Obtaining the CMS Ridge with Multiple Partonic Interactions

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- Introduction
 - Existing tunes used in this study
 - The CMS ridge effect
- Using Multiple Partonic interactions to describe the CMS ridge effect
 - Aligning the collision plane of individual MPIs to the event plane
 - Modification of $\operatorname{Pythia6}$
- Tuning
 - Tuning strategy
 - Case one: using a limited set of tuning observables to understand effects
 - Case two: two-step tune using more CMS minimum bias and underlying event data
- Results on two-particle angular correlations
 - Reminder: the ridge effect in CMS data
 - The ridge effect with modified $\operatorname{PythiA6}$
- Conclusions

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Disclaimer

- Only a limited set of CMS data was used
- \bullet All tuning activity was based on the Z2 tune for $\mathrm{PytHIA6}$
- More global tuning including other experiments' data was not within the scope of this study, but may be added later

Introduction		
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Existing tunes used in this study		

Tunes with **Pythia6**

- Z1 tune: PROFESSOR tunes to ATLAS minimum bias (MB) data, manually retuned multiple interaction parameters to CMS data (R. Field)
- Z2 tune: Manual retune of Z1 with CTEQ6L (R. Field)
- Z2* tune: PROFESSOR retune of Z2 to CMS underlying event (UE) data (A. Knutsson)



- Z2,Z2* describe increasingly well the CMS UE data
- Looking at multiplicity distributions in CMS MB data, there is room for improvement





 (a) Transverse charged multiplicity vs. leading jet-p_T

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Introduction			
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Existing tunes used in this	study		

Important parameters: effect of the p_T -cutoff

p^{min}_T: lower cutoff for which MPIs are generated (lower = more MPI activity)

 $p_{T}^{min}\left(E_{CM}\right) = PARP(82) \cdot \left(\frac{E_{CM}}{E_{REF}}\right)^{PARP(90)}$

- PARP(82): p_T^0 cutoff
- PARP(90): energy dependence



• 0.9 TeV:

- Z2 lowest p_T^{min} = highest activity
- Z2* highest $p_T^{min} =$ lowest activity
- 7.0 TeV:
 - Z2 highest $p_T^{min} = \text{lowest activity}$
 - Z2* lowest p_T^{min} = highest activity



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The CMS ridge effect			

Reminder on the CMS ridge effect

- Correlation function $R(\Delta \eta, \Delta \phi)$:
 - $\bullet\,$ data binned \sim track multiplicity
 - per bin:
 - signal: charged two-particle density
 - background: distribution of uncorrelated particle pairs = product of two single-particle distributions
 - $R(\Delta\eta, \Delta\phi)$: averaging $\left(\frac{S}{B} 1\right)$, weighted with bin multiplicity, over all bins

$$R(\Delta\eta,\Delta\phi) = \left\langle \left(\langle N \rangle - 1 \right) \left(\frac{S(\Delta\eta,\Delta\phi)}{B(\Delta\eta,\Delta\phi)} - 1 \right) \right\rangle_{bins}$$

- Effects explained by a single $2 \rightarrow 2$ partonic interaction:
- Effects requiring a different explanation:



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CMS Ridge with MP

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- Correlation function $R(\Delta \eta, \Delta \phi)$:

Introduction

 $R(\Delta\eta,\Delta\phi) = \left\langle (\langle N \rangle - 1) \left(\frac{S(\Delta\eta,\Delta\phi)}{B(\Delta\eta,\Delta\phi)} - 1 \right)
ight
angle_{bins}$

- Effects explained by a single $2{\rightarrow}\ 2$ partonic interaction:
 - near-side peak at $(\Delta \eta, \Delta \phi) = (0, 0)$:
 - jets, clustered in η and ϕ
 - away-side ridge at $(\Delta\eta,\Delta\phi)=(\Delta\eta,\pi)$:
 - away-side jets, back-to-back in $\boldsymbol{\phi}$
 - incoming p= variable ightarrow broad in $\Delta\eta$
 - Gaussian ridge at $(\Delta\eta,\Delta\phi)=(0,\Delta\phi)$:
 - decay of lower-p_T clusters
 - \rightarrow spreading out in $\Delta\eta$
- Effects requiring a different explanation:
 - near-side ridge at $(\Delta \eta, \Delta \phi) = (\Delta \eta, 0)$:
 - in high multiplicity events
 - at moderate p_T : $1 < p_T < 3$ GeV
 - not predicted by any Monte Carlo



