Small Kinetic Mixing in String Theory

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based on arXiv:2311.10817 with A. Hebecker & J. Jaeckel

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Outline

Punshline: Stringy brane world scenarios can realise small kinetic mixing

1. Motivation & Introduction
2. String Theory Perspective
3. D3-D3-Brane Kinetic Mixing
4. Summary & Outlook

Outline

Motivation & Introduction

- Gauge kinetic mixing (KM) represents one “portal” to a dark sector
- The KM operator mixes two different $U(1)$ gauge theories (say $U(1)_A$ and $U(1)_B$) [Okun, 1982, Holdom, 1986]

$$\mathcal{L} \supset -\frac{\chi_{AB}}{2} F_{(A)}^{\mu\nu} F_{\mu\nu}^{(B)}$$

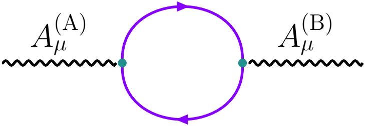
- Hidden photon could be massive and represent DM or mediate interaction to DM candidate
- KM is intensively researched and has to satisfy strong bounds:

$$\chi_{AB} < 10^{-17} - 10^{-5}$$

see e.g. [FIPs Report 2022]

Motivation & Introduction - II

- χ_{AB} can be generated by a heavy particle Φ running in a loop



$$= \frac{q_\mathrm{A} q_\mathrm{B}}{16\pi^2} \ln\left(\frac{\Lambda^2}{m_\Phi^2}\right) := \chi_\mathrm{AB}$$

$A_\mu^{(\mathrm{A})}$: visible photon, $A_\mu^{(\mathrm{B})}$: hidden photon

- Is there a UV motivation for these states?
- Pure loop suppression is insufficient

  $\Rightarrow$ Ideally, a (top down) mechanism should account for $\chi \ll 1$

Outline

Basics of String Phenomenology - Bulk Theory

- Focus of this talk: type IIB superstring
- At low energies IIB is described by a 10d sugra theory, "bulk theory"

$$S_{IIB} \sim \int_{\mathcal{M}^{10}} e^{-2\phi}\left(R + (\partial\phi)^2 + |\mathrm{d}B_2|^2\right) - \sum_{p=0,2,4} |\mathrm{d}C_p|^2 + C_4 \wedge \mathrm{d}B_2 \wedge \mathrm{d}C_2$$

- 6 extra dimensions are Kaluza-Klein (KK) compactified

$\mathcal{M}^{10} =$

$\mathcal{M}^{1,3}$

$\times$

$\mathcal{X}^6$

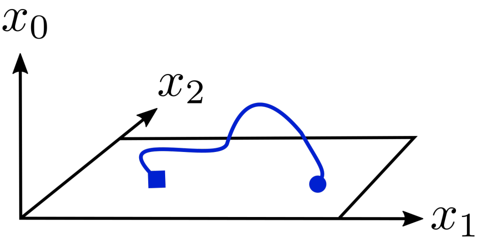
Basics of String Phenomenology - Bulk Theory - II

- Every 10d field is replaced by tower of ∞-many KK-modes
- Compactified 4d theory is dual to the 10d theory
  $\Rightarrow$ BUT turn to 4d EFT of the massless KK-modes, $m_{KK} \sim R^{-1}$
- Geometry of $\mathcal{X}^6$ is encoded in 4d massless scalar moduli fields
  $\Rightarrow$ Require "moduli stabilisation":
  - Flux-compactification, background of $\mathrm{d}B_2$, $\mathrm{d}C_2$ [Giddings,Kachru,Polchinski 2002]

  then:
  - KKLT-proposal [Kachru,Kallosh,Linde,Trivedi 2003]
  - Large Volume Scenario [Balasubramanian, Conlon, Quevedo 2005]

Basics of String Phenomenology - D-Branes

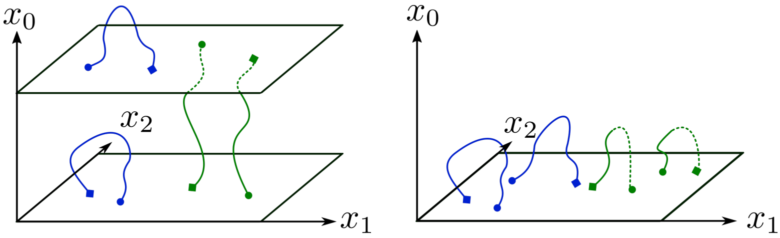
- D-branes are hypersurfaces where open strings can end



