# The QCD axion and inflation:
# Early vs late strings

**Marco Gorghetto**

Based on: [2311.09315] **MG, Hardy, Nicolaescu, Notari, Redi; JHEP**
**+ discussions with Giovanni Villadoro**

DESY.

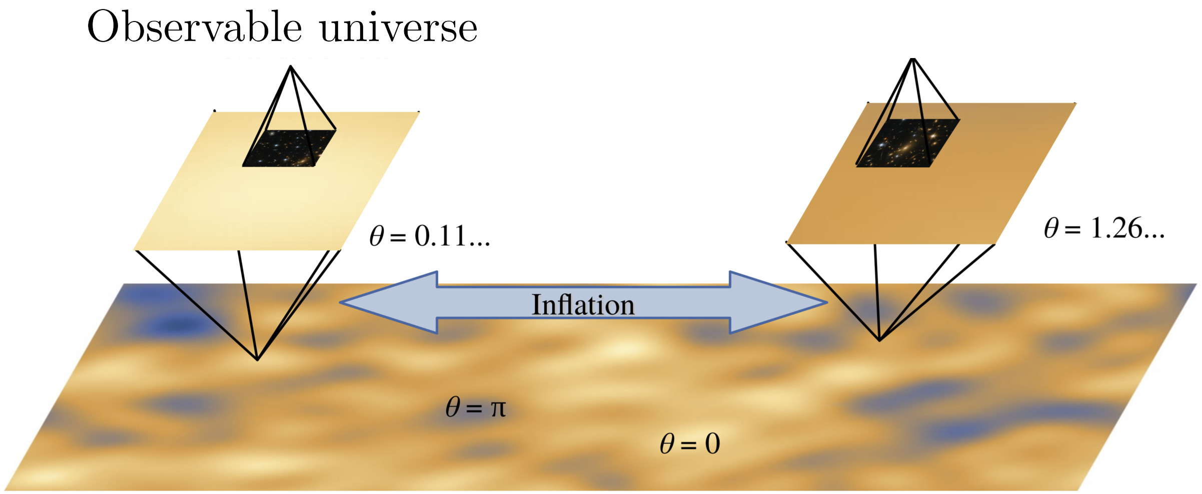
Alexander von Humboldt
Stiftung / Foundation

# Initial conditions from inflation

$(T_{\rm reheating} \ll f_a)$

## Pre-inflationary scenario

$H_I \lesssim f_a$



$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

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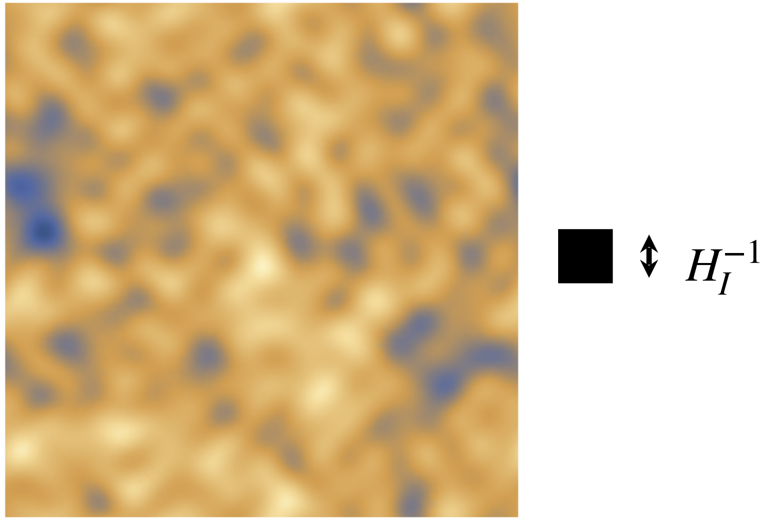

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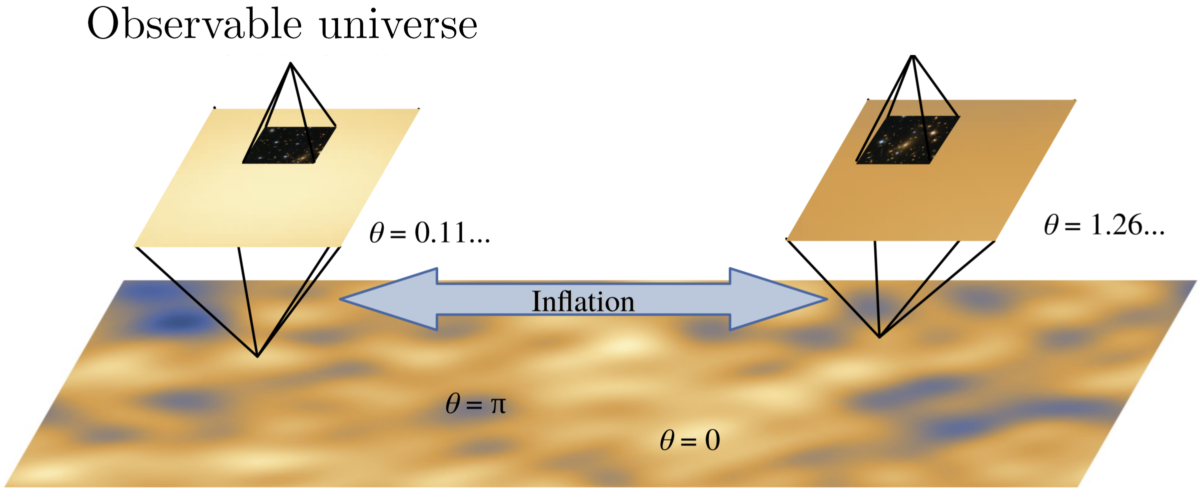

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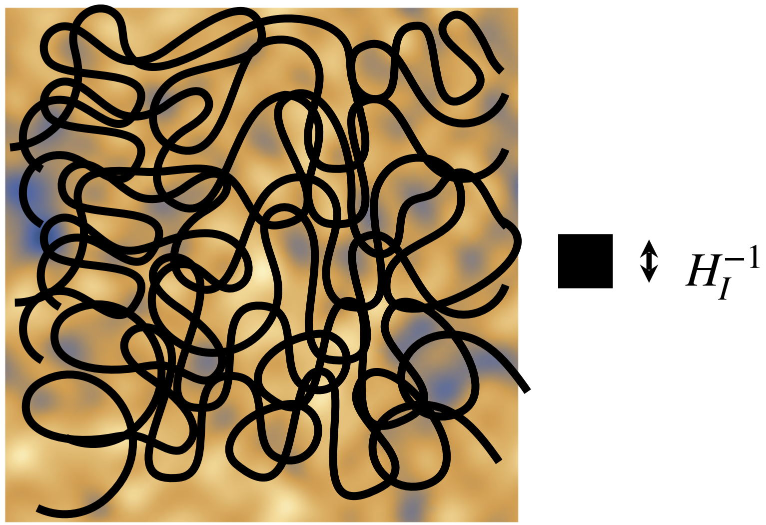

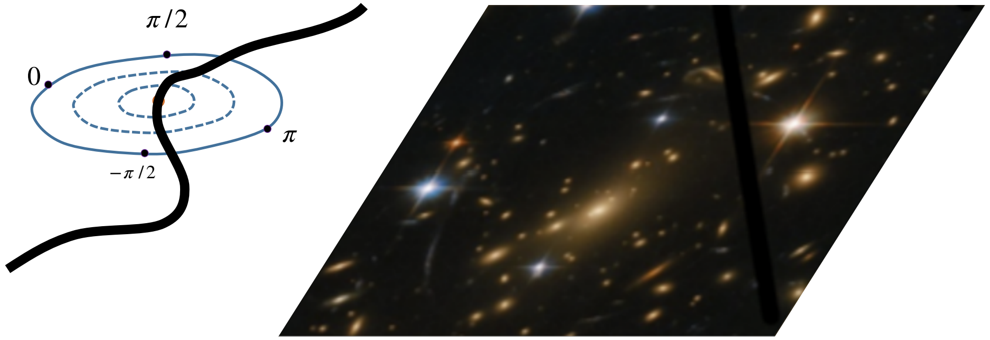

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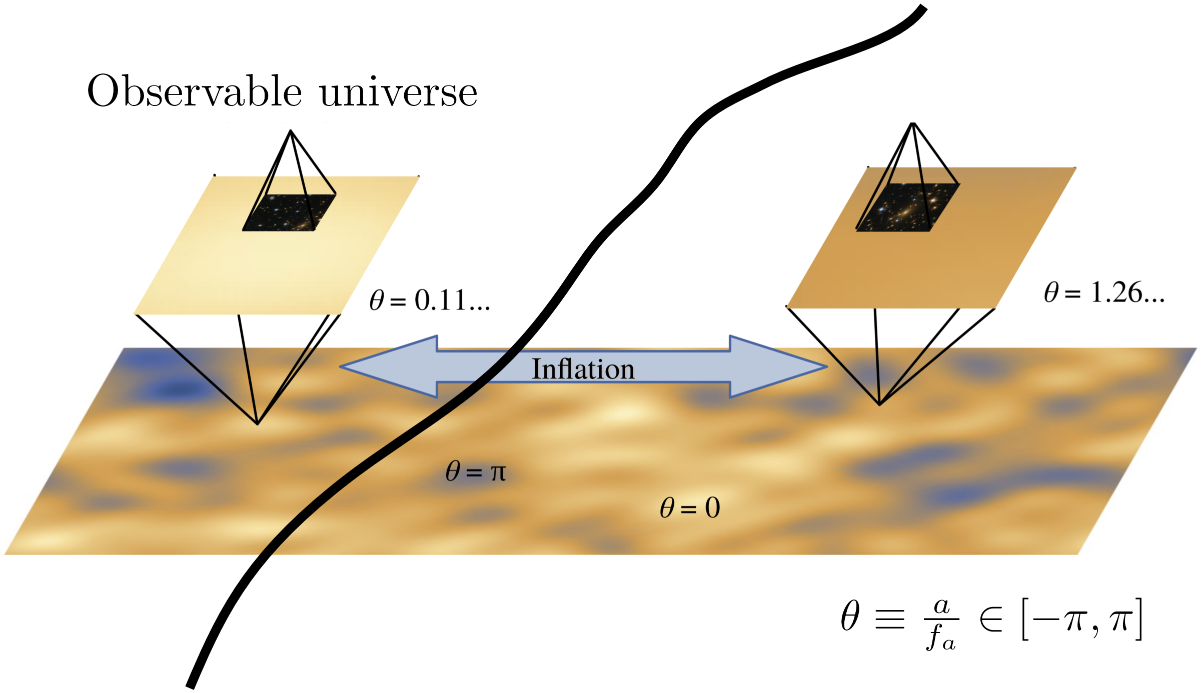

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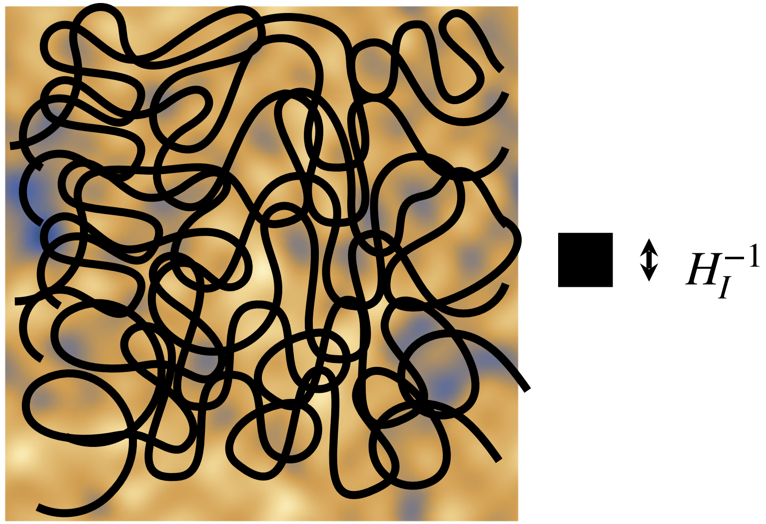

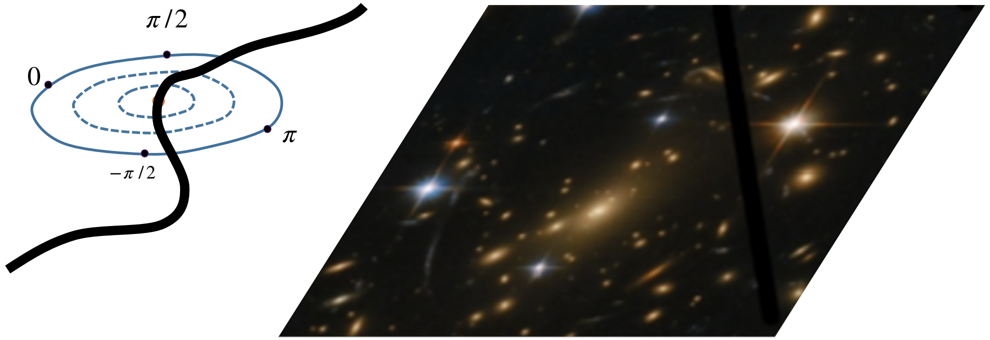

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$H_I/f_a \ll 1$

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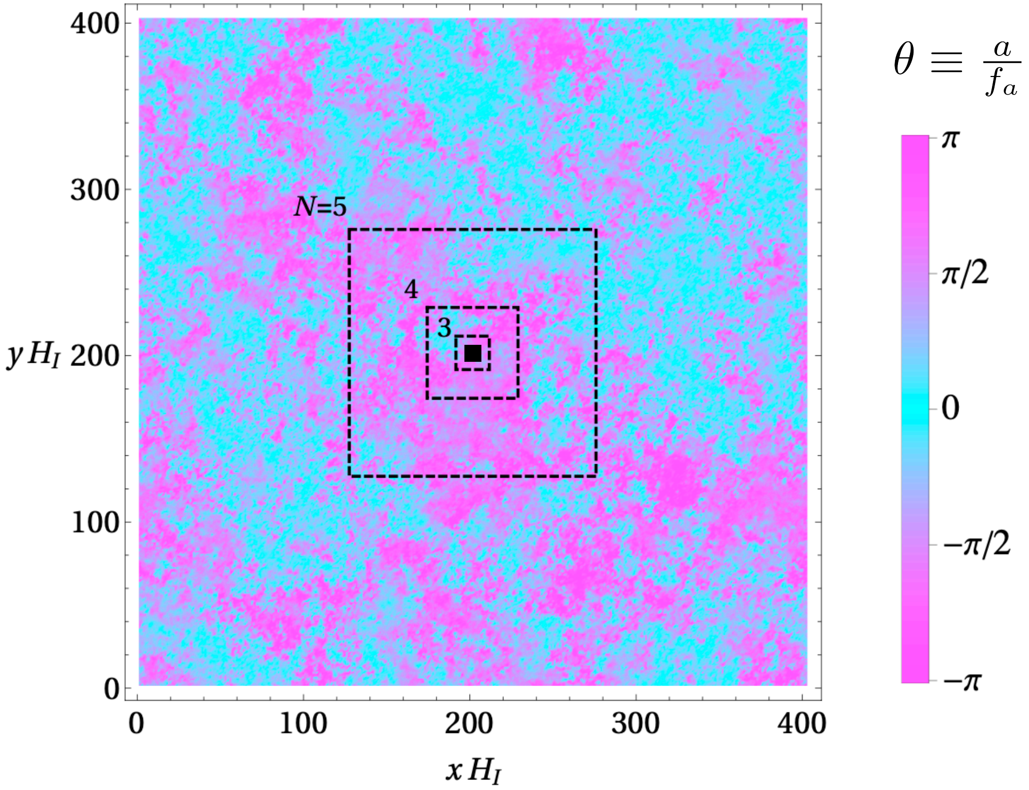

Inhomogeneities appear on scales $e^N H_I^{-1}$ , that can be parametrically larger than $H_I^{-1}$

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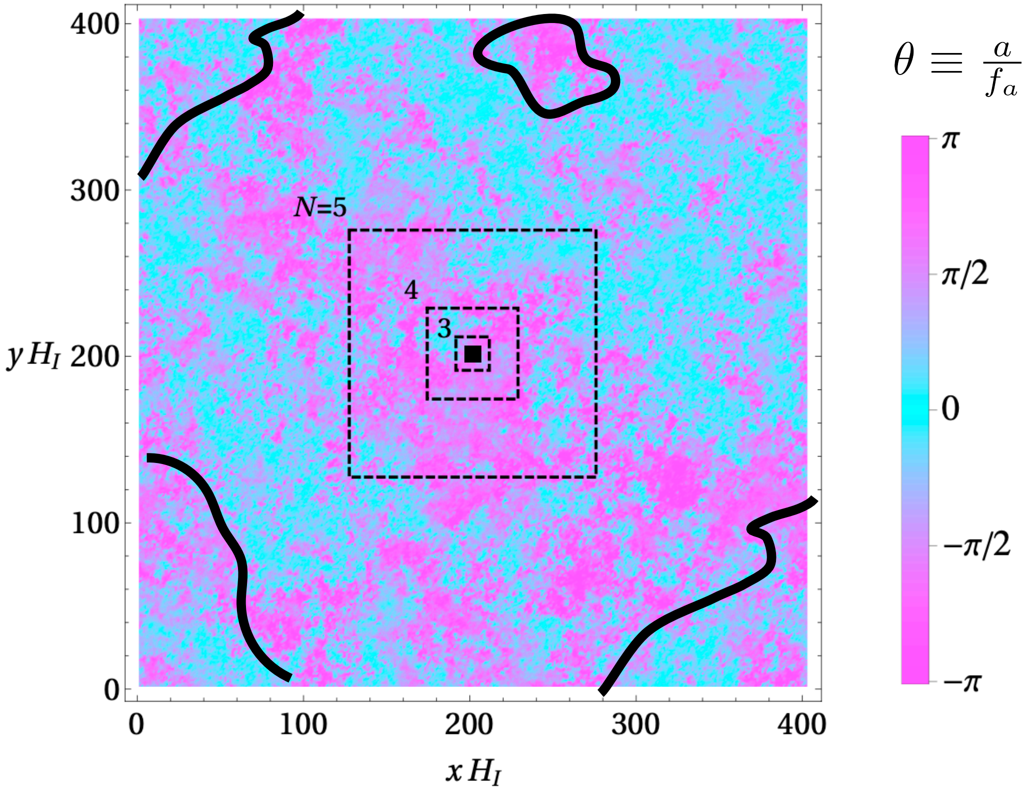

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→ Scalar $\mathbf{\Phi}$ with potential (e.g. KSVZ model)

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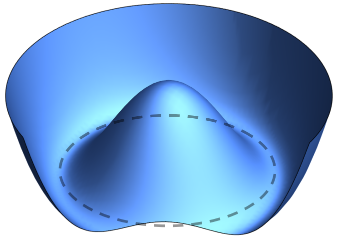
radial mode

axion

$$m_\rho^2 = 2\lambda f_a^2$$



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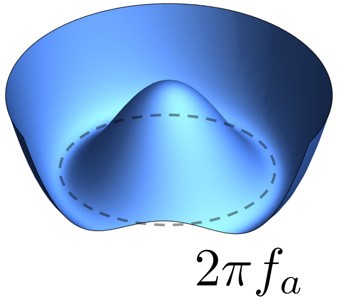

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→ Evolution during inflation with approximately constant $H_I$: free parameters: $(H_I/f_a, \lambda)$

