COSMIC WISPers, DESY, February 1st 2024

The QCD axion sum rule

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Motivation for ALPs

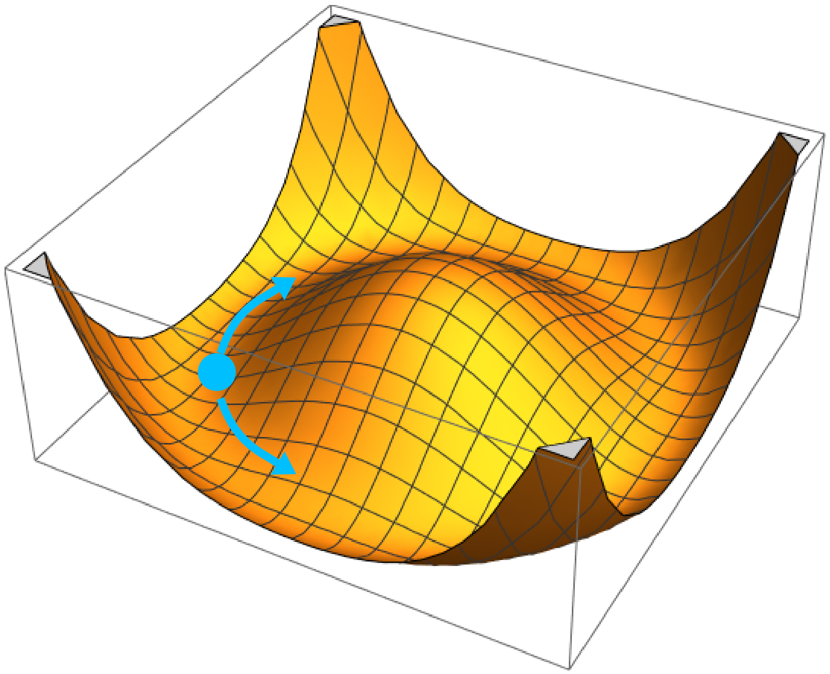
ALPs are primary examples of WISPs.

They arise commonly in BSM as pGBs.

$$V(\phi) = -\mu^2 \phi\phi^\dagger + \lambda(\phi\phi^\dagger)^2$$

$$\phi = (f+\rho)e^{ia/f}$$

$$U(1): \ \phi \to f e^{\underbrace{\mathrm{i}(a/f+\kappa)}_{\textbf{shift-symmetry}}}$$

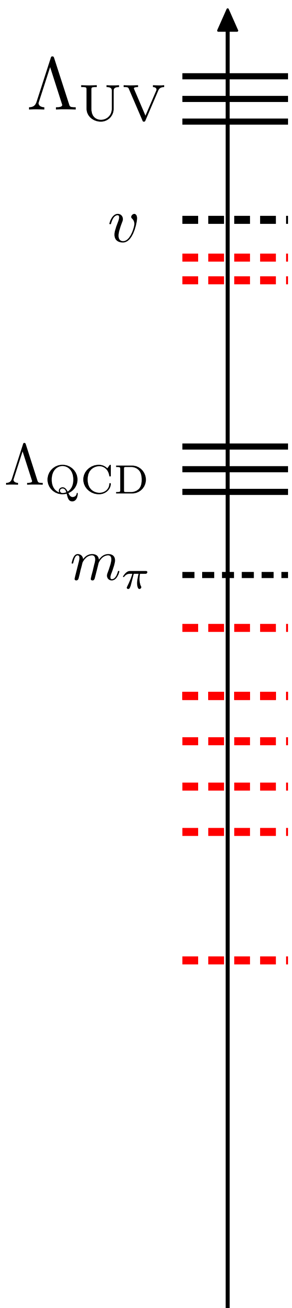


They are natural in QFT.

$$m_a^2 = \epsilon f^2 \ll m_\rho^2 = 4\lambda f^2$$

$$\mathcal{L}_a = \frac{1}{2}\left(\partial_\mu a\right)^2 + \mathcal{L}\left(\partial_\mu a, \mathcal{L}_{\mathrm{SM}}\right) + \mathcal{O}(\epsilon)$$

The scale of ALPs in Nature



$$m^2 \sim \frac{\gamma y_t^2}{(4\pi)^2} m_T^2$$

Composite Higgs models

WIMP dark matter?
Baryogenesis?

$$m^2 = \,?$$

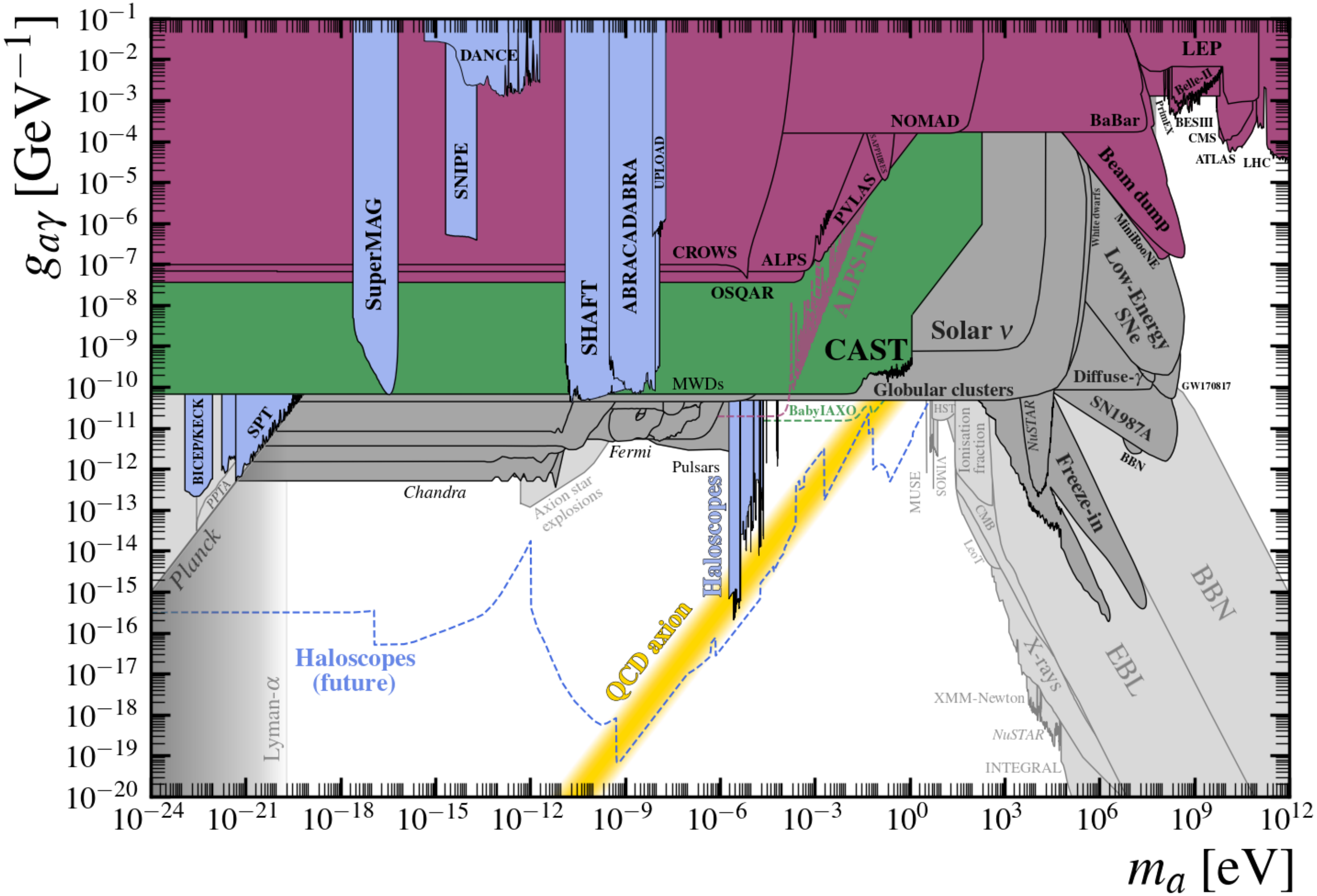
String axiverse
Extra-dim Kaluza-Klein modes

$$m^2 \sim \frac{m_\pi^2 f_\pi^2}{f^2}$$

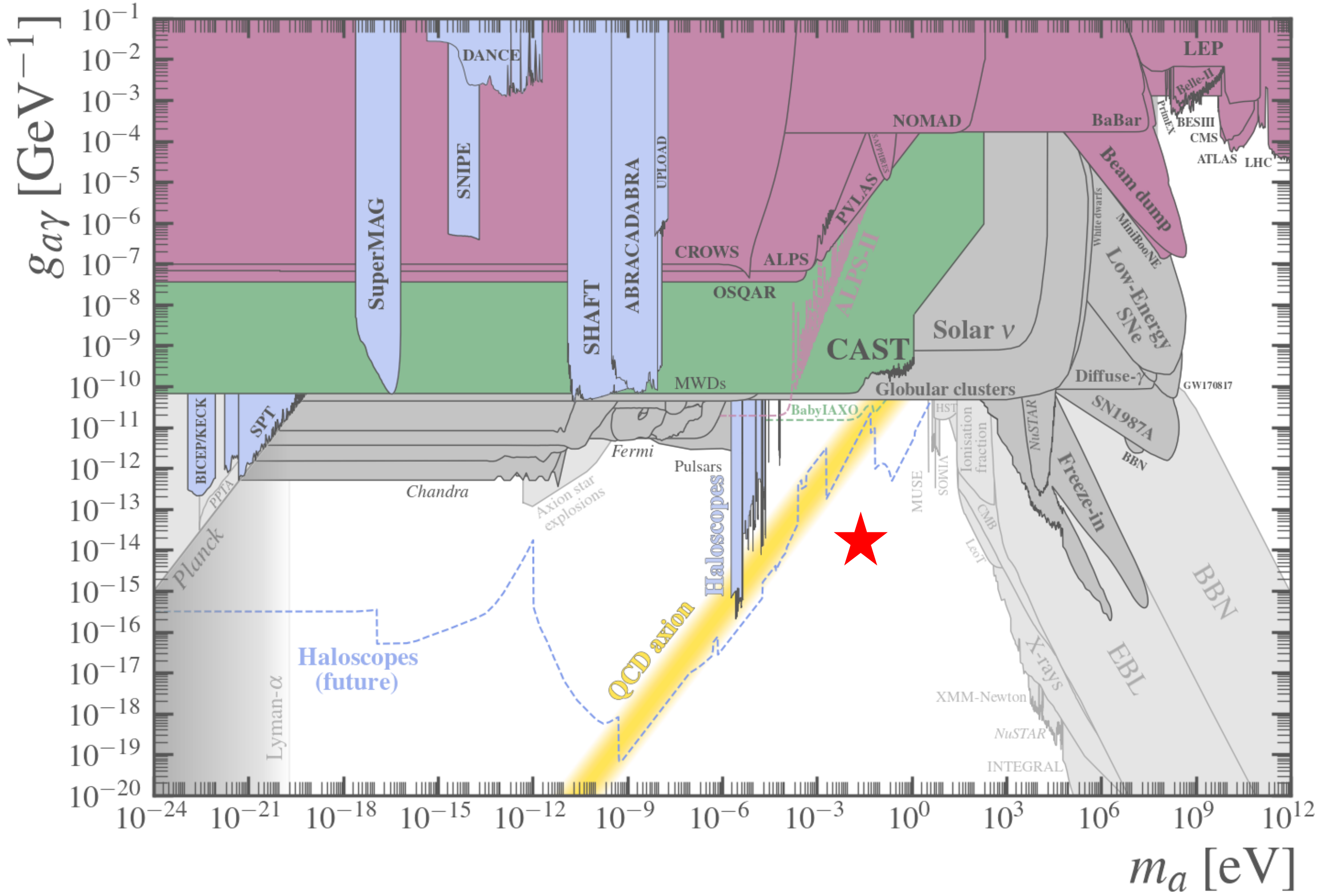
Peccei-Quinn model

Strong CP problem?
Non-thermal dark matter?

