Double Parton Interactions in Proton Deuteron Collisions

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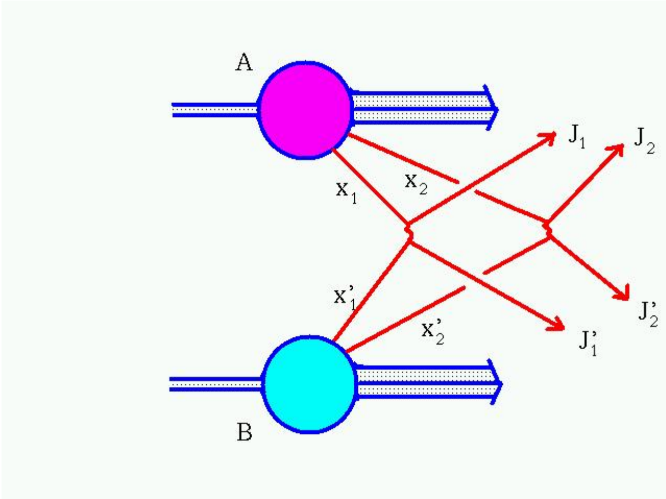
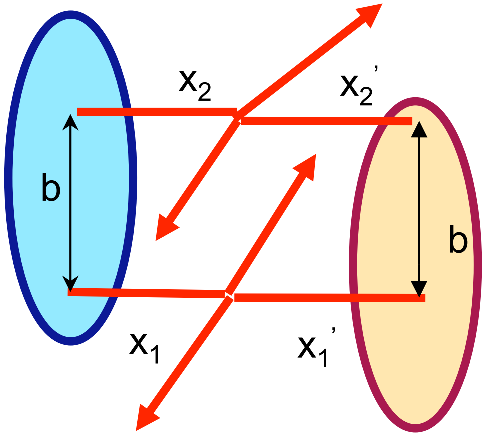




The inclusive double parton-scattering cross-section, for two parton processes A and B in a pp collision, is given by

$$\sigma_{double}^{pp\ (A,B)} = \frac{m}{2}\sum_{i,j,k,l}\int \Gamma_{i,j}(x_1, x_2;\ b)\hat{\sigma}^A_{i,k}(x_1, x_1')\hat{\sigma}^B_{j,l}(x_2, x_2')\Gamma_{k,l}(x_1', x_2';\ b)\ dx_1 dx_1' dx_2 dx_2' db$$

A reason of interest is that the non-perturbative input, $\Gamma_{i,j}(x_1, x_2;\ b)$ namely the double parton distribution function, contains ***information on the hadron structure*** which is ***not accessible in a single scattering large p_t processes***.

Factorizing the dependence in x and b of the double parton distributions, one obtains the simple expression of the double parton cross section used in the experimental search.

$$\sigma_{double}^{pp\ (A,B)} = \frac{m}{2}\frac{\sigma_A\sigma_B}{\sigma_{eff}}$$

The non perturbative input to the MPI depends linearly on multi-parton Correlations.

In the case of double parton interactions in *pp* collisions, all information concerning the correlations is however summarized in the value of a single quantity, the effective cross section.

Partons may be correlated in all degrees of freedom and all different correlation terms contribute to the cross section. The contributions due to the different parton flavors can be isolated, at least to some extent, by selecting properly the final state. In the case of high energy proton-proton collisions, the effects of correlations in the transverse coordinates and in fractional momenta are, on the contrary, unavoidably mixed in the final observables.

One may write the double parton distribution functions as

$$\Gamma(x_1, x_2;\ b) = G(x_1, x_2) f_{x_1 x_2}(b), \qquad G(x_1, x_2) = K(x_1, x_2) G(x_1) G(x_2)$$

where *f* is normalized to one and the transverse scales, characterizing *f*, may still depend on fractional momenta. In the simplest case one would have $K(x_1,x_2)=1$ which, after integrating on *b*, would be the case of a Poissonian multi-parton distribution. In *pp* one thus has

$$\begin{aligned}
\sigma^{pp\ (A,B)}_{double}(x_1, x_1', x_2, x_2') &= \frac{m}{2} K(x_1, x_2) K(x_1', x_2') G(x_1) \hat{\sigma}_A(x_1, x_1') G(x_1') \\
&\quad \times G(x_2) \hat{\sigma}_B(x_2, x_2') G(x_2') \int f_{x_1 x_2}(b) f_{x_1' x_2'}(b) db \\
&= \frac{m}{2} \frac{K(x_1, x_2) K(x_1', x_2')}{\pi \Lambda^2(x_1, x_1', x_2, x_2')} \sigma_A(x_1, x_1') \sigma_B(x_2, x_2')
\end{aligned}$$

where $\displaystyle \int f_{x_1 x_2}(b) f_{x_1' x_2'}(b) db = \frac{1}{\pi \Lambda^2(x_1, x_1', x_2, x_2')}$