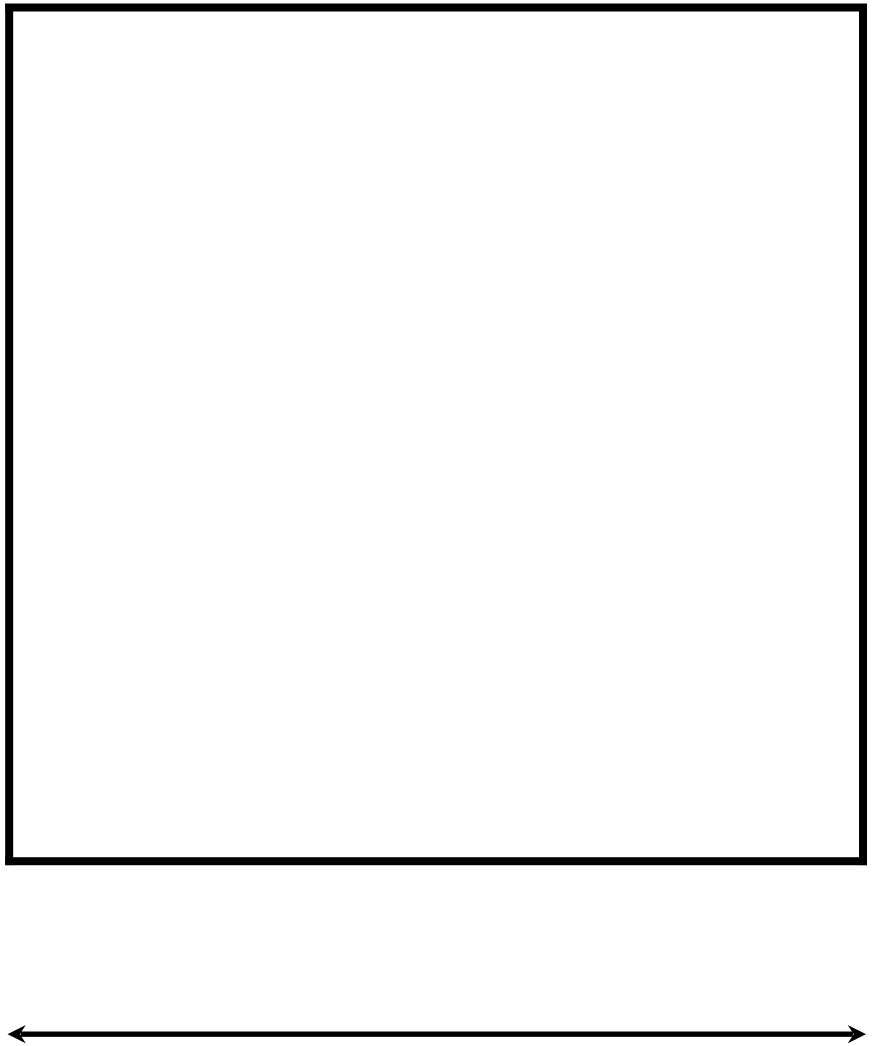
spatial average over Hubble of massless fields has de Sitter fluctuations $\simeq H_I/2\pi$

$$m^2 < 2H_I^2$$

initial condition

$\Phi = \Phi_0$

$|\delta\Phi(k)| \simeq H_I/(2\pi)$



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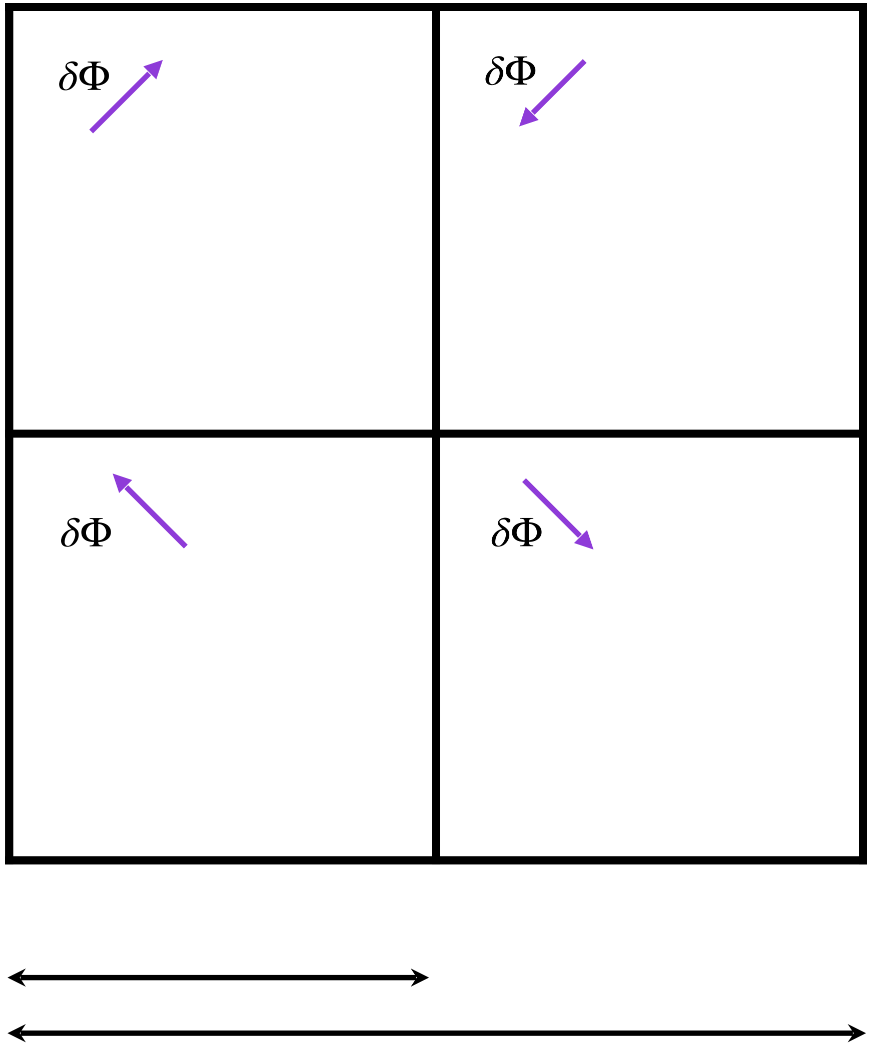

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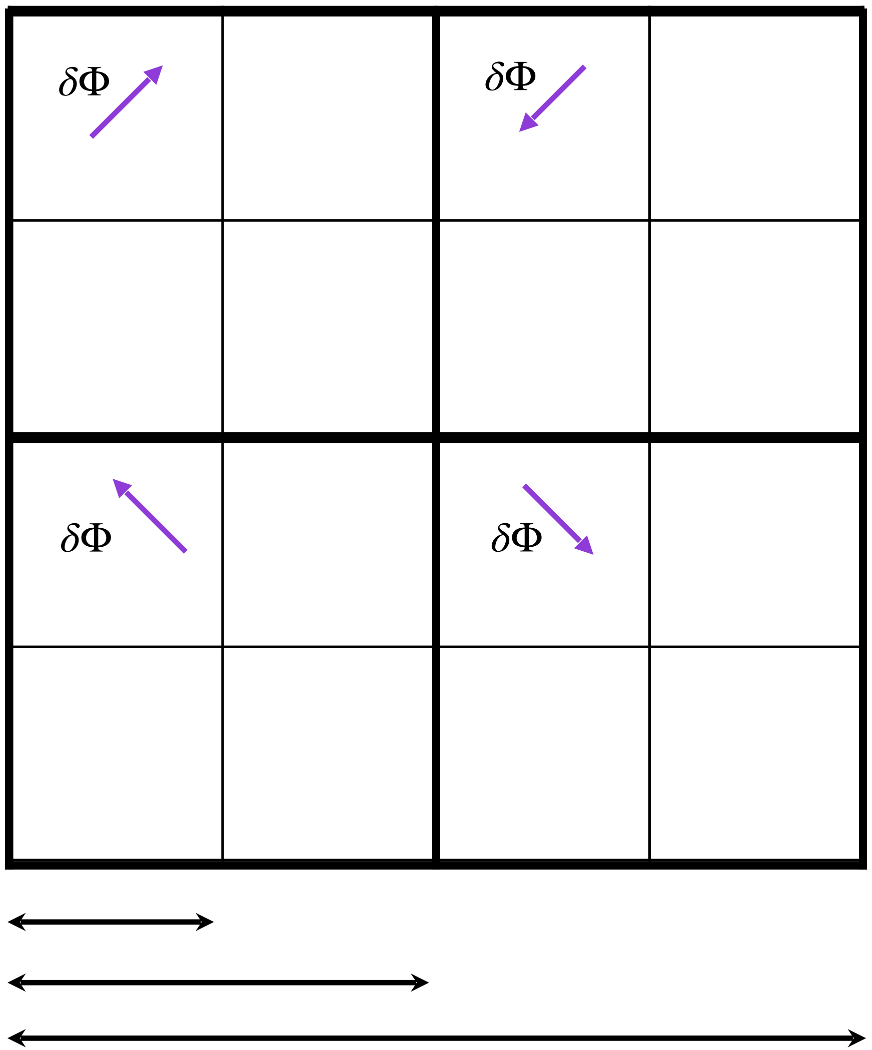

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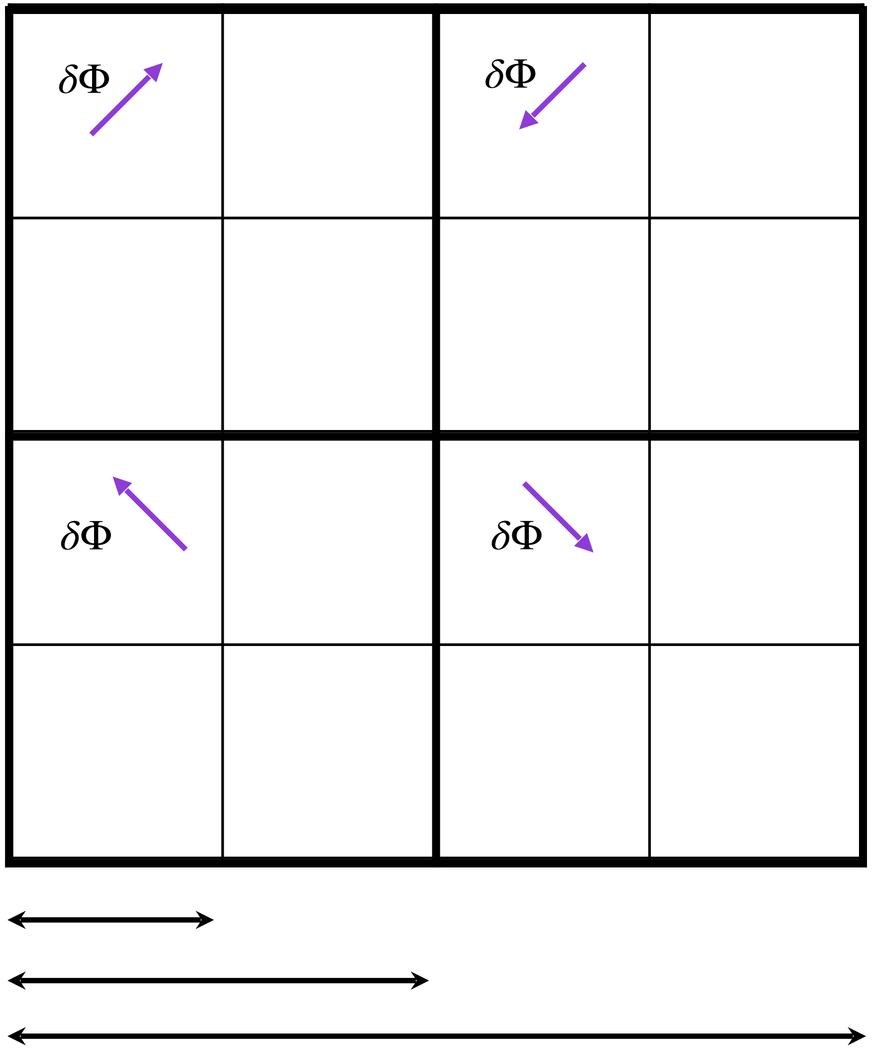

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$P(t, \Phi) =$ probability distribution

$$\int d\phi_1 d\phi_2 P = 1$$

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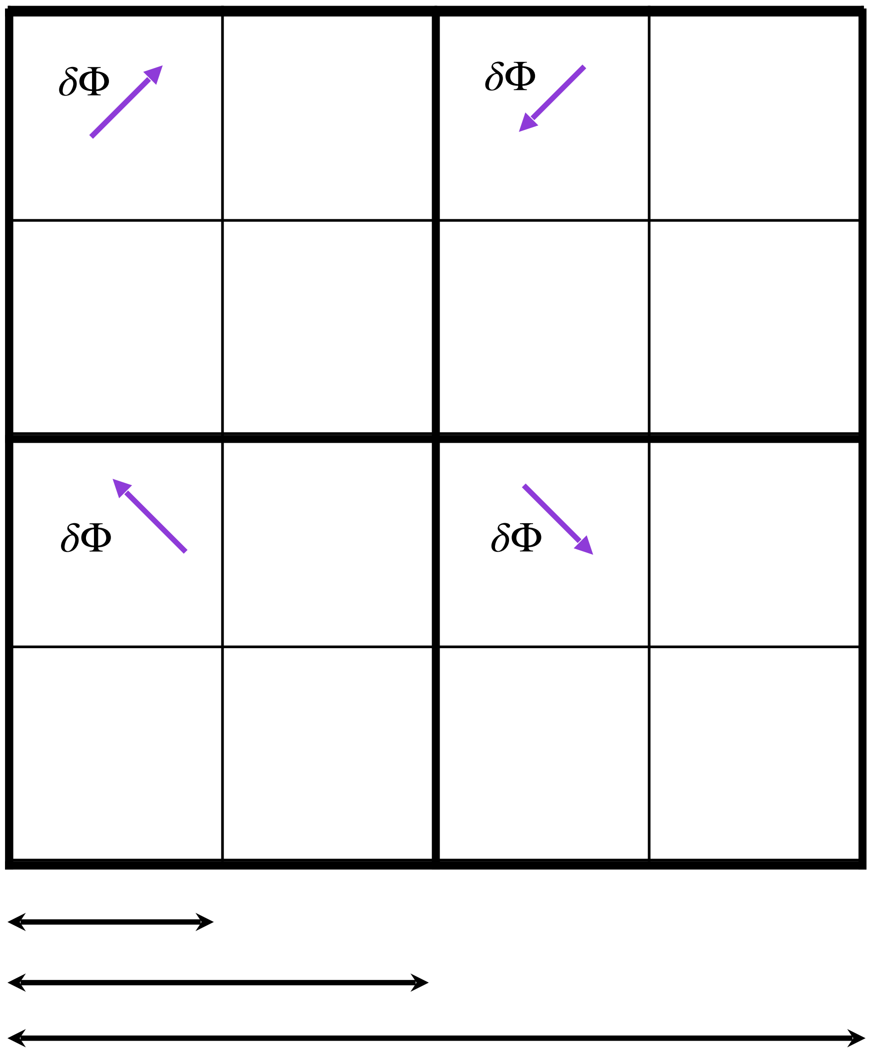

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Fokker-Planck

$$\frac{\partial P}{\partial t} = \frac{H_I^3}{8\pi^2}\sum_i \partial_i\partial_i P + \frac{1}{3H_I}\partial_i(P\partial_i V) \qquad \partial_i \equiv \frac{\partial}{\partial \phi_i}$$

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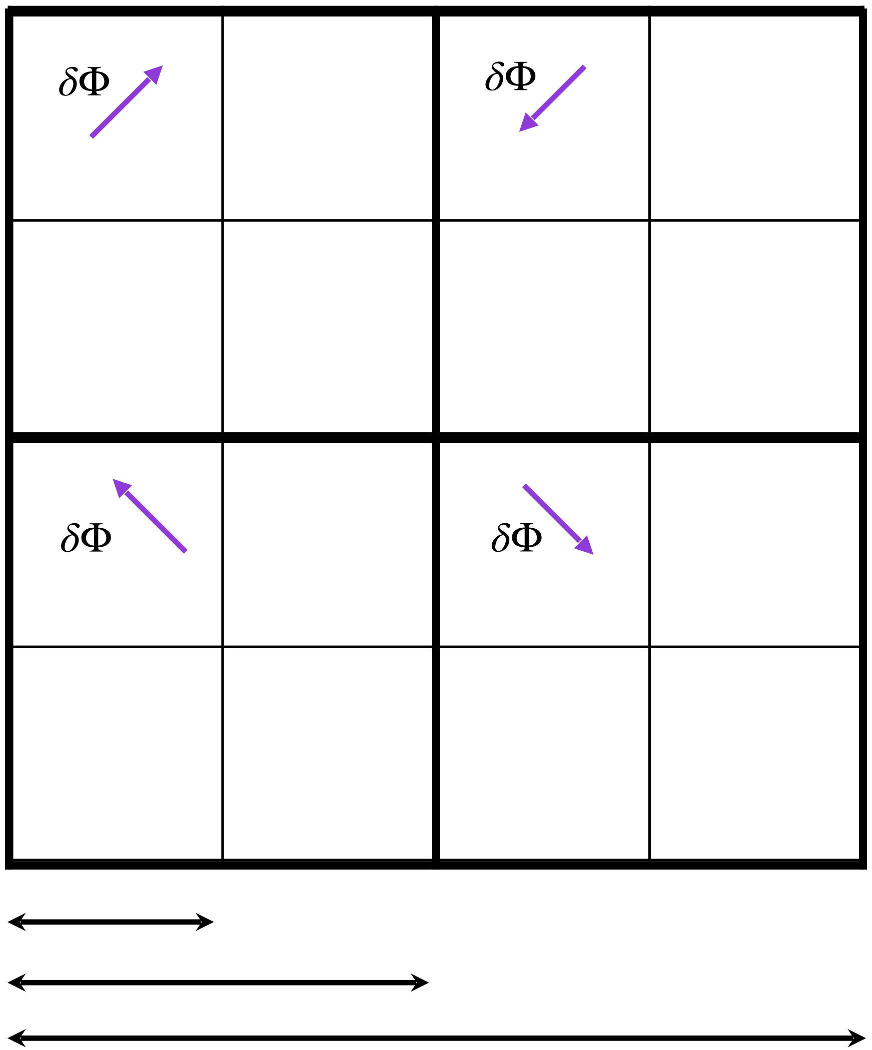

$(\boxed{R}H_I)^{-1}$

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$$\frac{\partial P}{\partial t} = \underbrace{\boxed{\frac{H_I^3}{8\pi^2}\sum_i \partial_i\partial_i P}}_{\text{diffusion from quantum fluctuations}} + \underbrace{\boxed{\frac{1}{3H_I}\partial_i(P\partial_i V)}}_{\text{classical motion}}$$

$$\partial_i \equiv \frac{\partial}{\partial\phi_i}$$

Equilibrium distribution: $$P_{\mathrm{eq}}(\Phi) \propto \exp\left[-\frac{8\pi^2 V(\Phi)}{3H_I^4}\right] \propto \exp\left[-\alpha^{-4}\left(\rho^2/f_a^2-1\right)^2\right]$$

