PDF Correlations- LHC Higgs Working Group

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PDF correlations

- Consider a cross section X(a), a function of the Hessian eigenvectors
- i^{th} component of gradient of X is

$$\frac{\partial X}{\partial a_i} \equiv \partial_i X = \frac{1}{2}(X_i^{(+)} - X_i^{(-)})$$

- Now take 2 cross sections X and Y
  - or one or both can be pdf's
- Consider the projection of gradients of X and Y onto a circle of radius 1 in the plane of the gradients in the parton parameter space
- The circle maps onto an ellipse in the XY plane
- The angle ϕ between the gradients of X and Y is given by

$$\cos\varphi = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X\,\Delta Y}\sum_{i=1}^{N}\left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right)$$

- The ellipse itself is given by

$$\left(\frac{\delta X}{\Delta X}\right)^2 + \left(\frac{\delta Y}{\Delta Y}\right)^2 - 2\left(\frac{\delta X}{\Delta X}\right)\left(\frac{\delta Y}{\Delta Y}\right)\cos\varphi = \sin^2\varphi$$

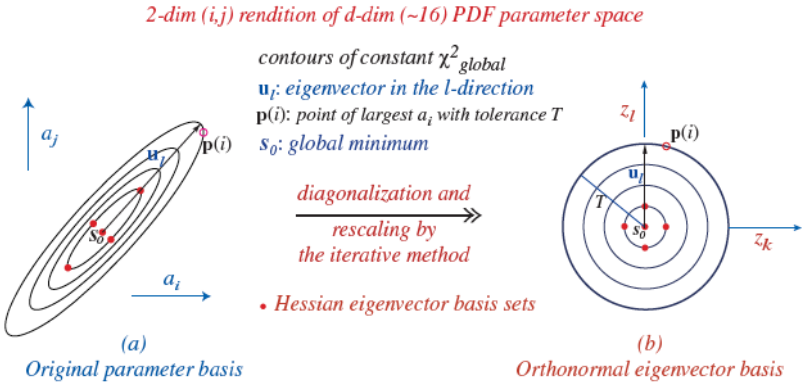

Figure 28. A schematic representation of the transformation from the pdf parameter basis to the orthonormal eigenvector basis.

- If two cross sections are very correlated, then cosϕ~1
- …uncorrelated, then cosϕ~0
- …anti-correlated, then cosϕ~-1

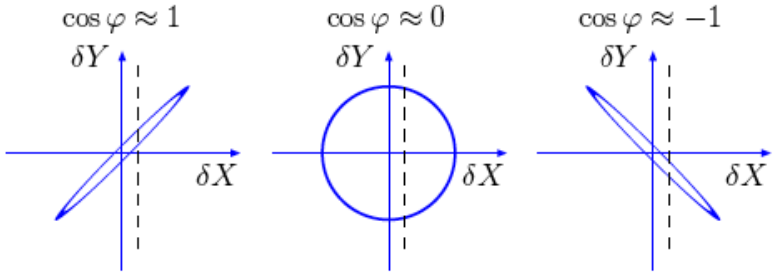

Figure 1: Dependence on the correlation ellipse formed in the $\Delta X - \Delta Y$ plane on the value of the correlation cosine $\cos\varphi$.

…from PDF4LHC report (CTEQ6.6)

Process	σ	PDF (asym)	PDF (sym)	$\alpha_s(m_Z)$ error	combined	orrelation
$\sigma_{W^+} * BR(W^+ \to l^+\nu)[nb]$	6.057	+0.123/-0.119	0.116	0.045	0.132	0.87
$\sigma_{W^-} * BR(W^- \to l^-\nu)[nb]$	4.106	+0.088/-0.091	0.088	0.029	0.092	0.92
$\sigma_{Z^o} * BR(Z^o \to l^+l^-)[nb]$	0.9469	+0.018/-0.018	0.018	0.006	0.0187	1.00
$\sigma_{t\bar{t}}[pb]$	156.2	+7.0/-6.7	6.63	4.59	8.06	-0.74
$\sigma_{gg\to Higgs}(120\ GeV)[pb]$	11.59	+0.19/-0.23	0.21	0.20	0.29	0.01
$\sigma_{gg\to Higgs}(180\ GeV)[pb]$	4.840	+0.077/-0.091	0.084	0.091	0.124	-0.47
$\sigma_{gg\to Higgs}(240\ GeV)[pb]$	2.610	+0.054/-0.058	0.056	0.055	0.078	-0.73