Introduction Using MPIs Tuning Results 0000 000000 000 Aligning the collision plane of individual MPIs to the event plane

Can multiple partonic interactions explain the ridge effect? (1)

- MPIs tend to lie in the 'collision plane' of the hardest interaction
 - \rightarrow for large enough impact parameter b
 - \rightarrow final state particles will have similar azimuthal angle ϕ
- Explanation with MPIs would require many MPIs:
 - \rightarrow we're discussing high-multiplicity events
- MPIs are semi-hard:
 - \rightarrow we're discussing moderate- p_T particles
- MPIs are spread in pseudo-rapidity η since incoming partons have different x_{bj}:
 → we're discussing long-range correlations

 \rightarrow All that matches with the observations by CMS

Still a problem

- High-multiplicity events are generally central collisions:
 - \rightarrow impact parameter $b\sim$ 0, while definition of 'collision plane' needs large b
 - \rightarrow possibly a small upward fluctuation in # MPI for moderate impact parameter suffices to explain the CMS ridge effect



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Can multiple partonic interactions explain the ridge effect? (2)

- Currently in Pythia 6.425:
 - \rightarrow #MPI \sim activity \sim 1/impact parameter *b* (parameter *b* rescaled to $b_{avg} = 1$ for MB)
 - \rightarrow azimuthal angle ϕ = random
 - \rightarrow With the random azimuthal angle, long-range near-side angular correlations would be missing in $\rm PythiA6$
- Proposed modification:

$$\phi_i = \phi_{hardest} + \arctan\left(\frac{y_2 - y_1}{(x_2 + \alpha \cdot b / b_{avg}) - x_1}\right)$$



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- sample random points (x_i, y_i) in Gaussian proton profiles
- separate protons by impact parameter b, with scalable unit α/b_{avg} (α introduced to allow some tuning freedom, ideally $\alpha = 1$)(b = VINT(139))
- calculate $\phi\text{-offset}$ from hardest interaction
- the modification is implemented for two different settings of the new MPI model in $\rm PYTHIA6~(set by MSTP(82))$: options 4 and 5 both use hadronic overlap according to Gaussian distributions, in which case the above ϕ -definition makes sense

		Using MPIs		
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	Aligning the collision plane of indiv	idual MPIs to the event plane		

Implications of the modification

Original situation:

- \bullet activity $\sim \# \mathsf{MPI} \sim 1/\mathsf{impact}$ parameter
- typical underlying event (UE) tunes use activity transverse to a jet: *N_{transverse}*
- minimum bias (MB) plots are integrated over azimuth $\Delta\phi$

Modified situation:

- \bullet MPIs get shifted to toward/away regions: plateau for $N_{transverse}$ drops
- · re-raising plateau to describe data requires retune
- important parameters include PARP(82) and PARP(90), describing the p_T-cutoff and thus the activity (N_{transverse})
- minimum bias plots are integrated over azimuth $\Delta \phi$ \rightarrow not sensitive to modification
- \bullet existing tension between UE and MB \rightarrow released by modification?

Conclusion:

• If the modification desccribes a real effect, existing tunes should be wrong?







• PARP(82) & PARP(90) affect all activity:

• Modification with α -parameter affects the transverse activity,

Distribution N_{ch} at 0.9 TeV, max $p_T > 3GeV/c$, same phase space

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(a) 0.9 TeV: both parameters affect transverse activity

Distribution N_{ch} at 7 TeV, max $p_T > 3GeV/c$, same phase space $/dN_{cl}$ 10 $(1/N_{ev}) dN$ 10^{-2} CMS data Z2 [1.83,0.28] [1.70,0.28] [1.95.0.28] 10 1.83.0.20 [1.83,0.35] 1.4 MC/data 10 15 20 Nch

(b) 7.0 TeV: both parameters affect transverse activity イロト イポト イヨト イヨ

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• PARP(82) & PARP(90) affect all activity:

