Plasma Instabilities of TeV Pair Beams induced by Blazars

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Astroparticle Physics Theory Group

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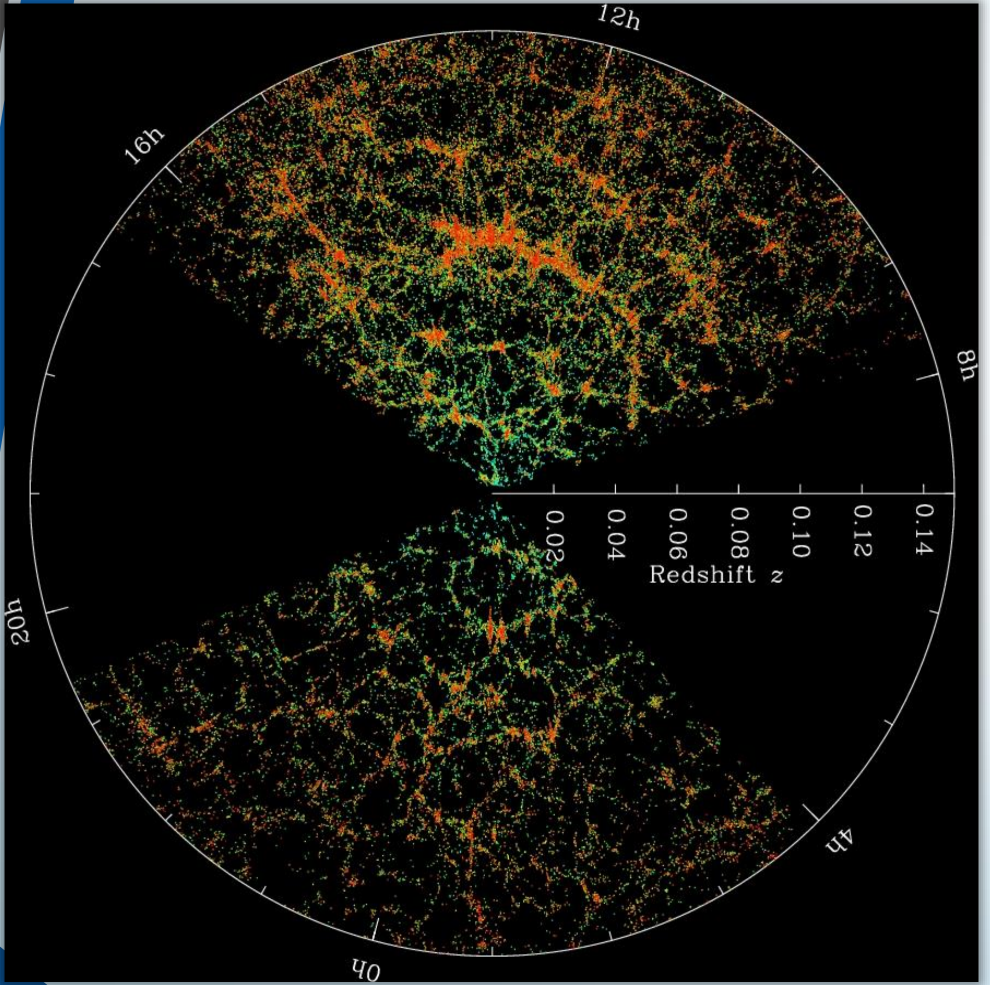
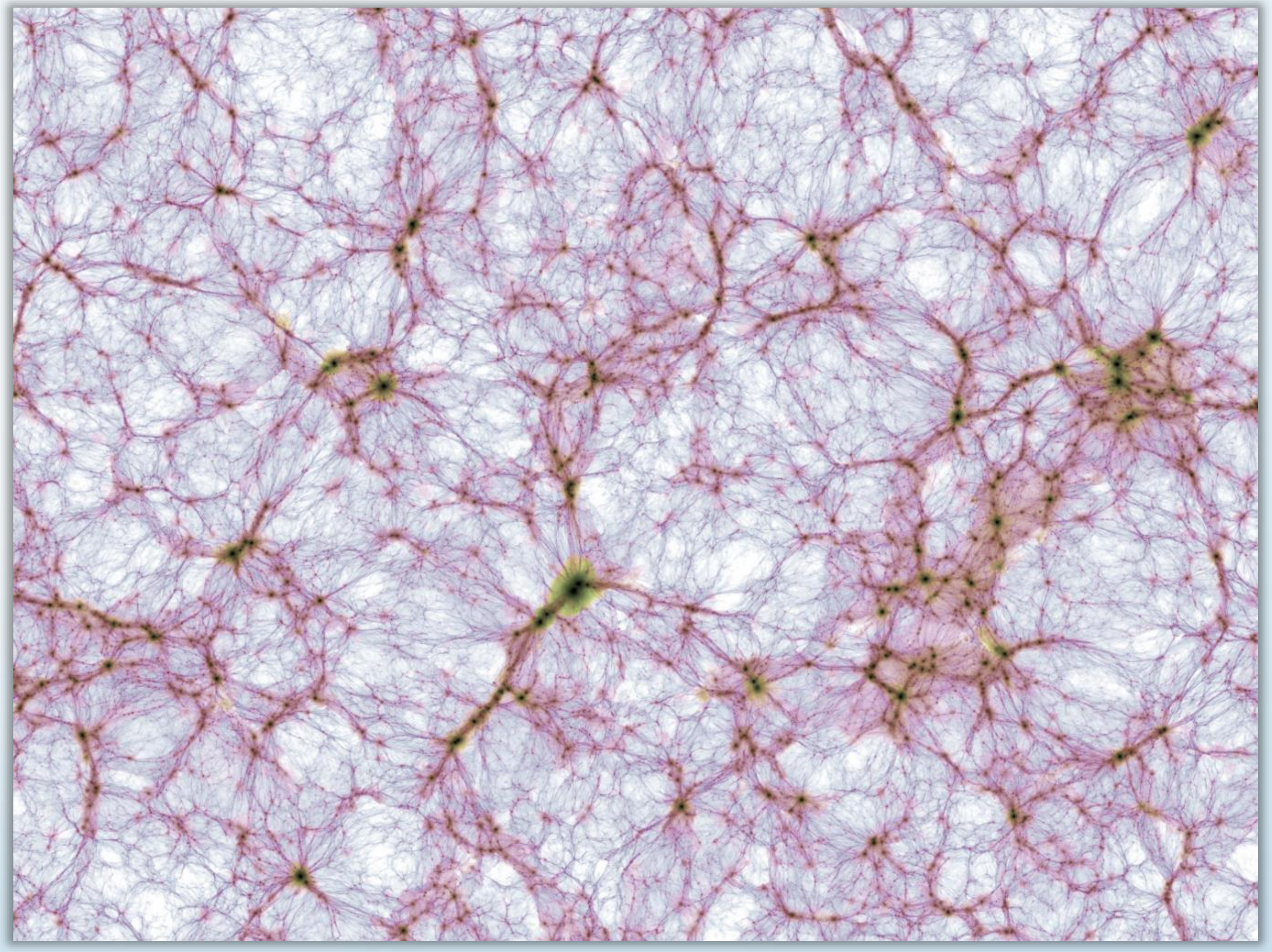
Cosmic Voids

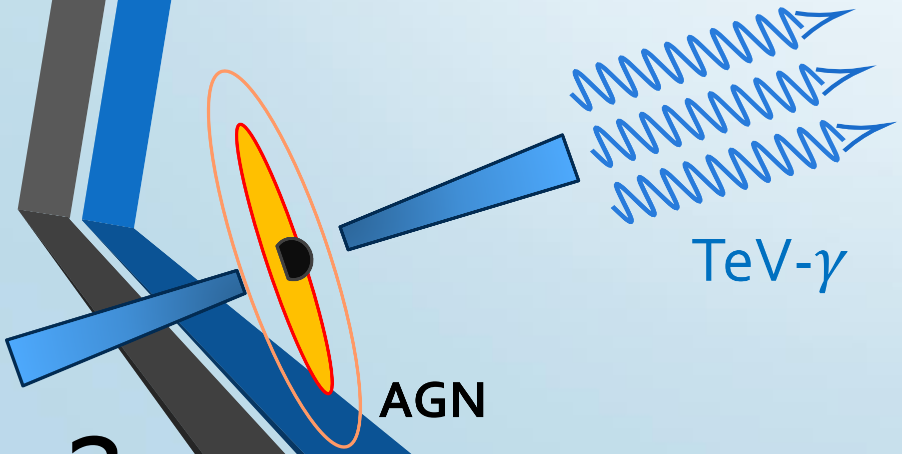


600 Mpc
Sloan Digital Sky Survey

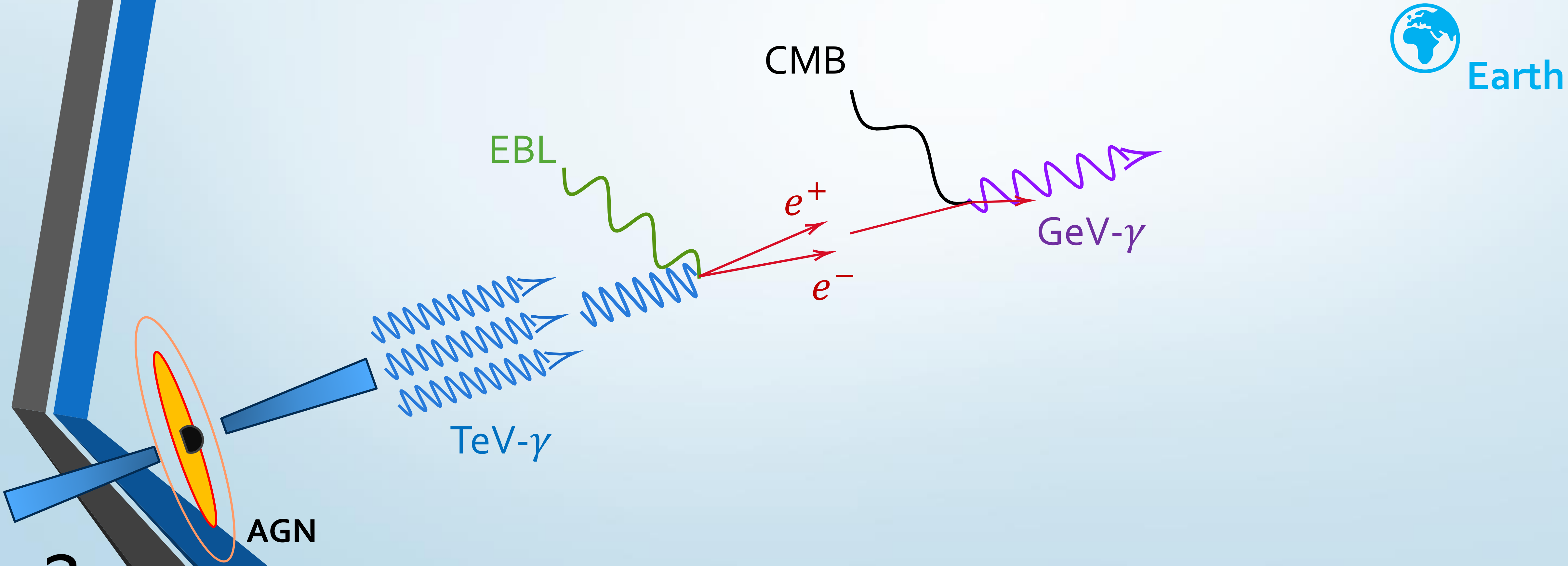


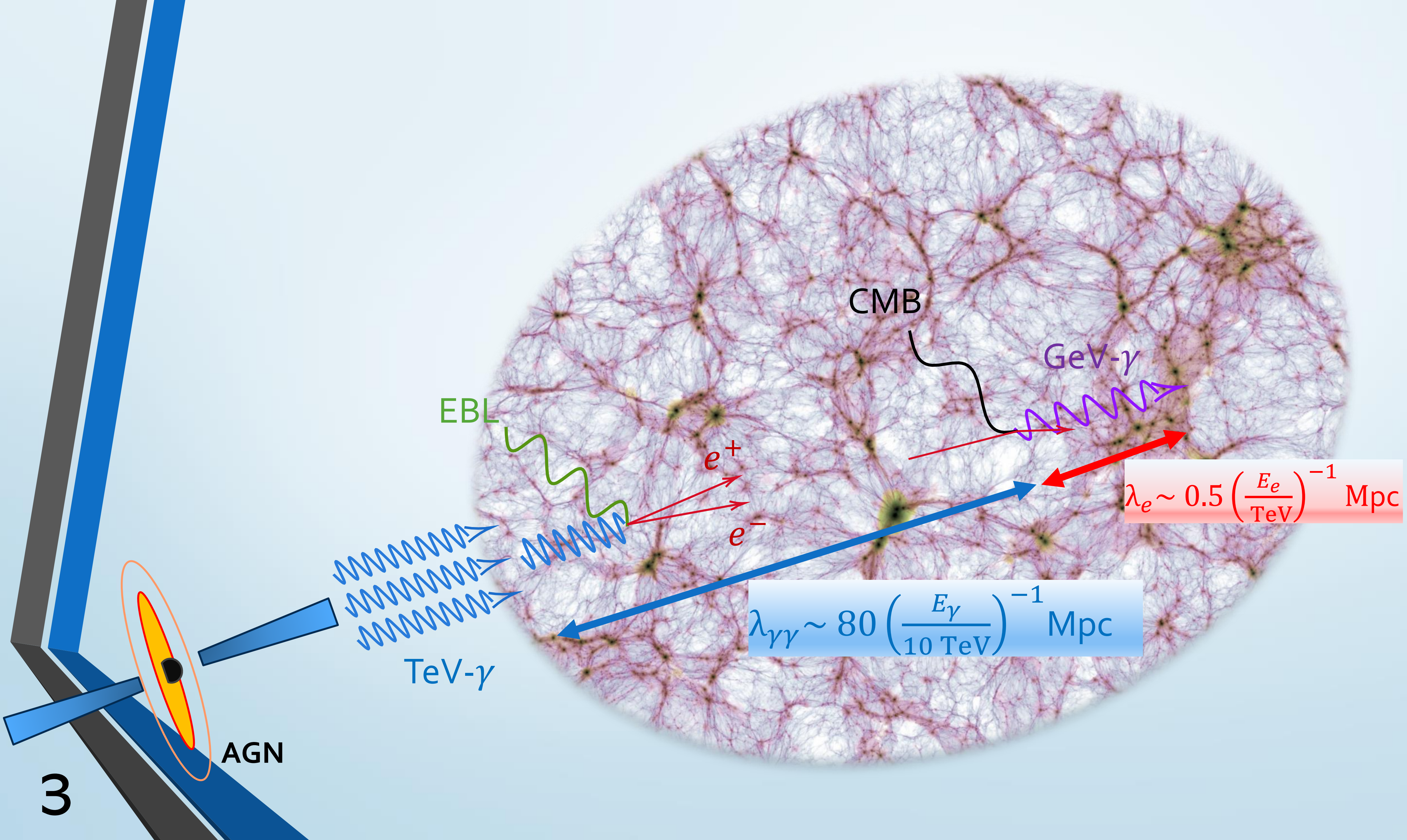
300MpcX300Mpc
TNG300 Simulation

- **Blazars are AGN with their jet pointing at us.**



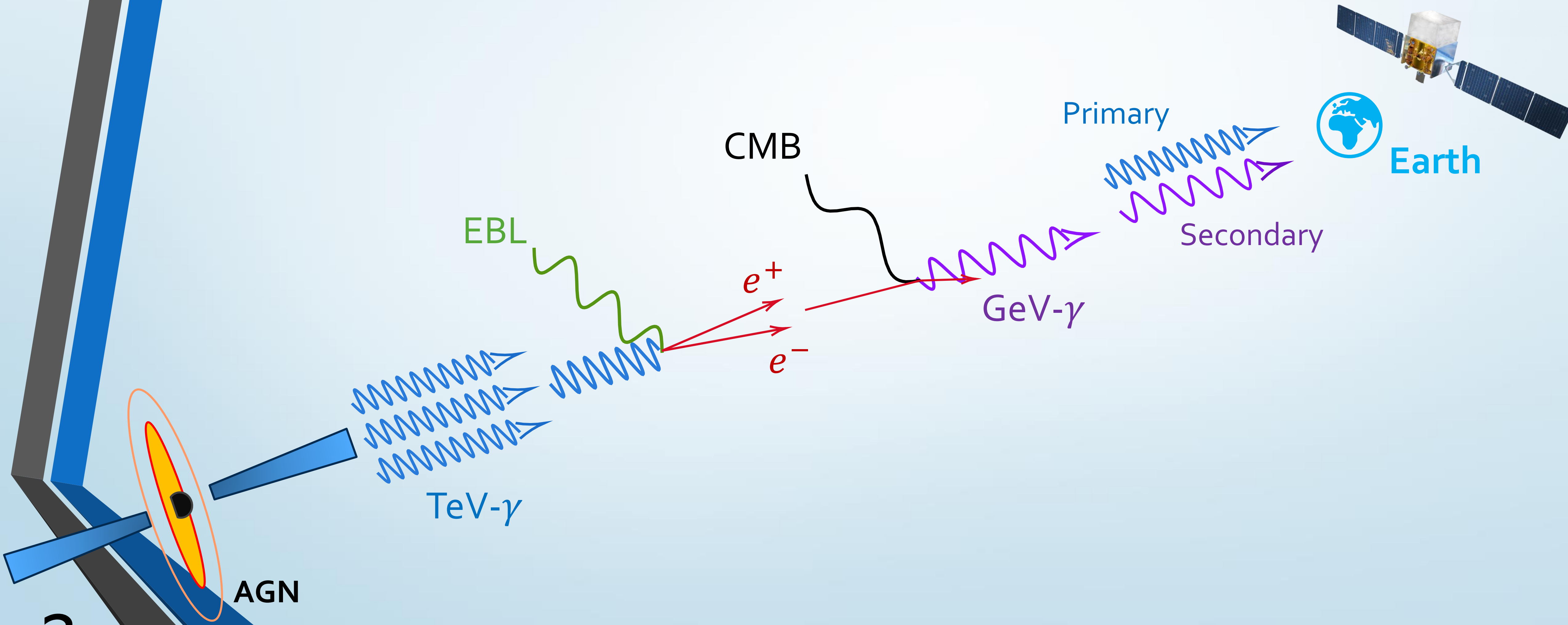
- **Blazars are AGN with their jet pointing at us.**
- **TeV gamma-rays attenuate in the cosmic voids giving GeV cascade.**



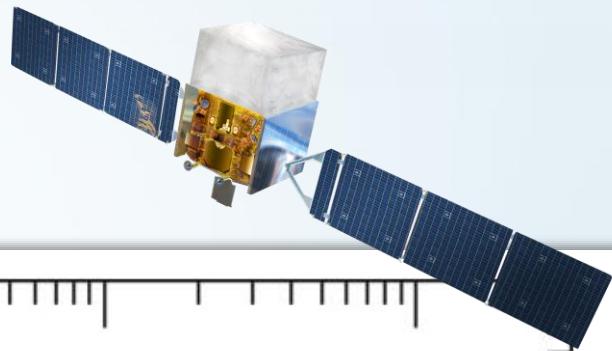

CMB
GeV-γ
EBL
e^+
e^-
$\lambda_e \sim 0.5 \left(\frac{E_e}{\text{TeV}}\right)^{-1}$ Mpc
$\lambda_{\gamma\gamma} \sim 80 \left(\frac{E_\gamma}{10\text{ TeV}}\right)^{-1}$ Mpc
TeV-γ
AGN

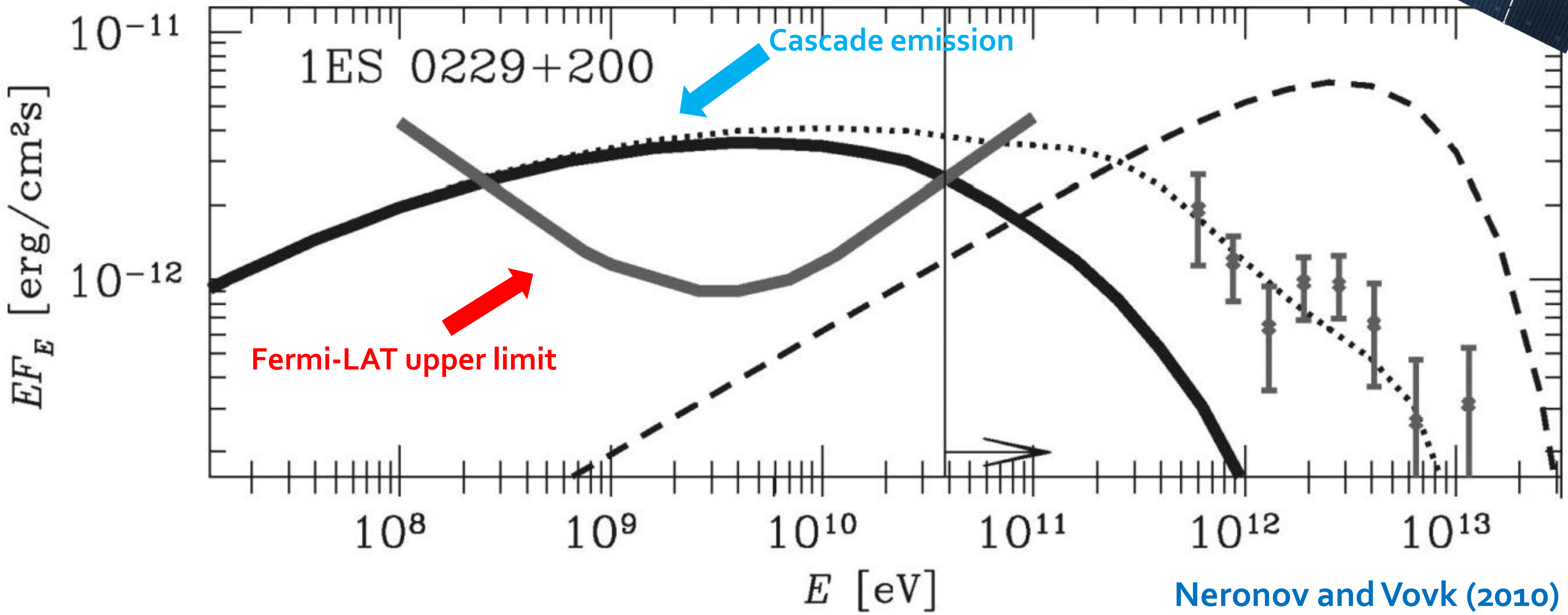
- **Blazars are AGN's with their jet pointing to us.**