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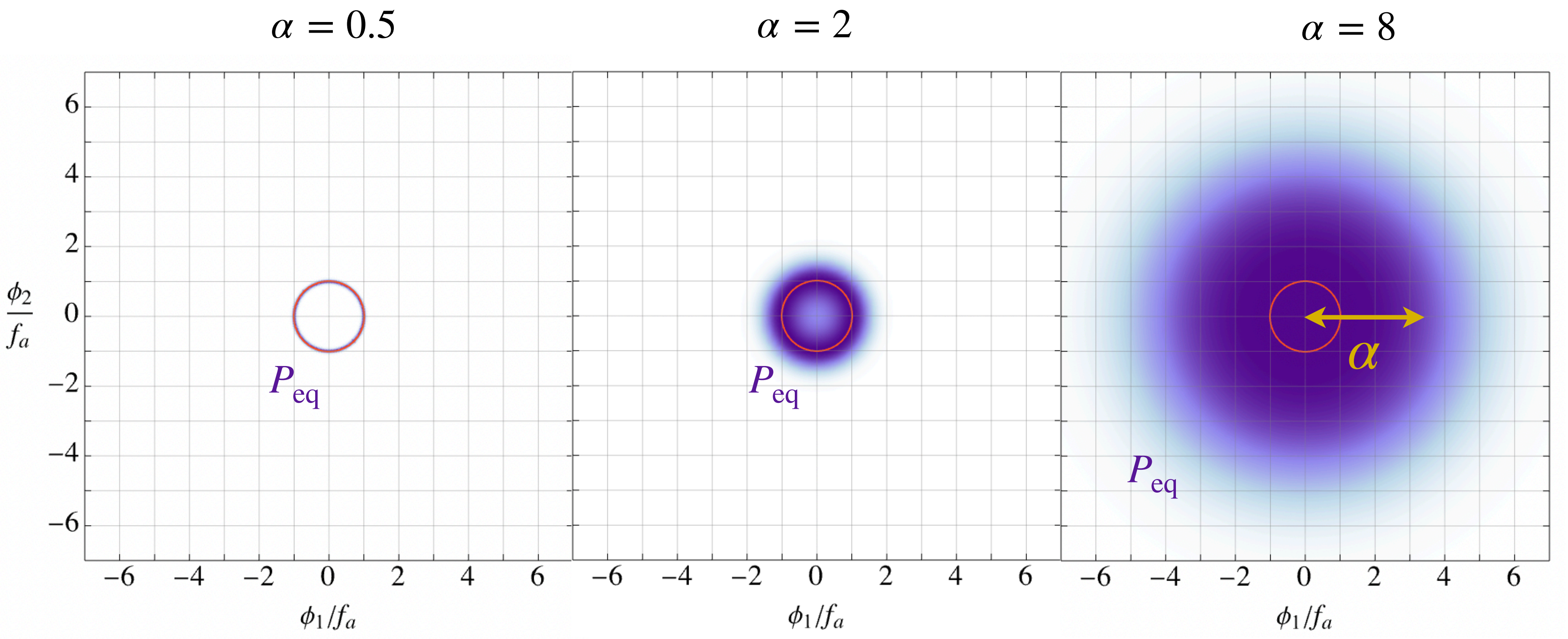


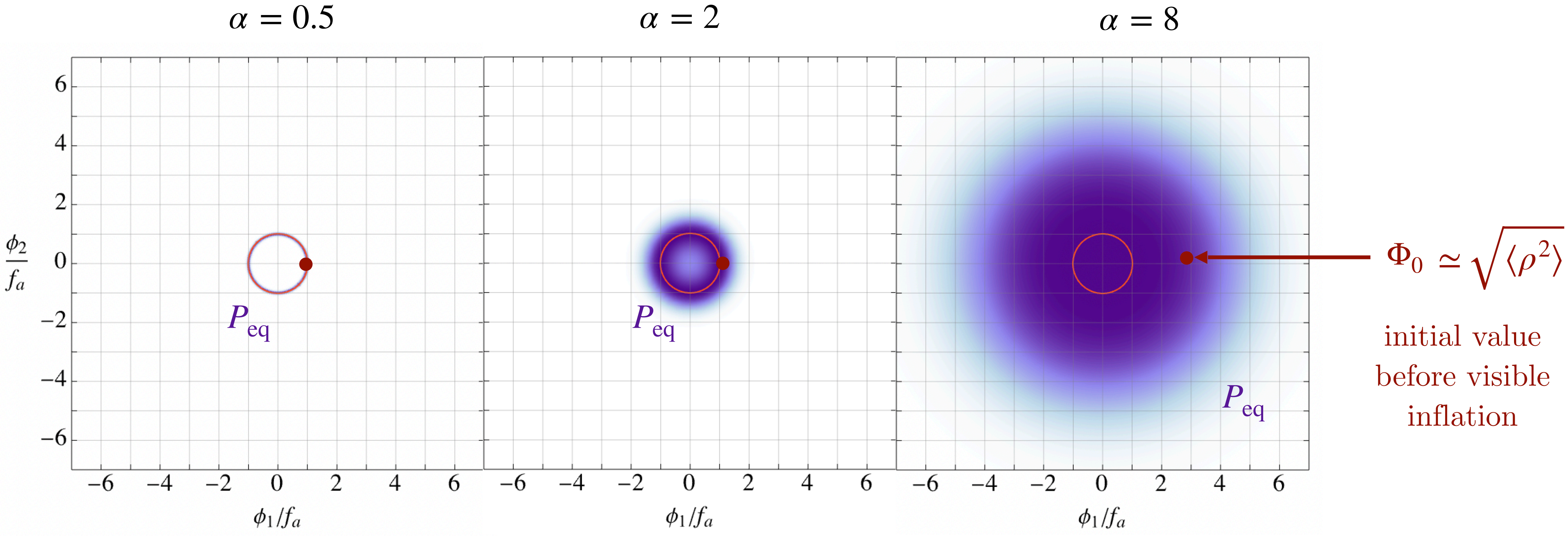
$\alpha \ll 1$

$\langle \rho^2 \rangle = f_a^2$

$\alpha \gg 1$

$\langle \rho^2 \rangle \simeq H_I^2/\lambda^{1/2} \simeq \alpha^2 f_a^2$

$N_s$ = number of e-folds during inflation needed to approach the equilibrium solution (reached asympt.)



$\Phi_0 \simeq \sqrt{\langle \rho^2 \rangle}$

initial value
before visible
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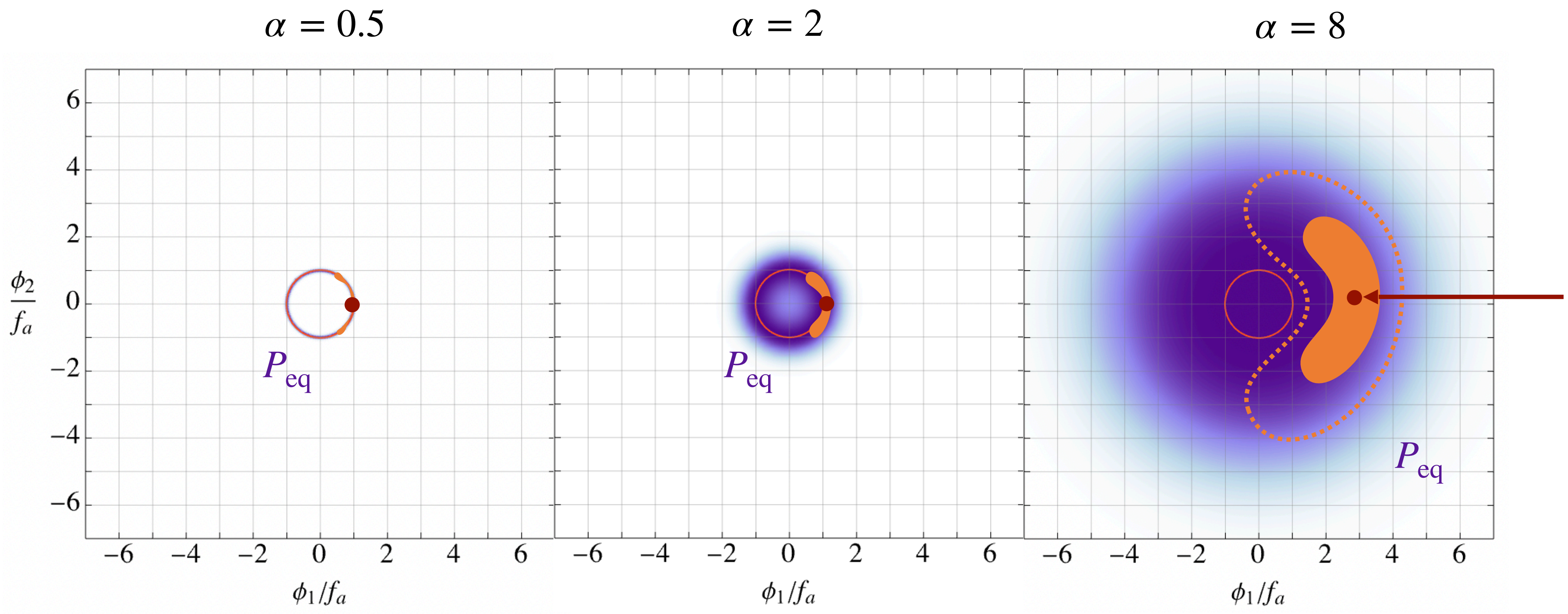

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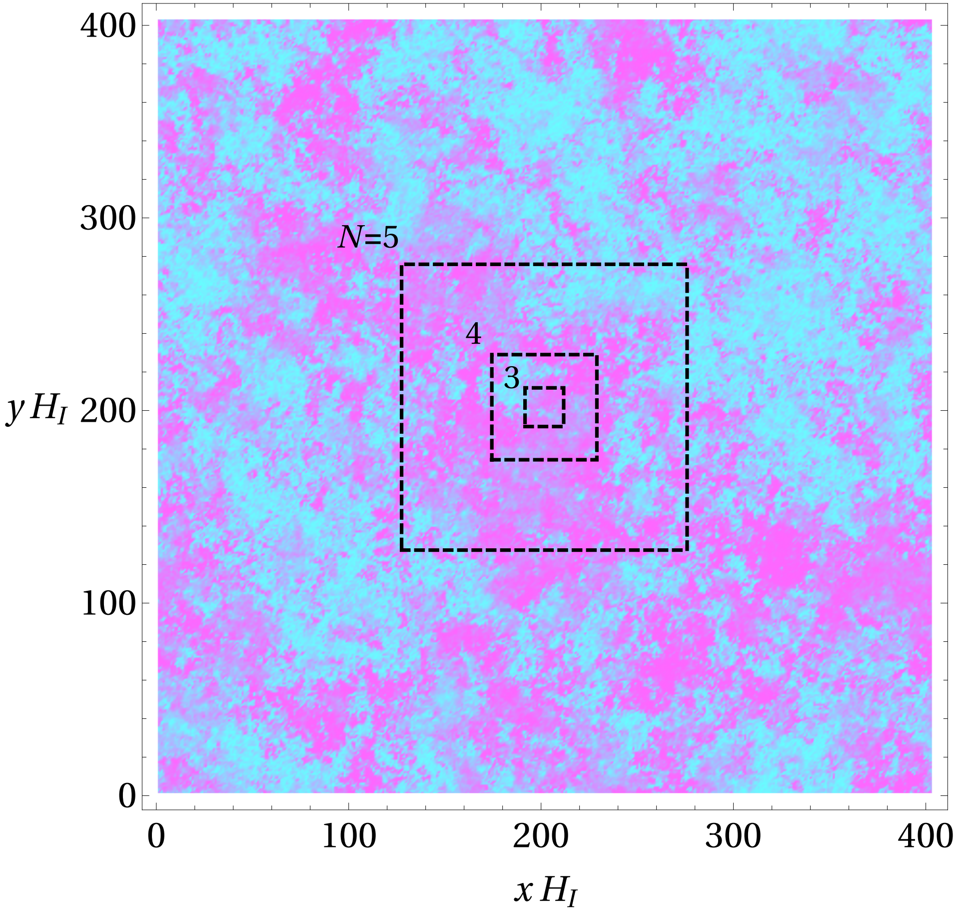
At the end of inflation, over length scales:

- $> e^{N_s} H_I^{-1}$ the angular field is random
- $< e^{N_s} H_I^{-1}$ the angular field is homogeneous



$H_I / f_a = \pi$

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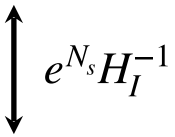

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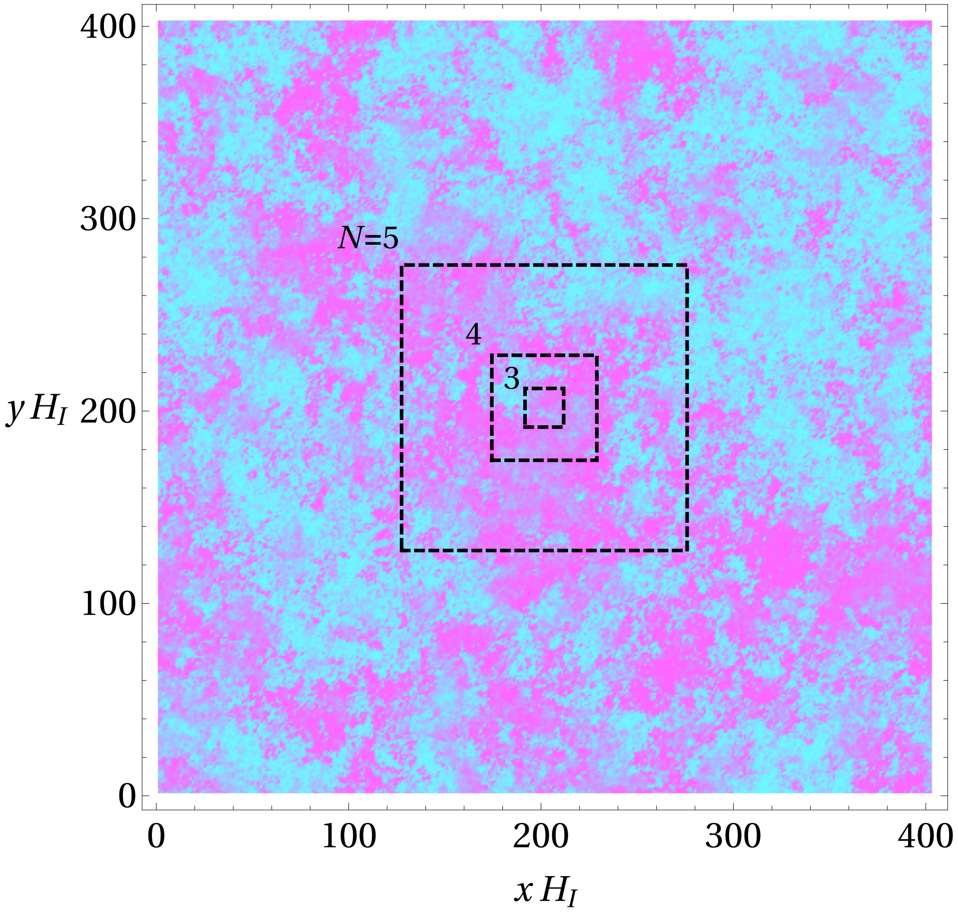


$H_I/f_a = \pi$

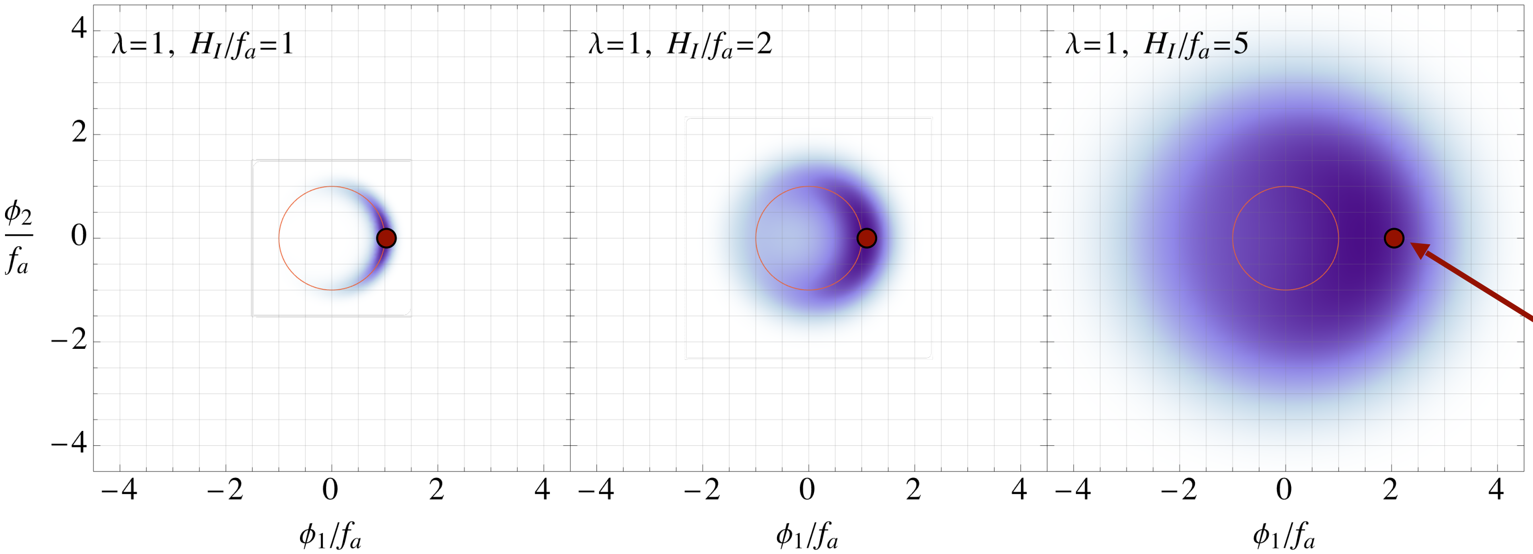
$\lambda \ll 1$

$\updownarrow$ $e^{N_s}H_I^{-1}$

Peccei-Quinn is restored (on average):

- in our Hubble patch, for $N_s \gtrsim 50-60$
- in a patch that re-enters the horizon after the QCD phase transition, for $N_s \gtrsim 25$

$P(t_{N=25}, \Phi)$ after $N = 25$ e-folds ( = field distribution over a patch that reenters at the QCDpt)