The ALP landscape



The ALP landscape



Which puzzle can this signal explain?

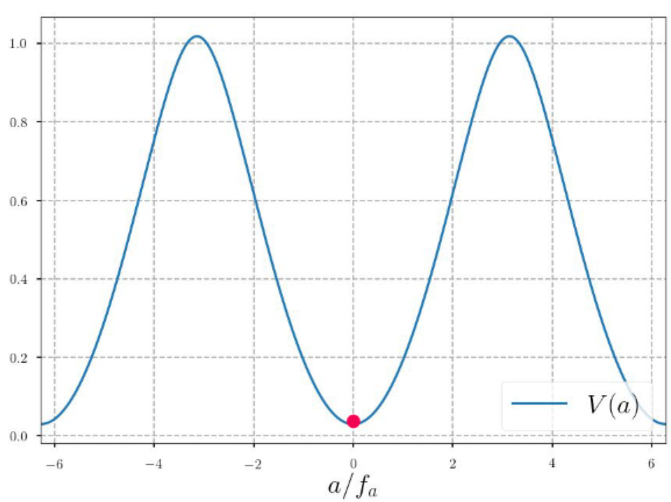
Let us revisit the canonical axion solution to the strong CP problem

Assuming the SM gauge group setting

The canonical QCD axion

The strong CP problem: $\mathcal{L} \supset \frac{\alpha_s}{8\pi} \bar{\theta} G\widetilde{G}$

A dynamical $U(1)_{PQ}$ solution: $$\mathcal{L} = \frac{\alpha_s}{8\pi}\Bigg(\underbrace{\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}}_{a/f_a}\Bigg) G\widetilde{G}$$



$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{1}{2}\frac{a}{f_a}\right)}$$

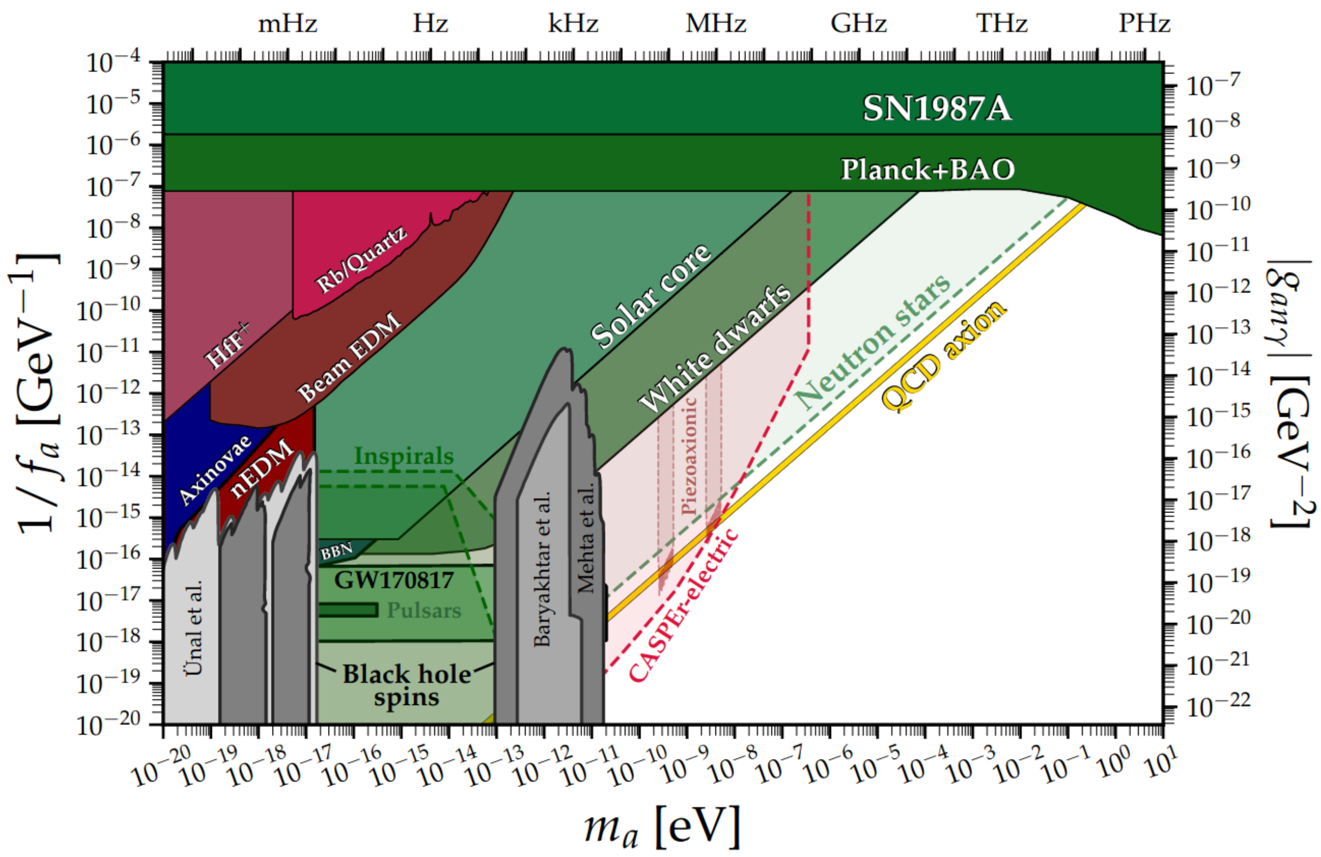
$$m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2 \frac{m_u\, m_d}{(m_u + m_d)^2}$$

assuming all PQ breaking comes from QCD

The canonical QCD axion

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G} \rightarrow$$

$$m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$



The canonical QCD axion

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G} \rightarrow \boxed{m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}}$$

[interaction basis = mass basis]

up to mixing with QCD resonances

Beyond the canonical QCD axion

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{a_{G\widetilde{G}}}{f_a} - \bar{\theta}\right) G\widetilde{G} \rightarrow \quad m_a^2\, f_a^2 = \chi_{\rm QCD} \simeq m_\pi^2\, f_\pi^2\, \frac{m_u\, m_d}{(m_u + m_d)^2}$$

[interaction basis = mass basis]

up to mixing with QCD resonances

<u>But the axion may not be the only singlet scalar in Nature</u>

e.g. "String axiverse"

Arvanitakia, Dimopoulos, Dubovskyc, Kalopere, Russell 09

Additional misalignement can lead to observable effects:

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V'(\hat{a}_{G\widetilde{G}}, \ldots, \hat{a}_N)$$

$$\Rightarrow m_i^2\, f_i^2 = g_i \chi_{\rm QCD}$$

PQ condition

$$\mathcal{L} = \frac{\alpha_s}{8\pi}\left(\sum_{k=1}^{N}\frac{\hat{a}_k}{\hat{f}_k} - \bar{\theta}\right) G\widetilde{G} - V_B(\hat{a}_1,\, \hat{a}_2, \ldots, \hat{a}_N) \rightarrow \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_{G\widetilde{G}}}{F} - \bar{\theta}\right) G\widetilde{G} - V_B^{\mathrm{R}}(\hat{a}_{G\widetilde{G}}, \ldots)$$

$$\frac{1}{F^2} = \sum_{k=1}^{N}\frac{1}{\hat{f}_k^2}$$

A preferred basis.

$$\boxed{\mathbf{M}^2 \equiv \mathbf{R}\,\hat{\mathbf{M}}^2\mathbf{R}^T}$$

$$\mathbf{M}^2 = \mathbf{M}_A^2 + \mathbf{M}_B^2 = \begin{pmatrix} b_{11} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix} = \frac{\chi_{\mathrm{QCD}}}{F^2}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{0} \end{pmatrix} + \begin{pmatrix} b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} & \mathbf{X}^\dagger \\ \mathbf{X} & \mathbf{M}_1^2 \end{pmatrix},$$

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$$\boxed{\exists\, U(1)_{PQ} \implies \lim_{\chi_{\mathrm{QCD}}\to 0} \det \mathbf{M}^2 = 0 \implies \det \mathbf{M}_B^2 = 0} \quad \langle \hat{a}_0 | a_{G\widetilde{G}} \rangle \neq 0$$

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Applying Schur's formula.

$$\det \mathbf{M}_1^2 \left(b_{11} - \frac{\chi_{\mathrm{QCD}}}{F^2} - \mathbf{X}^\dagger \mathbf{M}_1^{-2}\mathbf{X}\right) = 0$$

$$\Rightarrow \boxed{\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2}\mathbf{X}\right) = \frac{\chi_{\mathrm{QCD}}}{F^2}}$$

PQ condition

$$\Rightarrow \quad \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

Moving to the physical basis.

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \frac{a_i}{f_i} G\widetilde{G} \qquad \text{with} \qquad \frac{1}{f_i} = \frac{\langle \hat{a}_{G\widetilde{G}} | a_i \rangle}{F} \equiv \frac{v_{i1}}{F} \implies \sum_{i=1}^{N} \frac{1}{f_i^2} = \frac{1}{F^2}$$

PQ condition

$$\Rightarrow \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

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Eigenvector-eigenvalue Th.
(generic A matrix)

$$\frac{\det\left(\lambda \mathbb{I}_{N-1} - M_j\right)}{\det\left(\lambda \mathbb{I}_N - A\right)} = \sum_{i=1}^{N} \frac{|v_{ij}|^2}{\lambda(A) - \lambda_i(A)}$$

PQ condition

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$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|v_{1i}|^2}{m_i^2} = \frac{F^2}{\chi_{\rm QCD}} \sum_{i=1}^{N} \frac{1}{g_i}$$

$$\boxed{g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}}$$

The QCD axion sum rule

$$\Rightarrow \frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \left(b_{11} - \mathbf{X}^\dagger \mathbf{M}_1^{-2} \mathbf{X}\right) = \frac{\chi_{\rm QCD}}{F^2}$$

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$$\frac{\det \mathbf{M}_1^2}{\det \mathbf{M}^2} = \sum_{i=1}^{N} \frac{|v_{1i}|^2}{m_i^2} = \frac{F^2}{\chi_{\rm QCD}} \sum_{i=1}^{N} \frac{1}{g_i} \xrightarrow{\exists\, U(1)_{\rm PQ}} \boxed{\sum_{i=1}^{N} \beta_i = 1, \;\; \beta_i \equiv \frac{1}{g_i}}$$

$$\boxed{g_i = \frac{m_i^2 f_i^2}{\chi_{\rm QCD}}}$$

***axionness* is shared!**

The QCD axion sum rule

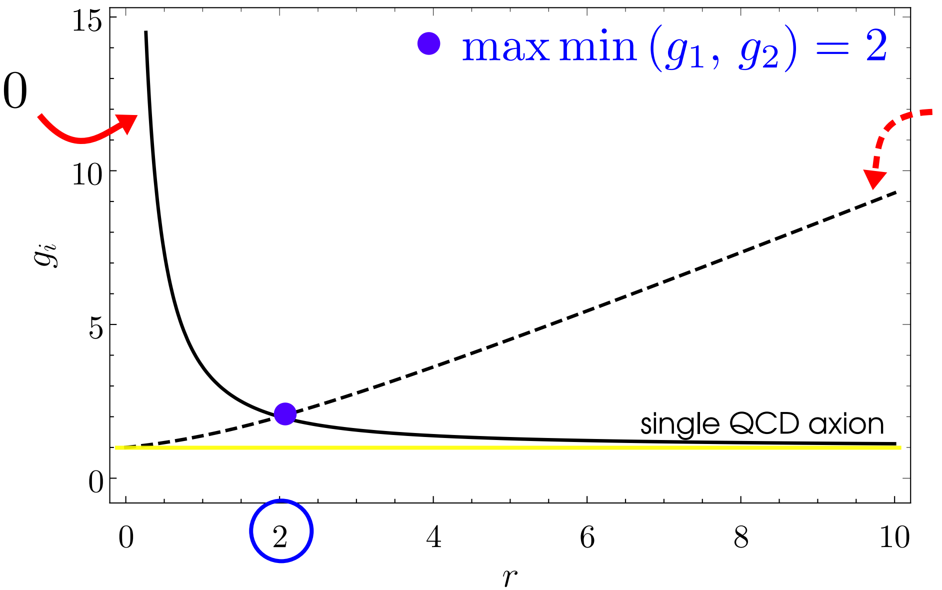
$$\sum_{i=1}^{N} \beta_i = 1, \;\; \beta_i \equiv \frac{1}{g_i} = \frac{\langle \hat{a}_{\rm PQ} \mid a_i \rangle \langle a_i \mid \hat{a}_{G\widetilde{G}} \rangle}{\langle \hat{a}_{\rm PQ} \mid \hat{a}_{G\widetilde{G}} \rangle}$$

