Unitarity and consistency in multiple hard collisions

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We address the question of how to simultaneously account for unitarity constraints in a variety of different types of very high energy cross sections wherein a description of multiple hard partonic collisions is important. It is shown how models and extrapolations that utilize the concept of multiple hard partonic scatterings can be made consistent with one another while still adequately describing existing experimental data.

1 Introduction

1.1 s-channel unitarity

The procedure to account for s-channel unitarity in very high energy hadronic collisions has been understood for many years now; one defines a profile function for the limit of $s >> -t$ in terms of the amplitude $A(s,t)$ for elastic hadron-hadron scattering in the high energy limit $t \approx -\mathbf{q}_t^2$:

$$\Gamma(s, b_t) \equiv \frac{1}{2is(2\pi)^2} \int d^2\mathbf{q}_t \, e^{i\mathbf{q}_t \cdot \mathbf{b}_t} A(s,t), \tag{1}$$

Imposing unitarity and analyticity leads to the following set of well known relations between the elastic, total, and inelastic cross sections:

$$\sigma_{tot}(s) = 2 \int d^2\mathbf{b}_t \, \mathrm{Re}\,\Gamma(s, b_t), \tag{2}$$

$$\sigma_{el}(s) = \int d^2\mathbf{b}_t \, |\Gamma(s, b_t)|^2, \tag{3}$$

$$\sigma_{inel}(s) = \int d^2\mathbf{b}_t \left(2\,\mathrm{Re}\,\Gamma(s, b_t) - |\Gamma(s, b_t)|^2\right). \tag{4}$$

Defining an inelastic profile function,

$$\Gamma^{\rm inel}(s, b_t) \equiv \left(2\,\mathrm{Re}\,\Gamma(s, b_t) - |\Gamma(s, b_t)|^2\right), \tag{5}$$

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a description of the total cross section must obey

$$\Gamma^{\text{inel}}(s,b), \Gamma(s,b) \leq 1. \tag{6}$$

(Here it is assumed that the amplitude is entirely imaginary which is appropriate in the $s >> -t$ limit.) Generally, the profile function grows with energy. When $\Gamma^{\text{inel}}(s,b_t) \approx 1$ in some region $b_t < b_{\max}$ then it is said to have reached the "black disk limit" (BDL).

1.2 Minijets and multiple hard collisions

It has become common to combine the s-channel picture with treatments of multiple hard partonic collisions (e.g., [1] and references therein) thereby relating descriptions of minimum bias events, the underlying event, and other complex aspects of hadron-hadron collisions to the treatment of the total cross section. One common approach is to describe the production of semi-hard minijets using the standard perturbative QCD expression while modeling the contribution from soft physics (using for example Regge theory) and using an eikonal model of multiple scattering to reconstruct from this the total cross section:

$$\Gamma(s,b) = 1 - e^{-\chi_h(s,b) - \chi_s(s,b_t)}, \tag{7}$$

where $\chi_h(s,b)$ and $\chi_s(s,b_t)$ are eikonals that describe the hard and soft partonic collisions respectively.

For the case of just one hard collision the inclusive cross section can be calculated directly from the standard perturbative QCD factorization formula:

$$\begin{aligned} \sigma^{\text{inc}}_{\text{pQCD}}(s;p_t^c) = &\sum_{i,j,k,l} \frac{K}{1+\delta_{kl}} \int d\,x_1 d\,x_2 \int d\,p_t^2 \times \\ &\times \frac{d\hat{\sigma}_{ij\to kl}}{dp_t^2} f_{i/p_1}(x_1;p_t)\, f_{j/p_2}(x_2;p_t)\, \theta(p_t - p_t^c)\ , \end{aligned} \tag{8}$$