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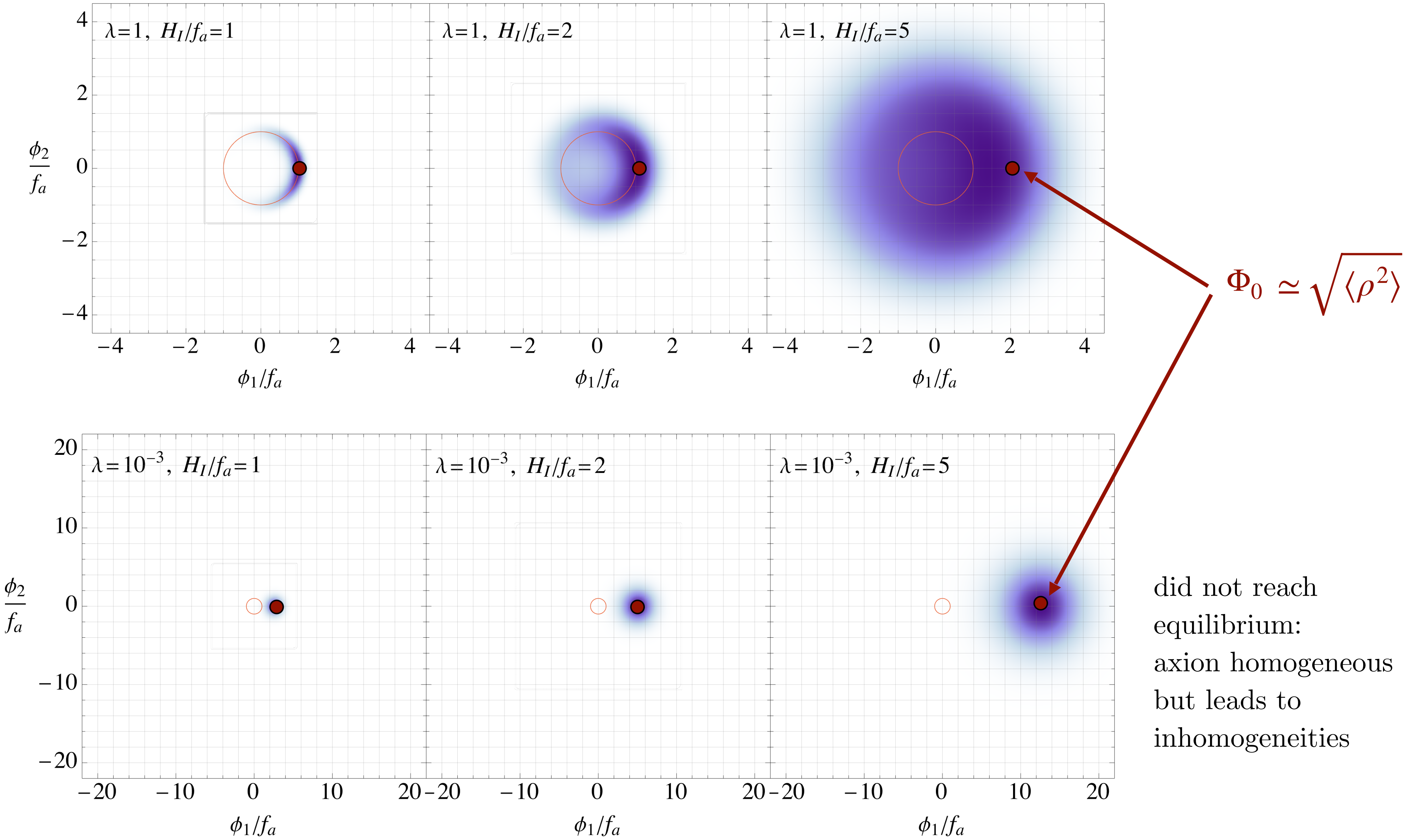

$\Phi_0 \simeq \sqrt{\langle \rho^2 \rangle}$

did not reach
equilibrium:
axion homogeneous
but leads to
inhomogeneities

# Evolution after inflation and string formation

If $N_s \gg 1$ we can neglect the gradients →

$$\ddot{\rho} + \underset{\uparrow \atop H(t)}{\frac{2}{t}}\dot{\rho} + \lambda(\underset{\uparrow \atop \partial V/\partial\rho}{\rho^2 - f_a^2})\rho = 0$$

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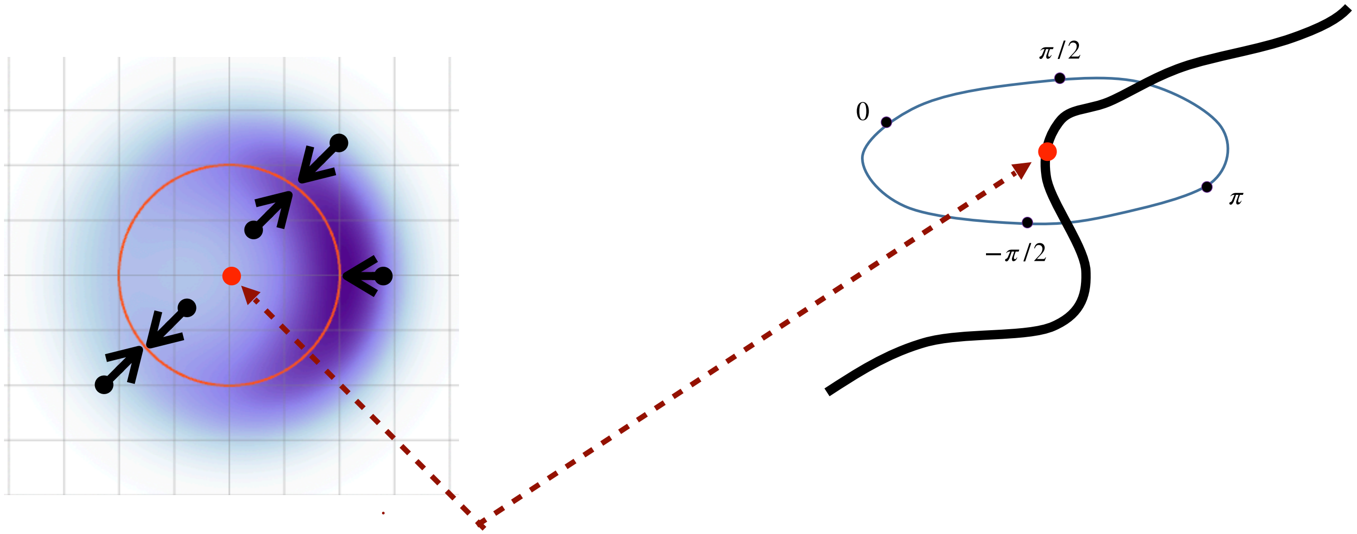
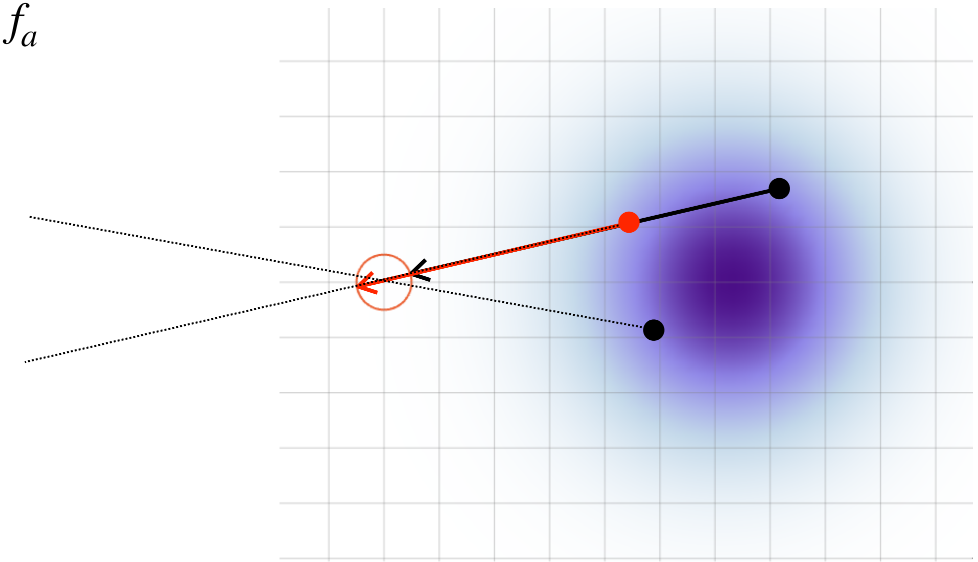
1) If $\rho \lesssim f_a$ , patches go to the nearest minimum (except at string centres)

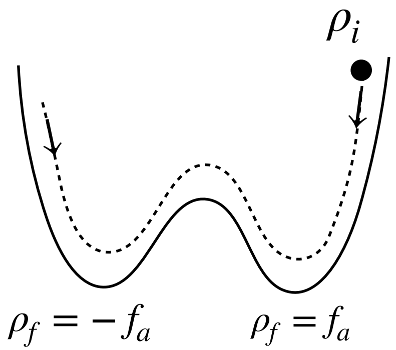


this patch becomes a string centre

**‘Inflationary formation’**

2) If $\rho \gg f_a$ , patches oscillate many times before settling down, to $\pm f_a$





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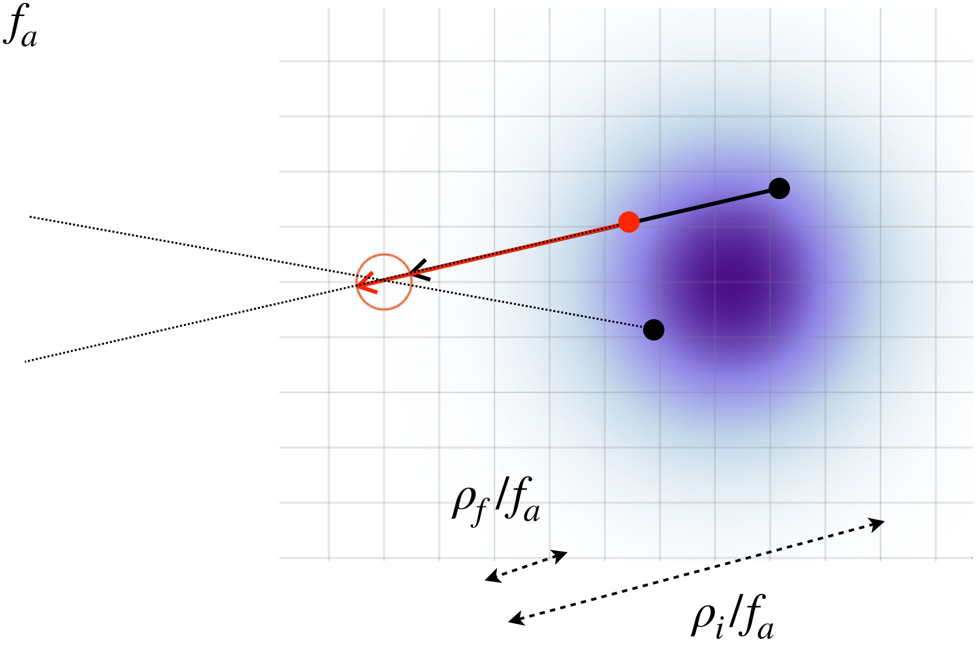
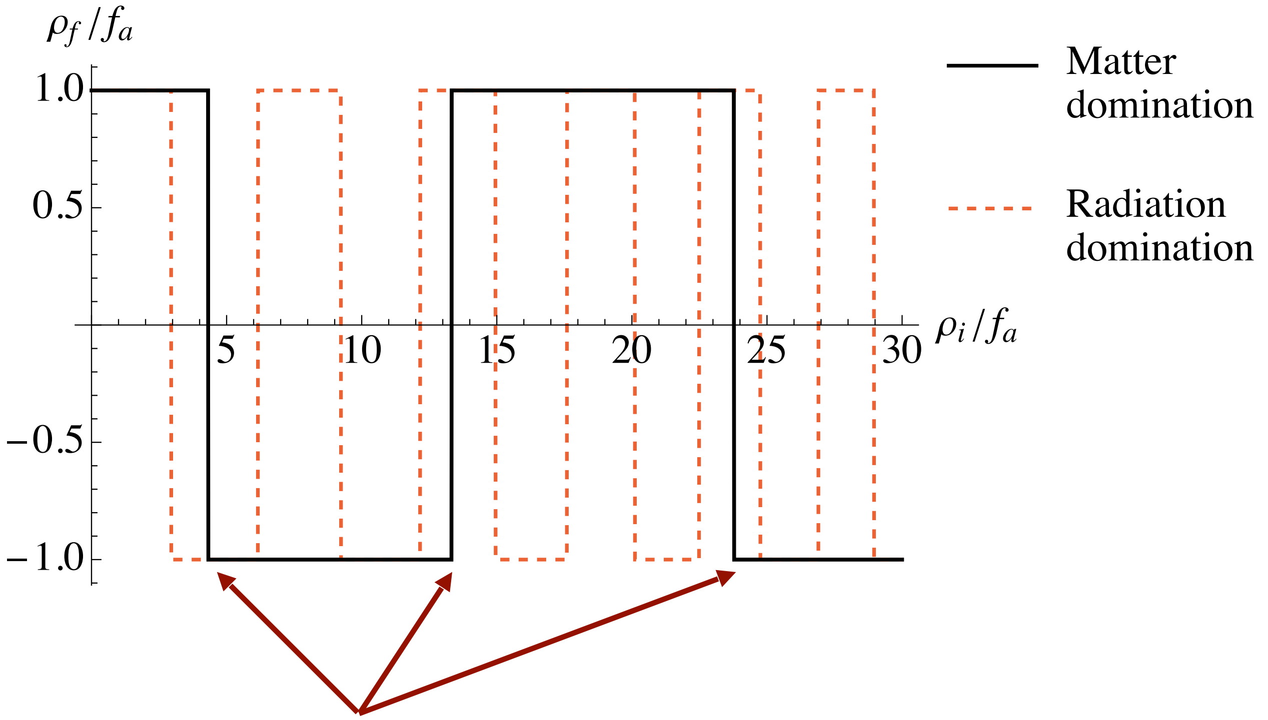
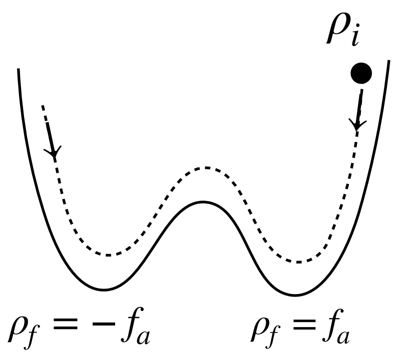
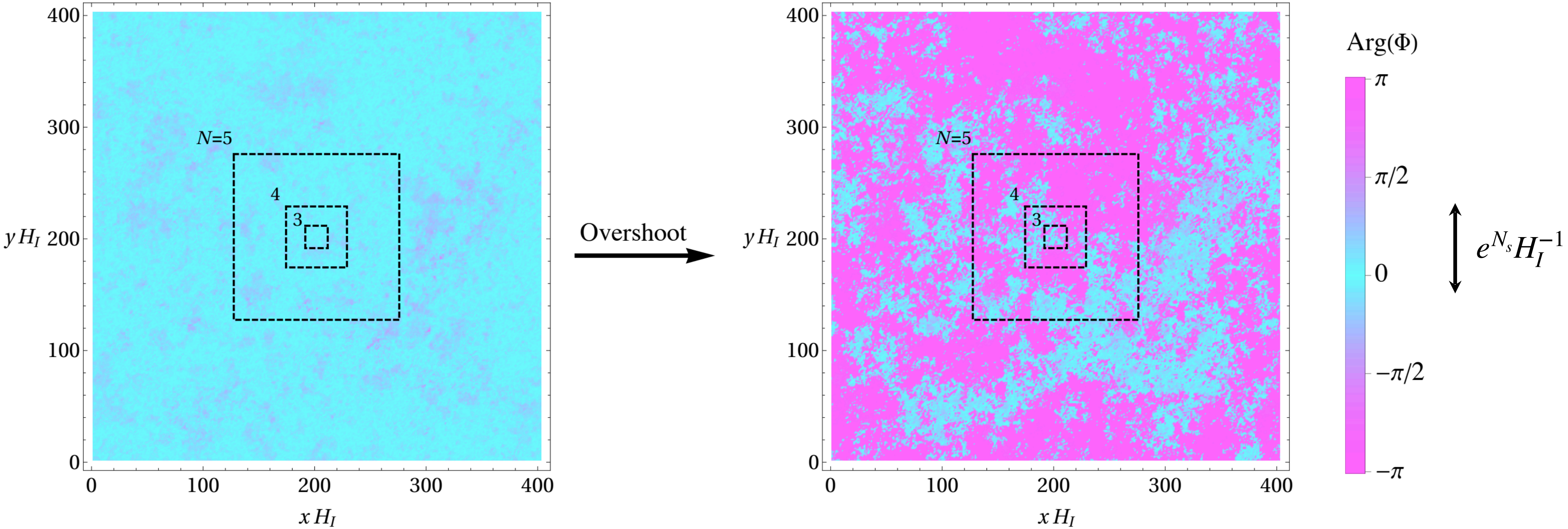


## **‘Overshoot mechanism’**



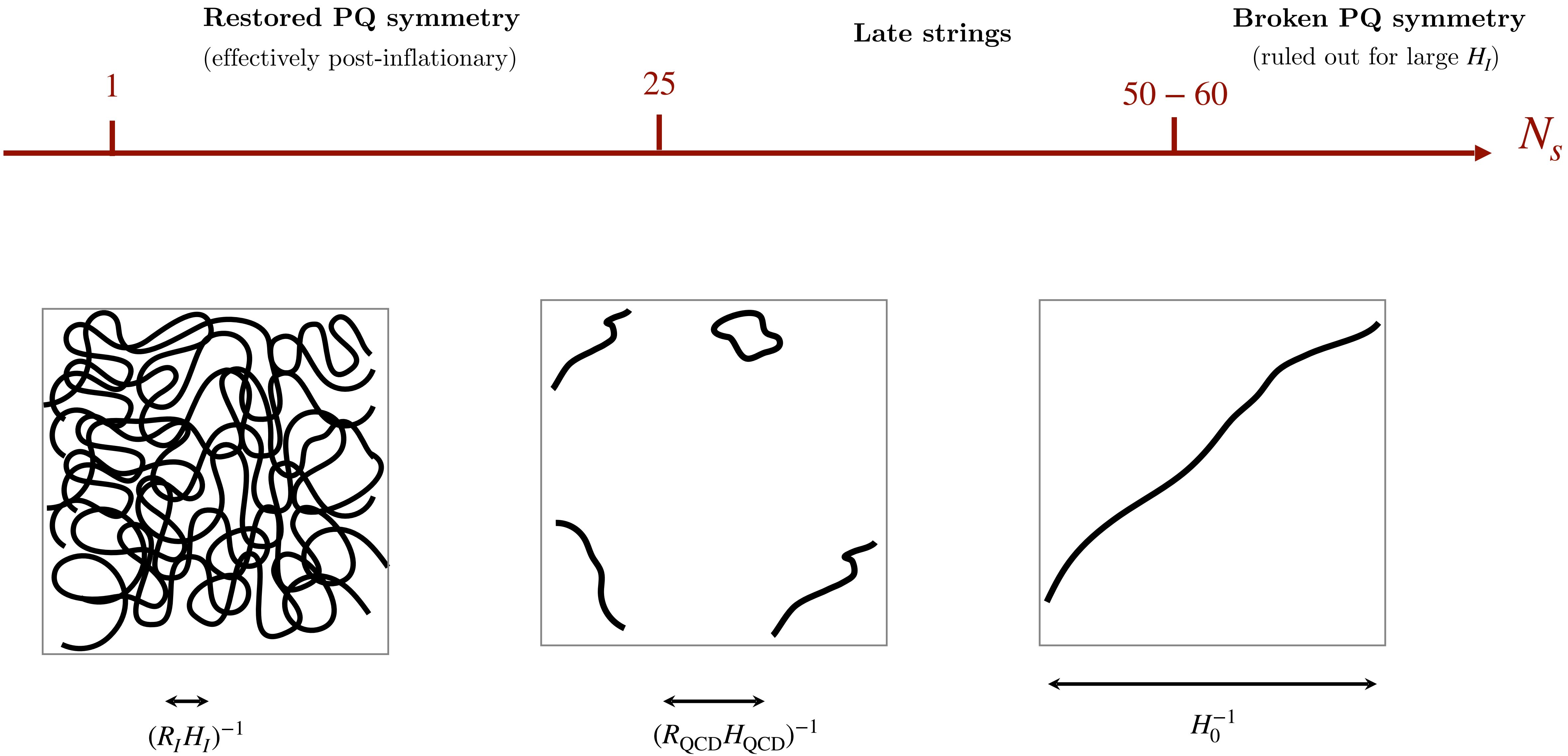
become string centres





The angular field is initially almost homogeneous and gets effectively randomised over $e^{N_s}H_I^{-1}$

# Landscape of inflationary QCD axion scenarios



**Restored PQ symmetry**

(effectively post-inflationary)