- D-brane are independent extended objects of string theory
- D-branes carry gauge theories from open string excitations

Basics of String Phenomenology - D-Branes II

- single D-brane: U(1) theory, stack of N D-branes: U(N) theory

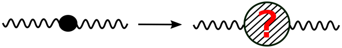


- D-branes also interact with closed string modes of bulk theory
- D-branes can be taken into account by adding a brane action!

$$S_{Dp} \sim \int_{D_p} e^{-\Phi}\sqrt{-\det\left(g_{ab} - B_{ab} + F_{ab}\right)} + \exp\left(F_2 - B_2\right) \wedge \sum_q C_q$$

Stringy Approach to KM

[Dienes et al., 1997]
[Abel and Schofield, 2004]
[Abel et al., 2008]

- In principle clear what to do: We have UV theory, so what is hidden in the KM vertex? 

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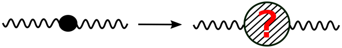
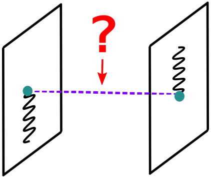
- In principle clear what to do: We have UV theory, so what is hidden in the KM vertex? 

- Gauge theories live on D-branes $\curvearrowright$ separated surfaces in 10d

$\Rightarrow$ KM appears only in 4d!



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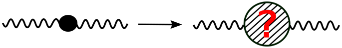
- In principle clear what to do: We have UV theory, so what is hidden in the KM vertex? 

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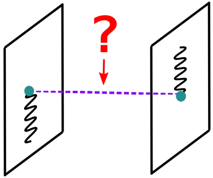

- To communicate D-branes need to exchange bulk fields $\Leftrightarrow$?

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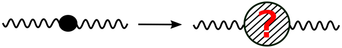
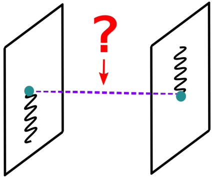
- In principle clear what to do: We have UV theory, so what is hidden in the KM vertex? 
- Gauge theories live on D-branes ↷ separated surfaces in 10d

⇒ KM appears only in 4d!



- To communicate D-branes need to exchange bulk fields ⇔ ?
- We can start top down in 10d supergravity + D-branes
  1. compactify to 4d, integrate out heavy KK modes
  2. compute exchange diagram via Greens functions in 10d
- Suppression from propagation over large distance

Outline

Extracting Kinetic Mixing

[Abel et al., 2008]

- Simple setup: 2 × D3-branes ⇒ What are the coupling terms?

$$S^{(\mathrm{i})}_{\mathrm{DBI}} \supset -\int_{\mathcal{M}^{1,3}} \mathrm{d}^4x \; \frac{1}{4g_\mathrm{s}} \left(F^{(i)}_{\mu\nu} F^{\mu\nu}_{(i)} + 2F^{(i)}_{\mu\nu} B^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right)$$

$$S^{(\mathrm{i})}_{\mathrm{CS}} \supset \int_{\mathcal{M}^{1,3}} \mathrm{d}^4x \left[\frac{1}{4} \left(F^{(i)} + B \right)_{\mu\nu} \; C_{\rho\sigma} + \frac{C_0}{2 \cdot 4} \left(F^{(i)} + B \right)^2_{\mu\nu\rho\sigma} \right] \epsilon^{\mu\nu\rho\sigma}$$

Extracting Kinetic Mixing

[Abel et al., 2008]

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$$S_{\mathrm{DBI}}^{(\mathrm{i})} \supset - \int_{\mathcal{M}^{1,3}} \mathrm{d}^4 x \, \frac{1}{4 g_s} \left(F_{\mu\nu}^{(i)} F_{(i)}^{\mu\nu} + 2 F_{\mu\nu}^{(i)} B^{\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right)$$

