Axion landscapes in string theory

Oliver Janssen, EPFL

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based on [2309.01831] and earlier work with
T. Bachlechner, K. Eckerle
N. Gendler, M. Kleban
J. La Madrid and V. M. Mehta

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The CC problem, the landscape, and axions

The axion could serve an even grander purpose than solving the strong CP problem via the PQ mechanism, including solving open problems in cosmology: it could be the dark matter, the inflaton, and might play an important role in solving the cosmological constant problem

Weinberg ['87] opened the door to an *anthropic* solution to the CC problem: assuming $\delta\rho/\rho \sim 10^{-5}$ in the early universe, he showed $\Lambda \lesssim 10^{-122} M_{\rm Pl}^4$ (provided $\Lambda > 0$)

A full solution would allow Λ to vary, and be $\sim$ uniformly distributed for small values. The anthropic argument would then predict $\Lambda \approx \Lambda_{\rm obs.}$

The string theory landscape is a terrain where Λ varies [BP '00, DD '04], with its vacua populated by first order phase transitions in eternal inflation

$\boxed{\text{String theory axions could provide an anthropic solution to the CC problem}}$

The Kreuzer-Skarke axiverse

We studied axion potentials in a corner of the string landscape, namely type IIB string theory compactified on (orientifolds of) CY 3-folds. The amount of axions in the 4D EFT is equal to $h^{1,1} \equiv N$, which ranges from 1 to 491 in our ensemble. Our CYs are derived from the Kreuzer-Skarke database of 4D reflexive polytopes [1808.01282]

Axion potentials in string theory

The axion potential is determined from geometric data of the CY via

$$
\begin{aligned}
V(\boldsymbol{\theta}) &= e^{\mathcal{K}} \left(K^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2 \right) \\
&\approx -\frac{8\pi}{\mathcal{V}^2} \sum_{\alpha} (\boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau})\, e^{-2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau}} \cos\left(2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\theta} + \delta_\alpha \right)
\end{aligned}
$$

Here

- $\boldsymbol{q}^{(\alpha)}$: instanton "charges", from E-$D3$ branes wrapping *holomorphic* 4-cycles of the CY
- $\boldsymbol{\tau}$: saxions = volumes of the 4-cycles, $\tau^i = (1/2) \int_{D_i} J \wedge J$, large in string units ($\ell_s = 1$ here). *Assumed* to be stabilized
- δ's: phases coming from phases in $W_0, A_\alpha \in \mathbb{C}$, chosen randomly, also *assumed* $|W_0| \sim |A_\alpha| = \mathcal{O}(1)$

In general determining the possible $\boldsymbol{q}^{(\alpha)}$ is difficult, but for particular CYs (constructed as hypersurfaces in toric varieties) some are guaranteed to exist, namely the charges associated with *prime toric divisors* (they are inherited from the ambient variety). If the CY is furthermore "favorable", there are $h^{1,1}+4$ of these. There may be others ("autochthonous divisors") not inherited from the ambient space; we will neglect them [2107.09064]

Approximation to "effective cone" $\mathcal{E}$ containing the charges that contribute: non-negative integer linear combinations of the following generators (themselves integer-valued)

$$\boldsymbol{p}^{(a)} = \begin{pmatrix} \mathbb{1}_N \\ \boldsymbol{q}^{(1)\top} \\ \boldsymbol{q}^{(2)\top} \\ \boldsymbol{q}^{(3)\top} \\ \boldsymbol{q}^{(4)\top} \end{pmatrix}, \quad \boldsymbol{q}^{(\alpha)} = \sum_{a=1}^{N+4} n_a^{(\alpha)} \boldsymbol{p}^{(a)}, \quad n_a^{(\alpha)} \in \mathbb{Z}_{\geq 0}$$

In a large set of CYs ($\sim 400,000$) derived from the Kreuzer-Skarke list, we used `CYTools` [cy.tools, 2211.03823] to efficiently compute the $\boldsymbol{\tau}$ and $\boldsymbol{q}^{(\alpha)}$

Our question: how many (distinct) minima does the axion potential

$$V(\boldsymbol{\theta}) = -\frac{8\pi}{\mathcal{V}^2}\sum_{\alpha=1}^{\infty}(\boldsymbol{q}^{(\alpha)}\cdot\boldsymbol{\tau})\,e^{-2\pi\boldsymbol{q}^{(\alpha)}\cdot\boldsymbol{\tau}}\cos\left(2\pi\boldsymbol{q}^{(\alpha)}\cdot\boldsymbol{\theta}+\delta_\alpha\right)$$

have in each realization? Is the structure rich enough to anthropically solve the CC problem?

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(No.)