Table 5: Benchmark cross section predictions and uncertainties for CTEQ6.6 for $W^{\pm}, Z, t\bar{t}$ and Higgs production (120, 180, 240 GeV) at 7 TeV. The central prediction is given in column 2. Errors are quoted at the 68% c.l.. Both the symmetric and asymmetric forms for the PDF errors are given. In the next-to-last column, the (symmetric) form of the PDF and $\alpha_s(m_Z)$ errors are added in quadrature. In the last column, the correlation cosine with respect to Z production is given.

The values of ΔX, ΔY, and $\cos\varphi$ are also sufficient to estimate the PDF uncertainty of any function $f(X,Y)$ of X and Y by relating the gradient of $f(X,Y)$ to $\partial_X f \equiv \partial f/\partial X$ and $\partial_Y f \equiv \partial f/\partial Y$ via the chain rule:

$$\Delta f = \left|\vec{\nabla} f\right| = \sqrt{(\Delta X\ \partial_X f\ )^2 + 2\Delta X\ \Delta Y\ \cos\varphi\ \partial_X f\ \partial_Y f + (\Delta Y\ \partial_Y f)^2}. \qquad (9)$$

Used for LHC Higgs searches

Procedure for the LHC Higgs boson search combination in summer 2011

(LHC Higgs Combination Group Report)

A. Armbruster[1], K. A. Assamagan[2], S. Banerjee[3], G. Gomez-Ceballos[4], M. Chen[5], G. Cowan[6], K. Cranmer[7], S. Ferrag[8], E. Gross[9], J. Huston[10], A. Korytov[5], C. Mariotti[11], L. Moneta[12], W. J. Murray[13], G. Petrucciani[14], J. Qian[2], G. Schott[15], V. Sharma[14], G. E. Steele[8], R. Tanaka[16], F. Tarrade[17], O. Vitells[9], and H. Wang[4]

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Correlations for Higgs Working Group

Backgrounds

	Z	W	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
Z	1	0.95	0.67	0.70	0.95	0.9	0.43/0.53	0.08	-0.67	-0.75	-0.74	-0.81	0.59	-0.29
W	0.95	1	0.52/0.69	0.60/0.71	0.88/1.0	0.90/0.80	0.39/0.50	0.08	-0.67	-0.74	-0.73	-0.8	0.57	-0.29
ZZ	0.67	0.52/0.69	1	0.97	0.54/0.73	0.62	0.78/0.87	-0.09	-0.36	-0.34	-0.17	-0.81	0.9	-0.23
WW	0.70	0.60/0.71	0.97	1	0.63/0.75	0.69	0.80/0.86	-0.02	-0.34	-0.33	-0.20	-0.33	0.94	-0.08
WZ	0.95	0.88/1.0	0.54/0.73	0.63/0.75	1	0.9	0.55	0.1	-0.64	-0.71	-0.71	-0.73	0.61	-0.34
Wγ	0.9	0.90/0.80	0.62	0.69	0.9	1	0.63/0.53	0.32	-0.44	-0.54	-0.68	0.61	0.61	0
WQQ	0.43/0.53	0.39/0.50	0.78/0.87	0.80/0.86	0.55	0.63/0.53	1	0.08	-0.12	-0.12	-0.05	-0.15	0.64	-0.32
ZQQ	0.08	0.08	-0.09	-0.02	0.1	0.32	0.08	1	0.54	0.36	-0.26	-0.05	-0.03	0.59
ggWW	-0.67	-0.67	-0.36	-0.34	-0.64	-0.44	-0.12	0.54	1	0.98	0.65	0.81	-0.28	0.63
ggZZ	-0.75	-0.74	-0.34	-0.33	-0.71	-0.54	-0.12	0.36	0.98	1	0.79	0.91	-0.27	0.55
ttbar	-0.74	-0.73	-0.17	-0.20	-0.71	-0.68	-0.05	-0.26	0.65	0.79	1	0.97	-0.12	0.17
tW	-0.81	-0.8	-0.81	-0.33	-0.73	0.61	-0.15	-0.05	0.65	0.91	0.97	1	-0.25	0.31
tb	0.59	0.57	0.9	0.94	0.61	0.61	0.64	-0.03	-0.28	-0.27	-0.12	-0.25	1	0.04
tbq	-0.29	-0.29	-0.23	-0.08	-0.34	0	-0.32	0.59	0.63	0.55	0.17	0.31	0.04	1

