UNIVERSITY OF CAMBRIDGE
Department of Physics
Cavendish Laboratory
Science & Technology Facilities Council

Double Parton Scattering Singularity in One Loop Integrals

Jo Gaunt

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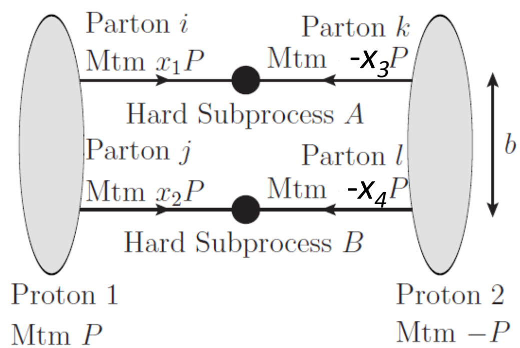
Based on JHEP 1106 (2011) 048 (JG, W J Stirling)

Outline

- Introduction to 'double PDF framework' for describing double parton scattering.
- Summary of our work showing that a compact analytic expression can be obtained for the 'DPS singular' part of a one-loop diagram. I use this expression to:
  - show that the treatment of 'double perturbative splitting' or '1v1' diagrams by the double PDF framework appears to be unsatisfactory.
  - explain behaviour of certain one-loop amplitudes near DPS singular points, that was not well-understood before.
- I explain what is wrong with the dPDF framework, and speculate as to how 1v1 diagrams might be correctly treated in the DPS cross section.
- Some brief discussion of how the 2v1 (or single perturbative splitting) diagrams should be incorporated in the DPS cross section. Discussion of full expression for the DPS cross section.
- Summary

Cross Section for Double Parton Scattering

Assuming only the factorisation of the two hard processes A and B, we can write the cross section for proton-proton DPS as follows:



Two-parton GPDs

Parton-level cross sections

$$\sigma^D_{(A,B)} \propto \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2) \hat{\sigma}_{ik\to A}(\hat{s} = x_1 x_3 s) \hat{\sigma}_{jl\to B}(\hat{s} = x_2 x_4 s)$$
$$\times \Gamma_{kl}(x_3, x_4, \boldsymbol{b}; Q_A^2, Q_B^2) dx_1 dx_2 dx_3 dx_4 d^2\boldsymbol{b}$$

Transverse parton pair separation

Cross Section for Double Parton Scattering

In many past studies of DPS, it has been assumed that the 2pGPD can be approximately factorised into a product of a longitudinal piece and a (typically flavour and scale independent) transverse piece:

$$\Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2) \simeq D_p^{ij}(x_1, x_2; Q_A^2, Q_B^2) F(\boldsymbol{b})$$

Smooth function of size R_p.

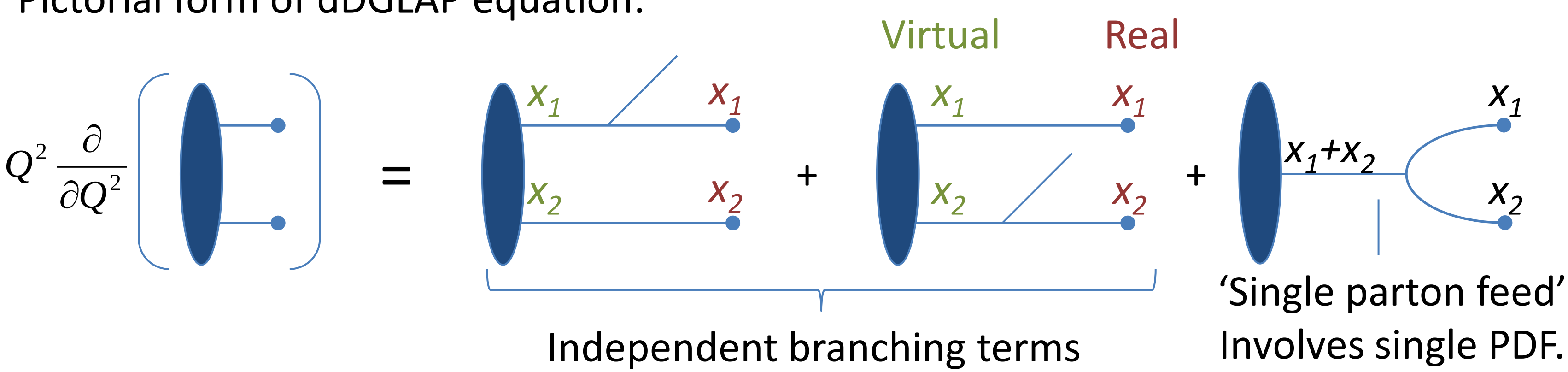
Then, introducing σ_{eff} via $\sigma_{eff} \equiv 1/\int F(\boldsymbol{b})^2 d^2\boldsymbol{b}$ we can write:

$$\sigma^D_{(A,B)} \propto \frac{1}{\sigma_{eff}} \sum_{i,j,k,l} \int \prod_{a=1}^{4} dx_a D_p^{ij}(x_1, x_2; Q_A^2, Q_B^2) D_p^{kl}(x_3, x_4; Q_A^2, Q_B^2) \times \hat{\sigma}_{ik\to A}(\hat{s} = x_1 x_3 s) \hat{\sigma}_{jl\to B}(\hat{s} = x_2 x_4 s)$$