- **TeV gamma-rays attenuate in the cosmic voids giving GeV cascade.**



The electromagnetic cascade is missing in the observations

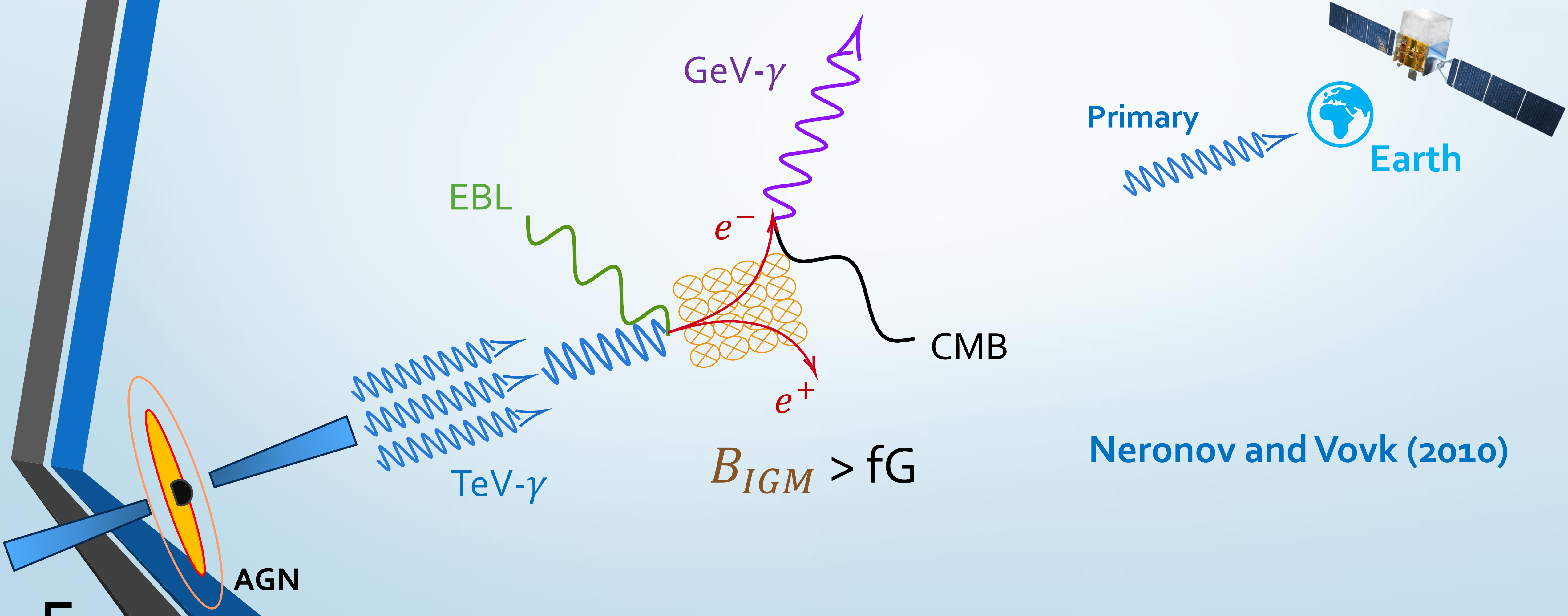




Neronov and Vovk (2010)

First possible solution

Pairs deflected by IGM magnetic fields



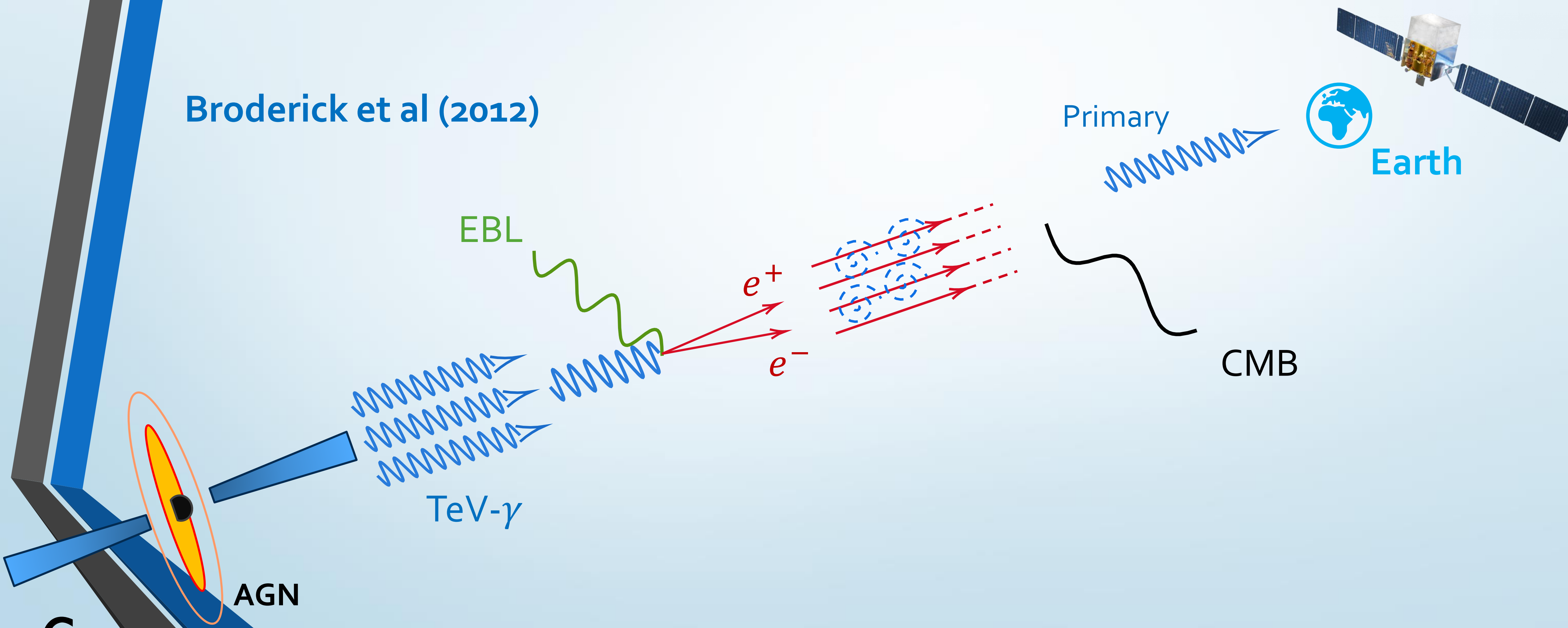
B_{IGM} > fG

Neronov and Vovk (2010)

Second possible solution

Energy loss by plasma instability before IC

Broderick et al (2012)



Second possible solution

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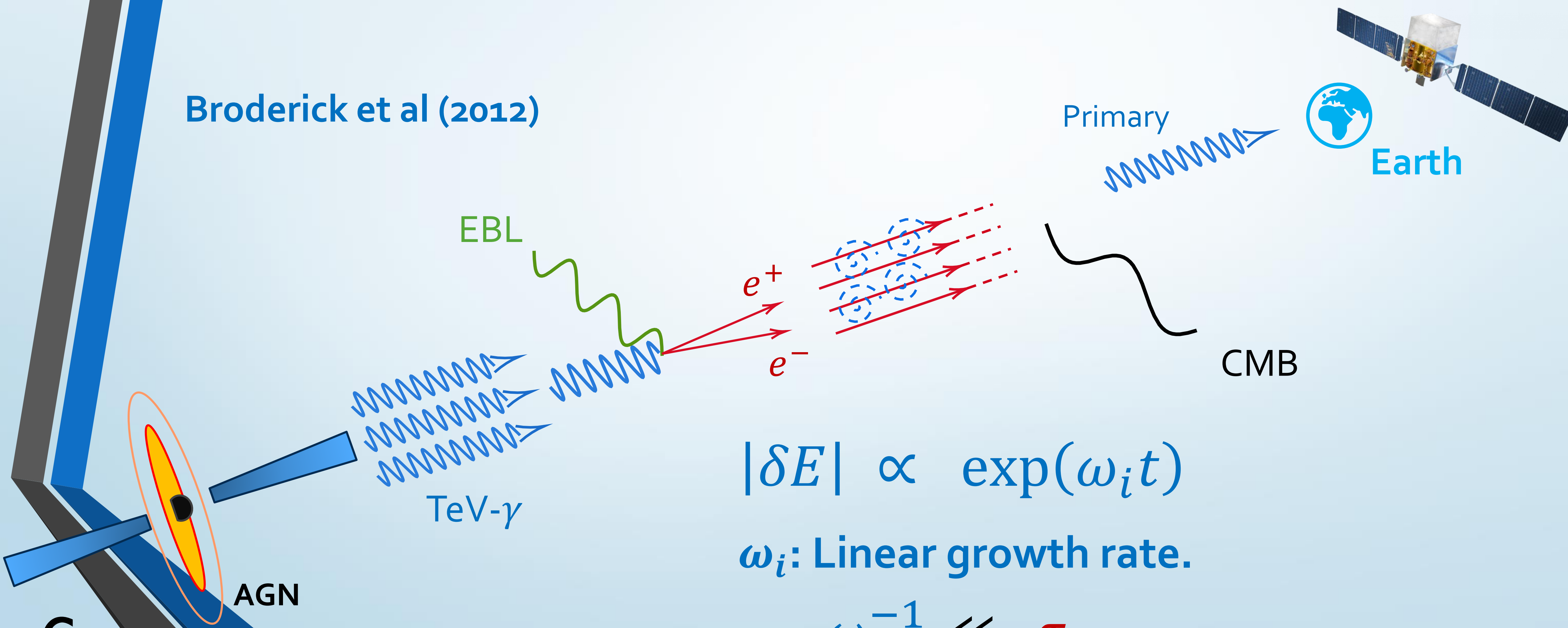


$$|\delta E| \propto \exp(\omega_i t)$$

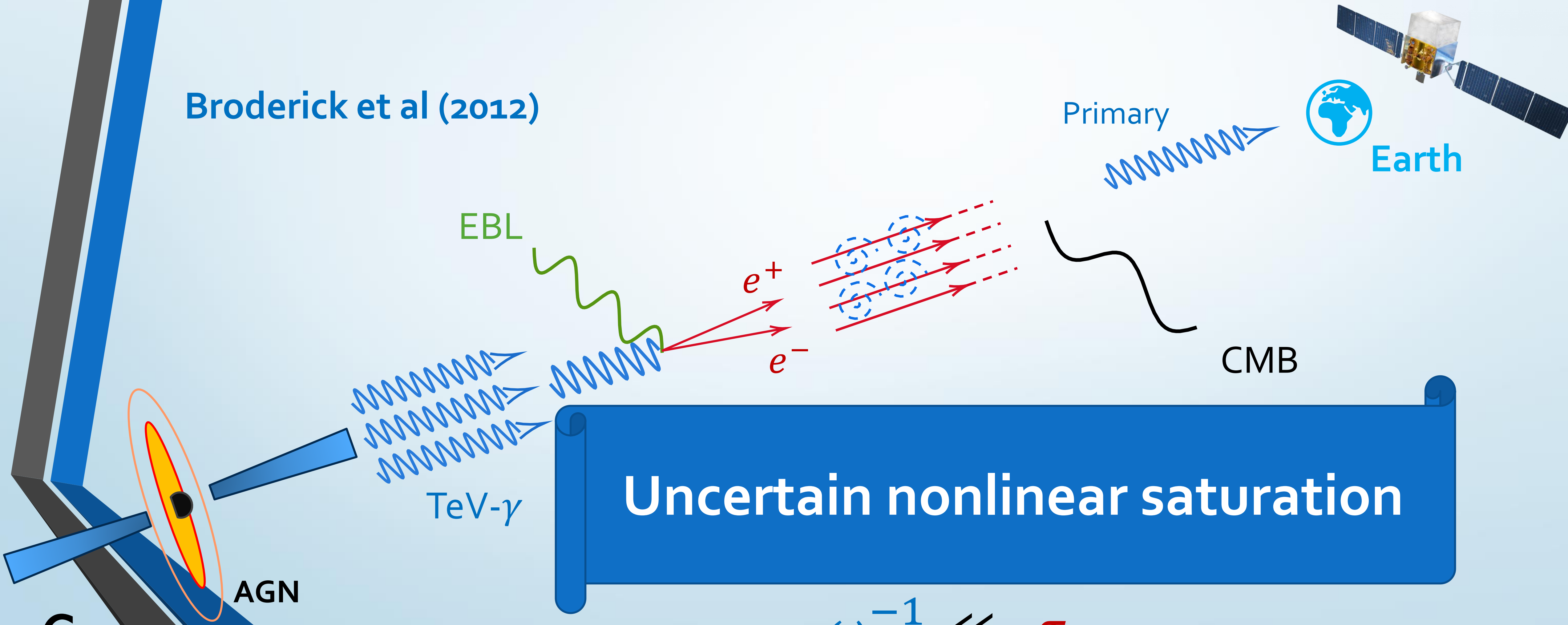
ω_i: Linear growth rate.

$$\omega_i^{-1} \ll \tau_{IC}$$

Second possible solution

Energy loss by plasma instability before IC

Broderick et al (2012)



Uncertain nonlinear saturation

$$\omega_i^{-1} \ll \tau_{IC}$$

Are the two solutions independent of each other?

Are the two solutions independent of each other?

NO, IGMFs impact on the instability.

Alawashra and Pohl (2022) *ApJ* 929 67

IGMFs impact the instability

- **Weak IGMFs with small correlation lengths, $\lambda_B \ll \lambda_e$, deflect the beam stochastically**

$$\Delta\theta = \frac{1}{\gamma}\sqrt{1 + \frac{2}{3}\lambda_e \lambda_B \left(\frac{eB_{IGM}}{m_e c}\right)^2}$$

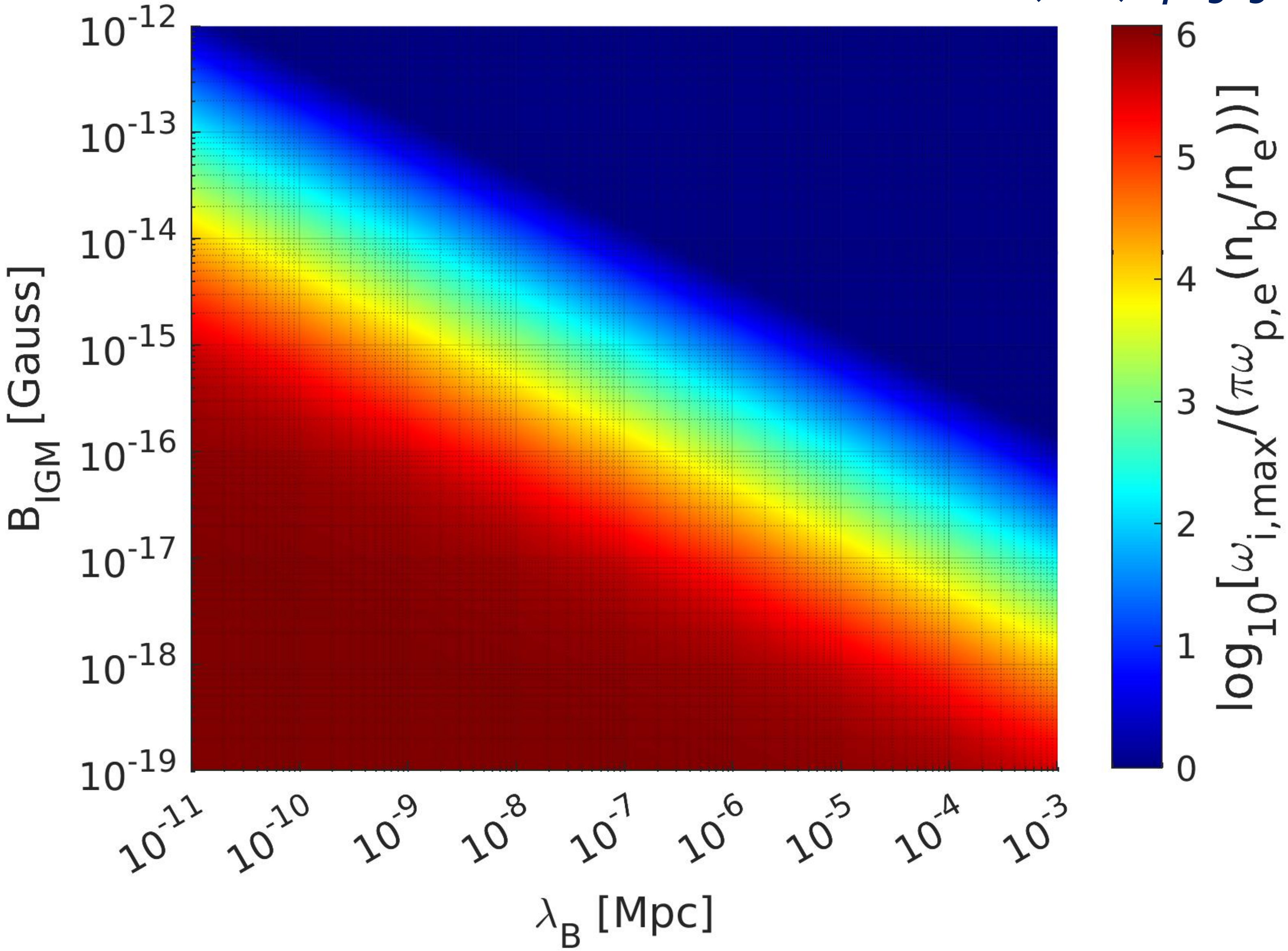
- **IGMFs widening of the beam impacts the instability growth:**

$$\omega_i \propto \frac{1}{\Delta\theta^2}$$

Alawashra and Pohl (2022) *ApJ* 929 67

Instability suppression by the IGMFs

Alawashra and Pohl (2022) *ApJ* 929 67



IGMFs impact the instability

- **Assume certain non-linear saturation of the waves**

$$\tau_{\rm loss}^{-1} = 2\ \delta\ \omega_{i,\rm max}$$

$$\delta = W_{\rm tot}/U_{\rm beam}$$

We consider the one found in Vafin et al. (2018)

$$\tau_{\rm loss}/\tau_{\rm IC} = 0.026$$

IGMFs impact the instability

- **Assume certain non-linear saturation of the waves**

$$\tau_{\rm loss}^{-1} = 2\ \delta\ \omega_{i,\rm max}$$

$$\delta = W_{\rm tot}/U_{\rm beam}$$

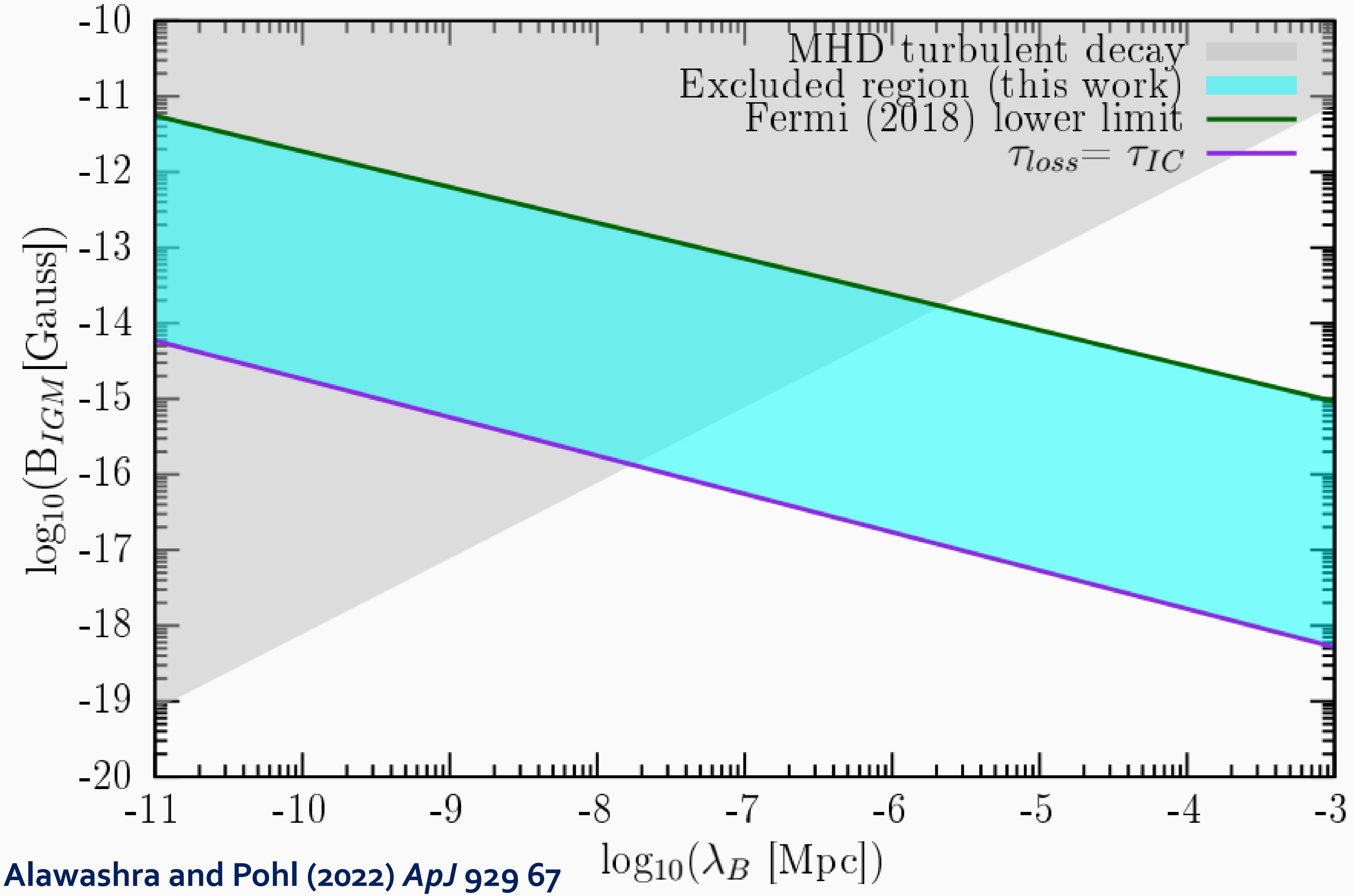
We consider the one found in Vafin et al. (2018)

$$\tau_{\rm loss}/\tau_{\rm IC} = 0.026$$

- **The instability is suppressed by the IGMFs when**

$$\tau_{\rm loss} = \tau_{\rm IC}$$

Instability suppression by the IGMFs



Alawashra and Pohl (2022) *ApJ* 929 67

Is there something else that can impact the instability?

