Electromagnetic Modes of the MAGO Cavity

Status Update

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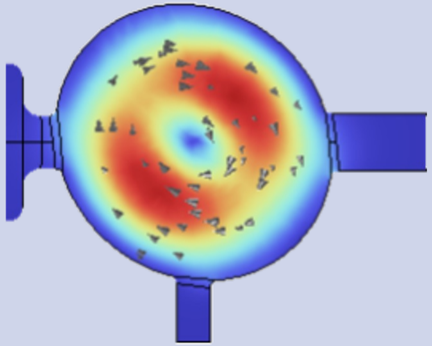
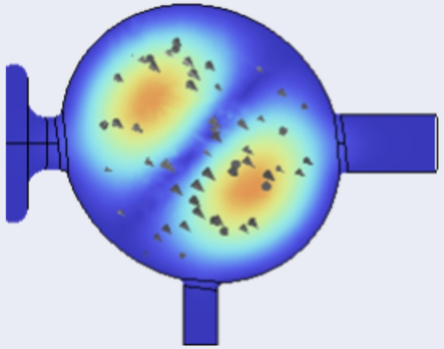
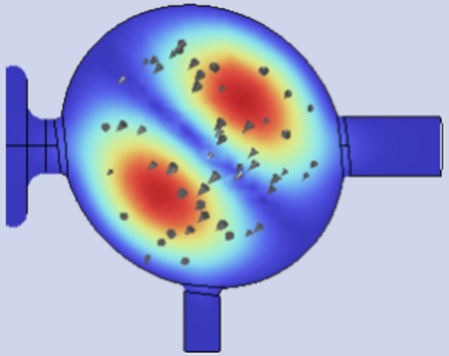
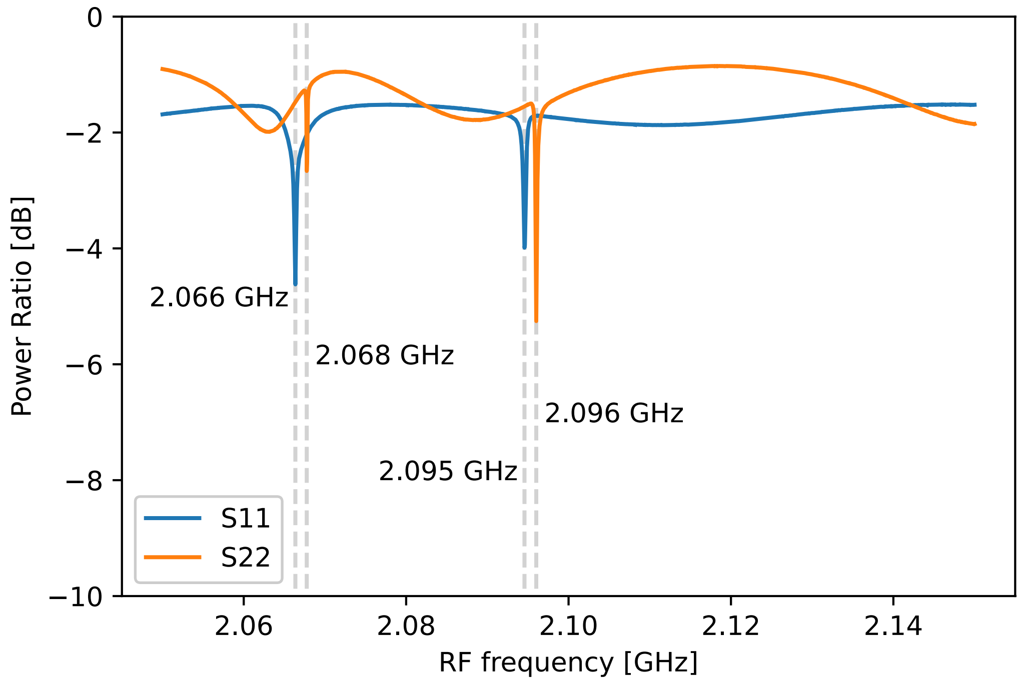
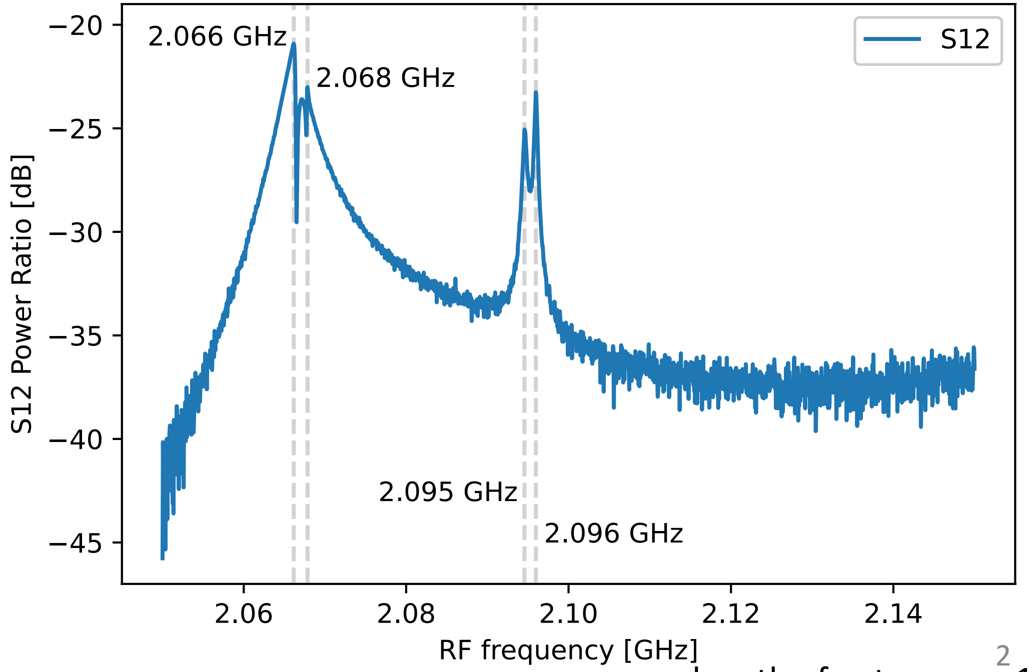
Warm Measurement of EM Modes in MAGO cavity