Two toy examples

(1) Consider

$$V(\boldsymbol{\theta}) = \sum_{\alpha=1}^{N} \Lambda_\alpha \cos\left(2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\theta} + \delta_\alpha\right)$$

for *any* $\{\Lambda_\alpha, \delta_\alpha\}$ and (full rank) matrix $\boldsymbol{Q} = (\boldsymbol{q}^{(\alpha)})$

Claim: this potential has a single distinct minimum

Argument: perform the (invertible) coordinate transformation $\boldsymbol{Q}\boldsymbol{\theta} = \boldsymbol{\theta}'$, in terms of which the potential reads

$$V(\boldsymbol{\theta}') = \sum_{\alpha=1}^{N} \Lambda_\alpha \cos\left(2\pi \theta'_\alpha + \delta_\alpha\right) \, .$$

There is 1 minimum in the fundamental domain $[0, 1)^N$

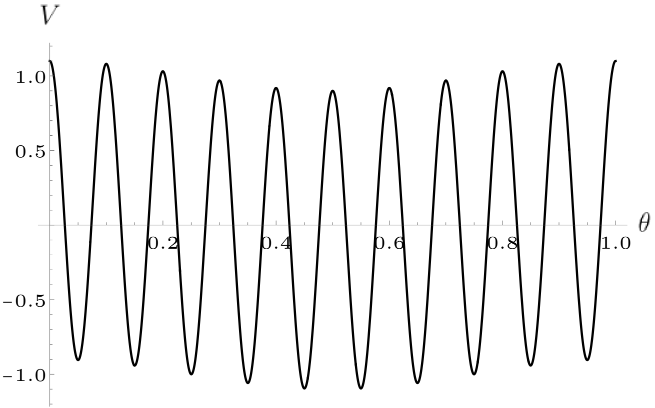
(1.5) Consider

$$V(\theta) = \cos(2\pi \cdot 10\theta) + \varepsilon \cos(2\pi\theta)$$

with $0 < \varepsilon \ll 1$

Claim: this potential has 10 distinct minima

Argument: before adding the correction, there was a single distinct minimum in the fundamental domain $[0, 1/10)$. With the correction, the fundamental domain is enlarged to $[0, 1)$, and what used to be exact copies of minima are now good approximations to new minima as long as we remain in $[0, 1)$


V
1.0
0.5
−0.5
−1.0
0.2
0.4
0.6
0.8
1.0
θ

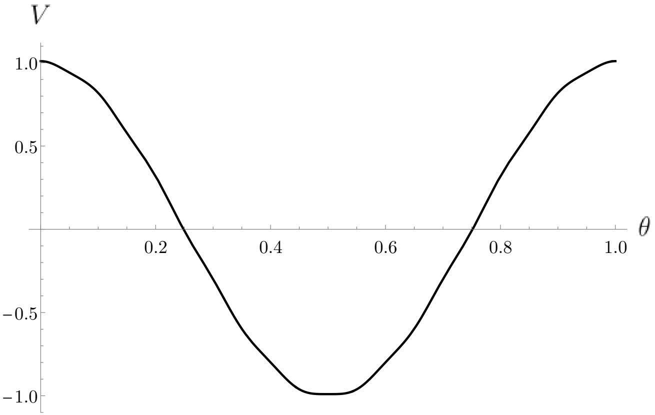
(1.75) Consider

$$V(\theta) = \cos(2\pi\theta) + \varepsilon \cos(2\pi \cdot 10\theta)$$

with $0 < \varepsilon \ll 1$

Claim: this potential has 1 distinct minimum

Argument: the fundamental domain is $[0, 1)$ both before and after adding the correction. The small amplitude, larger frequency correction is not sufficient to create new minima in $[0, 1)$


V
1.0
0.5
−0.5
−1.0
θ
0.2
0.4
0.6
0.8
1.0

(2) Consider

$$V(\boldsymbol{\theta}) = \sum_{\alpha=1}^{N} \Lambda_\alpha \cos\left(2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\theta} + \delta_\alpha\right) + \sum_{\beta=1}^{M} \tilde{\Lambda}_\beta \cos\left(2\pi \tilde{\boldsymbol{q}}^{(\beta)} \cdot \boldsymbol{\theta} + \tilde{\delta}_\beta\right)$$

for any $\{\Lambda_\alpha, \delta_\alpha, \tilde{\delta}_\beta\}$, full rank $\boldsymbol{Q} = (\boldsymbol{q}^{(\alpha)})$, $\tilde{\boldsymbol{q}}^{(\beta)} = \mathcal{O}(\boldsymbol{q}^{(\alpha)})$ and $\Lambda_\alpha \ggg \tilde{\Lambda}_\beta$

Claim: this potential has $|\det \boldsymbol{Q}|$ minima

Argument: a basis for the fundamental domain of the leading piece are the columns of $\boldsymbol{Q}^{-1}$. This parallellepiped has volume $1/|\det \boldsymbol{Q}|$, which is smaller than the volume of the full fundamental domain $[0,1)^N$ by a factor $|\det \boldsymbol{Q}|$. In the full potential there are exactly this many approximate copies of the original unique minimum of the truncated potential