Is there something else that can impact the instability?

Yes, Nonlinear feedback.

Perry and Lyubarsky (2021)

Alawashra and Pohl (2024) *ApJ 964 82*

Feedback of the instability on the pair beam

Breizman and Ryutov (1970)

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta p}\frac{\partial f}{\partial p}\right)$$

$$+\frac{1}{p^2}\frac{\partial}{\partial p}\left(p D_{p\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right)$$

f: Beam distribution

D_{ij}: Diffusion coefficients

W: Wave energy density

ω_i: Linear growth rate

$$D_{ij}(\boldsymbol{p}) = \pi e^2 \int d^3\boldsymbol{k}\, W(\boldsymbol{k},t)\frac{k_i k_j}{k^2}\delta\left(\boldsymbol{k}\cdot\boldsymbol{v} - \omega_p\right)$$

$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right)W(\boldsymbol{k},t)$$

$$\omega_i(\boldsymbol{k}) = \omega_p\frac{2\pi^2 n_b e^2}{k^2}\int d^3\boldsymbol{p}\left(\boldsymbol{k}\cdot\frac{\partial f(\boldsymbol{p})}{\partial \boldsymbol{p}}\right)\delta\left(\omega_p - \boldsymbol{k}\cdot\boldsymbol{v}\right)$$

Feedback of the instability on the pair beam

Breizman and Ryutov (1970)

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta p}\frac{\partial f}{\partial p}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p D_{p\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right)$$

The plasma waves impact the beam

$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right) W(\boldsymbol{k},t)$$

The beam impacts the plasma waves

Feedback of the instability in Perry and Lyubarsky (2021)

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p\ldots}\ldots\left(\ldots\frac{\partial f}{\partial p}\right)$$

$$+\frac{1}{p^2\ldots}\ldots\left(\ldots\frac{\partial f}{\partial\theta}\right) + \frac{1}{p^2\ldots}\ldots\left(\ldots\frac{\partial f}{\partial p}\right)$$

The significant feedback initially is the beam widening $\theta\theta$.

Feedback of the instability in Perry and Lyubarsky (2021)

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p\cdots}\cdots\frac{\partial f}{\partial p}\Big)$$

$$+\frac{1}{p^2}\cdots\frac{\partial f}{\partial\theta}\Big) + \frac{1}{p^2}\cdots\frac{\partial f}{\partial p}\Big)$$

Considered simplified 1D beam distribution.

$$g(\theta) = \int_0^\infty dp\, p\, f(p,\theta) \approx \exp(-0.2(\gamma\theta)^5)$$

$$\gamma = 10^6$$

Feedback of the instability in Perry and Lyubarsky (2021)

The beam widens by one order of magnitude, suppressing the instability energy loss of the beam.

$\gamma = 10^6$

Questions

- **What is the feedback impact on the GeV cascade?**
  Need the realistic 2D beam distribution.

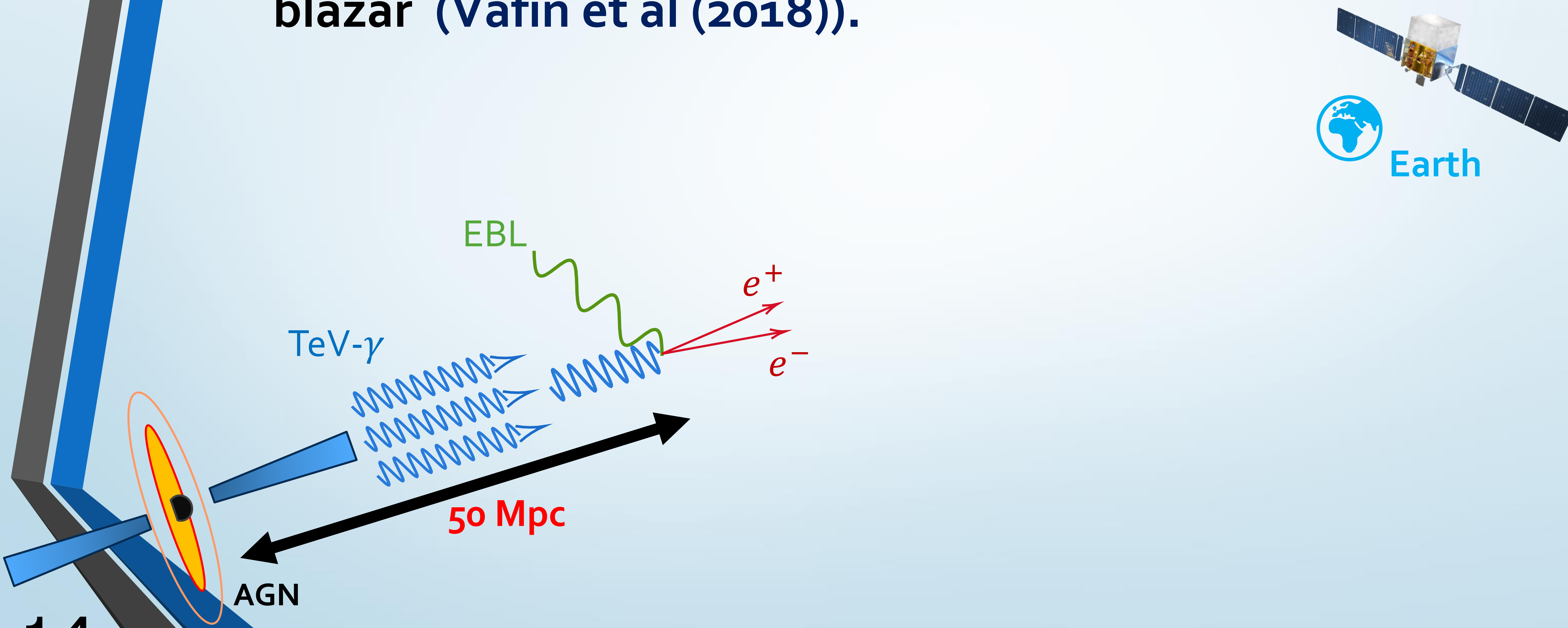
- **What is the impact of continuous pair production?**
  Need to include pair injection in the beam evolution equation.

Q1

What is the feedback impact on the GeV cascade?

2D simulation of the widening feedback

- Start with the realistic beam distribution at 50 Mpc from blazar (Vafin et al (2018)).



2D simulation of the widening feedback

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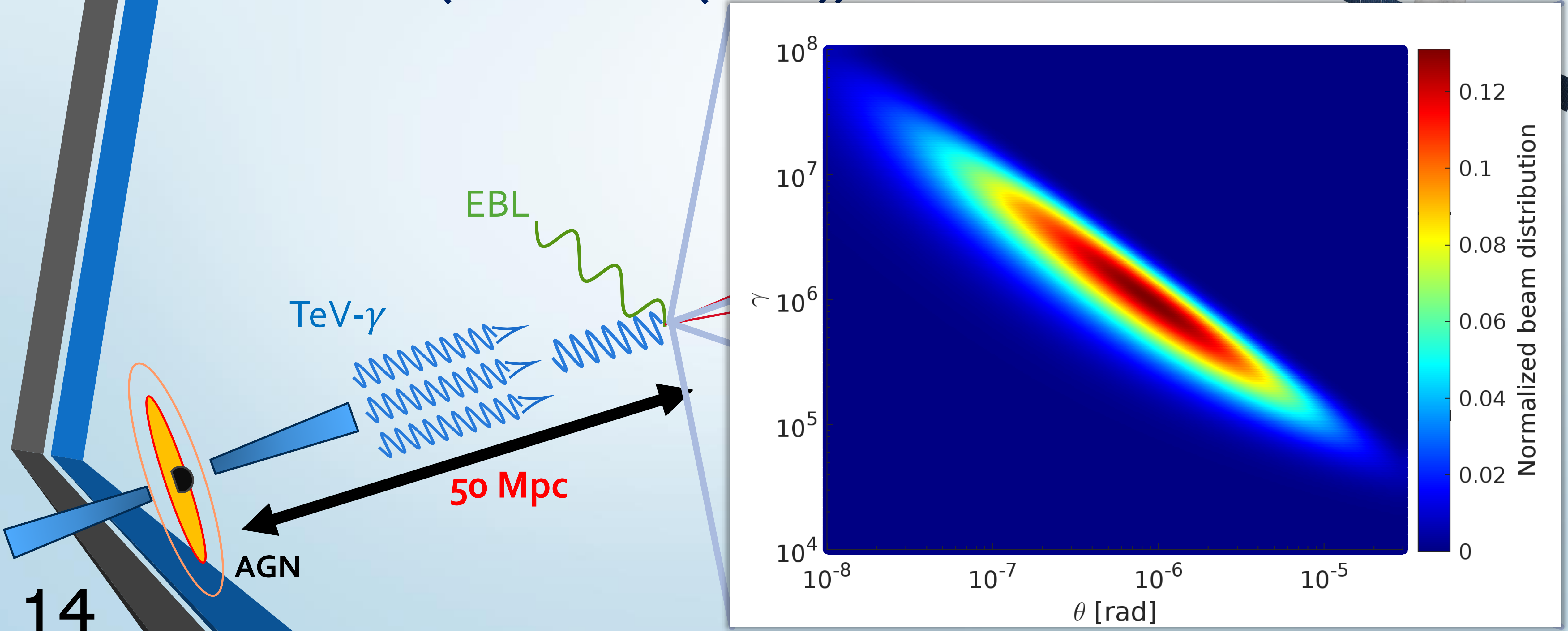
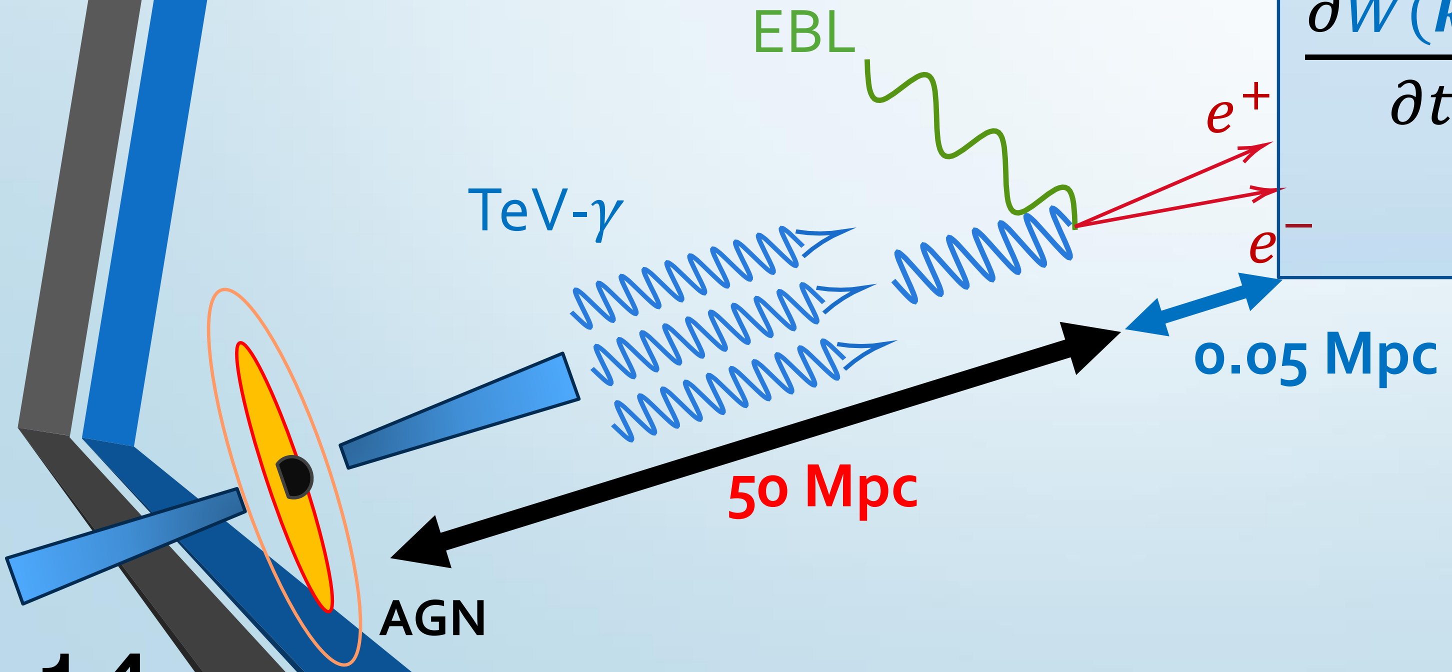


2D simulation of the widening feedback

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right)$$

$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right) W(\boldsymbol{k},t)$$



2D simulation of the widening feedback

Beam and IGM parameters

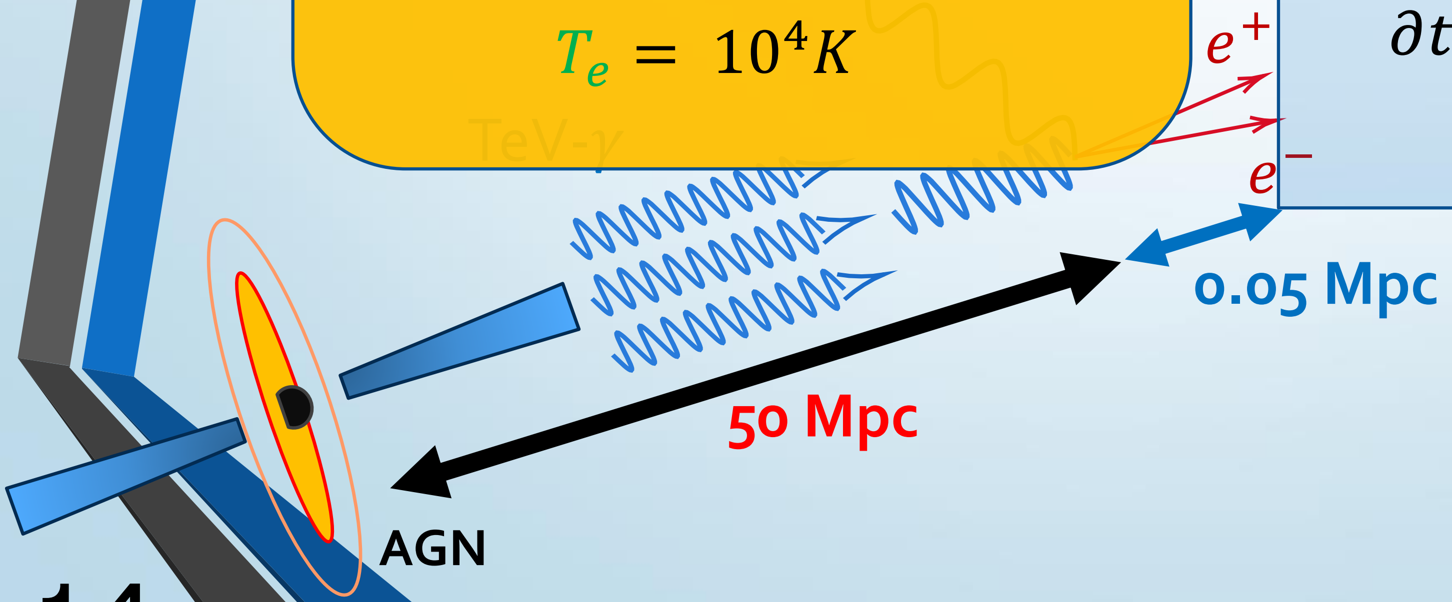
$n_b = 3 \times 10^{-22}\ \mathrm{cm}^{-3}$

$n_e = 10^{-7}\,(1+z)^3\,\mathrm{cm}^{-3}$

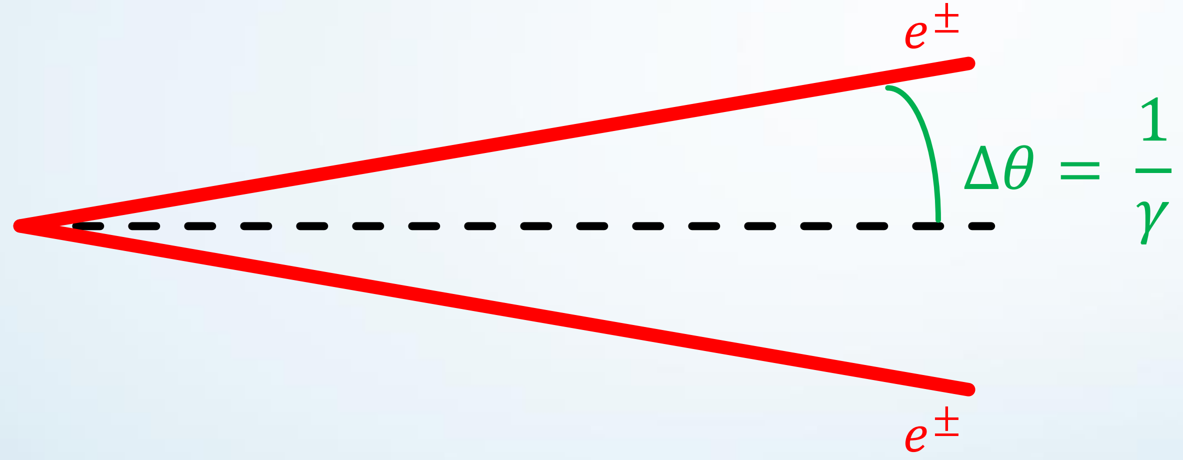
$T_e = 10^4 K$

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right)$$

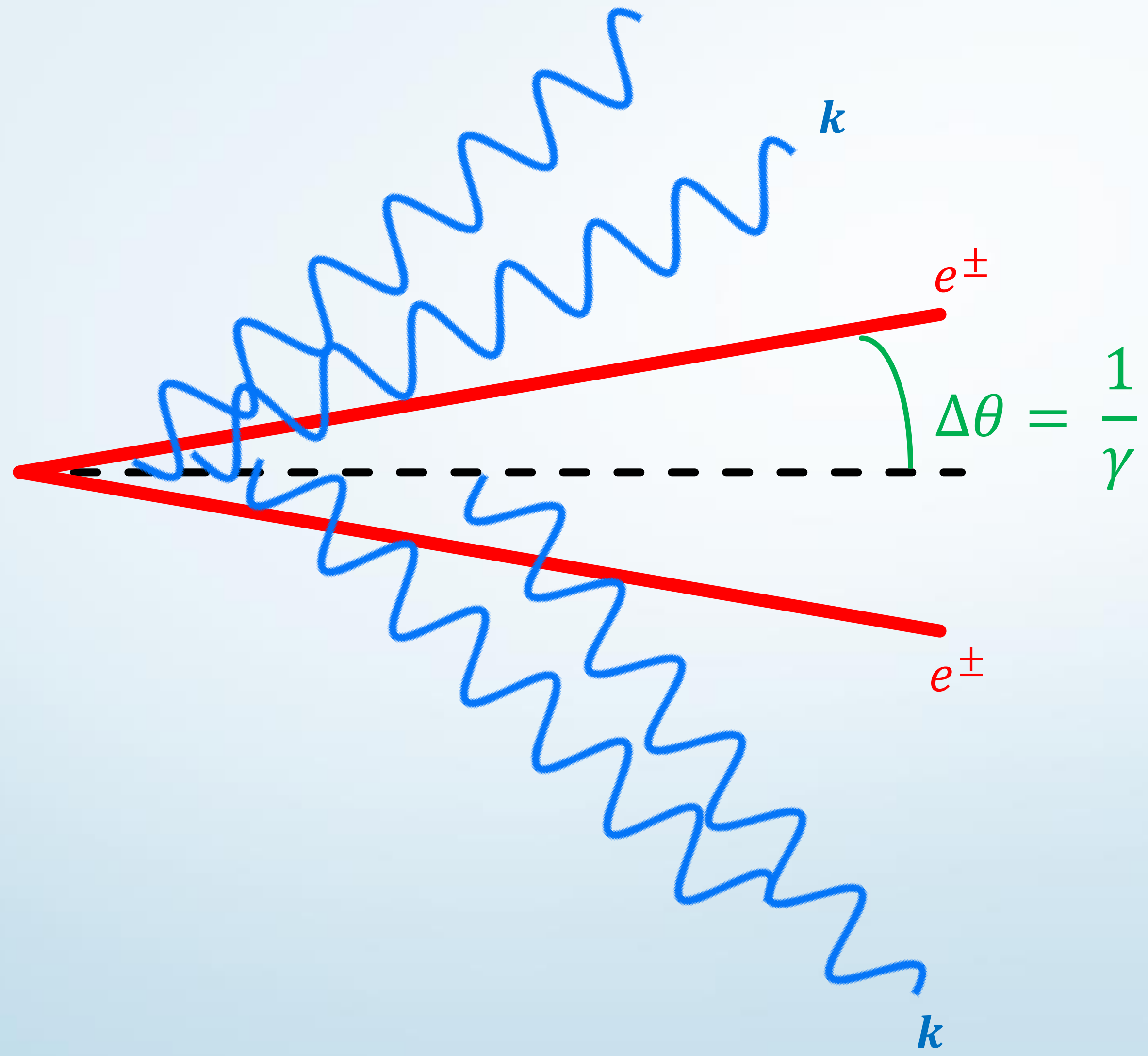
$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right) W(\boldsymbol{k},t)$$