…calculated with CTEQ6.6 and using MCFM

the use of correlations allows for PDF uncertainties to be reduced

Higgs correlations (using CTEQ6.6)

m_H=120

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.57	-0.23	-0.14	-0.6	0.01	0.03	0.02	-0.20	0.04	0.23	-0.14	0.95	0.47	0.28	-0.35	-0.12	-0.24	0.52
VBF	-0.57	1	0.63/0.73	0.76	0.09	0.43	0.26/0.41	0.79	0.72	0.28/0.43	0.28/0.37	0.52/0.71	-0.41	-0.47	-0.4	-0.10	-0.28	0.65	-0.25
WH	-0.23	0.63/0.73	1	0.93	0	0.62	0.52/0.64	0.92	0.93	0.65/058	0.65/0.56	0.79/0.95	-0.02	-0.29	-0.28	-0.15	-0.28	0.99/0.77	0.05/-0.30
ZH	-0.14	0.76	0.93	1	0.03	0.64	0.53/0.66	0.99	0.99	0.55/0.71	0.63	0.83	-0.07	-0.31	-0.3	-0.14	-0.28	0.93	-0.14
ttH	-0.6	0.09	0	0.03	1	-0.61	-0.6	0	-0.05	-0.58	-0.64	0.04	-0.5	0.03	0.56	0.94	0.84	0.02	-0.07

m_H=160

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.61	-0.29	-0.35	-0.24	-0.32	-0.32	-0.35	-0.29	-0.29	-0.06	-0.12	0.9	0.82	0.68	0.1	0.33	-0.27	0.67
VBF	-0.61	1	0.62	0.74	0.2	0.35	0.19/0.34	0.75	0.66	0.20/0.36	0.19/0.28	0.46/0.70	-0.47	-0.46	-0.37	-0.03	-0.22	0.6	-0.29
WH	-0.29	0.62	1	0.93	0.1	0.55	0.52	0.9	0.93	0.56	0.56	0.93	-0.07	-0.26	-0.23	-0.07	-0.21	1	0.03
ZH	-0.35	0.74	0.93	1	0.16	0.54	0.43/0.58	0.98	0.97	0.45/0.63	0.52	0.93	-0.14	-0.29	-0.25	-0.04	-0.2	0.91	-0.16
ttH	-0.24	0.2	0.1	0.16	1	-0.59	-0.58	0.03	-0.03	-0.56	-0.62	-0.05	-0.54	0.33	0.51	0.92	0.8	0.04	-0.12

m_H=200

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.5	-0.26	-0.3	0.13	-0.59	-0.59	-0.36	-0.32	-0.55	-0.33	-0.11	0.68	0.98	0.93	0.5	0.69	-0.27	0.67
VBF	-0.5	1	0.60/0.73	0.72	0.26	0.28	0.13/0.28	0.7	0.62	0.15/0.30	0.12/0.20	0.40/0.69	-0.52	-0.44	-0.34	0.02	-0.17	0.55	-0.32
WH	-0.26	0.60/0.73	1	0.92	0.2	0.44	0.44/0.38	0.89	0.86	0.48/0.41	0.47/0.36	0.78/0.74	-0.15	-0.24	-0.2	0	-0.15	0.98/0.69	0
ZH	-0.3	0.72	0.92	1	0.24	0.46	0.34/0.51	0.95	0.93	0.37/0.56	0.43	0.74/0.85	-0.19	-0.3	-0.22	0.02	-0.14	0.88	-0.2
ttH	0.13	0.26	0.2	0.24	1	-0.57	-0.57	0.03	-0.03	-0.55	-0.63	0.03	-0.56	0.29	0.48	0.9	0.78	0.03	-0.15