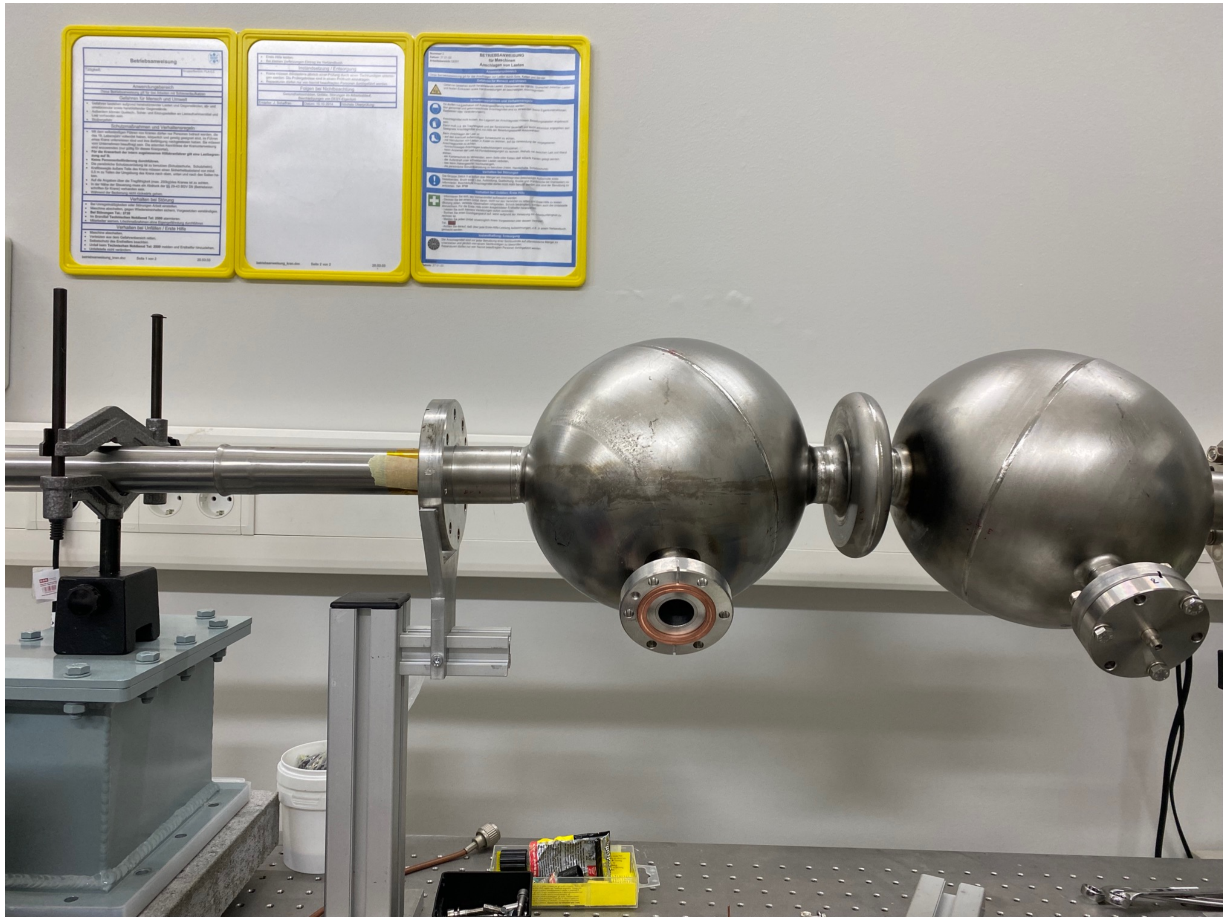
Simulated Eigenfrequency	Electric Field Distributions
(previous mode ~1.80 GHz) 2.07306 GHz (Pi-mode) 2.07311 GHz (0-mode)	
2.07710 GHz 2.07717 GHz (probably bad coupling to antenna)	
TE_{011} 2.10381 GHz 2.10390 GHz (next mode: ~2.26 GHz)	