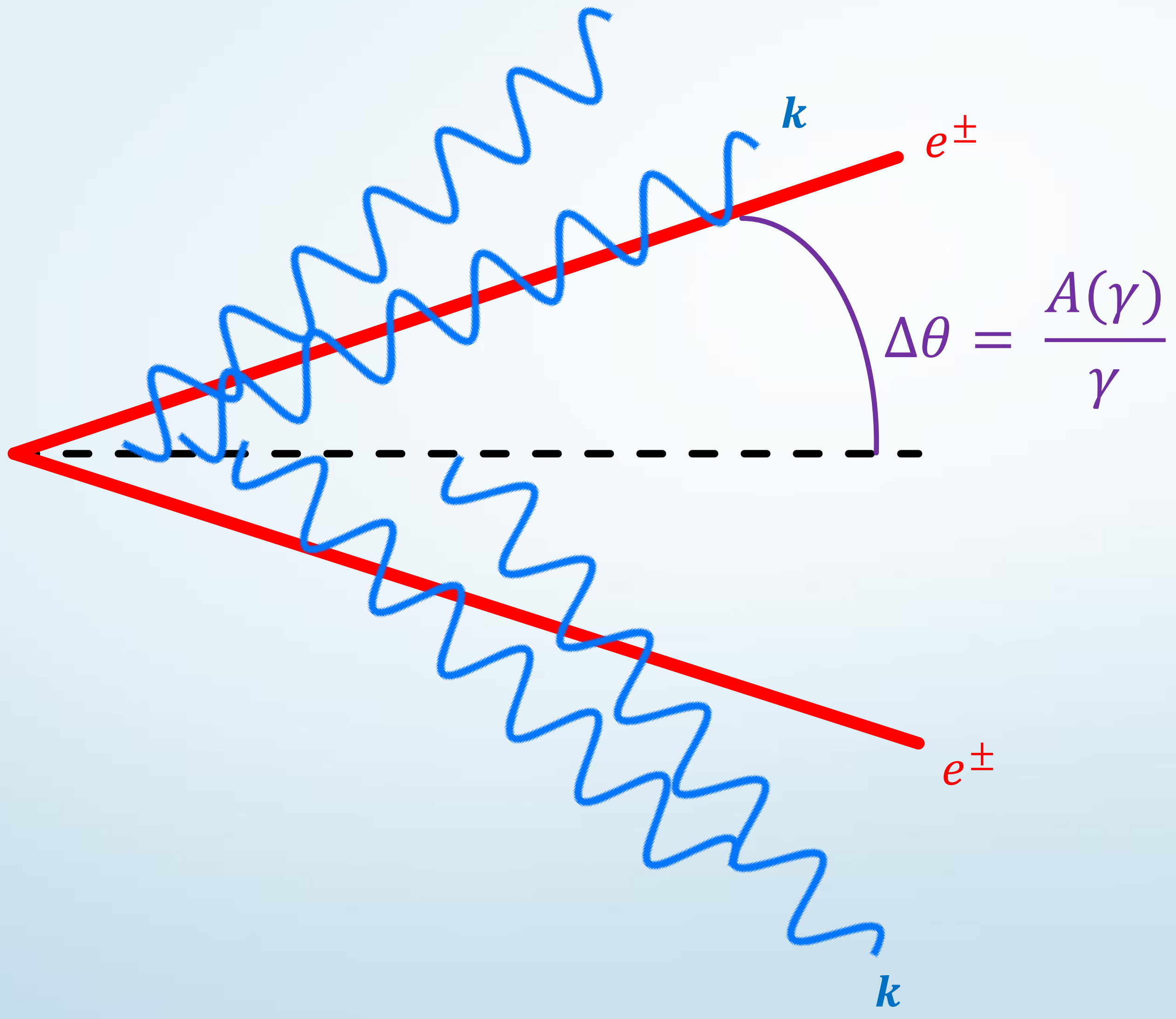
Initially focused beam



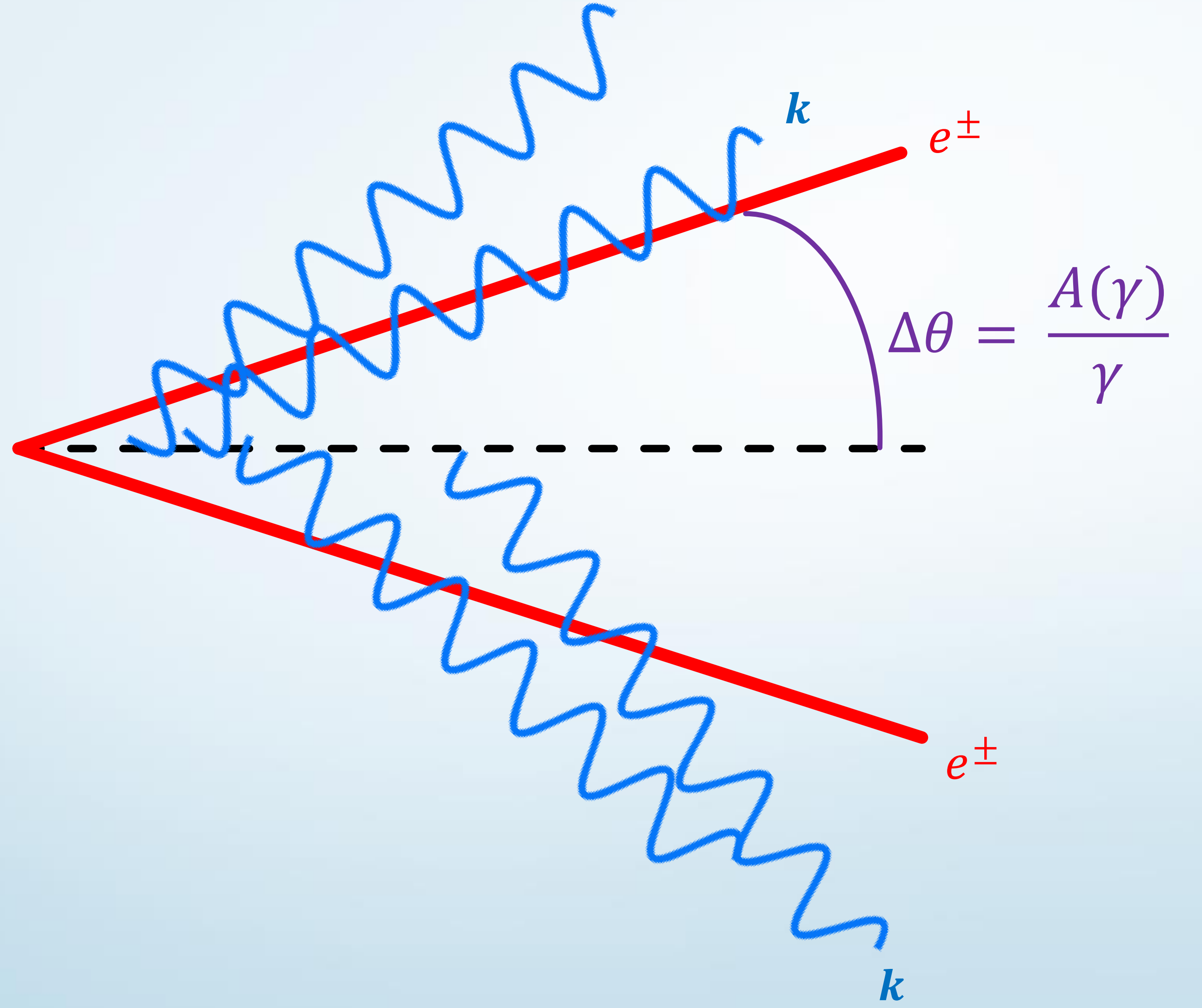
Plasma waves grow due to the focused beam



The feedback of the waves widens the beam

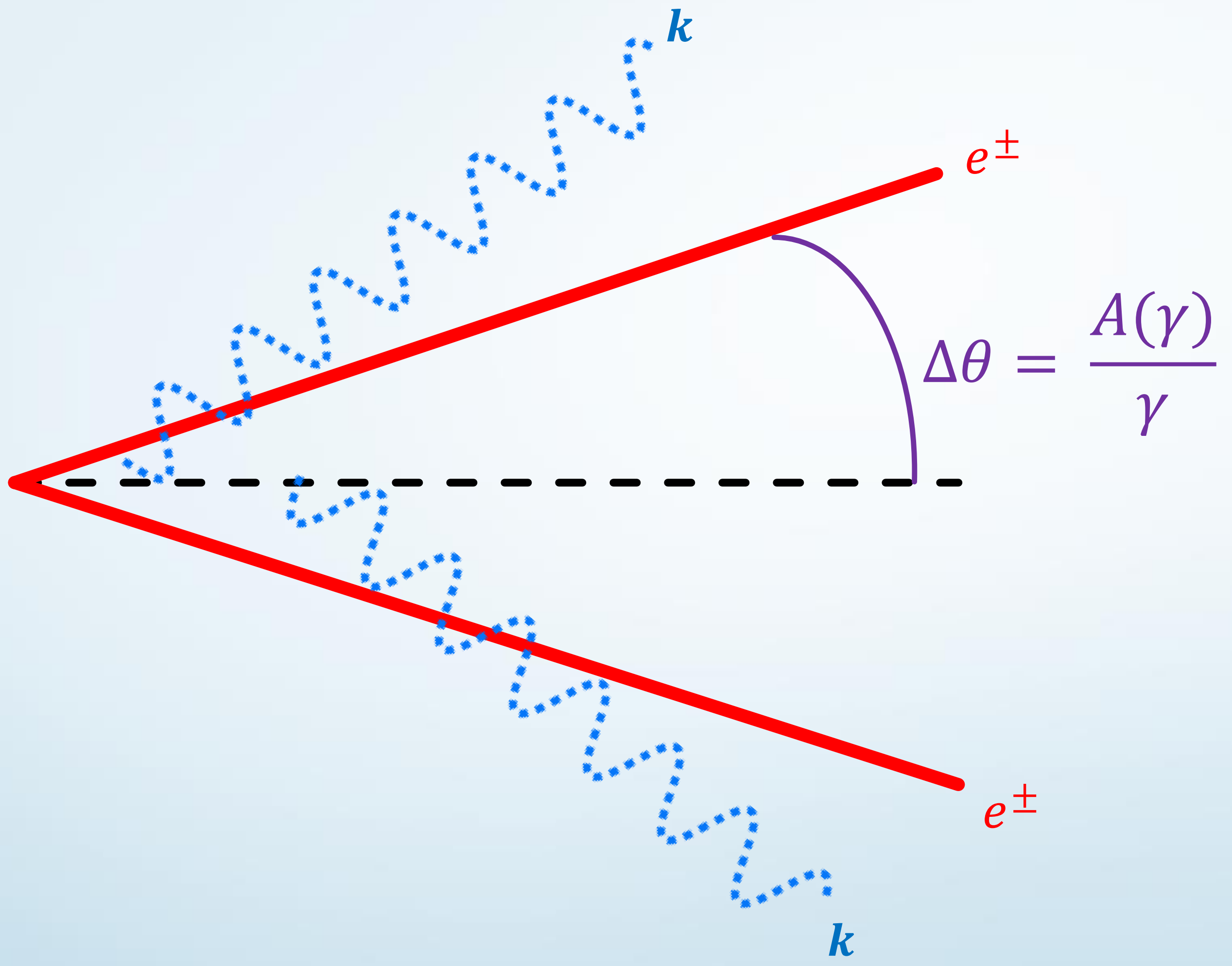


Waves growth is reduced



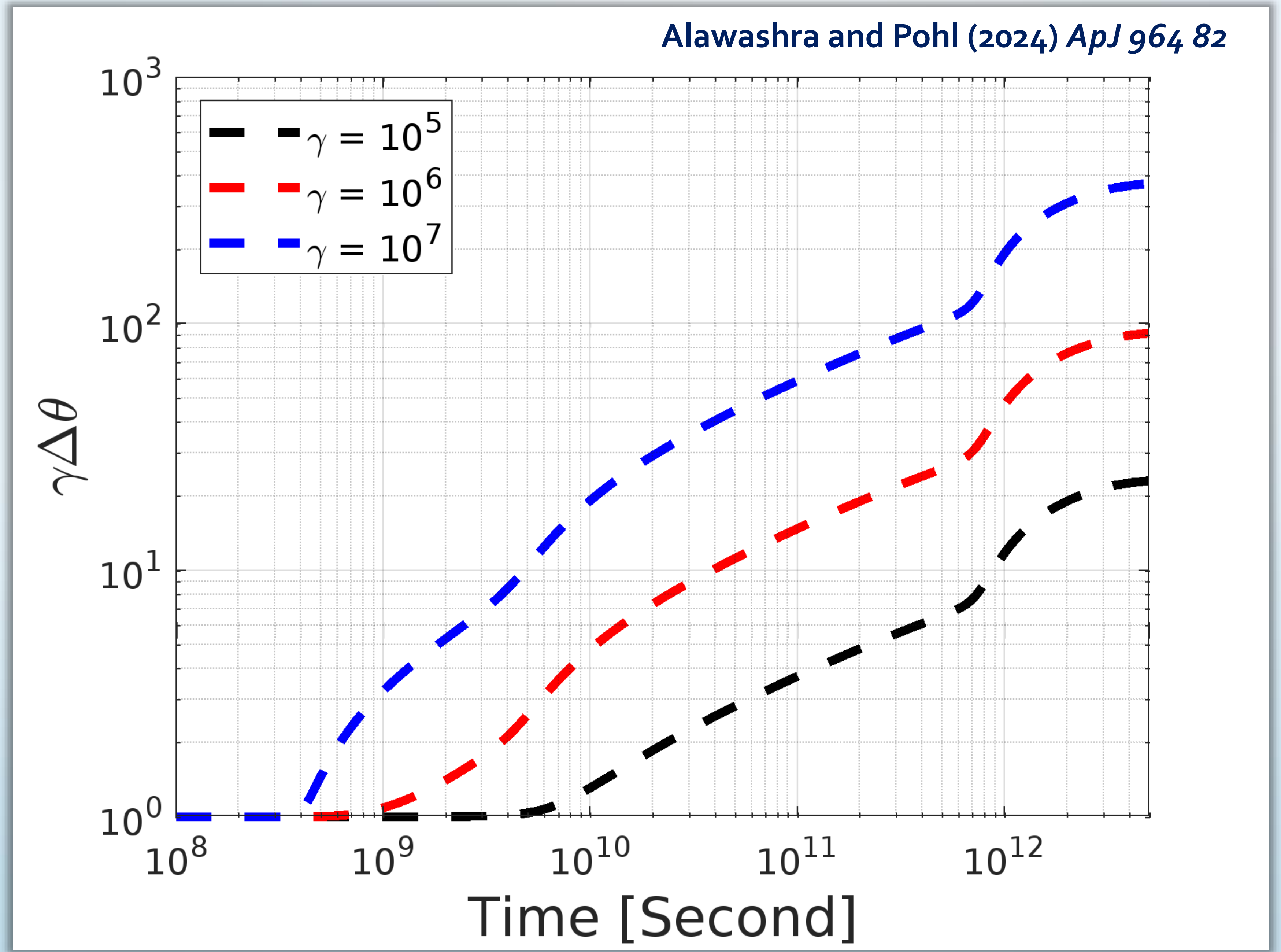
$$\omega_i \propto \frac{1}{\Delta\theta^2}$$

