Workshop on Multiple Parton Interactions, DESY, November 2011

Parton Showers, Forward Physics and Multiparton Interactions

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Thanks for collaboration to
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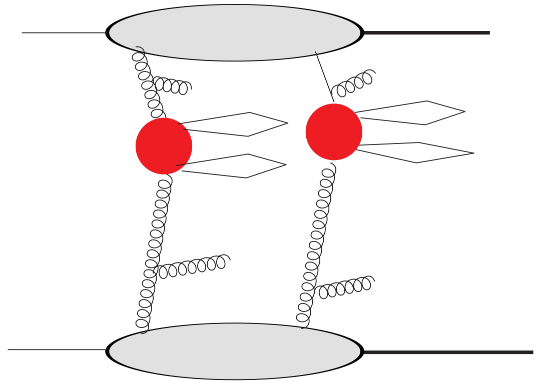
H. Jung (CERN/DESY), K. Kutak (Cracow)

• Multi-parton interactions increasingly important
as parton densities grow with energy



Multi-jet production by (left) multiple parton collisions; (right) single parton collision.

• Effective picture of parton density evolution based on
collinear DGLAP for inclusive observables

• MPI contribute primarily to highly differential cross section
probing detailed distribution of the states produced by parton evolution

- How do high-energy corrections to parton shower evolution affect treatment of MPI

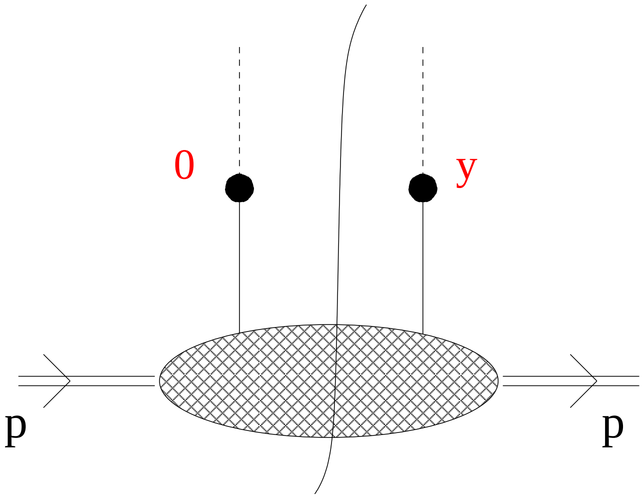
OUTLINE

I. Noncollinear corrections to shower evolution

II. Forward jets and energy flow

III. J/ψ and associated jet multiplicities

I.A TRANSVERSE MOMENTUM DEPENDENT INITIAL-STATE DISTRIBUTIONS



$$p = (p^+, m^2/2p^+, 0_\perp)$$

$$\widetilde{f}(y) = \langle \, P \mid \overline{\widetilde{\psi}}(y) \, \gamma^+ \, \widetilde{\psi}(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

correlation of quark fields (‘dressed’ with gauge links) at distances y, $y_\perp \neq 0$

i) Single-scale hadron scattering

$$\sigma(Q,m) = C(Q, \text{parton momenta } > \mu) \otimes f(\text{parton momenta } < \mu, m)$$

$$\text{RG invariance} \quad \frac{d}{d\ln\mu}\,\sigma = 0 \quad \Rightarrow \quad \frac{d}{d\ln\mu}\,\ln f = \gamma = -\frac{d}{d\ln\mu}\,\ln C$$

$\hookrightarrow$ DGLAP evolution equations [Altarelli-Parisi
Dokshitzer
Gribov-Lipatov]

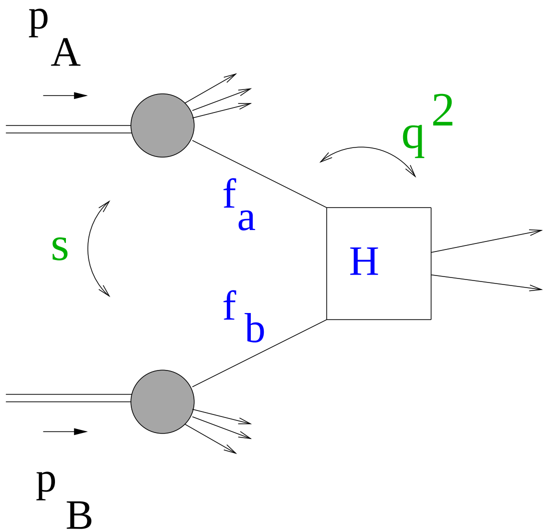
$$f = f_0 \times \exp \int \frac{d\mu}{\mu}\,\gamma(\alpha_s(\mu))$$

$\nearrow$ resummation of $(\alpha_s \ln Q/\Lambda_{\rm QCD})^n$ to all orders in PT

Expansions $\gamma \simeq \gamma^{(LO)}\left(1 + b_1\alpha_s + b_2\alpha_s^2 + \ldots\right)$, $C \simeq C^{(LO)}\left(1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots\right)$

give LO, NLO, ... logarithmic corrections

ii) Multiple-scale hard scattering at LHC energies



$s \gg q_1^2 \gg \cdots q_n^2 \gg \Lambda$

- more complex, potentially large corrections to all orders in α_s, $\sim \ln^k(q_i^2/q_j^2)$

$$e.g.\ \ \gamma \simeq \gamma^{(LO)}\left(1 + c_1\alpha_s + \ldots + c_{n+m}\alpha_s^m(\alpha_s\ L)^n + \ldots\right)\ ,\ L = \text{"large log"}$$

$\hookrightarrow$ yet summable by QCD techniques that

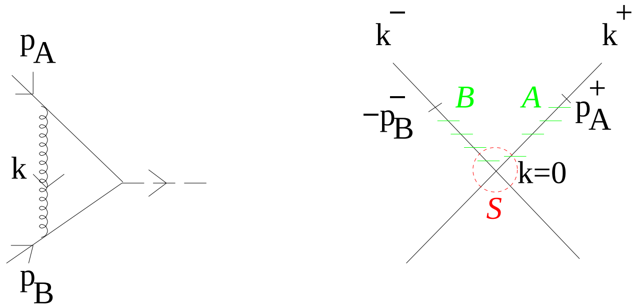
▷ generalize renormalization-group factorization

▷ extend parton correlation functions off the lightcone

$\Rightarrow$ unintegrated (or TMD) pdf's

Examples:

- Sudakov form factor S:



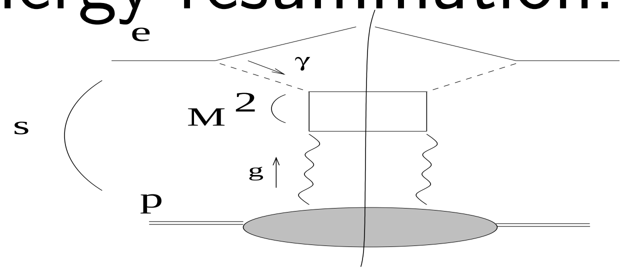
(a) (b)

▷ entering Drell-Yan production, W-boson $p_\perp$ distribution, etc.

$\Rightarrow \partial S/\partial \eta = K \otimes S$ CSS evolution equations [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

- High-energy resummation: $s \gg M^2 \gg \Lambda_{\rm QCD}^2$



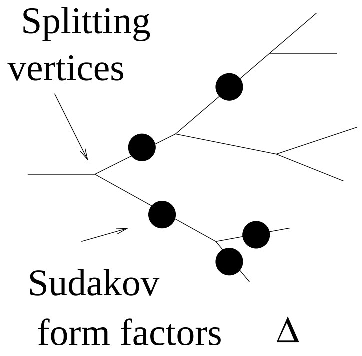
♢ energy evolution: BFKL equation [Balitsky-Fadin-Kuraev-Lipatov]

↪ corrections down by $1/\ln s$ rather than $1/M$

I.B FROM QCD TO MONTE CARLO EVENT GENERATORS

- Factorizability of QCD x-sections $\longrightarrow$ probabilistic branching picture

$\diamondsuit$ $A)$ QCD evolution by "parton showering" methods:

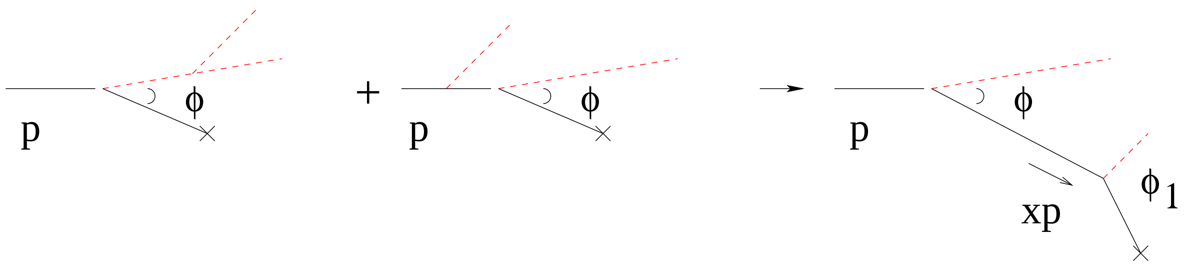


$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \; \alpha_S(q^2) \; P(z) \; \Delta(q^2, q_0^2)$$

$\hookrightarrow$ collinear, incoherent emission

$\diamondsuit$ $B)$ Soft emission $\longrightarrow$ interferences $\longrightarrow$ ordering in decay angles:

$\hookrightarrow$ gluon coherence for $x \sim 1$



$\diamondsuit$ $C)$ Gluon coherence for $x \ll 1$ $\Rightarrow$ corrections to angular ordering:

$\hookrightarrow$ MC based on $k_\perp$-dependent unintegrated pdfs and MEs

COHERENCE IN HIGH-ENERGY LIMIT

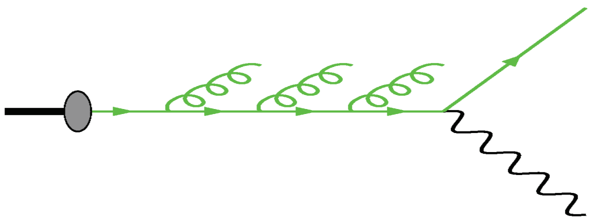
Soft vector-emission current from external legs $\rightarrow$

• leading IR singularities

[J.C. Taylor, 1980; Gribov-Low (QED)]

• fully appropriate in single-scale hard processes

Dokshitzer, Khoze, Mueller and Troian, RMP (1988); Webber, A. Rev. Nucl. Part. (1986)



multi-scale: $s = q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda^2$

[e.g.: LHC final states with multi-jets]

$\Downarrow$

▷ internal emissions non-negligible

▷ current also factorizable at high-energy: *[Ciafaloni 1998; 1988]*

$$\left|M^{(n+1)}(k,p)\right|^2 = \left\{ [M^{(n)}(k+q,p)]^\dagger \, [\mathbf{J}^{(R)}]^2 \, M^{(n)}(k+q,p) - [M^{(n)}(k,p)]^\dagger \, [\mathbf{J}^{(V)}]^2 \, M^{(n)}(k,p) \right\} .$$

BUT... ▷

▷ ... • **J** depends on total transverse momentum transmitted

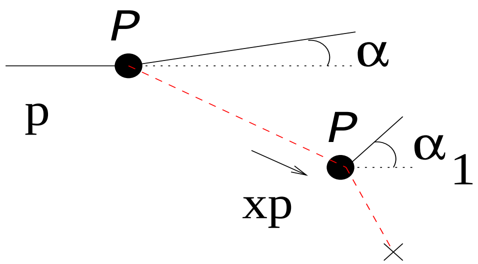
$\Rightarrow$ matrix elements and pdf at fixed $k_\perp$ ("unintegrated")

• virtual corrections not fully represented by Δ form factor

$\Rightarrow$ modified branching probability $P(z, k_\perp)$ as well

▷ $K_\perp$-DEPENDENT PARTON BRANCHING

$$\mathcal{G}(x, k_T, \mu) = \mathcal{G}_0(x, k_T, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2}\, \Theta(\mu - zq) \times \underbrace{\Delta(\mu, zq)}_{\text{Sudakov}} \; \underbrace{\mathcal{P}(z, q, k_T)}_{\text{unintegr. splitting}} \; \mathcal{G}(x/z, k_T + (1-z)q, q)$$



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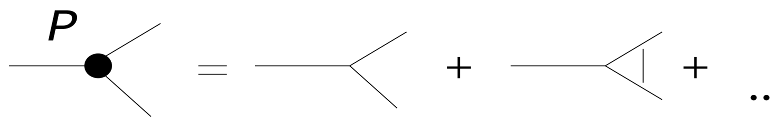
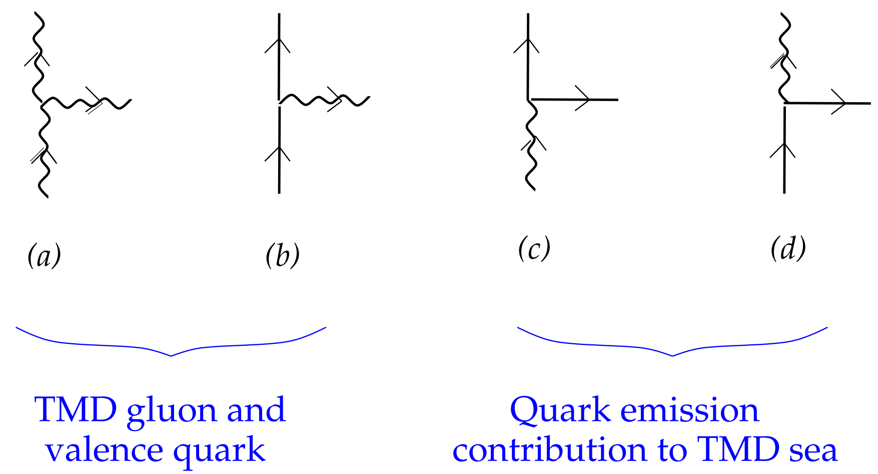


▷ CCFM evolution equation

▷ Monte Carlo implementations: CASCADE, LDC, ...