Double PDF framework for calculating DPS

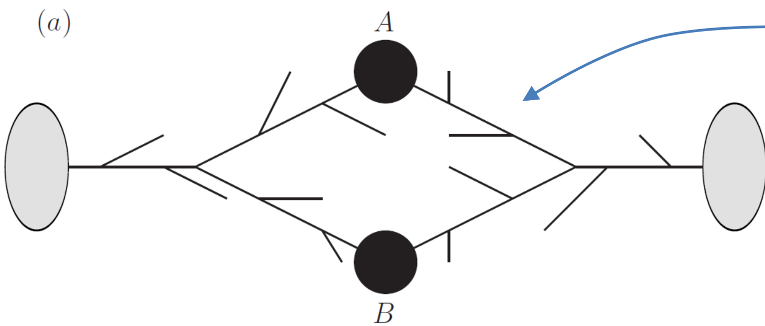
A quantity denoted as $D_h^{j1j2}(x_1,x_2,Q^2)$ (the double PDF, or dPDF) was introduced in 1982 by Shelest, Snigirev and Zinovjev [Phys. Lett. B 113:325], and an evolution equation for this quantity was given (dDGLAP equation). Subsequently suggested [see e.g. Snigirev, Phys.Rev. D68 (2003) 114012] that this quantity is equal to the factorised longitudinal piece of the 2pGPD for the case where $Q_A = Q_B \equiv Q$.

Pictorial form of dDGLAP equation:



Double PDF framework for calculating DPS

$Q_A{}^2 = Q_B{}^2 = Q^2 > 0$



Given the inclusion of single feed term, dPDF framework predicts that part of these 'double perturbative splitting' or '1v1' graphs should be included as DPS. At the cross section level the part that should be included is proportional to:

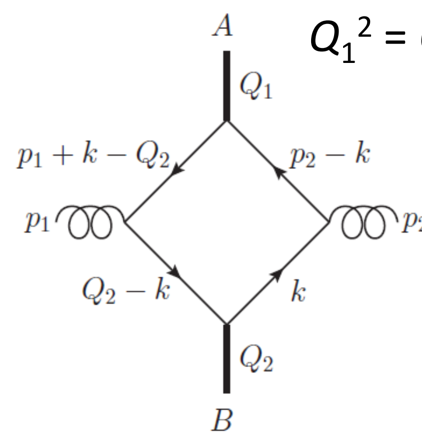
$$[\log(Q^2/\Lambda^2)]^n/\sigma_{eff}$$

n = total number of QCD branching vertices on either side of diagram.

This part should be associated with QCD branchings on either side of the diagram being strongly ordered in transverse momenta.

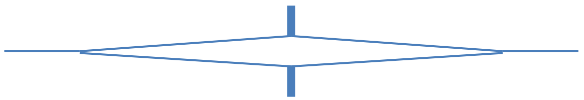
$Q_1{}^2 = Q_2{}^2 = Q^2 > 0$

Simplest example of a loop diagram with the given structure is the gg→AB crossed box diagram on the right. Does the cross section expression for this contain a piece proportional to $(\alpha_s \log(Q^2/\Lambda^2))^2/\sigma_{eff}$?



Double Parton Scattering Singularity

We would expect this piece to come from the portion of the external and loop integrations in which the transverse momenta of the outgoing particles are small, and all internal loop particles are almost on shell and collinear.



This is the region around a specific pinch singularity (Landau singularity) in the crossed box integral known as the double parton scattering singularity.

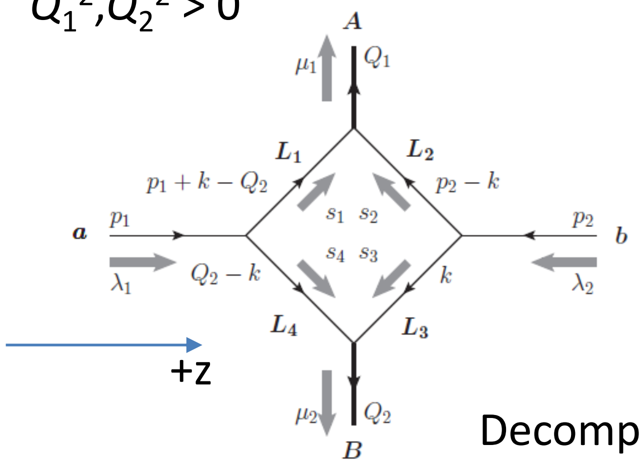
Nagy, Soper (Phys.Rev. D74 (2006) 093006)

DPS divergent part of a crossed box integral = expression for the part of the integral associated with the loop particles being almost on-shell and collinear, valid in the limit of small external transverse momenta. Expect this to diverge as external p_Ts → 0.