Toy example:

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \frac{1}{2}\hat{m}_2^2 \hat{a}_2^2 \Longrightarrow \quad \hat{a}_{G\widetilde{G}} = \frac{1}{2}(\hat{a}_1 + \hat{a}_2) \quad \text{and} \quad \hat{a}_{\rm PQ} = \hat{a}_1$$

$$\beta_1 = 1 \times 0$$

$$r = \frac{\hat{m}_2^2}{\chi_{\rm QCD}/\hat{f}^2}$$

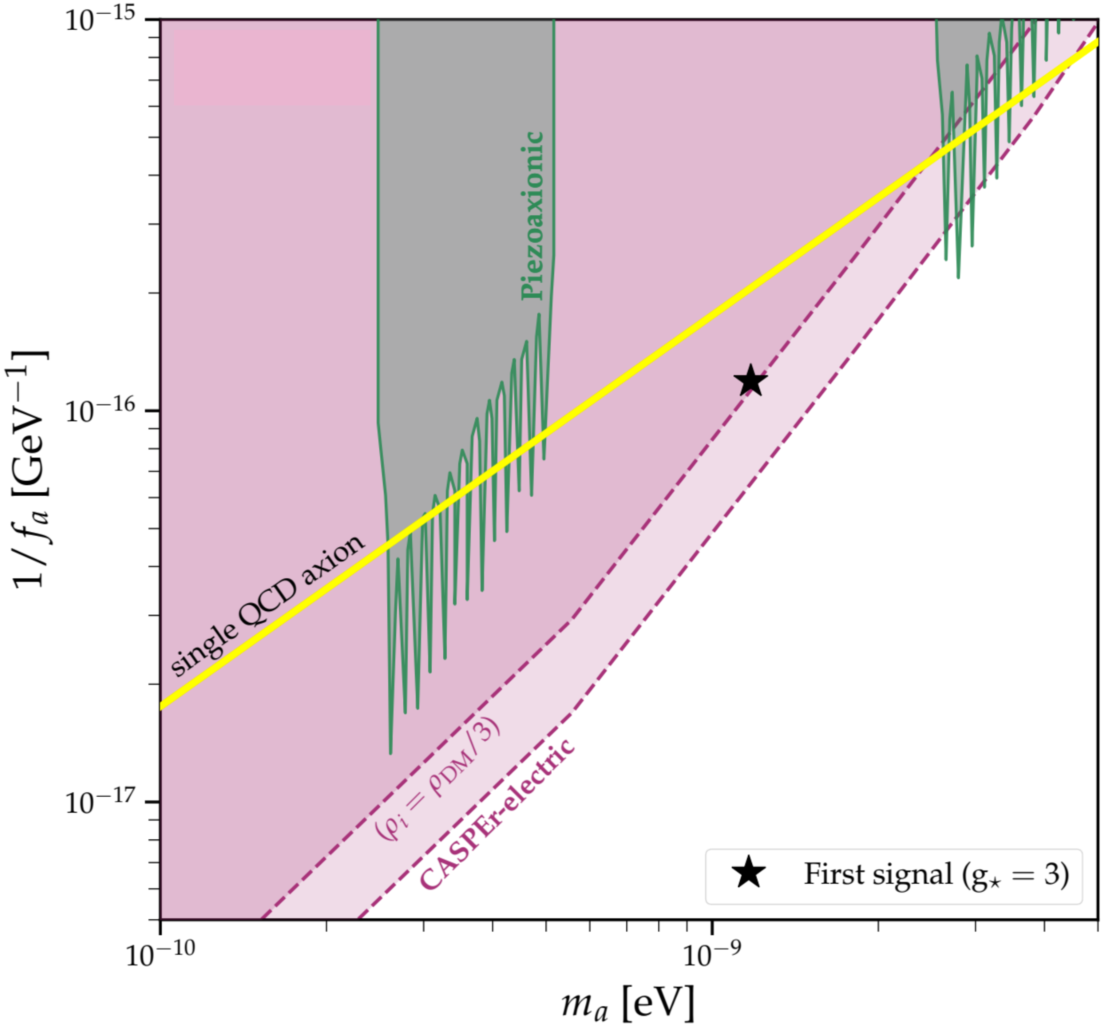


$$\beta_2 = 0 \times 1$$

Kim et al, 04
Agrawal et al, 17 and 22
Farina et al, 17
Coy, Frigerio, 17
Choe et al, 14 and 16
Kaplan et al, 16
Giudice et al, 16

Large deviations require new scales close to the QCD generated mass

An ALP or a true QCD axion?



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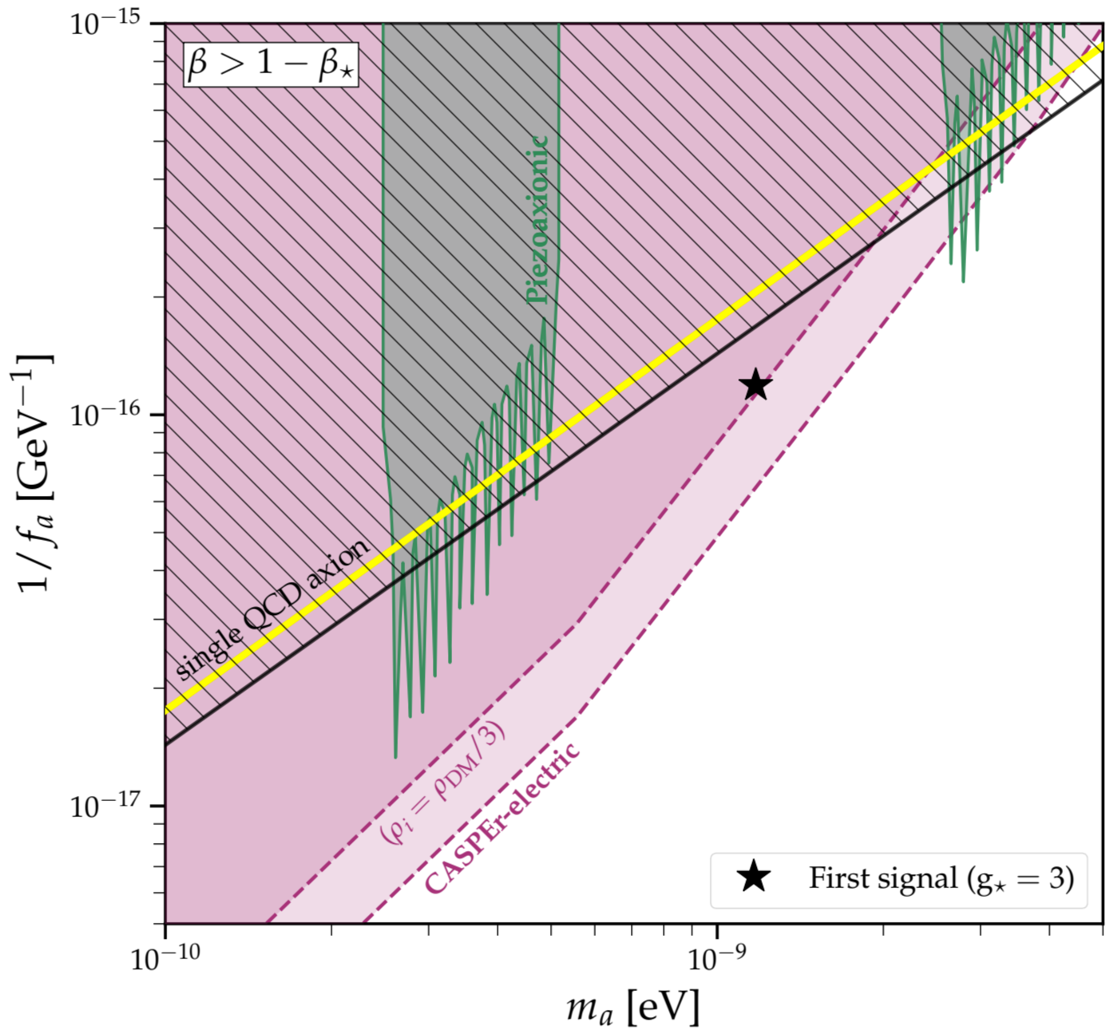
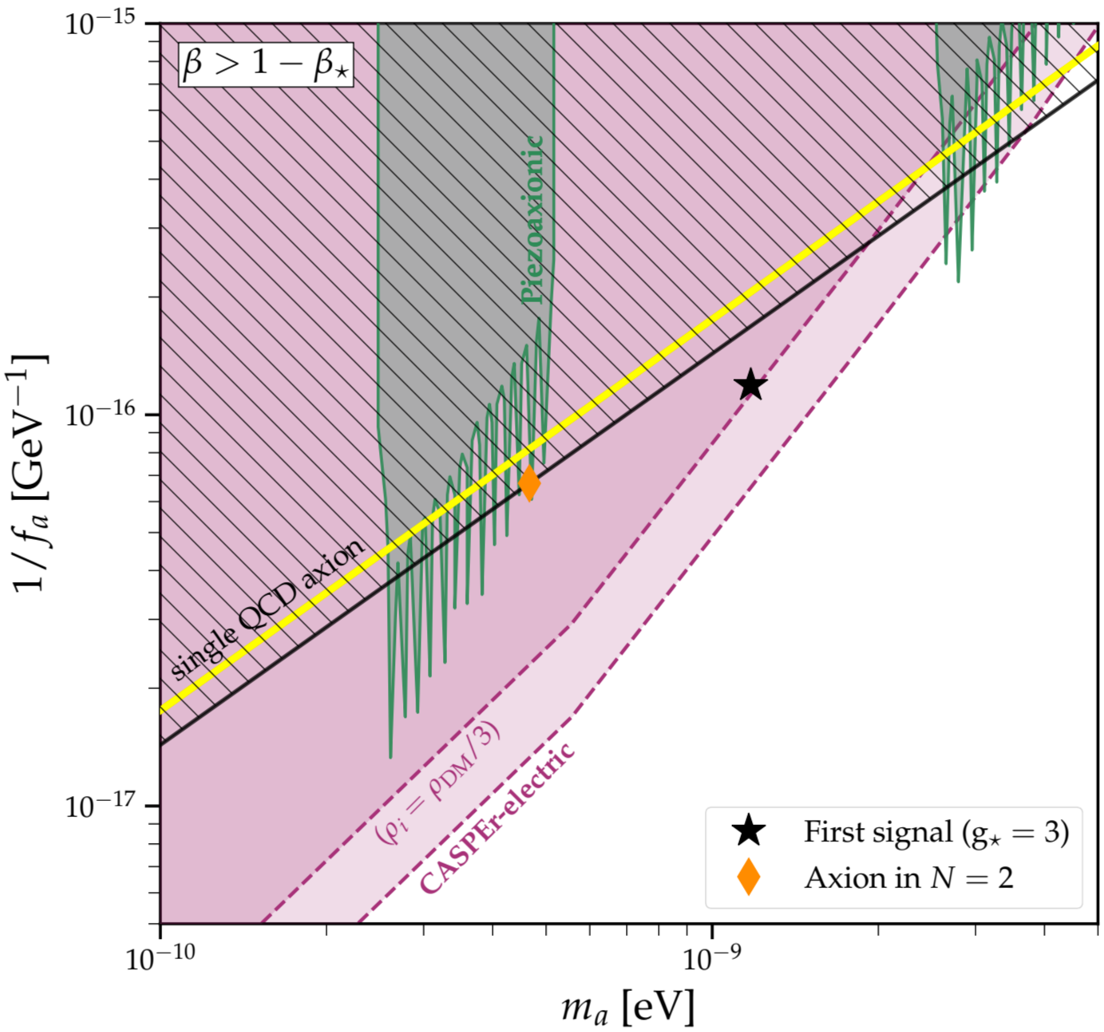