Beyond quenched approximation: unintegrated quark evolution

[Hentschinski, Jung & H, in progress]



- sea: flavor-singlet evolution coupled to gluons at small x via

$$\mathcal{P}_{g\to q}(z;q,k) = P_{qg,\mathrm{GLAP}}(z)\ \left(1 + \sum_{n=0}^{\infty} b_n(z)(k^2/q^2)^n\right)$$

all b_n known; $\mathcal{P}_{g\to q}$ computed in closed form (positive-definite)

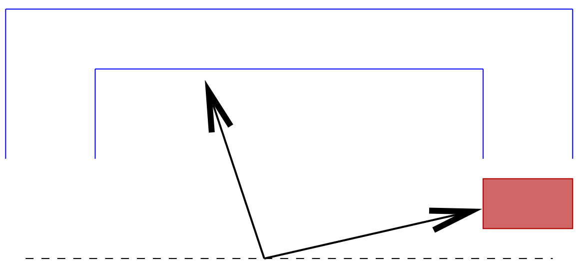
in *[Catani & H, 1994; Ciafaloni et al., 2005-2006]* by small-x factorization

- valence: independent evolution (dominated by soft gluons x $\to$ 1)

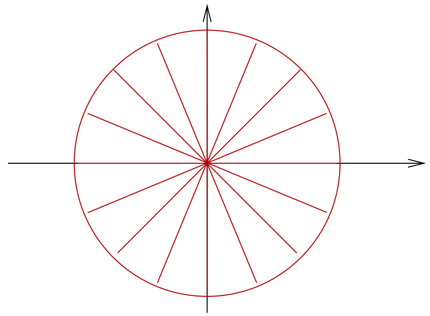
II. FORWARD JETS AT THE LHC

- polar angles small but far enough from beam axis
  - measure correlations in azimuth, rapidity, p_T

$$p_\perp \gtrsim 20 \text{ GeV} , \ \Delta\eta \gtrsim 4 \div 6$$

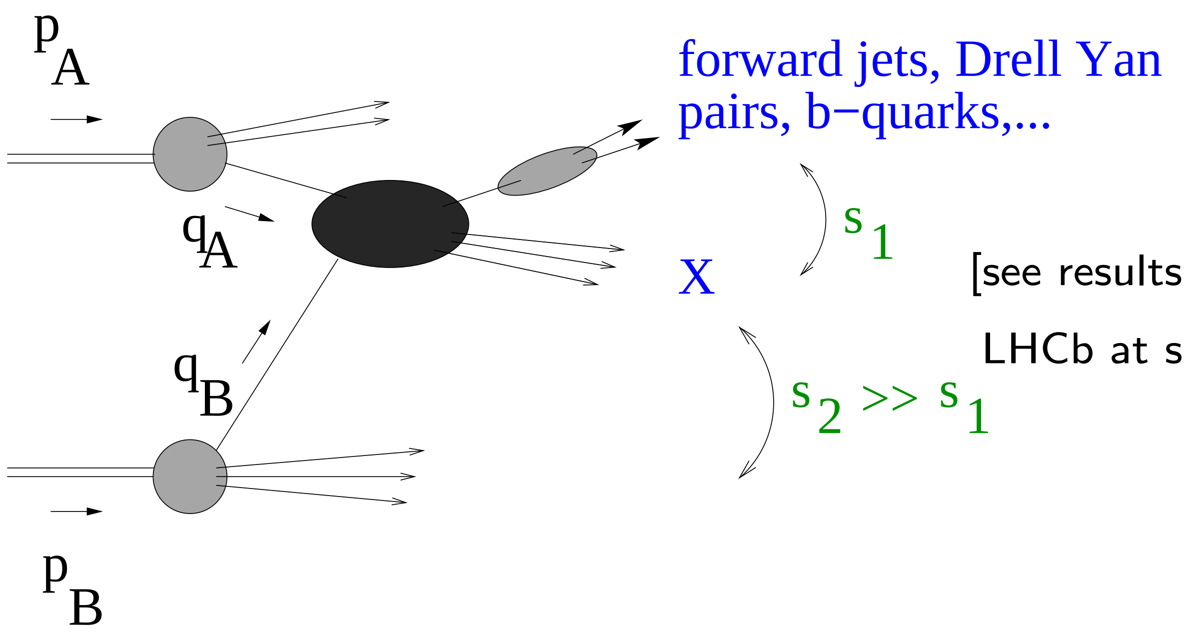


central + forward detectors



azimuthal plane

High-p_T production in the forward region



[see results by CMS,

LHCb at summer conf's]

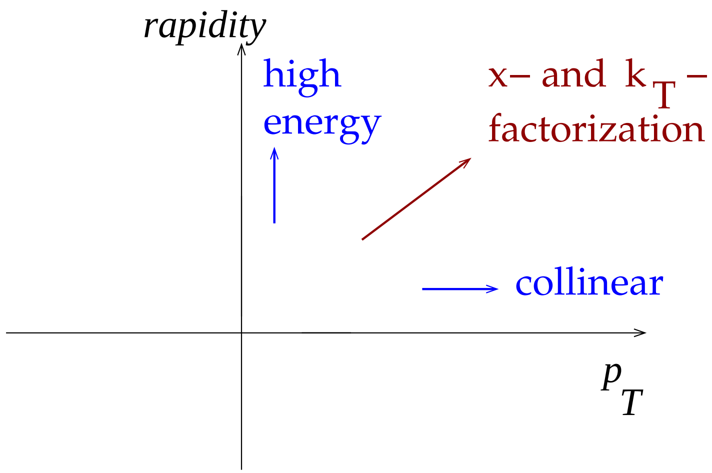
- multiple hard scales
- asymmetric parton kinematics $x_A \to 1, \; x_B \to 0$

Forward jet production as a multi-scale problem

- summation of high-energy logarithmic corrections long recognized to be necessary for reliable QCD predictions
  $\Rightarrow$ BFKL calculations

Mueller & Navelet, 1987; Del Duca et al., 1993; Stirling, 1994; Colferai et al., arXiv:1002.1365

- Large logarithmic corrections are present both in the hard p_T and in the rapidity interval



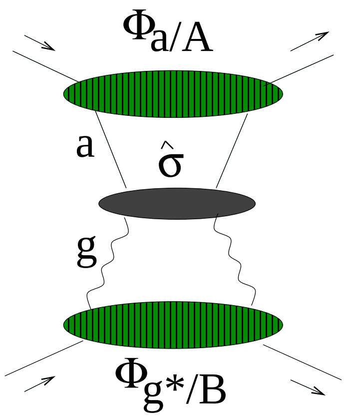
$\longrightarrow$ Both kinds of log contributions can be summed consistently to all orders of perturbation theory via QCD factorization at fixed k_T

Forward jets:

- High-energy factorization at fixed transverse momentum

$$\frac{d\sigma}{dQ_t^2 d\varphi} = \sum_a \int \phi_{a/A} \otimes \frac{d\widehat{\sigma}}{dQ_t^2 d\varphi} \otimes \phi_{g^*/B}$$

▷ needed to resum consistently both logs of rapidity and logs of hard scale *Deak, Jung, Kutak & H, JHEP 09 (2009) 121*



(a)

Figure 1: Factorized structure of the cross section.

♢ ϕ_a near-collinear, large-x; ϕ_{g^*} $k_\perp$-dependent, small-x

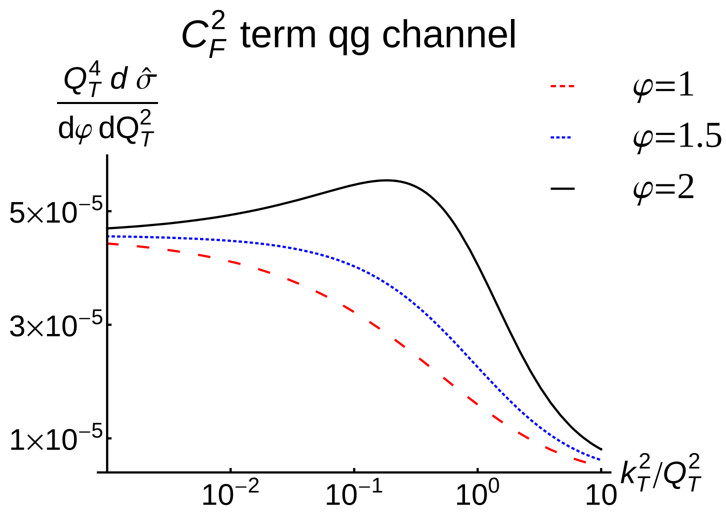
♢ $\widehat{\sigma}$ off-shell (but gauge-invariant) continuation of hard-scattering matrix elements *[Catani et al., 1991; Ciafaloni, 1998]*

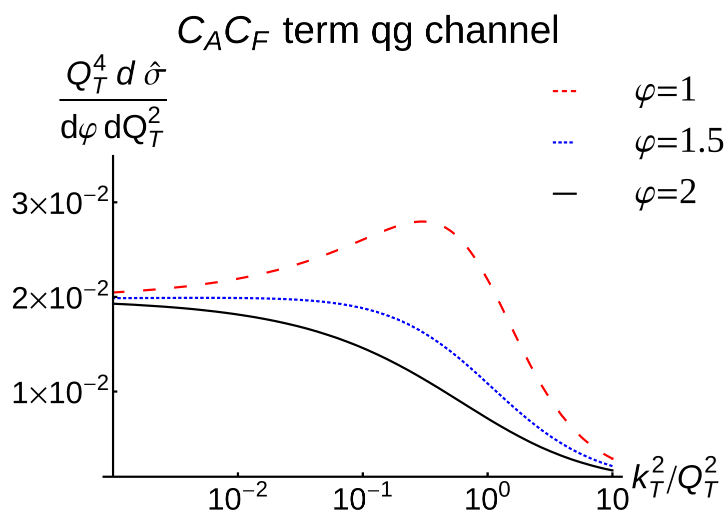
FULLY EXCLUSIVE MATRIX ELEMENTS: BEHAVIOR AT LARGE $K_\perp$

Deak, Jung, Kutak & H, JHEP 09 (2009) 121

Q_t = final-state transverse energy (in terms of two leading jets p_t's)

k_t = transverse momentum carried away by extra jets





- Matrix elements factorize for high energy

not only in collinear region but also at finite angle

$\Rightarrow$ effects of coherence across large rapidity intervals not associated with small angles

- Coupling to parton showers via merging scheme defined by factorization at high energy

Remarks

♢ Note difference from classic Mueller-Navelet approach

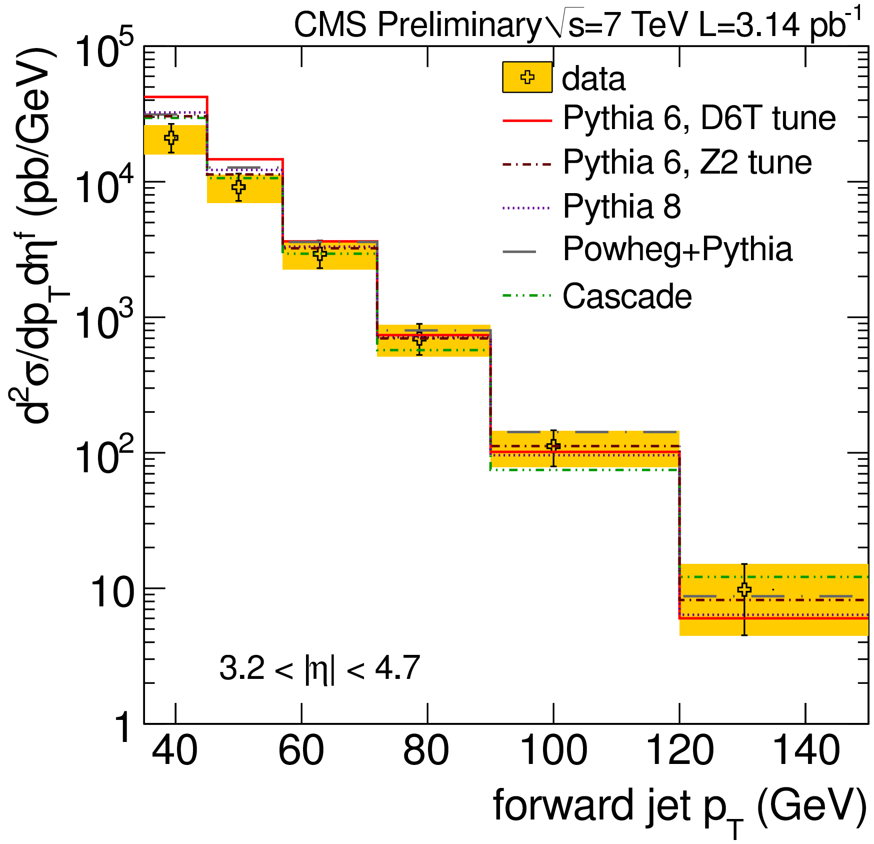
$$\sigma^{(MN)} = \sum_a \int \phi_{a/A} \otimes V_{jet1} \otimes \mathcal{G}_{gg} \otimes V_{jet2} \otimes \phi_{b/B}$$