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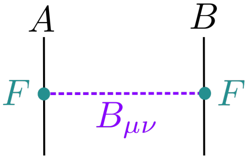
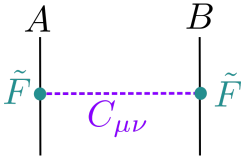
- Need to focus on $C_{\mu\nu}$ and $B_{\mu\nu}$!
- Have to compute diagrams like:





Extracting Kinetic Mixing - II

[Abel et al., 2008]

- To compute diagrams we need the propagators Δ_B^{-1} and Δ_C^{-1}

$$S_{IIB} = -\frac{1}{2}\int_{\mathcal{M}_{10}} \mathrm{d}^{10}x \left[B_{mn}(g_s^{-2} + C_0^2)\Box\, B^{mn} + C_{mn}\ \Box\ C^{mn} - B_{mn}\ 2C_0\Box\ C^{mn}\right]$$

Hence the propagators are given by $(C_0 = 0)$

$$\Delta_B^{-1} = g_s^2\Box^{-1}\ , \quad \Delta_C^{-1} = \Box^{-1}$$

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Hence the propagators are given by $(C_0 = 0)$

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- This gives naive result for KM:

$$\left.\begin{array}{ll} B_{\mu\nu}) & \left(\frac{1}{2\mathrm{g_s}}F^{(A)}_{\mu\nu}\right)\mathrm{g_s}^2\Box^{-1}(y_A,y_B)\left(\frac{1}{2\mathrm{g_s}}F^{\mu\nu}_{(B)}\right) \\ C_{\mu\nu}) & \left(\frac{\epsilon^{\mu\nu\alpha\beta}}{4}F^{(A)}_{\alpha\beta}\right)\Box^{-1}(y_A,y_B)\left(\frac{\epsilon_{\mu\nu}{}^{\rho\sigma}}{4}F^{(B)}_{\rho\sigma}\right)\end{array}\right\}\sum = F^{(A)}_{\mu\nu}\Box^{-1}F^{\mu\nu}_{(B)}\left(\frac{1}{4}-\frac{1}{4}\right) = 0$$

What should we expect?

- Full string theory diagram gives same result, but only calculable with very simple internal spaces [Dienes et al., 1997, Abel and Schofield, 2004]

- We didn't specify any internal space, so to find this result is very good for our EFT (good approximation)
  & interesting for pheno since we can expect small KM!

- BUT we used very crude simplifications!

  $\Rightarrow$ What happens when all generalizations are taken into account?

- Also, have $SL(2,\mathbb{R})$ doublet $\begin{pmatrix} C_{\mu\nu} \\ B_{\mu\nu} \end{pmatrix}$, should take this into account.

Full Treatment

- Include all terms of action:

$$S_{D3} = \int_{\mathcal{M}^{1,3}} \mathrm{d}^4 x \begin{pmatrix} C_{\mu\nu} \\ B_{\mu\nu} \end{pmatrix}^T \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & g_s^{-1} + C_0 \end{pmatrix} \begin{pmatrix} C^{\mu\nu} \\ B^{\mu\nu} \end{pmatrix} + \begin{pmatrix} -\tilde{F}_{\mu\nu} \\ g_s^{-1} F_{\mu\nu} + C_0 \tilde{F}_{\mu\nu} \end{pmatrix}^T \begin{pmatrix} C^{\mu\nu} \\ B^{\mu\nu} \end{pmatrix}$$

$$S_{IIB} = \int_{\mathcal{M}^{10}} \mathrm{d}^{10} x \begin{pmatrix} C_{\mu\nu} \\ B_{\mu\nu} \end{pmatrix}^T \begin{pmatrix} 1 & -C_0 \\ -C_0 & g_s^{-2} + C_0^2 \end{pmatrix} \Box \begin{pmatrix} C^{\mu\nu} \\ B^{\mu\nu} \end{pmatrix}$$

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- Get a new vertex on the branes...

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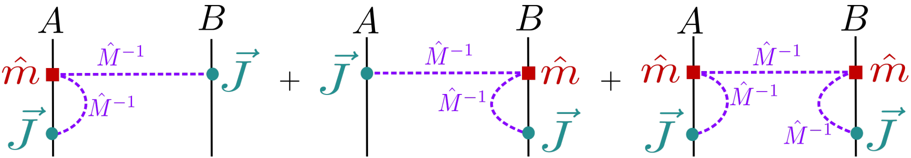
- Get a new vertex on the branes...