The effective cross section is thus $\boxed{\sigma_{eff}(x_1, x_1', x_2, x_2') = \dfrac{\pi \Lambda^2(x_1, x_1', x_2, x_2')}{K(x_1, x_2) K(x_1', x_2')}}$

$$\sigma_{eff}(x_1, x_1', x_2, x_2') = \frac{\pi\Lambda^2(x_1, x_1', x_2, x_2')}{K(x_1, x_2)K(x_1', x_2')}$$

Notice that the experimental indication is that the effective cross section depends only weakly on fractional momenta.

→ weak dependence of $\boldsymbol{\Lambda}$ and K on fractional momenta

As the experimental information is summarized by a single quantity, the effective cross section, *pp* collisions alone do not provide enough information to discriminate between the two different sources.

Additional information on multi-parton correlations can be obtained by studying double parton interactions in *pD* collisions.

The effects of longitudinal and transverse correlations are in fact different when a single nucleon or both target nucleons participate in the hard process.

Double parton interactions in *pd* collisions

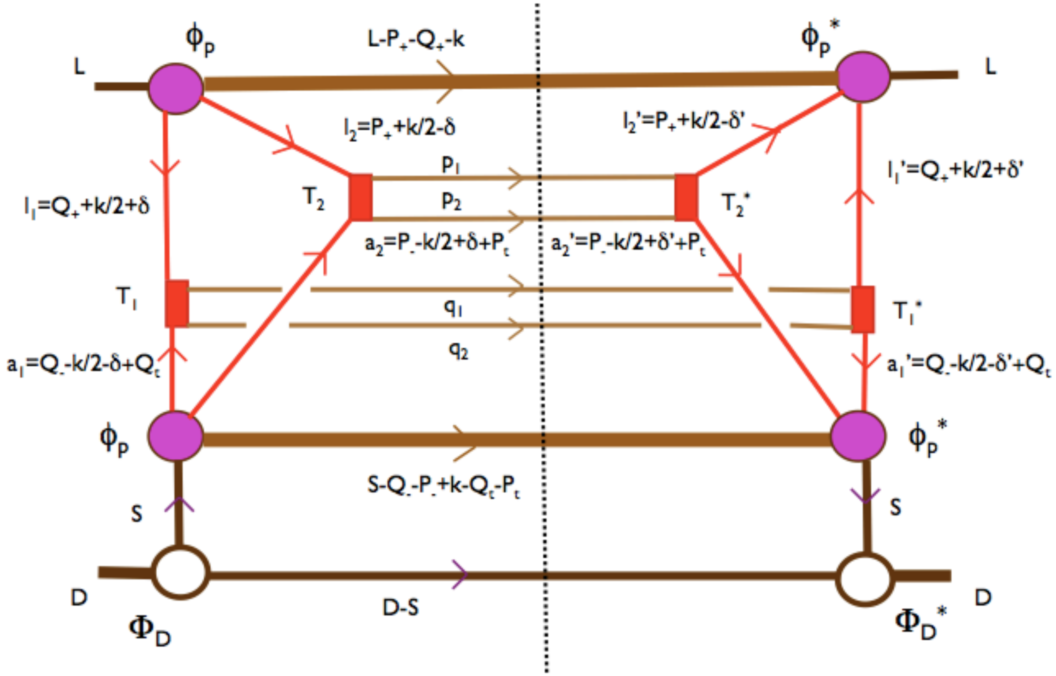
The double parton scattering cross section in proton-deuteron collisions σ^{pD}_{double} is given by

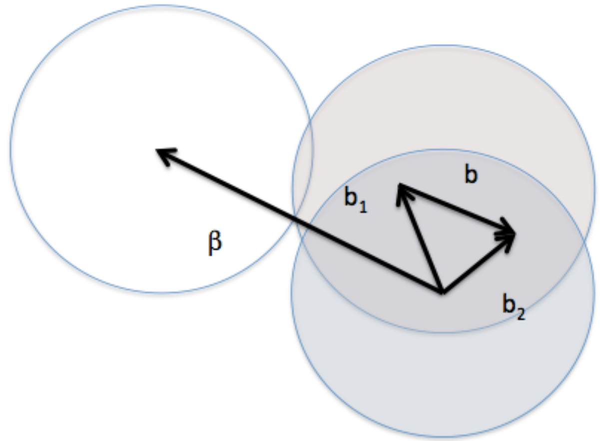
$$\sigma^{pD}_{double} = \sigma^{pD}_{2,0} + \sigma^{pD}_{1,1;\mathcal{D}} + \sigma^{pD}_{1,1;\mathcal{I}}$$