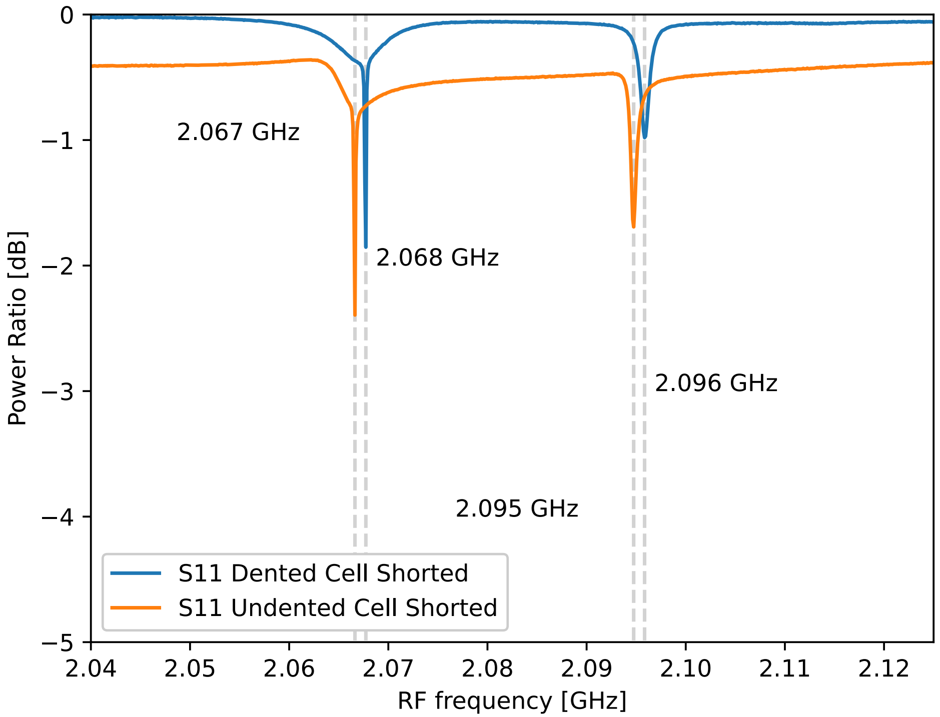
length of antennas ~ 11 cm

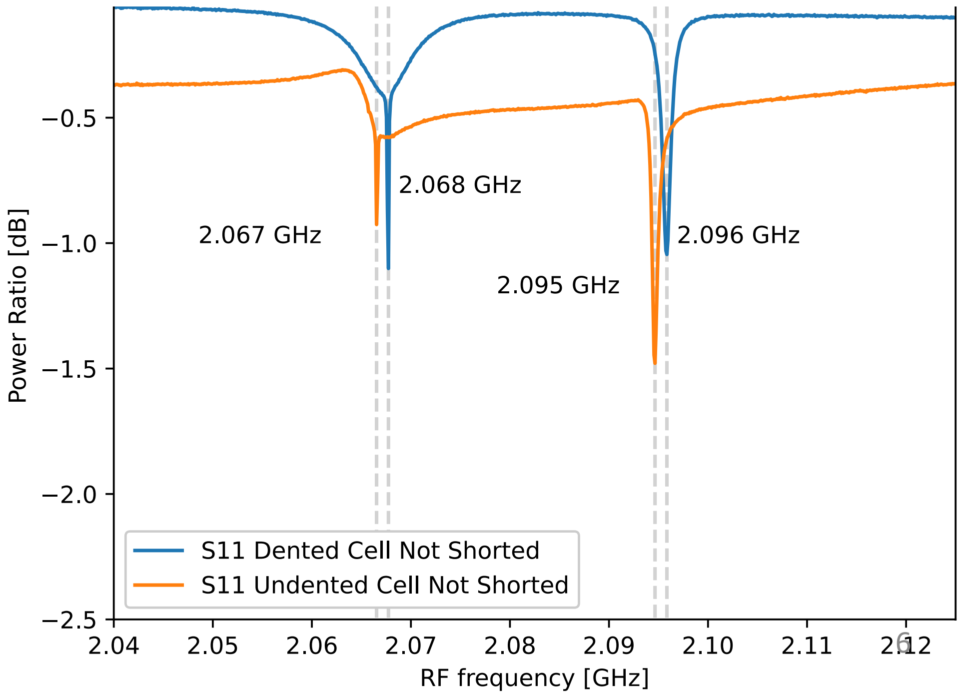
Shorting the Cavity: Pictures



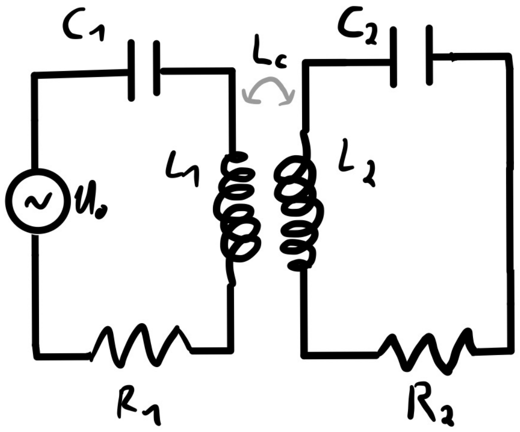
Shorting the Cavity: Results

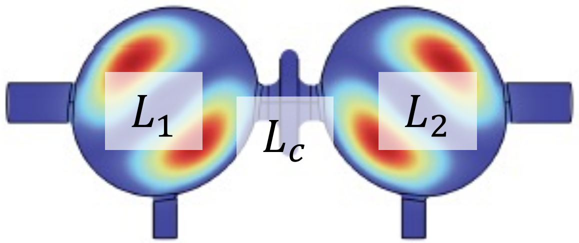
- The Eigenfrequencies of the single cells are identical to the non-shorted case
- They also (almost) match with the previous results (slide 1)





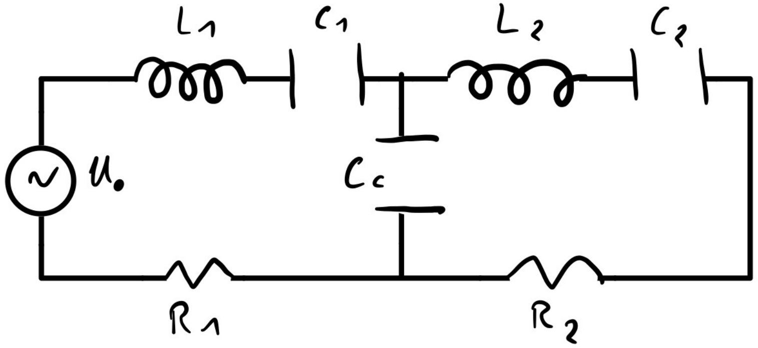
Explanation through Equivalent Circuits



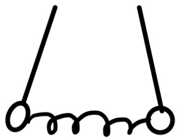


- Model the two MAGO cells as inductively coupled RLC circuits
- External oscillator U(t) in circuit 1
- Mechanical analogue: double pendulum (small angles)
- E.O.Ms:

$$L_1\ddot{I}_1 + R_1\dot{I}_1 + \frac{1}{C_1}I_1 = -L_c\ddot{I}_2 + \dot{U}$$
$$L_2\ddot{I}_2 + R_2\dot{I}_2 + \frac{1}{C_2}I_2 = -L_c\ddot{I}_1$$



- Capacitively coupled circuits yield same qualitative results
- Mechanical analogue: coupled pendula (small angles)
- E.O.Ms: replace $L_c\frac{d^2}{dt^2} \rightarrow 1/C_c$

