}$ between the two hard interactions, i.e. from interaction 2 to 1.

3 Parton distributions

Interferences and spin correlations in double parton interactions are described by the DPDs and give rise to a large number of different double parton distributions.

The DPDs depend on the momentum fractions x_1 and x_2 carried by the partons in the two collisions, their average transverse momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and the transverse distance $\boldsymbol{y}$. Integrating over the transverse momenta yields collinear double parton distributions [11, 12, 13, 14]. The correlation between the spin of the two colliding quarks is reflected by parton distributions describing the polarization of quarks inside a proton, similar to those in single parton distributions with polarized protons [15]. We denote unpolarized quarks by q, longitudinally polarized quarks by Δq and transversely polarized quarks by δq. We take the DPDs, in equations (1)-(3), in a right moving proton from [5]. For unpolarized and longitudinally polarized quarks the possible combinations are

$$\begin{aligned} F_{qq} &= f_{qq}(x_1, x_2, \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{y}) \\ F_{\Delta q \Delta q} &= f_{\Delta q \Delta q}(x_1, x_2, \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{y}) \\ F_{q \Delta q} &= g_{q \Delta q}(x_1, x_2, \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{y}) \\ F_{\Delta q q} &= g_{\Delta q q}(x_1, x_2, \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{y}), \end{aligned} \tag{1}$$

where f's are scalar- and g's are pseudo scalar-functions. For transverse polarization the parton distributions carry an open, transverse, index which corresponds to the transverse spin vector

$$\begin{aligned} F^i_{\Delta q \delta q} &= M \left(y^i f_{\Delta q \delta q} + \tilde{y}^i g_{\Delta q \delta q} \right) \\ F^i_{q \delta q} &= M \left(\tilde{y}^i f_{q \delta q} + y^i g_{q \delta q} \right). \end{aligned} \tag{2}$$

M is the proton mass and $\tilde{y}^i = y^j \epsilon^{ij}$, $i = 1, 2$ is a transverse vector orthogonal to y^i. In a left moving proton the sign changes for the pseudo scalar functions g and also of $\tilde{y}$, due to the change of plus/minus components in the epsilon tensor. When both interactions contain transversely polarized quarks the two open indices make the structure more involved

$$\begin{aligned} F^{ij}_{\delta q \delta q} = \delta^{ij} f_{\delta q \delta q} &+ \left(2 y^i y^j - y^2 \delta^{ij} \right) M^2 f^t_{\delta q \delta q} + \left(y^i \tilde{y}^j + \tilde{y}^i y^j \right) M^2 g^s_{\delta q \delta q} \\ &+ \left(y^i \tilde{y}^j - \tilde{y}^i y^j \right) M^2 g^a_{\delta q \delta q}. \end{aligned} \tag{3}$$

The color interference doubles the number of DPDs since they appear as color singlet distributions ${}^1 f_{qq}$ and as color octets ${}^8 f_{qq}$. Further, when the flavors of the two quarks are different, there are both flavor square $f^{S_f}_{qq}$ and flavor interference distributions $f^{I_f}_{qq}$. The DPDs for the left moving protons are defined analogously and will be denoted by a bar, i.e. $\bar{f}_{qq}$ for the unpolarized DPD. This is not to be confused with the bar appearing over subscripts which indicate anti-particles.

4 Differential cross section

The different parton distributions are contracted with the appropriate parts of a polarization dependent partonic cross section, yielding the final differential cross section for the double Drell-Yan process. We will present the results of the calculation with focus on the angular structure, and discuss how the cross section is affected by the correlations between the quarks.

To describe the final state kinematics we use a modified version of the Collins-Soper frame [16], where the arbitrary x-axis is, for definiteness, chosen to point towards the center of the LHC ring, see figure 2. The cross section is split into three parts, the first containing no transverse polarization $\sigma^{(0)}$, the second $\sigma^{(1)}$ and third $\sigma^{(2)}$ containing one and two interactions with transversely polarized quarks. The cross section with only unpolarized and longitudinally

