Axions, the path to the hot dark matter bound

Working Group Meeting COSMIC WISPers

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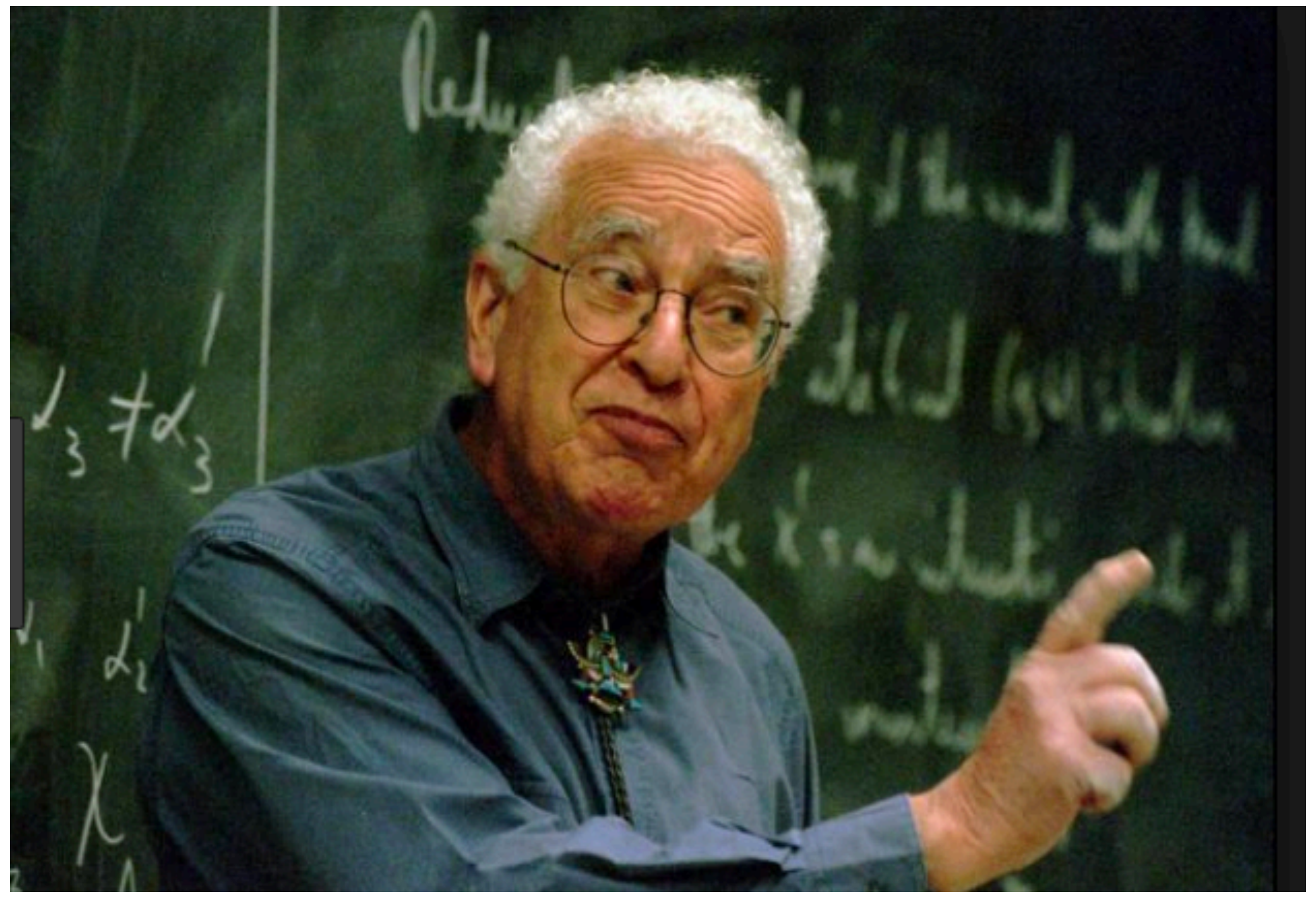
Based on:

L. Di Luzio, G. Martinelli, **GP** [arXiv: 2101.10330]
L. Di Luzio, **GP** [arXiv: 2206.04061]
L. Di Luzio, G. Martinelli, J.M. Camalich, J.A. Oller, **GP** [arXiv: 2211.05073]

Strong CP puzzle & Axion



"Any process which is not forbidden by a conservation law actually does take place with appreciable probability" [M. Gell-Mann,1956]

$$\mathcal{L}_{\rm QCD}^{\theta} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

$$\bar{\theta} = \arg\det(Y_u Y_d e^{i\theta})$$

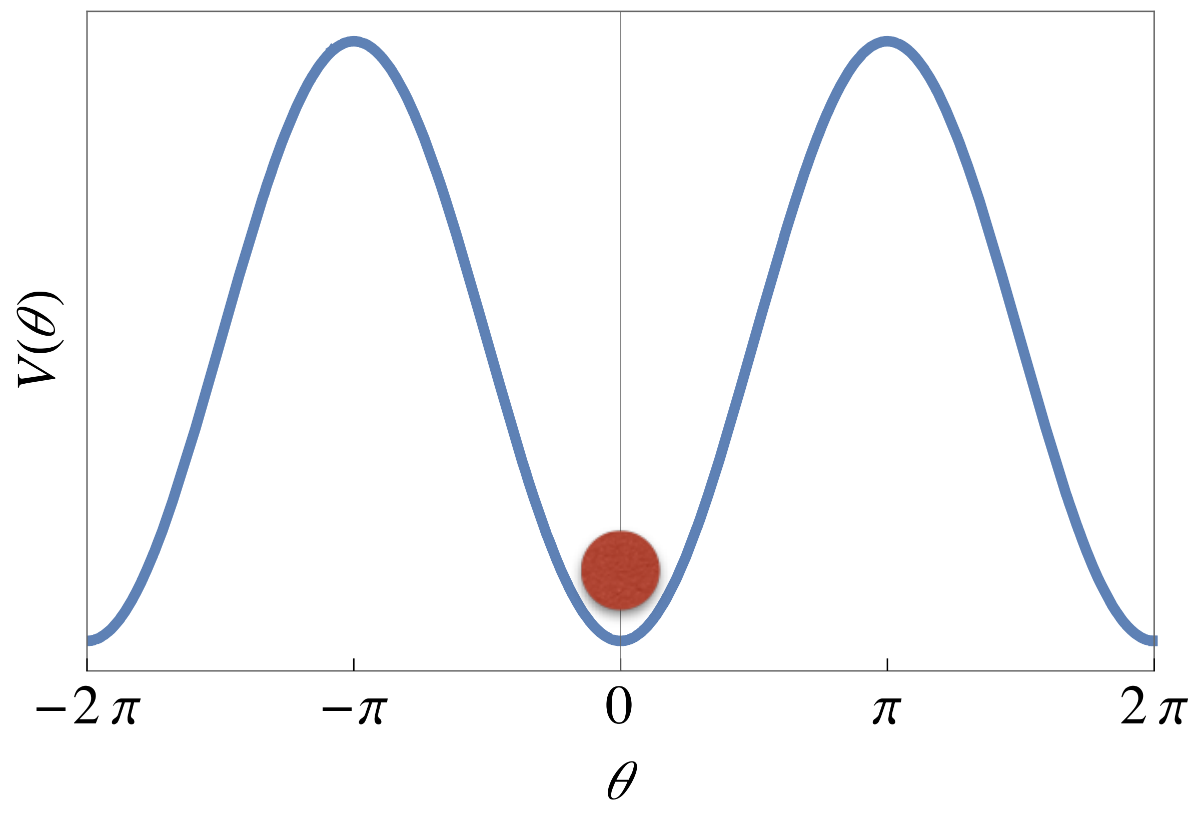
$|\bar{\theta}| \lesssim 10^{-10}$ from nEDM

✦ Spontaneously broken global symmetry, with QCD anomaly, solves the strong CP problem

$$\mathcal{L}_a = \left[\frac{a}{f_a} - \theta\right] \frac{\alpha_s}{4\pi} G\tilde{G} \qquad a \to a + \alpha f_a \qquad \text{with} \quad \langle a \rangle = 0$$

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Light



[Peccei, Quinn '77]
[Wilczek '78]
[Weinberg '78]
[Preskill, Wise, Wilczek '82]
[Dine, Fischler, Srednicki '81]

A possible discovery channel for the axion

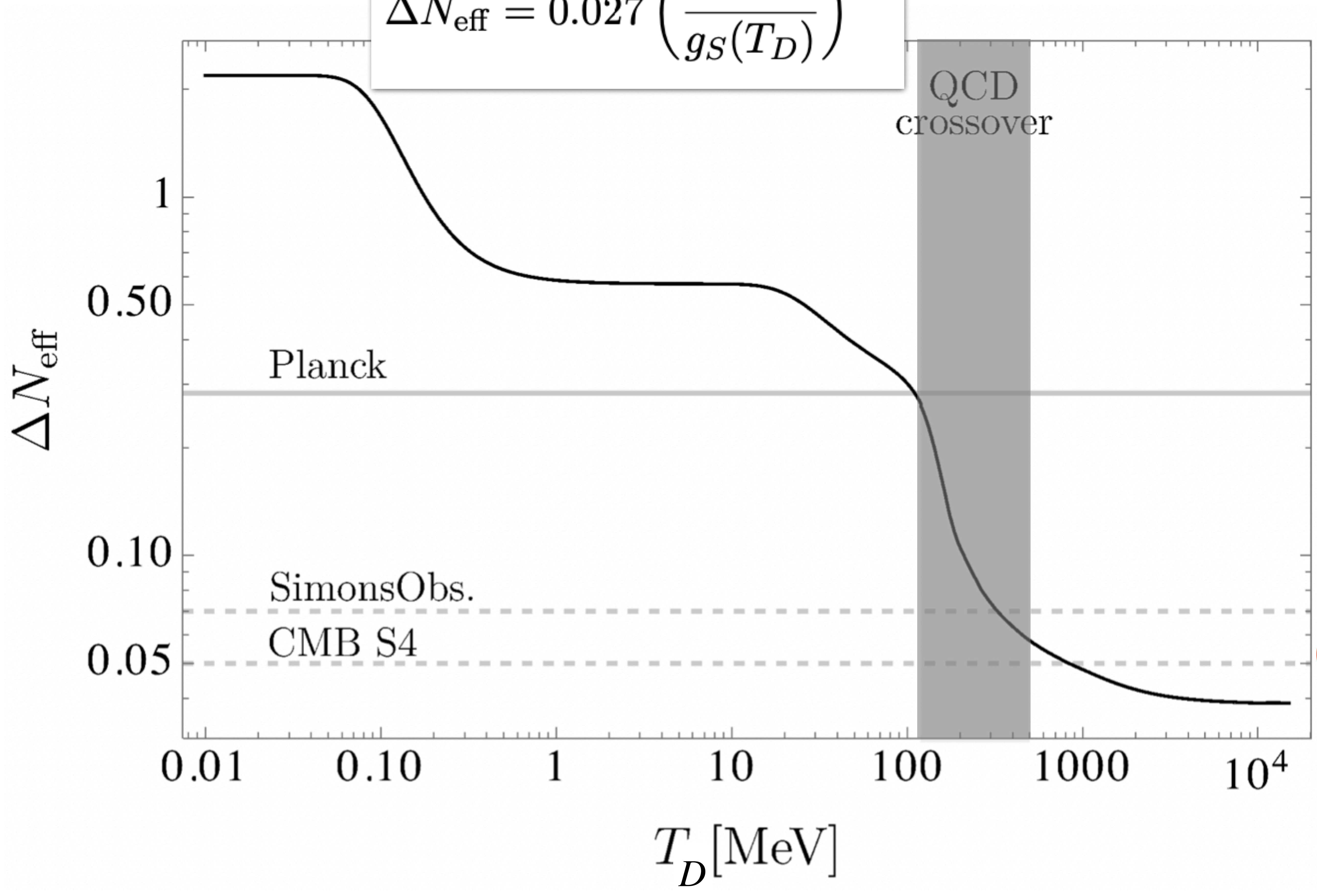
- Axions once in equilibrium with SM thermal bath contribute to the radiation density of the Universe ($\Delta N_{\rm eff}$)

$$\rho = \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^4 N_{\rm eff} \right]$$

- $T_{\rm Decoupling}$ depends on the strength of the axion interactions set by f_a

- Range of $\Delta N_{\rm eff}$ will be covered by CMB-S4

$$\Delta N_{\rm eff} = 0.027 \left(\frac{106.75}{g_S(T_D)} \right)^{4/3}$$



(2029 ?)

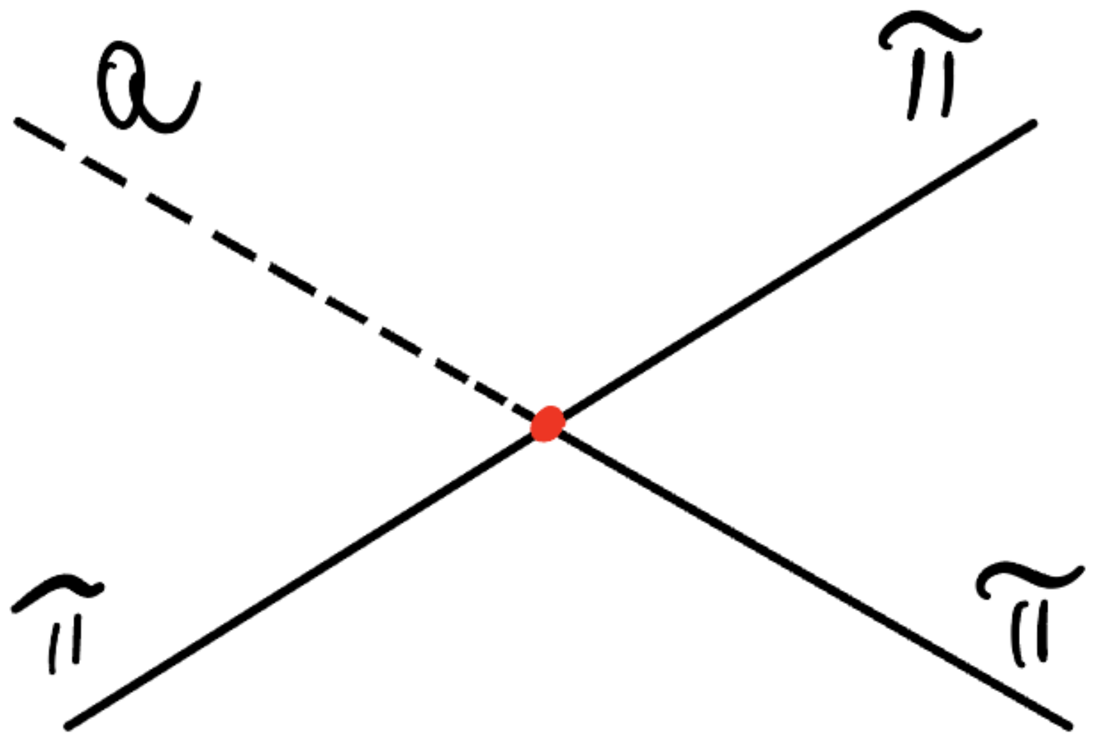
A possible discovery channel for the axion