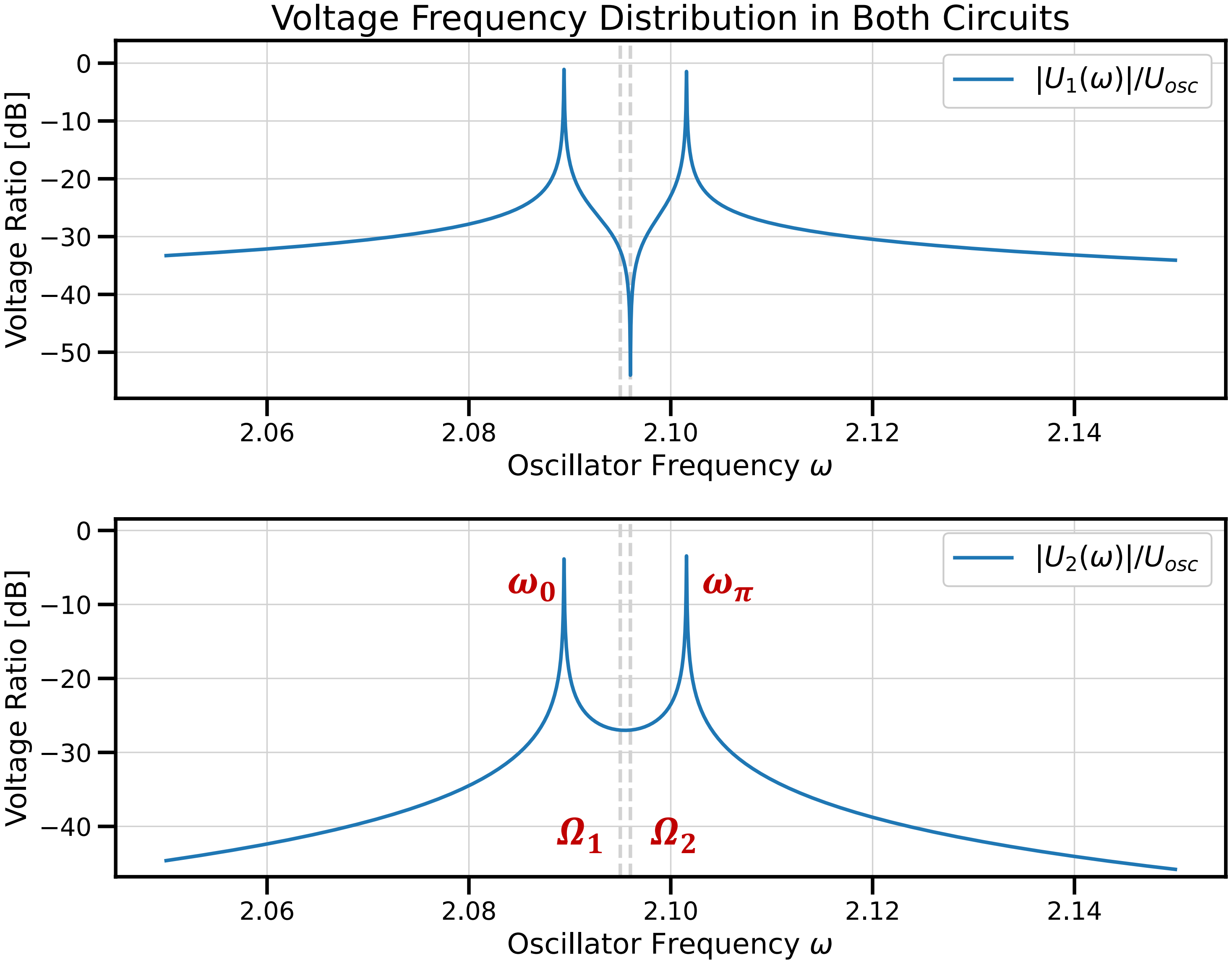


Solution of the coupled E.O.Ms

- Define parameters
  - $k = L_c / \sqrt{L_1 L_2}$ (coupling strength)
  - $\Delta\Omega = \Omega_2 - \Omega_1,\ \Omega_i = \frac{1}{\sqrt{L_i C_i}}$ (single cell Eigenfrequencies)

- $\omega_\pi - \omega_0 \approx k \frac{\Omega_1 + \Omega_2}{2} \approx k \frac{\omega_\pi + \omega_0}{2}$

valid for $1 \gg k \gg \frac{\omega_2 - \omega_1}{(\omega_1 + \omega_2)/2}$ and $R_1 = R_2 \approx 0$