$\sigma^{pD}_{2,0}$ is the contribution to the cross section where only a single target nucleon undergoes a double parton collision, while there is no large momentum transfer exchange with the second nucleon

$\sigma^{pD}_{1,1;\mathcal{D}}$ and $\sigma^{pD}_{1,1;\mathcal{I}}$ are respectively the diagonal and the off diagonal contributions to the cross section, where both target nucleons interact with large momentum exchange

Only a single target nucleon interacts with large momentun exchange





Configuration in transverse space

$$\sigma_{2,0}^{pD} = 2\int \Gamma(x_1, x_2;\ b)\hat{\sigma}(x_1x_1')\hat{\sigma}(x_2x_2')\Gamma\Big(\frac{x_1'}{Z}, \frac{x_2'}{Z};\ b\Big)\frac{|\Psi_D(Z,\beta)|^2}{Z(2-Z)}dZ\ d\beta\ db$$

By looking at the non relativistic limit of the Bethe-Salpeter equation in the deuteron c.m. system one obtains the expression of the covariant Deuteron vertex in terms of the non-relativistic Deuteron wave function as a function of the c.m. three momentum $\vec{p}^2$:

$$\Phi_D(p) = -i\big(\vec{p}^2 + m^2\big)^{1/4}\big(M_D^2 - 4m^2 - 4\vec{p}^2\big)\psi_{NR}(\vec{p}^2)(2\pi)^{3/2}$$

Keeping into account that one of the two nucleons is on shell, the c.m. quantities are expressed in terms of invariants as follows:

$$E_p = \frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big), \qquad p_z = \frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 - \frac{2}{Z}m_t^2\Big)$$

And the function $\Psi_D(p)$ in the cross section is finally expressed in terms of light cone variables through the non-relativistic Deuteron wave function $\psi_{NR}(\vec{p}^2)$ as

$$\begin{aligned}\Psi_D(Z;p_t) \;&=\; (2\pi)^{3/2}\sqrt{\frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big)} \\ &\times\;\psi_{NR}\left(\frac{1}{4M_D^2}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big)^2 - m^2\right)\end{aligned}$$

$\Psi_D(Z;\beta)$ is the Fourier transform of $\Psi_D(Z;p_t)$ and the normalization is

$$\int |\Psi_D(Z;\beta)|^2 \frac{1}{Z(2-Z)} dZ d\beta = 1$$

In this contribution to the cross section, the integration on the relative transverse distance between the two nucleons β is decoupled from the integration on the relative transverse distance between the two interacting partons b.

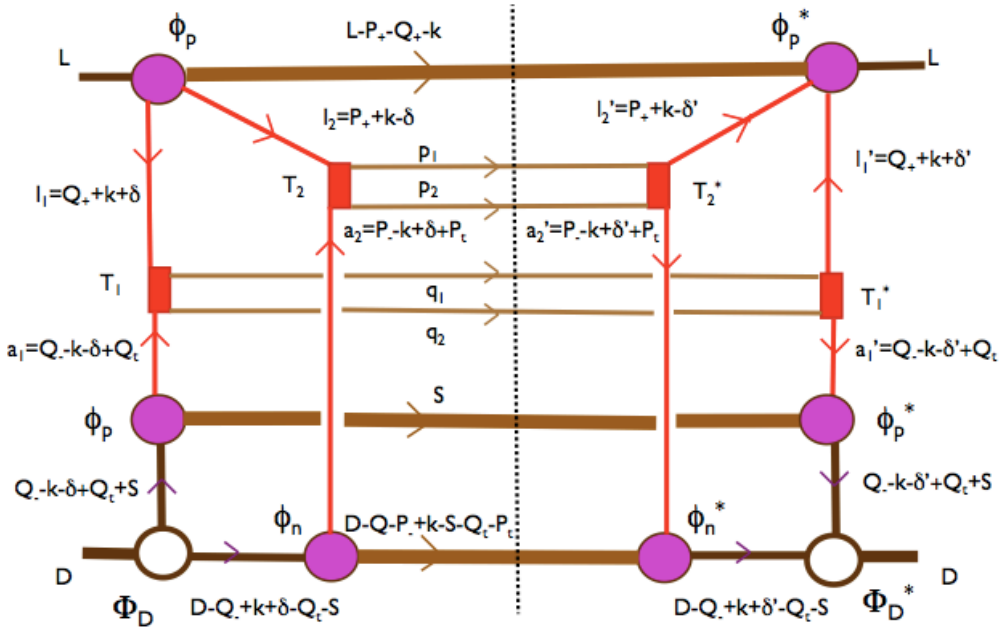
$$\sigma_{2,0}^{pD} = 2\int \Gamma(x_1, x_2;\ b)\hat{\sigma}(x_1 x_1')\hat{\sigma}(x_2 x_2')\Gamma\Big(\frac{x_1'}{Z}, \frac{x_2'}{Z};\ b\Big)\frac{|\Psi_D(Z,\beta)|^2}{Z(2-Z)} dZ\ d\beta\ db$$