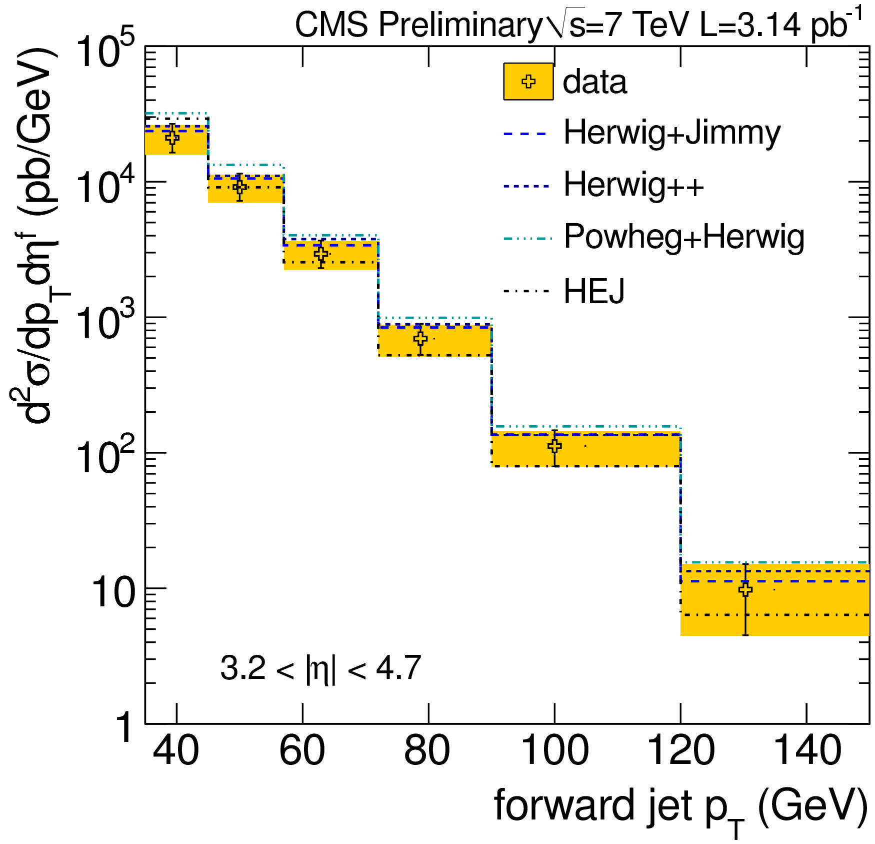
[Colferai, Schwennsen, Szymanowski and Wallon, JHEP 12 (2010) 026]

[D'Enterria, arXiv:0911.1273]

- non-collinear corrections to ϕ distributions
- no "vertex jet function" V_{jet}
- jets produced by either hard ME or parton shower (taking account of $k_\perp$)

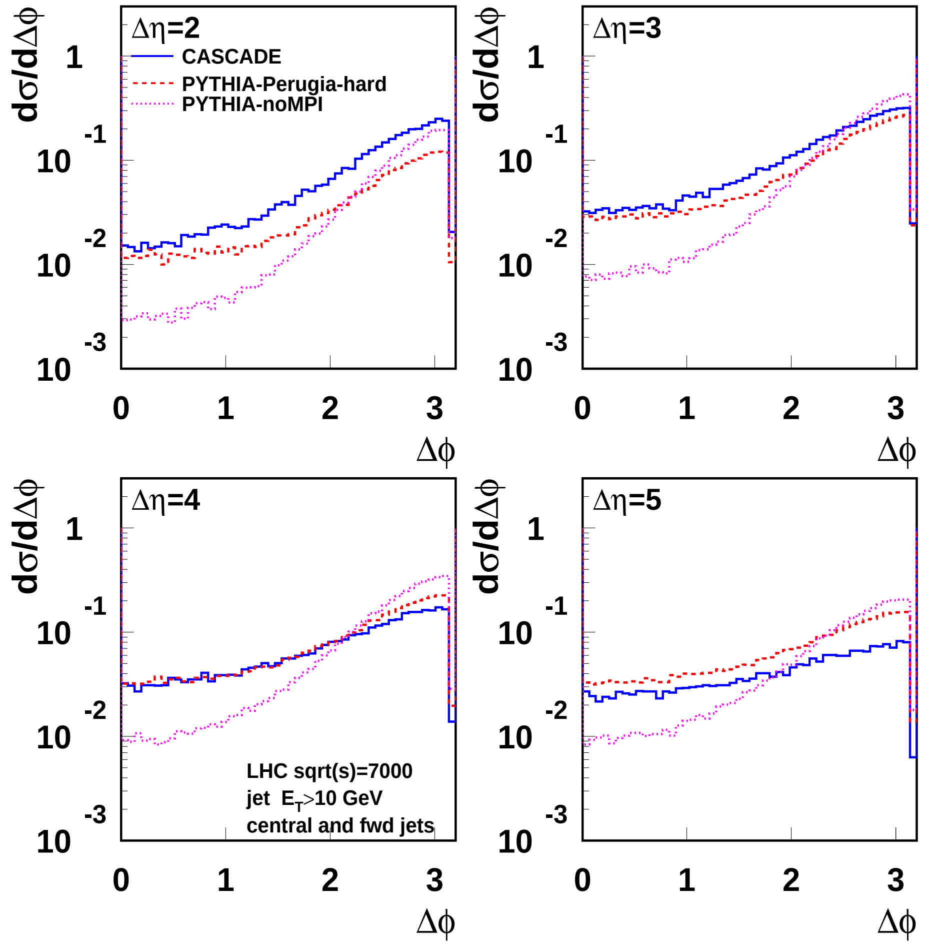
Forward jet spectrum *[CMS PAS FWD-10-006 (April 2011)]*

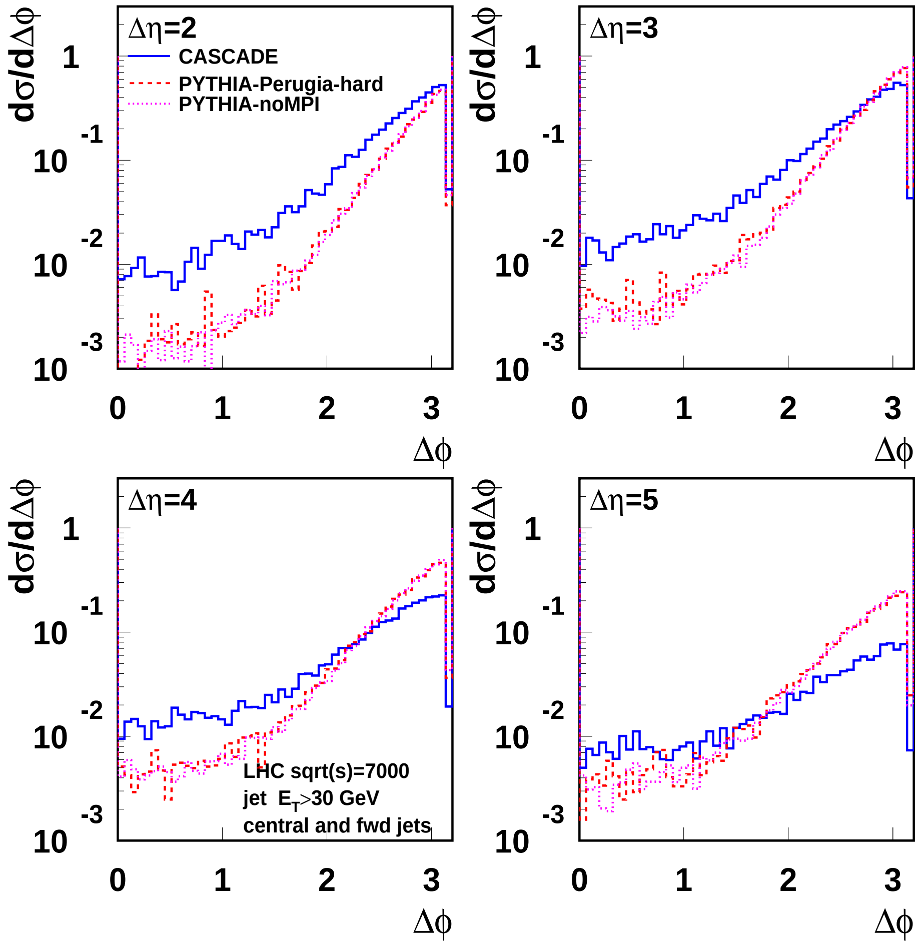




Cross section as a function of the azimuthal difference $\Delta\phi$ between central and forward jet for different rapidity separations

[Deak et al., arXiv:1012.6037]





MC models:
- CASCADE: non-collinear radiative corrections to single parton chain
- PYTHIA: multiple parton interactions, no corrections to collinear approximation