Waves get damped

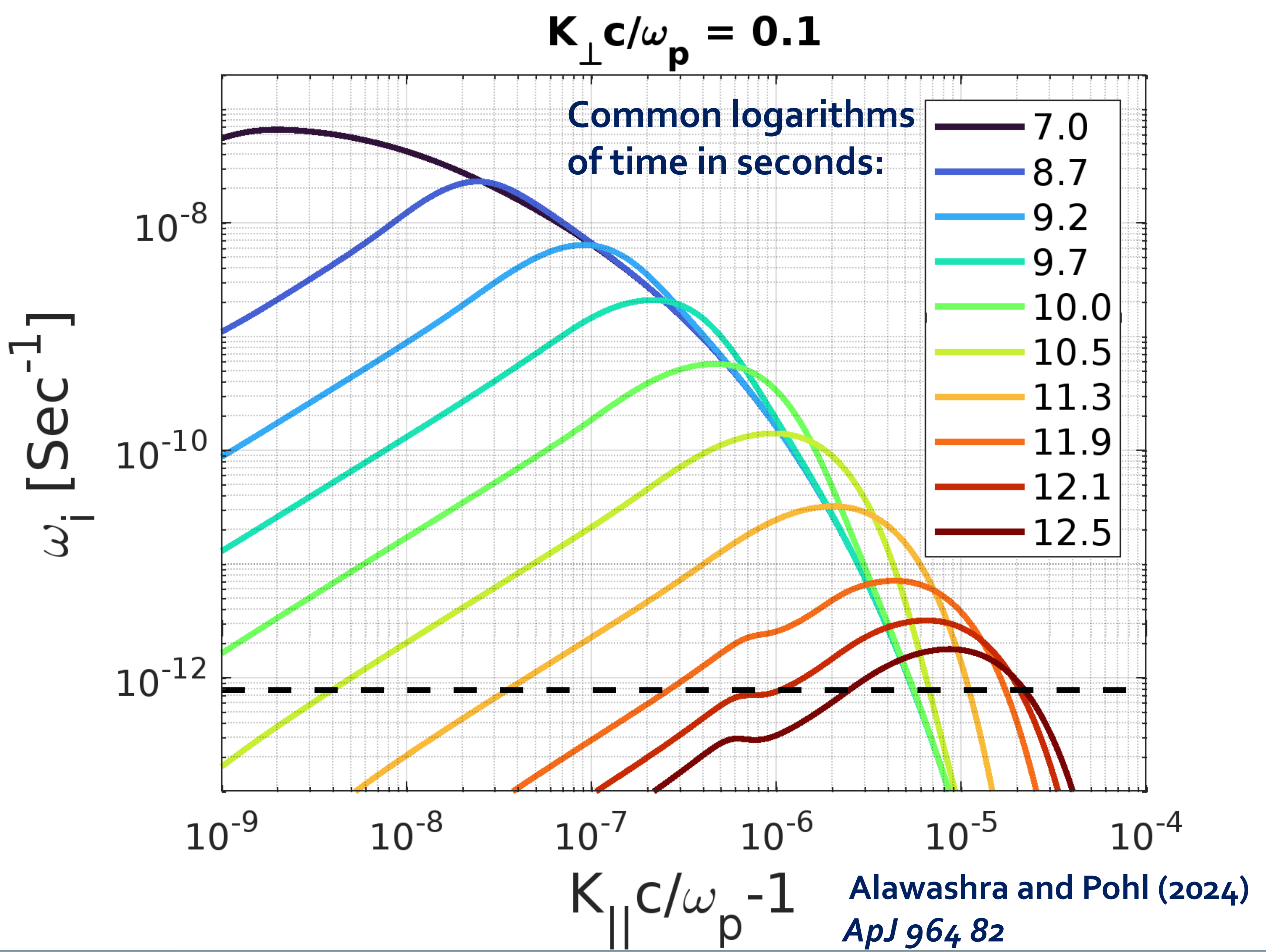


$\omega_i < |\omega_c|$

Significant widening of the beam

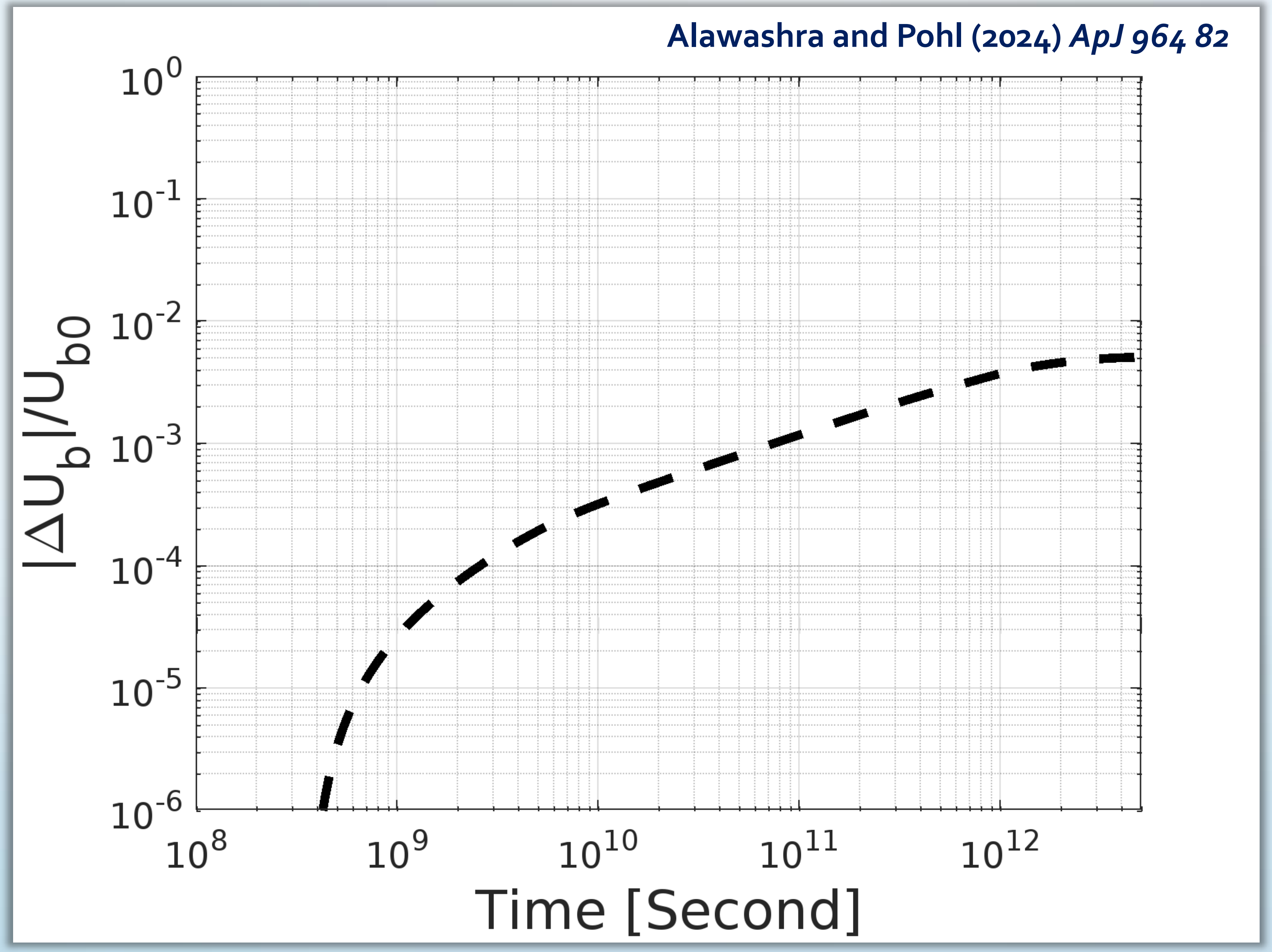


The instability is suppressed by the widening



Alawashra and Pohl (2024)
ApJ 964 82

Beam energy loss is subdominant



Q2

What is the impact of continuous pair production?

What is the impact of pairs continuous production ?

Continuous production of new pair due to the gamma-rays annihilation with EBL

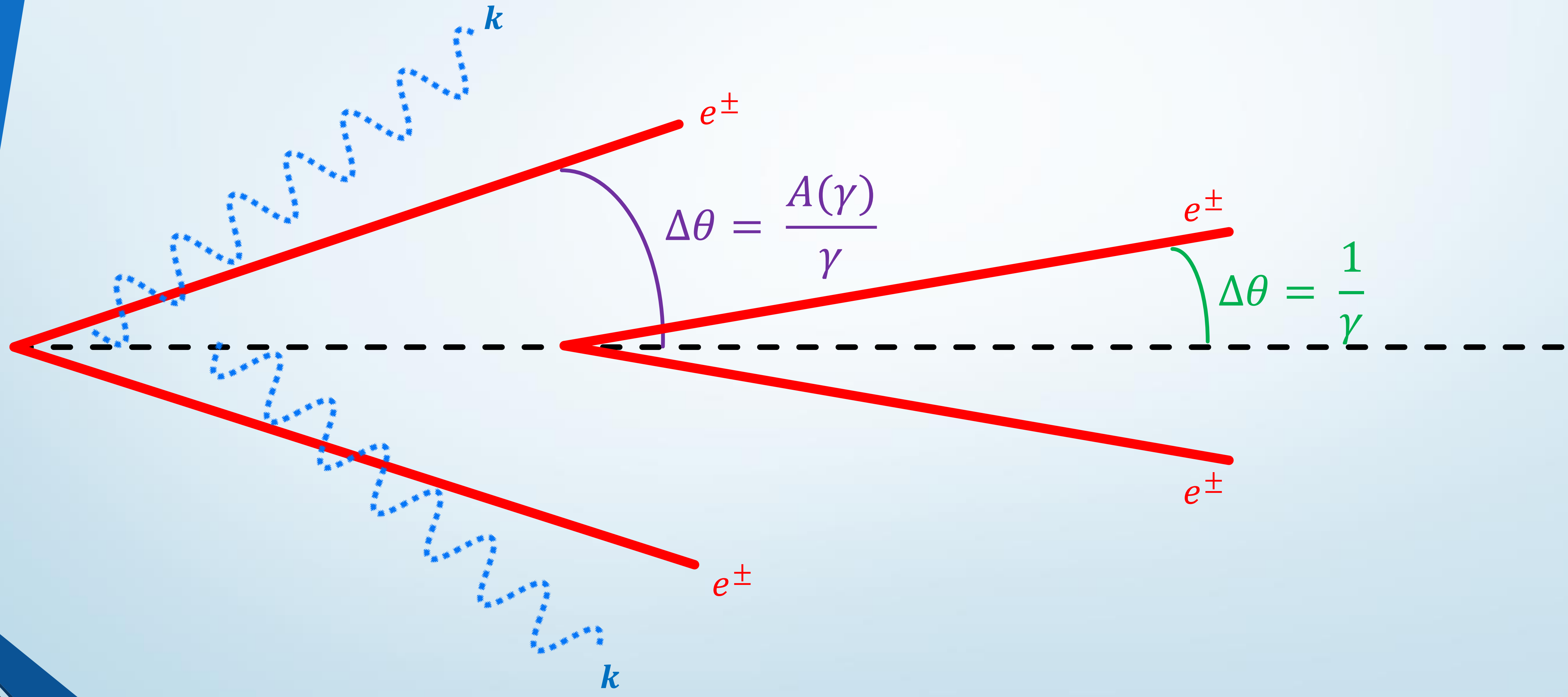
We just need to add a constant source term, Q_{ee}.

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + Q_{ee}$$

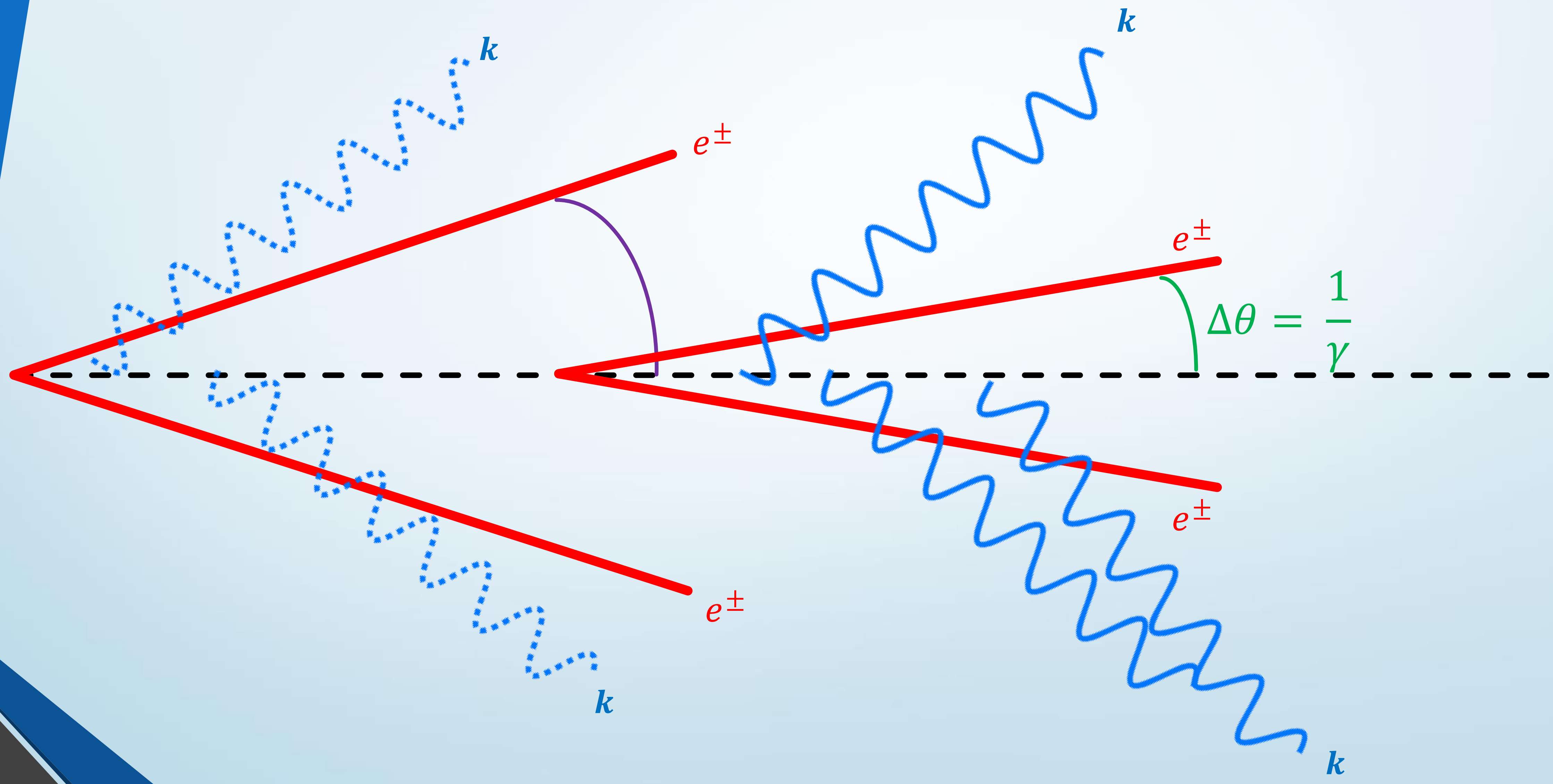
$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right) W(\boldsymbol{k},t)$$

We used the production rate found by Vafin et. al (2018).

New focused pairs get produced

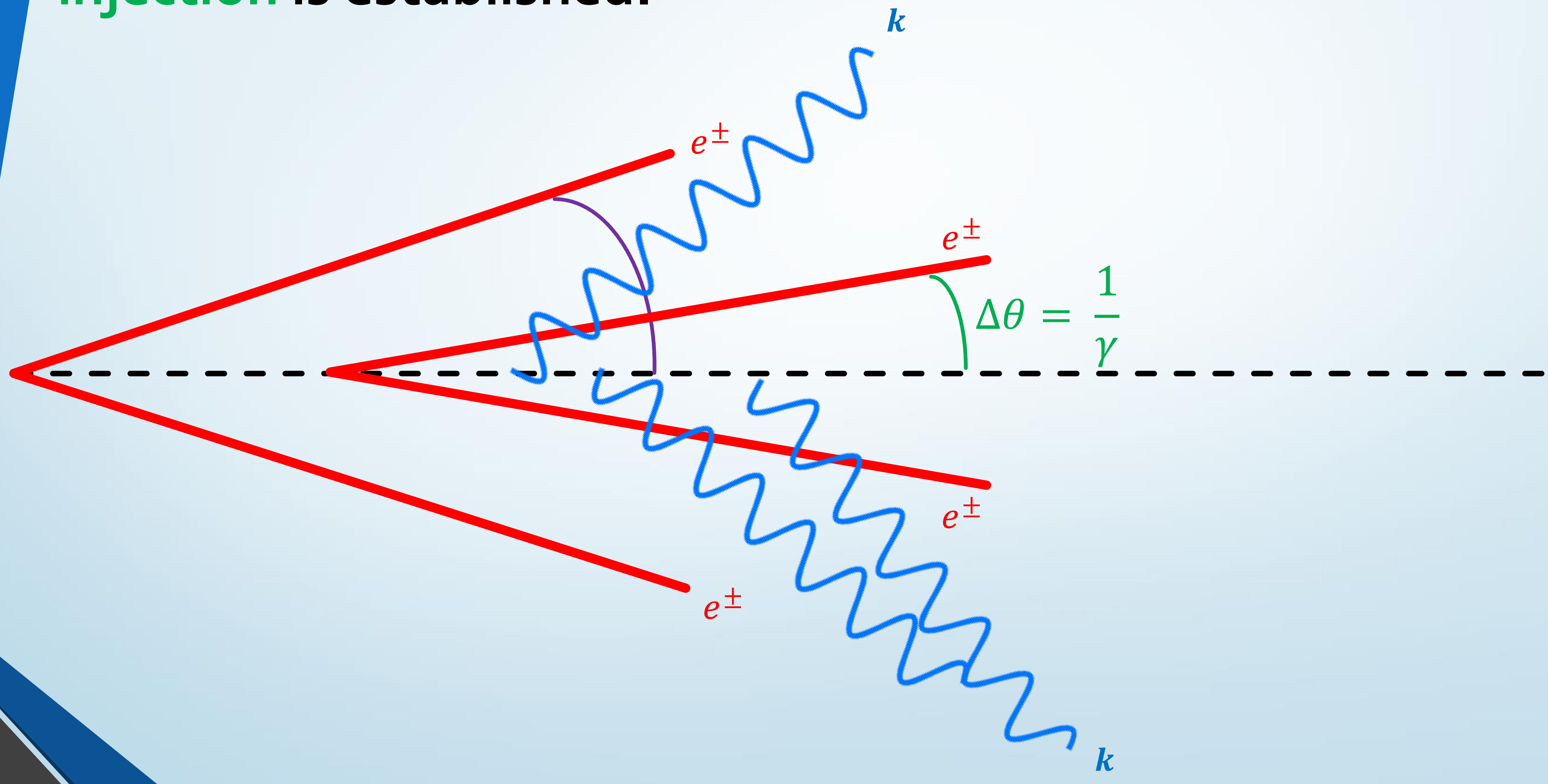


New waves get generated

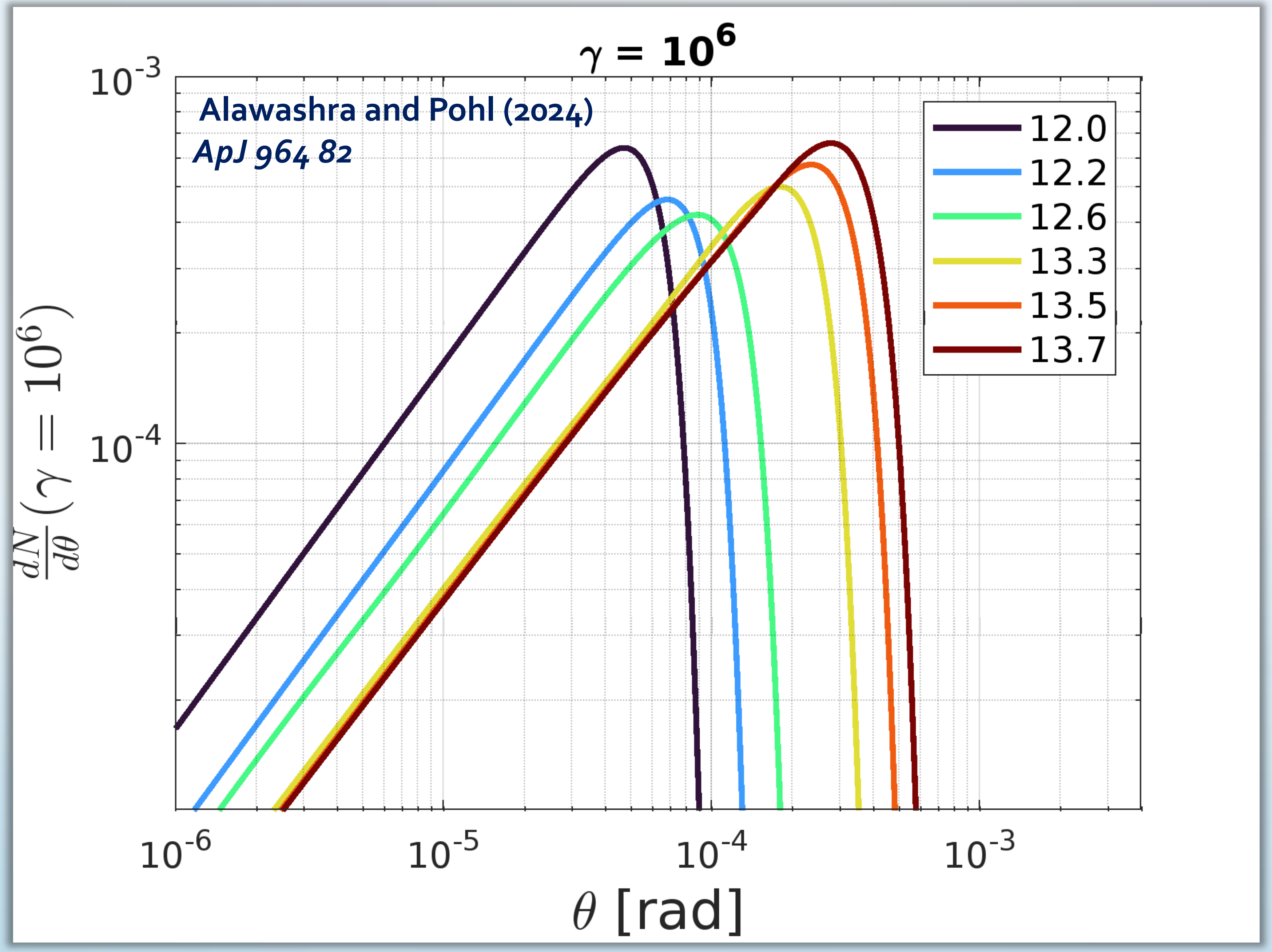


A new quasi-steady state is established

A balance between the instability widening and the injection is established.