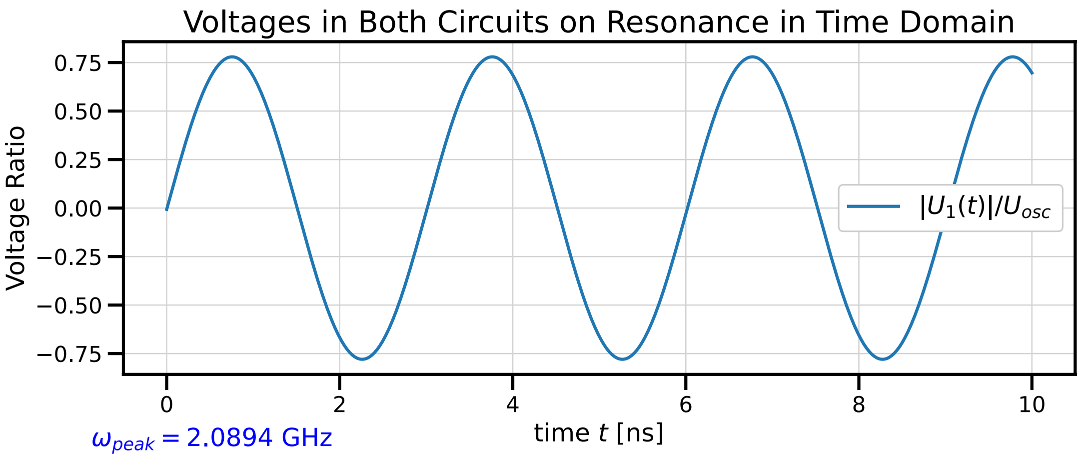


$k = 10^{-2}, \qquad \Delta\Omega = 1\ \mathrm{MHz}$

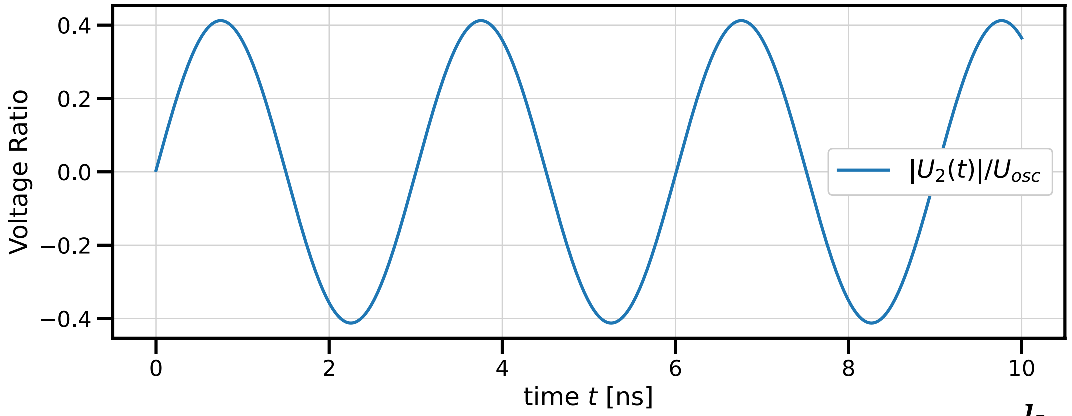
Solution in The Time Domain

→ Fourier transform spectrum assuming $U_0 = U_{osc}\sin(\omega_{peak} t)$

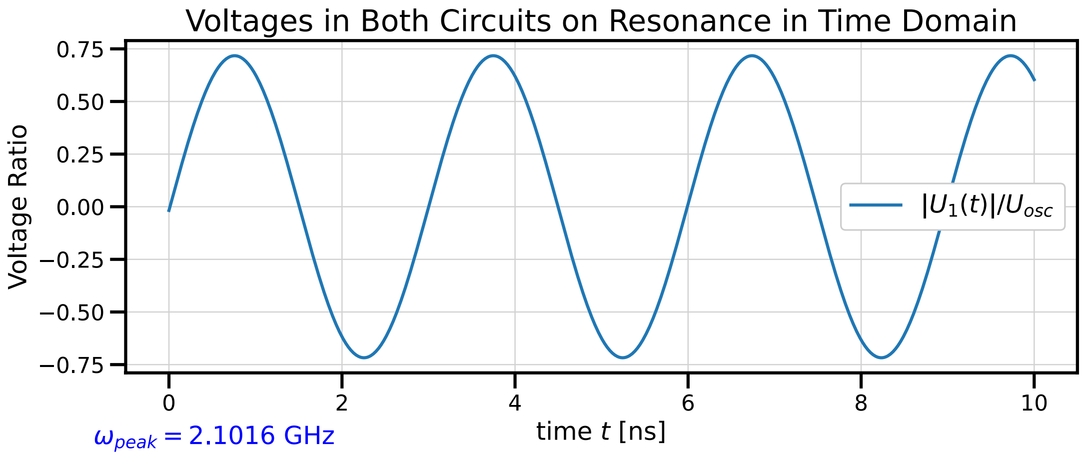
Symmetric (0) Mode $\boldsymbol{\omega_0}$