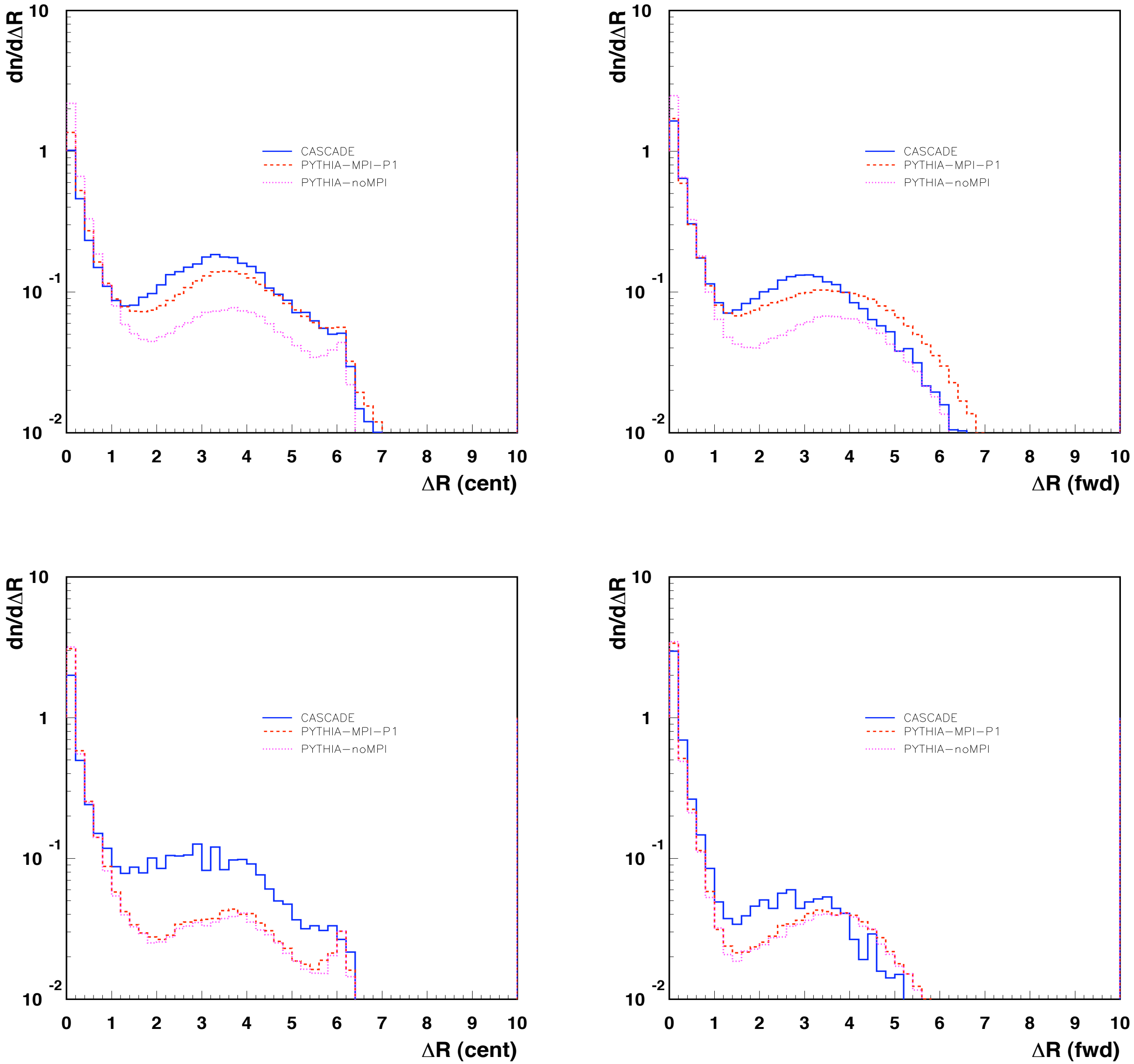
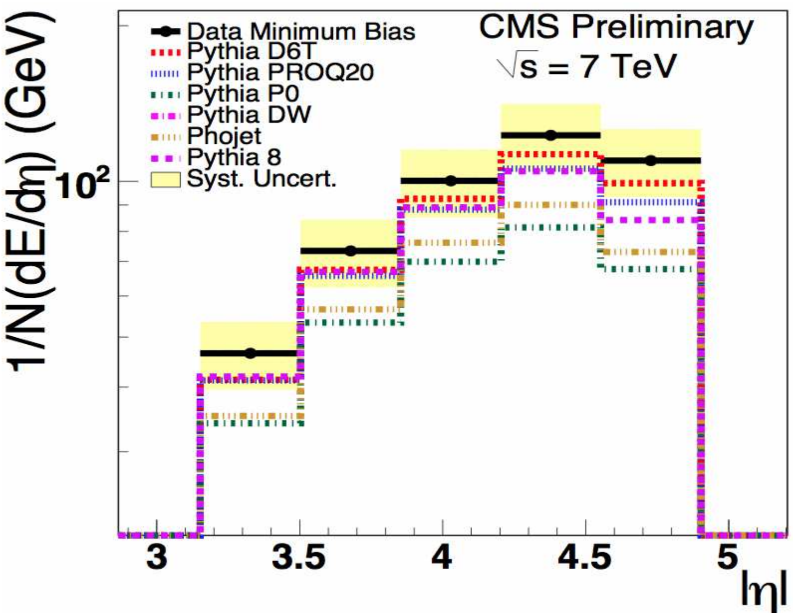
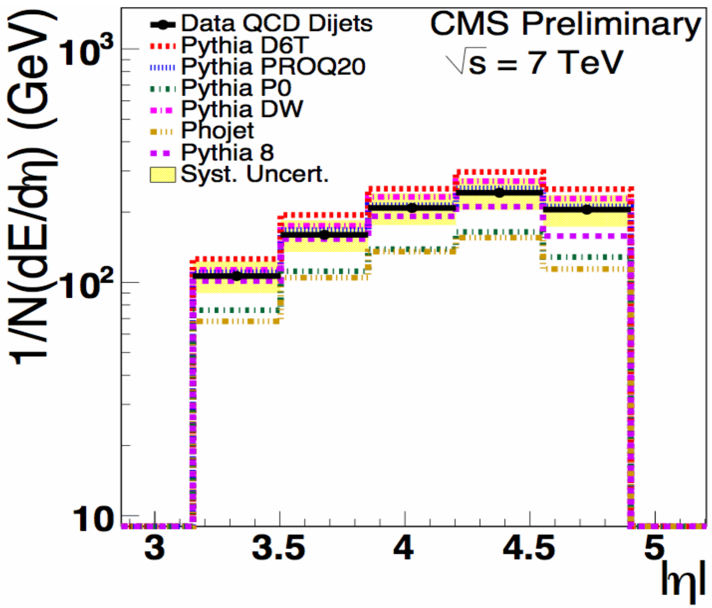


Figure 5: ΔR distribution of the central ($|\eta_c| < 2$, left) and forward jets ($3 < |\eta_f| < 5$, right) for $E_T > 10$ GeV (upper row) and $E_T > 30$ GeV (lower row). The prediction from the $k_\perp$ shower (Cascade) is shown with the solid blue line; the prediction from the collinear shower (Pythia) including multiple interactions and without multiple interactions is shown with the red and purple lines. $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$, where $\Delta\phi = \phi_{jet} - \phi_{part}$, $\Delta\eta = \eta_{jet} - \eta_{part}$

- MPI contribute significantly to forward energy flow.

Forward energy flow in minimum bias and central dijet sample:



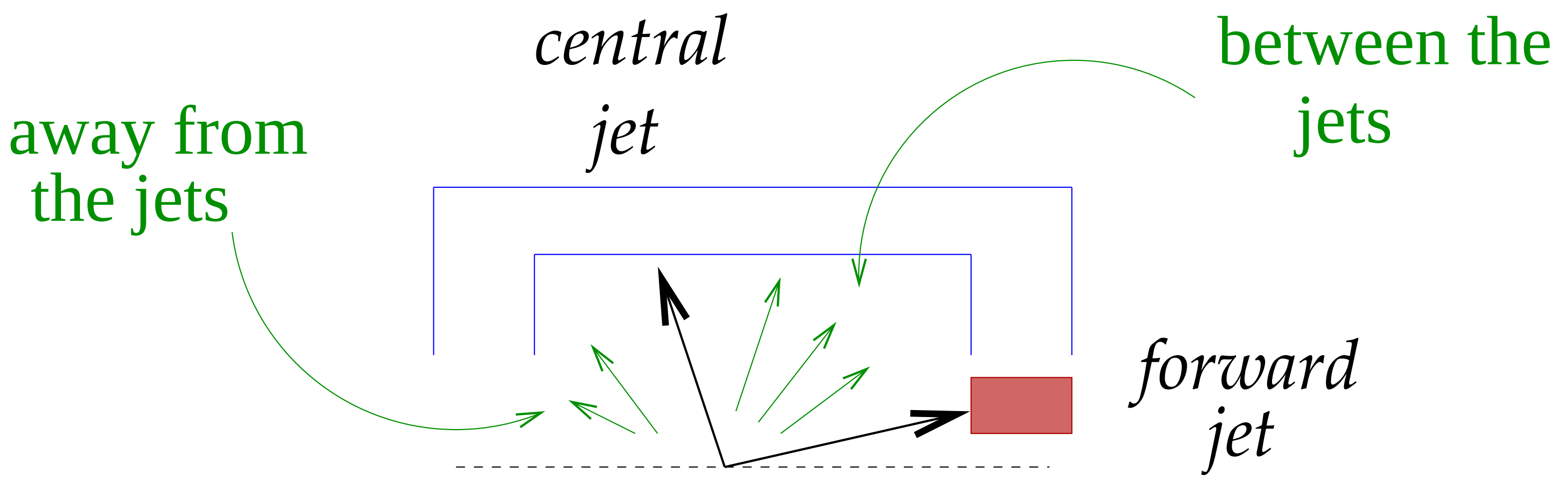


- observed increase with increasing $\sqrt{s}$
- energy flow in forward region not well described by PYTHIA tunes based on charged particle spectra in central region, especially for minimum bias

[Bartalini & Fanò, arXiv:1103.6201]

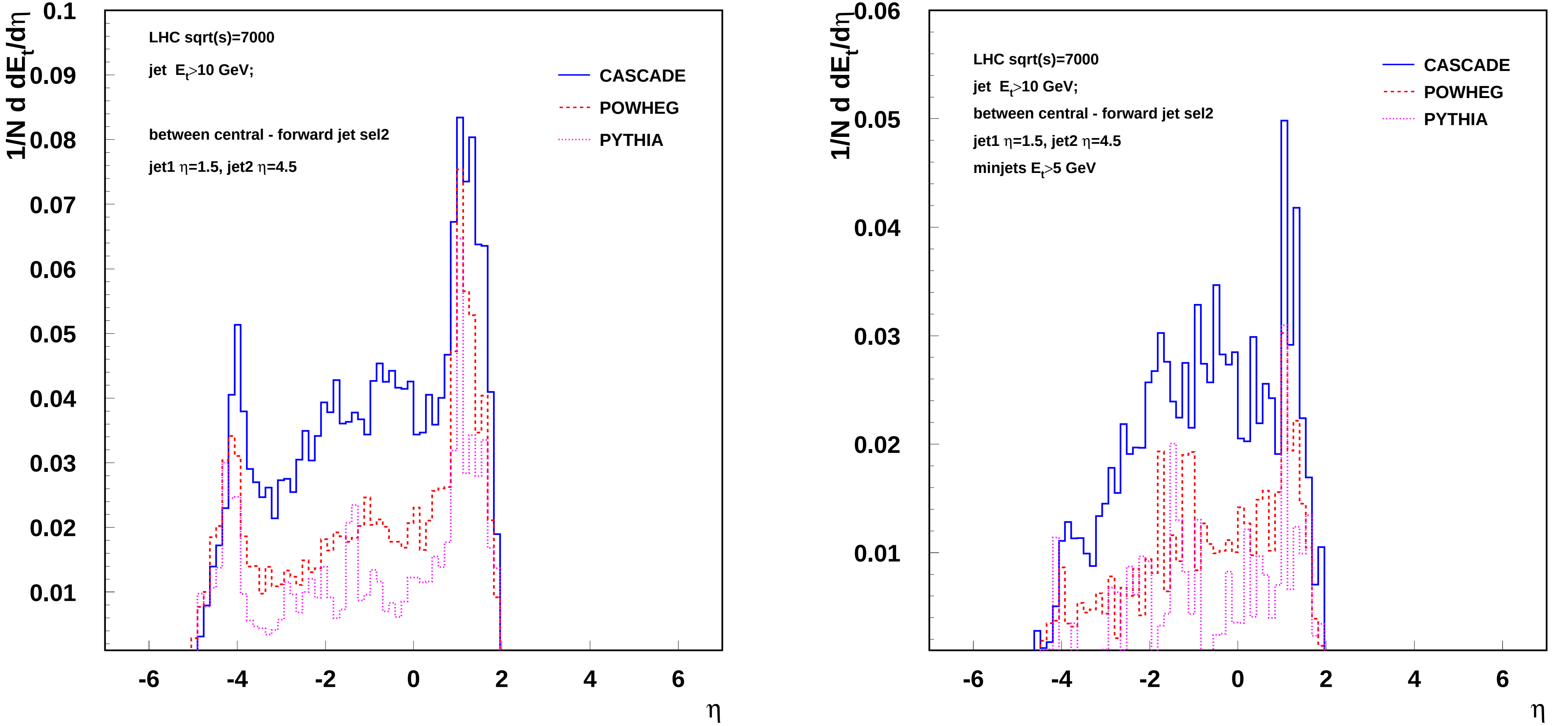
1 central + 1 forward jet:

particle and energy flow in the inter-jet and outside regions



Transverse energy flow in the inter-jet region

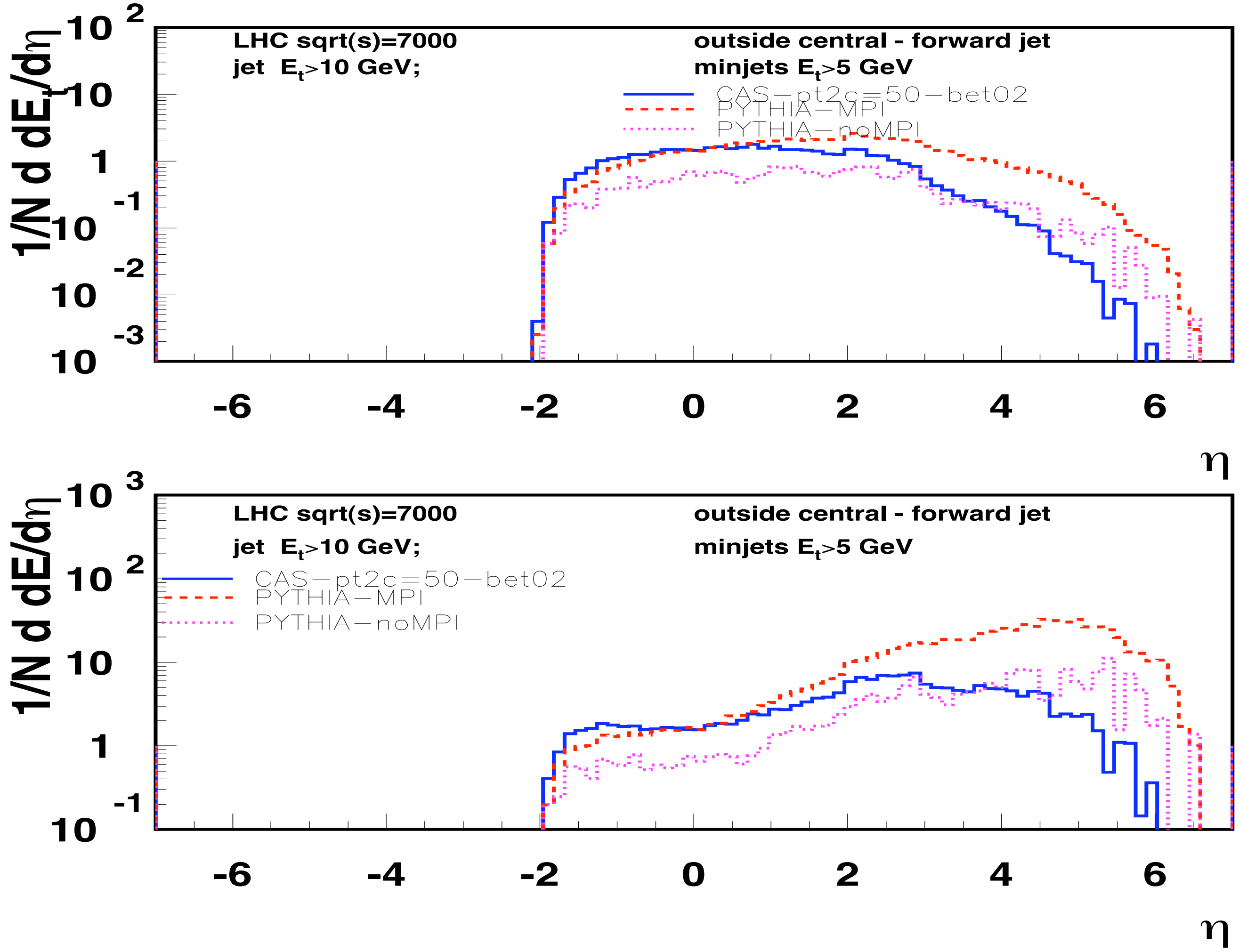
[Deak et al., in progress]