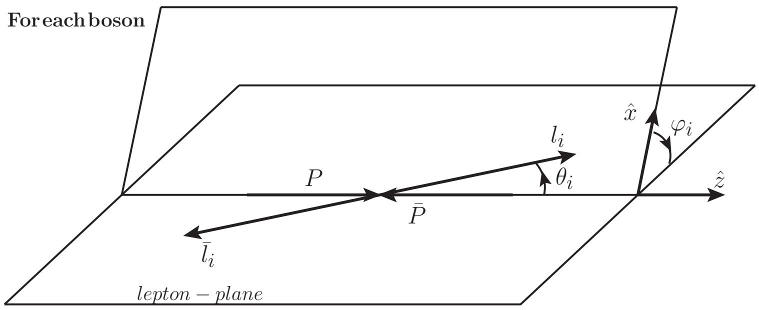

Figure 2: Reference frame for each boson, for simplicity displayed when the transverse momenta of the vector bosons are zero. P is the momentum of the right moving proton while l_i is the momentum of the lepton in interaction i. The x-axis is an arbitrary reference axis which we chose to point towards the center of the LHC ring.

polarized quarks is

$$\begin{aligned} \frac{d\sigma^{(0)}}{dx_1 dx_2 d\bar{x}_1 d\bar{x}_2 d^2\mathbf{q}_1 d^2\mathbf{q}_2 d\Omega_1 d\Omega_2} &= \frac{\alpha^4 Q_1^2 Q_2^2}{S} \sum_{\substack{q_1,\bar{q}_1,q_2,\bar{q}_2 \\ V_1,V_1',V_2,V_2'}} \sum_{I} \sum_{\substack{a_1\bar{a}_1=\{q_1\bar{q}_1,\Delta q_1\Delta\bar{q}_1,q_1\Delta\bar{q}_1,\Delta q_1\bar{q}_1\} \\ a_2\bar{a}_2=\{q_2\bar{q}_2,\Delta q_2\Delta\bar{q}_2,q_2\Delta\bar{q}_2,\Delta q_2\bar{q}_2\}}} \\ &\times \frac{\left[K^I_{a_1\bar{a}_1}(1+\cos^2\theta_1) - K'^I_{a_1\bar{a}_1}\cos\theta_1\right]}{\left(Q_1^2 - m^2_{V_1} + i m_{V_1}\Gamma_{V_1}\right)\left(Q_1^2 - m^2_{V_1'} - i m_{V_1'}\Gamma_{V_1'}\right)} \\ &\times \frac{\left[K^I_{a_2\bar{a}_2}(1+\cos^2\theta_2) - K'^I_{a_2\bar{a}_2}\cos\theta_2\right]}{\left(Q_2^2 - m^2_{V_2} + i m_{V_2}\Gamma_{V_2}\right)\left(Q_2^2 - m^2_{V_2'} - i m_{V_2'}\Gamma_{V_2'}\right)} \\ &\times \int d^2y\, \mathcal{I}\left[F^I_{a_1a_2}\bar{F}^I_{\bar{a}_1\bar{a}_2} + F^I_{a_1\bar{a}_2}\bar{F}^I_{\bar{a}_1a_2} + F^I_{\bar{a}_1a_2}\bar{F}^I_{a_1\bar{a}_2} + F^I_{\bar{a}_1\bar{a}_2}\bar{F}^I_{a_1a_2}\right] \end{aligned} \tag{4}$$

where we have used the short hand notation

$$\mathcal{I}\left[F^I_{a_1a_2}\bar{F}^I_{\bar{a}_1\bar{a}_2}\right] \equiv \int d^2k_1 d^2\bar{k}_1 \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{k}_1 - \bar{\boldsymbol{k}}_1) \int d^2k_2 d^2\bar{k}_2 \delta^{(2)}(\boldsymbol{q}_2 - \boldsymbol{k}_2 - \bar{\boldsymbol{k}}_2) F^I_{a_1a_2}\bar{F}^I_{\bar{a}_1\bar{a}_2} \tag{5}$$

for the momentum integrals over the DPDs. Q_i^2 is the squared momentum of vector boson i, S is 2 when the final states of the two hard interactions are equal and 1 otherwise. The first sum is over the flavors of quark q_i and anti-quark $\bar{q}_i$ for the two interactions and over the allowed vector bosons in the amplitude V_i and in the conjugated amplitude V_i', with mass m_{V_i} and $m_{V_i'}$. The sum over I sums the color (singlet 1 and octet 8) and flavor (S_f and I_f) squares and interference terms. $a_i\bar{a}_i$ label the different combinations of unpolarized and longitudinally polarized quarks which affect not only the parton distributions but also the combinations of coupling constants K and K', which also depend on the flavor of the quarks and vector bosons. K' is zero for photons but for a more detailed discussion we refer to [17]. Instead we want to focus upon the angular structure. θ_i is the angle between the z-axis and the momentum of the outgoing lepton from hard interaction i.