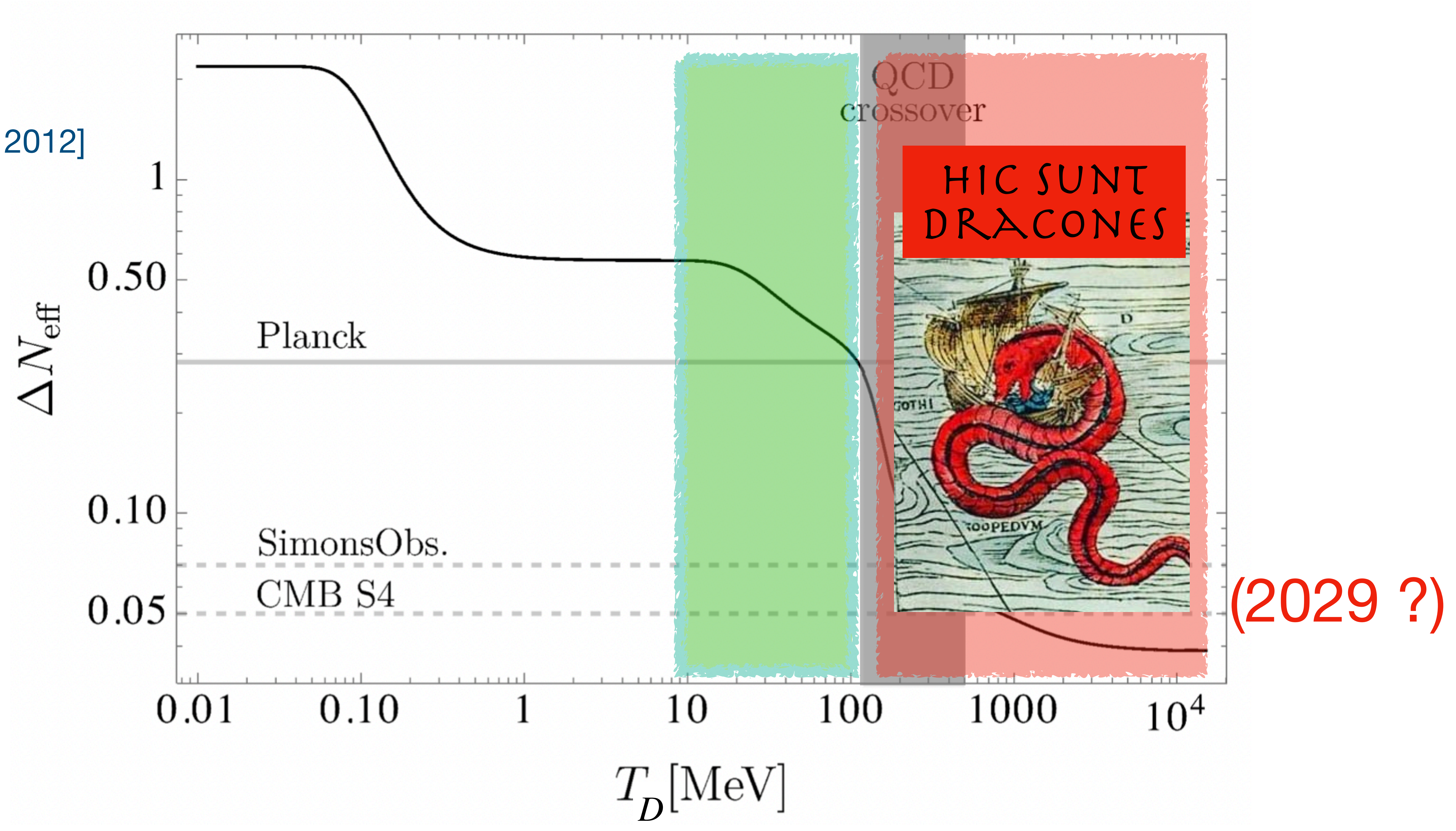
- Axions once in equilibrium with SM thermal bath contribute to the radiation density of the Universe ($\Delta N_{\rm eff}$)

$$T_{\rm Decoupling} \lesssim 155\ \mathrm{MeV} = \mathrm{T_c}$$

[Bazavov et al. 2012]

- Below QCD deconfinement the main thermalization channel is





Axion-Pion Effective Lagrangian: Leading Order

$$\frac{m_\pi^2}{\Lambda_{\rm QCD}^2} \ll 1$$

$$\mathcal{L}_a^\chi = \frac{f_\pi^2}{4} Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2} Tr\left[c_q \sigma^a\right] J_\mu^a$$

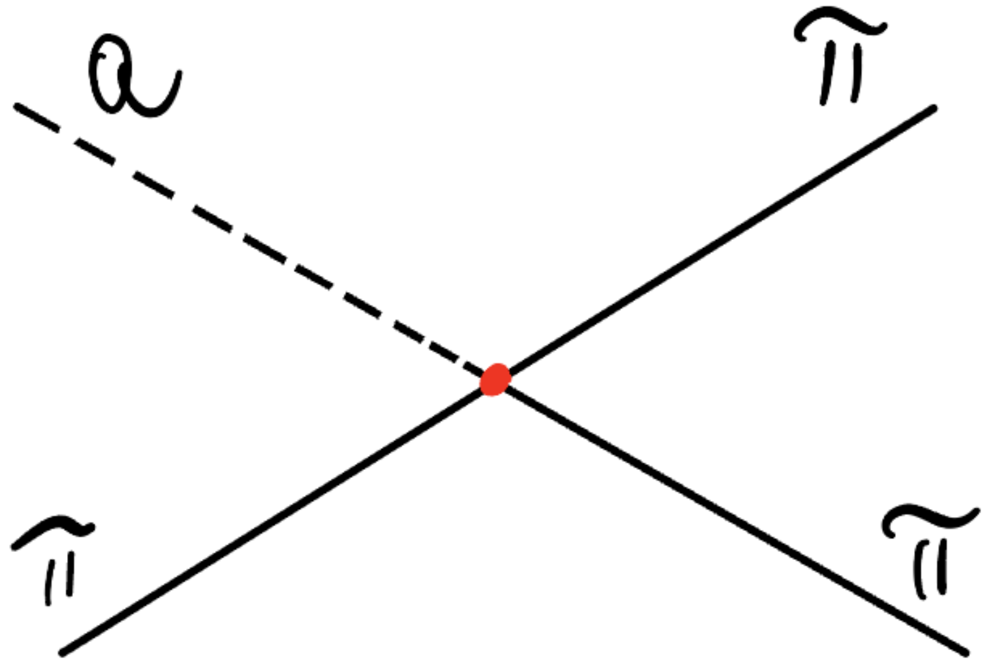
[Georgi, Kaplan, Randall, Phys. Lett. B **169** (1986)]

$$\begin{cases} U = e^{i\pi^a\sigma^a/f_\pi} \\ \chi = 2B_0\, e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \end{cases}$$

$$J_\mu^a = \frac{i}{4} f_\pi^2 Tr\left[\sigma^a\{U, (D^\mu U)^\dagger\}\right]$$

$$\mathcal{L}_{a\pi}^{(\rm LO)} = \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a\left(2\partial_\mu\pi_0\pi_+\pi_- - \pi_0\partial_\mu\pi_+\pi_- - \pi_0\pi_+\partial_\mu\pi_-\right)$$

$$C_{a\pi} = \frac{1}{3}\left(\frac{m_d - m_u}{m_u + m_d} + c_d^0 - c_u^0\right)$$



Axion thermal production in the Early Universe

To extract the HDM bound we compute the axion decoupling temperature T_D via the freeze-out condition* (conservative estimate)

$$\Gamma_a(T_D) = H(T_D)$$

Rate of reactions keeping the axions in thermal equilibrium

$$\Gamma_a = \frac{1}{n_a^{\rm eq}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p_4) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

Hubble Rate

$$H(T) = \sqrt{4\pi^3 g_\star(T)/45}\, T^2/m_{\rm pl}$$

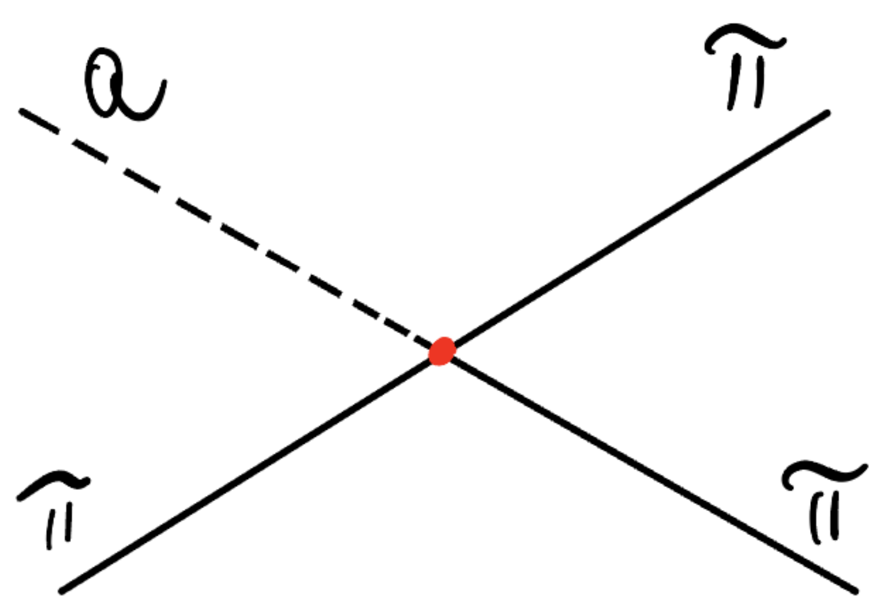
* For improved treatment of freeze-out see *Notari, Rompineve, Villadoro* [2211.03799]

Common Lore circa 2021:

Chang, Choi [hep-ph/9306216]
Hannestad, Mirizzi, Raffelt [hep-ph/0504059]

Axion HDM bound computed via LO ChPT

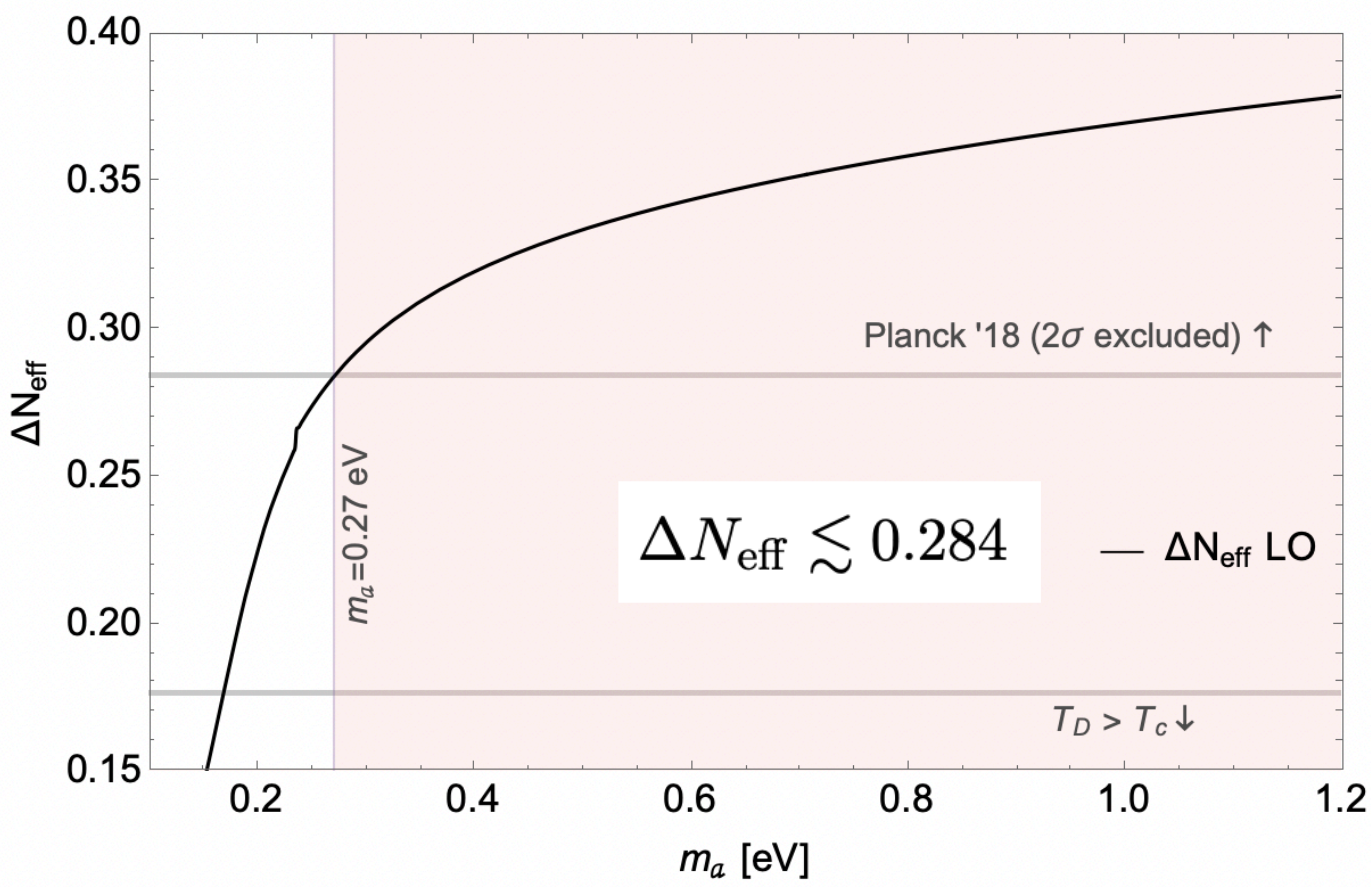
$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$



LO ChPT set the bound

$$m_a < 0.27\ \text{eV}$$

$$\Delta N_{\rm eff} = 0.027\left(\frac{106.75}{g_S(T_D)}\right)^{4/3}$$



See further:
Melchiorri, Mena, Slosar [0705.2695]
Hannestad, Mirizzi, Raffelt, Wong [0803.1585]
Hannestad, Mirizzi, Raffelt, Wong [1004.0695]
Di Valentino, Giusarma, Lattanzi, Mena, Melchiorri, Silk [1507.08665]