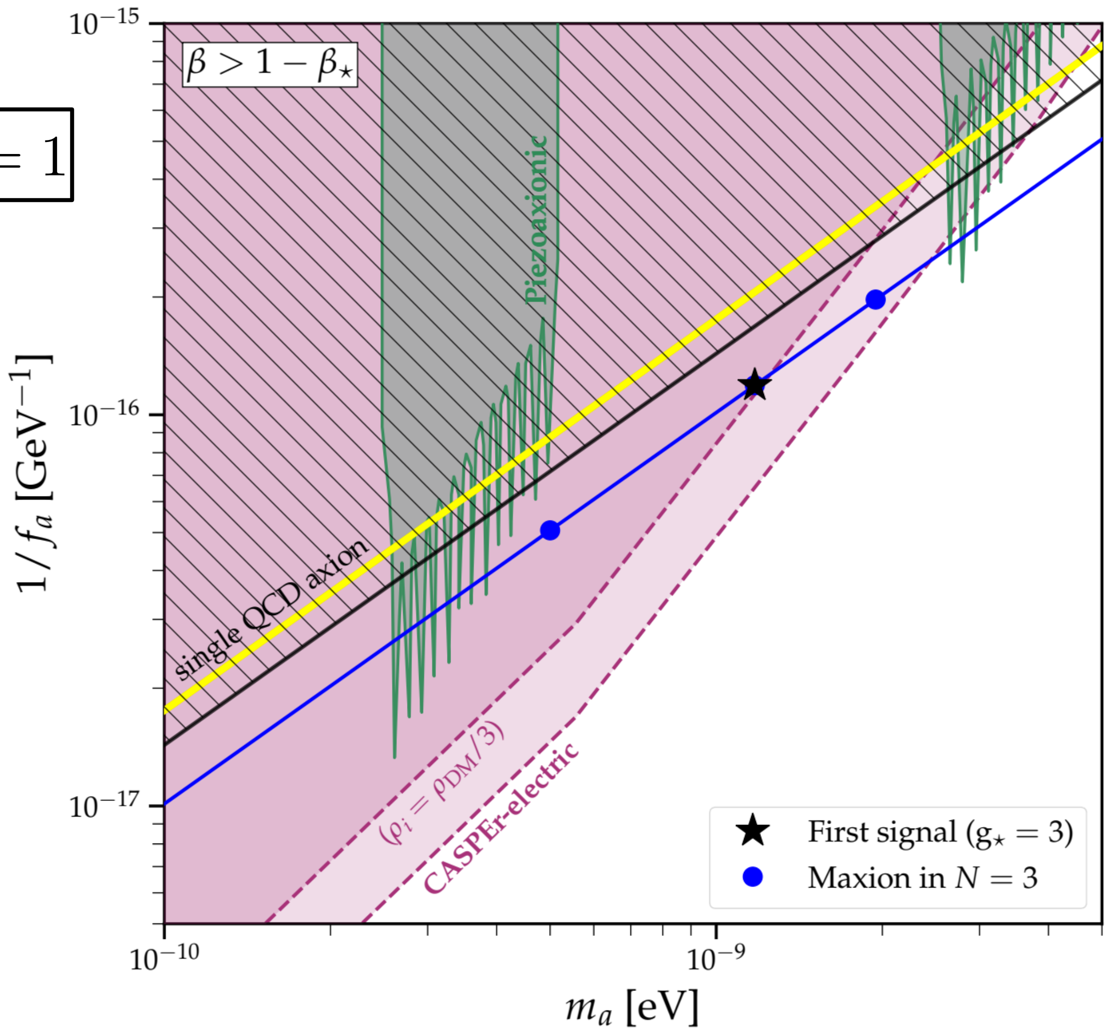
An ALP or a true QCD axion?

$$\boxed{\beta_\star + \beta_2 = 1}$$



An ALP or a true QCD axion?

$$\beta_\star + \beta_2 + \beta_3 = 1$$



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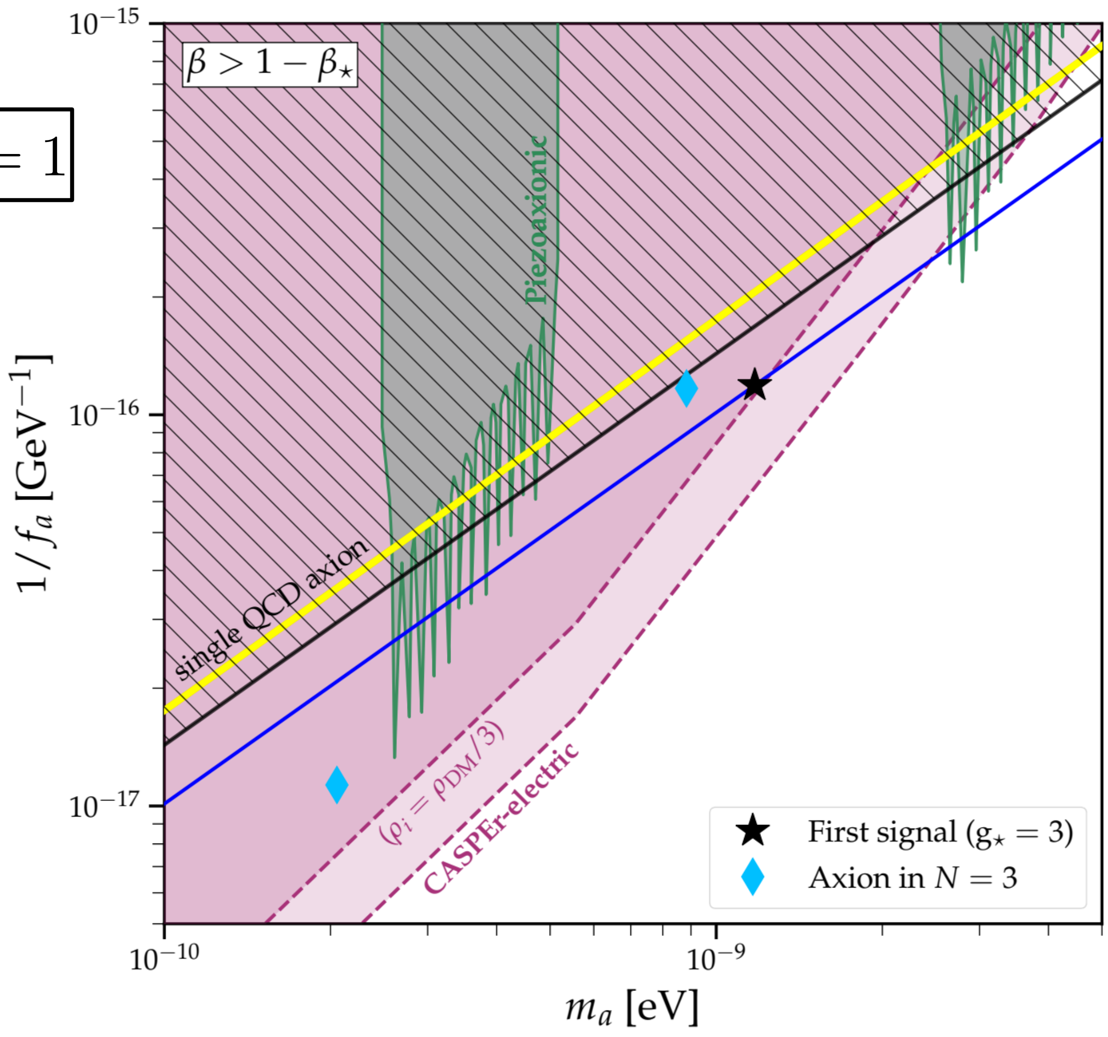
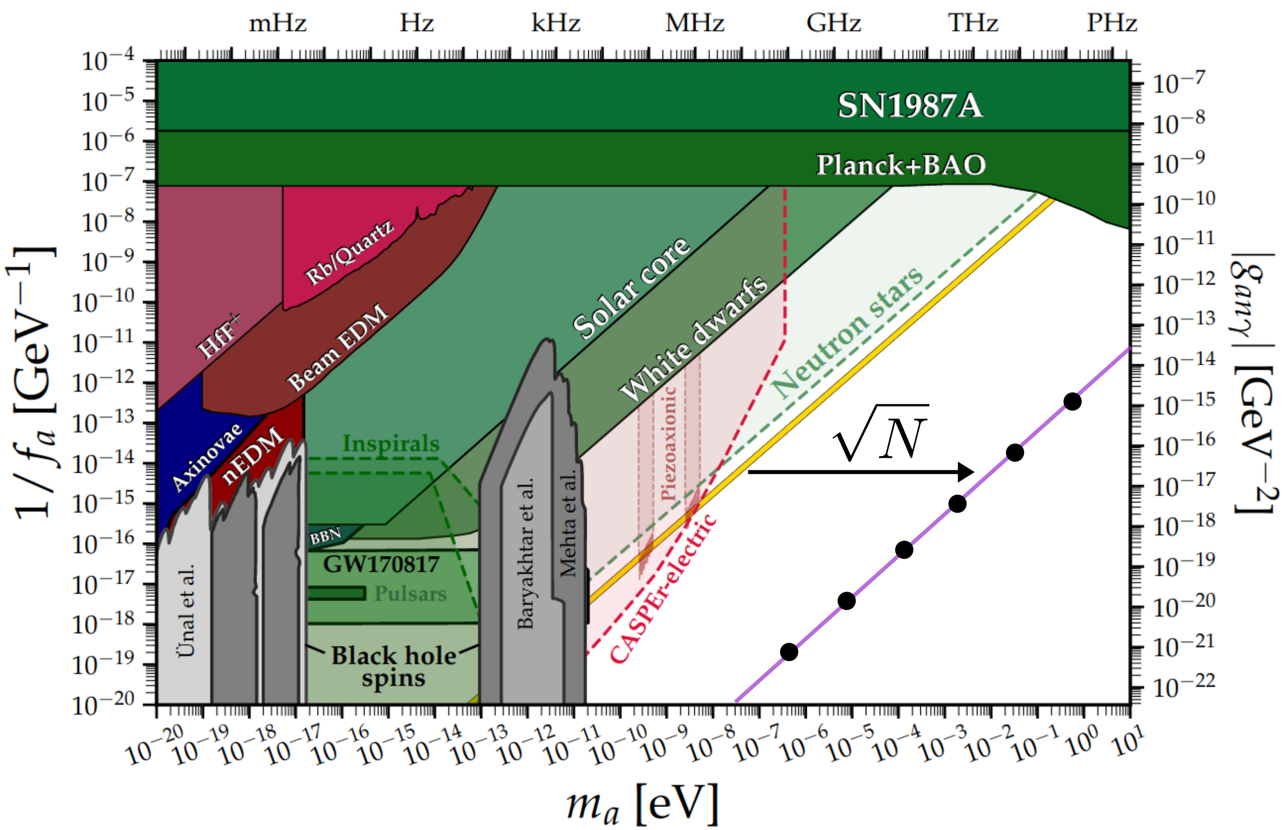