Higgs correlations

m_H=300

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	-0.16	-0.08	-0.09	0.66	-0.8	-0.79	-0.31	-0.31	-0.76	-0.64	-0.11	0.12	0.9	0.97	0.92	0.98	-0.23	0.43
VBF	-0.16	1	0.53/0.72	0.68	0.29	0.16	0.04/0.19	0.6	0.51	0.05/0.20	0.03	0.27/0.65	-0.57	-0.42	-0.31	0.09	-0.11	0.44	-0.39
WH	-0.08	0.53/0.72	1	0.92	0.23	0.32	0.20/0.36	0.82	0.80/0.71	0.34/0.37	0.30/0.20	0.68/0.64	-0.24	-0.22	-0.16	0.1	-0.06	0.89	-0.06
ZH	-0.09	0.68	0.92	1	0.27	0.32	0.20/0.38	0.87	0.82	0.21/0.44	0.26	0.61/0.81	-0.29	-0.25	-0.18	0.11	-0.07	0.79	-0.28
ttH	0.66	0.29	0.23	0.27	1	-0.6	-0.59	-0.05	-0.12	-0.58	-0.65	-0.04	-0.58	0.28	0.47	0.9	0.78	-0.04	-0.17

m_H=500

	ggH	VBF	WH	ZH	ttH	Z	W+/W-	ZZ	WW	WZ	Wγ	WQQ	ZQQ	ggWW	ggZZ	ttbar	tW	tb	tbq
ggH	1	0.09	0.05	0.05	0.91	-0.78	-0.76	-0.25	-0.28	-0.75	-0.73	-0.13	-0.3	0.63	0.78	0.99	0.97	-0.2	0.15
VBF	0.09	1	0.38/0.70	0.6	0.24	0.073	0.0/0.12	0.47	0.37	0/0.12	-0.08	0.11/0.59	-0.58	-0.4	-0.29	0.1	-0.08	0.29	-0.48
WH	0.05	0.38/0.70	1	0.9	0.16	0.19	0.09/0.26	0.69	0.64	0.20/0.20	0.14/0.09	0.55/0.53	-0.3	-0.21	-0.14	0.14	-0.02	0.73	-0.12
ZH	0.05	0.6	0.9	1	0.16	0.22	0.09/0.29	0.77	0.68	0.10/0.34	0.12	0.44/0.74	-0.35	-0.27	-0.19	0.13	-0.05	0.65	-0.37
ttH	0.91	0.24	0.16	0.16	1	-0.63	-0.61	-0.18	-0.23	-0.61	-0.69	-0.14	-0.57	0.3	0.48	0.89	0.79	-0.15	-0.14