KS axion potentials have $\mathcal{O}(1)$ minima

Now consider

$$V(\boldsymbol{\theta}) = \sum_{\alpha=1}^{\infty} (\boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau})\, e^{-2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau}} \cos\left(2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\theta} + \delta_\alpha\right)$$

where $\boldsymbol{q}^{(\alpha)}$ are non-negative integer linear combinations of

$$\boldsymbol{p}^{(a)} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}, \boldsymbol{q}^{(1)}, \boldsymbol{q}^{(2)}, \boldsymbol{q}^{(3)}, \boldsymbol{q}^{(4)} \right\}$$

Claim: this potential has $|\det \boldsymbol{Q}_{\text{red.}}| = \mathcal{O}(1)$ minima

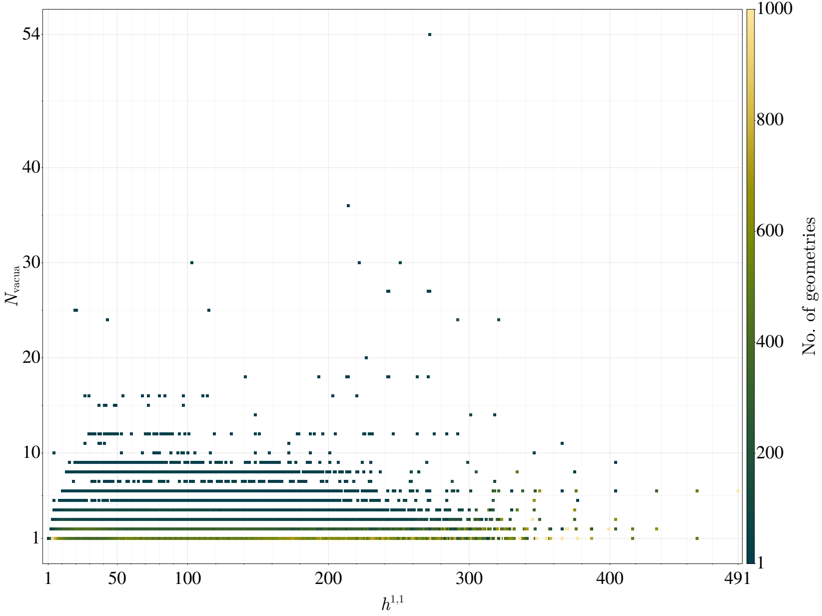
$$V(\boldsymbol{\theta}) = \sum_{\alpha=1}^{\infty} (\boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau})\, e^{-2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\tau}} \cos\left(2\pi \boldsymbol{q}^{(\alpha)} \cdot \boldsymbol{\theta} + \delta_\alpha\right)$$

Argument: we may define a "reduced" or truncated potential

$$V_{\text{red.}}(\boldsymbol{\theta}) = \sum_{\gamma=1}^{N} (\boldsymbol{q}^{(\gamma)} \cdot \boldsymbol{\tau})\, e^{-2\pi \boldsymbol{q}^{(\gamma)} \cdot \boldsymbol{\tau}} \cos\left(2\pi \boldsymbol{q}^{(\gamma)} \cdot \boldsymbol{\theta} + \delta_\gamma\right)$$

where the $\{\boldsymbol{q}^{(\gamma)}\} \subset \{\boldsymbol{q}^{(\alpha)}\}$ are chosen according to an iterative procedure [2309.01831]. Since all $\tau^i \gg 1$ to have control of the instanton expansion, the (infinitely many) terms we have neglected in V are very small and can only lift degeneracies of copies of the single minimum of $V_{\text{red.}}$. The amount of distinct minima is exactly $|\det \boldsymbol{Q}_{\text{red.}}| = \mathcal{O}(1)$ because $\boldsymbol{Q}_{\text{red.}}$ is sparse

Results



54
40
30
20
10
1
N_{vacua}
1
50
100
200
300
400
491
$h^{1,1}$
1000
800
600
400
200
1
No. of geometries

Conclusions

- We showed that in a large class of controlled axion EFTs descending from string theory, each axion potential has a simple structure with an $\mathcal{O}(1)$ amount of minima. This should be contrasted with random axion landscapes where all entries in $\boldsymbol{Q}$ are $\mathcal{O}(1)$: then $\mathcal{N}_{\text{vac}} \sim |\det \boldsymbol{Q}| \sim \sqrt{N!} \approx 10^{555}$ for $N = 491$
- To potentially get a rich structure in individual axion potentials we require tools that can go beyond the controlled limit we have taken (e.g. large non-perturbative corrections to $\mathcal{K}$, large corrections in the α' expansion)
- Each axion potential may be simple, but there are *many*: one may change the CY topology (in or out of the KS list), or for fixed geometry change the flux. Transitions between topologies or flux configurations may populate a sufficiently complex landscape that solves the CC problem anthropically
- In random axion landscapes, there are phenomenologically interesting inflationary patches. What about in the KS axiverse?