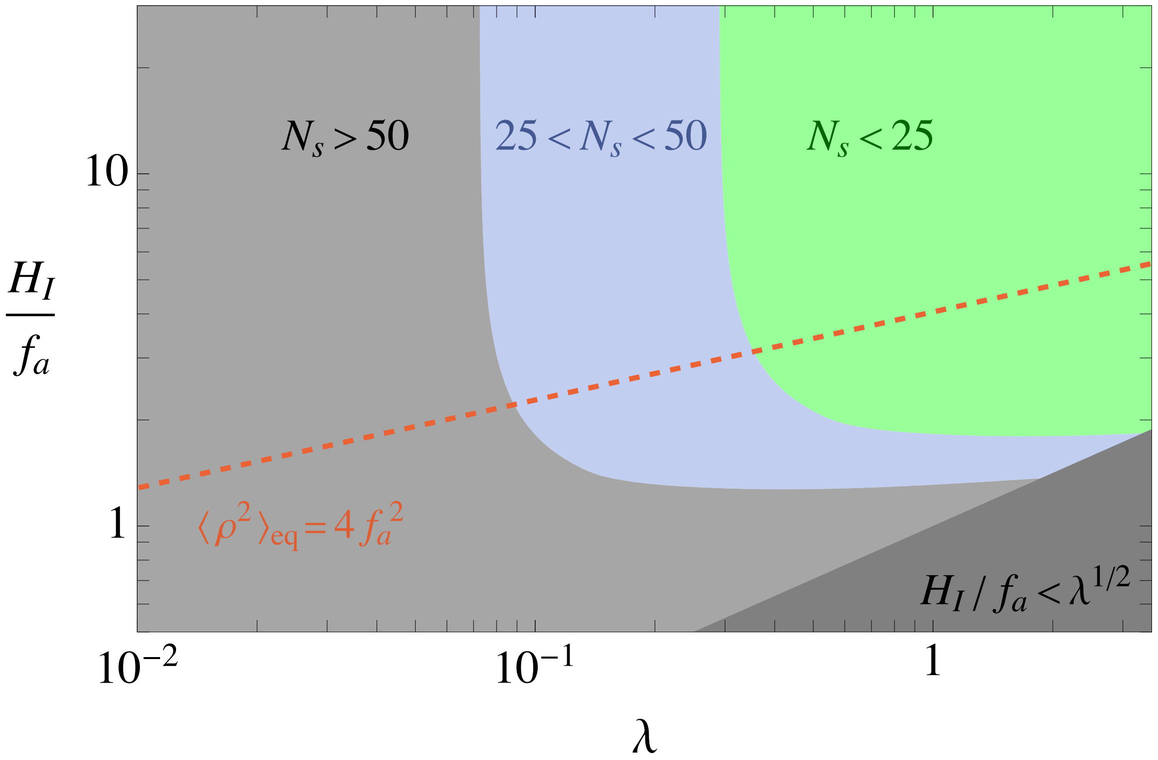
**Late strings**

**Broken PQ symmetry**

(ruled out for large $H_I$)

$$\alpha = \frac{H_I}{\lambda^{1/4} f_a}$$

Inflationary production



$$N_s \simeq \frac{80}{(H_I/f_a)^2}$$

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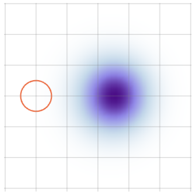

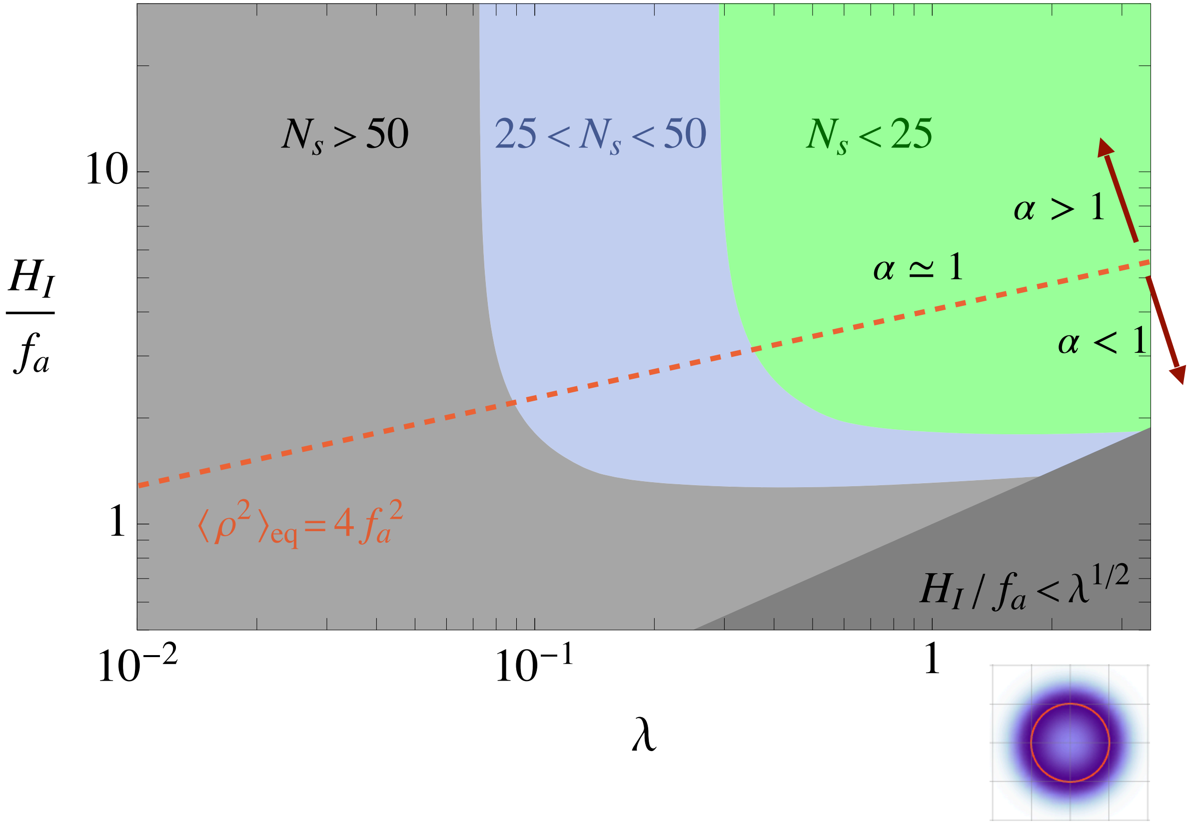

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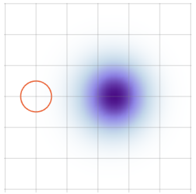

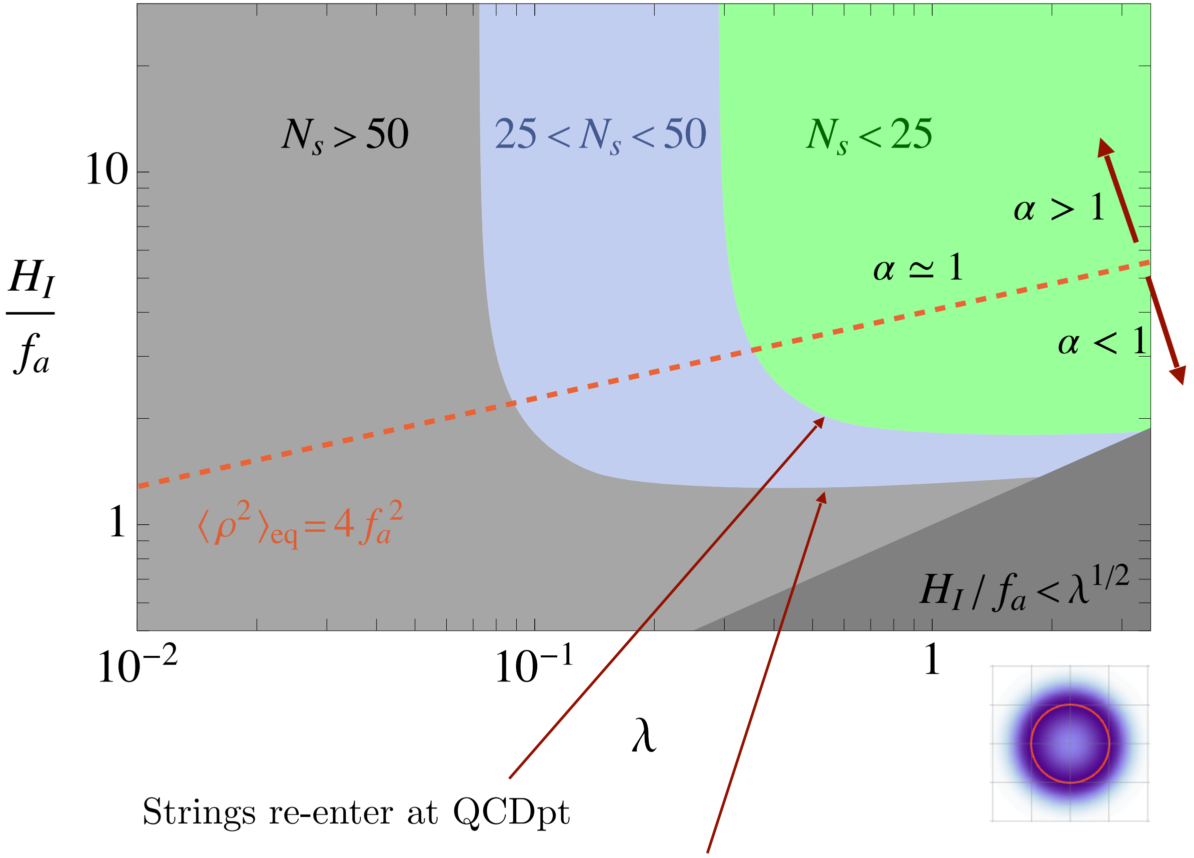

Strings re-enter at QCDpt

Strings re-enter ∼ today

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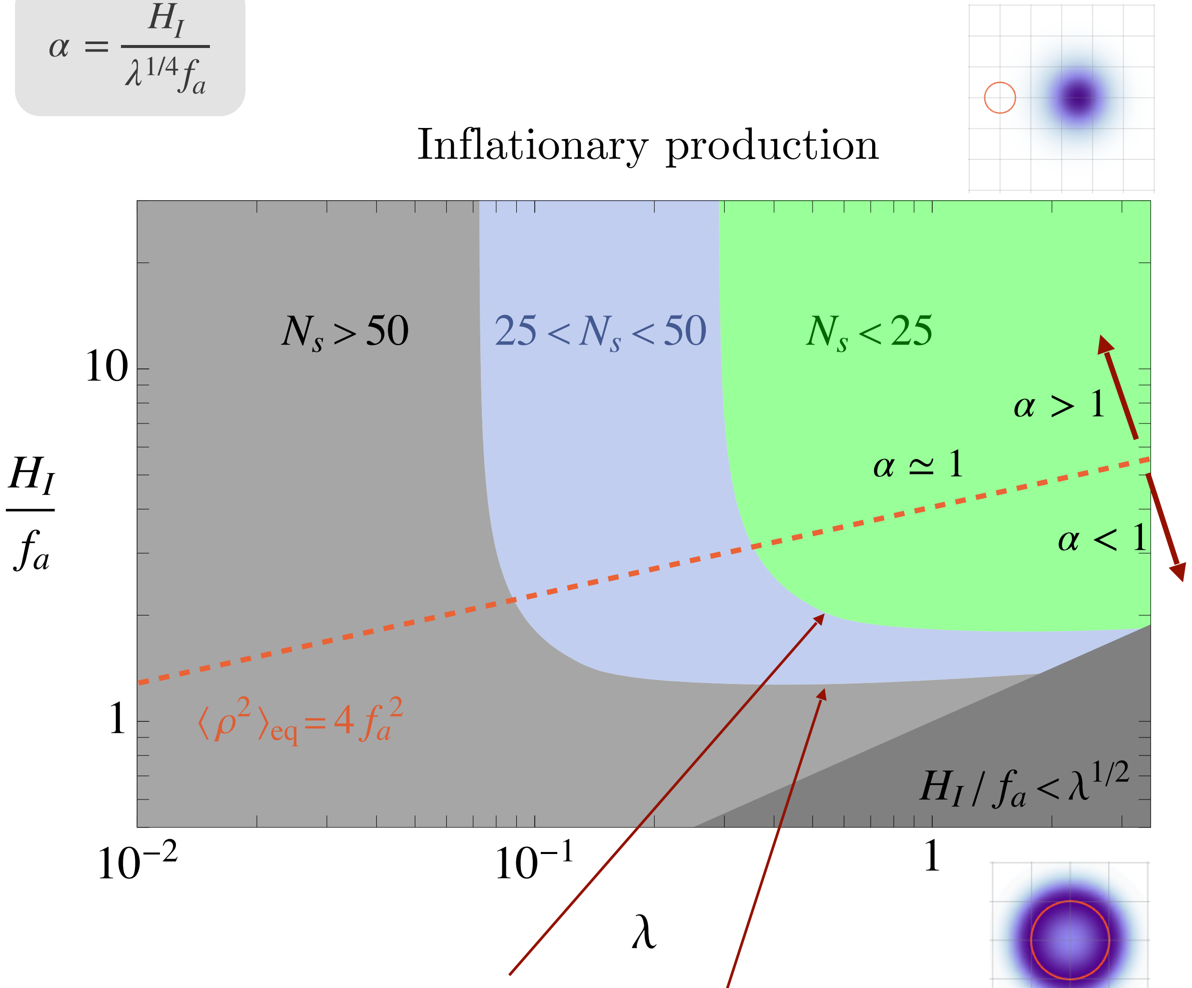

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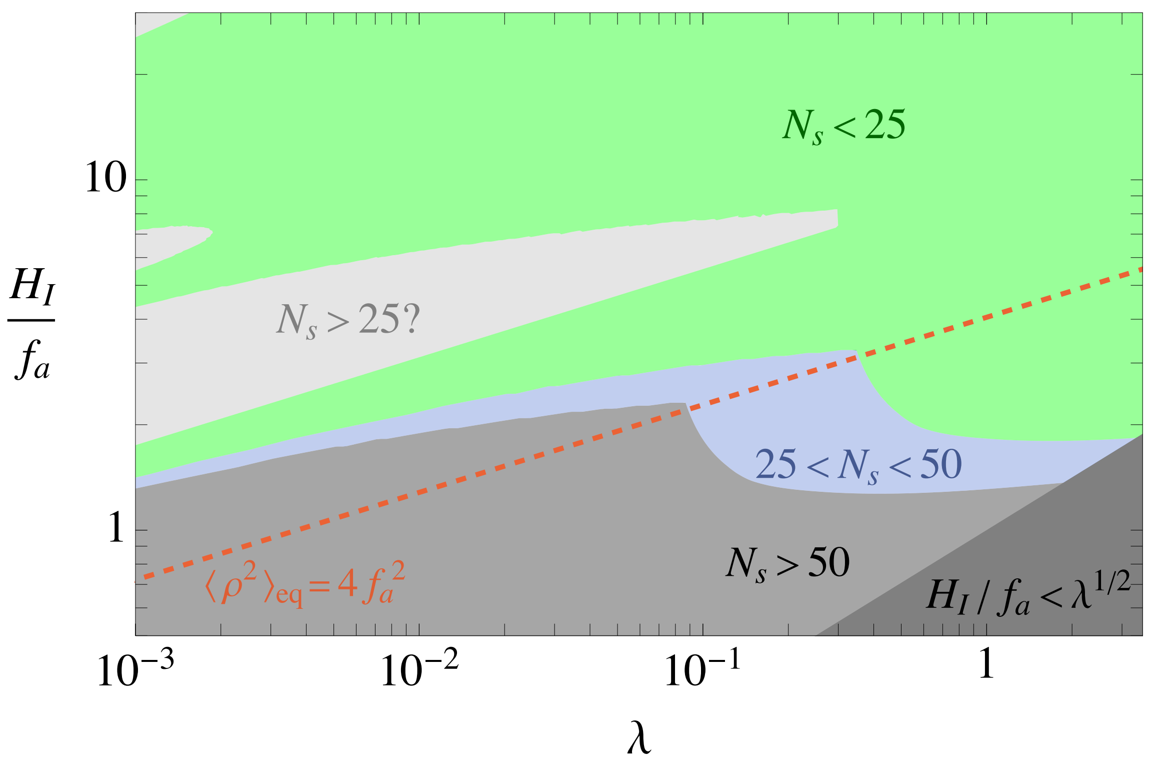
Including overshoot



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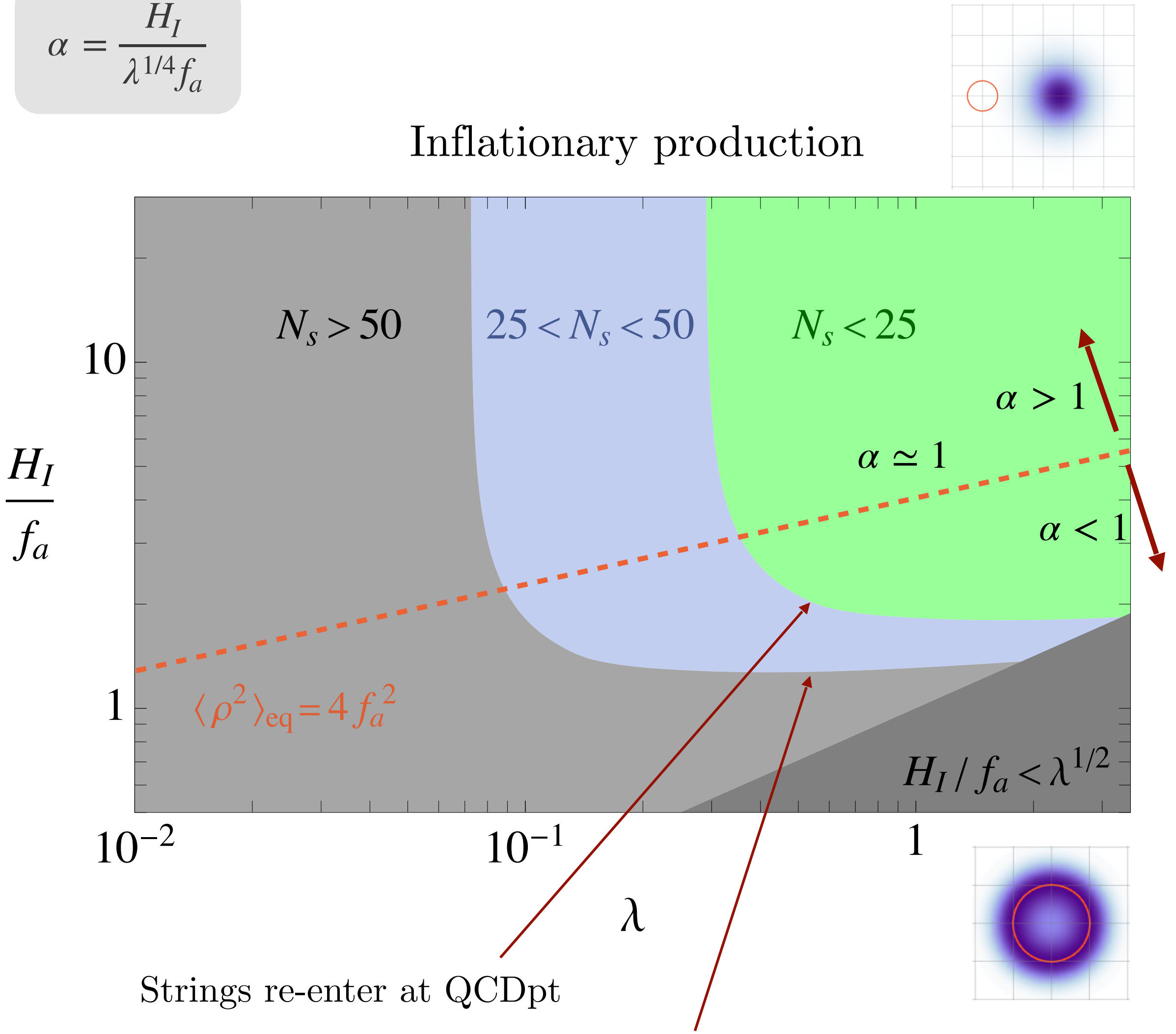