All the different terms have the same angular structures, thus, despite the proliferation of DPDs they all contribute to the same structures in the cross section. However, the relative factors are different and hence longitudinal polarization as well as interferences in flavor and color affects both the overall rate and the angular distribution.

We now turn towards the part where one interaction involves quarks with transverse polarizations. For the production of W bosons the transversely polarized part of the cross section is zero since they only couple to left-handed particles and hence the only nonzero contribution is for γ/Z bosons in the interaction with transverse polarizations. The cross section is

$$
\begin{aligned}
\frac{d\sigma^{(1)}}{dx_i d\bar{x}_i d^2\mathbf{q}_i d\Omega_i} &= \frac{\alpha^4 Q_1^2 Q_2^2}{S} \sum_{\substack{q_1,\bar{q}_1,q_2,\bar{q}_2\\ V_1,V_1',V_2,V_2'}} \sum_I \frac{1}{\left(Q_1^2 - m_{V_1}^2 + im_{V_1}\Gamma_{V_1}\right)\left(Q_1^2 - m_{V_1'}^2 - im_{V_1'}\Gamma_{V_1'}\right)} \\
&\times \frac{1}{\left(Q_2^2 - m_{V_2}^2 + im_{V_2}\Gamma_{V_2}\right)\left(Q_2^2 - m_{V_2'}^2 - im_{V_2'}\Gamma_{V_2'}\right)} M^2 \sin^2\theta_2 \int d^2y\, \boldsymbol{y}^2 \\
&\times \Bigg\{ \sum_{a_1\bar{a}_1 = \{q_1\bar{q}_1, \Delta q_1 \Delta\bar{q}_1\}} \left[K^I_{a_1\bar{a}_1}(1+\cos^2\theta_1) - K'^I_{a_1\bar{a}_1}\cos\theta_1\right] \\
&\quad \times \bigg(\left[C^I \cos 2(\varphi_2 - \varphi_y) - C'^I \sin 2(\varphi_2 - \varphi_y)\right] \mathcal{I}\left[g^I_{a_1\delta q_2}\bar{g}^I_{\bar{a}_1\delta\bar{q}_2} - f^I_{a_1\delta q_2}\bar{f}^I_{\bar{a}_1\delta\bar{q}_2} + \text{perm.}\right] \\
&\quad - \left[C^I \sin 2(\varphi_2 - \varphi_y) + C'^I \cos 2(\varphi_2 - \varphi_y)\right] \mathcal{I}\left[g^I_{a_1\delta q_2}\bar{f}^I_{\bar{a}_1\delta\bar{q}_2} + f^I_{a_1\delta q_2}\bar{g}^I_{\bar{a}_1\delta\bar{q}_2} + \text{perm.}\right] \bigg) \\
&+ \sum_{a_1\bar{a}_1 = \{q_1\Delta\bar{q}_1, \Delta q_1 \bar{q}_1\}} \left[K^I_{a_1\bar{a}_1}(1+\cos^2\theta_1) - K'^I_{a_1\bar{a}_1}\cos\theta_1\right] \\
&\quad \times \bigg(\left[C^I \cos 2(\varphi_2 - \varphi_y) - C'^I \sin 2(\varphi_2 - \varphi_y)\right] \mathcal{I}\left[g^I_{a_1\delta q_2}\bar{f}^I_{\bar{a}_1\delta\bar{q}_2} - f^I_{a_1\delta q_2}\bar{g}^I_{\bar{a}_1\delta\bar{q}_2} + \text{perm.}\right] \\
&\quad - \left[C^I \sin 2(\varphi_2 - \varphi_y) + C'^I \cos 2(\varphi_2 - \varphi_y)\right] \mathcal{I}\left[g^I_{a_1\delta q_2}\bar{g}^I_{\bar{a}_1\delta\bar{q}_2} + f^I_{a_1\delta q_2}\bar{f}^I_{\bar{a}_1\delta\bar{q}_2} + \text{perm.}\right] \bigg) \\
&+ \{1 \leftrightarrow 2\} \Bigg\}
\end{aligned} \tag{6}
$$