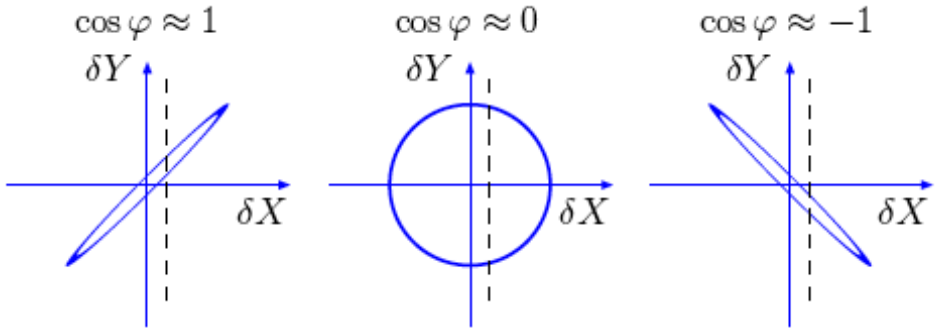
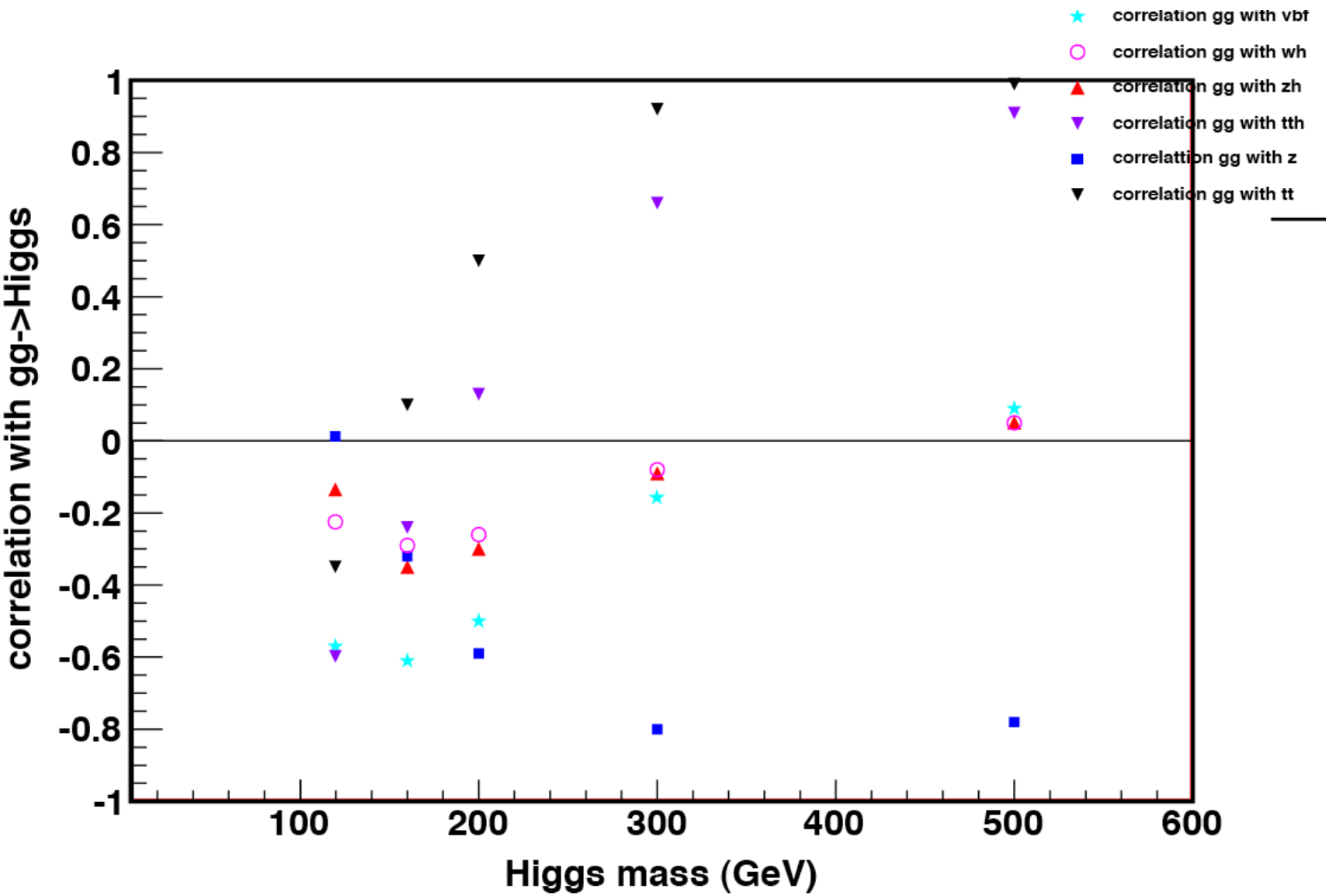


Figure 1: Dependence on the correlation ellipse formed in the $\Delta X - \Delta Y$ plane on the value of the correlation cosine $\cos\varphi$.

Some results



Extension

- The correlations should be similar for all NLO PDFs, but we would like to include correlation information from the different PDFs in the next Higgs CERN Yellow Report, as well as in future updates for the PDF4LHC working group documents
- I've done the same thing for the Tevatron; it would also be interesting to look at the correlations between the cross sections at the two accelerators