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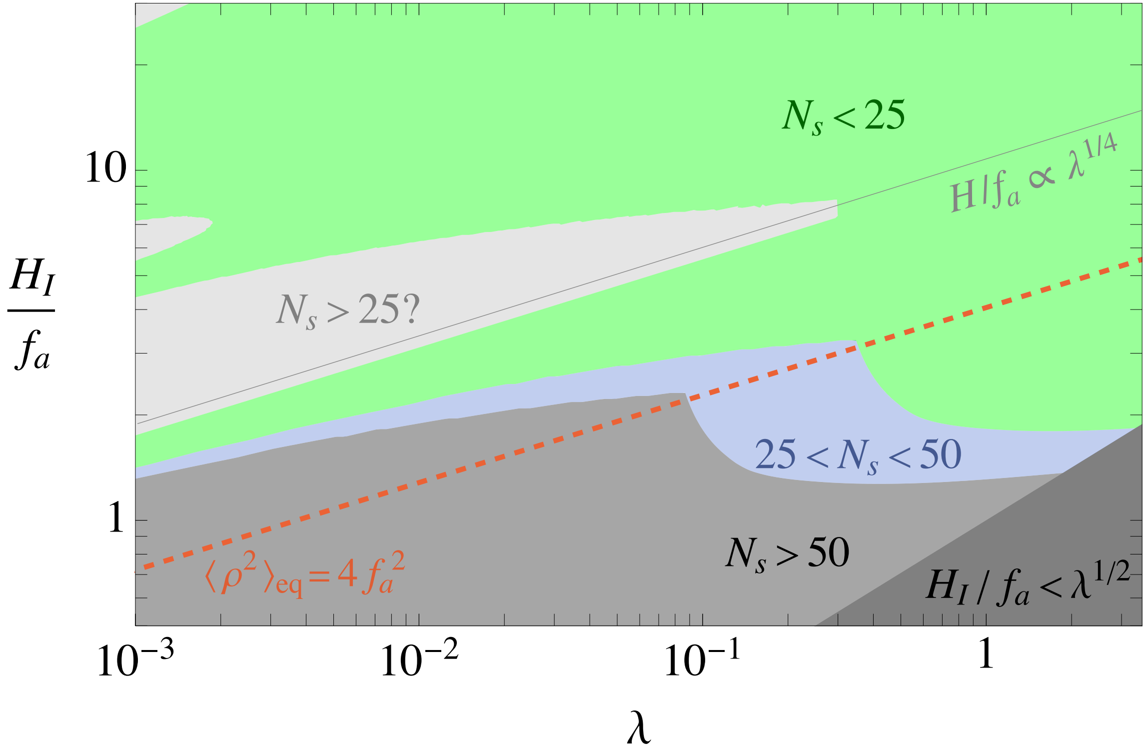

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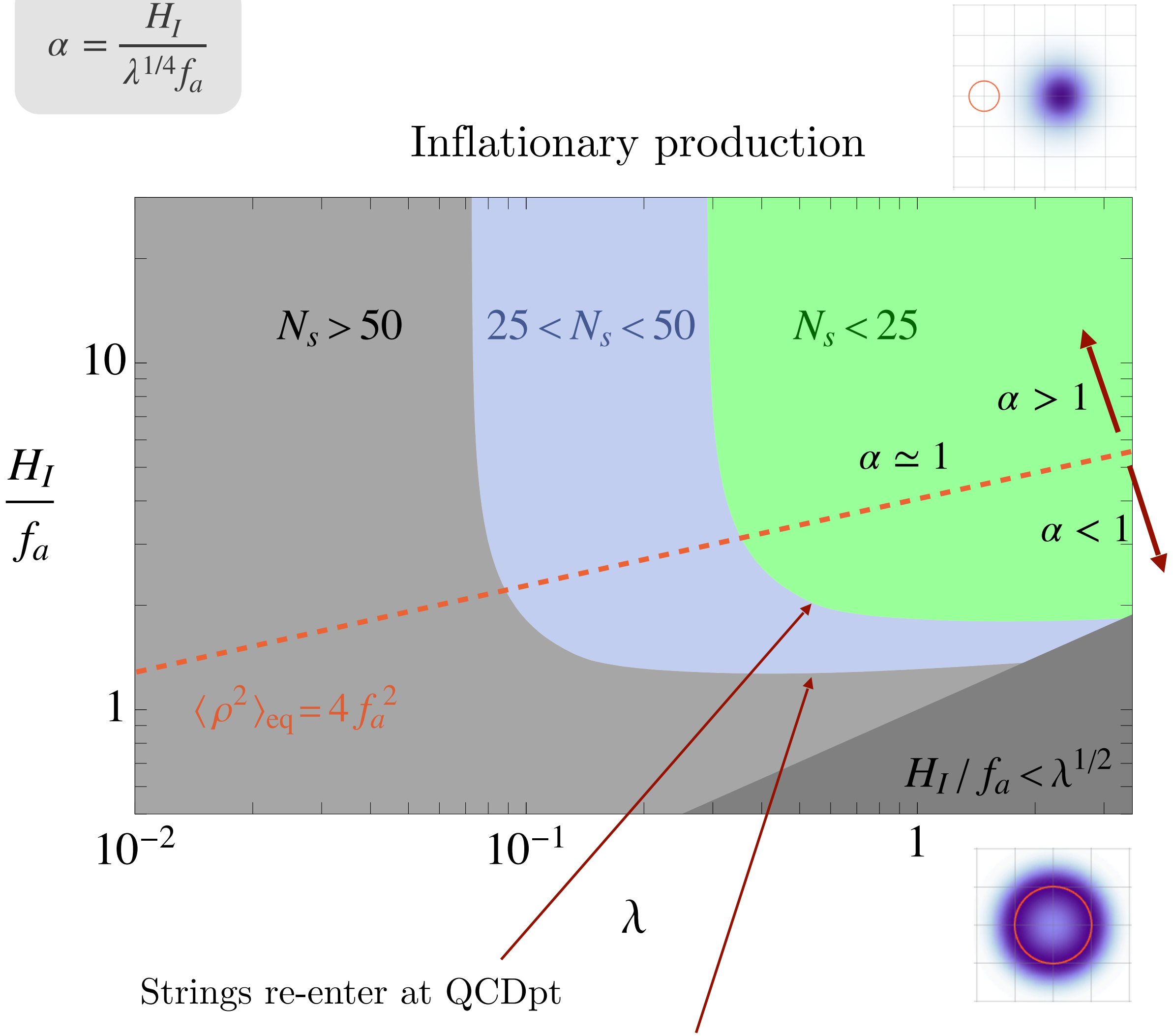

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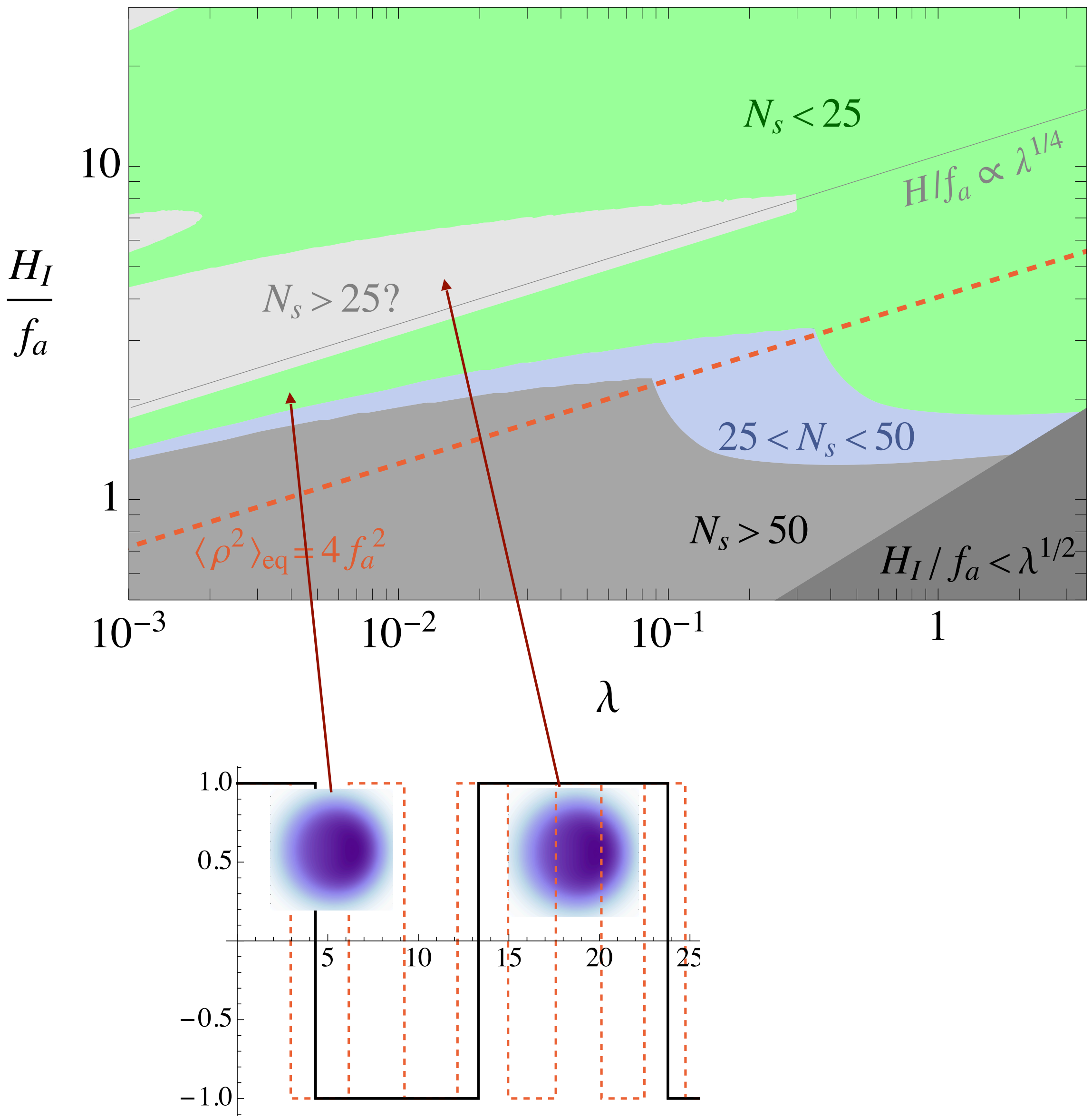
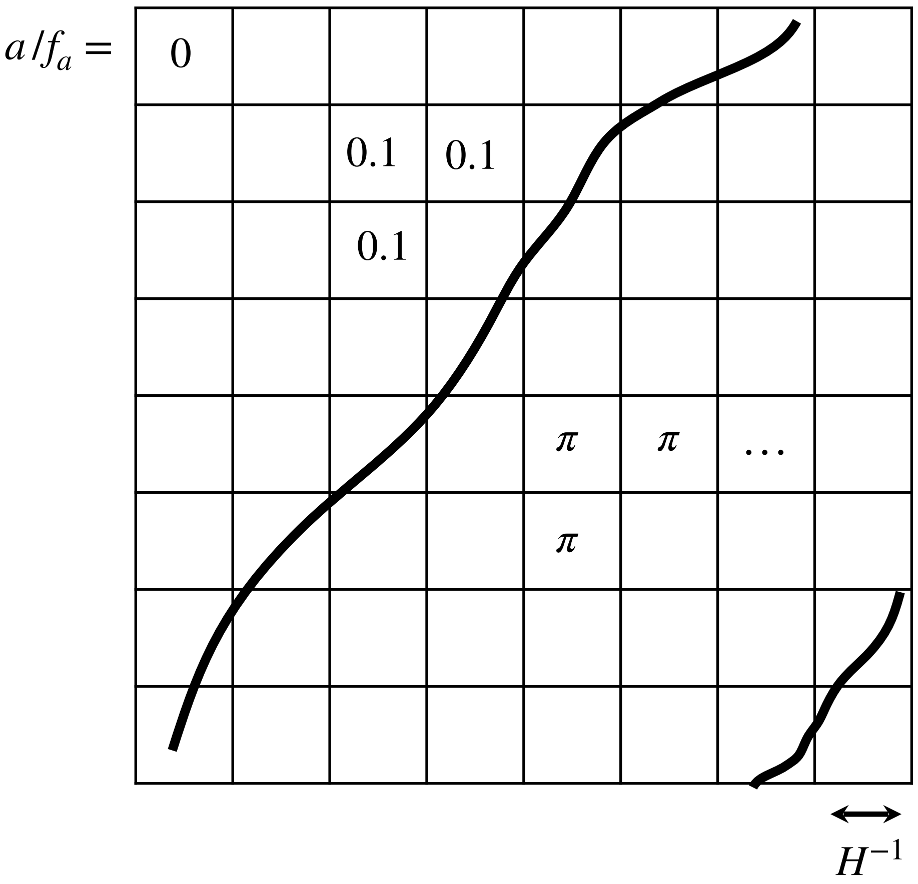


# Late strings

$@H = m_a \qquad (T \simeq \Lambda_{\text{QCD}})$



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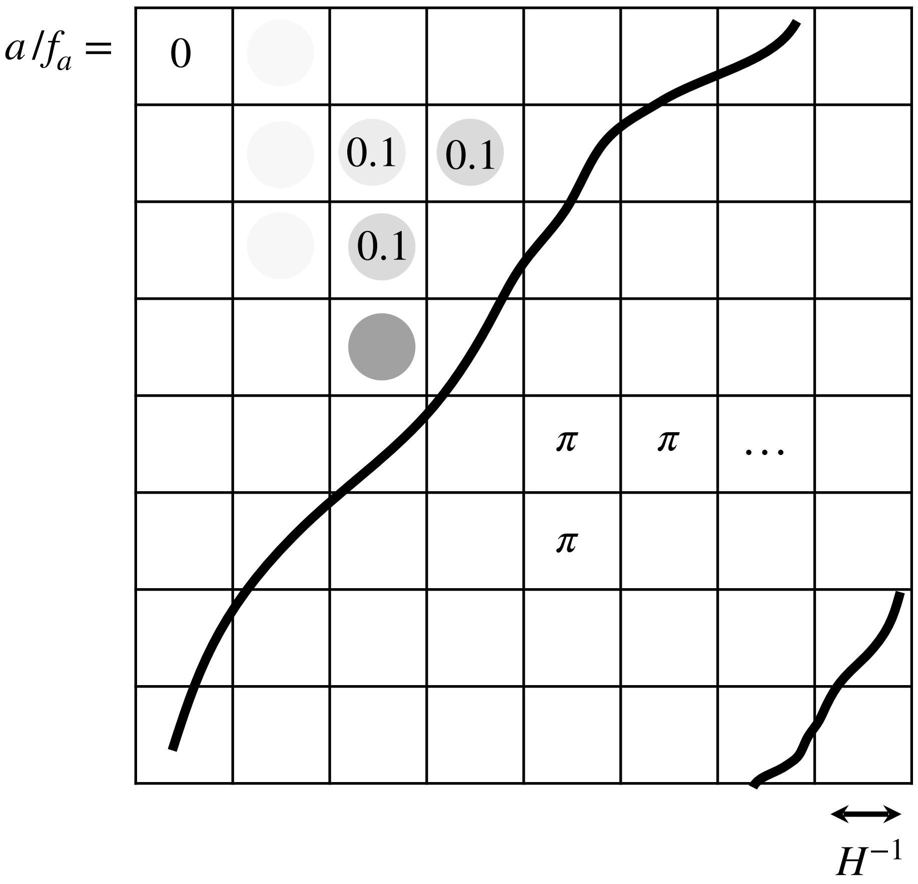

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average-angle misalignment

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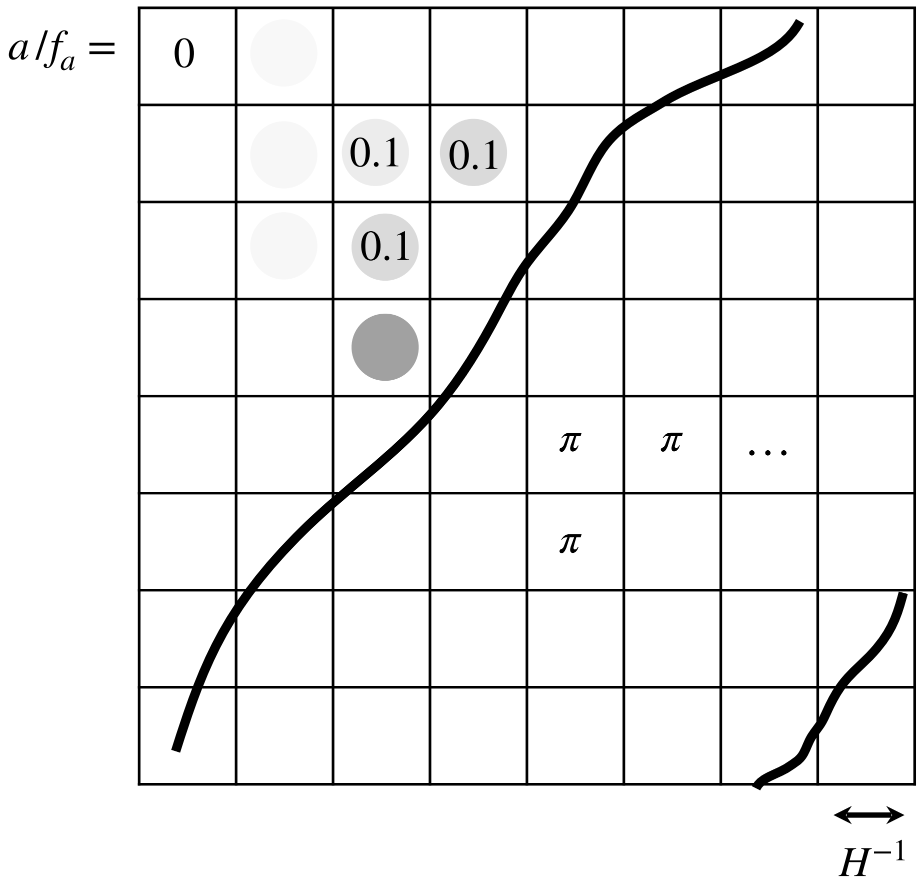
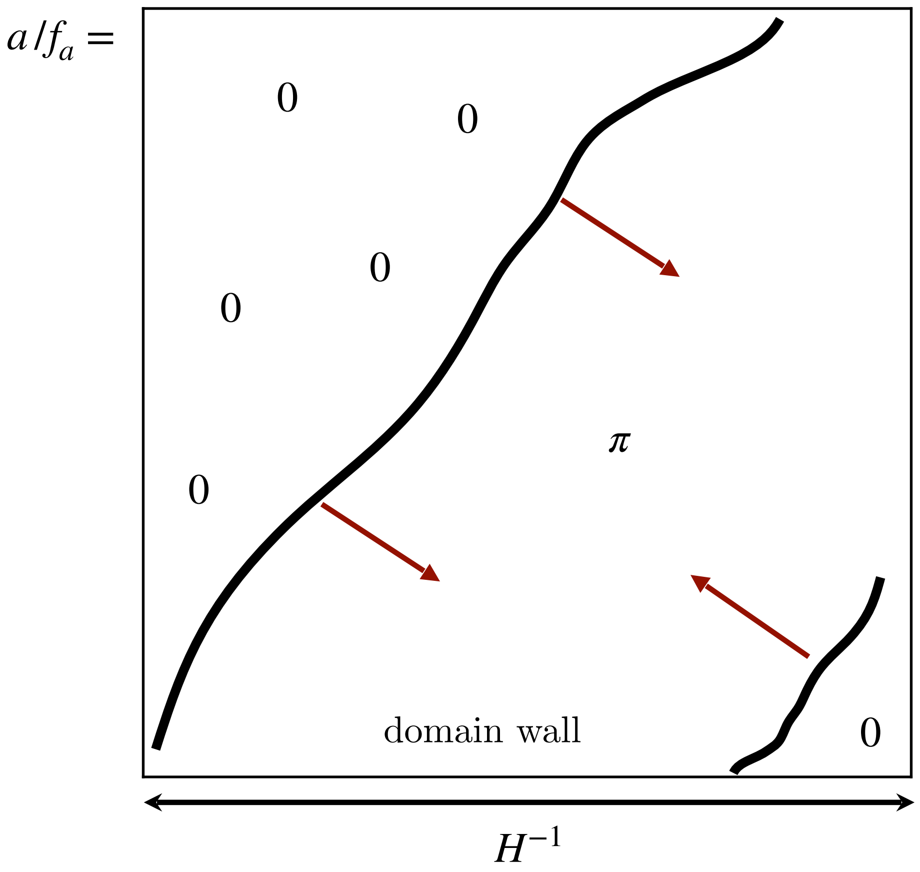