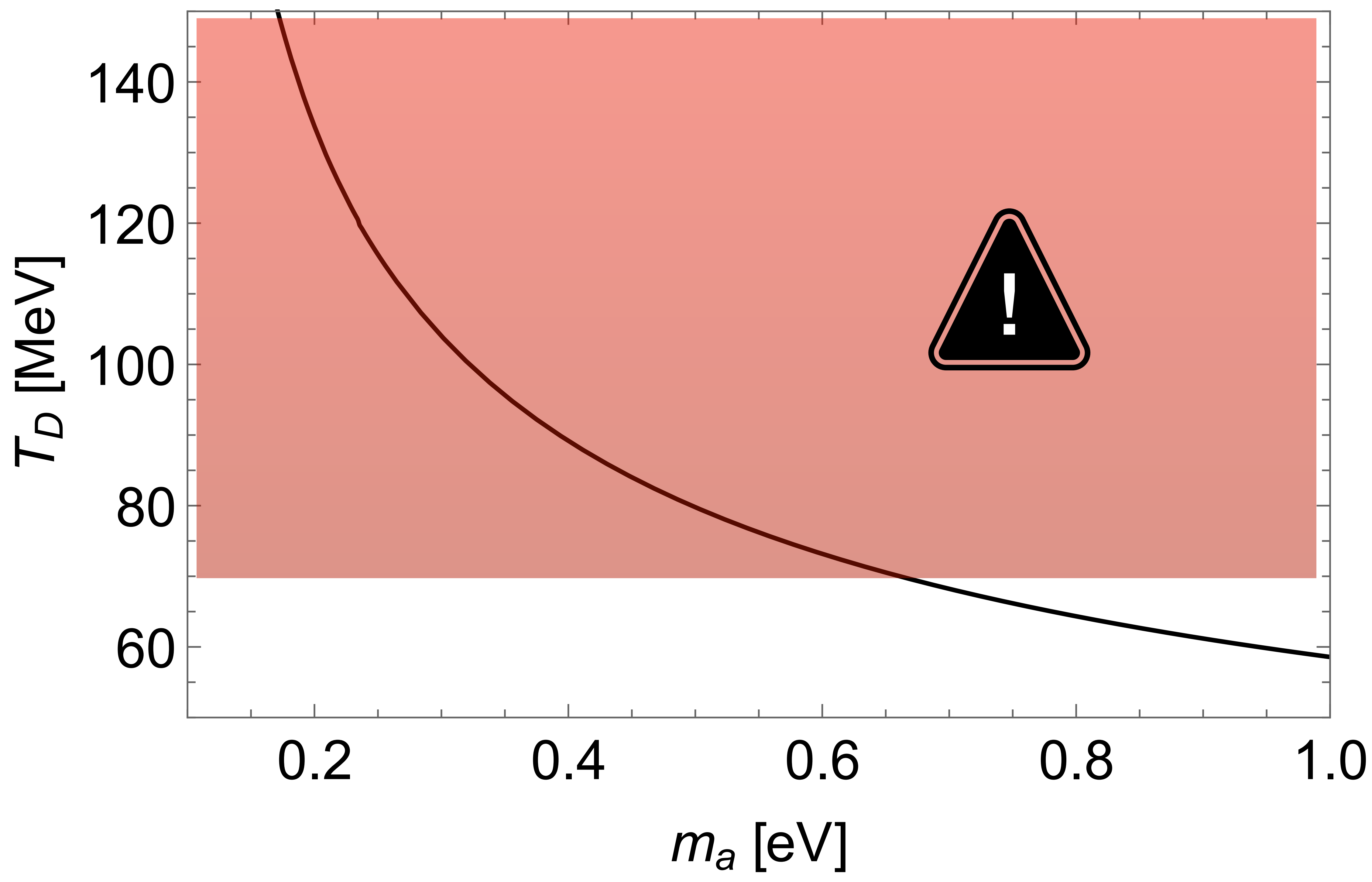
But… is ChPT valid?

E.g. at $T \simeq 70$ MeV, $\sqrt{s}_{a\pi} \simeq 550$ MeV

BUT

ChPT violates perturbative unitarity* for $E \gtrsim 460$ MeV

Is ChPT reliable?



NLO axion production rate

Breakdown of Chiral Perturbation Theory for the Axion Hot Dark Matter Bound

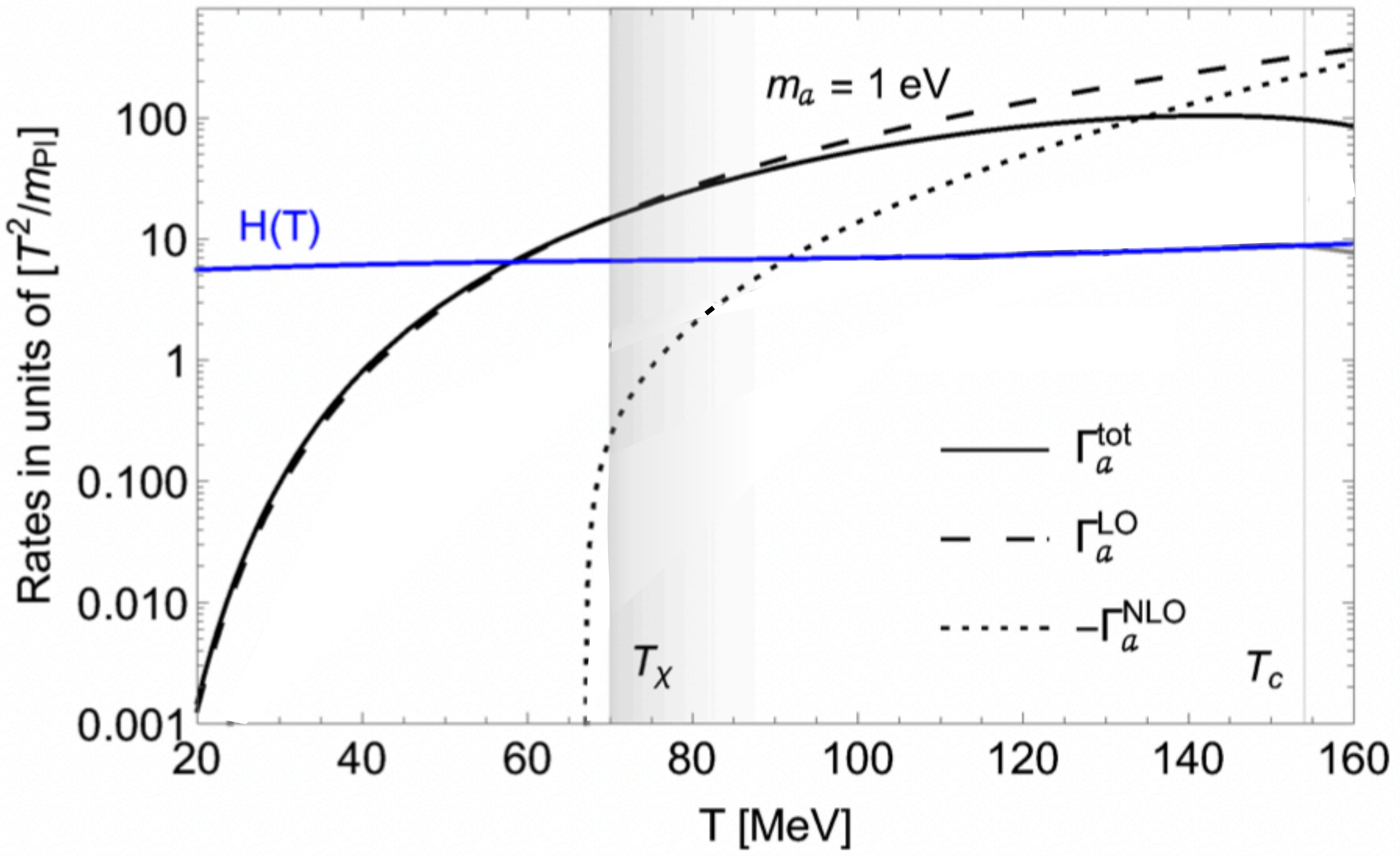
Luca Di Luzio[1,2,3,*], Guido Martinelli[4,†] and Gioacchino Piazza[5,‡]

*see e.g. [Donoghue et al., PhysRevD.86.014025]
For $\pi\pi$ scattering see also [Schenk 1993]

NLO Thermalization rate

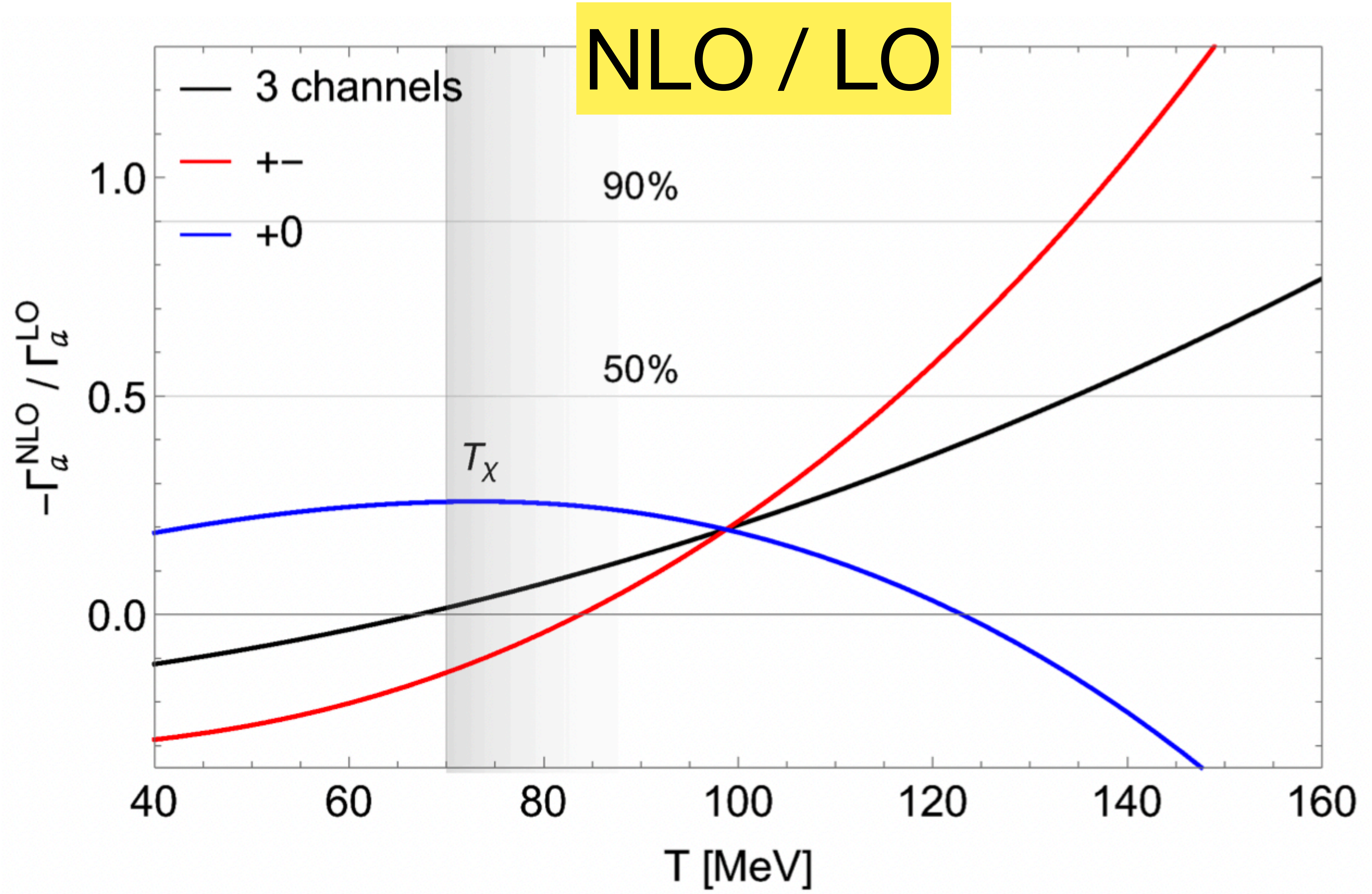
$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO}]$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.163\, T^5 \left[h_{\rm LO}(m_\pi/T) - 0.251\frac{T^2}{f_\pi^2}\, h_{\rm NLO}(m_\pi/T)\right]$$



Di Luzio, Martinelli, **Piazza** [2101.10330]

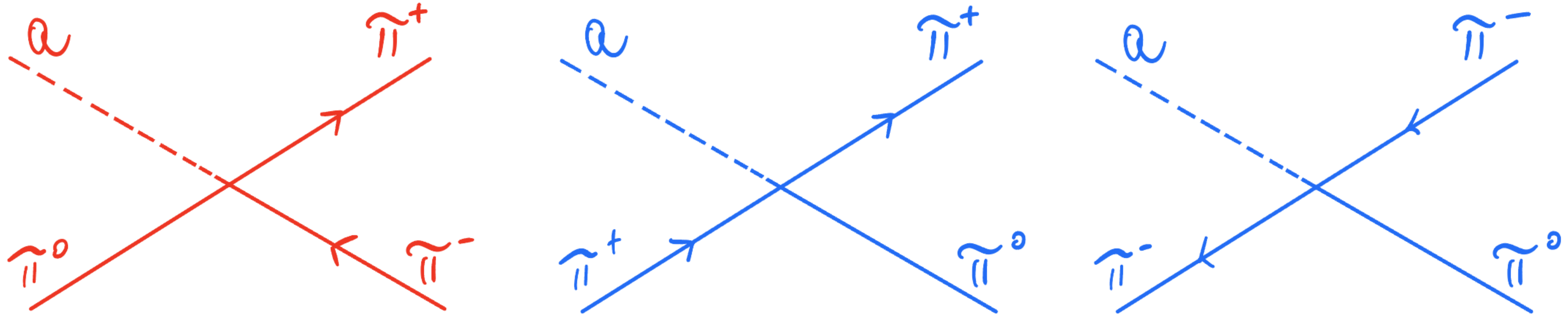
Breakdown of ChPT : Γ



✦ The NLO corrections to total Γ reach 50% x LO at $T \simeq 135$ MeV, due to accidental cancellations;

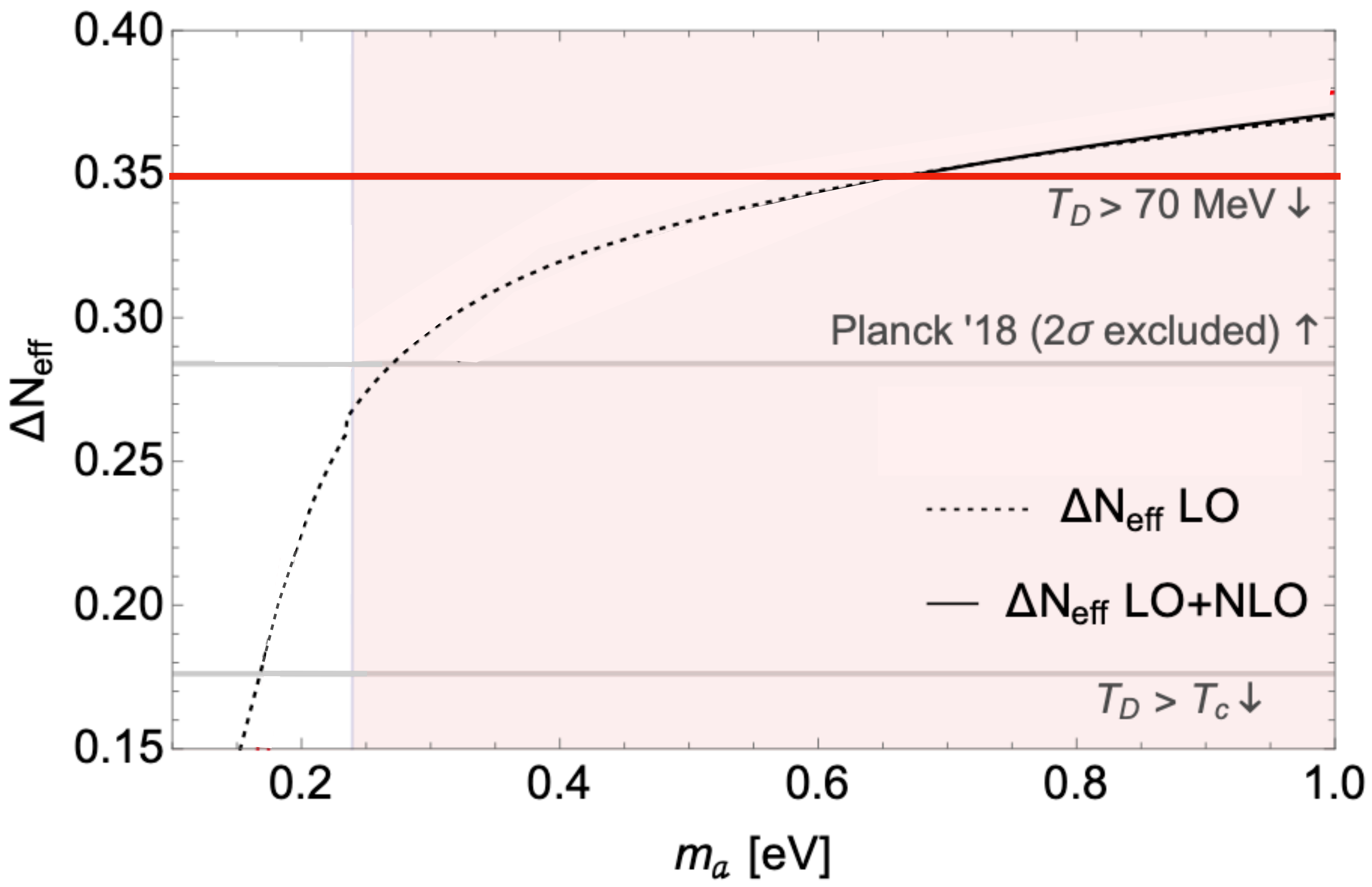
✦ A more realistic estimate of T_χ by looking at the first exclusive channels with large NLO correction.