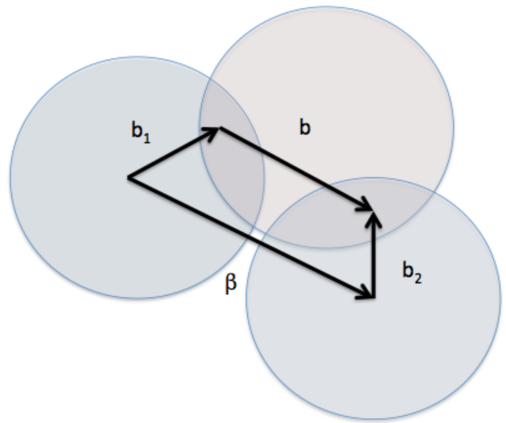
By neglecting the smearing in the longitudinal nuclear fractional momentum Z one thus obtains

$$\sigma_{2,0}^{pD} \simeq 2\ \sigma_{double}^{pp}$$

which shows that this term does not contain much new information as compared to the case of *pp* interactions. In particular ***the transverse scale characterizing the process is the same as in*** *pp* ***interactions***.

Both target nucleons interacting with large momentun exchange.
Diagonal contribution:

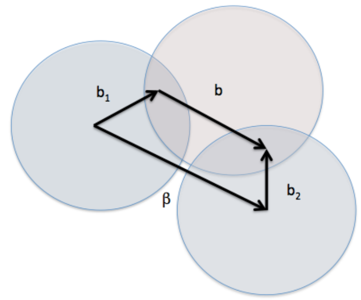




Configuration in transverse space

$$\sigma^{pD}_{1,1;\mathcal{D}} = \int \Gamma(x_1,x_2;\ b)\hat\sigma(x_1x'_1)\hat\sigma(x_2x'_2)\Gamma\Big(\frac{x'_1}{Z};\ b_1\Big)\Gamma\Big(\frac{x'_2}{2-Z};\ b_2\Big)$$
$$\times\frac{|\Psi_D(Z,\beta)|^2}{Z(2-Z)}\delta(\beta-b_1+b_2-b)dZ\ d\beta\ db_1\ db_2\ db$$

$$\sigma^{pD}_{1,1;\mathcal{D}} = \int \Gamma(x_1, x_2;\ b)\hat{\sigma}(x_1 x_1')\hat{\sigma}(x_2 x_2')\Gamma\Big(\frac{x_1'}{Z};\ b_1\Big)\Gamma\Big(\frac{x_2'}{2-Z};\ b_2\Big)$$
$$\times \frac{|\Psi_D(Z,\beta)|^2}{Z(2-Z)}\delta(\beta - b_1 + b_2 - b)dZ\ d\beta\ db_1\ db_2\ db$$



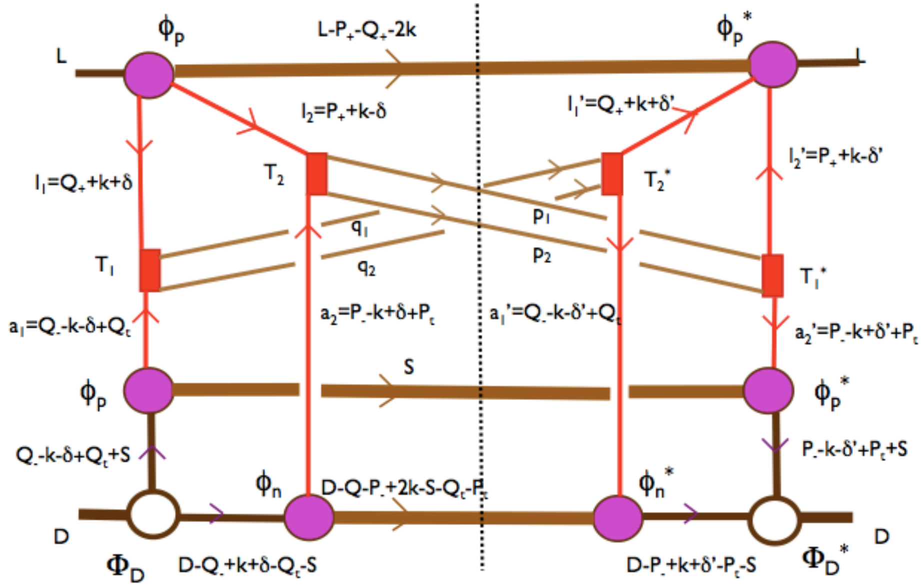
Differently from the previous case the integrations on the transverse coordinates are now connected by the δ-function. The dominant contribution can be obtained by neglecting the hadronic scale as compared with the deuteron radius. In such a case the δ-function becomes $\delta(\beta)$ and one obtains