Maximally deviated QCD axions=**Maxions**

$$\max \left\{ \min_i \{ g_i \} \right\} = N \qquad \Longrightarrow \quad g_i = N, \ \forall \, i$$



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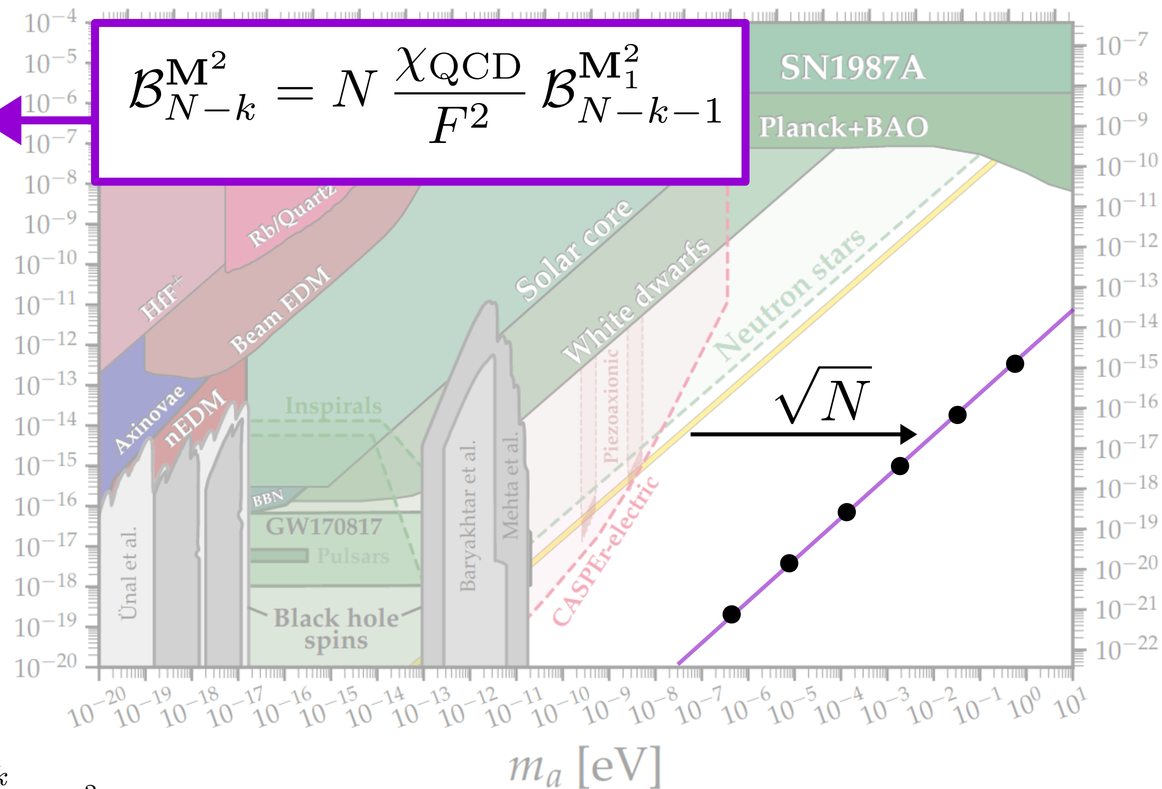
$$\max_{} \left\{ \min_i \{g_i\} \right\} = N \quad \Longrightarrow \quad g_i = N, \ \forall\, i$$

$$\mathcal{B}^{\mathbf{M}^2}_{N-k} = N \, \frac{\chi_{\rm QCD}}{F^2} \, \mathcal{B}^{\mathbf{M}_1^2}_{N-k-1}$$

$$\mathrm{tr}\, \mathbf{M}^2 = N \frac{\chi_{\rm QCD}}{F^2}$$

$$\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$$

$$\dots$$



$$p_{\mathbf{M}^2}(\lambda) = \sum_{k=0}^{N} \frac{(-1)^{N-k}}{(N-k)!} \, \mathcal{B}^{\mathbf{M}^2}_{N-k} \, \lambda^k$$

UV completion: extended KSVZ

$$\mathcal{L}_{\rm UV} \supset -\left[y_1 \overline{\Psi}_1 \Psi_1 S_1 + y_2 \overline{\Psi}_2 \Psi_2 S_2 + \text{h.c.}\right] - V(S_{1,2})$$

$$U(1)_{\rm PQ}: \quad \Psi_{j,L} \to e^{i\alpha_j/2}\Psi_{j,L}\,, \;\; \Psi_{j,R} \to e^{-i\alpha_j/2}\Psi_{j,R}\,, \;\; S_j \to e^{i\alpha_j} S$$

After SSB : $$S_{1,2} = \frac{1}{\sqrt{2}}\left(\hat{f}_{1,2} + \rho_{1,2}\right) e^{i\hat{a}_{1,2}/\hat{f}_{1,2}}$$

Explicit breaking effects can reduce the symmetry:

- $\mathcal{L}_{\rm UV} \supset \frac{\mu^2}{2} S_2 S_2 + \text{h.c.}$ —— 2 maxions ——→ $\mu^2 = \chi_{\rm QCD}/\hat{f}^2$

$$\mathcal{L}_{\rm eff} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}} + \frac{\hat{a}_2}{\hat{f}} + \bar{\theta}\right) G\widetilde{G} - \mu^2 \hat{a}_2^2 \implies \mathbf{M}^2 = \begin{pmatrix} 2\frac{\chi_{\rm QCD}}{\hat{f}^2} + \mu^2 & \mu^2 \\ \mu^2 & \mu^2 \end{pmatrix}$$

- $\mathcal{L}_{\rm UV} \supset \mu \overline{\Psi}_1 \Psi_1$ —— 2 maxions ——→ $\mu M_1^3 = 8\pi^3 \chi_{\rm QCD}$

UV completion: Extra dimensions?!

$$S \supset \int \mathrm{d}^4x \int_{-\pi R}^{\pi R} \mathrm{d}y \sqrt{g} \left(\frac{1}{2} g^{AB} \partial_A a \partial_B a - \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu} G^{\mu\nu} \delta^n(\vec{y} - \pi R) \right)$$

Dienes, Dudas, Gherghetta 99

Integrating over a fifth dimension:

$$\mathcal{L} \supset \sum_n \left(\frac{1}{2} (\partial_\mu a_n)^2 - \frac{1}{2} \frac{n^2}{R^2} a_n^2 \right) + \frac{1}{\hat{f}} \frac{\alpha_s}{8\pi} \sum_n (a_n \psi_n^\pi) \tilde{G}_{\mu\nu} G^{\mu\nu}$$

WORK IN PROGRESS with Belén Gavela, Arturo De Giorgi, Pablo Quílez

$$\mathcal{M}^2 = m_{\rm PQ}^2 \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \cdots \\ \sqrt{2} & 2+y^2 & 2 & 2 & \cdots \\ \sqrt{2} & 2 & 2+4y^2 & 2 & \cdots \\ \sqrt{2} & 2 & 2 & 2+9y^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad m_{\rm PQ}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}, \quad y \equiv \frac{R^{-1}}{m_{\rm PQ}} \lesssim 1$$

Mixing structure in LED can produce large deviations!

UV completion: Extra dimensions?!

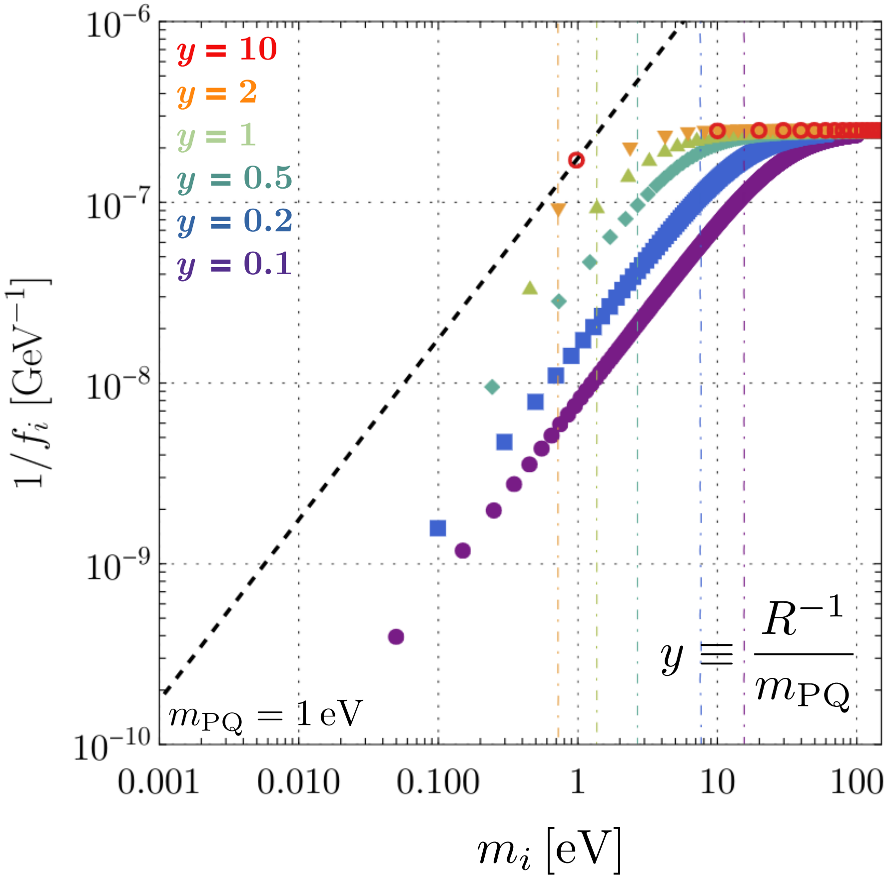
WORK IN PROGRESS with
Belén Gavela, Arturo De Giorgi, Pablo Quílez

The eigenvalues must satisfy

$$\frac{\pi\lambda_i}{y}\cot\left(\frac{\pi\lambda_i}{y}\right) = \lambda_i^2\,, \quad \lambda_i = \frac{m_i}{m_{PQ}}$$

Knowing the eigenvectors as well,

$$g_i^{\rm LED} = \frac{1}{2}\left[\lambda^2 + 1 + \frac{\pi^2}{y^2}\right]$$



Model has some pseudo-maxions!

Some heavier axions decouple, leading to a plateau that gives directly access to the effective PQ scale.

Currently studying other couplings and patterns in different ED models!