➡ In $\pi^+\pi^0$ big corrections at $T_\chi \simeq 70$ MeV



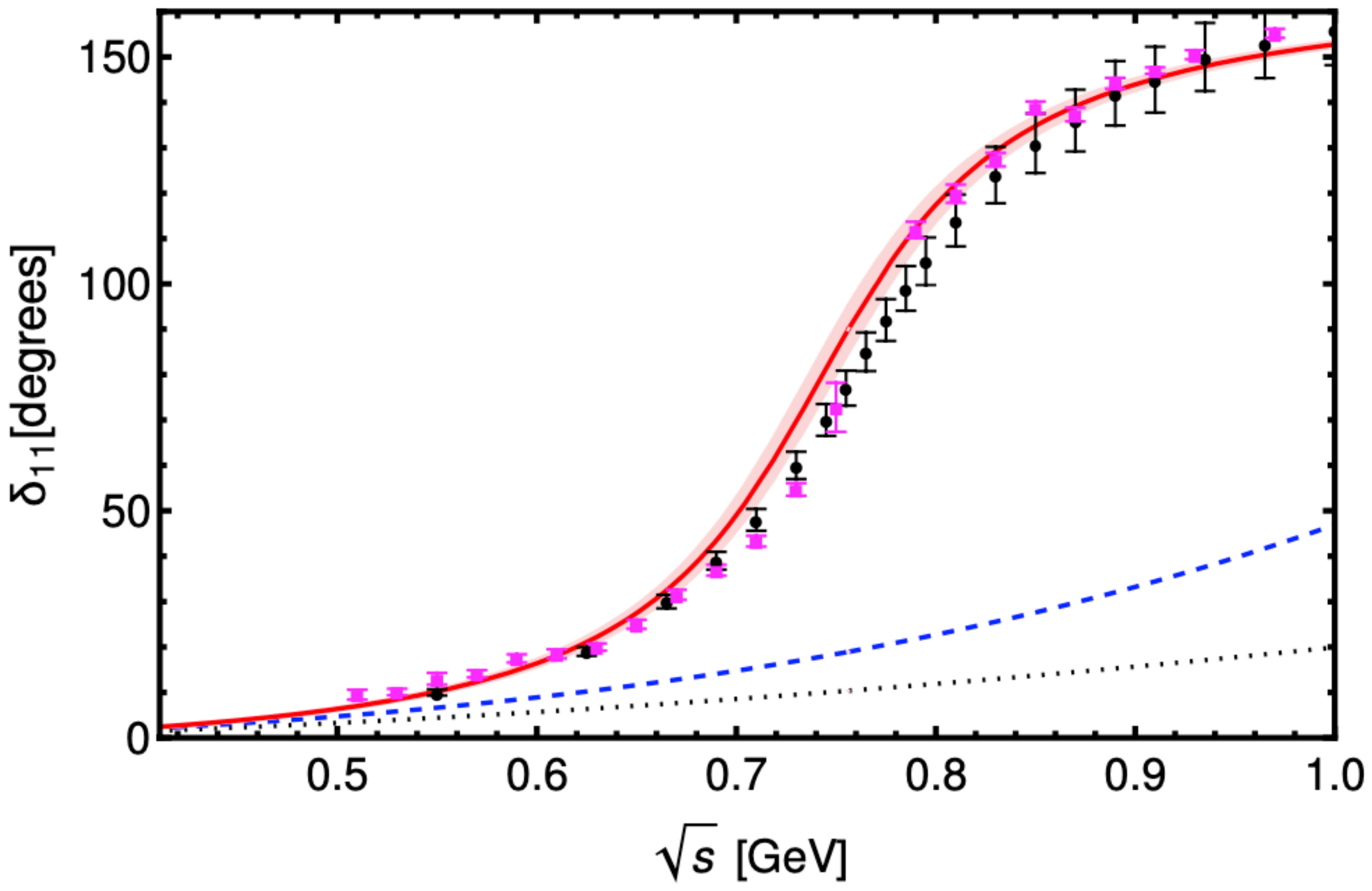
ΔN_{eff} including NLO correction

T_{D} cannot be extracted in the region of interest since the *NLO ~ LO for* $T > 70$ MeV



Di Luzio, Martinelli, **Piazza** [2101.10330]

Extending the validity of ChPT: Unitarization

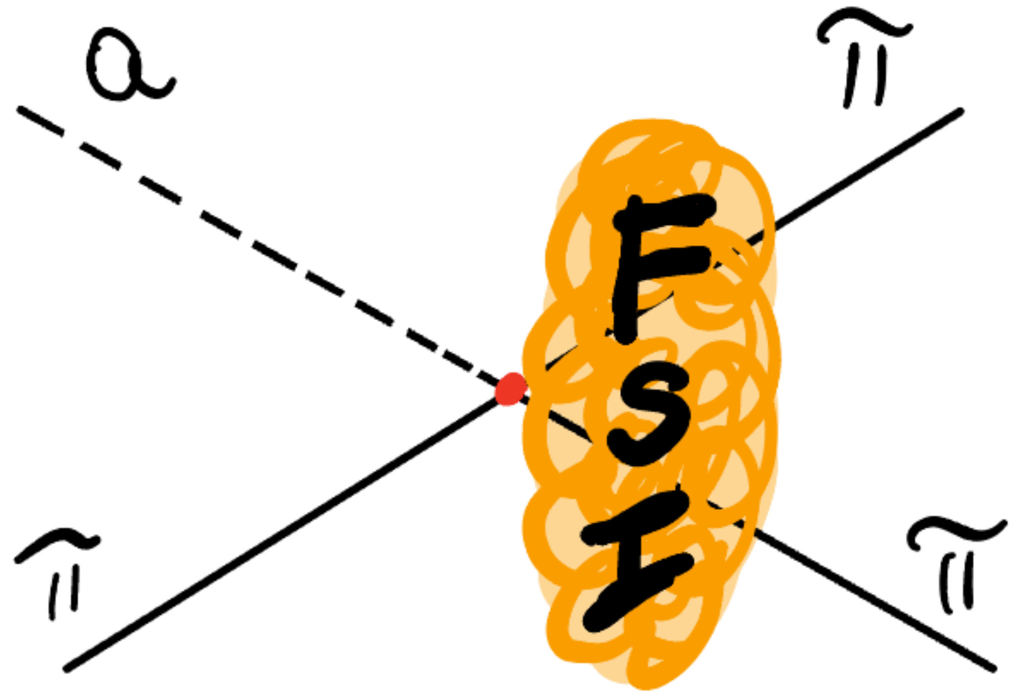


Di Luzio, Camalich, Martinelli, Oller, **Piazza** [2211.05073]

$\pi\pi$ final-state interactions (FSI) are resonant

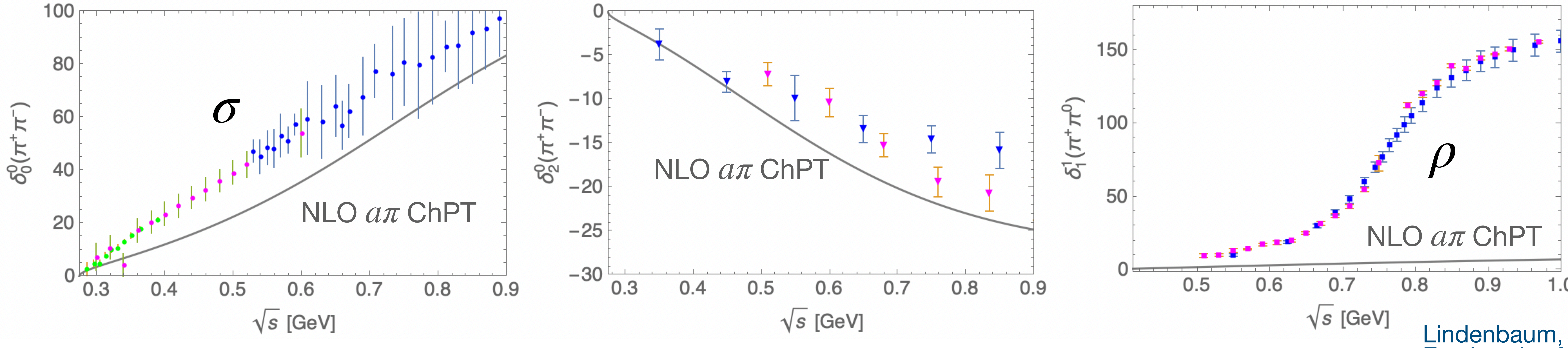
<u>ChPT cannot produce resonances</u>

$$\begin{cases} \sigma \text{ or } f_0(500) \text{ in } I = L = 0 \\ \rho(770) \text{ in } I = L = 1 \end{cases}$$



<u>Unitarity</u> (Watson theorem) $\Rightarrow (\delta_{a\pi})^{\ell}_{I} = (\delta_{\pi\pi})^{\ell}_{I}$ [K. M. Watson (1952)]

Comparing NLO $a\pi$ ChPT to $\pi\pi$ data: $(\delta_a)^{\ell}_{I} \neq (\delta_{\pi-\text{scatt}})^{\ell}_{I}$



☹

Lindenbaum, Longacre '92
Estabrooks, Martin '74
....

Unitarization to extend the validity of ChPT

✤Inverse Amplitude Method (IAM): [Truong, PRL **61**, 2526]

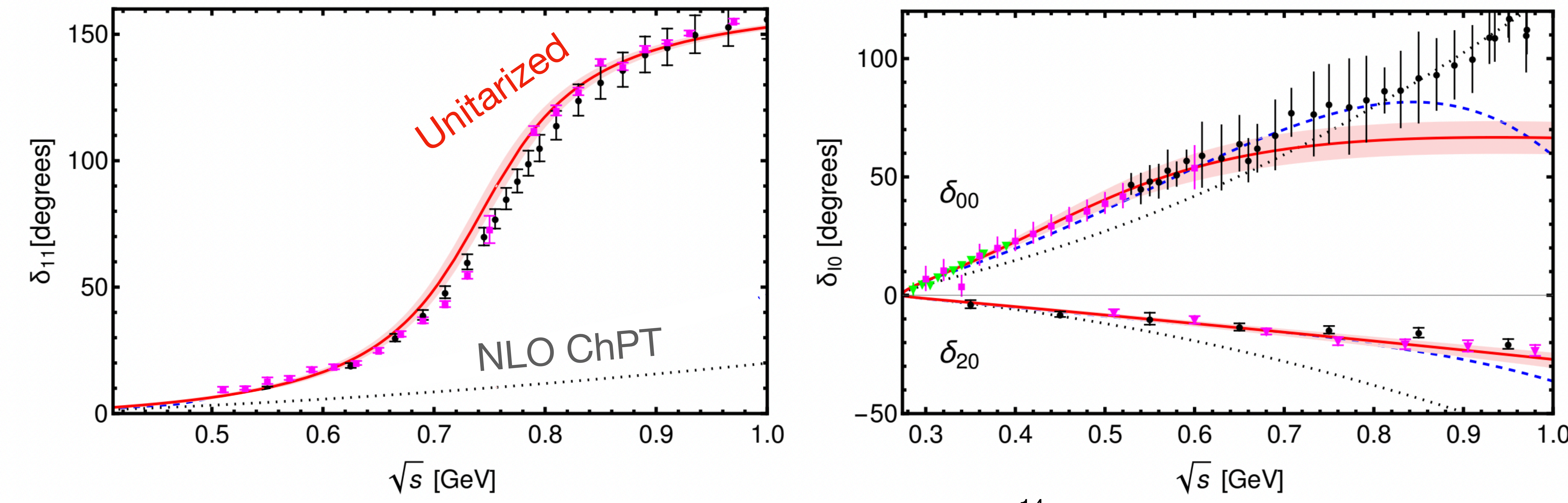
The IAM amplitude satisfies exactly unitarity, includes the resonances, and reproduces ChPT at low-energies

Definite I, J amplitudes

$$A_{IJ}(s) = A_{IJ}^{(2)}(s) + A_{IJ}^{(4)}(s) + \dots \quad \Rightarrow \quad A_{IJ}^{\mathrm{IAM}}(s) = \frac{A_{IJ}^{(2)}(s)}{1 - A_{IJ}^{(4)}(s)/A_{IJ}^{(2)}(s)}$$

IAM LECs from fit to $\pi\pi$ scatt. [Dobado, Pelaez 1997]

✓ Phases obtained in IAM correspond to phases of $\pi\pi$ scattering: Watson th. !