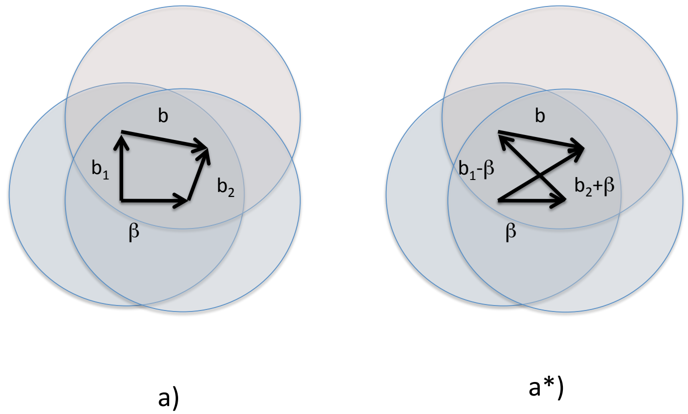
$$\sigma^{pD}_{1,1;\mathcal{D}} \simeq G(x_1, x_2)\hat{\sigma}(x_1 x_1')\hat{\sigma}(x_2 x_2')G(x_1')G(x_2') \times |\Psi_D(1,0)|^2$$

$$G(x_1, x_2) = \int \Gamma(x_1, x_2;\ b)db$$

which shows that the deuteron radius gives the dimensionality to the dominant contribution to $\sigma^{pD}_{1,1;\mathcal{D}}$, which thus ***does not depend on the correlation between partons in transverse space*** $\boldsymbol{\lambda}$***.***

Both target nucleons interacting with large momentum exchange.
Off-diagonal contribution:

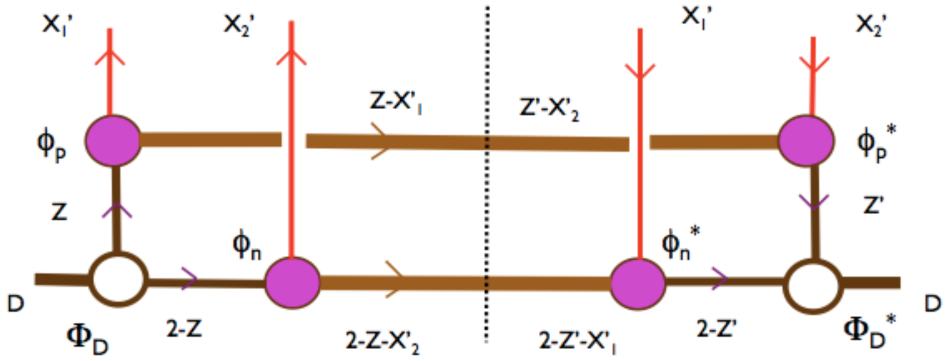


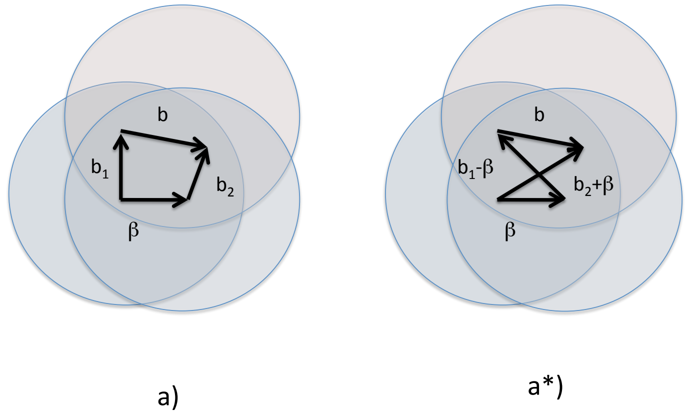


Interfering configurations a) and a*) in the off-diagonal contribution

$$\sigma^{pD}_{1,1;\mathcal{I}} = \int \Gamma(x_1,x_2;\ b)\hat{\sigma}(x_1x_1')\hat{\sigma}(x_2x_2')\tilde{H}\left(\frac{x_1'}{Z}\ ,\frac{x_2'}{Z'};\ b_1,b_2-\beta\right)\tilde{H}\left(\frac{x_2'}{2-Z}\ ,\frac{x_1'}{2-Z'};\ b_2,b_1-\beta\right)$$
$$\times\frac{\Psi_D(Z,\beta)}{Z}\frac{\Psi_D^*(2-Z',\beta)}{2-Z'}\delta(\beta+b_2-b-b_1)\delta(Z'-Z+x_1'-x_2')dZ\ dZ'\ d\beta\ db_1\ db_2\ db$$