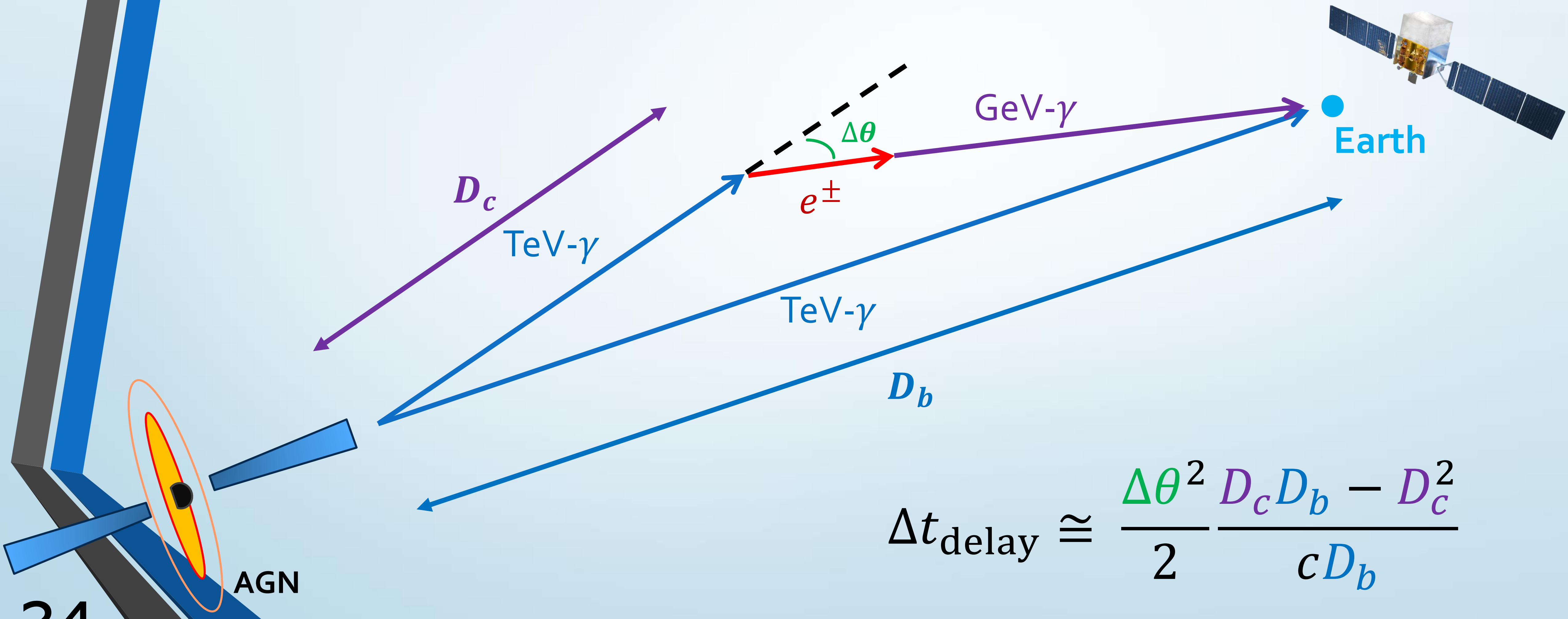


The beam keeps widening



Observational implications

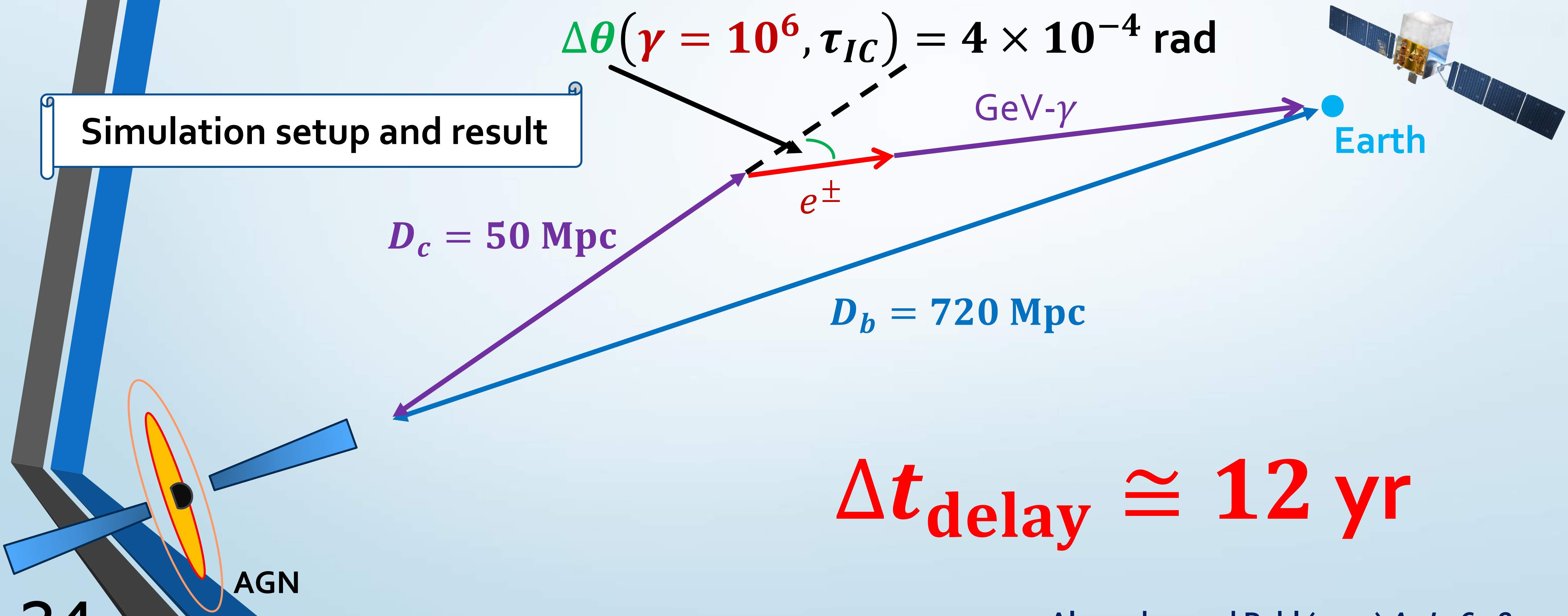
Cascade arrival time delay due to the instability broadening



$$\Delta t_{\mathrm{delay}} \cong \frac{\Delta\theta^2}{2}\frac{D_c D_b - D_c^2}{c D_b}$$

Cascade arrival time delay due to the instability broadening

Simulation setup and result



$$\Delta t_{\mathrm{delay}} \cong 12 \text{ yr}$$

Alawashra and Pohl (2024) *ApJ 964 82*

Conclusions

- IGMFs suppress the instability.
- Widening feedback is the dominant instability feedback.
- New quasi-steady state with continues pairs production.

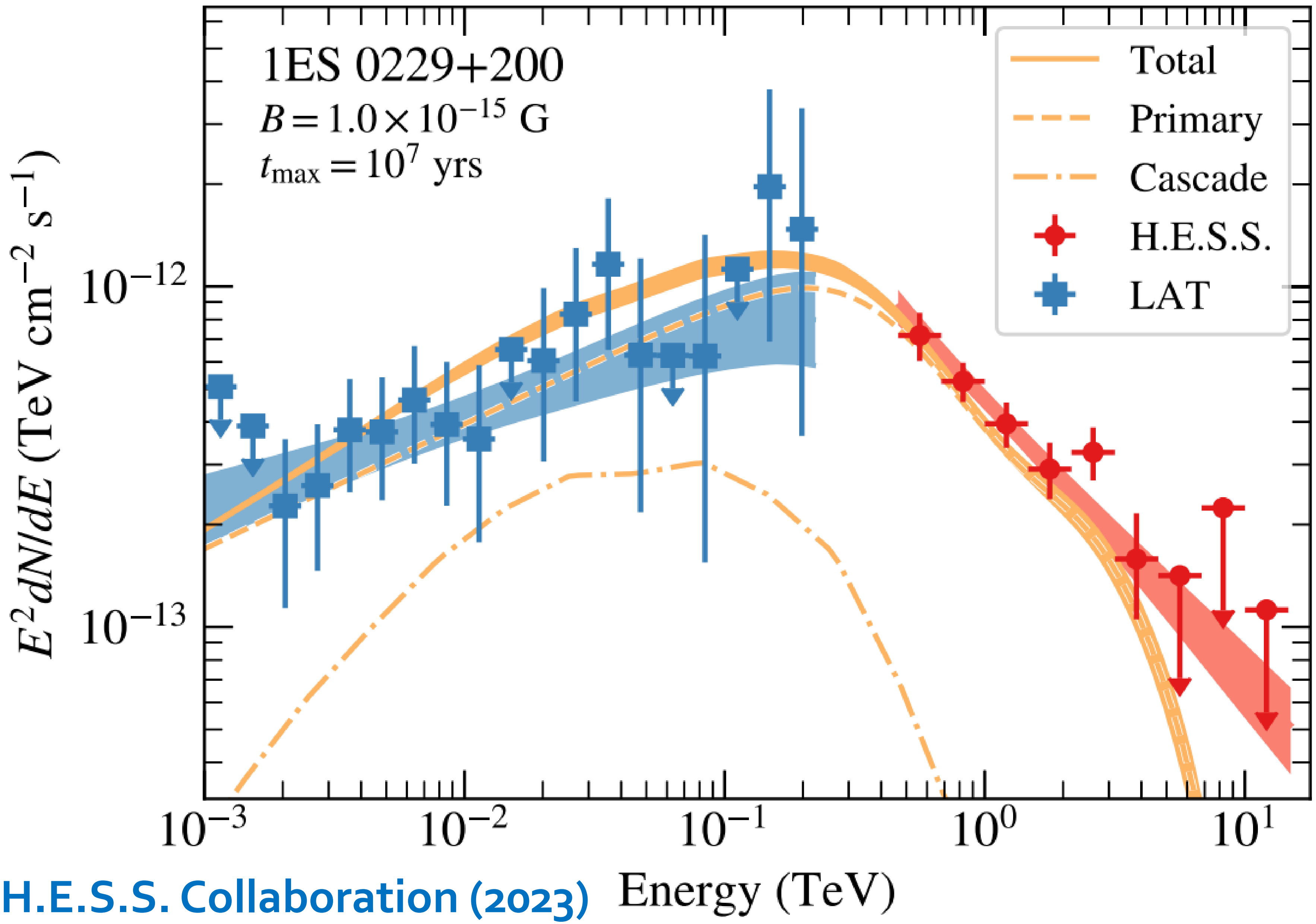
Outlook

- Calculating the instability broadening at different distances in the IGM.

Thank you

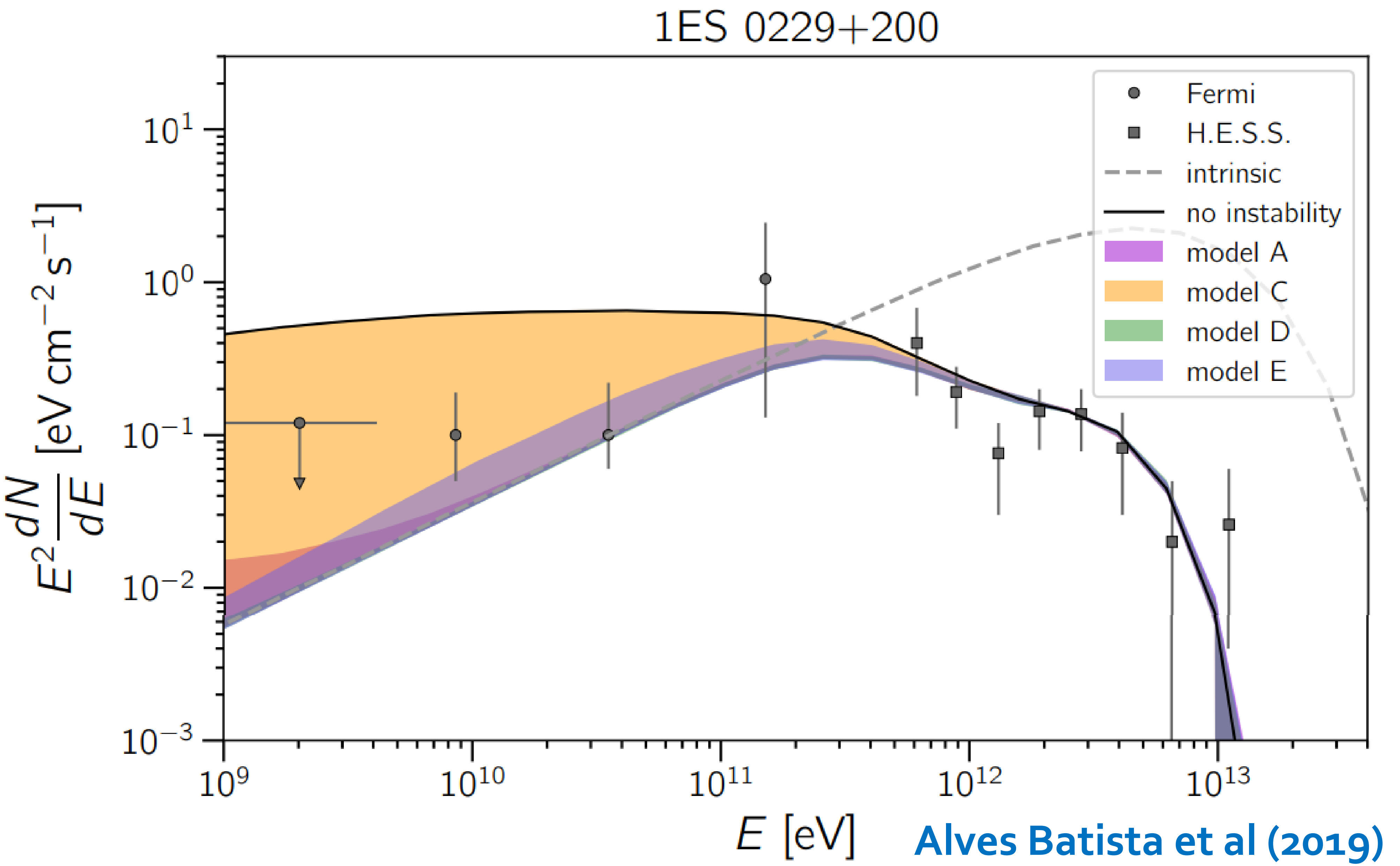
Back up slides

Suppression of the cascade emission by IGMFs



H.E.S.S. Collaboration (2023)

Suppression of the cascade by instability energy loss



Alves Batista et al (2019)

What about the other angular diffusion term $\boldsymbol{\theta p}$?

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta p}\frac{\partial f}{\partial p}\right)$$

[redacted]

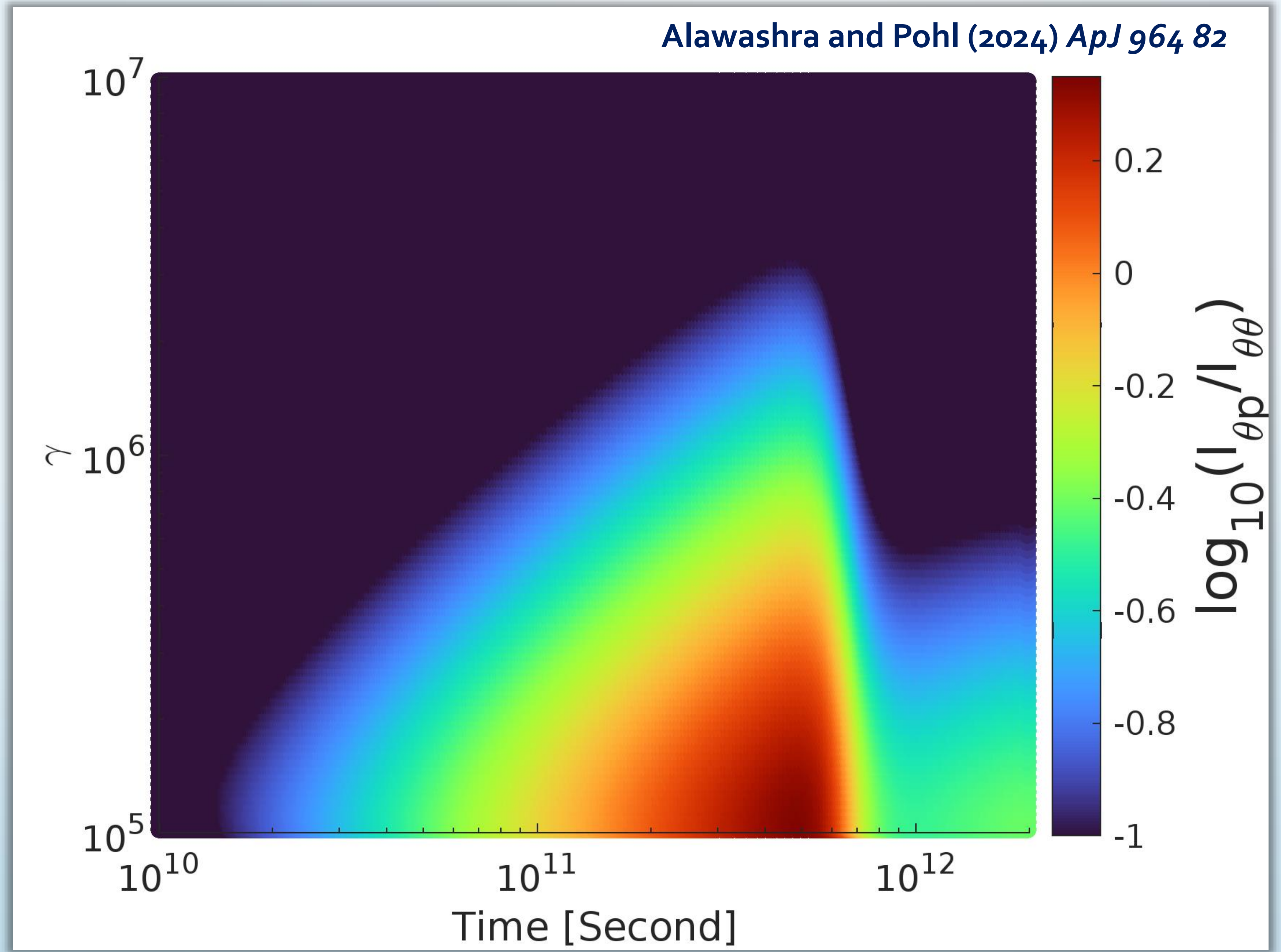
With time: **Decreases** **Constant**

We need to compare:

$$I_{\theta p} = \int d\cos\theta \left| \frac{\partial f}{\partial t} \Big|_{\theta p} \right| = \int d\cos\theta \left| \frac{1}{p\theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta p} \frac{\partial f}{\partial p} \right) \right|$$

$$I_{\theta \theta} = \int d\cos\theta \left| \frac{\partial f}{\partial t} \Big|_{\theta \theta} \right| = \int d\cos\theta \left| \frac{1}{p^2\theta} \frac{\partial}{\partial \theta} \left(\theta D_{\theta \theta} \frac{\partial f}{\partial \theta} \right) \right|$$

Relevant for pairs with Lorentz factors less than 10^6



Can we quantify the energy loss/gain in the momentum diffusion terms?

$$\frac{\partial f(p,\theta)}{\partial t} = \text{[redacted]}$$

$$+\frac{1}{p^2}\frac{\partial}{\partial p}\left(p D_{p\theta}\frac{\partial f}{\partial \theta}\right)+\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right)$$

Can we quantify the energy loss/gain in the momentum diffusion terms?

$$\frac{df}{dt}\Big|_{p\theta} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(pD_{p\theta}\frac{\partial f}{\partial \theta}\right)$$

$$\frac{dU_b}{dt}\Big|_{p\theta}(t) = 2\pi m_e c^2 \int d\theta\, \theta \int dp\, p^2\, \gamma\, \frac{df}{dt}\Big|_{p\theta}(p,\theta)$$

Can we quantify the energy loss/gain in the momentum diffusion terms?

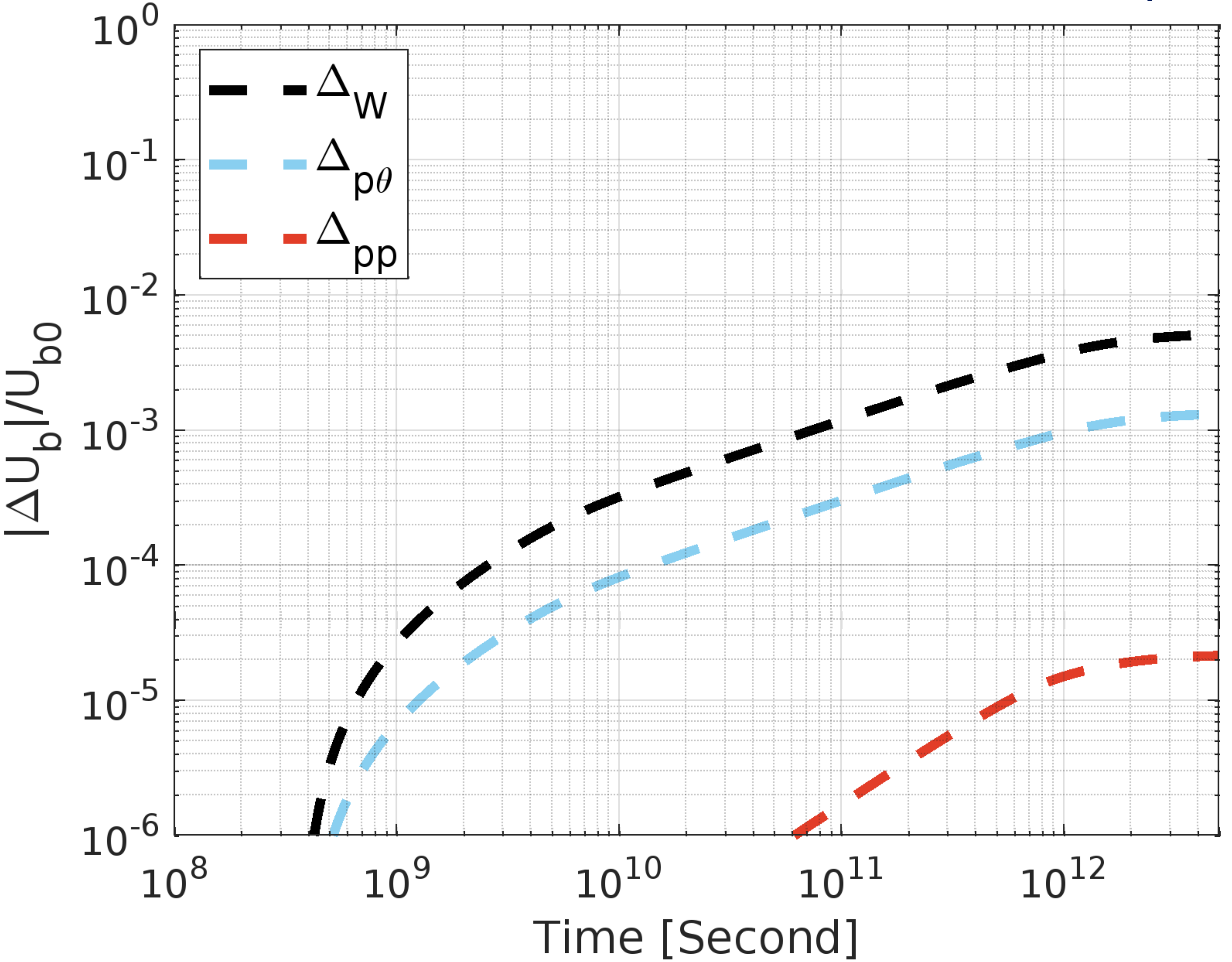
$$\frac{df}{dt}\Big|_{p\theta} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(pD_{p\theta}\frac{\partial f}{\partial \theta}\right)$$

$$\frac{dU_b}{dt}\Big|_{p\theta}(t) = 2\pi m_e c^2 \int d\theta\, \theta \int dp\, p^2\, \gamma\, \frac{df}{dt}\Big|_{p\theta}(p,\theta)$$