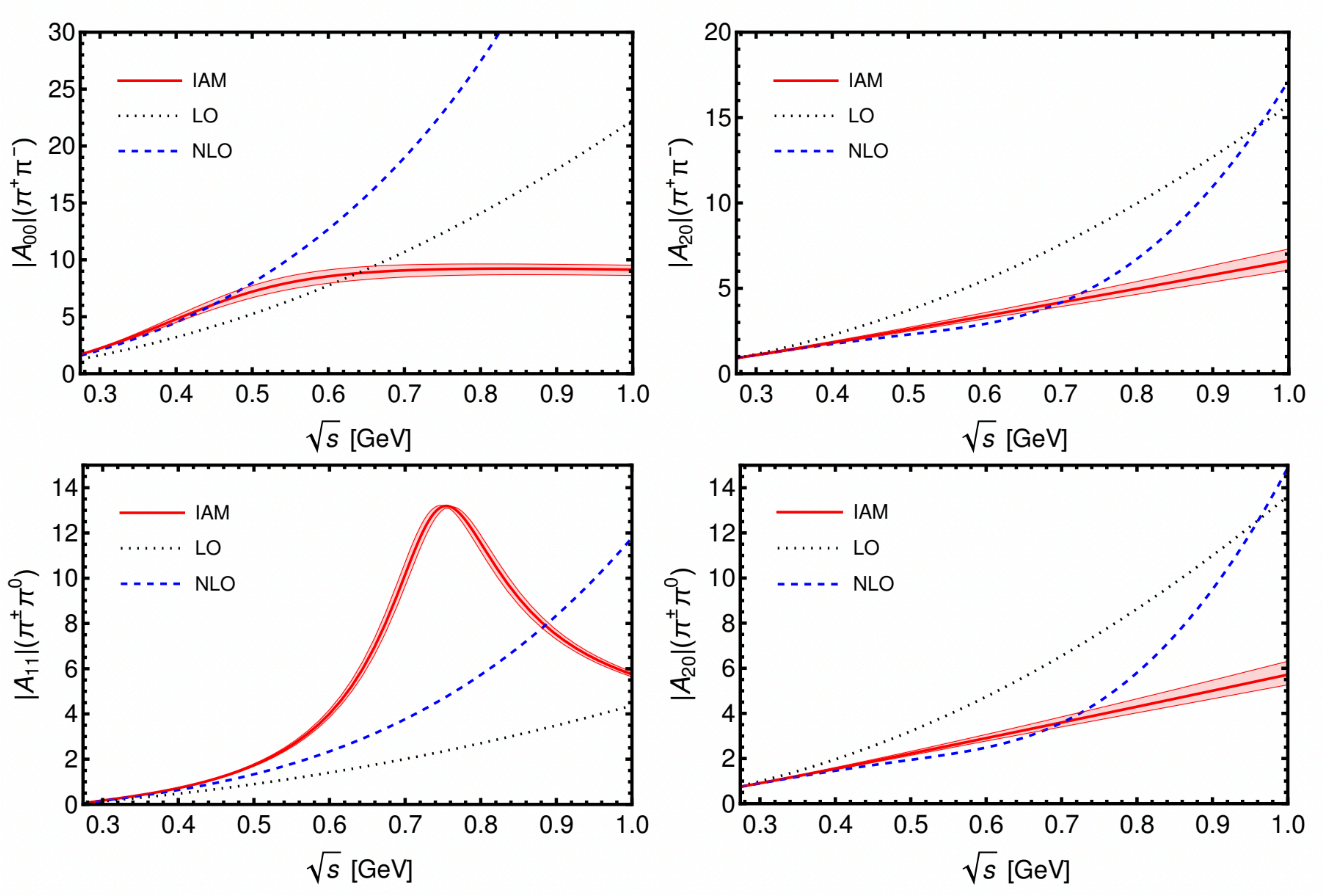


ChPT extended up to ~ 800 MeV

For $\pi\pi$ scattering see also [Schenk 1993]

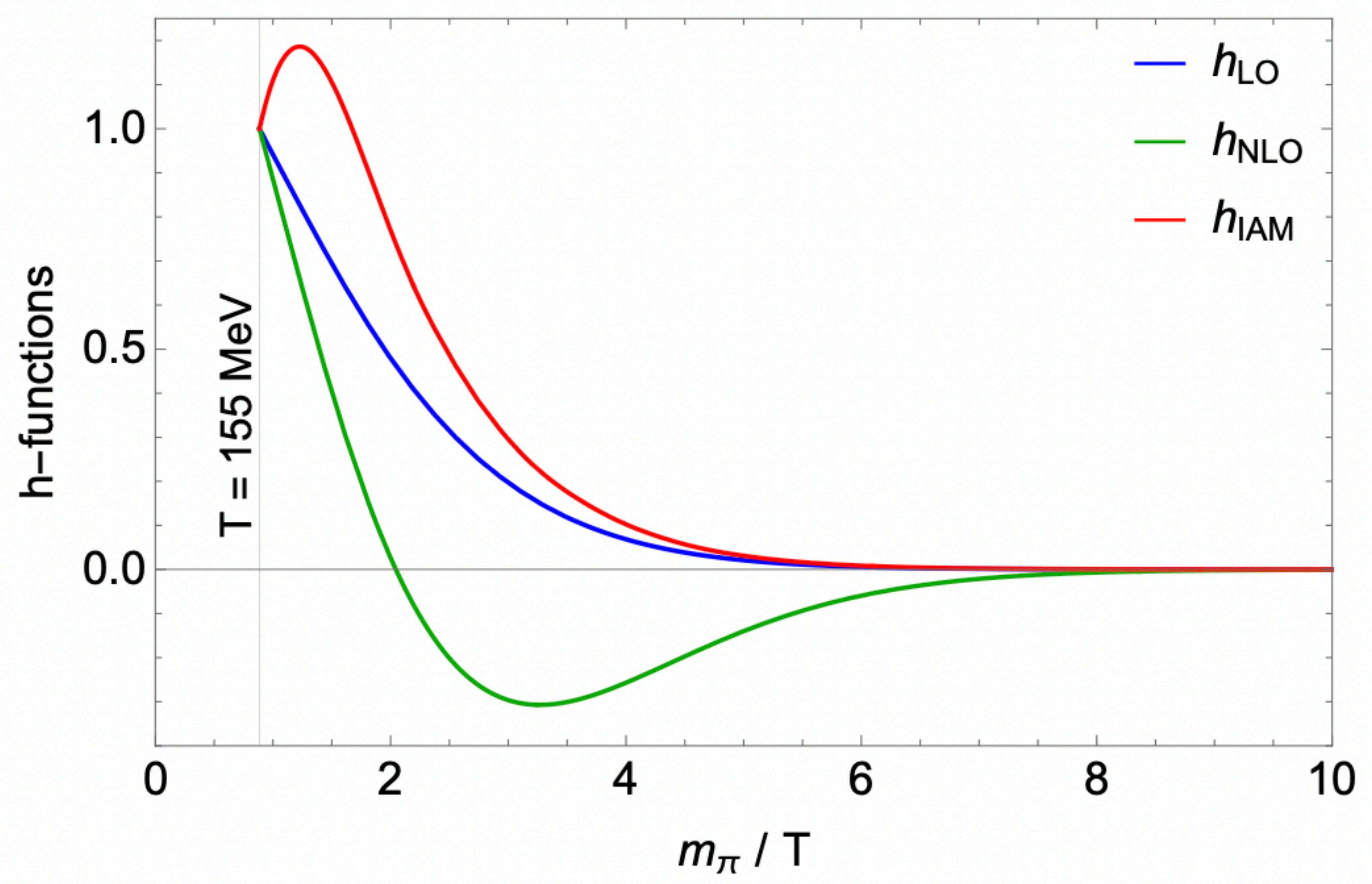
Partial wave amplitudes

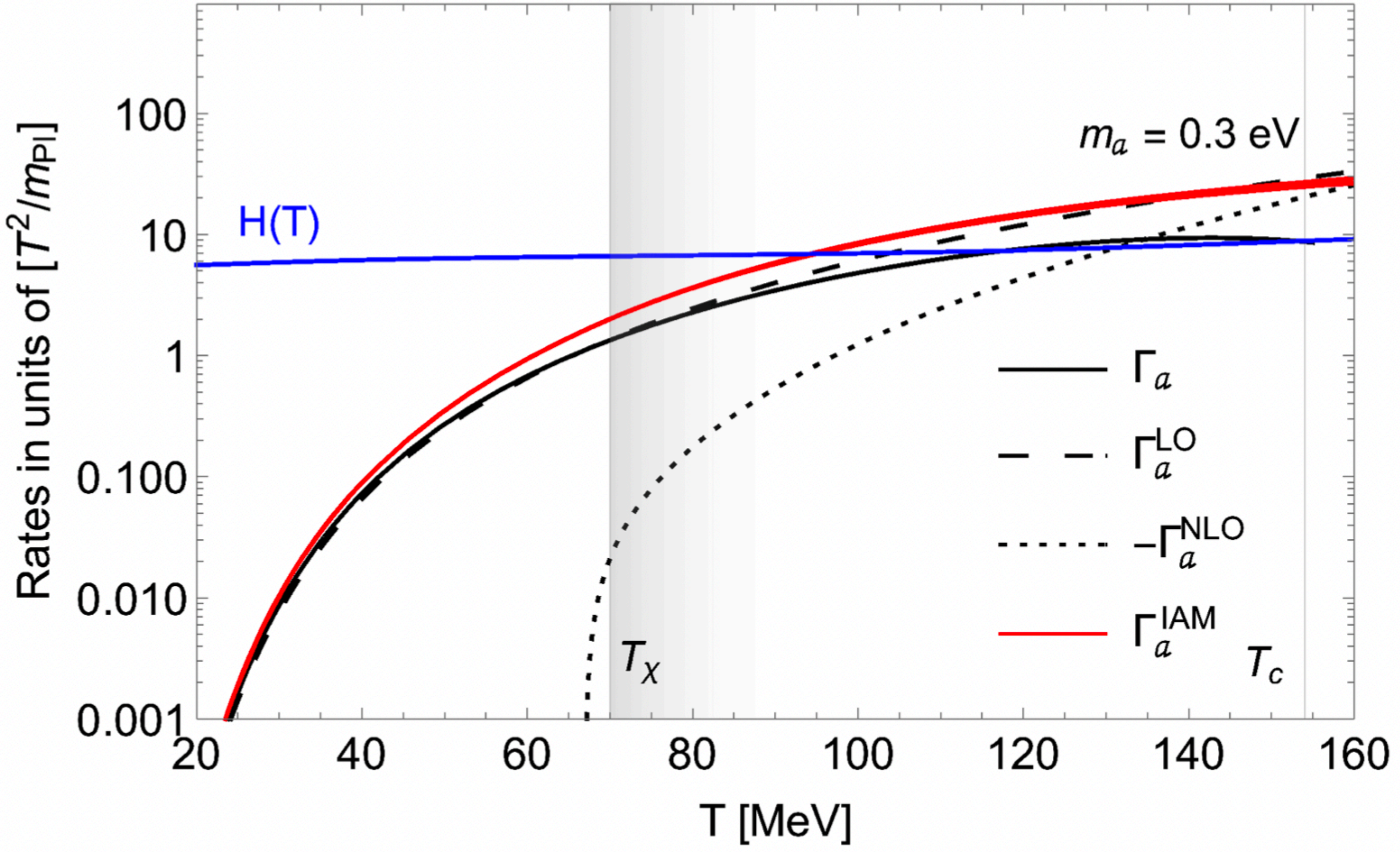
Growth with energy of ChPT amplitudes is tamed by unitarization!



Thermal rate

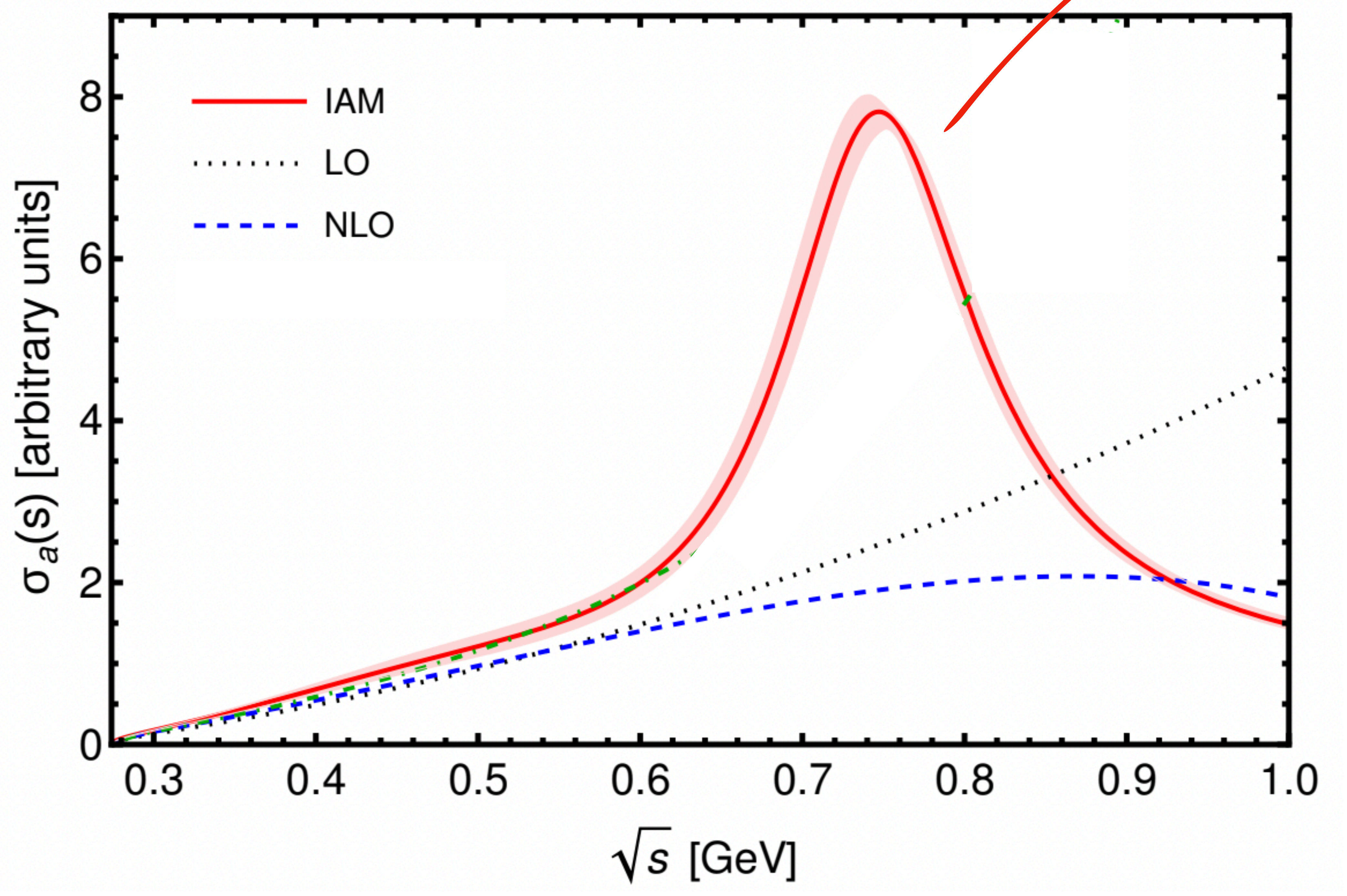
$$\Gamma_a^{\rm IAM}(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.137\, T^5\, h_{\rm IAM}(m_\pi/T)$$