where $f_{i/p_1}(x_1;p_t)$ and $f_{j/p_2}(x_2;p_t)$ are the ordinary parton distribution functions. The perturbative expression is only valid for sufficiently large jet transverse momentum p_t, so a lower cutoff p_t^c must be imposed on the integral in Eq. 8. In practice, the value of $\sigma^{\text{inc}}_{\text{pQCD}}(s;p_t^c)$ is quite sensitive to the precise choice of p_t^c, and the cross section grows rapidly with energy [2]. This has been a persistent complication in attempts to incorporate Eq. (8) into complete descriptions of multiple partonic scattering, such as the eikonal description in Eq. (7). One naturally hopes to be guided by considerations like unitarity to determine the most appropriate value for p_t^c. However, constraints like the Froissart bound do not apply directly to inclusive cross sections like Eq. (8) which are proportional to particle multiplicity.

One way to tame the rapid growth of the cross section while lowering p_t^c is to adjust the width of the distribution of hard partons in impact parameter space, so that the unitarization effects built into the eikonal description of Eq. (7) become stronger. However, this does not actually increase the range of validity of the perturbative expression, as illustrated in Fig. 1. Moreover, it conflicts with direct measurements of the impact parameter distribution of hard partons, as we will discuss later.

2 Total cross section from multiple hard collisions

Given a description of hard multiple partonic scatterings, one can directly reconstruct their contribution to the total inelastic cross section from basic combinatorial arguments [3]. Following

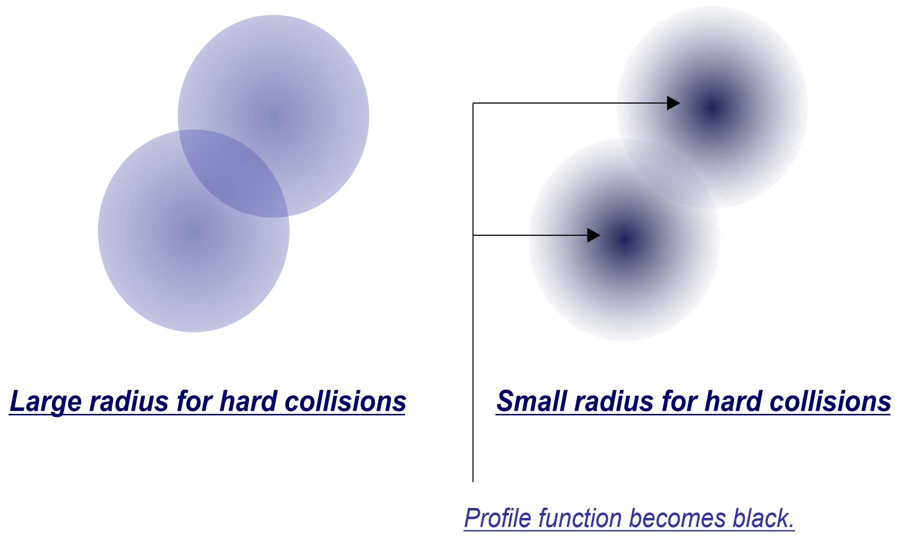

Figure 1: If the distribution of hard partons in transverse coordinate space is narrow, the cross section is tamed, but the approach to the black disk limit is faster, signaling a more rapid breakdown of normal QCD factorization.

the reasoning of Ref. [4], one finds

$$\Gamma^{\rm inel}_{\rm dijets}(s, b_t; p^c_t) = \sum_{n=1}^{\infty} (-1)^{n-1} \chi_{2n}(s, b_t; p^c_t), \tag{9}$$

where $\chi_{2n}(s, b_t; p^c_t)$ is the impact parameter dependent probability for n hard scatterings at impact parameter b_t. Overall consistency with Sect. 1.1 therefore requires that

$$\Gamma^{\rm inel}_{\rm dijets}(s, b; p^c_t) \leq \Gamma^{\rm inel}_{\rm actual}(s, b). \tag{10}$$