• Modification with α -parameter affects the transverse activity,





(e) 0.9 TeV: higher α = lower transverse activity



(f) 7.0 TeV: higher α = lower transverse activity

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- PARP(82) & PARP(90) affect all activity:
- Modification with α-parameter affects the transverse activity, but not the total:





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	Tuning	
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Tuning strategy		

Tuning: used datasets

- CMS_QCD_10_010 (CMS_2011_S9120041): CMS, $\sqrt{s} = 0.9 \& 7.0 \text{ TeV}$ [CMS UE data
 - traditional UE measurement in the central region
 - · measurement of activity transverse to leading charged particle jet
- CMS_QCD_10_004 (CMS_2011_S8884919): CMS, $\sqrt{s} = 0.9, 2.36 \& 7.0 \text{ TeV}$

[CMS MB data]

- measurement of NSD charged particle multiplicity
- five η regions

Tuning strategy

- Case one:
 - try to understand effects
 - limit to just 4 plots: $\textit{N}_{ch,transv}$ & $\textit{N}_{ch,total}(|\eta| < 2.4)$ for both 0.9 and 7.0 TeV



- Case two:
 - fix PARP(82) and PARP(90) using MB plots: integrated over ϕ = insensitive to α
 - tune α -parameter using UE plots: transverse activity fixes α

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(a) 0.9 TeV UE data







Tuning: Case one (Z2R) (2)

• Tuning with PROFESSOR:

	PARP(82)	PARP(90)	α
Z2	1.83	0.28	0.0
Z2*	1.93	0.23	0.0
Z2R	1.87	0.23	4.15

 UE improvement over Z2 remains, extra parameter α allows to slightly improve MB (without degrading UE):



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CMS Ridge with MPI

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 Case one: using a limited set of tuning observables to understand effects
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Tuning: Case one (Z2R) (2)

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Tuning 0000000 Case one: using a limited set of tuning observables to understand effects Tuning: Case one (Z2R) (3) p^{min}_T: lower cutoff for which MPIs are generated p_T^{min} before (lower = more MPI activity) $p_{T}^{min}\left(E_{CM}\right) = PARP(82) \cdot \left(\frac{E_{CM}}{E_{PEE}}\right)^{PARP(90)}$ Charged hadron multiplicity, $|\eta| < 2.4$, $\sqrt{s} = 0.9$ TeV • PARP(82): p_T^0 cutoff 10-PARP(90): energy dependence 10-3 - CMS data - Z2 p_{_{T}}^{min} (E_{_{CM}} , PARP(82) , PARP(90)) Z2 () 3.0 () 2.6 10-2.6 VIC/data



- Z2 lowest p_T^{min} = highest activity
- Z2* highest $p_T^{min} = \text{lowest activity}$
- 7.0 TeV:
 - Z2 highest $p_{T}^{min} =$ lowest activity
 - Z2* lowest p_T^{min} = highest activity

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Charged hadron multiplicity, $|\eta| < 2.4$, $\sqrt{s} = 7 \text{ TeV}$

CMS data

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 $\frac{\text{Introduction}}{\text{Case one: using a limited set of tuning observables to understand effects}}{\text{Tuning: Case one (Z2R) (3)}}$ $p_T^{min: lower cutoff for which MPIs are generated (lower = more MPI activity) p_T^{min} looking at Z2R$



• PARP(82): p_T^0 cutoff

PARP(90): energy dependence



 $p_{T}^{min}\left(E_{CM}\right) = PARP(82) \cdot \left(\frac{E_{CM}}{E_{PEE}}\right)^{PARP(90)}$

- Z2 lowest p_T^{min} = highest activity
- Z2* highest $p_T^{min} =$ lowest activity
- 7.0 TeV:
 - Z2 highest $p_T^{min} =$ lowest activity
 - Z2R lowest p_T^{min} = highest activity



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CMS Ridge with MI

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	Case two: two-step tune using more	e CMS MB and UE data		

Tuning: Case two (1)

- Observables:
 - include both 0.9 and 7.0 TeV data
 - tune-step one: use all CMS_2011_S8884919 MB observables to tune PARP(82)/PARP(90)
 - \rightarrow these observables are insensitive to α
 - \rightarrow this will disregard any match with UE data
 - tune-step two: use all CMS_2011_S9120041 UE observables to tune α
 - \rightarrow this allows to fix the damage done to the match with UE
- Tuning with PROFESSOR:

	PARP(82)	PARP(90)	α
Z2	1.83	0.28	0.0
Z2*	1.93	0.23	0.0
Z2R	1.87	0.23	4.15
Z2R'	1.90	0.23	2.67

- \rightarrow The step-one result lies between Z2* and Z2R for PARP(82): very clear in the MB observable plots, but also causes non-optimal UE results
- \rightarrow The step-two result will restore the non-optimal UE results to something similar to Z2* and Z2R



• Larger set of observables: MB fixed in first tune-step: $N_{ch,tot}$ ($p_T > 500$) which was not tuned in case one, looks slightly better **TOTAL**





Tuning: Case two (2)

• Larger set of observables: after first tune-step UE is worse than Z2*/Z2R, after the second it's better again **TRANSVERSE**



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CMS Ridge with MP

		Results	
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Ridge plot: CMS data			

Ridge plot: CMS data

• CMS data, high-multiplicity moderate- p_T events (arXiv:1009.4122v1, 21 Sep 2010): \rightarrow long-range near-side correlations only at moderate p_T and high-multiplicity



Monte Carlo, existing tunes (arXiv:1009.4122v1, 21 Sep 2010):
 → no long-range near-side correlations

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0000 Ridge plot: modified Pyr	0000 HIA6		000	
Didro plati madif	iad Manta Carla (1)		

- Modified Monte Carlo, Z2R tune ($\alpha = 4.15$):
 - large amount of statistics needed
 - high-multiplicity, moderate-p_T: near-side ridge is visible, as in data
 - moderate-multiplicity, moderate- p_T : near-side ridge is visible but too strong
 - minimum bias: ridge disappears (not shown)
 - high-multiplicity, all p_T : near-side ridge is not there, same as in data \rightarrow away-side ridge changes shape



0000 Ridge plot: modified PyTHIA6			000	
Pidro plati modified	Aonto Carlo (1)		

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6	0000 Ridge plot: modified P	0000 YTHIA6		000	
			1)		
Rid	ge plot: mod	ified Monte Carlo (1)		

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		Results	
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Ridge plot: modified PYTHIA6			

Ridge plot: modified Monte Carlo (2)

- Modified Monte Carlo, Z2R' tune ($\alpha = 2.67$):
 - high-multiplicity, moderate- p_T : near-side ridge is not visible \rightarrow higher scaling parameter α = necessary?
 - moderate-multiplicity, moderate- p_T : near-side ridge is not visible
 - high-multiplicity, all p_T : near-side ridge is not there, same as in data \rightarrow shape change in away-side ridge seems to be still there



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			Results	
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	Ridge plot: modified Pythia6			

Ridge plot: modified Monte Carlo (2)

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			Results	
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	Ridge plot: modified Pythia6			

Ridge plot: modified Monte Carlo (2)

- Modified Monte Carlo, Z2R' tune ($\alpha = 2.67$):
 - high-multiplicity, moderate-p_T: near-side ridge is not visible
 → higher scaling parameter α = necessary?
 - moderate-multiplicity, moderate- p_T : near-side ridge is not visible
 - $\bullet\,$ high-multiplicity, all $p_T\colon$ near-side ridge is not there, same as in data
 - → shape change in away-side ridge seems to be still there



6			Conclusions
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Conclusions

- A modification of **Pythia6** was proposed, explaining the ridge effect with MPIs:
 - a correlation is introduced between the azimuth of the event planes of individual MPIs and the event plane of the hardest interaction
 - such a correlation can be naturally explained in a physical picture based on the impact parameter between the protons (*b* as defined by PYTHIA6 needs to be scaled by a factor \sim 4)
- Implications of the modification:
 - using the modification, the ridge effect in high-multiplicity moderate- $p_{\mathcal{T}}$ events becomes visible
 - using the modification, activity gets shifted away from the transverse region, the total activity remains the same, requiring a retune to UE observables
- Tune Z2R was obtained retuning to CMS UE and MB data

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CMS FWD data



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Effect on the amount of MPIs

• The MPI based modification doesn't touch the amount of MPIs, it only reorganises them in ϕ :



	average #MPI		
Z2	2.78		
Z2*	2.86		
mod	2.86		
(d) average #MPI			

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