We have derived a simple analytical expression for the DPS divergent part of a crossed box diagram with arbitrary external and loop particles [JG and Stirling, JHEP 1106 048 (2011)]. Let us show how it is derived.

DPS Divergence in Crossed Box

$Q_1^2, Q_2^2 > 0$



Numerator – depends on nature of particles in diagram

$$L = \int d^d k \frac{\mathcal{N}}{[k^2 + i\epsilon][(k - Q_2)^2 + i\epsilon][(p_1 + k - Q_2)^2 + i\epsilon][(p_2 - k)^2 + i\epsilon]}$$

Loop propagator denominators – universal to all crossed boxes

Decompose all vectors in terms of a light cone basis defined using p_1 and p_2 as basis vectors.

Perform k^- integral followed by k^+ integral using contour methods, throwing away terms that are negligible in region around DPS singularity $|k^- - Q_2^-|, k^+, |\mathbf{k} - \mathbf{Q_2}|, |\mathbf{k}| << Q_i^+, Q_i^-$

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\, \mathcal{N}\, |_{k^- = Q_2^-, k^+ = 0}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

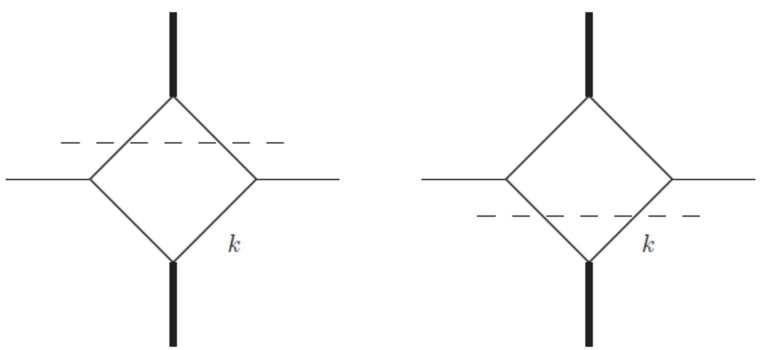
Compact expression!

Cutkosky cuts of the box

$$L_{DPS} \simeq \frac{(2\pi i)^2}{2s} \int_{|\boldsymbol{k}| \ll Q_i^+, Q_i^-} \frac{d^{d-2}\boldsymbol{k}\, \mathcal{N}\, |_{k^- = Q_2^-, k^+ = 0}}{(\boldsymbol{k} - \boldsymbol{Q_2})^2 \boldsymbol{k}^2}$$

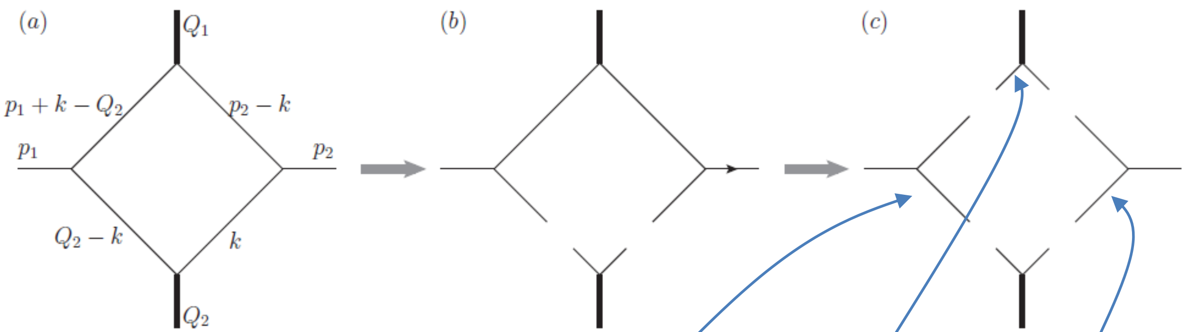
DPS divergence is in the real part of box integral – i.e. imaginary part of box amplitude since $i\mathcal{M} \propto L$

⇒ DPS divergent part of loop integral can also be found by taking sum of cuts in limit where external transverse momenta are small and internal particles are almost on shell.



Two cuts give the same contribution.