If it is assumed that the partons are identical and correlations are totally ignored, then

$$\chi_{2n}(s, b_t; p^c_t) = \frac{1}{n!} \chi_2(s, b_t; p^c_t)^n, \tag{11}$$

and the contribution to the total inelastic cross section in Eq. (9) becomes

$$\Gamma^{\rm inel}_{\rm dijets}(s, b_t; p^c_t) = 1 - \exp\left[-\chi_2(s, b_t; p^c_t)\right] \ . \tag{12}$$

Note that Eq. (12) has the same form as the eikonal expression in Eq. (7), though the reasoning that leads to it is quite different. $\chi_2(s, b_t; p^c_t)$ represents the distribution in transverse coordinate space of hard partons. Therefore, a description of the b_t-dependence of $\chi_2(s, b_t; p^c_t)$ is required to completely reproduce the total inelastic profile function.

3 Two-gluon form factor

Fortunately, the b_t-dependence in $\chi_2(s, b_t; p_t^c)$ can be extracted directly from experiments that probe impact parameter dependence. In Ref. [5], for instance, the following form was fitted to the two-gluon form factor in exclusive deeply inelastic vector meson production:

$$F_g(x, t; \mu) = \frac{1}{\left(1 - \frac{t}{m_g(x,\mu)^2}\right)^2}, \tag{13}$$

where the x and μ dependence in the parameter $m_g(x, \mu)$ account for some of the effects of evolution. Using this result allows $\chi_2(s, b_t; p_t^c)$ to be written as

$$\chi_2(s, b_t; p_t^c) = \sigma_{\rm pQCD}^{\rm inc}(s; p_t^c) P_2(s, b_t; p_t^c) \tag{14}$$

where

$$P_2(s, b_t; p_t^c) \equiv \frac{m_g^2(x; p_t^c)}{12\pi} \left(\frac{m_g(x; p_t^c) b_t}{2}\right)^3 K_3(m_g(x; p_t^c) b_t). \tag{15}$$

Expressed in this way, the total inclusive dijet cross section $\sigma_{\rm pQCD}^{\rm inc}(s; p_t^c)$ is

$$\sigma_{\rm pQCD}^{\rm inc}(s; p_t^c) = \int d^2\mathbf{b}_t \, \chi_2(s, b_t; p_t^c). \tag{16}$$

Using Eq. (14) in Eq. (12) then gives the contribution from dijet production to the left side of Eq. (10). The comparison in Eq. (10) was performed in Ref. [6] against typical extrapolations of $\Gamma_{\rm actual}^{\rm inel}(s, b)$ to high energy, and the inequality was found to be violated even for $b_t \gtrsim 1$ fm where multiple hard collisions are expected to be rarer. Since the impact parameter distribution is fixed by other measurements, this suggests that there is a problem with the uncorrelated scattering ansatz of Eq. (11).

4 General correlations

Reconciling the descriptions of the total cross section from Sect. 1.1 and 1.2 with the requirements of unitarity requires an account of non-perturbative correlations between the initial state partons. We note that the sizes of correlations can be extracted from measurements of observables like σ_{eff} [7] (Also, see E. Dobson, these proceedings). Therefore, in Ref. [6] the role of correlations was organized so that their effect on $\Gamma_{\rm dijets}^{\rm inel}(s, b; p_t^c)$ is easy to analyze.

Starting with the $n = 2$ contribution in the uncorrelated ansatz Eq. (11), a shift parametrized by $\eta_2(s, b)$ is introduced to account for correlations between two initial state partons. That is, we write

$$\chi_4(s, b; p_t^c) \to \frac{1}{2}\left(1 + \eta_4(s, b_t)\right) \chi_2(s, b; p_t^c)^2, \tag{17}$$

and similarly for larger n to account for triple and higher correlations. We call $\eta_{2n}(s, b_t)$ the n-correlation correction. Using Eq. (9) then gives

$$\Gamma_{\rm jets}^{\rm inel}(s, b_t; p_t^c) = 1 - \exp\left[-\chi_2(s, b_t; p_t^c)\right] - \sum_{n=2}^{\infty} \frac{(-1)^n \eta_{2n}(s, b_t)}{n!} \chi_2(s, b_t; p_t^c)^n \exp\left[-\chi_2(s, b_t; p_t^c)\right]. \tag{18}$$