Coupling to photons

Assuming **universal** anomaly factors,

$$\mathcal{L} \supset \frac{\alpha_{em}}{8\pi} \sum_{k=1}^{N} \frac{E_k}{\mathcal{N}_k} \frac{\hat{a}_k}{\hat{f}_k} F\widetilde{F} \implies \frac{\alpha_{em}}{8\pi} \frac{E}{\mathcal{N}} \frac{a_{G\tilde{G}}}{F} F\widetilde{F}$$

Making an axion-dependent rotation, $q = \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{i\gamma_5 a_{G\tilde{G}}/(2F) Q_a} \begin{pmatrix} u \\ d \end{pmatrix}$:

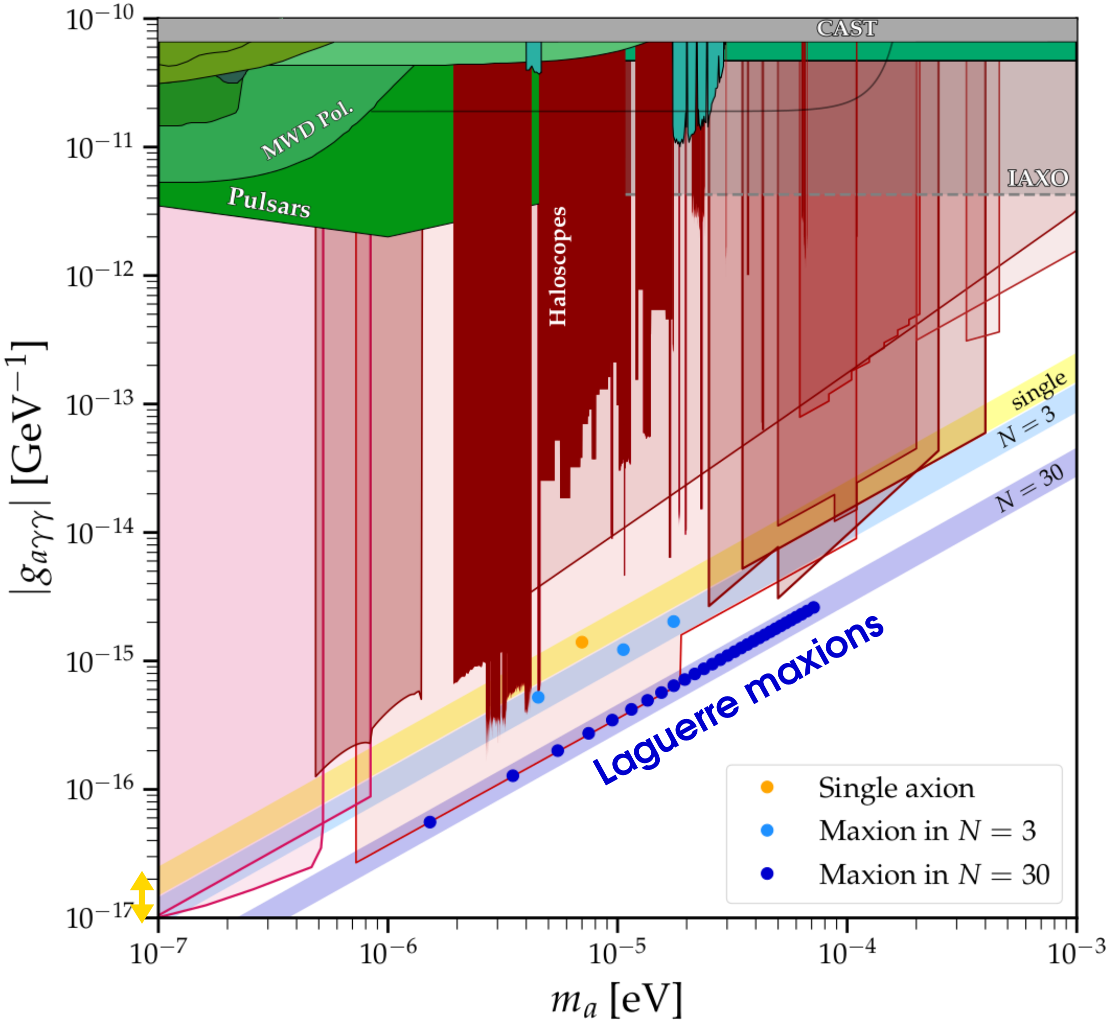
Di Cortona, Hardy, Vega, Villadoro 15

$$\mathcal{L} \supset \frac{\alpha_{em}}{2\pi} \left[\frac{E}{\mathcal{N}} - 1.92 \right] \sum_i \frac{a_i}{f_i} F\widetilde{F}$$

$$\boxed{\frac{m_i^2}{g_{a_i\gamma\gamma}^2} = \frac{m_a^2}{g_{a\gamma\gamma}^2}\Bigg|_{\text{single QCD axion}} \times g_i}$$

$$\frac{(2\pi)^2}{\alpha_{em}^2} \left[\frac{E}{\mathcal{N}} - 1.92 \right]^{-2} \sum_{i=1}^{N} \frac{g_{a_i\gamma\gamma}^2}{m_i^2} = 1$$

Coupling to photons

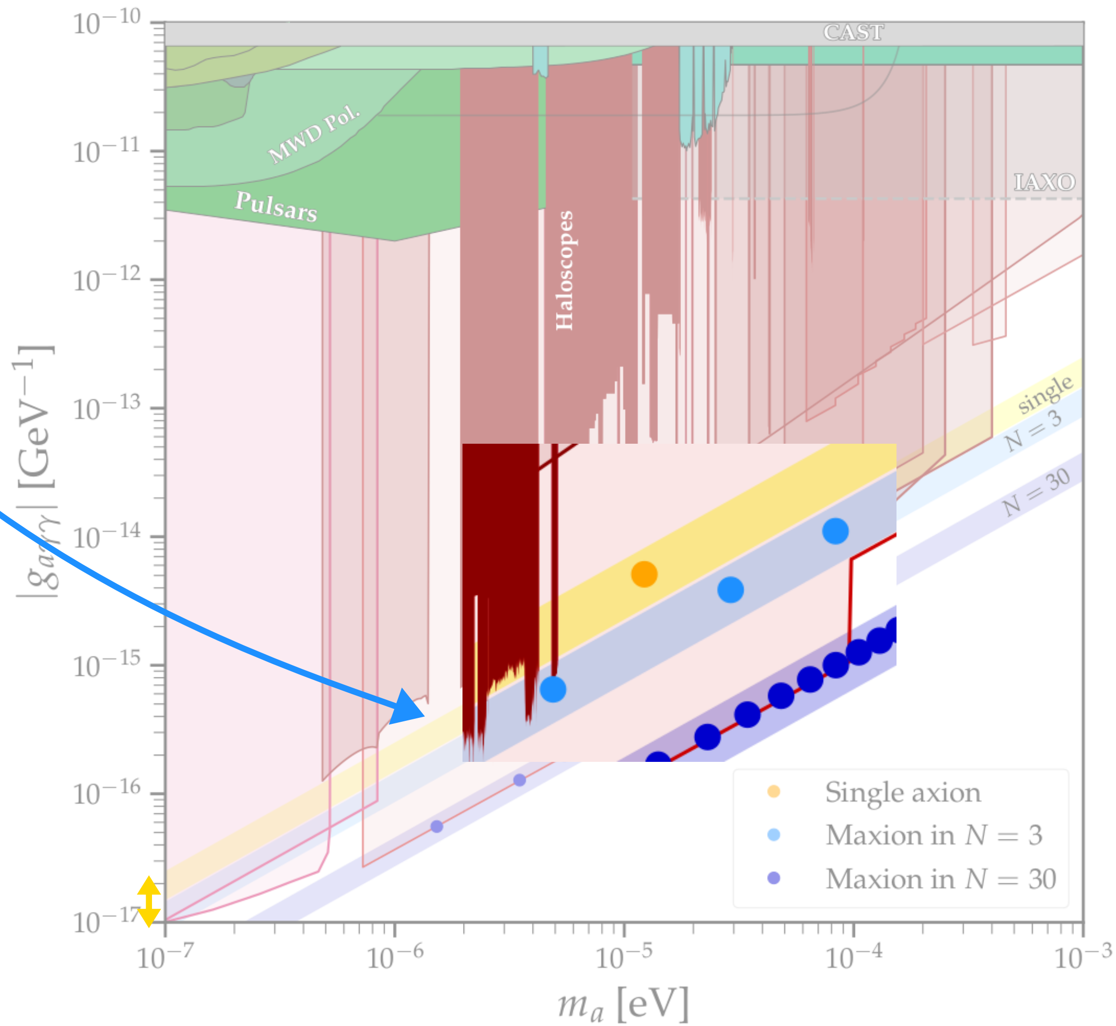


Band width spans:

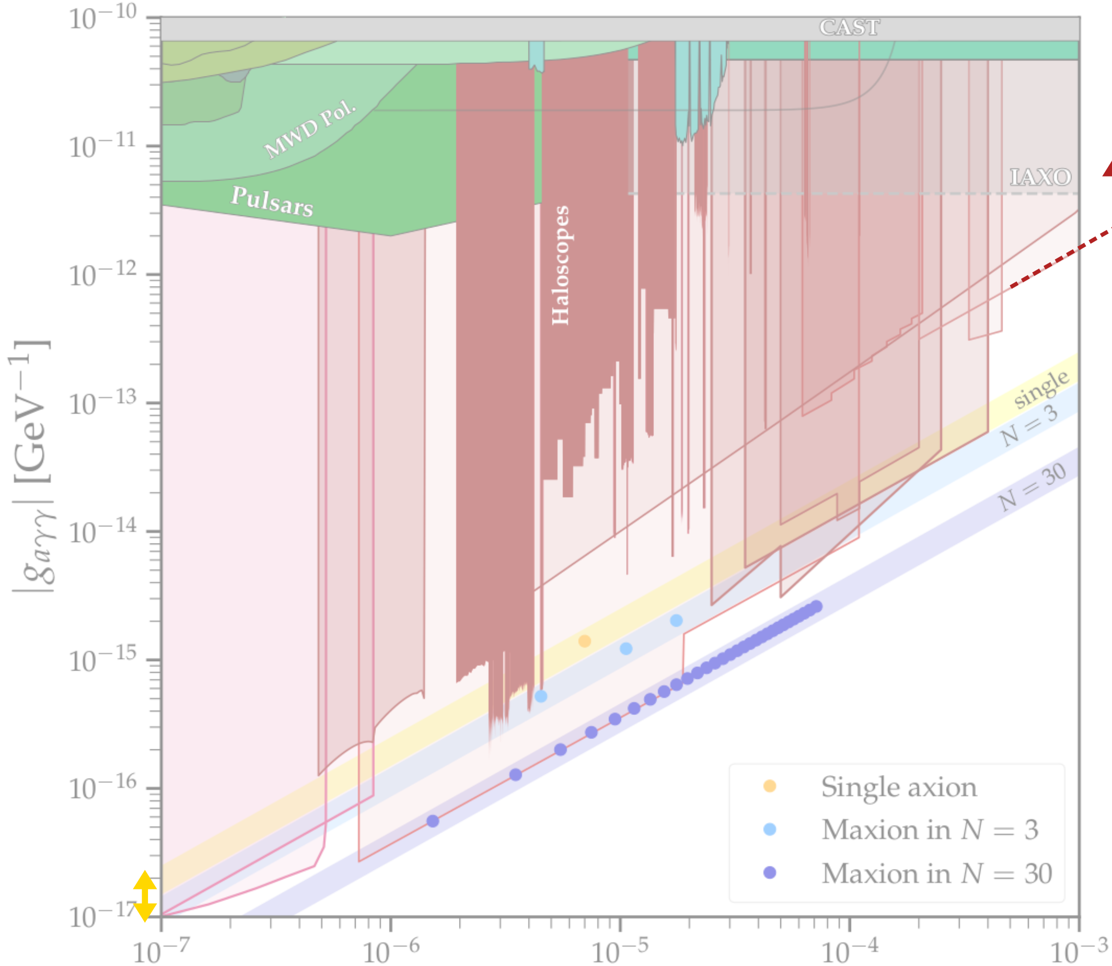
$$E/\mathcal{N} \in [2/3, 8/3]$$

Coupling to photons

Multiplicity of signals might be the smoking gun



Coupling to photons



$\sqrt{N}$ *if* $\rho_i = \rho_{\rm DM}/N$

How does each scalar contribute to the **dark matter** abundance?

WORK IN PROGRESS with David Dunsky, Claudio Manazari, Pablo Quílez, Philip Sorensen

Take home messages

1. Any signal to the right of the canonical axion band can indicate a multiple QCD axion solution to the strong CP problem!