Decomposition of DPS divergent part of Crossed Box



$$
\begin{aligned}
L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i,L_i} \int d^dk\,\delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2\to s_2 s_3}_{b\to L_2 L_3}(p_2;p_2-k,k) \\
\times\Phi^{\lambda_1\to s_1 s_4}_{a\to L_1 L_4}(p_1;p_1+k-Q_2,Q_2-k)\mathcal{M}^{s_3 s_4\to\mu_2}_{L_3 L_4\to B}(k,Q_2-k;Q_2) \\
\times\mathcal{M}^{s_1 s_2\to\mu_1}_{L_1 L_2\to A}(p_1+k-Q_2,p_2-k;Q_1)\left(\times\sqrt{Q_1^2/Q_2^2}\right)
\end{aligned}
$$

Hard matrix elements – can evaluate with incoming off-shellness and transverse momentum = 0

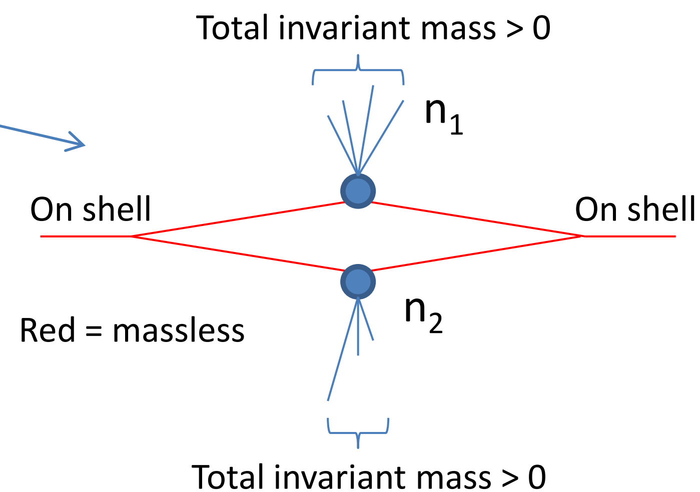
‘Light-cone wavefunction to find L_2L_3 in b’

DPS divergent part of arbitrary one-loop integral

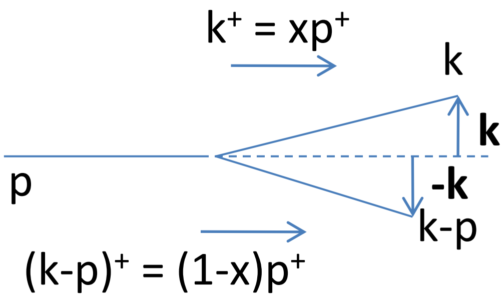
For crossed box: $$L_{DPS}(\lambda_1\lambda_2\mu_1\mu_2) = \sum_{s_i,L_i}\int d^dk\,\delta(k^2)\delta((k-Q_2)^2)\Phi^{\lambda_2\to s_2s_3}_{b\to L_2L_3}(p_2;p_2-k,k)$$
$$\times\Phi^{\lambda_1\to s_1s_4}_{a\to L_1L_4}(p_1;p_1+k-Q_2,Q_2-k)\mathcal{M}^{s_3s_4\to\mu_2}_{L_3L_4\to B}(k,Q_2-k;Q_2)$$
$$\times\mathcal{M}^{s_1s_2\to\mu_1}_{L_1L_2\to A}(p_1+k-Q_2,p_2-k;Q_1)\left(\times\sqrt{Q_1^2/Q_2^2}\right)$$

Any one-loop diagram of this structure also has a DPS divergence.

To obtain DPS divergent part of an arbitrary one-loop diagram (of the appropriate character), replace 2⟶1 matrix elements by 2⟶n_1, 2⟶n_2 matrix elements above.



Light-cone wavefunctions



$$\Phi_{a\to bc}^{\lambda_1\to s_1 s_2}(p;k,k-p)=X_{a\to bc}^{\lambda_1\to s_1 s_2}(x)K_{a\to bc}^{\lambda_1\to s_1 s_2}(\mathbf{k})$$

Square root of helicity dependent splitting function!

Transverse momentum dependent factor *K* contains a 1/$\mathbf{k}^2$ factor from propagator denominator, multiplied by a further factor coming from splitting matrix element.

Scalar ϕ^3 theory : splitting matrix element doesn't depend on **k.** For an arbitrary loop:

$$L_{DPS,\phi^3}\sim\int d^{d-2}\mathbf{k}\,K(\mathbf{k}-\mathbf{Q_2})K(\mathbf{k})\propto\int\frac{d^{d-2}\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{Q_2})^2}\propto\frac{1}{\mathbf{Q}_2^2}\quad \text{when d=4}$$

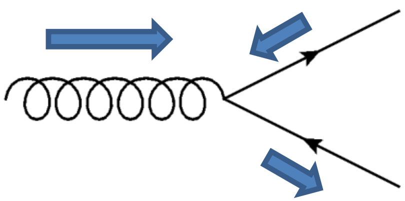
Power divergence at the DPS singular point which is unintegrable at the cross section level.

$$\int\frac{d^2\mathbf{Q}}{\mathbf{Q}^4}\to\infty$$

Light-cone wavefunctions

For any Standard Model massless particle splitting, matrix element is proportional to **k**.