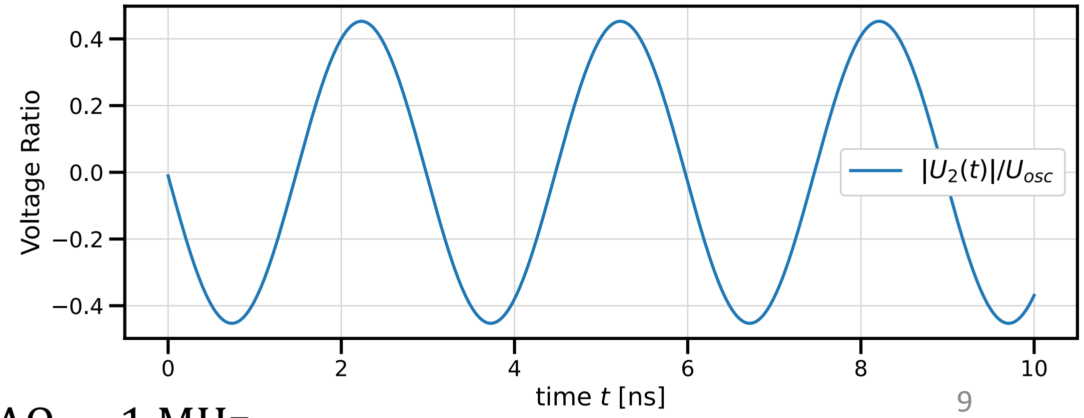
$\omega_{peak} = 2.0894$ GHz



Antisymmetric (π) Mode $\boldsymbol{\omega_\pi}$



$\omega_{peak} = 2.1016$ GHz

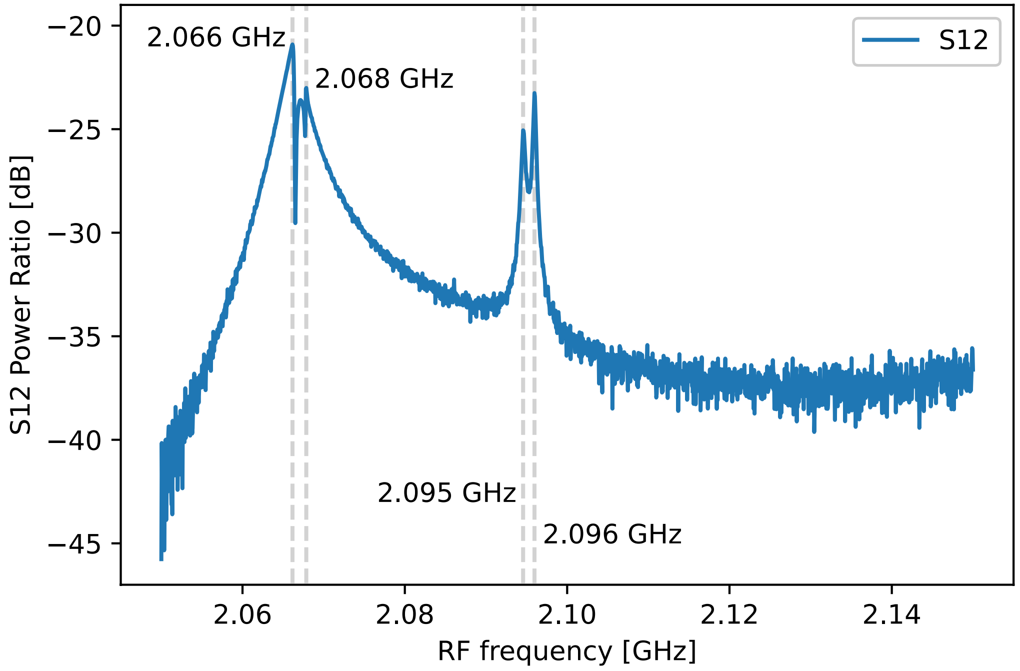


Circuit 1

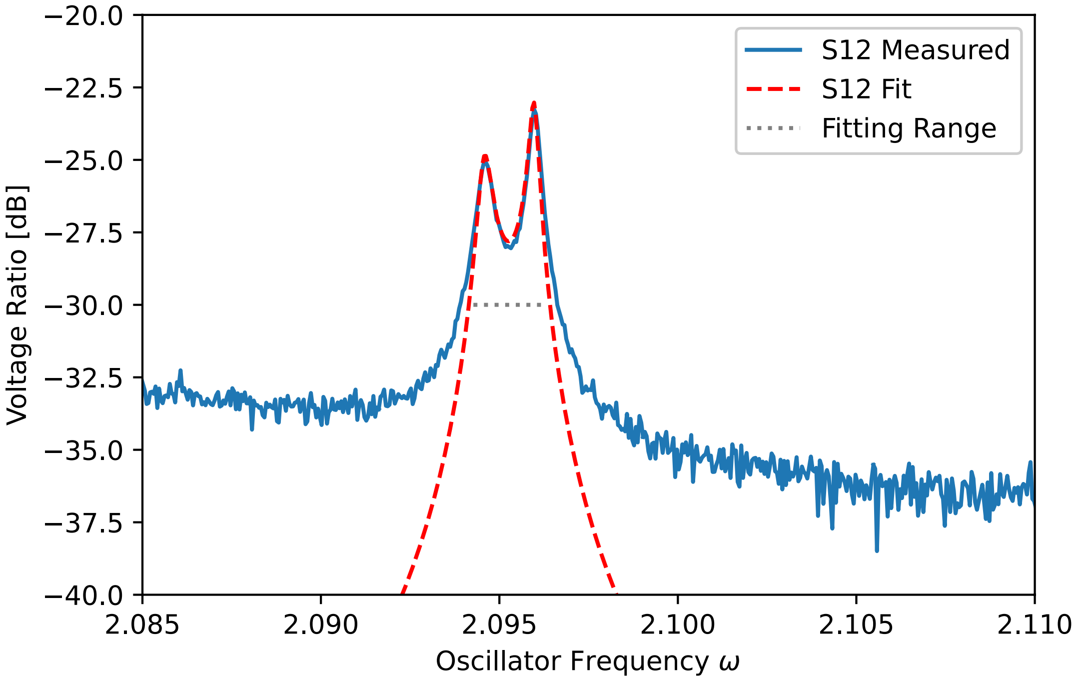
Circuit 2

$k = 10^{-2}, \qquad \Delta\Omega = 1\ \text{MHz}$

Comparison to Measurement



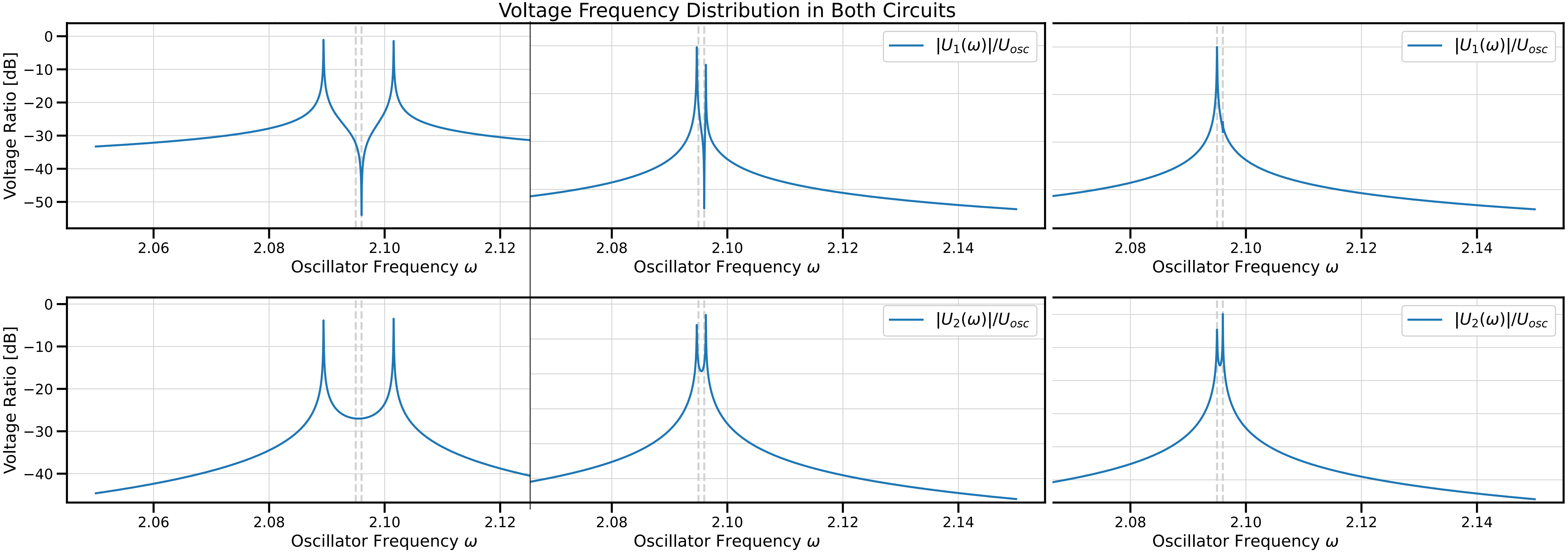
→



tentative results: ($S12$ and $|U_2/U_0|$ are not exactly the same)

$k = 0.5 \cdot 10^{-5}$, $\Delta\Omega = 1.4$ MHz, $Q_1 = 0.5 \cdot 10^4$, $Q_2 = 0.8 \cdot 10^4$