where C and C' are coupling factors for transverse quarks ($\delta q_2 \delta\bar{q}_2$), similar to the K's in the previous part and 'perm.' stands for permutations of the quark/anti-quark labels. The angular dependence from interaction 1 is unchanged compared to $\sigma^{(0)}$ but for interaction 2 we now get dependence on azimuthal angles. The angular structure however stays simple and we only have dependence on one new angle, between the transverse momentum of the outgoing lepton from interaction 2 and the vector $\boldsymbol{y}$ between the two hard interactions. The transverse dependence originates in the breaking of the rotation symmetry around the z-axis caused by the transverse spin of the partons. $\{1 \leftrightarrow 2\}$ represents the contribution in which interaction 1 is transversely polarized. It can be obtained by interchanging the labels for the two interactions and at the same time swapping the positions of the subscripts on the DPDs (For example: $f_{a_1\delta q_2}$ then becomes $f_{\delta q_1, a_2}$). $\boldsymbol{y}$ cannot be measured and performing the d^2y integral relates the correlations to the transverse momenta of the vector bosons. The cross section can also be displayed with the quark and final state lepton dependencies separated by aid of an arbitrary x-axis, but we have chosen to display the results in the above form since it gives formulas of shorter lengths.

The doubly transversely polarized cross section is richer in structure

$$
\begin{aligned}
\frac{d\sigma^{(2)}}{dx_i d\bar{x}_i d^2\mathbf{q}_i d\Omega_i} =& \frac{\alpha^4 Q_1^2 Q_2^2}{S} \sum_{\substack{q_1,\bar{q}_1,q_2,\bar{q}_2\\ V_1,V_1',V_2,V_2'}} \sum_I \frac{1}{\left(Q_1^2 - m_{V_1}^2 + i m_{V_1}\Gamma_{V_1}\right)\left(Q_1^2 - m_{V_1'}^2 - i m_{V_1'}\Gamma_{V_1'}\right)} \\
&\times \frac{1}{\left(Q_2^2 - m_{V_2}^2 + i m_{V_2}\Gamma_{V_2}\right)\left(Q_2^2 - m_{V_2'}^2 - i m_{V_2'}\Gamma_{V_2'}\right)} \sin^2\theta_1 \sin^2\theta_2 \int d^2y \\
&\times \Big\{ \left[A^I \cos 2(\varphi_1 - \varphi_2) - A'^I \sin 2(\varphi_1 - \varphi_2)\right] \\
&\qquad\qquad \times \mathcal{I}\left[f^I_{\delta q_1 \delta q_2} \bar{f}^I_{\delta \bar{q}_1 \delta \bar{q}_2} - \boldsymbol{y}^4 M^4 g^{aI}_{\delta q_1 \delta q_2} \bar{g}^{aI}_{\delta \bar{q}_1 \delta \bar{q}_2} + \text{perm.}\right] \\
&\quad + \left[B^I \cos 2(\varphi_1 + \varphi_2 - 2\varphi_y) - B'^I \sin 2(\varphi_1 + \varphi_2 - 2\varphi_y)\right] \\
&\qquad\qquad \times \boldsymbol{y}^4 M^4 \mathcal{I}\left[f^{tI}_{\delta q_1 \delta q_2} \bar{f}^{tI}_{\delta \bar{q}_1 \delta \bar{q}_2} - g^{sI}_{\delta q_1 \delta q_2} \bar{g}^{sI}_{\delta \bar{q}_1 \delta \bar{q}_2} + \text{perm.}\right] \\
&\quad + \left[A^I \sin 2(\varphi_1 - \varphi_2) + A'^I \cos 2(\varphi_1 - \varphi_2)\right] \\
&\qquad\qquad \times \boldsymbol{y}^2 M^2 \mathcal{I}\left[f^I_{\delta q_1 \delta q_2} \bar{g}^{aI}_{\delta \bar{q}_1 \delta \bar{q}_2} + g^{aI}_{\delta q_1 \delta q_2} \bar{f}^I_{\delta \bar{q}_1 \delta \bar{q}_2} + \text{perm.}\right] \\
&\quad - \left[B^I \sin 2(\varphi_1 + \varphi_2 - 2\varphi_y) + B'^I \cos 2(\varphi_1 + \varphi_2 - 2\varphi_y)\right] \\
&\qquad\qquad \times \boldsymbol{y}^4 M^4 \mathcal{I}\left[f^{tI}_{\delta q_1 \delta q_2} \bar{g}^{sI}_{\delta \bar{q}_1 \delta \bar{q}_2} + g^{sI}_{\delta q_1 \delta q_2} \bar{f}^{tI}_{\delta \bar{q}_1 \delta \bar{q}_2} + \text{perm.}\right] \Big\},
\end{aligned}
\tag{7}
$$