(left) particle flow; (right) minijet flow

- higher mini-jet activity in the inter-jet region from corrections to collinear ordering

Transverse energy flow in the outside region

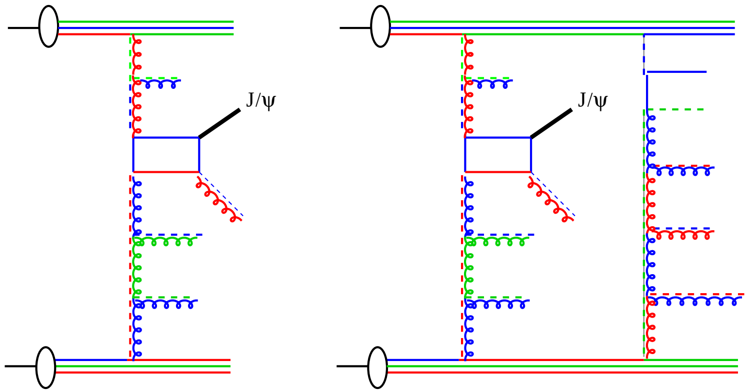


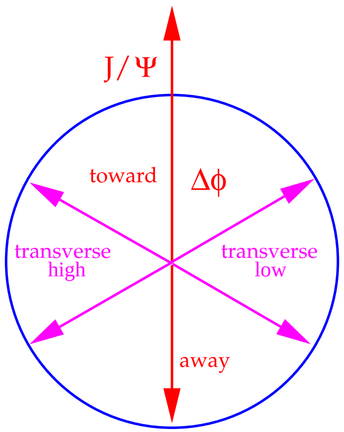
- at large (opposite) rapidities, full branching well approximated by collinear ordering
- higher energy flow only from multiple interactions

III. J/ψ PRODUCTION AND ASSOCIATED JET MULTIPLICITIES

▷ underlying event analysis using gluonic probe [cfr. $Z+$ jets]

▷ perturbative calculation down to p$_\perp$ of order m_ψ



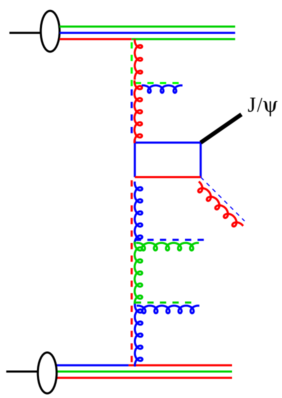


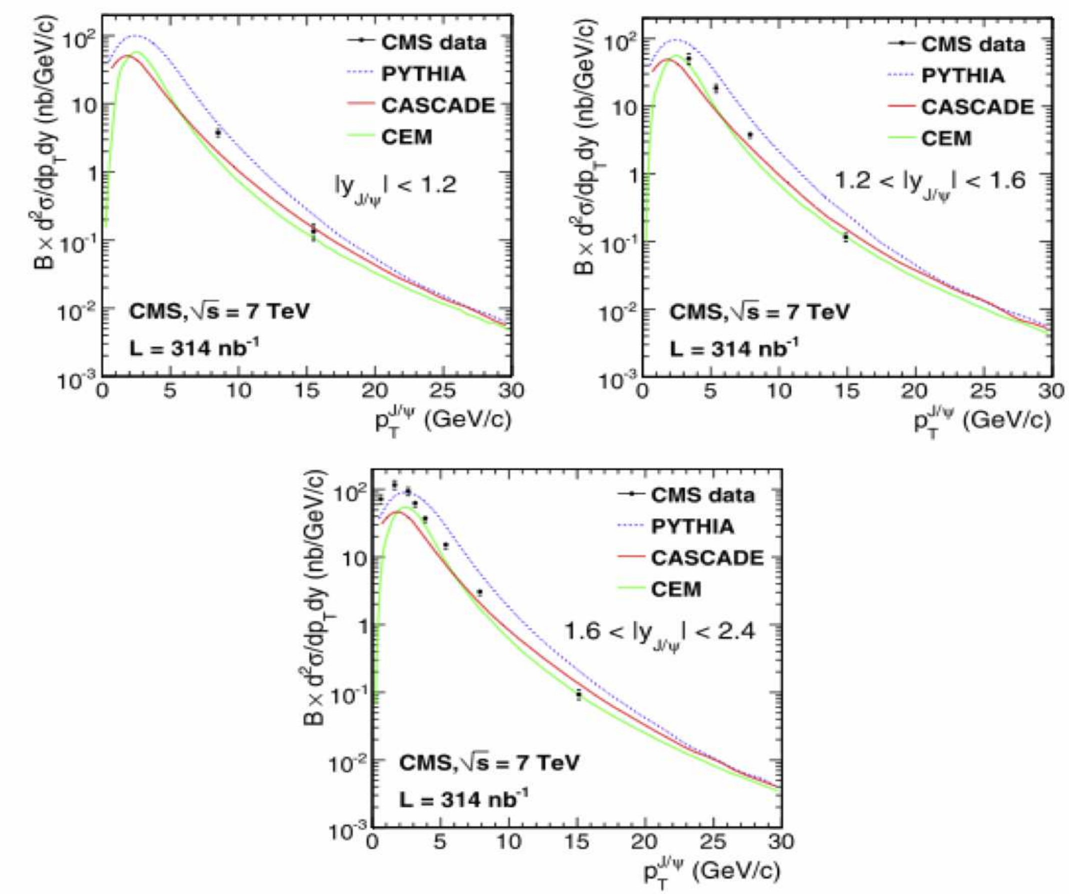
▷ See also: J/ψ vs. charged particle multiplicity *[Portebeuf & Granier, arXiv:1012.0719]*

▷ J/ψ pairs as a probe of DPI *[Kom, Kulesza & Stirling, arXiv:1105.4186]*

[LHCb Coll., LHCb-Conf-2011-009]