- ... a corrected propagator ... generalized sources ...

$$\Delta^{-1} = \hat{M}^{-1}\Box^{-1}\,, \qquad \vec{J} = \begin{pmatrix} -\tilde{F}_{\mu\nu} \\ g_s^{-1}F_{\mu\nu} + C_0\tilde{F}_{\mu\nu} \end{pmatrix}$$

Full Treatment - II

- … and a bunch of new diagrams



- Sum of all diagrams yields again zero, to all orders of perturbation theory

- To show this it is crucial to exploit $SL(2,\mathbb{R})$ structure of the theory

Modifications to yield KM

- We assumed $g_s = \text{const.}$ & $C_0 = \text{const.}$
- This is only valid in flux compactifications, g_s and C_0 are actually dynamical moduli fields and get fixed by fluxes

$$\text{Flux compactification} \Leftrightarrow \text{background for} \begin{array}{l} \bar{H}_{\mu\nu\rho} = \partial_{[\mu}\bar{B}_{\nu\rho]} \\ \bar{F}_{\mu\nu\rho} = \partial_{[\mu}\bar{C}_{\nu\rho]} \end{array}$$

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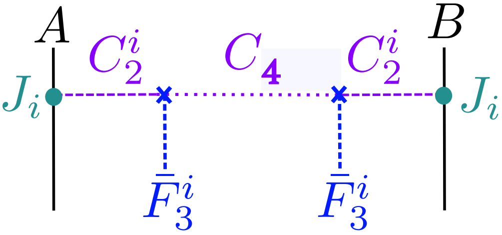
- This leads to even more vertices, breaks $SL(2,\mathbb{R})$ and yields KM
- The leading diagram is



Estimates of KM

Computing diagrams and extracting KM yields for χ:

$$\chi = \int_{y',y''} \left(\frac{-1}{4!} \Delta_6^{-1}(y_A, y') \; F^i(y')_{[abc} \; \partial_{d]}^{(y')} \; \left[\Delta_6^{-1}(y', y'')\right] \right.$$

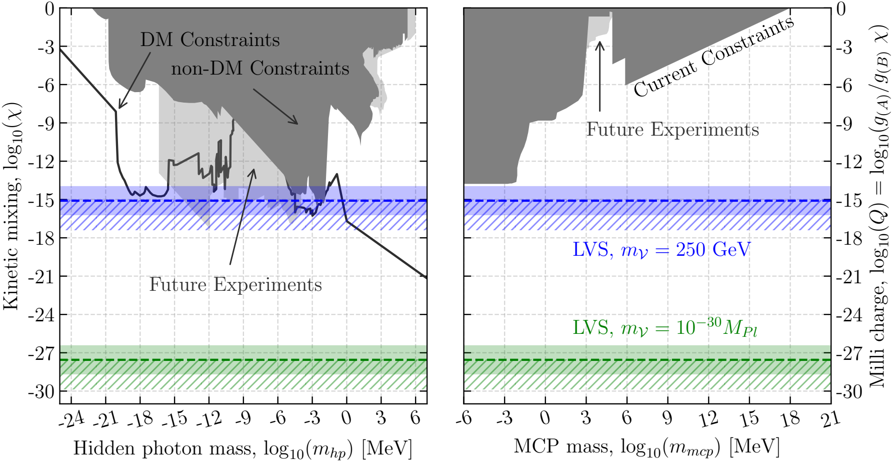
$$\left. \times F^j(y'')^{abc} [\Delta_6^{-1}(y'', y_B)] \right) + (A \leftrightarrow B)$$

Parametrically χ scales with the volume $\mathcal{V}$ of $\mathcal{X}$ (in string units):

$$\chi \sim \frac{1}{\mathcal{V}^{4/3}}$$

$\Rightarrow$ This is obviously constrained!

Estimates of KM



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Summary & Outlook

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- Parametric estimate: $\chi \gtrsim 10^{-15}$
- Open issue with 4d sugra embedding
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Thank you!

Backup - Small Gauge Couplings

Weak Gravity Conjecture: $g \geq \Lambda / M_{\text{Pl}}$ [Arkani-Hamed,Motl,Nicolis,Vafa 2007]