Can show where this comes from for e.g. $g \to q\bar{q}$ graph:



Helicity conservation ➔ J_z of final state = 0 in collinear limit

J_z of initial state $=\pm 1$ ➔ splitting must be suppressed in collinear limit.

$$L_{DPS,SM} \sim \int d^{d-2}\mathbf{k}\, K(\mathbf{k}-\mathbf{Q_2}) K(\mathbf{k}) \sim \int_0^{Q^2} \frac{d^{d-2}\mathbf{k}\; \mathbf{k}\cdot(\mathbf{k}-\mathbf{Q_2})}{\mathbf{k}^2(\mathbf{k}-\mathbf{Q_2})^2} \sim \log\left(\frac{\mathbf{Q}_2^2}{Q^2}\right)$$

Strongly related to logarithmic scaling violations of parton distributions

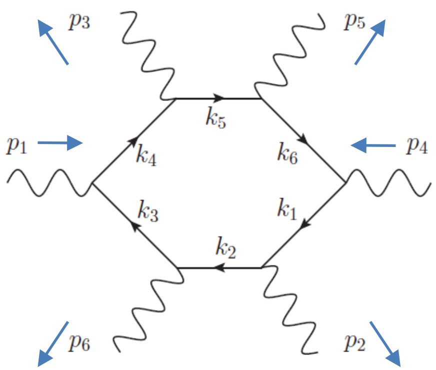
➔ DPS divergence in SM graphs cannot be stronger than a logarithm of ***Q*₂**.

Aside: DPS divergences in Six Photon Amplitude

Our analytical expression for the DPS divergence of a one-loop diagram can be used to **explain** interesting behaviour of amplitudes around DPS singular points that has been observed using 'traditional' NLO multileg integration techniques.

e.g. Six photon amplitude

Take all helicities as incoming, label helicity amplitude as $\lambda_1\lambda_2\ \lambda_3\lambda_4\ \lambda_5\lambda_6$



This is just one diagram contributing to the amplitude, which has a DPS singularity at $\mathbf{P}_\Sigma = \mathbf{p}_3 + \mathbf{p}_5 = 0$

Detailed numerical studies of specific MHV and NMHV amplitude by Bernicot and Guillet revealed the following properties of these amplitudes:

1. Neither helicity amplitude diverges as $\mathbf{P}_\Sigma \rightarrow 0$ like $1/\mathbf{P}_\Sigma^2$, as was expected by some authors.
2. The NMHV $- - - +++$ amplitude is finite at $\mathbf{P}_\Sigma = 0$.
3. The MHV $- ++ - ++$ amplitude is also finite at $\mathbf{P}_\Sigma = 0$.

Bernicot, arXiv:0804.1315,
Bern et. al., arXiv:0803.0494

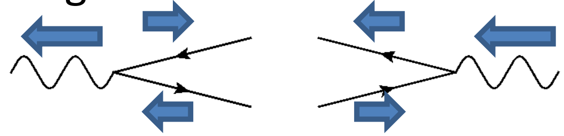
Aside: DPS divergences in Six Photon Amplitude

1. No helicity amplitude diverges at $\mathbf{P}_\Sigma \to 0$ like $1/\mathbf{P}_\Sigma^2$, as was expected by some authors.

Already explained. Associated with angular momentum nonconservation at both $\gamma \to q\bar{q}$ vertices in collinear limit.

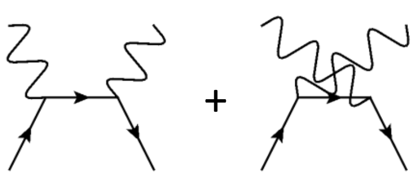
2. The NMHV $---+++$ amplitude is finite at $\mathbf{P}_\Sigma = 0$.

Overall J_z nonconservation between $\gamma\gamma$ initial state and $q\bar{q}q\bar{q}$ intermediate state in collinear limit weakens DPS divergence such that it is finite.