In the interference term the cross section depends on the off-diagonal parton distributions $\tilde{H}$

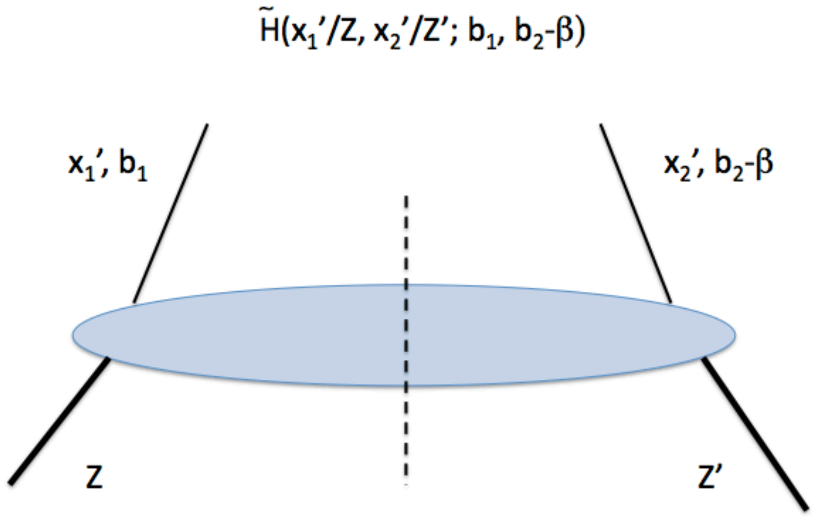




In the interference term the nucleon's fractional momenta are different in the right and in the left hand side of the cut: $Z\text{-}Z'=x'_1\text{-}x'_2$

x'_1 and x'_2 are measured in the final state. When $x'_1\text{-}x'_2$ is large the contribution of the interference term is small. In most cases of interest the contribution of the interference term is however sizable.

The two interactions are localized in two points in transverse space. Given two interaction points the parton with fractional momentum x'_1 may be provided by the proton and the parton with fractional momentum x'_2 by the neutron (configuration a) or vice-versa (configuration a*)



The interference term depends on the off diagonal parton distributions.

When neglecting the hadronic scale as compared with the deuteron radius, the deuteron radius gives the dimensionality to the cross section also in the case of the interference term.

To estimate the dominant contribution, one may neglect the hadronic scale as compared with the deuteron radius. Using

$$\Gamma(x_1, x_2;\ b) = G(x_1, x_2) f_{x_1 x_2}(b), \qquad G(x_1, x_2) = K(x_1, x_2) G(x_1) G(x_2)$$

and introducing

$$\hat{H}(x_1', x_2';\ b) \equiv \int \tilde{H}(x_1', x_2';\ b_1, b_1 - b) db_1 \qquad \hat{H}(x_1', x_1';\ 0) = G(x_1')$$

the expression of the interference term is given by

$$\begin{aligned} \sigma^{pD}_{1,1;\mathcal{I}}(x_1, x_2, x_1', x_2') \ \simeq\ & K(x_1, x_2) G(x_1) G(x_2)\ \hat{\sigma}(x_1 x_1') \hat{\sigma}(x_2 x_2') \int f_{x_1 x_2}(b) \hat{H}\Big(\frac{x_1'}{Z}, \frac{x_2'}{Z'};\ b\Big) \\ & \times \hat{H}\Big(\frac{x_2'}{2-Z}, \frac{x_1'}{2-Z'};\ b\Big) db \times \frac{\Psi_D(Z,0)\Psi_D^*(2-Z',0)}{Z(2-Z')} \delta(Z' - Z + x_1' - x_2') dZ dZ' \end{aligned}$$

As the Deuteron wave function is evaluated at β=0, **the cross section is proportional to $1/R^2_{\mathrm{D}}$. The Deuteron radius provides the dimensionality to the cross section.**

$$\sigma^{pD}_{1,1;\mathcal{I}}(x_1,x_2,x'_1,x'_2) \simeq K(x_1,x_2)G(x_1)G(x_2)\ \hat\sigma(x_1x'_1)\hat\sigma(x_2x'_2)\int f_{x_1x_2}(b)\hat H\Big(\frac{x'_1}{Z}\ ,\frac{x'_2}{Z'};\ b\Big)$$
$$\times\hat H\Big(\frac{x'_2}{2-Z}\ ,\frac{x'_1}{2-Z'};\ b\Big)db\times\frac{\Psi_D(Z,0)\Psi^*_D(2-Z',0)}{Z(2-Z')}\delta(Z'-Z+x'_1-x'_2)dZdZ'$$

Notice that although the interference term is proportional to $1/R_D^2$, it still depends on the correlation in transverse space λ through the overlap of *f* with the off diagonal parton distributions $\hat H$

The interference term can be nevertheless separated from the diagonal term since the two terms have a very different dependence on x'_1-x'_2.