2. Our sum rule links the possible mass-scale values of the different axions, and allows us to count how many axions may exist in Nature

3. All axions can be maximally deviated from the QCD line, by a factor of $\sqrt{N}$

4. The main experimental impact is from scales not far from the QCD contribution

5. The complete reconstruction of the multiple QCD axion may require a complementary search between different experiments

Exciting times ahead in the ALPs!

Thank you!

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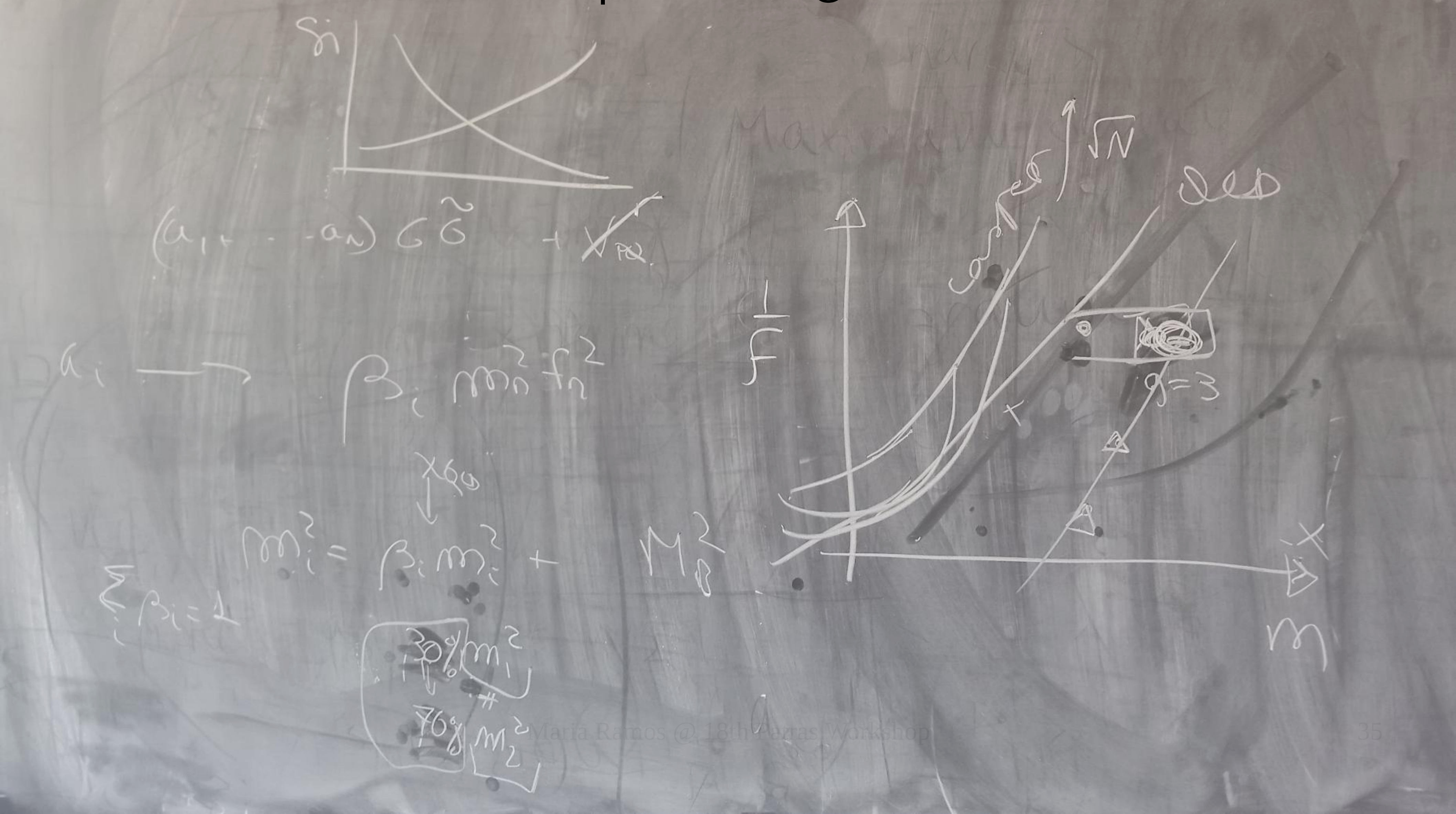

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Axion couplings

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{\mathrm{i}\gamma_5 \frac{a}{2f_a} Q_a} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \mathrm{Tr}(Q_a) = 1 \quad \Longrightarrow \quad M_a = e^{\mathrm{i}\frac{a}{2f_a}Q_a} M_q e^{\mathrm{i}\frac{a}{2f_a}Q_a}$$

$$\mathcal{L}_{p^2} \supset 2B_0 \frac{f_\pi^2}{4} U M_a^\dagger + \mathrm{h.c.} \quad U = e^{\mathrm{i}\Pi/f_\pi}$$

$$\Longrightarrow \quad V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{1}{2}\frac{a}{f_a}\right)}$$

Di Cortona, Hardy, Vega, Villadoro 15

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{\alpha_s}{8\pi}\frac{a}{f_a} G\widetilde{G} + \frac{1}{4} a g^0_{a\gamma\gamma} F\widetilde{F} + \frac{\partial_\mu a}{2f_a} j^\mu_{a,0} \to \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{4} a g_{a\gamma\gamma} F\widetilde{F} + \frac{\partial_\mu a}{2f_a} j^\mu_a$$

$$g_{a\gamma\gamma} = \frac{\alpha_{\mathrm{em}}}{2\pi f_a}\left[\frac{E}{\mathcal{N}} - 6\mathrm{Tr}(Q_a Q^2_{\mathrm{em}})\right], \quad j^\mu_a = j^\mu_{a,0} - \overline{q}\gamma^\mu\gamma^5 Q_a q\,, \quad Q_a = \frac{M_q^{-1}}{\langle M_q^{-1}\rangle}$$

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Toy model: minima conditions

$$\mathcal{L}_{N=2} = \frac{\alpha_s}{8\pi}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2} + \bar{\theta}\right) G\widetilde{G} - \frac{\mu^2}{2}\hat{a}_2^2$$

Below confinement:

$$V_{N=2} \supset \frac{\chi_{\rm QCD}}{2}\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right)^2 - V(\hat{a}_2)$$

Minima

$$\frac{\chi_{\rm QCD}}{\hat{f}_1}\sin\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right) = 0 \quad \text{and} \quad \frac{\chi_{\rm QCD}}{\hat{f}_2}\sin\underbrace{\left(\frac{\hat{a}_1}{\hat{f}_1} + \frac{\hat{a}_2}{\hat{f}_2}\right)}_{v_1+v_2=0} - \underbrace{\frac{\partial V(\hat{a}_2)}{\partial \hat{a}_2}}_{v_2=0} = 0$$

$$\mathbf{M^2} = \chi_{\rm QCD}\begin{pmatrix} 1/\hat{f}_1^2 & 1/(\hat{f}_1\hat{f}_2) \\ 1/(\hat{f}_1\hat{f}_2) & (1+r)/\hat{f}_2^2 \end{pmatrix}$$

$$r \equiv \mu^2\frac{\hat{f}_2^2}{\chi_{\rm QCD}}$$

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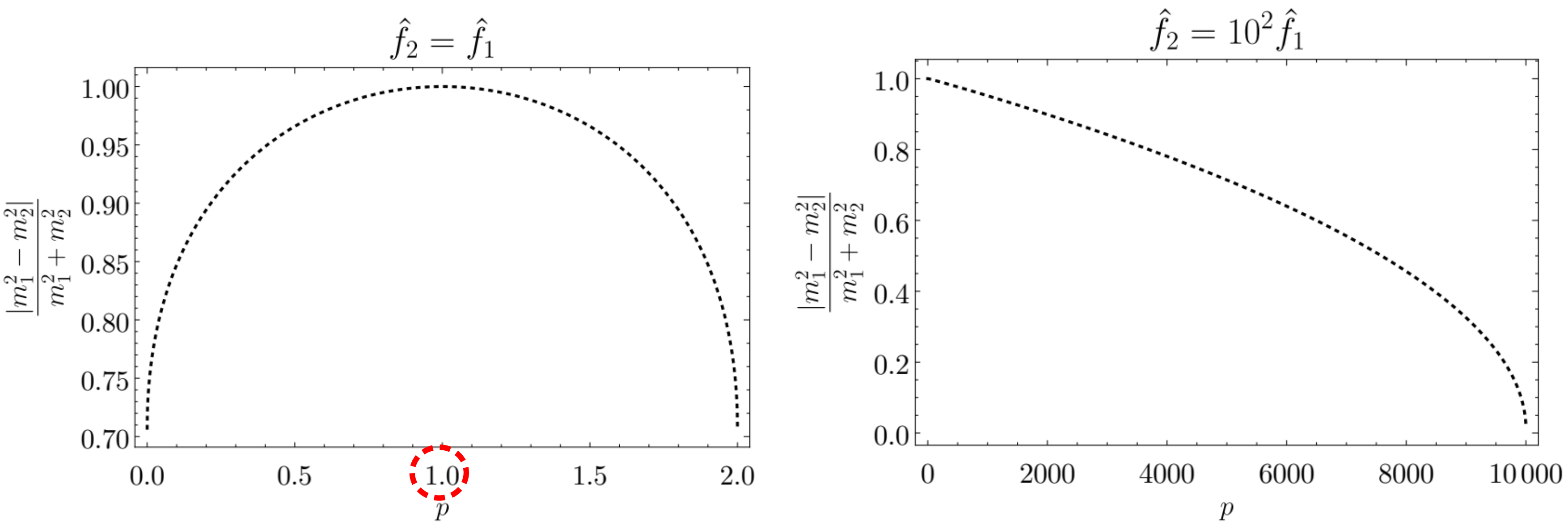
Eigenvalues dispersion

All families of maxions (with same scale) for N=2:

$$\mathbf{M}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2} \begin{pmatrix} 2-p & 1+\sqrt{p(2-p)} \\ 1+\sqrt{p(2-p)} & 1+p \end{pmatrix}$$



Limiting case: Massless state has no mixing with gluons, the heavy one with mass $\sim 4\frac{\chi_{\rm QCD}}{\hat{f}^2}$

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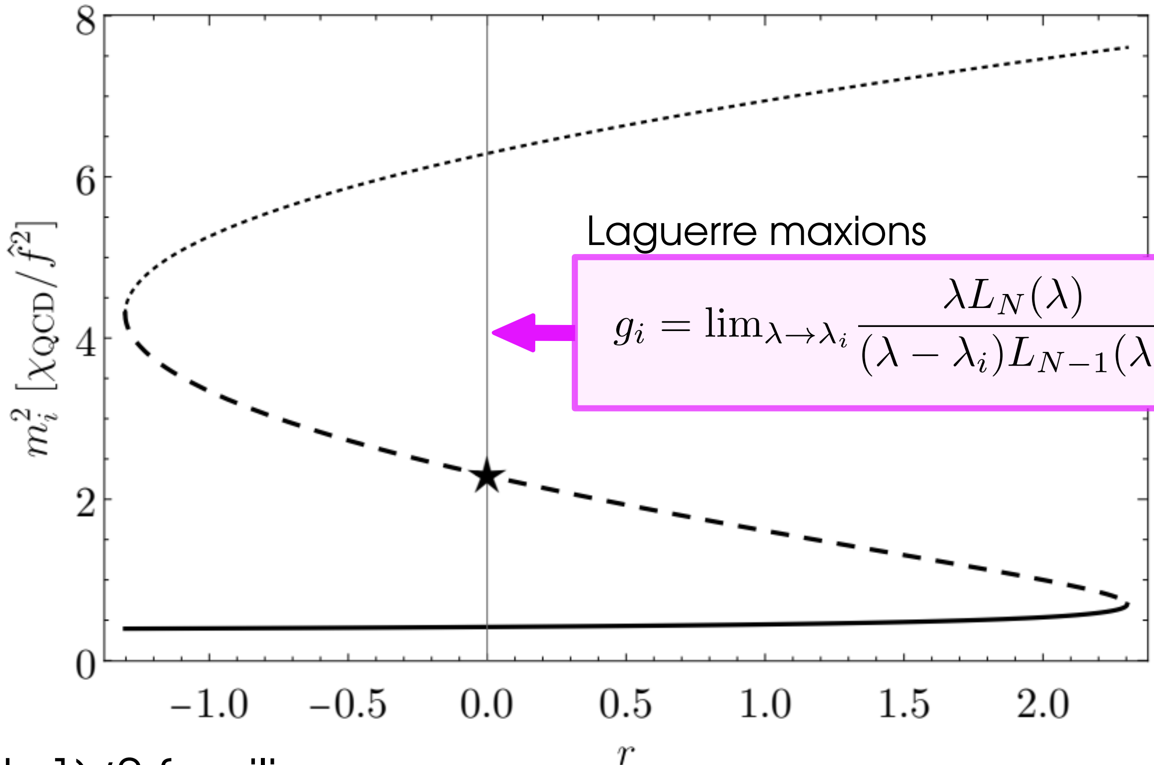
Eigenvalues dispersion