@$H = H_{PQ}$ $(T = T_{PQ})$ $T_{PQ}(N_s) \simeq T_0 e^{54-N_s}$



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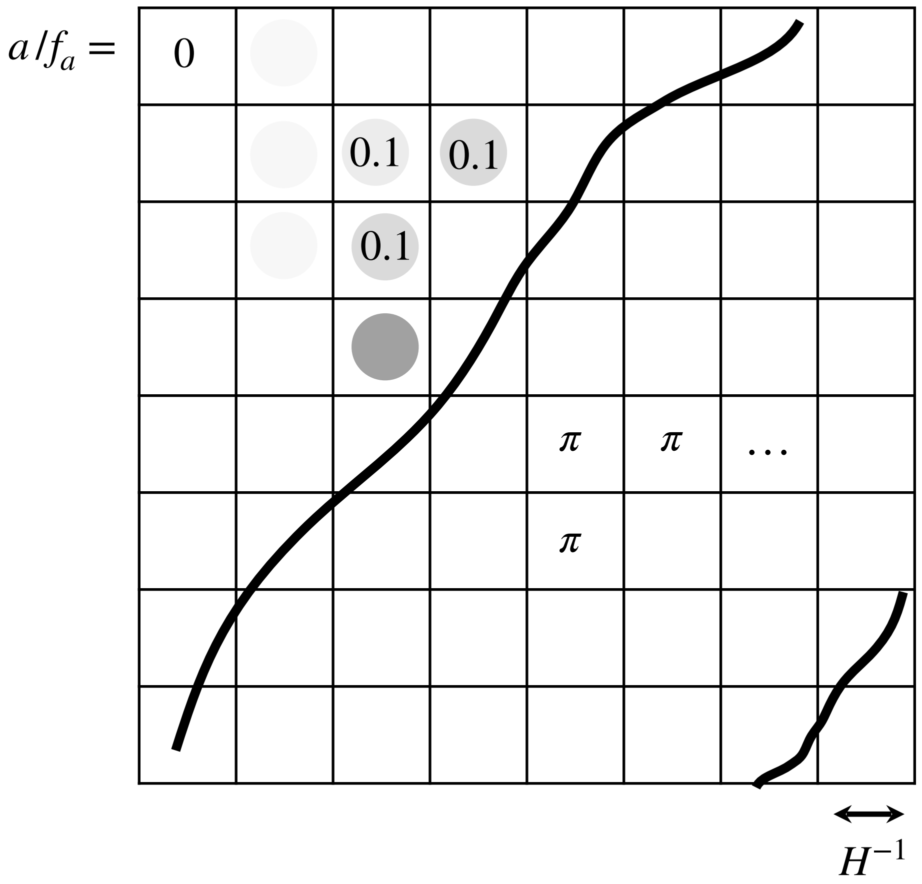

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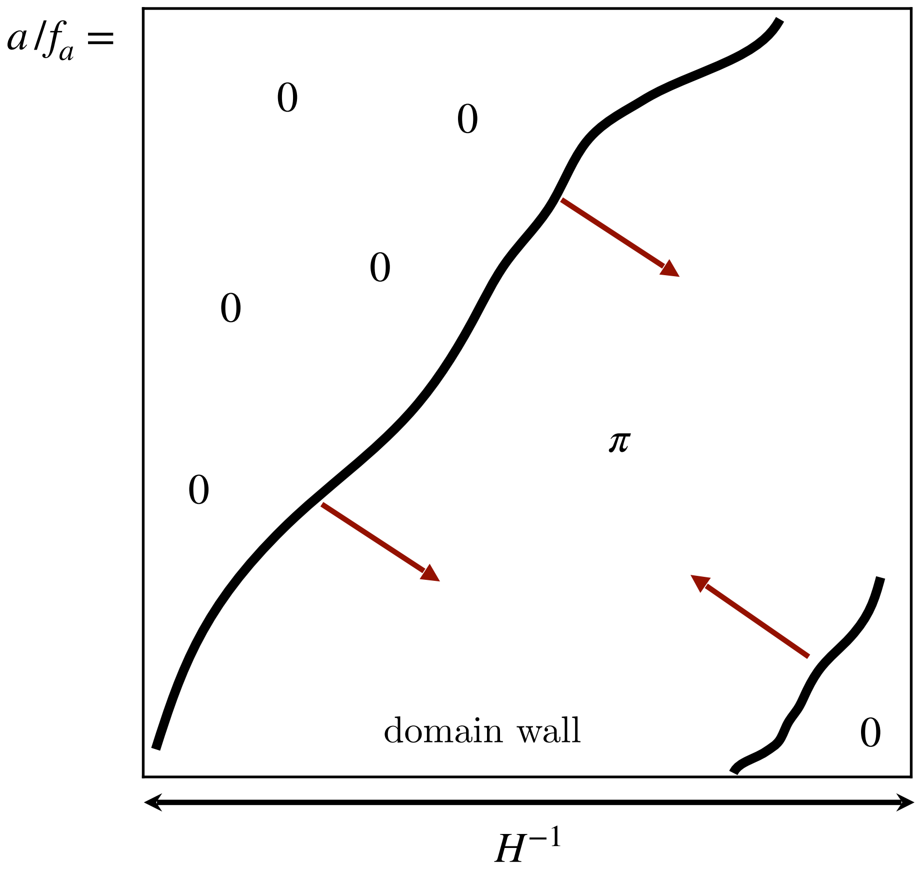

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DM for all allowed $f_a \gtrsim O(10^8)$ GeV

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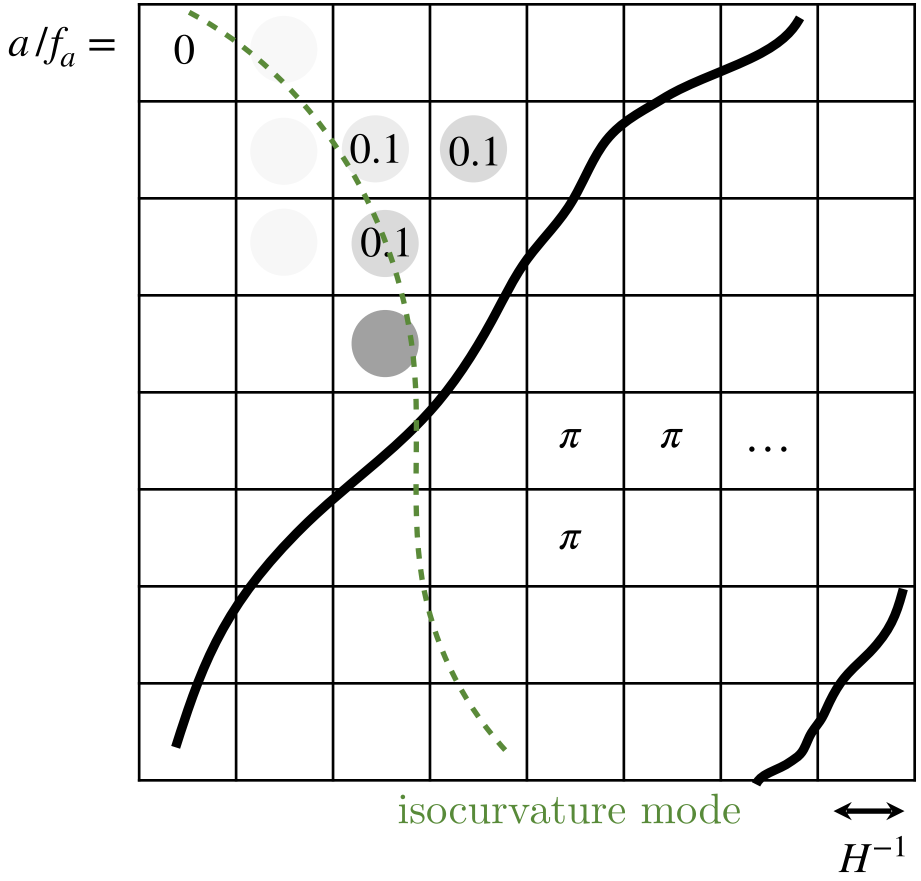

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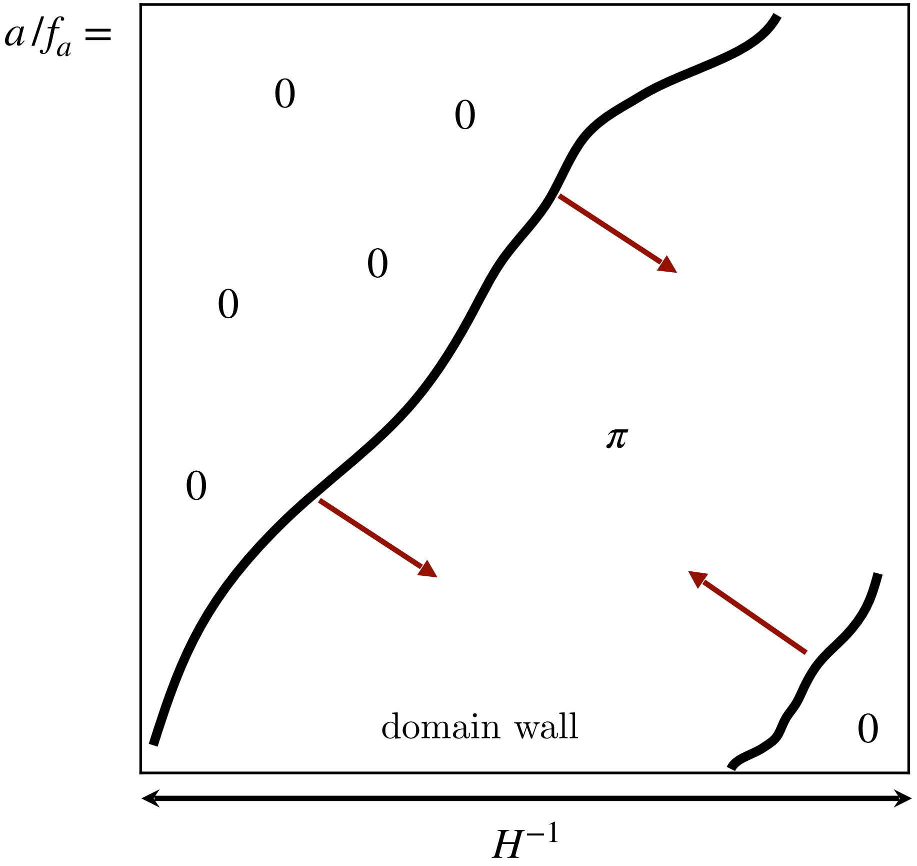

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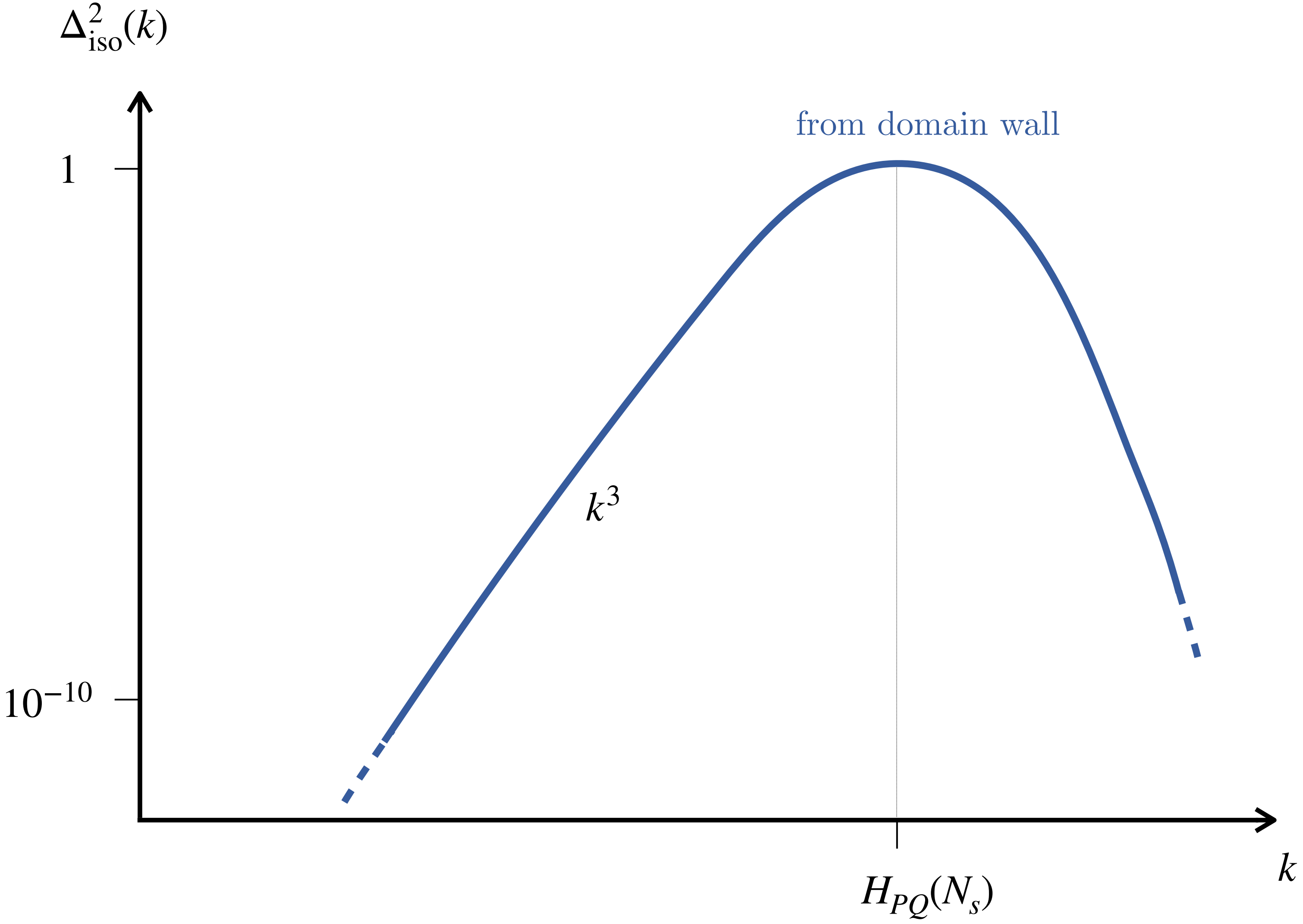

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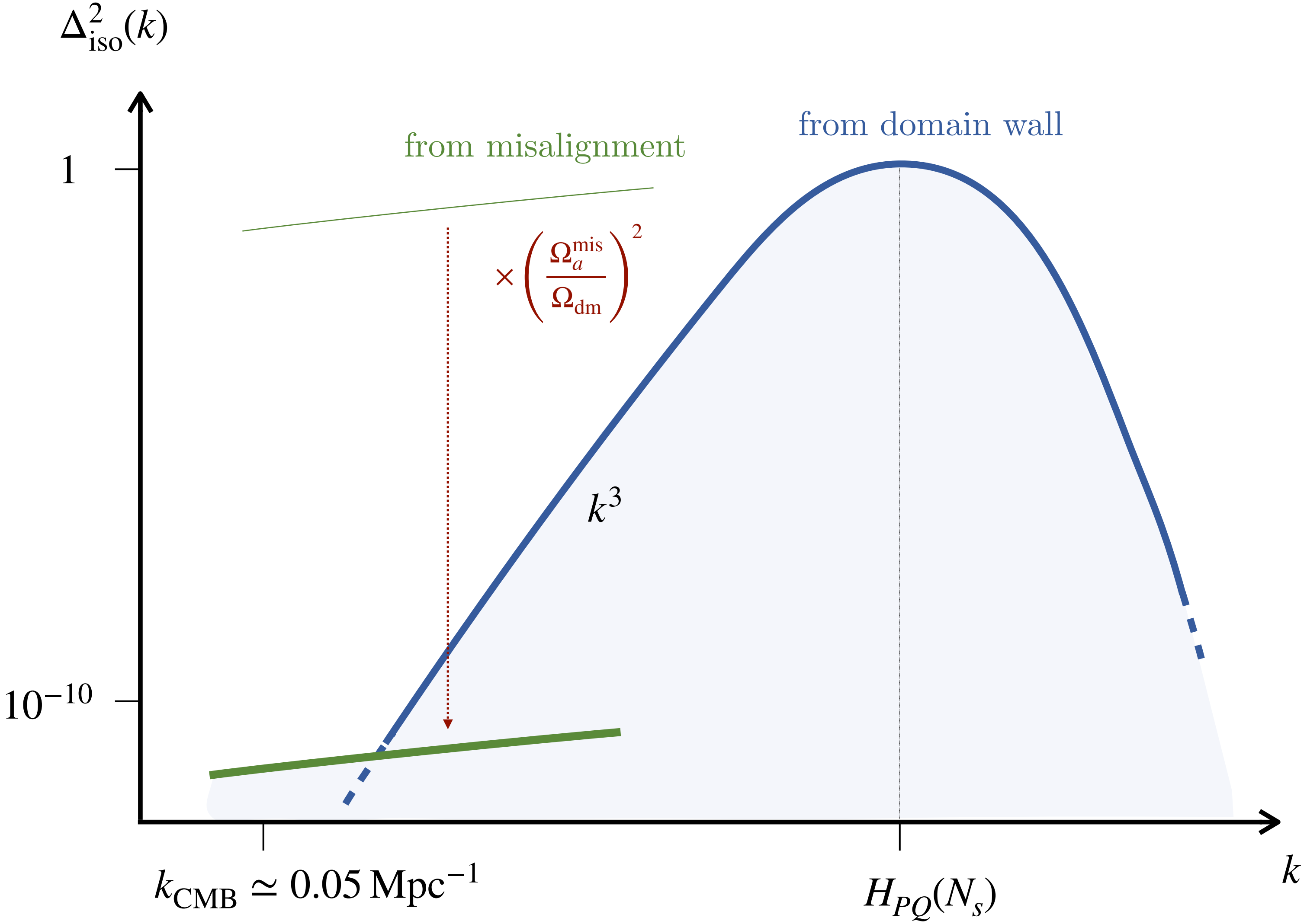