Differently from the diagonal term, the interference term has in fact a strong dependence on x'_1-x'_2 because of the overlap of the Deuteron wave function.

Strong dependence on x'_1-x'_2

$$\frac{\sigma^{pD}_{1,1;\mathcal{I}}(x_1,x_2,x'_1,x'_2)}{\sigma^{pD}_{1,1;\mathcal{D}}(x_1,x_2,x'_1,x'_2)} = \frac{\int \frac{\Psi_D(Z,0)\Psi^*_D(2-Z',0)}{Z(2-Z')}\delta(Z'-Z+x'_1-x'_2)dZdZ'}{G(x'_1)G(x'_2)|\Psi_D(1,0)|^2}$$
$$\times \int f_{x_1x_2}(b)\hat{H}\Big(\frac{x'_1}{Z},\frac{x'_2}{Z'};\ b\Big)\hat{H}\Big(\frac{x'_2}{2-Z},\frac{x'_1}{2-Z'};\ b\Big)db$$

The ratio of the interference term with the diagonal term has a known dependence on x'_1-x'_2, which is dominated by the Deuteron wave function.

The dominant terms to the different contributions to the cross section are thus

$$\sigma^{pD}_{2,0}(x_1,x_2,x'_1,x'_2) \simeq 2\ \sigma^{pp}_{double}(x_1,x_2,x'_1,x'_2)$$

$$\sigma^{pD}_{1,1;\mathcal{D}}(x_1,x_2,x'_1,x'_2) \simeq K(x_1,x_2)\sigma^{pp}_{single}(x_1,x'_1)\sigma^{pp}_{single}(x_2,x'_2) \times |\Psi_D(1,0)|^2$$

$$\sigma^{pD}_{1,1;\mathcal{I}}(x_1,x_2,x'_1,x'_2) \simeq K(x_1,x_2)G(x_1)G(x_2)\ \hat\sigma(x_1x'_1)\hat\sigma(x_2x'_2)\int f_{x_1x_2}(b)\hat H\Big(\frac{x'_1}{Z},\frac{x'_2}{Z'};\ b\Big) \times\hat H\Big(\frac{x'_2}{2-Z},\frac{x'_1}{2-Z'};\ b\Big)db \times \frac{\Psi_D(Z,0)\Psi^*_D(2-Z',0)}{Z(2-Z')}\delta(Z'-Z+x'_1-x'_2)dZdZ'$$

Linear in K

One has $$\sigma^{pD}_{double} - \sigma^{pD}_{2,0} = \sigma^{pD}_{1,1;\mathcal{D}} + \sigma^{pD}_{1,1;\mathcal{I}}$$

By studying the left hand side of the equation as a function of x'_1-x'_2, one can separate the contribution of the interference term and obtain an indication both on $K(x_1,x_2)$ and on the overlap

$$\int f_{x_1x_2}(b)\hat H\Big(\frac{x'_1}{Z},\frac{x'_2}{Z'};\ b\Big)\hat H\Big(\frac{x'_2}{2-Z},\frac{x'_1}{2-Z'};\ b\Big)db$$

By combing the information of σ^{pD}_{double} and of σ^{pp}_{double} one may thus obtain an indication on $K(x_1,x_2)$ and on the overlaps

$$\int f_{x_1x_2}(b)\hat{H}\left(\frac{x'_1}{Z},\frac{x'_2}{Z'};\ b\right)\hat{H}\left(\frac{x'_2}{2-Z},\frac{x'_1}{2-Z'};\ b\right)db$$

$$\int f_{x_1x_2}(b)f_{x'_1x'_2}(b)db=\frac{1}{\pi\Lambda^2(x_1,x'_1,x_2,x'_2)}$$

which provide an indication on the size of the transverse correlation λ

Conclusions

Double parton interactions in *pp* collisions do not provide enough information to disentangle the different partonic correlations in the hadron structure.