Example for N=3:

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4-\sqrt{3+r-r^2} & 1+r \\ 1 & 1+r & 4+\sqrt{3+r-r^2} \end{pmatrix}$$



Laguerre maxions

$$g_i = \lim_{\lambda\to\lambda_i} \frac{\lambda L_N(\lambda)}{(\lambda-\lambda_i)L_{N-1}(\lambda)} = N$$

In general, N(N+1)/2 families

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Potential scales

In the basis where the extra potential is diagonal, $\mathbf{M}_B^2 = \mathrm{diag}(\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N)$

$$g_i = \frac{m_i^2\, F^2}{\left|\langle a_{G\tilde{G}}|a_i\rangle\right|^2 \chi_{\rm QCD}} = \frac{m_i^2}{\left|\langle a_{\rm PQ}|a_i\rangle/f_{\rm PQ} + \sum_j^{N-1}\langle\tilde{a}_j|a_i\rangle/\tilde{f}_j\right|^2 \chi_{\rm QCD}}$$

For $\tilde{\lambda}_j \gg \chi_{\rm QCD}/F^2$:

$$\frac{1}{g_j} \sim \frac{\left|\langle a_{G\tilde{G}}|\tilde{a}_j\rangle\right|^2 \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} = \frac{(F/\tilde{f}_j)^2\, \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \leq \frac{\chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \longrightarrow 0$$

For $\tilde{\lambda}_j \ll \chi_{\rm QCD}/F^2$:

$$a_\varepsilon = \frac{a_{\rm PQ}}{f_{\rm PQ}} - \frac{\tilde{a}_j}{\tilde{f}_j} + \mathcal{O}(\varepsilon), \quad m_\epsilon^2 \sim \tilde{\lambda}_j = \varepsilon\, \chi_{\rm QCD}/F^2$$

$$\frac{1}{g_j} \sim \frac{\left|\langle a_{G\tilde{G}}|\tilde{a}_\varepsilon\rangle\right|^2 \chi_{\rm QCD}}{\tilde{\lambda}_j\, F^2} \sim \frac{\varepsilon^2}{\varepsilon} \longrightarrow 0$$

Whenever one scale is very different from the QCD induced mass, one state decouples.

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Clockwork scenario

Farina, Pappadopulo, Rompineve, Tesi 17

$$\hat{\mathbf{M}}^2 = \frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r\frac{\chi_{\rm QCD}}{\hat{f}^2}\begin{pmatrix} 1 & -q & 0 \\ -q & 1+q^2 & -q \\ 0 & -q & q^2 \end{pmatrix}$$

Correspondingly, $v_{j0} \propto \frac{1}{q^j}$ *leads* to decay constant exponentially enhanced

PQ: $\frac{\det \mathbf{M}^2}{\det \mathbf{M}_1^2} = \frac{\chi_{\rm QCD}}{F^2}$ ✓

Maxions:

$$\begin{cases} \mathrm{tr}\, \mathbf{M}^2 = N\frac{\chi_{\rm QCD}}{F^2} \Leftrightarrow r = \frac{1}{10} \\ \mathrm{tr}^2\, \mathbf{M}^2 - \mathrm{tr}\, \mathbf{M}^2 \cdot \mathbf{M}^2 = N\frac{\chi_{\rm QCD}}{F^2}\mathrm{tr}\, \mathbf{M}_1^2 \Leftrightarrow r = 0 \vee r = \frac{11}{182} \end{cases}$$

✗

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Dark matter abundance

in the presence of mixing

Introducing thermal effects:

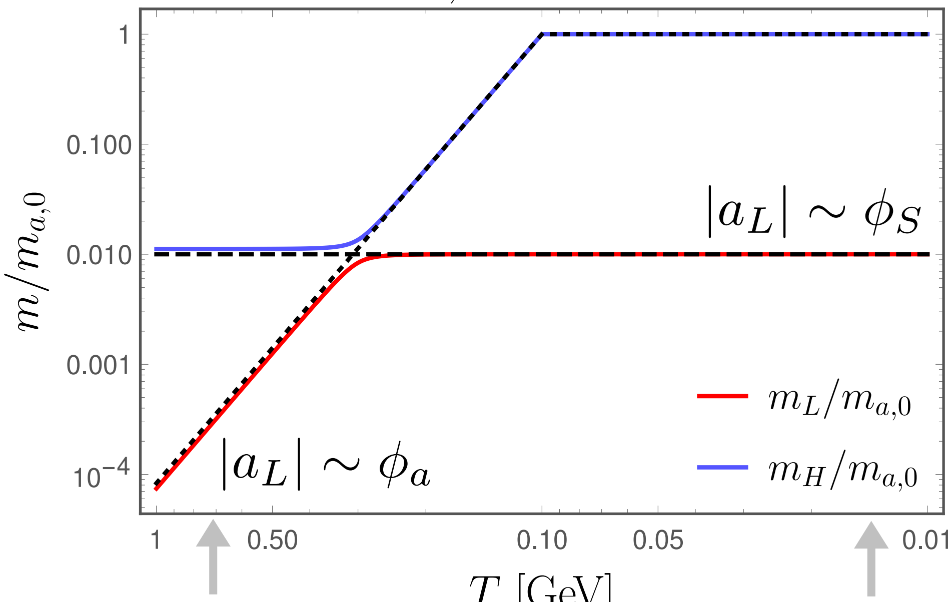
e.g. N. Kitajima, F. Takahashi 14

$$V(\phi_a, \phi_S) = m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi_a}{f_a}\right)\right] + m_S^2 f_S^2 \left[1 - \cos\left(\frac{\phi_S}{f_S} + \frac{\phi_a}{f_a}\right)\right]$$

$$m_a^2 = m_{a,0}^2 \max\left\{1, \left(\frac{T}{T_{\rm QCD}}\right)^{-n}\right\}$$

$$\mathcal{M}^2(T) = \begin{pmatrix} m_S^2 & m_S^2 R_f \\ m_S^2 R_f & m_{a,0}^2 + m_S^2 R_f^2 \end{pmatrix}$$

$$R_m \equiv \frac{m_S}{m_{a,0}} = 10^{-2}\,,\ R_f \equiv \frac{f_S}{f_a} = \frac{1}{2}$$



$\rho \propto n_L m_a$ $\rho \propto n_L m_S$

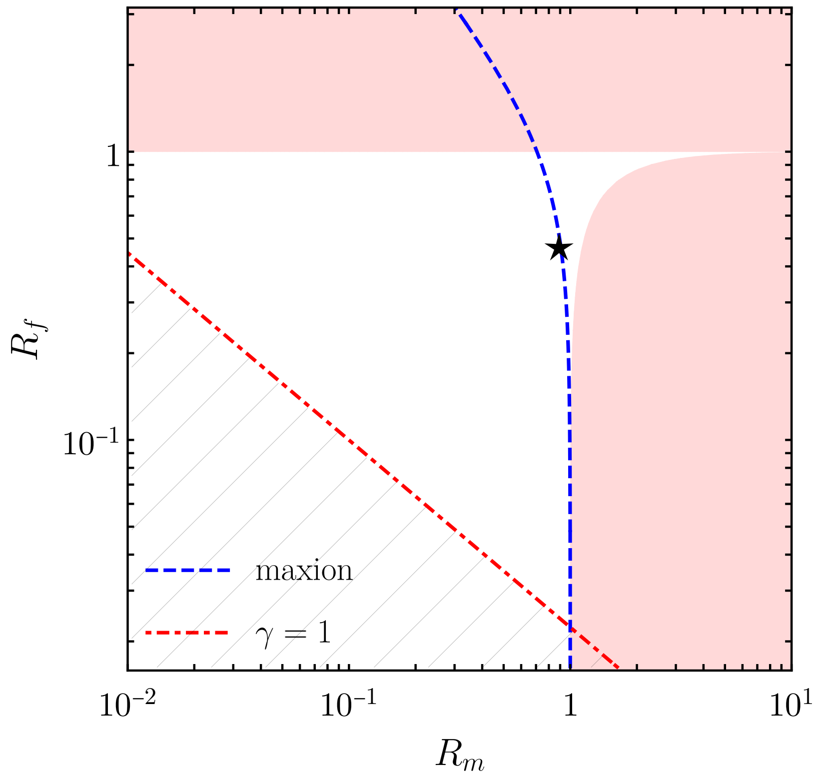
The prediction for the dark matter abundance can be significantly altered!

backup

Dark matter abundance

in the presence of mixing

Regions for level crossing



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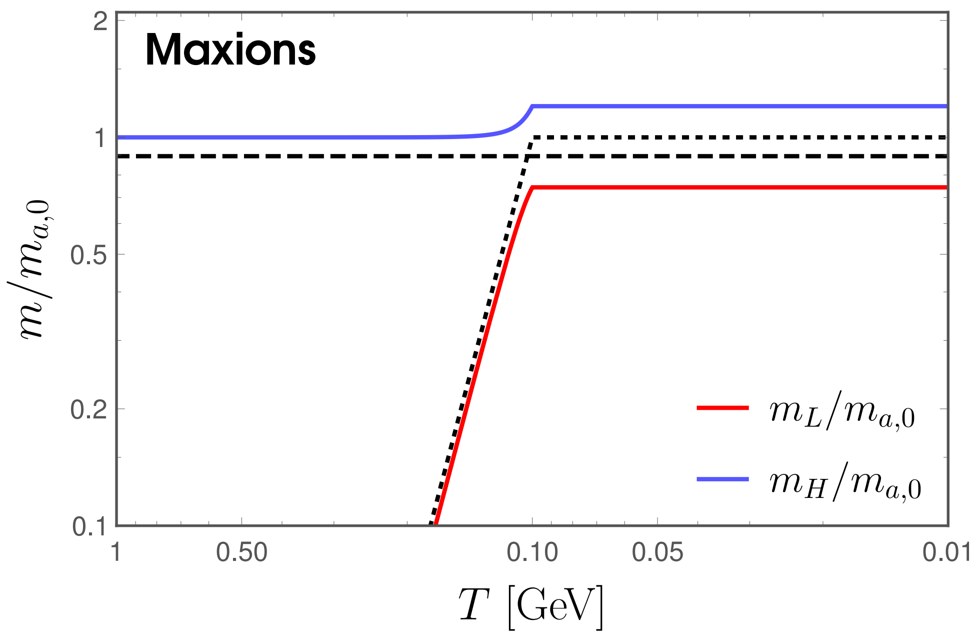

Goal is to also obtain analytical results for non-adiabatic transitions.