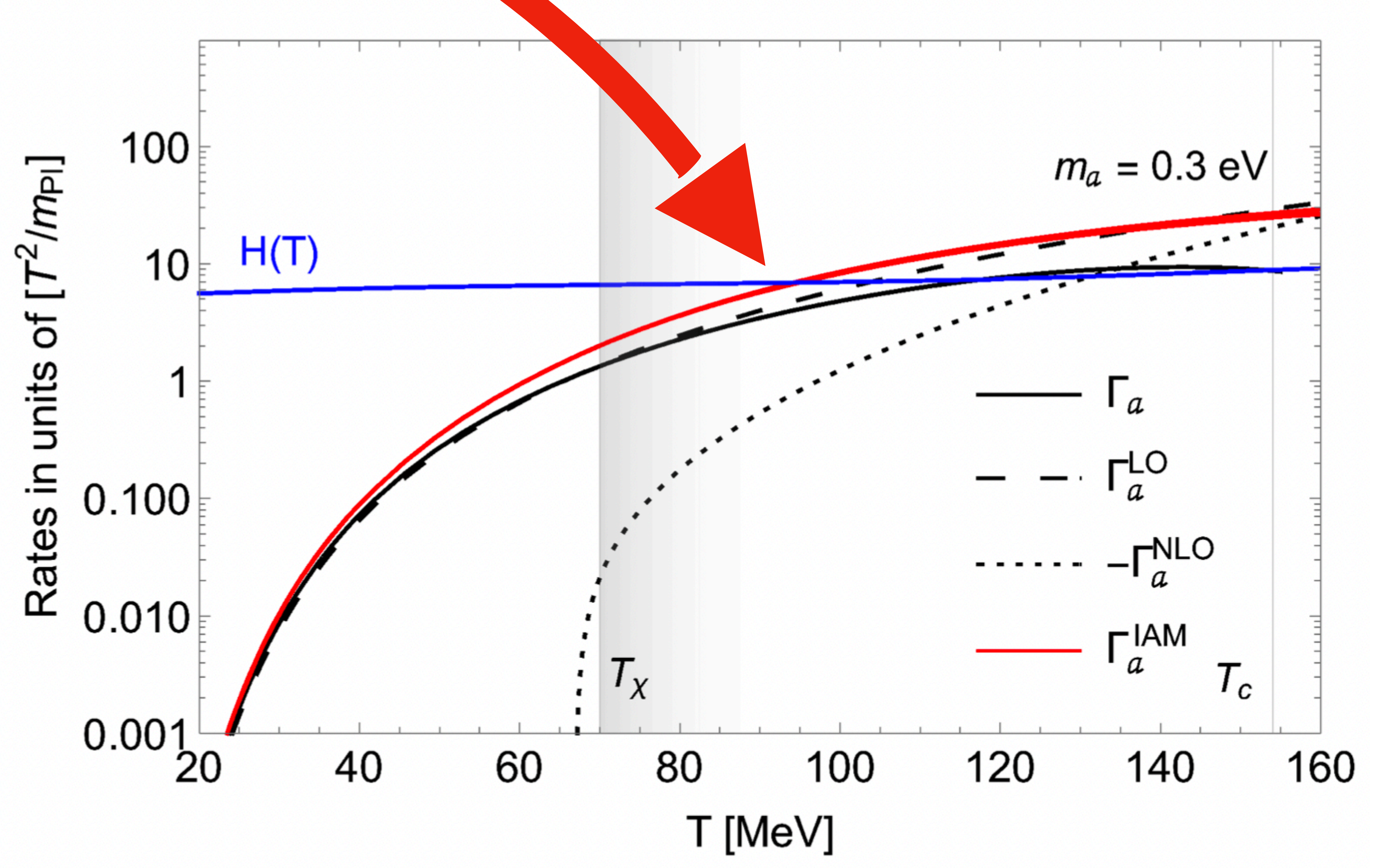




Di Luzio, Camalich, Martinelli, Oller, **Piazza** [2211.05073]

Thermal rate

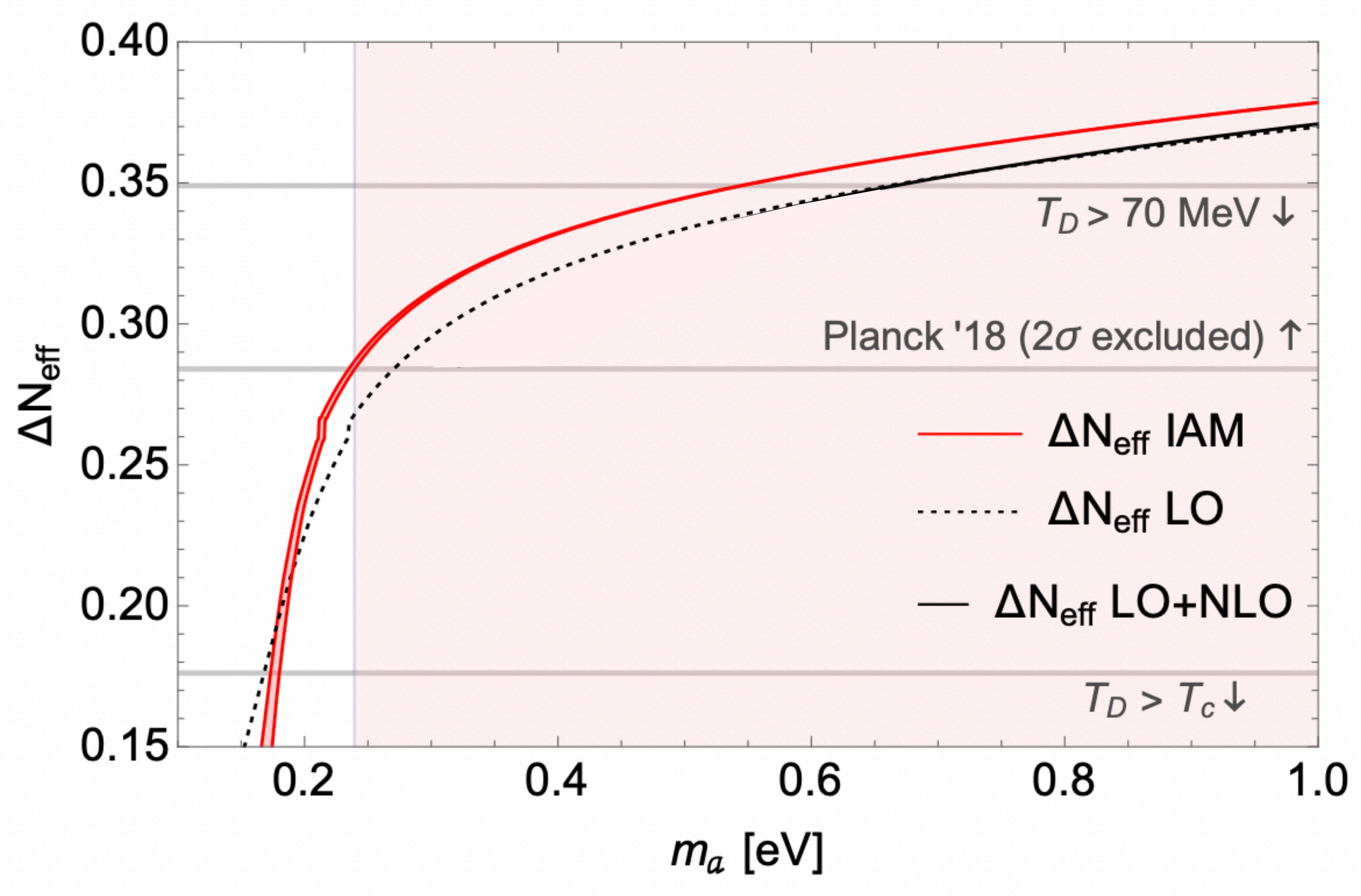




ρ resonance appears at $\sqrt{s} \sim 750$ MeV $->$ $T \sim 100$ MeV

Similar results in *Notari, Rompineve, Villadoro* [2211.03799]

ΔN_{eff} : IAM vs LO

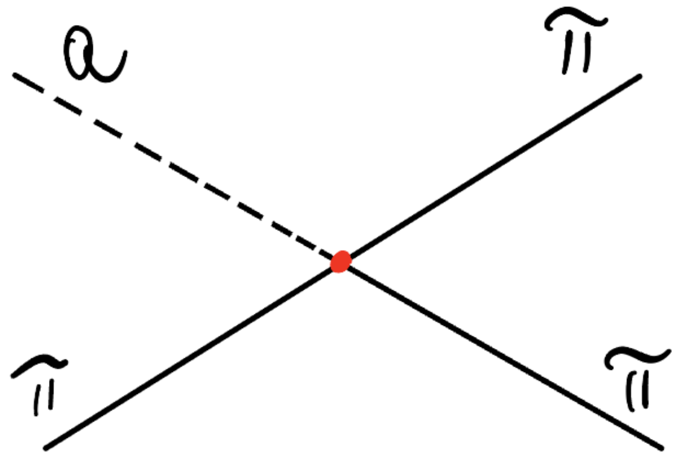


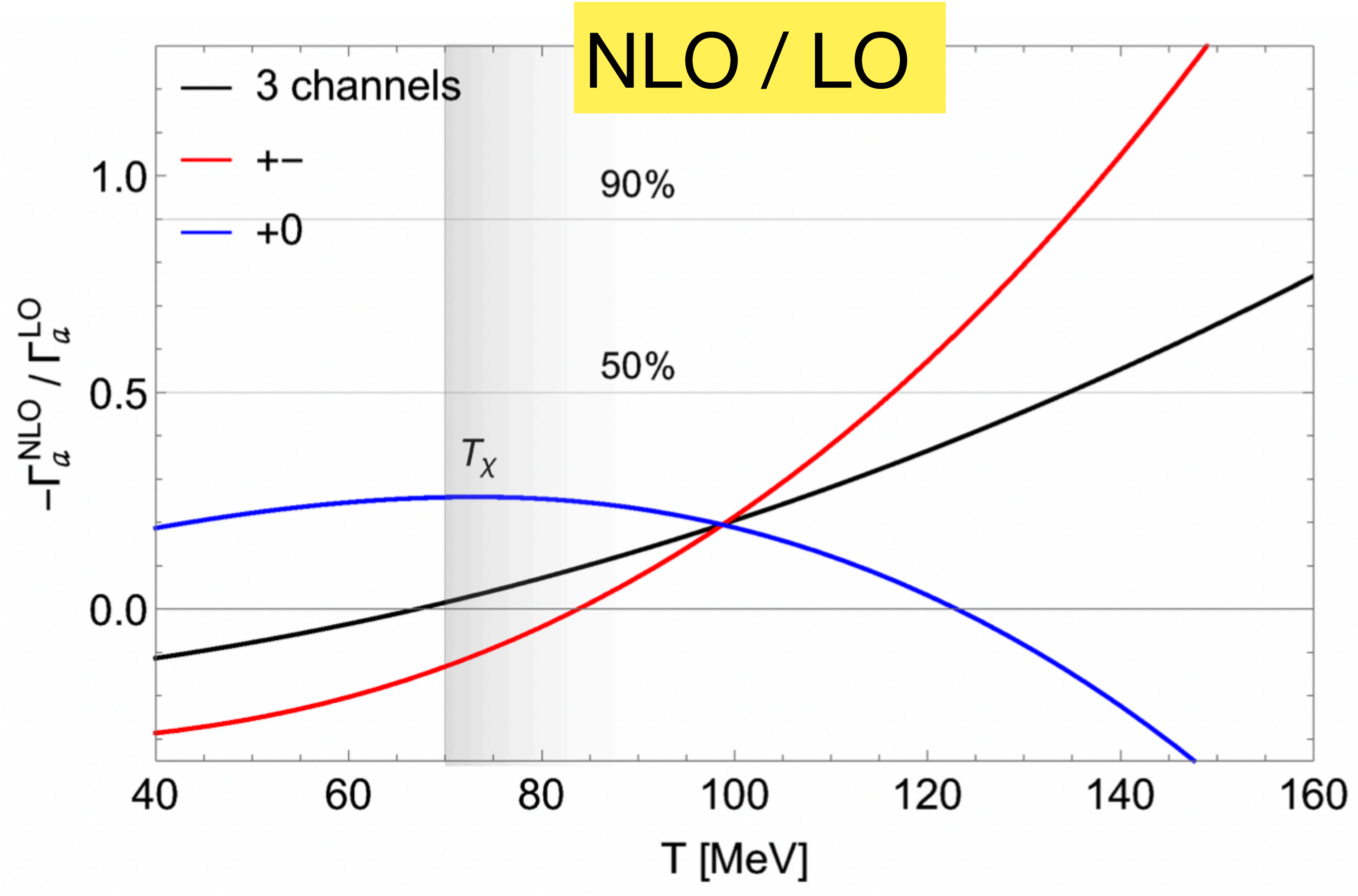
- ✦ LO and NLO ChPT reliable up to $T_D \sim 70$ MeV
- ✦ IAM **conservative** upper bound: $m_a \leq 0.24$ eV

Thermal corrections amount to 10% increase on the mass upper bound (see 2312.15240)

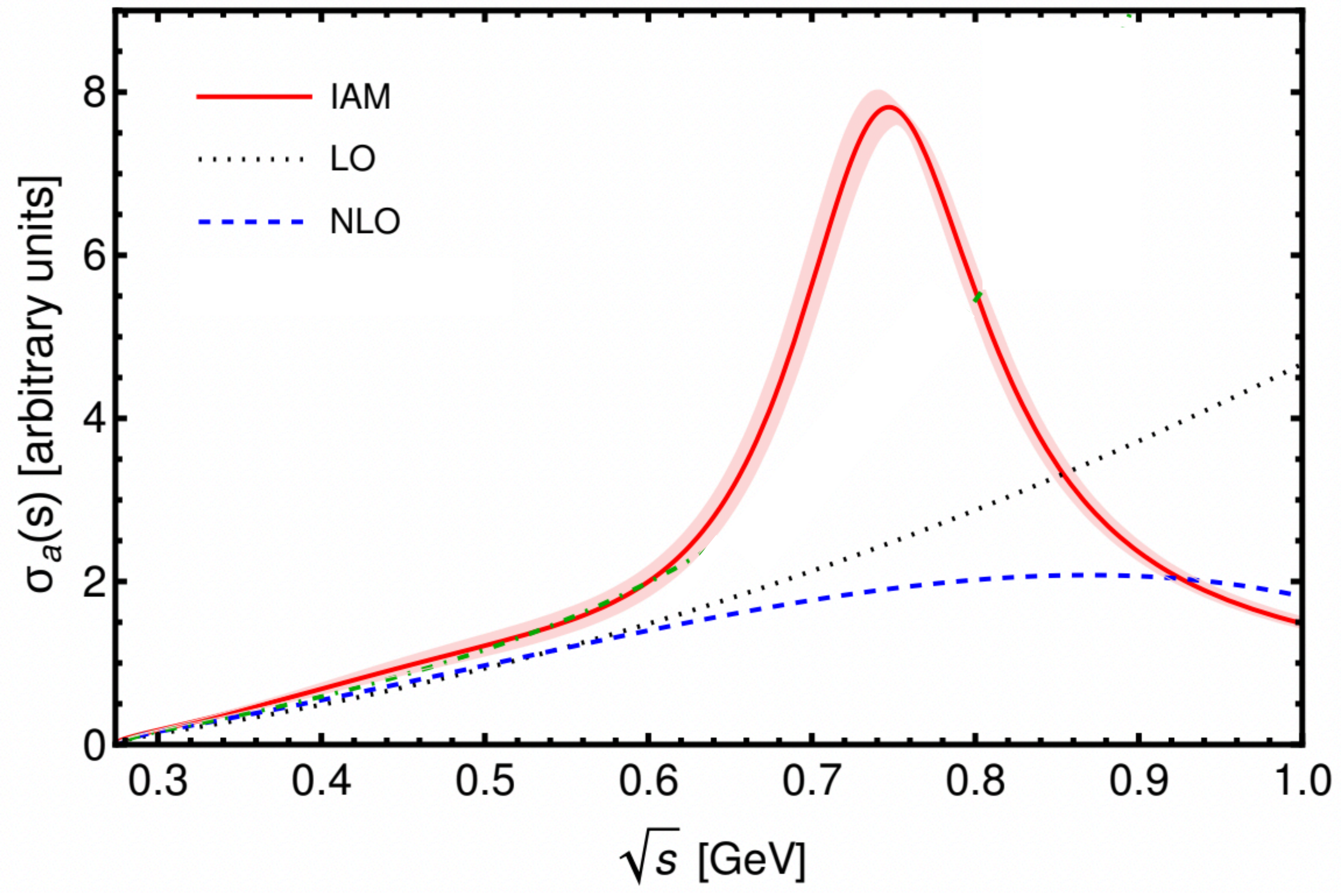
Di Luzio, Camalich, Martinelli, Oller, **Piazza** [2211.05073]

Conclusions





- ChPT **above $T \sim 70$ MeV is unreliable**
- **Unitarization** provides a way to **extend ChPT** including resonances and satisfying unitarity
- IAM **Hot Dark Matter bound** $m_a \leq 0.24$ eV



Huge effort from different groups leads to concordant results, although using different methods and cosmological analysis

[2211.03799] Notari, Rompineve, Villadoro $m_a \leq 0.24$ eV
[2212.11926] Di Valentino, Gariazzo, Giarè, Melchiorri, Mena, Renzi $m_a \leq 0.21$ eV
[2310.08169] Bianchini, di Cortona, Valli $m_a \leq 0.18$ eV
[2312.15240] Wang, Guo , Zhou $m_a \leq 0.25$ eV

Thanks for the attention!