3. The MHV $-++-++$ amplitude is perfectly finite at $\mathbf{P}_\Sigma = 0$.

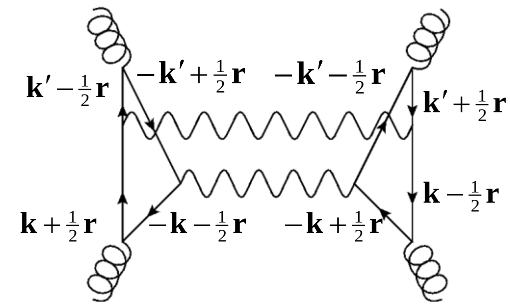
There are four graphs giving a DPS divergence at the point $\mathbf{P}_\Sigma = 0$. The matrix elements to be used in the calculation of the DPS divergent parts of the **sum** of these graphs are the sum of the following two graphs:



= full matrix element for $q\bar{q} \to \gamma\gamma$. For MHV amplitude studied, photons have same helicity in both matrix elements, and go to zero by MHV rules for QED.

‘Double Perturbative Splitting’ graphs

Insert our analytic expression for DPS singular part of loop into standard 2 → 2 cross section formula:



$$\sigma_{DPS,\text{fig 1(b)}} \propto \int \prod_{i=1}^{2} dx_i d\bar{x}_i \hat{\sigma}_{q\bar{q}\to A}(\hat{s}=x_1\bar{x}_1 s)\hat{\sigma}_{q\bar{q}\to B}(\hat{s}=x_2\bar{x}_2 s)$$

$$\times \int \frac{d^2\boldsymbol{r}}{(2\pi)^2}\Gamma_{g\to q\bar{q}}(x_1,x_2,\boldsymbol{r})\Gamma_{g\to\bar{q}q}(\bar{x}_1,\bar{x}_2,-\boldsymbol{r})$$

‘O(α_s) g → q$\bar{\text{q}}$ 2pGPD’

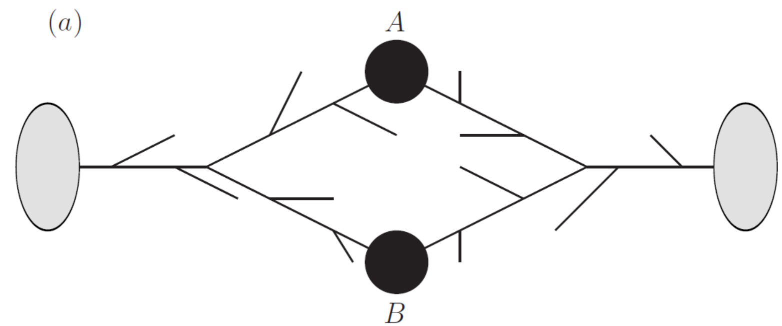
1 → 2 splitting function

$$\Gamma_{g\to q\bar{q}}(x_1,x_2,\boldsymbol{r}) \propto \frac{\alpha_S}{2\pi}\delta(1-x_1-x_2)T^{ij}(x_1,x_2)\int^{\tilde{\boldsymbol{k}}^2<\mathcal{O}(Q^2)} d^2\tilde{\boldsymbol{k}}\frac{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^i[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^j}{[\tilde{\boldsymbol{k}}+\frac{1}{2}\boldsymbol{r}]^2[\tilde{\boldsymbol{k}}-\frac{1}{2}\boldsymbol{r}]^2}.$$

r is Fourier conjugate of parton pair separation ***b***

Obtain a result that is consistent with the double PDF framework if one considers the portion of the integral with $|\boldsymbol{r}| < \Lambda_S$ as DPS, where Λ_S is a specific choice of cut-off of the order of Λ_{QCD}. But why should we consider this piece specifically as DPS?

‘Double Perturbative Splitting’ graphs



Same issues are encountered for an arbitrary double perturbative splitting graph. There is no distinct piece of the arbitrary double splitting graph that contains a natural scale of order Λ_{QCD} and is associated with the transverse momenta inside the loop being strongly ordered on either side of the diagram. Most of the contribution to the cross section expression for this graph comes from the region of integration in which the transverse momenta of particles inside the loop are of $O(\sqrt{Q^2})$.

Perhaps, then, we shouldn’t include any of this graph as DPS. This has the advantage of avoiding potential double counting between DPS and SPS.

‘Double Perturbative Splitting’ graphs

There are clearly theoretical issues with the double PDF framework. The source of these problems can be exhibited by Fourier transforming the ***r***-space perturbative splitting 2pGPD we obtained before into ***b*** space. We find:

$$\Gamma_{qq}\Big|_{g\to q\bar{q}}\left(x_1, x_2, \mathbf{b}\right) \sim 1/\mathbf{b}^2$$