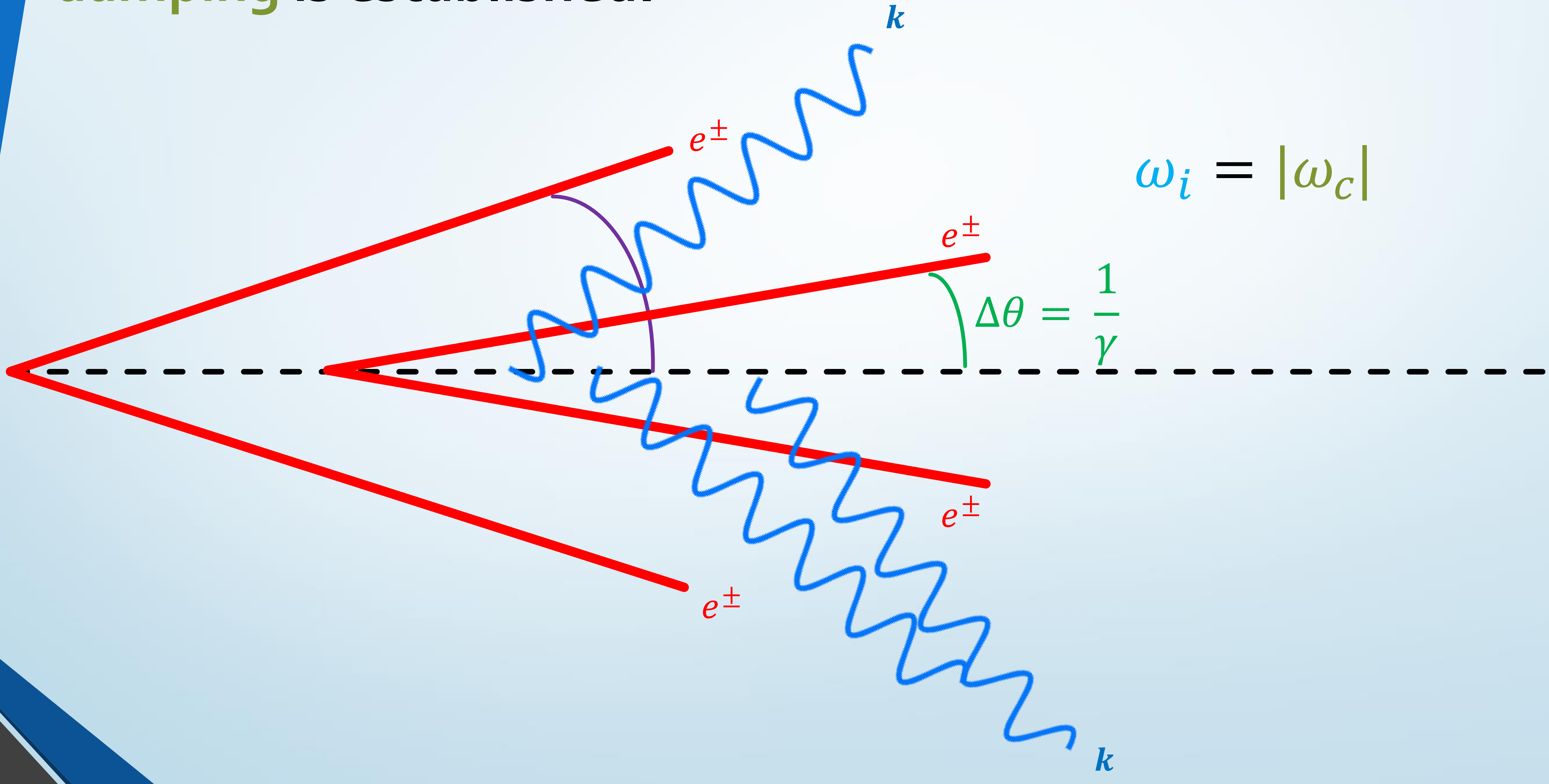
$$\Delta_{p\theta} \equiv \frac{\Delta U_b}{U_{b0}}\Big|_{p\theta}(t_s) = \frac{1}{U_{b0}}\int_{t_0}^{t_s} dt\, \frac{dU_b}{dt}\Big|_{p\theta}(t)$$

Momentum diffusion and energy loss are subdominant

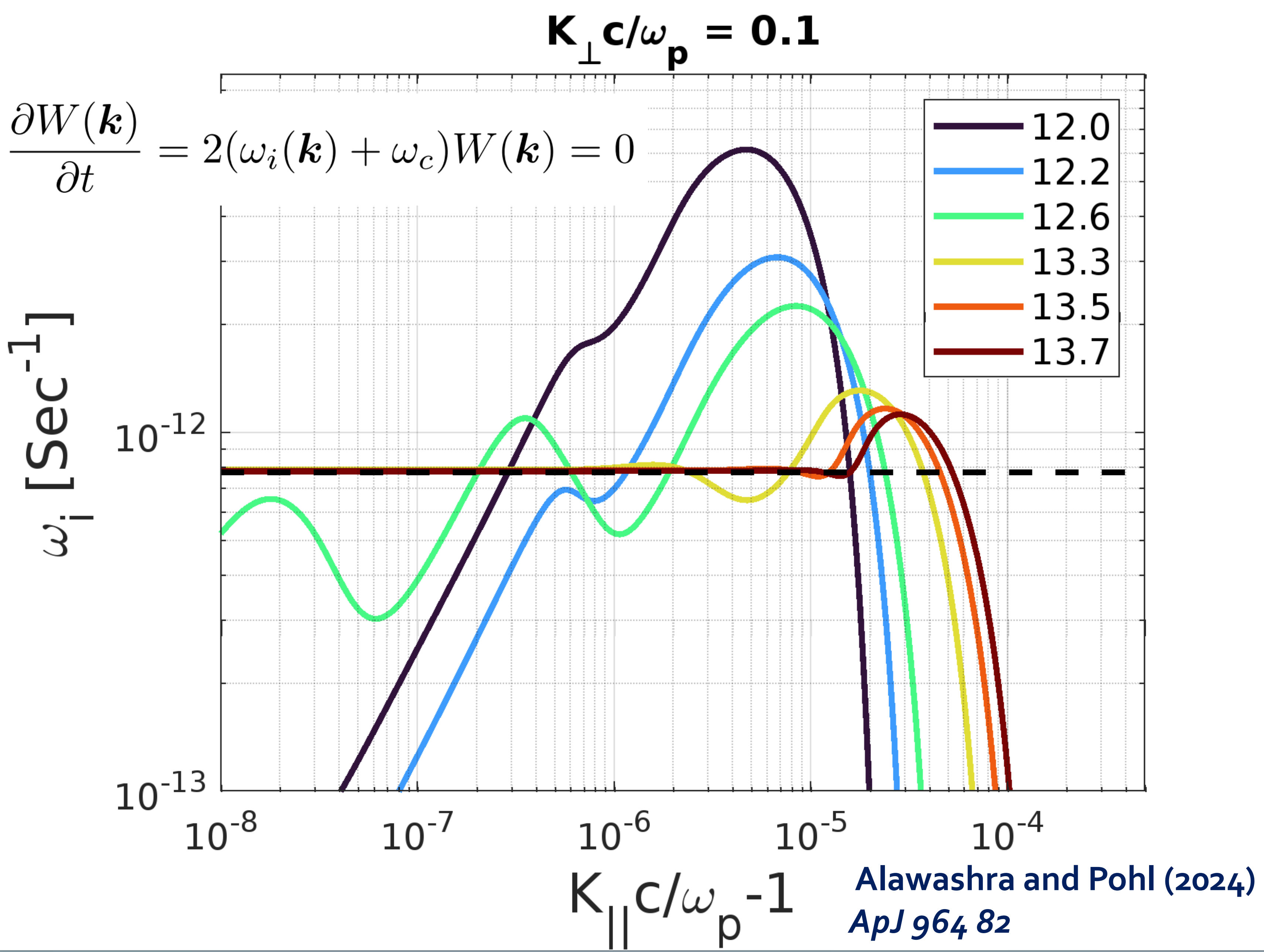
Alawashra and Pohl (2024) accepted in *ApJ*



On the waves side: A balance between the growth and the damping is established.



The linear growth rate balances the damping rate



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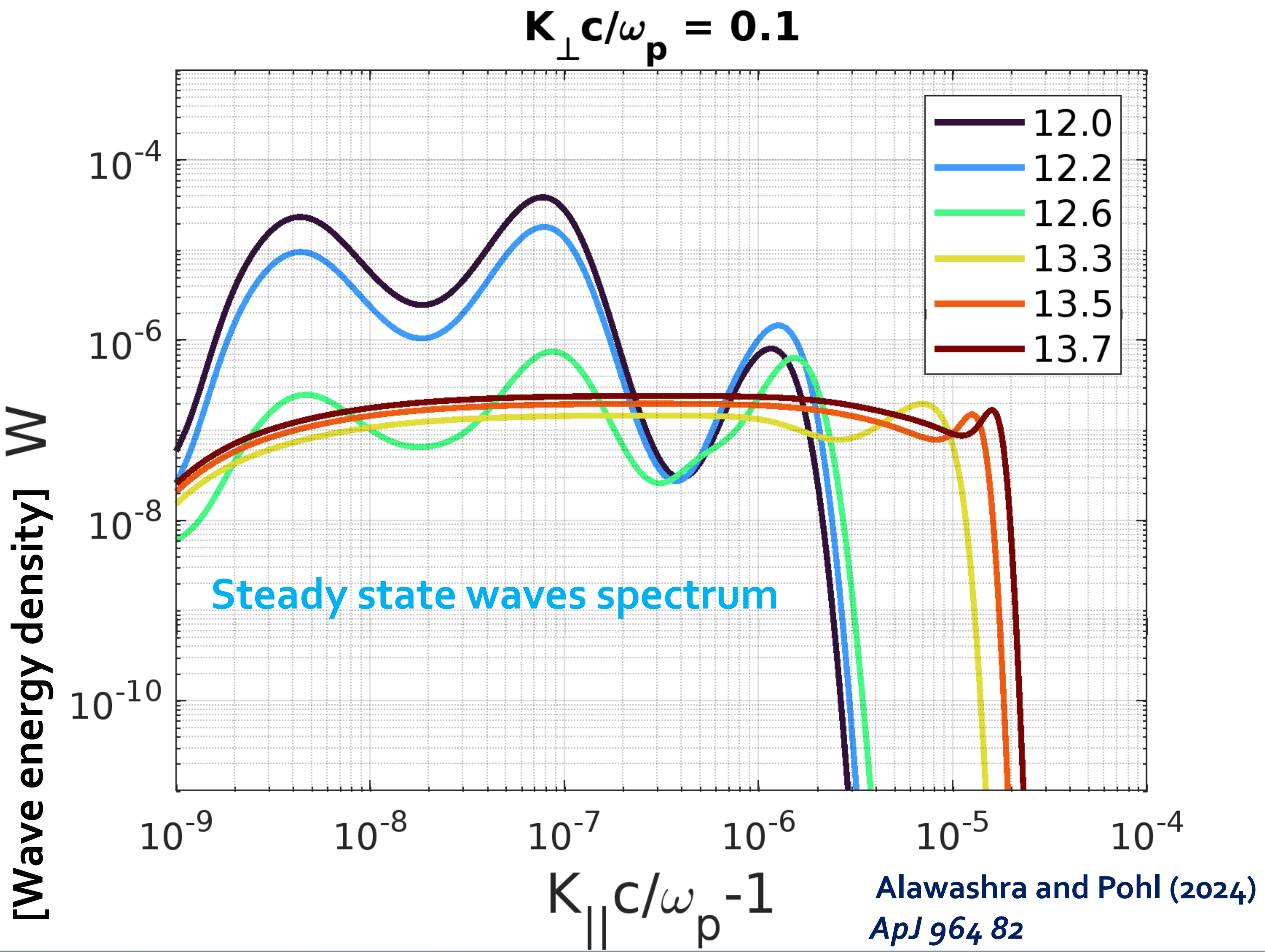
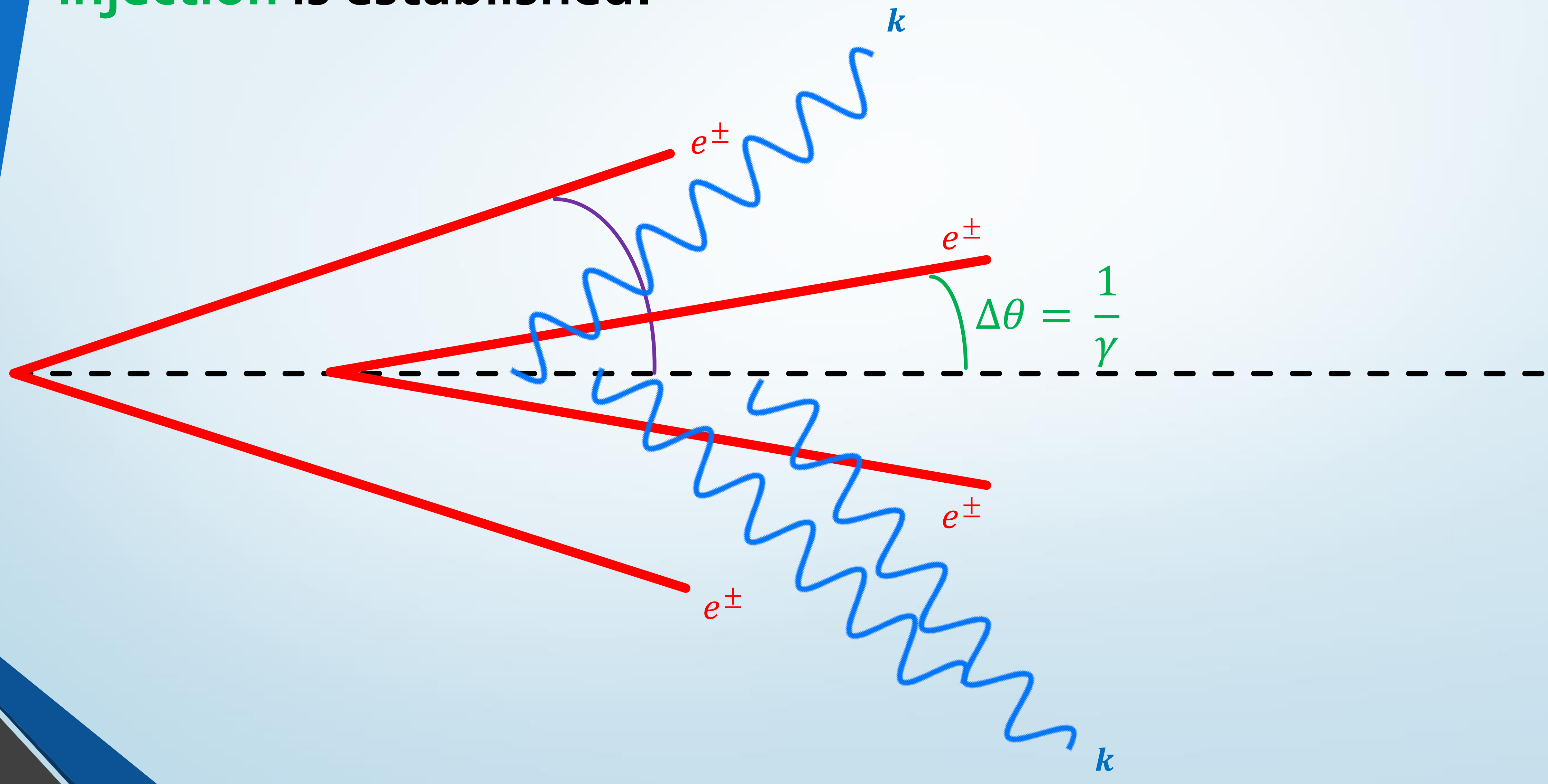
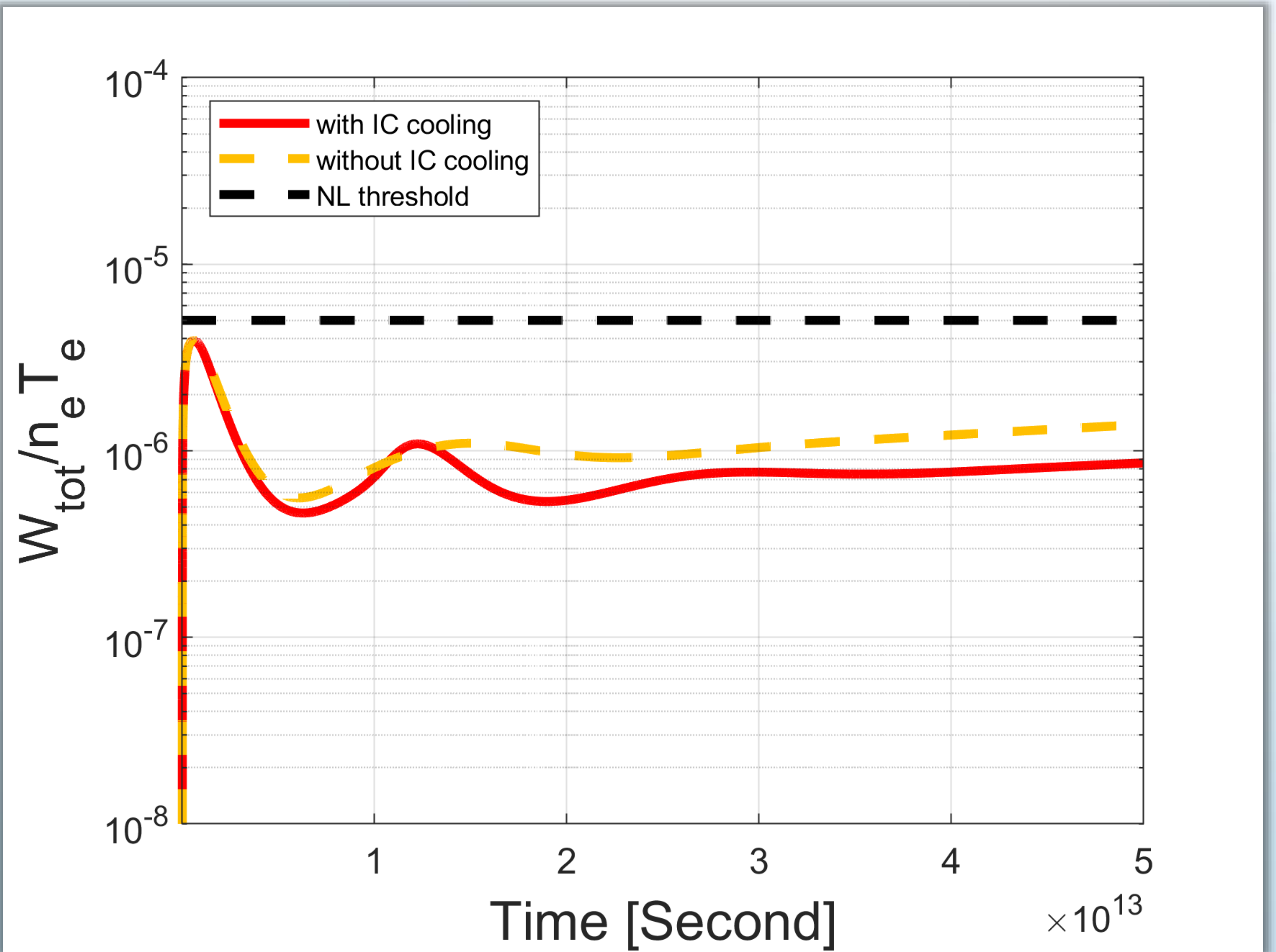


Alawashra and Pohl (2024)
ApJ 964 82

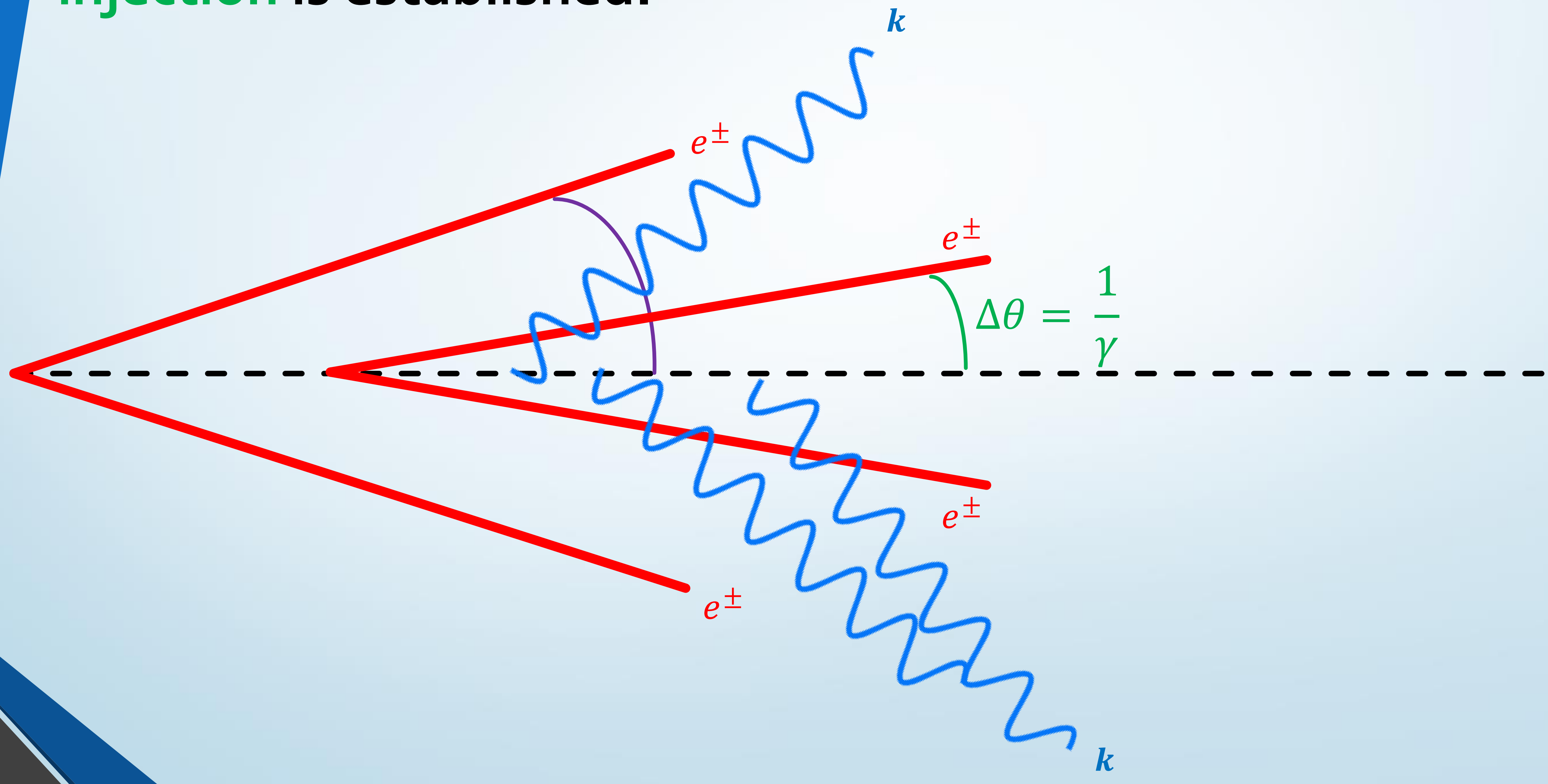
On the beam side: A balance between the widening and the injection is established.