Backup

Axion-Pion scattering: Next-to-Leading Order

Ingredients

Tree-level graph from NLO Lagrangian and loop amplitudes from LO Lagrangian contributes to the same Order

$$
\begin{aligned}
\mathcal{L}_{\rm NLO} = & \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\
& + \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right] \\
& + l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu}U f^{\mu\nu}_L U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(f^L_{\mu\nu}f^{\mu\nu}_L + f^R_{\mu\nu}f^{\mu\nu}_R\right)\right] \\
& + i\frac{l_6}{2}\mathrm{Tr}\left[f^R_{\mu\nu}D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\
& - \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{4}\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1-h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\
& \left. + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu}f^{\mu\nu}_L + f^R_{\mu\nu}f^{\mu\nu}_R\right)
\end{aligned}
$$

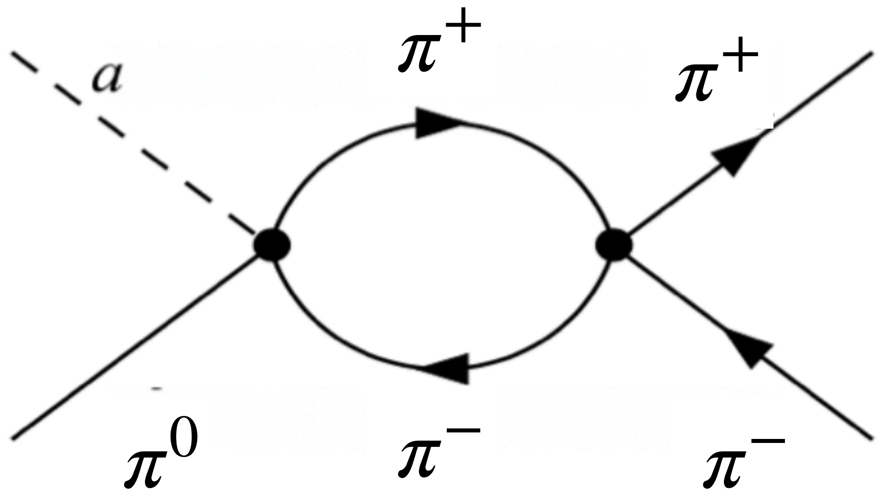
NLO Lagrangian

+

NLO chiral axial current J^a_μ

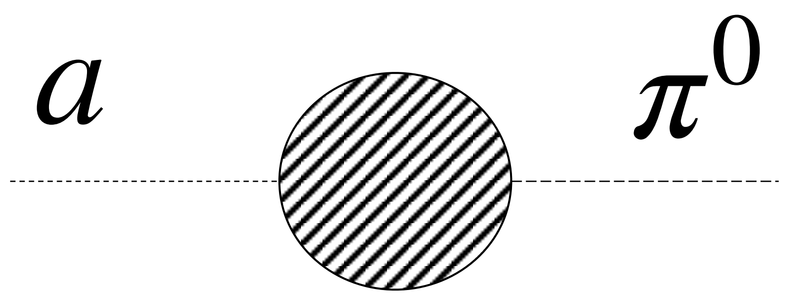
$$\mathcal{L}^\chi_a \supset \frac{\partial^\mu a}{f_a}\mathrm{Tr}\frac{1}{2}\left[c_q\sigma^a\right]J^a_\mu$$

+



1-loop amplitudes from LO

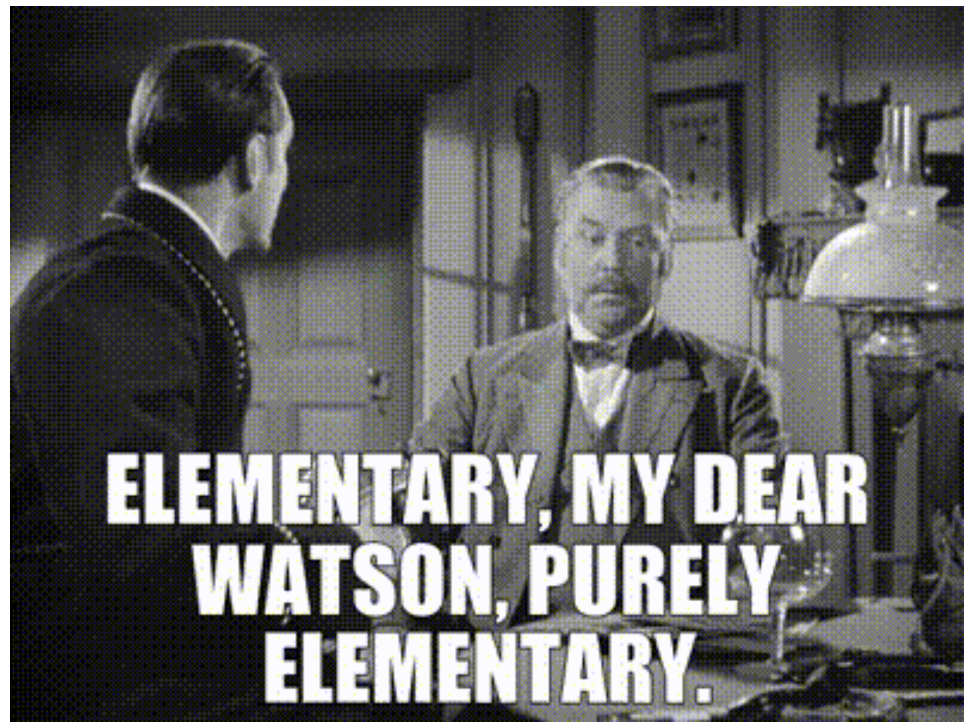
+



$a - \pi_0$ diagonalization

Watson Theorem

[K. M. Watson, Phys. Rev. 88, 1163 (1952)]



The axion interacts weakly, but $\pi\pi$ final-state interactions are strong and resonant

$$\rightarrow T_I^\ell = |T_I^\ell| e^{i(\delta_\pi)_I^\ell}$$

$$A_I^\ell = |A_I^\ell| e^{i(\delta_a)_I^\ell}$$

$$T^\dagger T = i(T^\dagger - T)$$

$$\boxed{\mathrm{Im}\, A_I^\ell(s) = \frac{\sigma(s)}{32\pi} A_I^\ell(s) T_I^\ell(s)^* \theta(s - 4m_\pi^2)}$$

$$\sigma = \sqrt{1 - 4m^2/s}$$

<u>Unitarity</u> $\Rightarrow (\delta_a)_I^\ell = (\delta_\pi)_I^\ell$

IAM "derivation" [Truong, PRL **61**, 2526]

$$\mathrm{Im}\, t(s) = \sigma(s)|t(s)|^2 \Rightarrow \mathrm{Im}\frac{1}{t(s)} = -\sigma(s) \qquad t(s) = \frac{1}{\mathrm{Re}t^{-1}(s) - i\sigma(s)}$$

$$\sigma = \sqrt{1 - 4m^2/s}$$

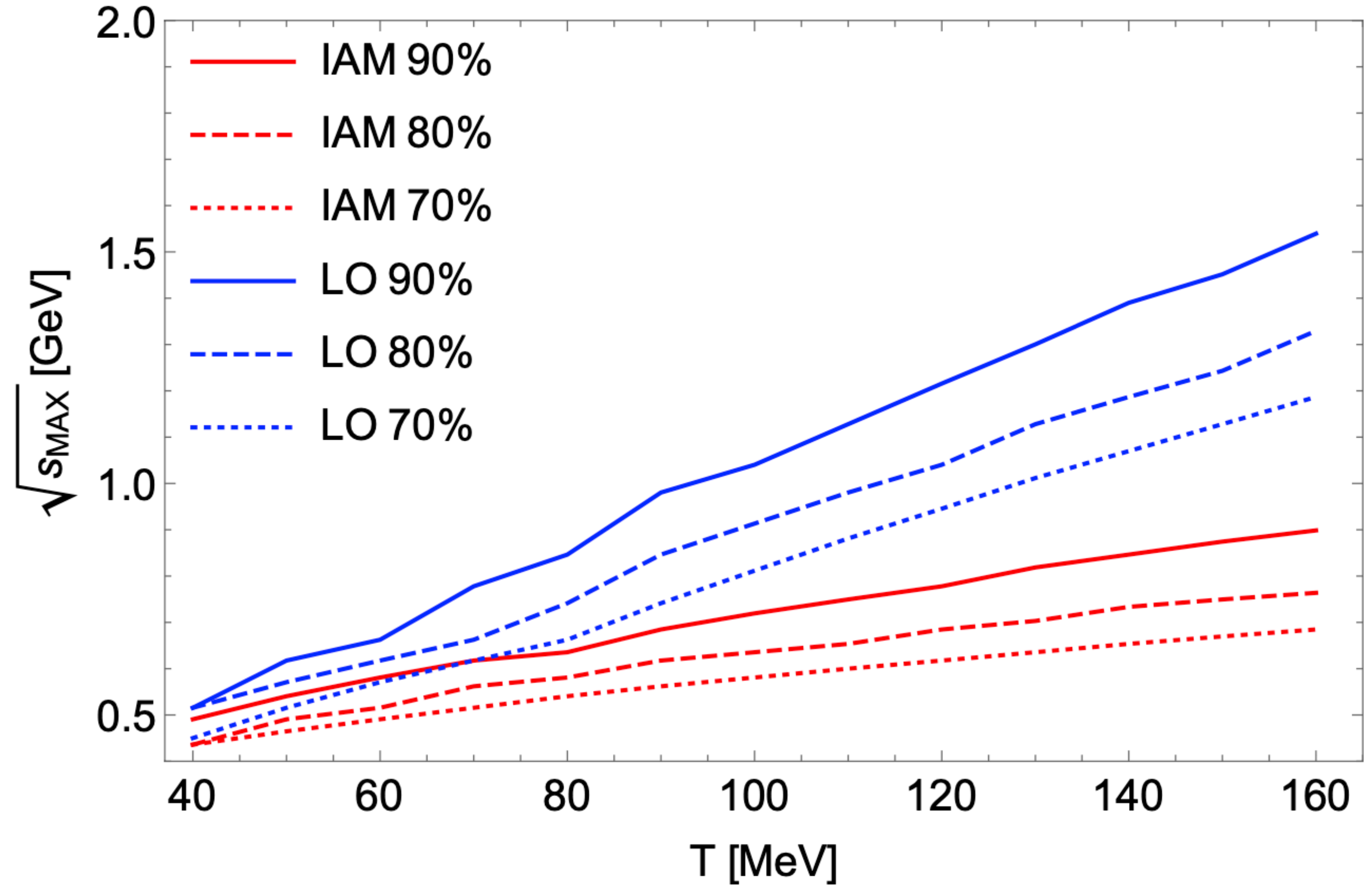
Replace Re t^{-1} by $\mathcal{O}(p^4)$ ChPT expansion

$$t^{\mathrm{IAM}}(s) \simeq \frac{t_2^2(s)}{t_2(s) - t_4(s)}$$

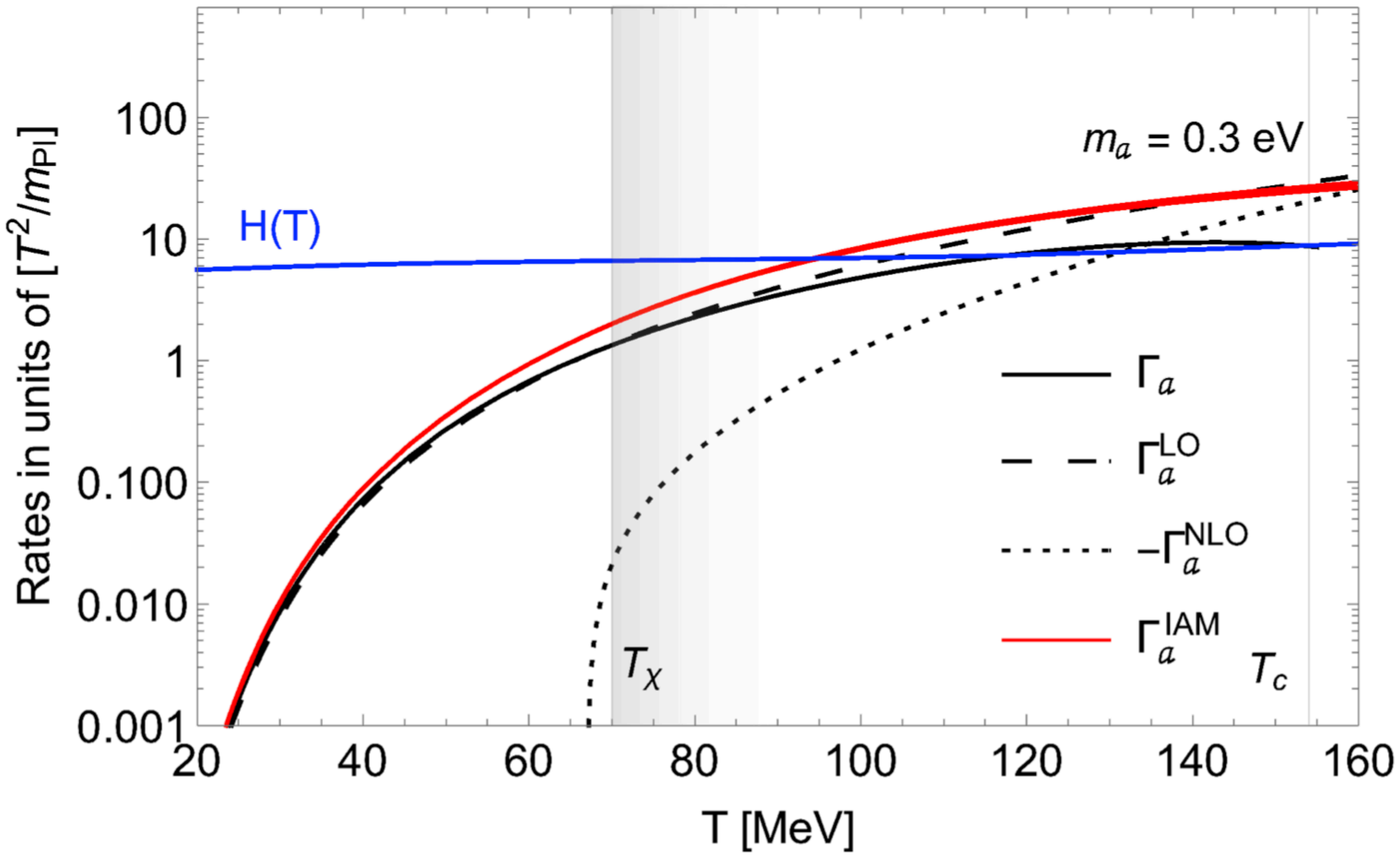
satisfies unitarity

Reproduces simultaneously the low-energy expansion (Padé approx.) and the lightest resonances without including them explicitly in the Lagrangian