Inclusive J/ψ spectra: comparison with CMS measurement

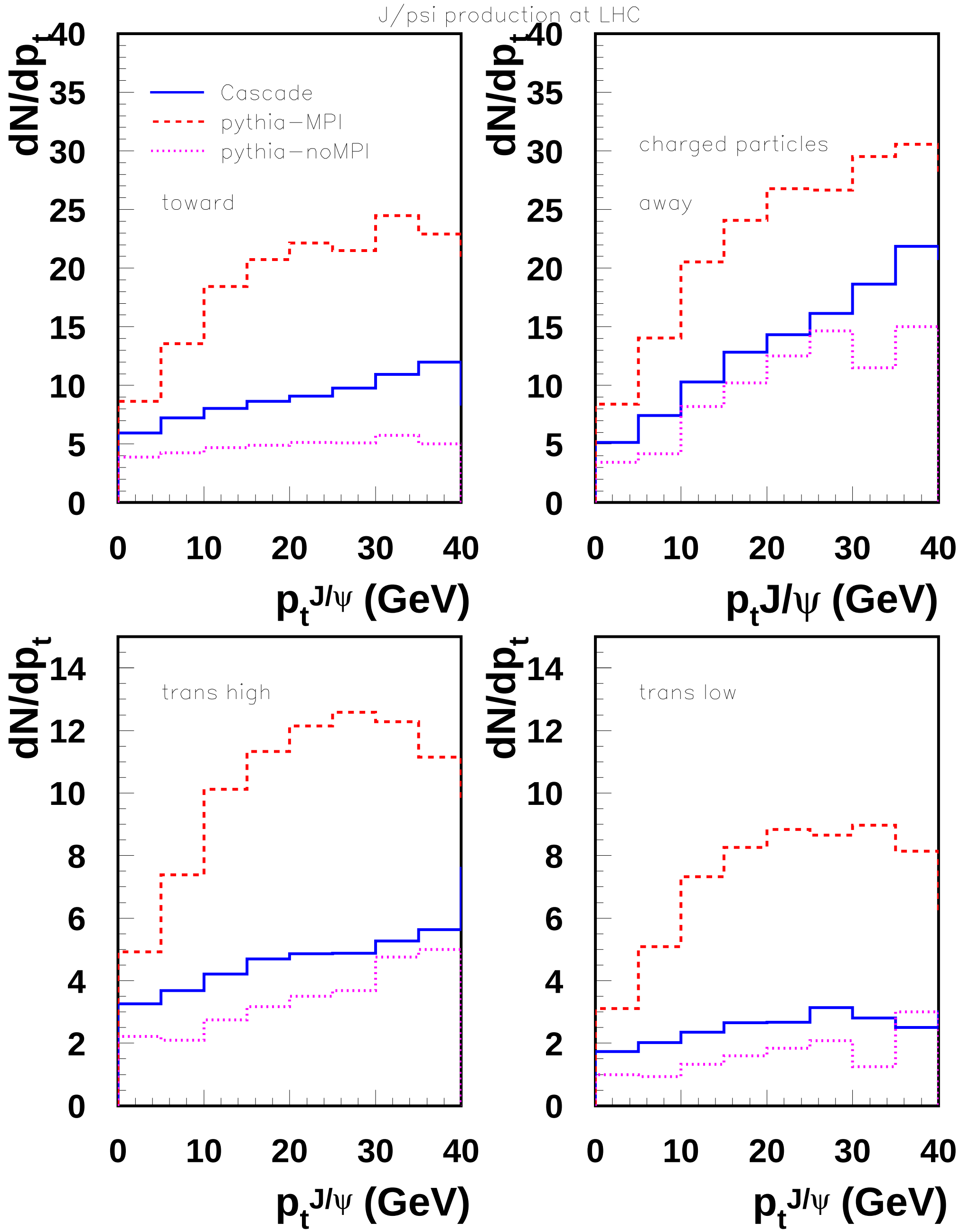




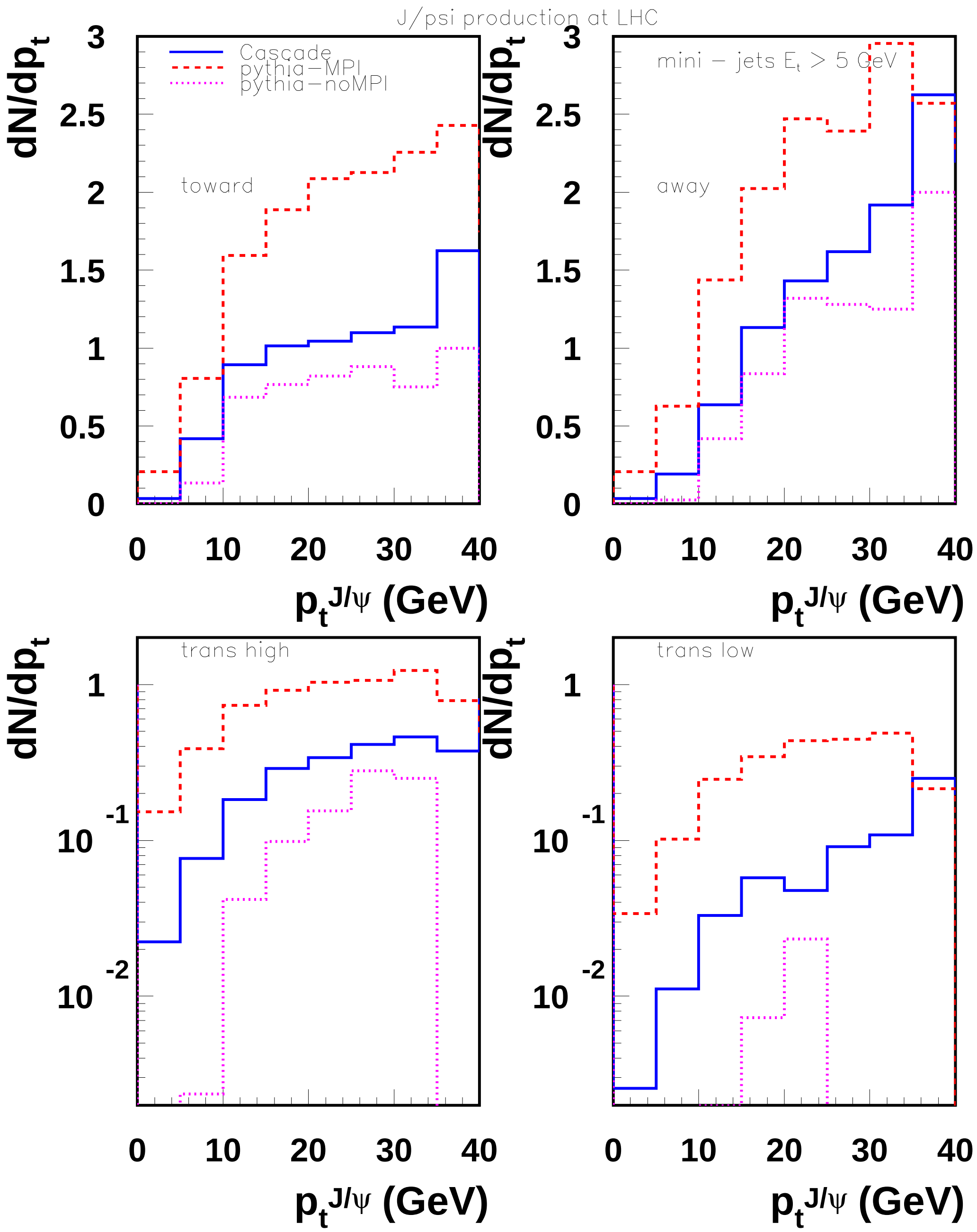
CMS Coll, Eur.Phys.J. C71 (2011) 1575, e-Print: arXiv:1011.4193

Charged particle multiplicity associated with J/ψ

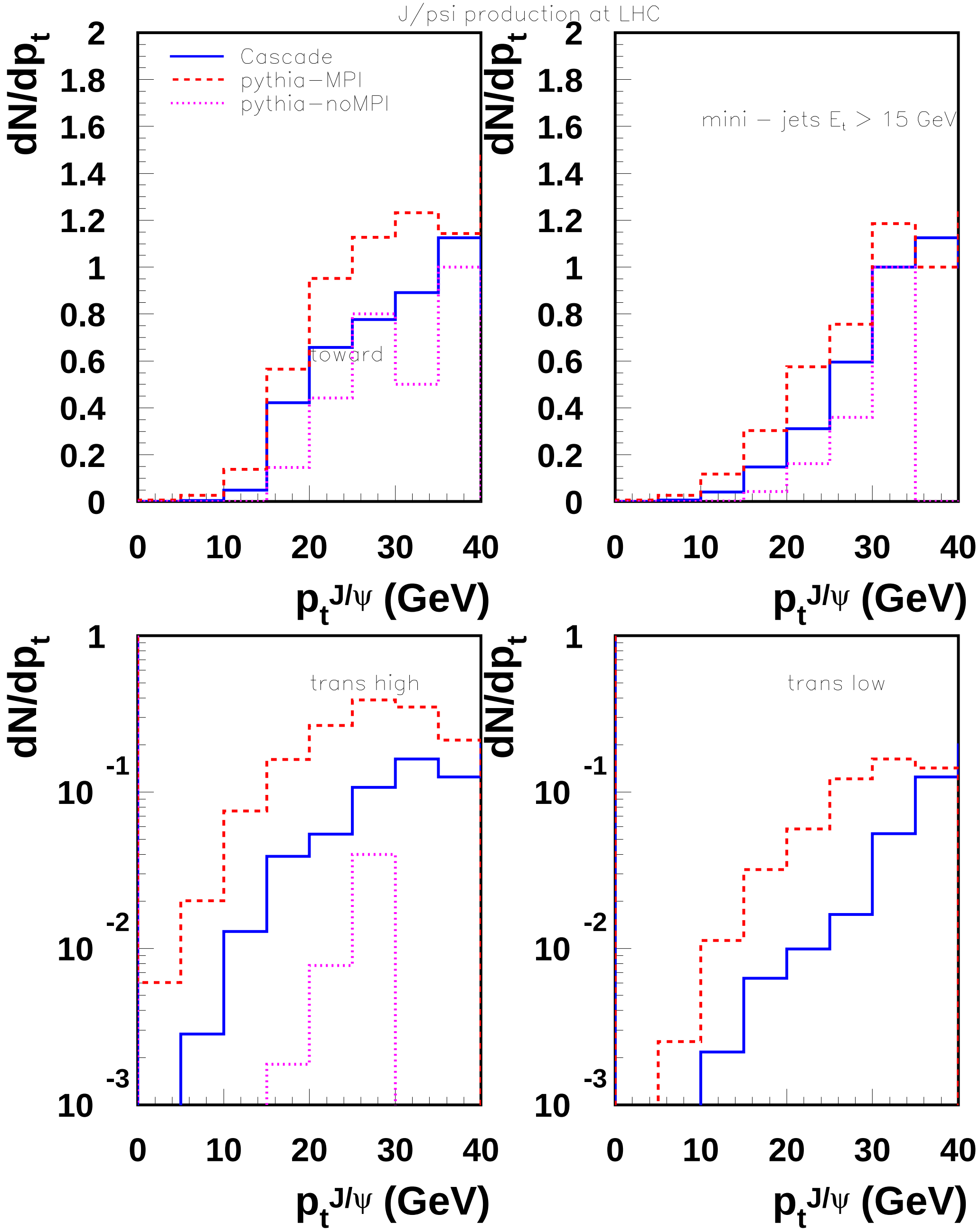
[Jung & H]



Mini-jet spectra $E_t > 5$ GeV



Mini-jet spectra $E_t > 15$ GeV



SUMMARY

- MPI increasingly important as parton densities grow with energy
- sensitive to detailed structure of final states produced by shower evolution

$\Rightarrow$ what level accuracy required in parton branching algorithms?

- finite-$k_\perp$ corrections to parton branching

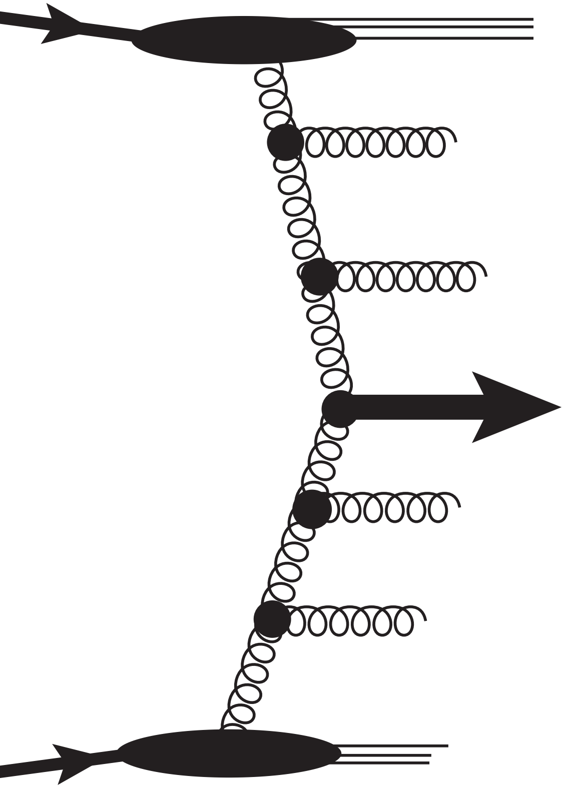
$\triangleright$ forward jets, angular correlations, energy flow

$\Rightarrow$ amount of MPI reduced by inclusion of non-collinear-ordered effects to showers?

- J/ψ associated multiplicities probe gluonic jets at low but perturbative $p_\perp$
- complementary to underlying event studies in Z + jets

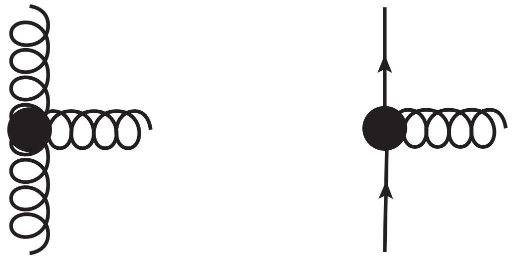
EXTRA SLIDES

CCFM evolution and quark emission

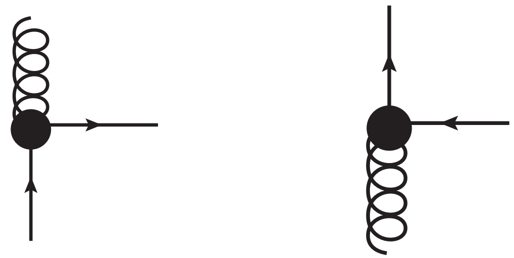


CCFM evolution based on principle of color coherence

→ emissions of **gauge bosons**



unintegrated gluon and
valence quark



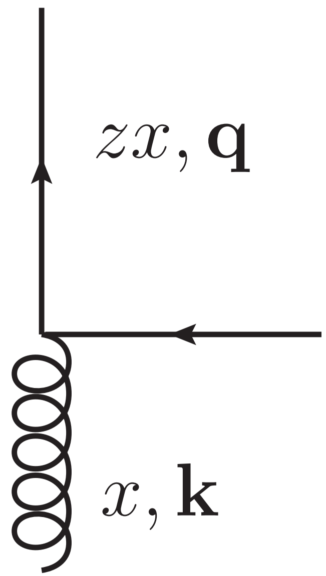
not present

Consequences: (A) Evolution (exclusive radiative corrections!):

- only gluonic emissions, no quark → jets purely gluonic
- DGLAP: naturally contained
- BFKL: through NLO corrections, not contained in (LO) CCFM evolution

Goal of this study: gluon $\longrightarrow$ quark splitting (P_{qg})

- supplement CCFM evolution by gluon $\rightarrow$ quark splitting
- restrict to splitting in the last evolution step
- keep finite transverse quark momentum q_T
  ➡ k_T factorized seaquark
- correct high energy & collinear limits,
  ➡ similar to CCFM evolution

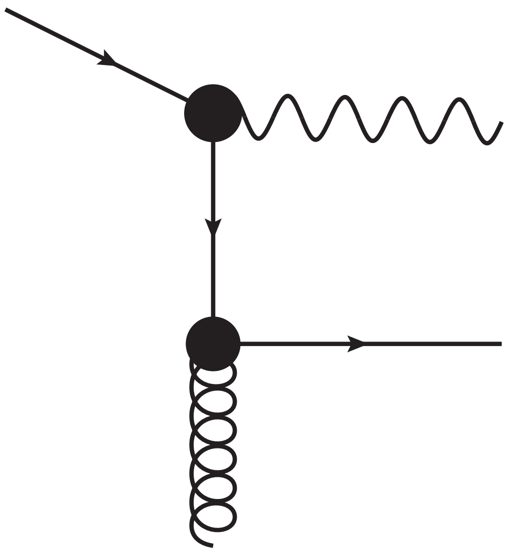


+ test accuracy of (formal) factorization numerically

Process of interest at LHC: **forward Drell-Yan** production (γ^*, Z, W)

- probe proton at very small x, up to $3 \cdot 10^{-6}$
- investigate small x dynamics: BFKL, saturation, ...
- allows to compare exact versus factorized expression

Quark-gluon splitting and collinear factorization



- **DGLAP:** contains naturally splitting function
$$P_{qg}(z) = Tr(z^2 + (1-z)^2)$$
- no k_T dependence for seaquark distribution $q(x, \mu^2)$ and partonic cross-section $\sigma_{q\bar{q}\to Z}$
- no small x dynamics included

$$\hat{\sigma}_{q\bar{q}\to Z}(\nu = \hat{s}) = \underbrace{\sqrt{2}G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c}\delta(\nu - M_Z^2)$$

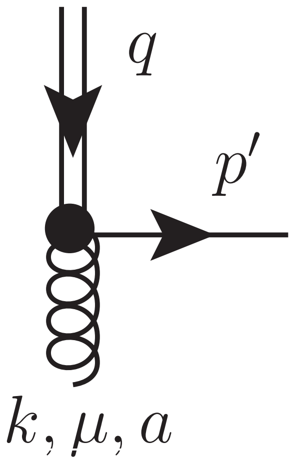
[Catani, Hautmann '94] : high energy resummation within collinear factorization: $\mathbf{k_T}$**-dependent splitting function**

$$P_{qg}^{\text{CH}}(z, \boldsymbol{k}^2, \boldsymbol{q}^2) = T_R \left(\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[P_{qg}(z) + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\boldsymbol{q}^2} \right]$$