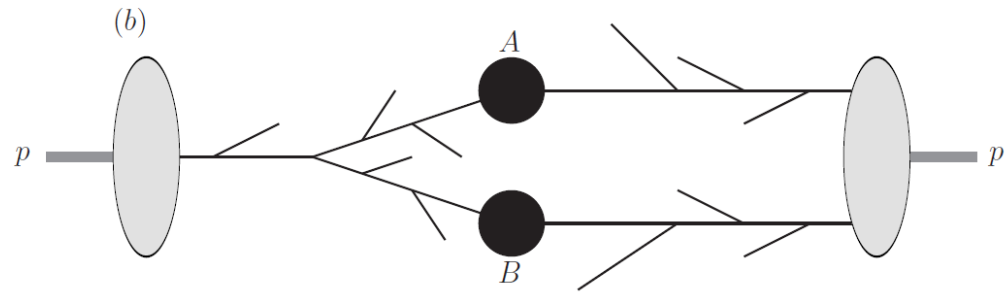
Power law behaviour – very different from smooth function of size R_p expected from double PDF framework. A key error in the formulation of the dPDF framework is the assumption that all 2pGPDs can be approximately factorised into dPDFs and smooth transverse functions of size R_p (as has been emphasised in previous talks in this session).

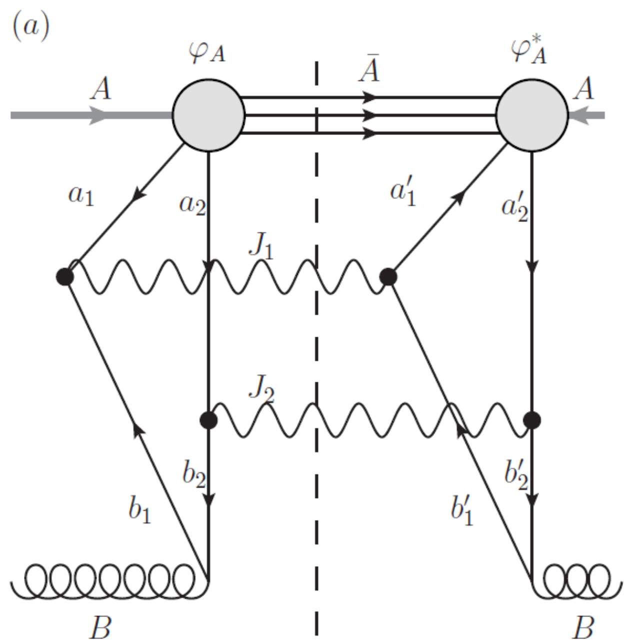
A sound theoretical framework for describing proton-proton DPS needs to carefully take account of the different ***b*** dependence of pairs of partons emerging from perturbative splittings, whilst simultaneously avoiding double counting between SPS and DPS.

See also Diehl and Schafer (Phys.Lett. B698 (2011) 389-402), Diehl, Ostermeier and Schafer (arXiv:1111.0910).

What about the 2v1 contribution?

This is where one proton provides one parton to the double scattering, and the other two, at the nonperturbative level.





Take a similar approach as we did for the 1v1 graphs. Look at the simplest graph in which a single parton splits and then interacts with two 'nonperturbatively generated' partons from a proton, and see if there is a structure in the cross section formula ~ $\log(Q^2/\Lambda^2)/R_P^2$

Need to use a wavefunction on the side with the two nonperturbative partons to represent the fact that the two partons are tied together in the same proton (see talk by Blok/Dokshitzer). I used formalism of Paver and Treleani (Nuovo Cim. A70 (1982) 215).

What about the 2v1 contribution?

Result:

Required large logarithm

$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}^{s_1,t_1;s_1',t_1';\mu_1}_{\bar{q}q\to\gamma*}(\hat{s}=x_1y_1s)\hat{\sigma}^{s_2,t_2;s_2',t_2';\mu_2}_{q\bar{q}\to\gamma*}(\hat{s}=x_2y_2s)$$

$$\times \quad \Gamma_A^{s_1 s_2, s_1' s_2'}(x_1,x_2;\boldsymbol{b}=\boldsymbol{0}) \quad \left[\underbrace{\frac{\alpha_s}{2\pi} P^{\lambda\to t_2 t_1, t_2' t_1'}_{g\to q\bar{q}}(y_2)\,\delta(1-y_1-y_2)}_{1 \longrightarrow 2 \text{ splitting function}} \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2}\right]$$

2pGPD of nonperturbatively generated parton pair evaluated at **b** = **0**

Summing leading logarithmic parts of all 2v1 graphs (diagonal unpolarised contribution):

$$\sigma^{D,1v2}_{(A,B)}(s) = 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s}=x_1y_1s)\hat{\sigma}_{jl\to B}(\hat{s}=x_2y_2s)$$

$$\times \breve{D}^{ij}_p(x_1,x_2;Q^2)\Gamma^{kl}_{p,indep}(y_1,y_2,\boldsymbol{b}=\boldsymbol{0};Q^2)$$

'sPDF feed' part of dPDF

'Independent branching' 2pGPD

Agrees with 2v1 contribution to DPS cross section recently proposed by Ryskin and Snigirev (Phys.Rev. D83 (2011) 114047), and 2v1 contribution in equation (11) of Blok et al., [arXiv:1106.5533].