Energy contributions to thermal rate



- At least 90% of the contribution to the thermal rates in IAM stems from $\sqrt{s} < 1$ GeV, where IAM is under control



- Error: difference between the total thermal rate in IAM and the one cut at $\sqrt{s_{\text{MAX}}} < 1$ GeV

$\Delta N_{\rm eff}$, the origins

$$
\begin{aligned}
\rho =&\rho_\gamma + \rho_\nu + \rho_a \\
=&\rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}^{\rm SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4\right] \\
=&\rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]
\end{aligned}
$$

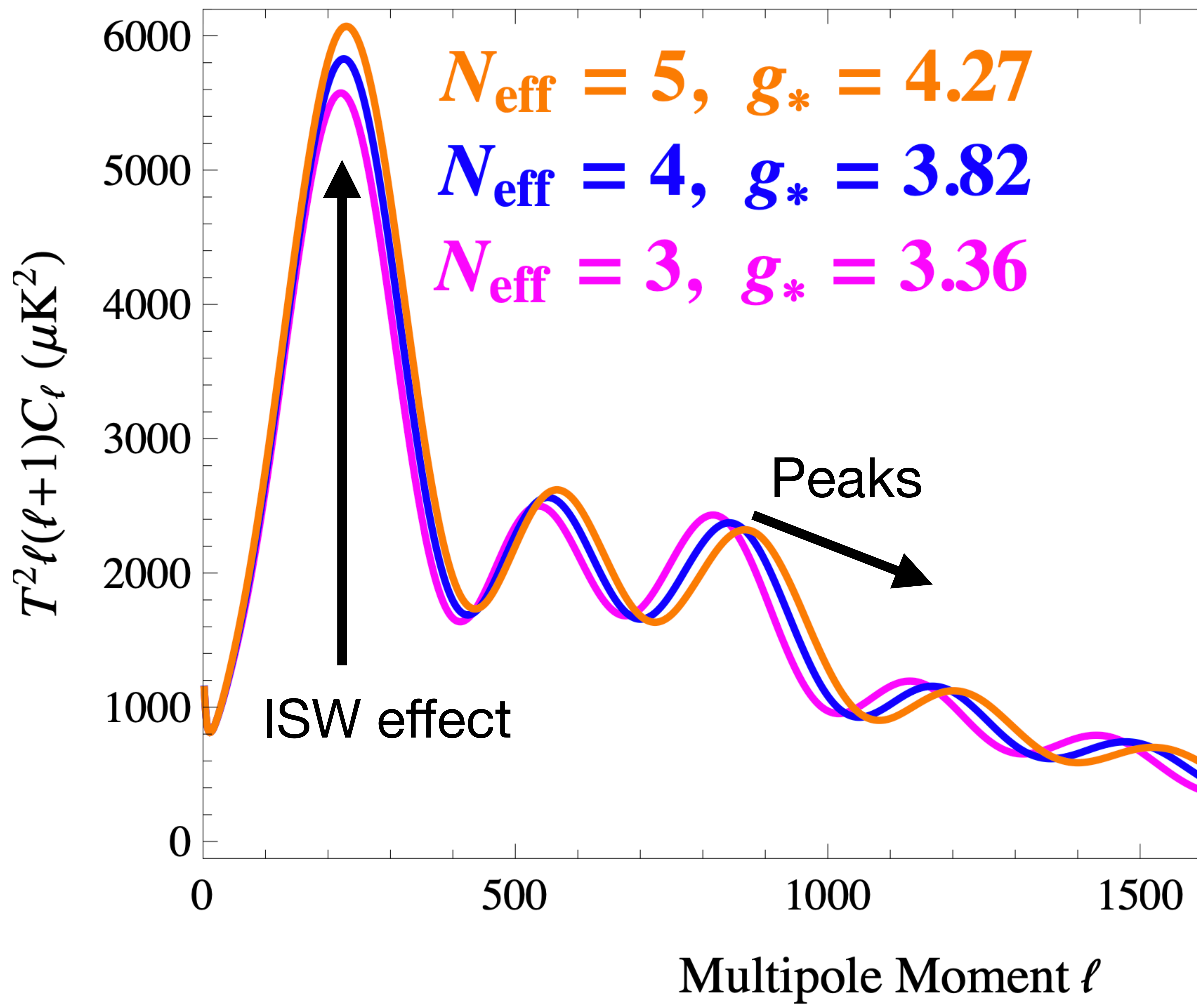
$$\Delta N_{\rm eff} = N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4$$

$$\frac{T_a}{T_\nu} = \left(\frac{43}{4g_S(T_D)}\right)^{1/3}$$

$$\Delta N_{\rm eff} = 0.027\left(\frac{106.75}{g_S(T_D)}\right)^{4/3}$$

Effects of $N_{\rm eff}$ on the CMB

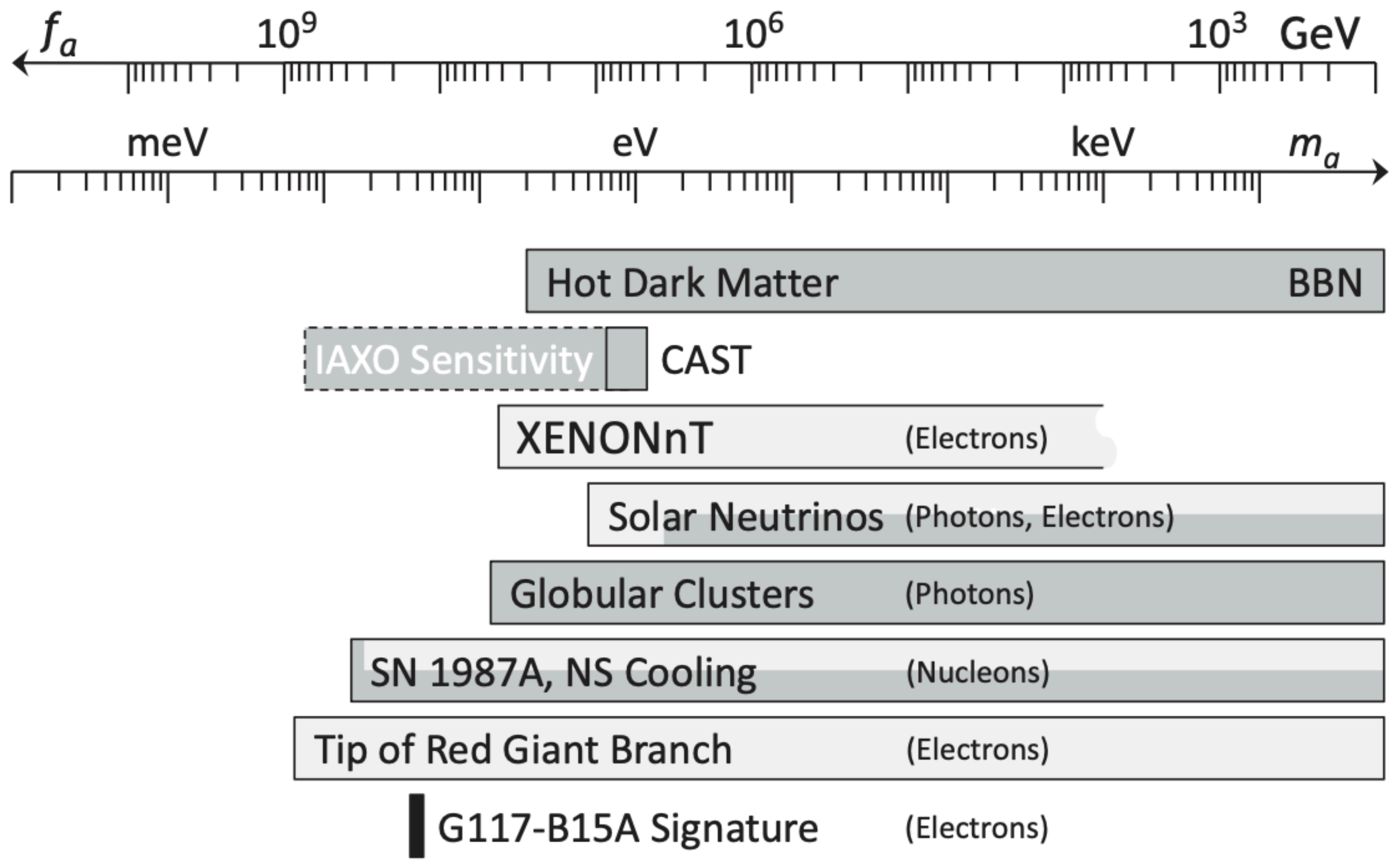
- $N_{\rm eff} \uparrow \Rightarrow H \uparrow$, time for photons diffusion in the plasma decreases, reducing Silk damping and restricting it to higher ℓ. $\ell_{\rm dump} \uparrow$
- $H \uparrow$ Acoustic oscillation length scale decreases, increasing the sound horizon. $\ell_{\rm sound} \uparrow$
- Overall less dumping but more peaks dumped. $H \uparrow \Rightarrow \ell_s/\ell_d \uparrow$
- Also, gravitational red/blue shift increased on 1st peak scales (ISW)



[Brust, Kaplan, Walters, arXiv:1303.5379]

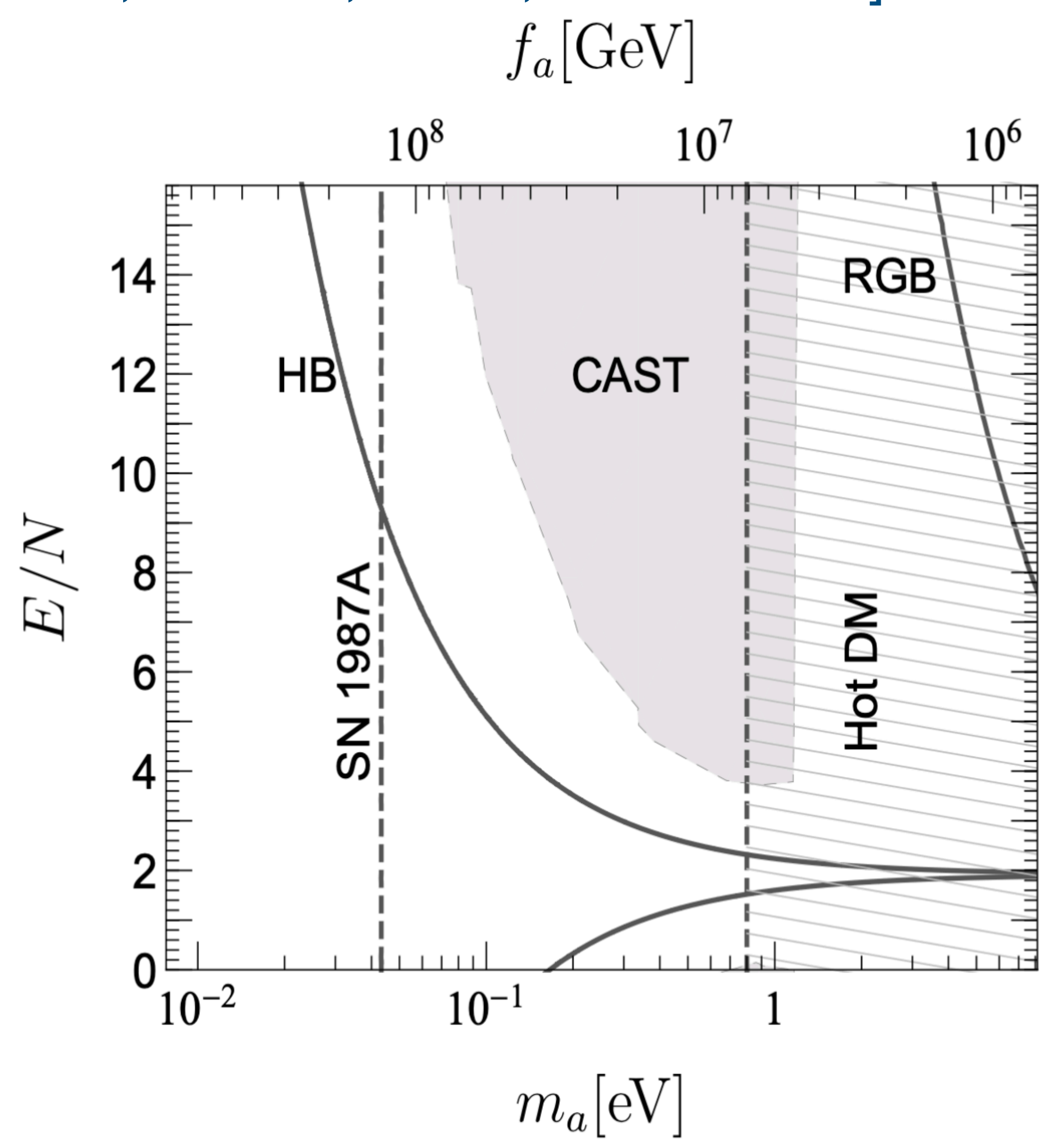
[Silk, *Astrophys.J.* 151 (1968)]
[Sachs, Wolfe, *Astrophys. J.* 147 (1967)]
[Bowen, Hansen, Melchiorri, Silk, Trotta, arXiv: astro-ph/0110636]
[Brust, Kaplan, Walters, arXiv:1303.5379]

ASTRO Bounds



From [Caputo, Raffelt (2024)]

- $g_{ae}^0 = 0$ in KSVZ models
- SN bound - large astrophysical uncertainties
  see e.g. [Bar, Blum, D'Amico, 1907.05020]
- $g_{a\gamma}$ can be accidentally suppressed
  [Di Luzio, Mescia, Nardi, 1705.05370]



[Di Luzio et al., Phys. Rept. **870** (2020)]