- $\otimes$ gluon Green's function: high energy resummed splitting
- universal ➡ defines small x-resummed seaquark distribution
- full k_T (gluon) dependence, but integrate out q_T (quark)

gauge invariant off-shell factorization: reggeized quarks

- **reggeized quarks** (in analogy to reggeized gluons for BFKL):
  - at high energies, effective d.o.f. in t-channel processes with quark exchange [Fadin,Sherman, 76,77], [Lipatov,Vyazovsky,'00], [Bogdan, Fadin, 06],
  - here applied to $qg^* \to Zq$ process at Born level
- **effective vertices**: re-arrangment of QCD diagrams



$$= igt^a \left(\gamma^\mu - \not{q} \frac{(n^+)^\mu}{k^+} \right) \qquad \text{etc.}$$

→ gauge invariant definition of off-shell Matrix Elements

$$\hat{\sigma}_{q\bar{q}^* \to Z}(\nu, \boldsymbol{q}^2) = \underbrace{\sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2)}_{\text{Z-coupling}} \times \frac{\pi}{N_c} \delta(\nu - M_Z^2 - \boldsymbol{q}^2)$$

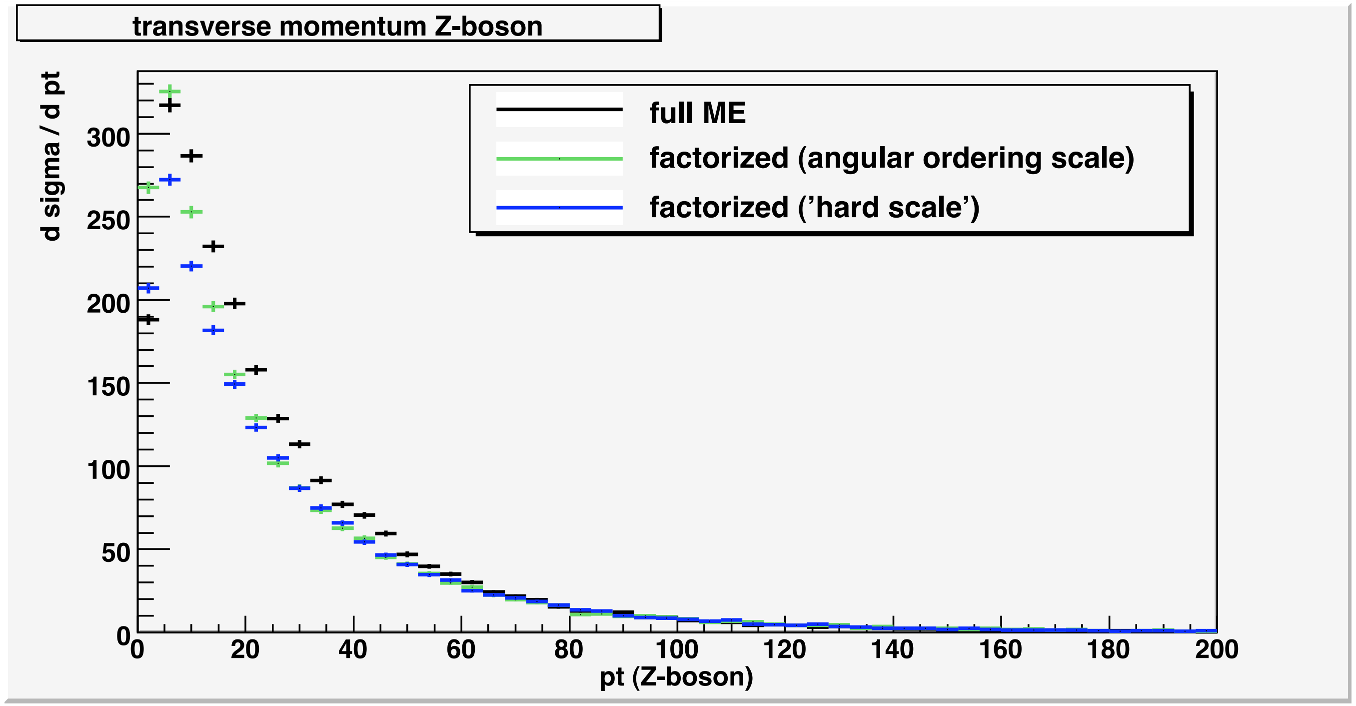
- gluon-quark splitting $= T_R$: Multi-Regge-Kinematics sets $z = 0$

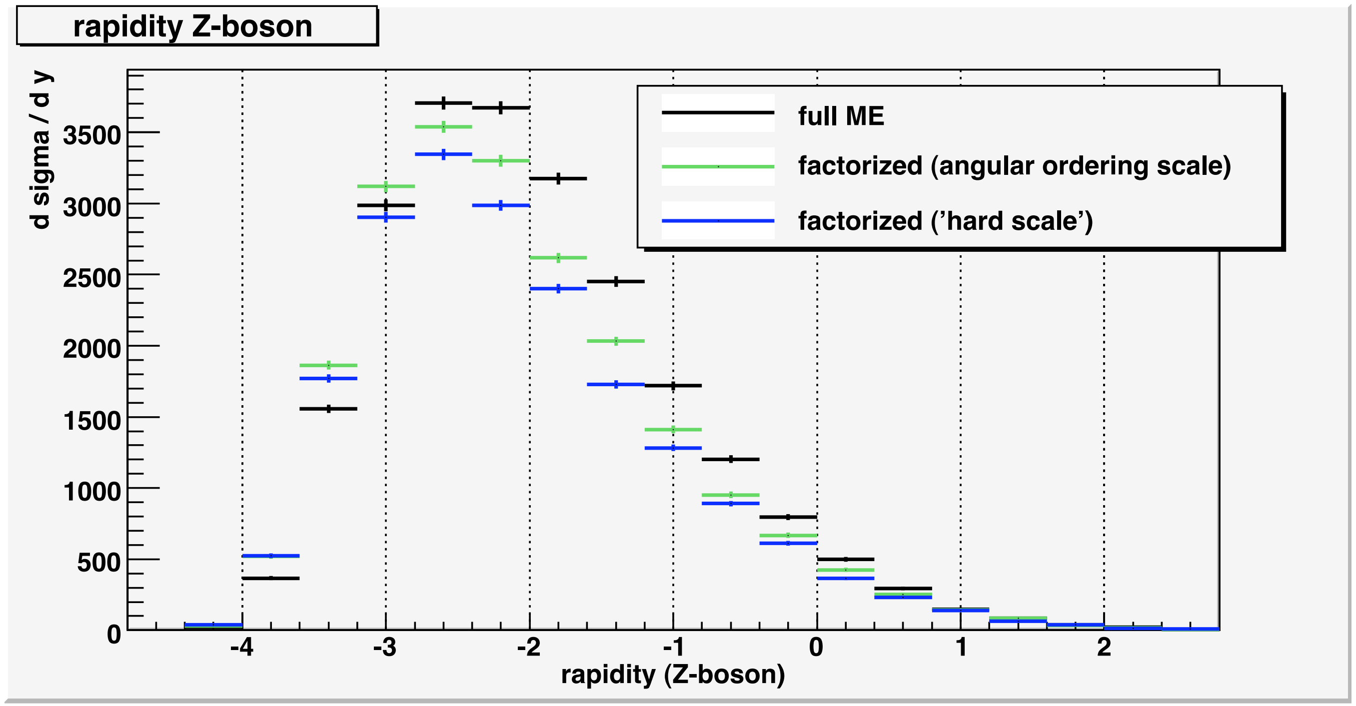
Forward DY from

Cascade MC implementation of TMD sea quark distribution

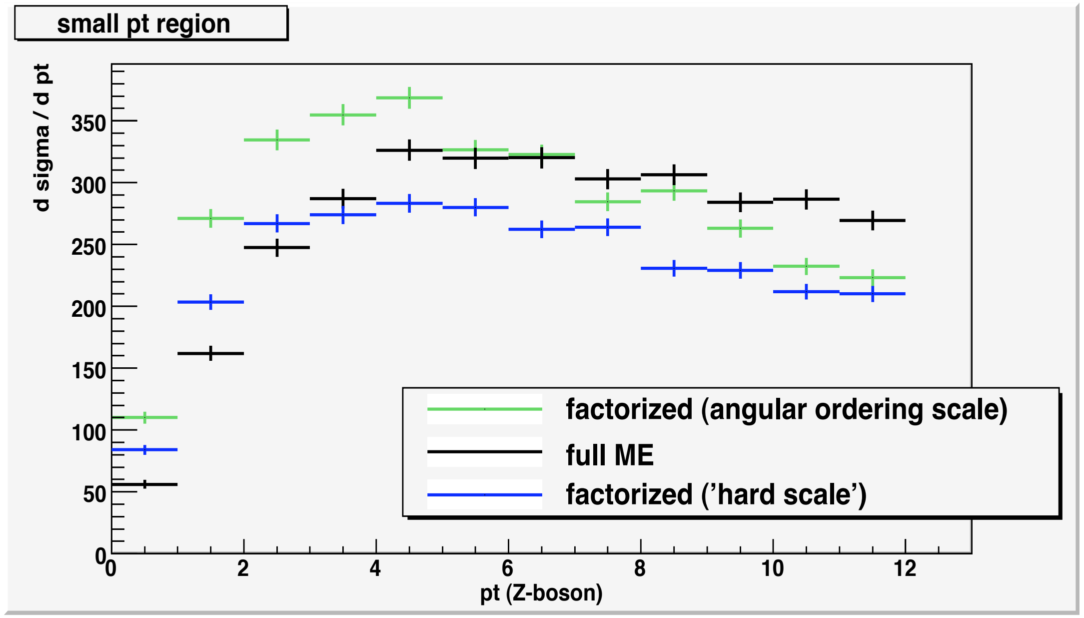
$$\text{angular ordering scale :} \quad \mu^2 = \frac{\mathbf{q}^2 + (1-z)\mathbf{k}^2}{(1-z)^2}$$

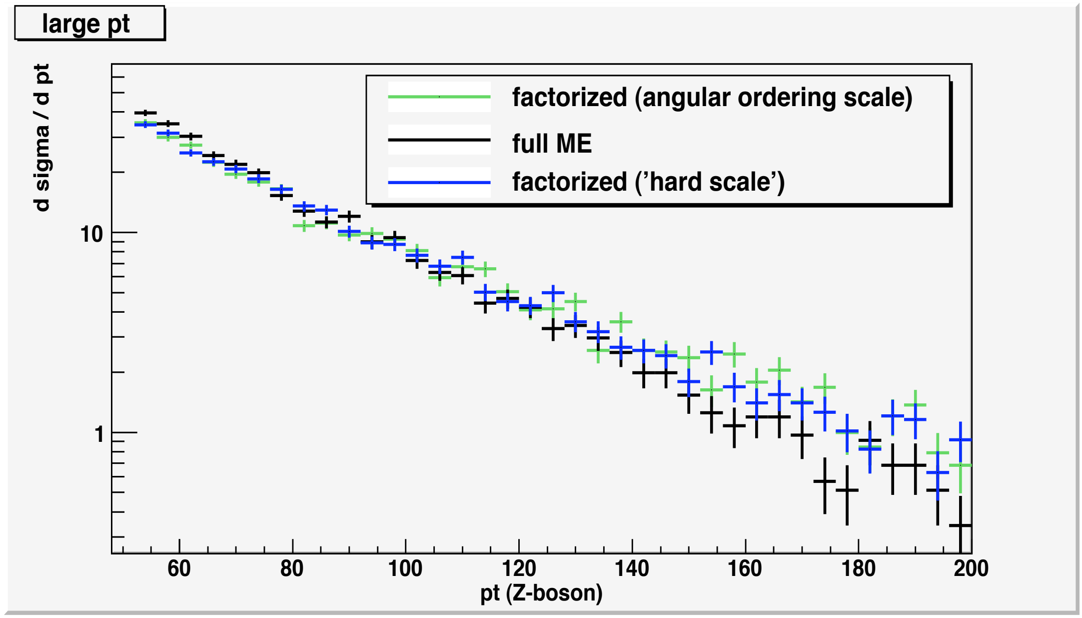
$$\text{hard scale :} \quad \mu^2 = \mathbf{p}^2 + M^2$$


transverse momentum Z-boson
d sigma / d pt
full ME
factorized (angular ordering scale)
factorized ('hard scale')
0
50
100
150
200
250
300
0
20
40
60
80
100
120
140
160
180
200
pt (Z-boson)


rapidity Z-boson
d sigma / d y
full ME
factorized (angular ordering scale)
factorized ('hard scale')
0
500
1000
1500
2000
2500
3000
3500
-4
-3
-2
-1
0
1
2
rapidity (Z-boson)