What about the 2v1 contribution?

The critical requirement for the validity of the derivation on the previous page is that parton pairs connected only via nonperturbative interactions should have an ***r*** distribution that is cut off at values of order Λ_{QCD} (or a ***b*** distribution that is smooth on scales of size $\ll R_p$). That is, the ***r*** profile of $\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{\Delta}; Q^2)$ should have a width of order Λ_{QCD} .

The results of the previous slide are potentially misleading, in that they appear to indicate that 2v1 contribution to DPS probes independent branching 2pGPDs at zero parton separation. In fact, the results correspond to a broad logarithmic integral over values of $\boldsymbol{b}^2$ that are $\ll R_p^2$ but $\gg 1/Q^2$.

If we assume $\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2) = \tilde{D}^{ij}(x_1, x_2; Q^2) F(\boldsymbol{b})$ then 2v1 contribution to DPS cross section is similar to that predicted by dPDF framework, except with a different 'σ_{eff}' :

$$\frac{1}{\sigma_{eff,2v2}} \equiv \int d^2\boldsymbol{b}[F(\boldsymbol{b})]^2$$

$$\frac{1}{\sigma_{eff,1v2}} \equiv \quad F(\boldsymbol{b} = \boldsymbol{0})$$

Naive Gaussian for F(**b**) gives factor of two enhancement for 2v1. F(**b**) is nonperturbative however – don't really know a lot about it.

The DPS Cross Section

Combining suggestions for 1v1 and 2v1 graphs, we obtain the following formula for the DPS cross section:

$$\sigma^{D}_{(A,B)}(s) = \sigma^{D,2v2}_{(A,B)}(s) + \sigma^{D,1v2}_{(A,B)}(s)$$

$$\begin{aligned}\sigma^{D,1v2}_{(A,B)}(s) =& 2 \times \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s) \\ & \times \breve{D}^{ij}_{p}(x_1, x_2; Q^2)\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{b} = \boldsymbol{0}; Q^2)\end{aligned}$$

$$\begin{aligned}\sigma^{D,2v2}_{(A,B)}(s) =& \frac{m}{2} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{ik\to A}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{jl\to B}(\hat{s} = x_2 y_2 s) \\ & \times \int d^2\boldsymbol{b}\Gamma^{ij}_{p,indep}(x_1, x_2, \boldsymbol{b}; Q^2)\Gamma^{kl}_{p,indep}(y_1, y_2, \boldsymbol{b}; Q^2)\end{aligned}$$

The DPS Cross Section

Comments on this formula:

1. We were originally expecting to get a formula for the DPS cross section looking something like:

$$\sigma^D_{(A,B)} \propto \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, \boldsymbol{b}; Q_A^2, Q_B^2) \hat{\sigma}_{ik\to A}(\hat{s} = x_1 x_3 s) \hat{\sigma}_{jl\to B}(\hat{s} = x_2 x_4 s)$$
$$\times \Gamma_{kl}(x_3, x_4, \boldsymbol{b}; Q_A^2, Q_B^2) dx_1 dx_2 dx_3 dx_4 d^2\boldsymbol{b}$$

with the 2pGPDs being expressible in terms of hadronic matrix elements. What we have got does not seem to look like this.

2. In this formula, we have made a sharp distinction between perturbatively and nonperturbatively generated parton pairs. Is there some scale at which we can regard all parton pairs in the proton as being ‘nonperturbatively generated’, and if so what is the appropriate choice for this scale? (Presumably something rather close to Λ_{QCD}).

3. We have ignored all of the interesting correlated parton and interference contributions pointed out by Mekhfi (Phys.Rev. D32 (1985) 2380, Phys.Rev. D32 (1985) 2371), and Diehl, Ostermeier and Schafer (arXiv:1111.0910).

Summary

- We have derived a compact analytical expression for the DPS divergence in an arbitrary one-loop diagram. Using this expression, we have:
  - Shown that the DPS divergent part of a one-loop diagram does not behave as is anticipated by the 'double PDF framework'.
  - Explained the behaviour of various one-loop amplitudes near points that correspond to a DPS singularity for a subset of the contributing graphs.
- The majority of the contribution to a 1v1 loop graph comes from the region in which the particles inside the loop have virtualities and transverse momenta of order of the hard scale. Maybe we should consider all of such graphs as SPS?
- Calculation of a simple 2v1 graph seems to indicate that 2v1 diagrams should be included in DPS cross section, but with a different σ_{eff}.
- Is the total cross section then just a sum of 2v2 and 2v1 contributions, with different σ_{eff} for each?