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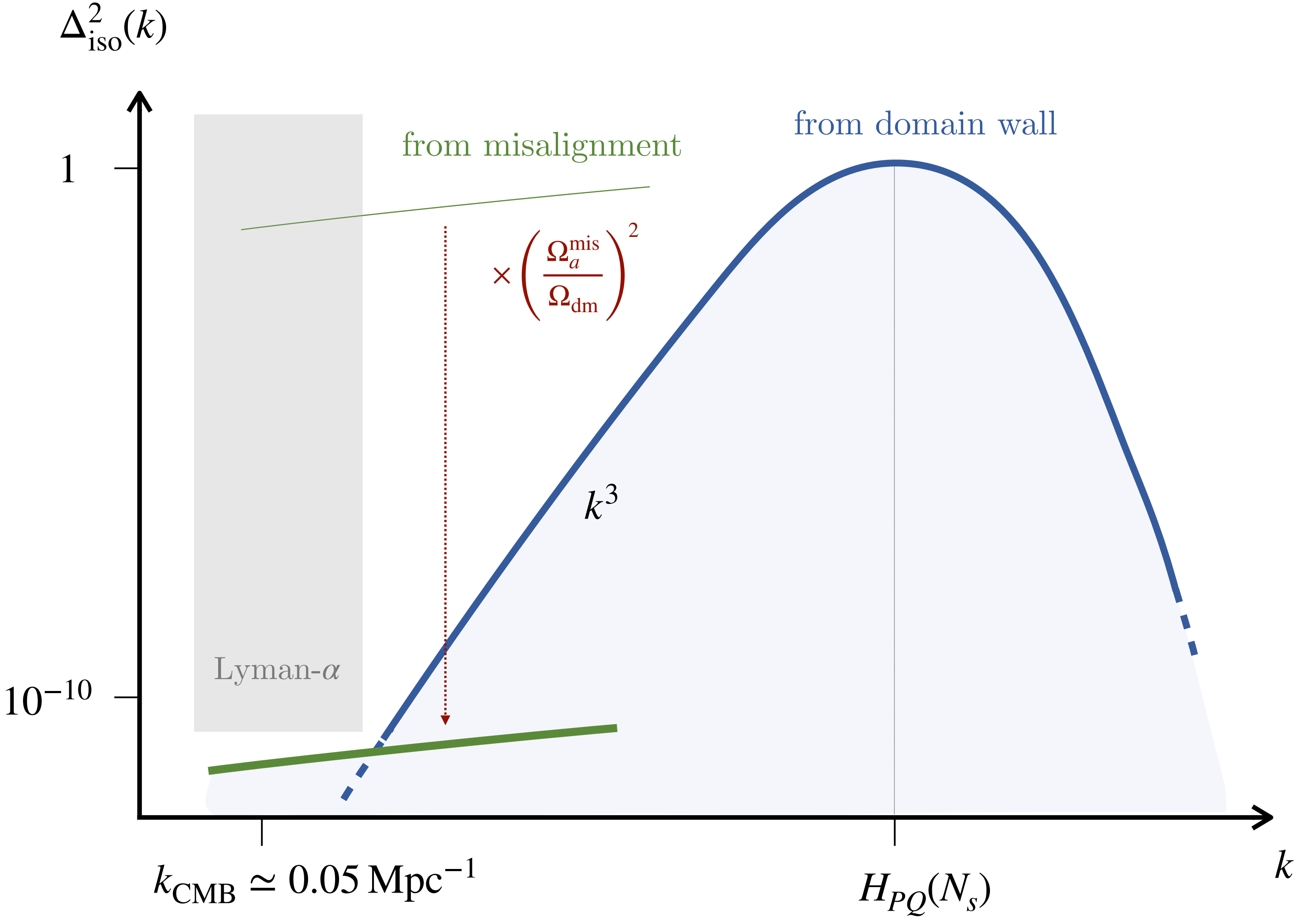

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$$\frac{\Delta_{\rm iso}}{\Delta_{\mathscr{R}}} \simeq 4\cdot 10^{-4}\,\mathscr{A}\left[\frac{f_a}{10^{8}\,\mathrm{GeV}}\right]\left[\frac{1\,\mathrm{MeV}}{T_{\rm PQ}}\right]^{\frac{5}{2}} \lesssim 4\cdot 10^{-3}$$

@$k = k_{\rm CMB}$, from Lyman-$\alpha$ observations

# Summary

New QCD axion dark matter scenario between pre- and post-inflationary

- Occurs for order one values of $H_I/f_a$ and $\lambda$
- Late string/wall produce dark matter for all allowed $f_a \gtrsim O(10^8)$ GeV, or $m_a \lesssim O(0.05)\,\mathrm{eV}$
- Potentially observable via dark matter isocurvature measurements and substructure (mini-clusters)

# Thanks!

# Backup

# Approaching the equilibrium distribution

$$\frac{\partial P}{\partial t} = \frac{H_I^3}{8\pi^2}\sum_i \partial_i\partial_i P + \frac{1}{3H_I}\partial_i(P\partial_i V)$$

$$\tilde{P} \equiv e^{\frac{4\pi^2 V}{3H_I^4}} P$$

$$\frac{\partial \tilde{P}}{\partial t} = \frac{H_I^3}{8\pi^2}\left[\partial_\rho^2 + \frac{1}{\rho}\partial_\rho + \frac{1}{\rho^2}\partial_\theta^2 - \left(\left(\partial_\rho v\right)^2 - \frac{1}{\rho}\partial_\rho v - \partial_\rho^2 v\right)\right]\tilde{P}$$

Ansatz: $\tilde{P} \propto \psi_m(t,\rho)e^{im\theta}$

$$v \equiv \frac{4\pi^2}{3H_I^4}V$$

$$\frac{\partial \psi_m}{\partial t} = O_m\psi_m$$

$$O_m = \frac{\sqrt{\lambda}H_I}{8\pi^2}\left[\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2} - \frac{8\pi^2}{9}(2x^2(\pi^2(x^2-\alpha^{-2})^2-3)+3\alpha^{-2})\right]$$

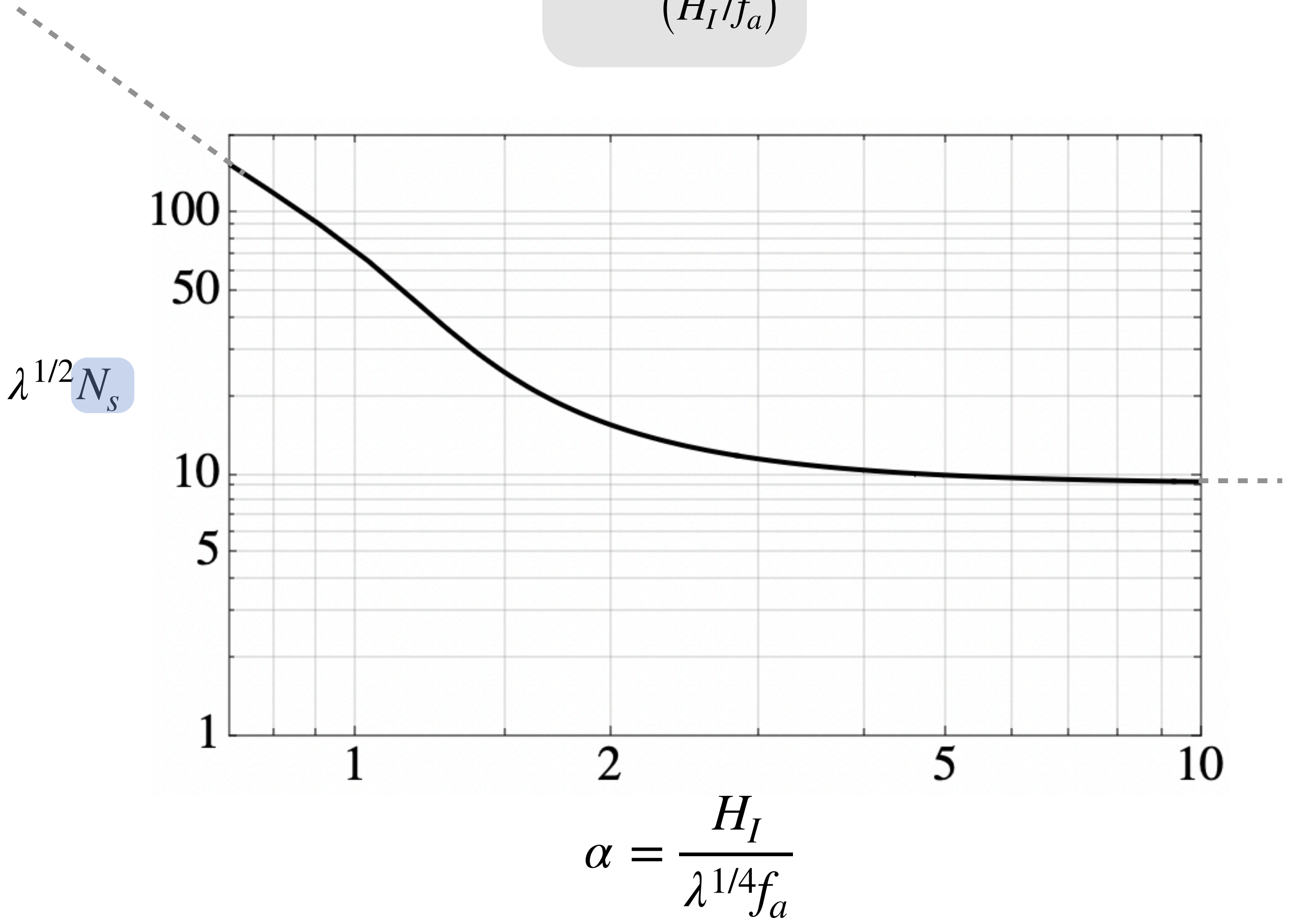
$$x \equiv \lambda^{1/4}\rho/H_I$$

$$P(t,x,\theta) = P_{\text{eq}}(x) + \sum_{(n,m)\neq(1,0)} a_{nm}e^{-v(x)}\psi_{nm}(x)\frac{e^{im\theta}}{\sqrt{2\pi}}e^{-\Gamma_{nm}t}$$

Eigenvalues of $O_m$

$$N_{nm} = \frac{H_I}{\Gamma_{nm}} \qquad N_s = N_{11}$$

$\simeq (\alpha/9)^{-2}$ $\longrightarrow$ $N_s \simeq \dfrac{80}{(H_I/f_a)^2}$ (independent of $\lambda$)



$\simeq 10$ $\longrightarrow$ $N_s \simeq \dfrac{10}{\sqrt{\lambda}}$

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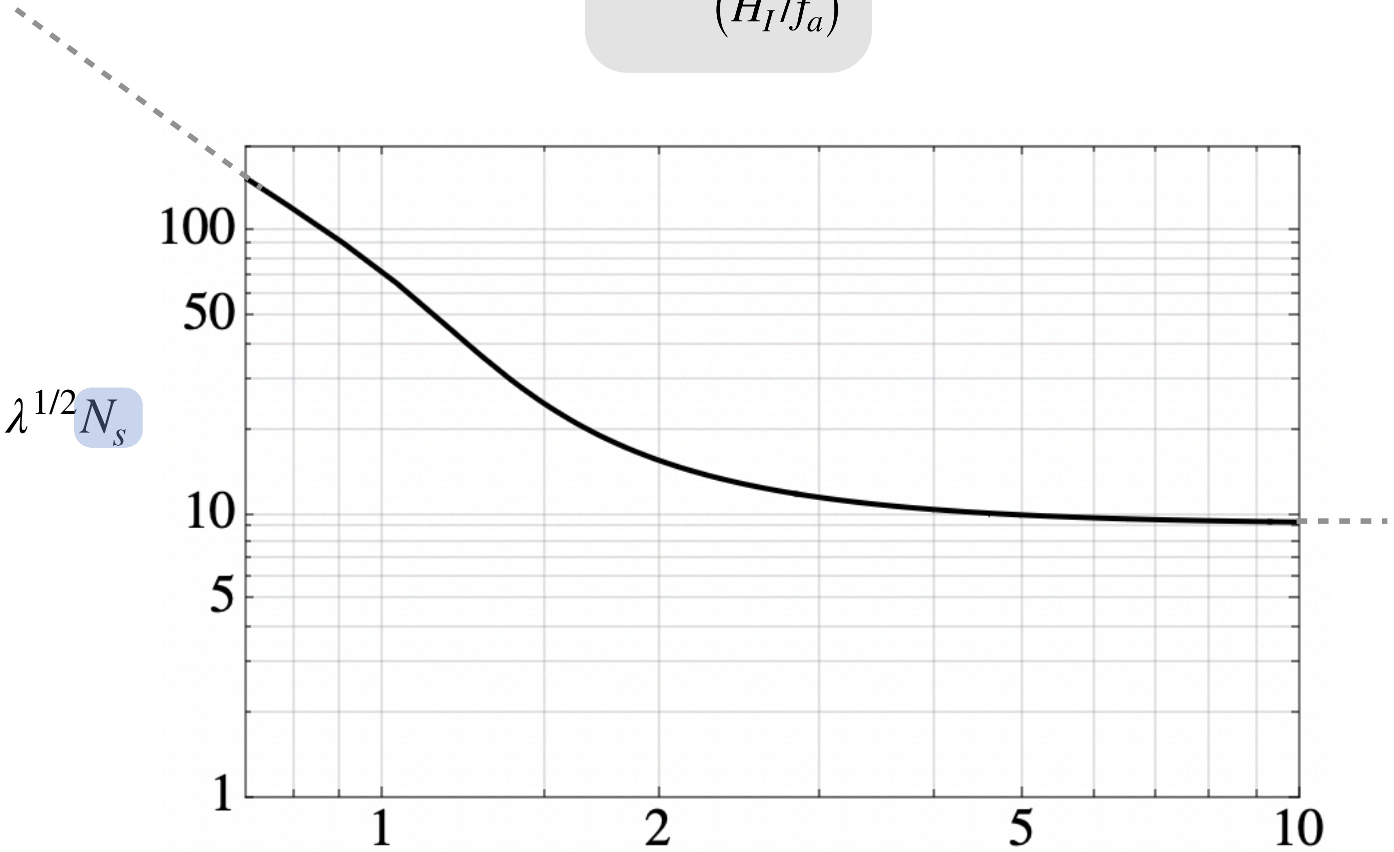
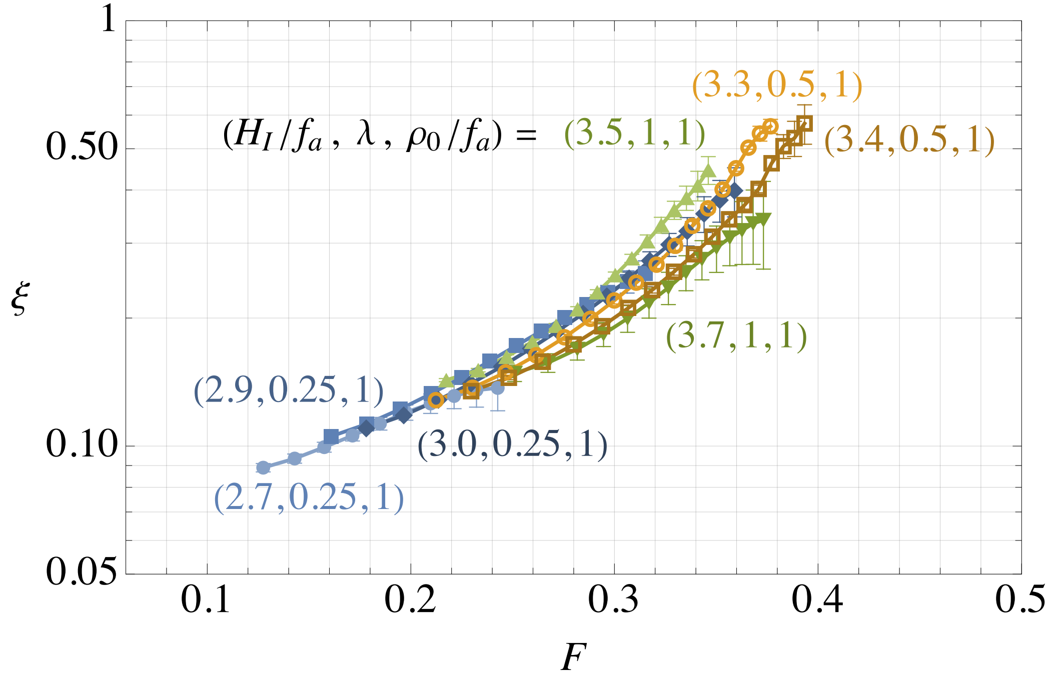


$\simeq 10$ $\longrightarrow$ $\boxed{N_s \simeq \dfrac{10}{\sqrt{\lambda}}}$

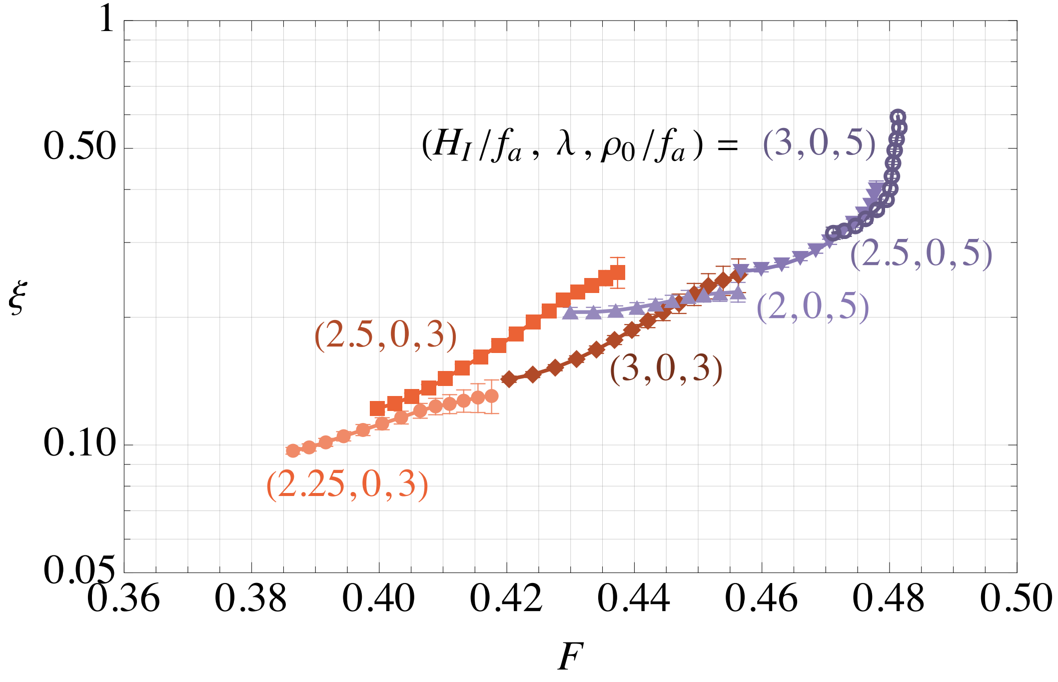
$$\alpha = \frac{H_I}{\lambda^{1/4} f_a}$$

For $\alpha \gg 1$ : $\sigma_\Phi = \dfrac{\sqrt{N} H_I}{2\pi}$ so to reach $\sigma_\Phi \simeq \dfrac{H_I}{\lambda^{1/4}}$ $\rightarrow$ $N_s \simeq \dfrac{10}{\sqrt{\lambda}}$

Initial conditions leading to inflationary production



Initial conditions leading to overshoot



$$N(F) = \frac{H_I}{\Gamma_{11}} \log\left[\frac{2a_{11}b_{11}}{2F-1}\right]$$