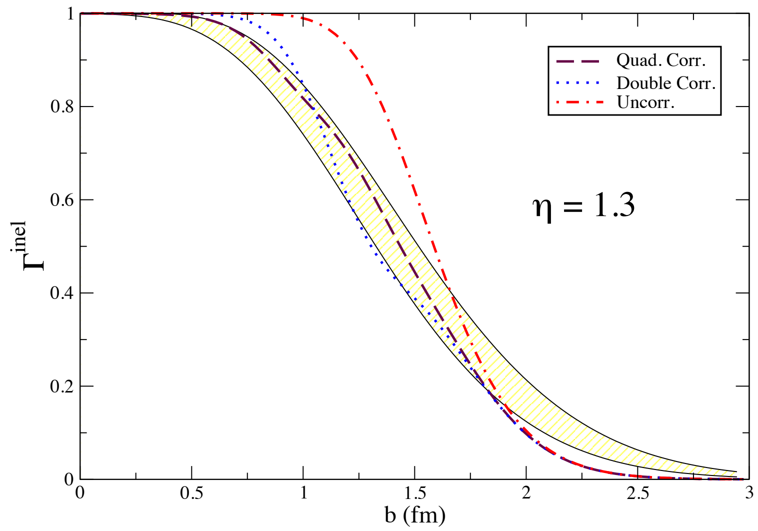

Figure 2: $\Gamma^{\text{inel}}_{\text{dijets}}(s, b_t; p_t^c)$ with correlation corrections imposed.

The $1 - \exp\left[-\chi_2(s, b; p_t^c)\right]$ part of Eq. 18 is the uncorrelated approximation. It also matches the commonly used eikonal model, though the reasoning used to arrive at it is rather different. Accounting for the $\eta_{2n}(s, b_t)$ functions for increasingly large n allows one to extend $\Gamma^{\text{inel}}_{\text{jets}}(s, b; p_t^c)$ to larger b_t before encountering a problem with Eq. (10).

The effect of double correlation corrections was estimated in Ref. [6], using the measured value of $\sigma_{eff} \approx 14.5$ mb from [8] which suggests a value for η_2 around ~ 1.3. As a first try, we ignore the impact parameter dependence of the correction. Using this in Eq. (18) gives the $\Gamma^{\text{inel}}_{\text{jets}}(s, b_t; p_t^c)$ shown in Fig. 2. For $\Gamma^{\text{inel}}_{\text{actual}}(s, b_t)$ we use a collection of Regge-like extrapolations [9], indicated in the figure by the yellow band. The dashed curve approximates all correlations by $\eta \approx 1.3$, while the dotted curve keeps only the double correlations. The energy is chosen to be the upper limit for the LHC, $\sqrt{s} = 14$ TeV and the transverse momentum cutoff is a typical value of $p_t^c = 2.5$ GeV. From the figure, it is clear that the large impact parameter region becomes much more consistent with Eq. (10) when the effect of non-perturbative correlations is included.

5 Conclusions

Taken together, existing measurements of inclusive cross sections and the impact parameter dependence of exclusive processes imply that non-perturbative correlations are needed in models of multiple hard scatterings in order to maintain reasonable consistency with unitarity while describing the growth of the total cross section, particularly at large impact parameters. The next step is to determine a method for estimating or calculating the correlation corrections in Eq. (18). Ideally, this will follow from a complete perturbative QCD factorization treatment that describes scattering with multi-parton correlation functions. Promising work in this direction has recently been presented in Ref. [10, 11]. General considerations of how to extract the sizes of correlations from observables in high energy collisions (e.g., Ref. [12]) are also needed.

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