where A, A', B, and B' are coupling factors. This part of the cross section depends on θ_1 and θ_2 but also on the transverse angle between the two outgoing leptons and the angles between them and the direction $\boldsymbol{y}$. These describe transverse correlations between the decay planes of the two vector bosons and between the decay planes and the direction between the hard collisions.

5 Transverse dependence of DPDs

We employ a simple Gaussian model for the transverse dependence of the double parton distributions, and study how the interplay of longitudinal and transverse dependence affects the cross section. Even though the transverse structure of the proton is much more complicated than the model suggests, it can still provide some useful insights into what might be expected in a more complete treatment.

The transverse dependence of the proton is approximated as a three quark Fock state [18] and the wave function is separated into a purely longitudinal part and a mixed longitudinal and transverse part

$$
\Psi(\hat{x}_i, \hat{\boldsymbol{k}}_i) = \phi(\hat{x}_i)\Omega(\hat{x}_i, \hat{\boldsymbol{k}}_i). \tag{8}
$$

We take the Gaussian ansatz [19]

$$
\Omega(\hat{x}_i, \hat{\boldsymbol{k}}_i) = \frac{(16\pi^2 a^2)^2}{\hat{x}_1 \hat{x}_2 \hat{x}_3} \exp\left\{-a^2 \left(\frac{\hat{\boldsymbol{k}}_1^2}{\hat{x}_1} + \frac{\hat{\boldsymbol{k}}_2^2}{\hat{x}_2} + \frac{\hat{\boldsymbol{k}}_3^2}{\hat{x}_3}\right)\right\}, \tag{9}
$$

where a is a positive constant of dimension GeV^{-1}, which allows us to carry out the integrations analytically. The $\hat{x}_i$ are the longitudinal momentum fractions and $\hat{\boldsymbol{k}}_i$ transverse momenta of the three quarks in the Fock state. After performing the transverse integrals, the transverse dependence of the DPDs renders the unpolarized cross section proportional to

$$\exp\left\{-\frac{1}{8a^2}\left(b\,\boldsymbol{q}_1^2 + c\,\boldsymbol{q}_1\boldsymbol{q}_2 + d\,\boldsymbol{q}_2\right)\right\}. \tag{10}$$

b, c and d are positive functions of $x_1, x_2, \bar{x}_1, \bar{x}_2$. The cross section decreases with transverse momenta of the two vector bosons, as expected since the quarks are more likely to be collinear to their proton. But due to the $\boldsymbol{q_1 q_2}$ term equation (10) also shows that the cross section increases when the two bosons have opposite transverse momentum. Since already this simple model causes dependence on the azimuthal angle between the vector boson momenta, there is reason to expect such effects to be present also in more realistic descriptions.

6 Conclusions

Spin correlations, color and flavor interference proliferate the number of double parton distributions. Nevertheless, for the double Drell-Yan process many of them contribute to the same angular structures in the differential cross section. Longitudinal polarization, color and flavor interference does not introduce any new angular structures but affects both the overall rate and the angular distribution. The spin vectors of the transversely polarized quarks break the rotational invariance around the z-axis. This leads to a dependence of the cross section on the transverse angle between the decay planes of the vector bosons, different from the dependence when the two bosons are produced in one hard collision. The cross section also depends on the angles between the decay planes and the transverse direction between the two collisions. This direction is not measurable and integrating over it causes correlations with the transverse momenta of the vector bosons. Effects appearing for double Drell-Yan process are also expected in other types of processes, for example double dijet production should display similar features, but the color structure of these processes increases their complexity dramatically. Finally, we showed that even a simple model for the transverse dependence of the double parton distributions causes a dependence on the azimuthal angle between the momenta of the two vector bosons.

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