Varying coupling parameter k

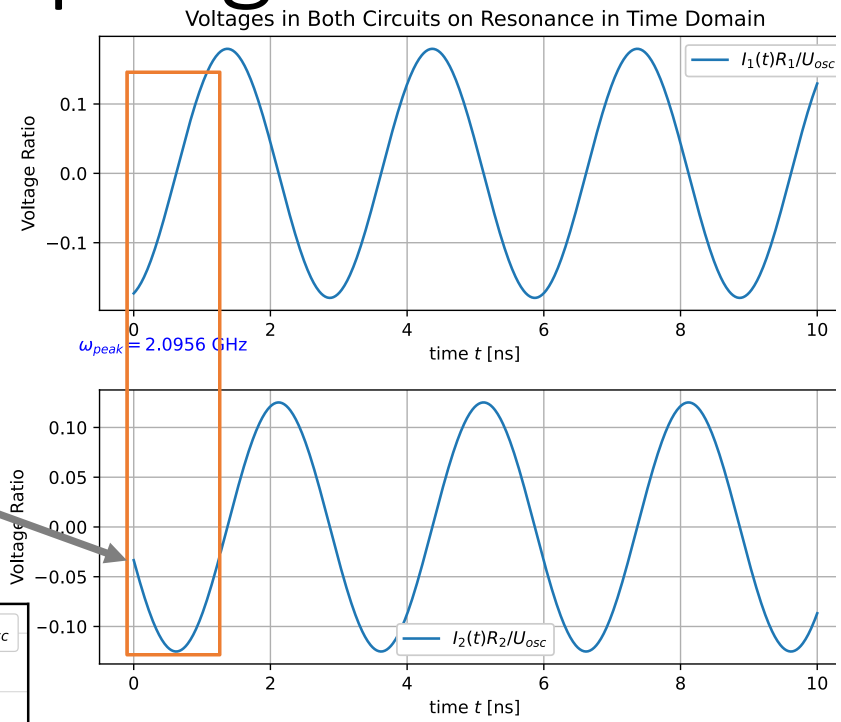


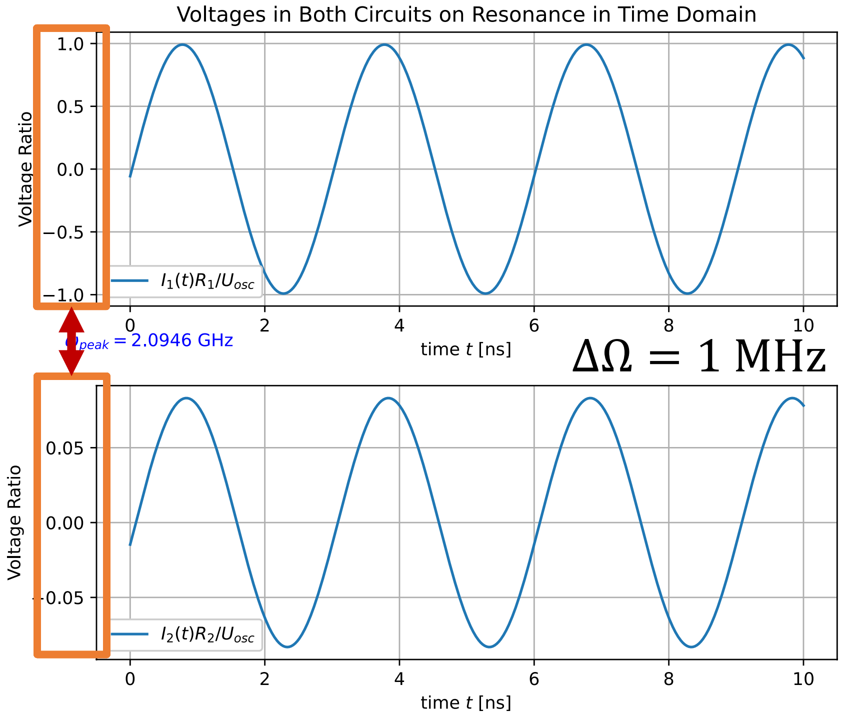
$k = 10^{-2}$, $\Delta\Omega = 1$ MHz $k = 10^{-3}$, $\Delta\Omega = 1$ MHz $k = 10^{-4}$, $\Delta\Omega = 1$ MHz

$Q_1 = 5 \cdot 10^{-4}$, $Q_2 = 3Q_1$ (leads to different peak sizes)

Effect of bad coupling $k = 10^{-4}$

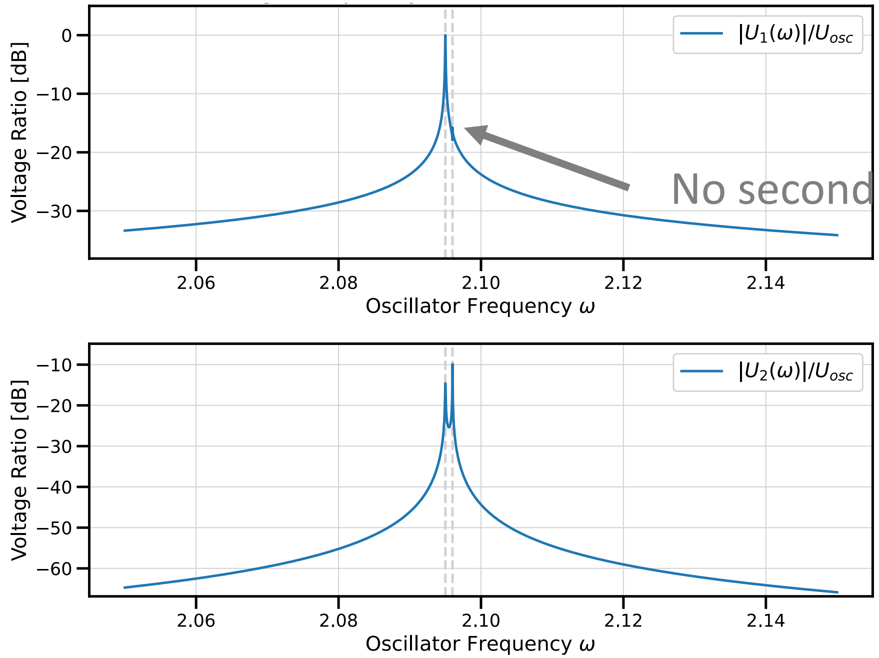
- No 0 and π modes anymore
- Large Amplitude Difference





$\Delta\Omega = 1$ MHz

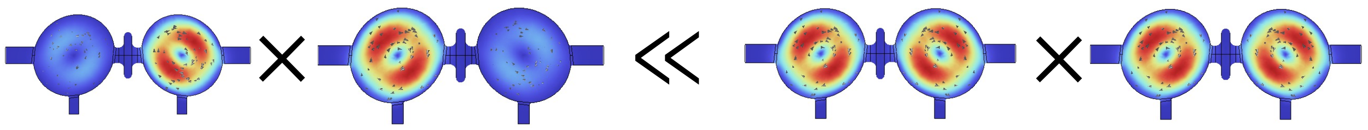
Arbitrary Phase Shift



No second resonance in circuit 1 anymore

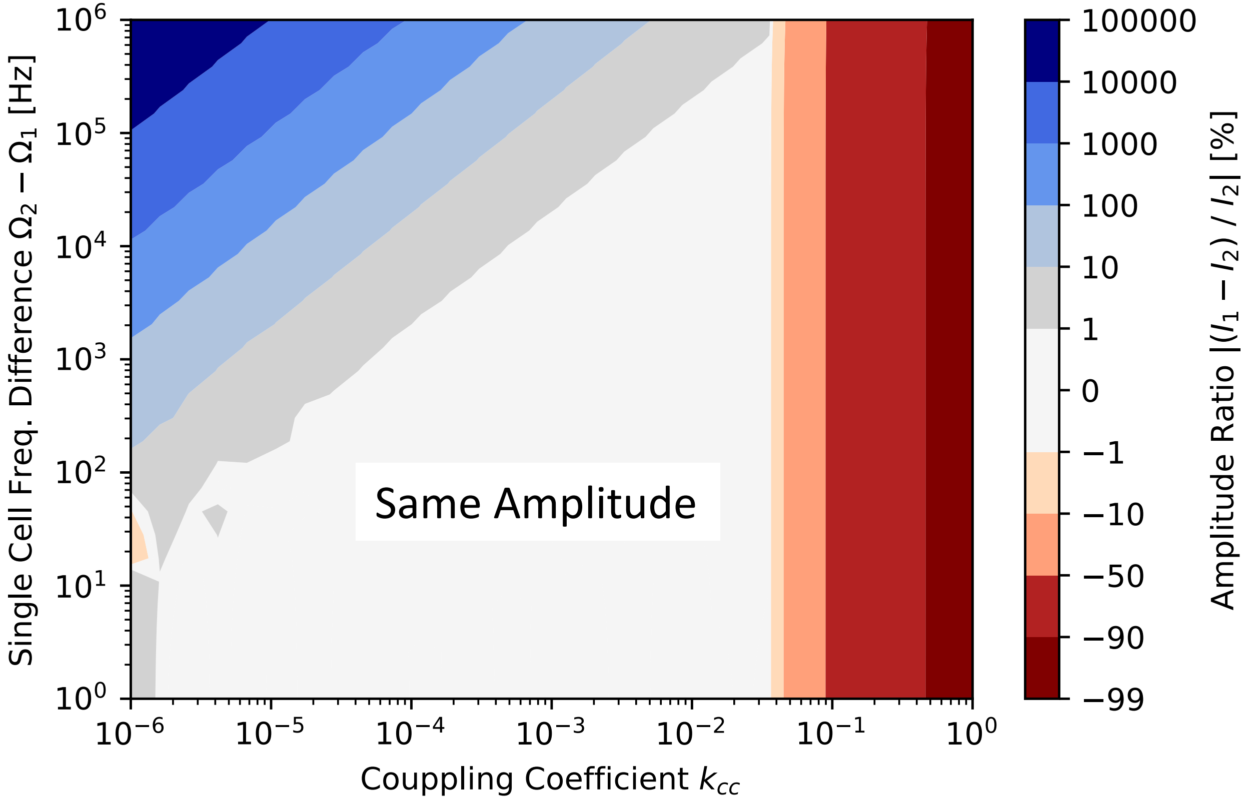
⇒ **Problem in Mech-EM-coupling:** $C_{01} \propto \int (dS_{cavity} \cdot \xi_{mech}) \cdot (E_0 \cdot E_1)$