Agreement best for large p_T region



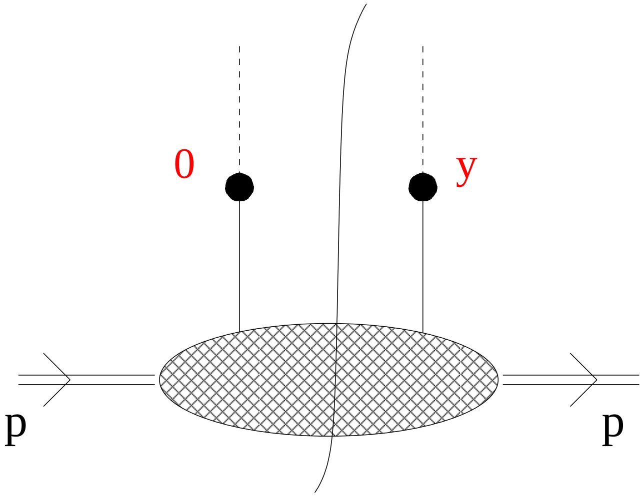


'Renormalized' $qg^* \to Zq$ cross-section

$$\bar{\sigma}(\nu, \boldsymbol{k}^2) \equiv \hat{\sigma}(\nu, \boldsymbol{k}^2) - \int_x^1 \frac{dz}{z} \int \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \hat{\sigma}_{q\bar{q}^* \to Z} P_{qg}^{\mathrm{CH}}$$

yields finite $(7\% - 16\%)$ correction to factorized expression, free of large collinear logarithms

II. Gauge links, lightcone divergences and TMD factorization



$p = (p^+, m^2/2p^+, 0_\perp)$

$$\widetilde{f}(y) = \langle\ P\ |\ \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0)\ |\ P\ \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp\left(ig_s \int_0^\infty d\tau\ n \cdot A(y + \tau\ n)\right)$ eikonal Wilson line in direction n

- works at tree level [Mulders, 2002; Belitsky et al., 2003]
- subtler at level of radiative corrections [Collins, Zu; H; Cherednikov et al.]

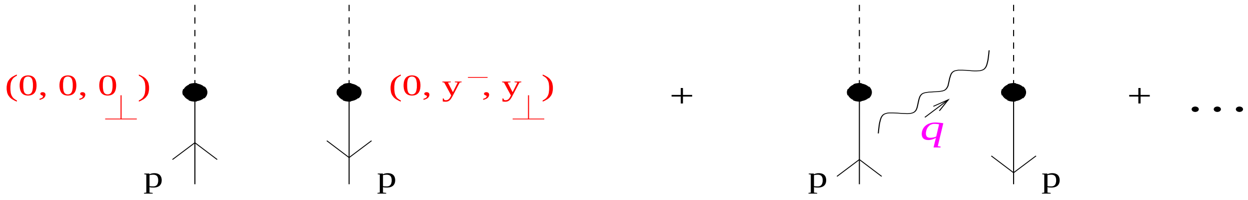
  $\hookrightarrow x \to 1 \Rightarrow$ explicit regularization method (unlike inclusive case)
- non-abelian Coulomb phase $\to$ spectator effects possibly non-decoupl.

  [Rogers, Mulders, Bomhof; Collins, Qiu; Vogelsang, Yuan; Brodsky et al]

LIGHTCONE DIVERGENCES

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx'\, dk'_\perp\, P_R(x', k'_\perp)$$

where $$P_R = \frac{\alpha_s\, C_F}{\pi^2} \left[\frac{1}{1-x}\; \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right]$$ ρ=IR regulator

↑ *endpoint singularity* $(q^+ \to 0, \; \forall \; k_\perp)$

[Brodsky et al, 2001; Collins, 2002]

◇ Physical observables:

$$\mathcal{O} = \int dx\; dk_\perp\; f_{(1)}(x, k_\perp)\; \varphi(x, k_\perp)$$

$$= \int dx\; dk_\perp\; [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\; P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

- Distributions at fixed $k_\perp$ are no longer protected by KLN against uncancelled lightcone divergences

- Only after supplying matrix element with a regularization prescription is distribution well defined.

$\Longrightarrow$ Need for infrared subtraction factors

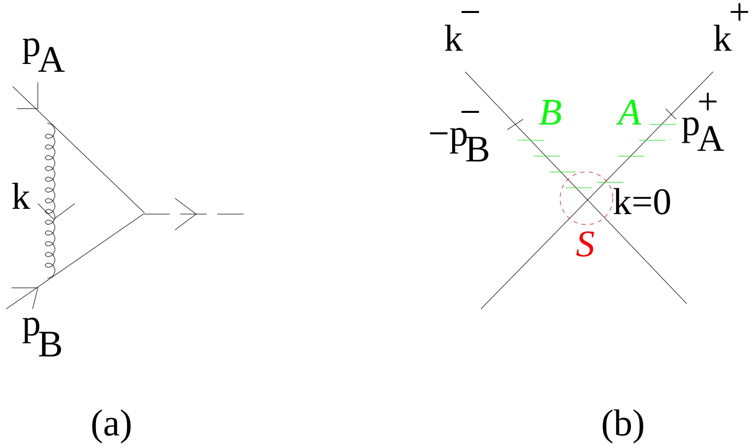
[more on this in Cherednikov's talk]

Example: Sudakov form factor of quarks

Collins & H, PLB 472 (2000) 129

Soft collinear effective theory (SCET): Hoang, Manohar et al., arXiv:0901.1332

- Theory well-known. Enters Drell-Yan production, W-boson $p_\perp$ distribution, etc.



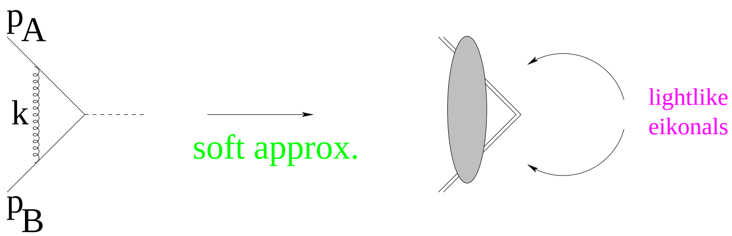
Look for decomposition of the amplitude Γ

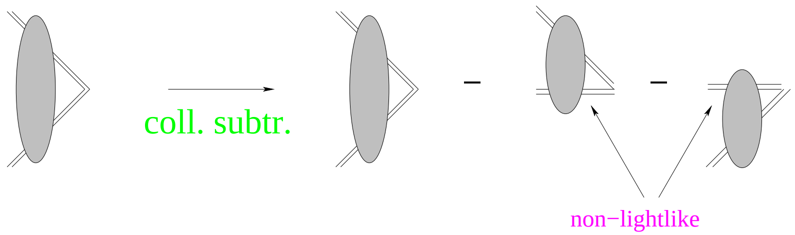
$$\Gamma = \sum_{\text{regions } R} M_\Gamma(R) \ + \ \text{nonleading}$$

such that i) term for hard region be integrable; ii) splitting be defined gauge-invariantly

$$\sigma[\Gamma] = \int [dk] \ S \ \otimes \ C_A \ \otimes \ C_B \ \otimes \ H \ + \ \text{nonleading}$$

Example: Soft-region term S





$$u_A = (u_A^+, {\color{red}u_A^-}, 0_\perp),\ u_B = ({\color{red}u_B^+}, u_B^-, 0_\perp) \quad (\eta = u_A^+/u_A^-)$$

$$S = \frac{\overbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}(\hat{p}_B)|0\rangle}^{\text{unsubtracted soft}}}{\underbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}({\color{red}u_B})|0\rangle \, \langle 0|V_q({\color{red}u_A})\, V_{\bar{q}}(\hat{p}_B)|0\rangle}_{\text{collinear subtractions}}} \; \overbrace{\langle 0|V_q(u_A)|0\rangle \, \langle 0|V_{\bar{q}}(u_B)|0\rangle}^{\text{residual external lines}}$$

$$\text{with} \quad V_q(n) = \mathcal{P} \exp\left(ig \int_{-\infty}^{0} dz\, A(z\,n) \cdot n \right) \quad , \quad V_{\bar{q}}(n) = \mathcal{P} \exp\left(-ig \int_{-\infty}^{0} dz\, A(z\,n) \cdot n \right)$$

Note: need for IR subtractions also in SCET (but counterterms not automatically gauge-invariant) [Manohar & Stewart, 2007]

II.A CUT-OFF APPROACH

▷ cut-off in Monte-Carlo generators using u-pdf's

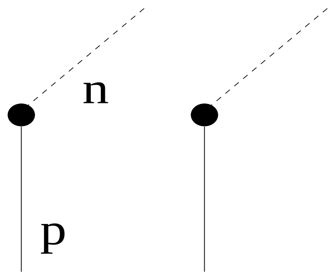
S. Jadach and M. Skrzypek, arXiv:1002.0010; arXiv:0905.1399 (DGLAP)

S. Höche, F. Krauss and T. Teubner, EPJC 58 (2008) 17 (KMR/BFKL)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, PRD 77 (2008) 085009

Ji, Ma & Yuan, PRD 71 (2005) 034005; JHEP 0507 (2005) 020

earlier work from 80's and 90's: Collins et al; Korchemsky et al

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

∗ Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

II.B UPDF's BY SUBTRACTIVE APPROACH

• Endpoint divergences $x \to 1$ from incomplete KLN cancellation

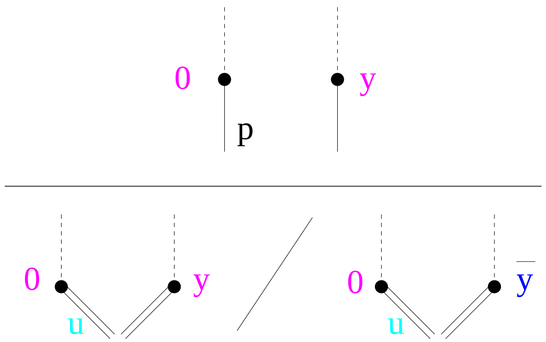
Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

[See also "SCET" analog: Manohar and Stewart, 2007; J. Chiu et al, arXiv:0905.1141]

• gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \ / \ \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, PLB 655 (2007) 26

◇ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$

FULL TMD FACTORIZATION IS YET TO BE ACHIEVED

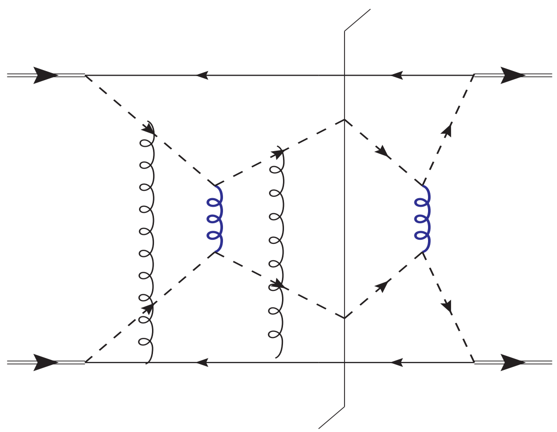
Mulders & Rogers, arXiv:1102.4569; arXiv:1001.2977; Xiao & Yuan, arXiv:1003.0482

- soft gluon exchange with spectator partons

Mert Aybat & Sterman, PLB671 (2009) 46

Boer, Brodsky & Hwang, PRD 67 (2003) 054003

$\Rightarrow$ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

$\diamondsuit$ likely suppressed for small-x, small-$\Delta\phi$

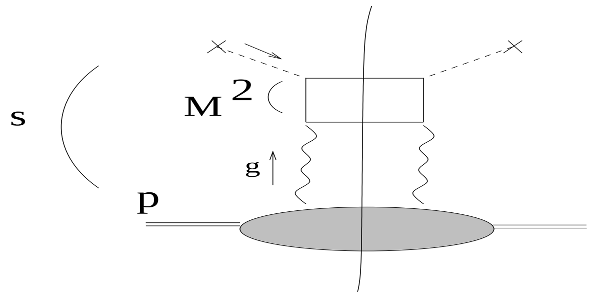
$\diamondsuit$ could affect physical picture near large x, back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity *[Forshaw & Seymour, arXiv:0901.3037]*

TMD FACTORIZATION AT SMALL X

A physical probe: *(in analogy with DIS/inclusive pdfs)*

TMD pdf factorization from heavy quark photo-production in high-energy limit:



♢ single gluon polarization dominates $s \gg M^2 \gg \Lambda^2_{\rm QCD}$

$\hookrightarrow$ gauge invariance rescued (despite gluon off-shell)

[Lipatov; Ciafaloni; Catani, H; ...]

♢ energy evolution equations / corrections down by $1/\ln s$ rather than $1/Q$

$\hookrightarrow$ BFKL (+ its variants)

♢ Note: it works to arbitrarily high $k_\perp$ in the UV $\Rightarrow$

- suitable for simulations of jet physics at the LHC
- well-defined summation of higher-order radiative corrections