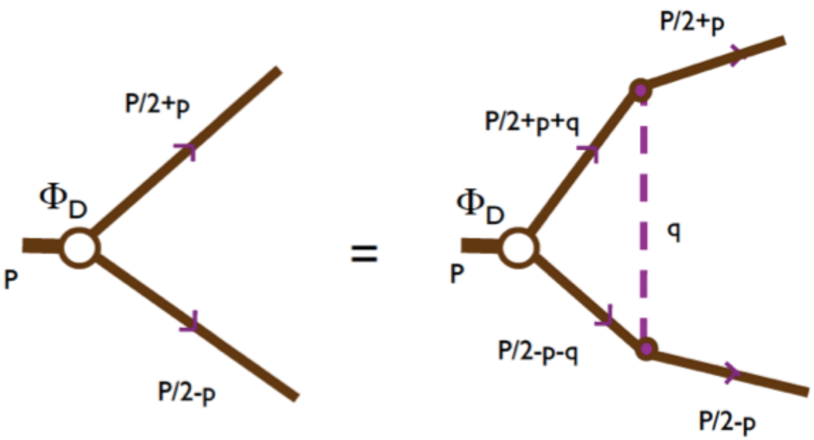
Additional information is provided by studying double parton interactions in *pD* collisions.

Double parton interactions in *pD* collisions are given by diagonal and off diagonal contributions.

The contributions to the *pD* cross section, were both target nucleons interact with large momentum exchange, are proportional to $K(x_1,x_2)$, which measures the longitudinal parton correlation.

By measuring double parton interactions in *pp* and *pD* collisions, one will obtain reliable indications on the size of $K(x_1,x_2)$ and on the correlation length in transverse space λ.

The Deuteron wave function



Bethe-Salpeter equation

$$\chi_D(P,p) = \frac{1}{\left[\left(\frac{P}{2}+p\right)^2 - m^2 + i\epsilon\right]\left[\left(\frac{P}{2}-p\right)^2 - m^2 + i\epsilon\right]} \int \frac{ig^2}{q^2-\mu^2}\chi_D(P,p+q)\frac{d^4q}{(2\pi)^4}$$

In the Deuteron rest frame one has:
and one can thus integrate on q_0

$$\frac{ig^2}{q^2-\mu^2} \to \frac{-ig^2}{\vec{q}^2+\mu^2}$$

Introducing $\phi_D(\vec{p}) = \int \chi_D(M_D, p)\frac{dp_0}{2\pi}$ one obtains

$$\phi_D(\vec{p}) = \frac{1}{\left(\vec{p}^2 + m^2\right)^{1/2}\left(M_D^2 - 4m^2 - 4\vec{p}^2\right)} \int \frac{-ig^2}{\vec{q}^2 + \mu^2}\phi_D(\vec{p}+\vec{q})\frac{d^3q}{(2\pi)^3}$$

which, in the non-relativistic limit, becomes the Schroedinger equation

$$\frac{\vec{p}^2}{2m_R}\phi_D(\vec{p}) + \frac{1}{m_R^2}\int \frac{g^2}{\vec{q}^2 + \mu^2}\phi_D(\vec{p}+\vec{q})\frac{d^3q}{(2\pi)^3} = -B\phi_D(\vec{p})$$

whre m_R is the reduced mass and B the binding energy. The relation with the usual non-relativistic Deuteron wave function (normalized to 1 after integrating on $\vec{p}$) is

$$\psi_{NR}(\vec{p}) = \frac{\phi_D(\vec{p})}{\sqrt{E_p}(2\pi)^{3/2}}$$

$$\Phi_D(p) = -i\big(\vec{p}^2 + m^2\big)^{1/4}\big(M_D^2 - 4m^2 - 4\vec{p}^2\big)\psi_{NR}(\vec{p})(2\pi)^{3/2}$$

Which gives the expression of the covariant Deuteron vertex in terms of the non-relativistic Deuteron wave function as a function of the c.m. three momentum $\vec{p}$ which has to be expressed in terms of boost invariant quantities:

$$E_p = \frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big), \qquad p_z = \frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 - \frac{2}{Z}m_t^2\Big)$$

The function $\Psi_D(p)$ is hence finally expressed in terms of light cone variables through the non-relativistic Deuteron wave function $\psi_{NR}(\vec{p})$ as

$$\begin{aligned}\Psi_D(zD_-;p_t) \;&=\; (2\pi)^{3/2}\sqrt{\frac{1}{2M_D}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big)} \\ &\times\; \psi_{NR}\left(\frac{1}{4M_D^2}\Big(\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\Big)^2 - m^2\right)\end{aligned}$$

At first sight σ_{eff} represents the transverse area in the overlap region of the two interacting hadrons, which is effective to originate double collisions. A small value of σ_{eff} would hence be originated by the correlation in transverse space in the parton pairs.

This is not the only possibility however. Configurations with high parton multiplicities may be frequent in the hadron structure, which would produce many double collisions and therefore a small value of the effective cross section.