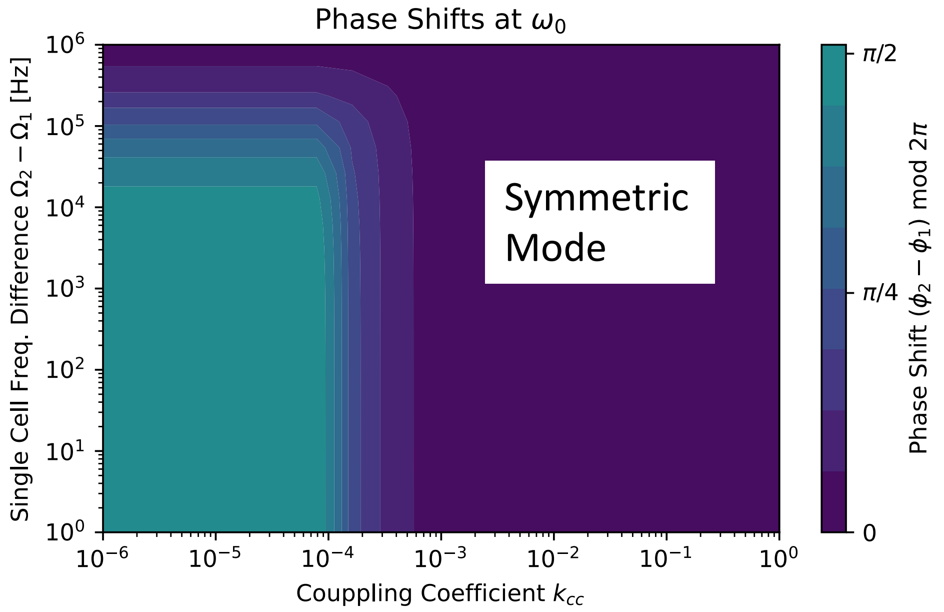
- **Phase shift of mechanical mode needs to match phase shift of EM-mode**
- **Low amplitude ratio enters** C_{01}

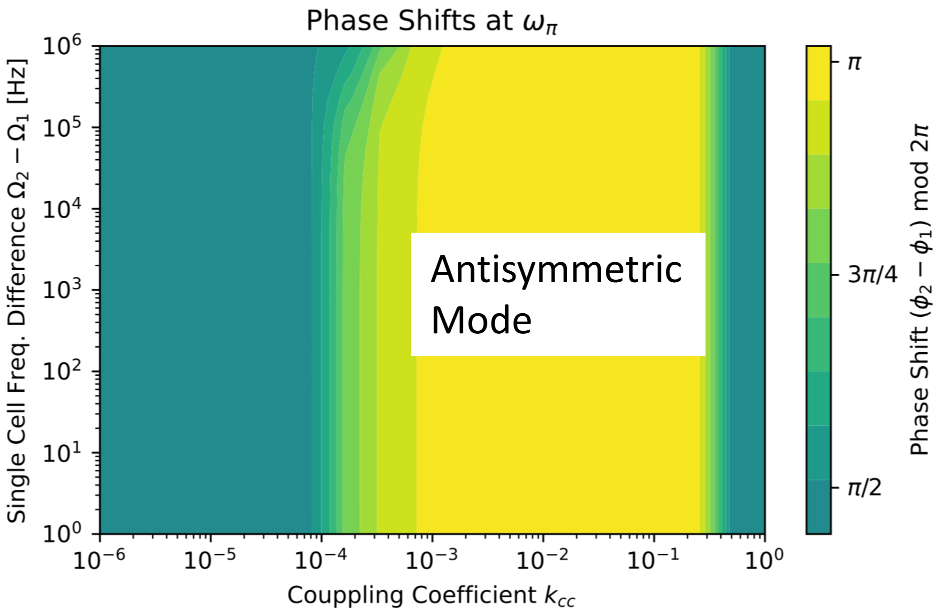


Design Requirements

$Q_1 = Q_2 = 10^9$







$Q_1 = Q_2 = 10^4$

For $Q_1, Q_2 \mapsto 10^{10}$: $\Delta\phi \mapsto 0 / \pi$

- (k, Ω_1, Ω_2) determine GW Frequency ω_g sensitivity through $k(\Omega_2 - \Omega_1) = \omega_\pi - \omega_0 \approx \omega_g$
- k and $\Omega_2 - \Omega_1$ should be chosen to create same Amplitude in both cells
- Q_1, Q_2 need to be $\gg 10^9$ to avoid arbitrary phase shifts

Conclusion

- The coupling of the two MAGO cavity cells is likely too bad to form the predicted mode patterns
- We expect the field strength in both cells to differ significantly (confirmed by COMSOL simulation)
  → Reduction of Mech-EM Coupling coefficient
- The predicted symmetric (0) and antisymmetric (pi) modes should still appear in the superconducting state

REMAINING QUESTION: Will the MAGO cavity need to be reshaped?