Plasma waves energy density evolution



A balance between the instability widening and the injection is established.



Add more Physics

IC cooling

Beam evolution including the full Physics (almost)

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}\left(-\dot{p}_{IC}p^2 f\right) + Q_{ee}$$

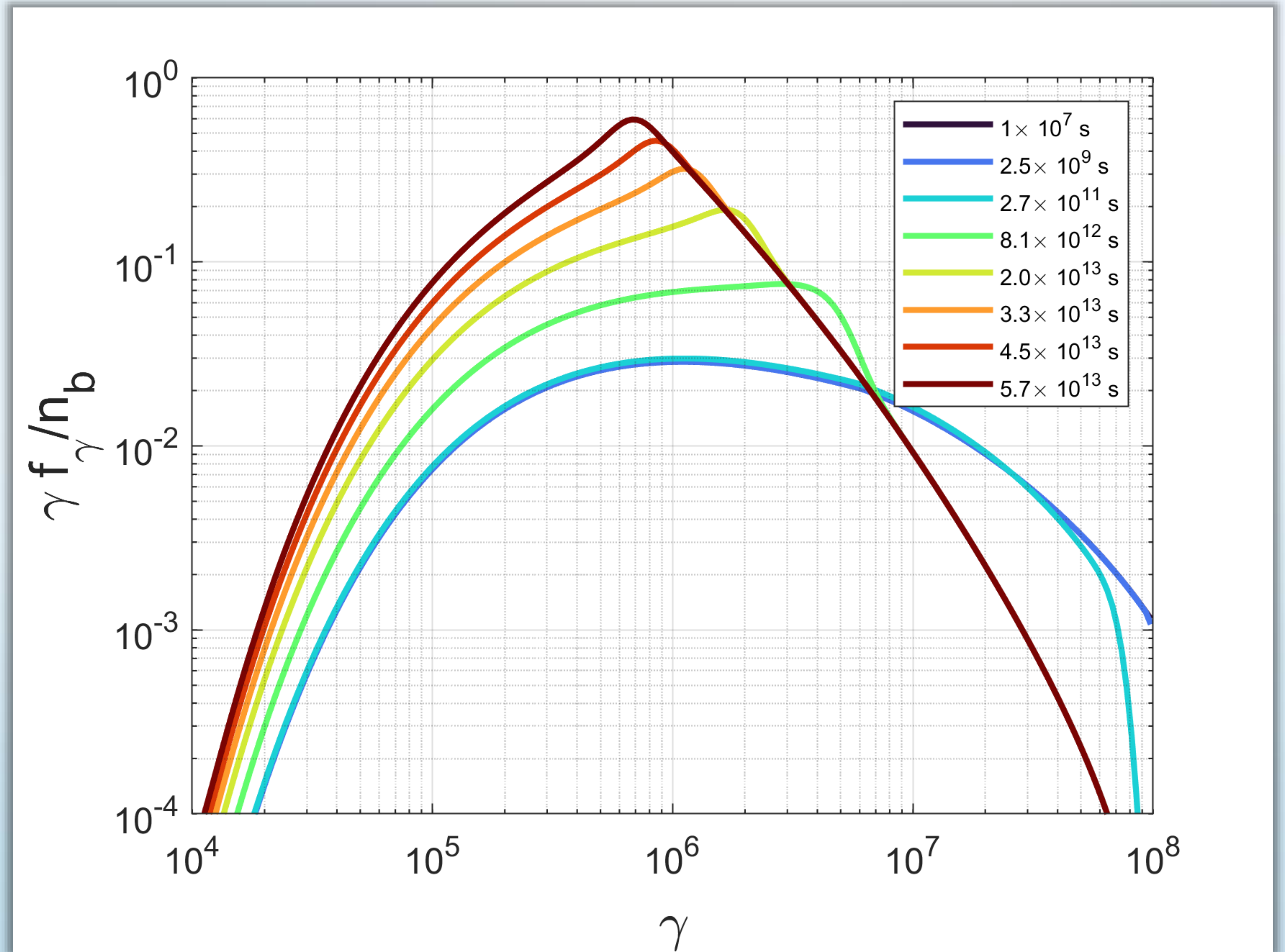
The IC cooling is only relevant for particle momentum

$$\dot{p}_{IC} = -\frac{4}{3}\sigma_T u_{CMB}\gamma^2$$

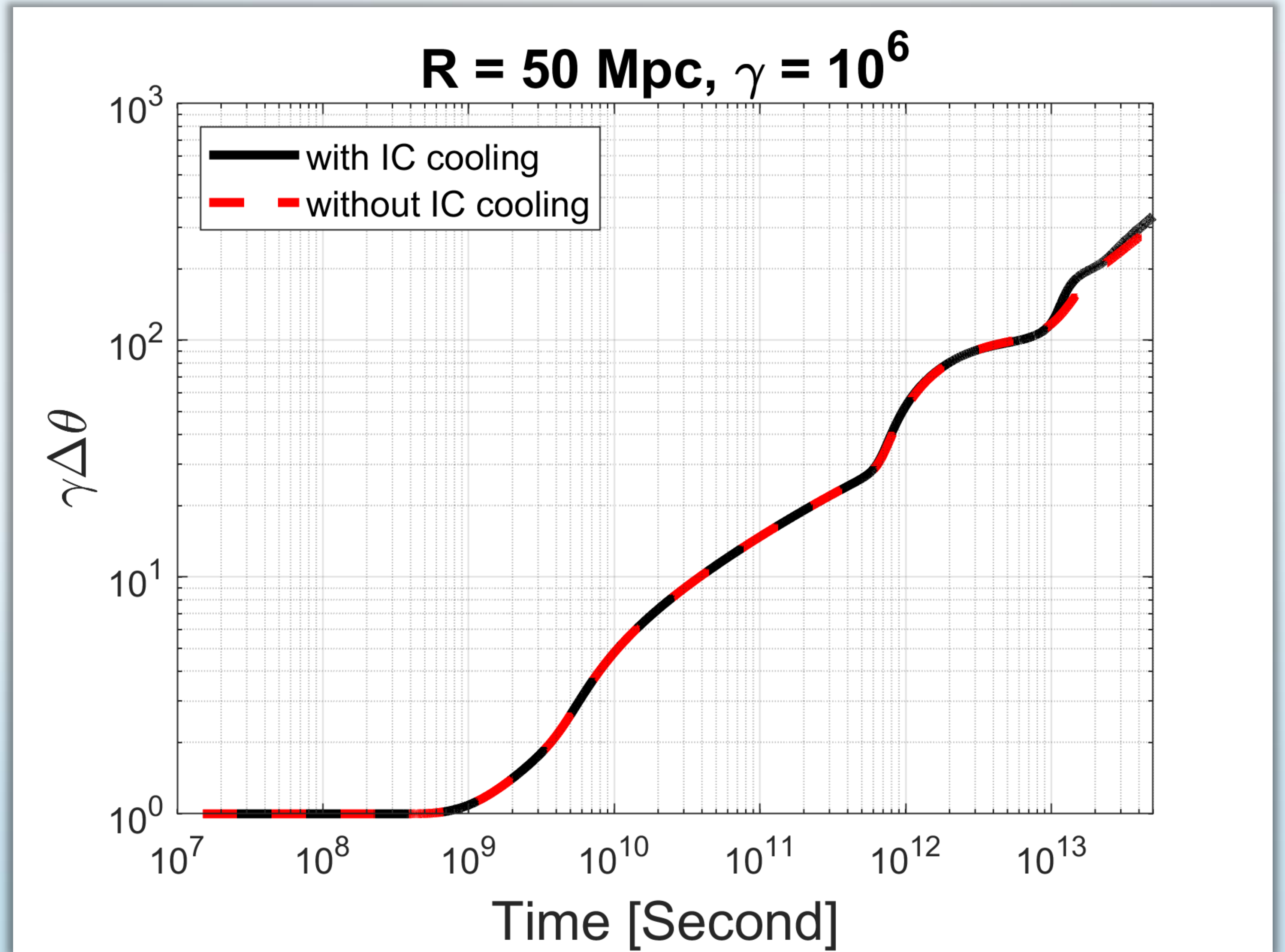
We use the same linear evolution of the plasma waves

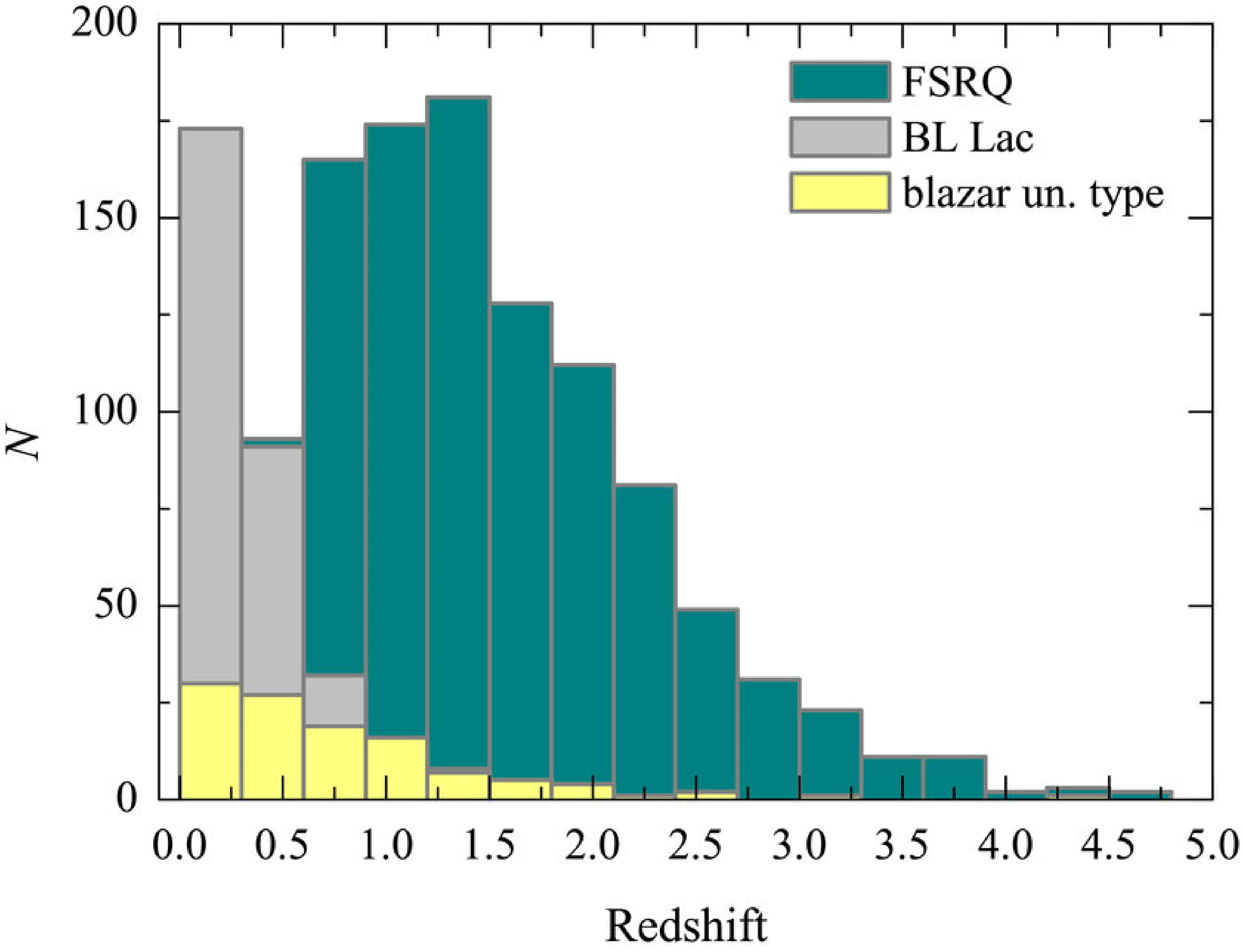
$$\frac{\partial W(\boldsymbol{k},t)}{\partial t} = 2\left(\omega_i(\boldsymbol{k}) + \omega_c\right)W(\boldsymbol{k},t)$$

Momentum beam distribution evolution

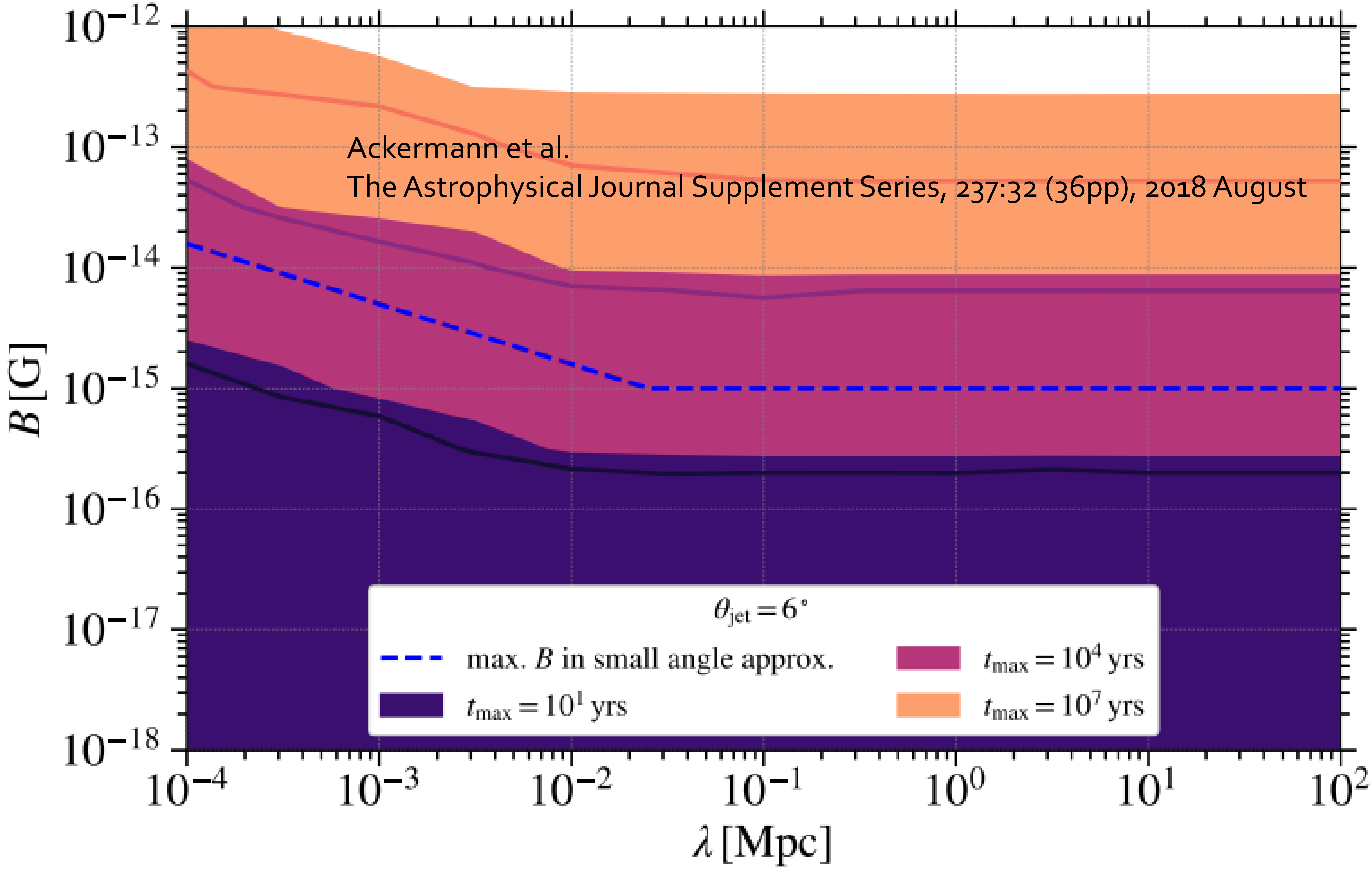


Beam broadening is almost unaffected



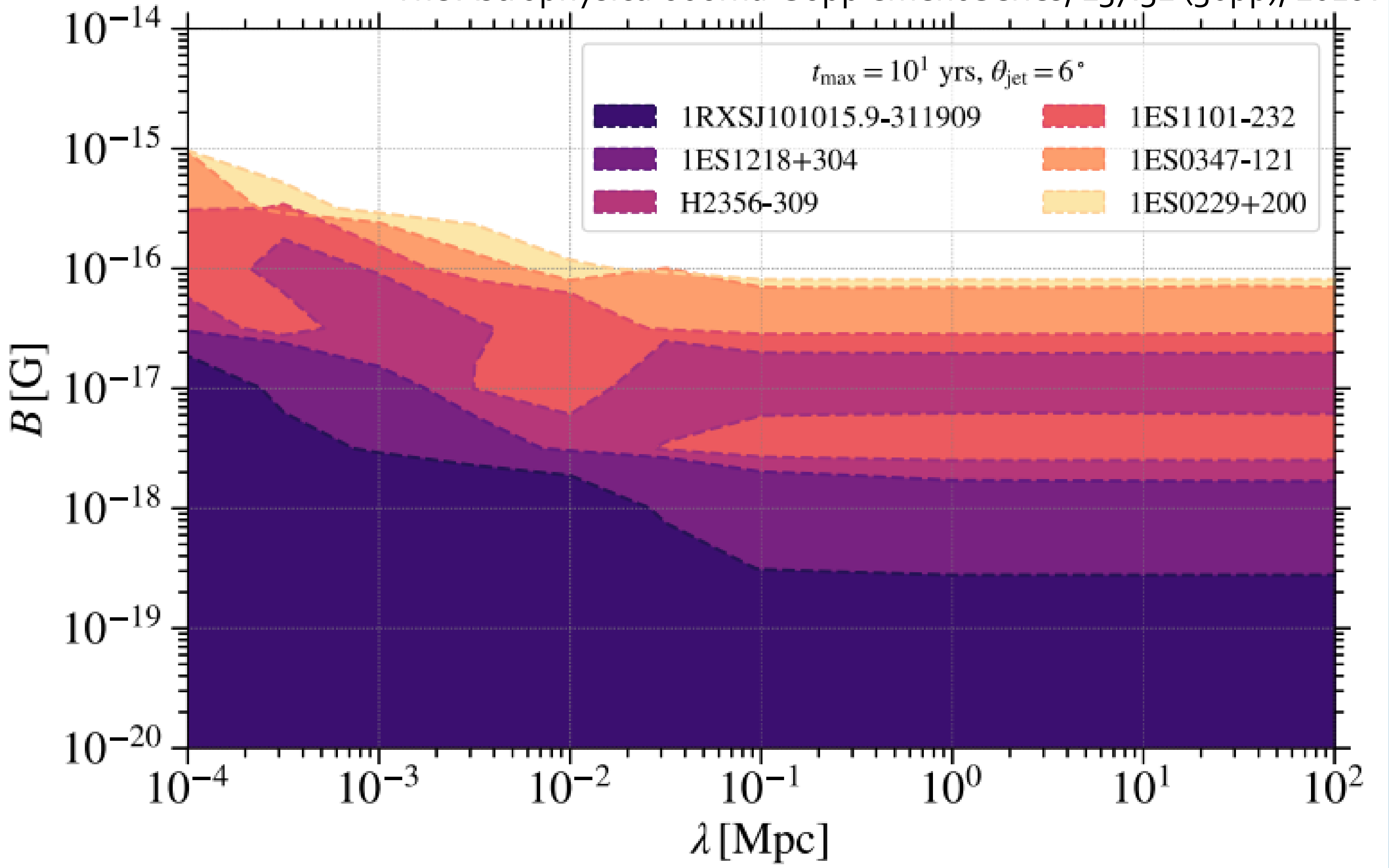


200
150
100
50
0
N
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Redshift
FSRQ
BL Lac
blazar un. type



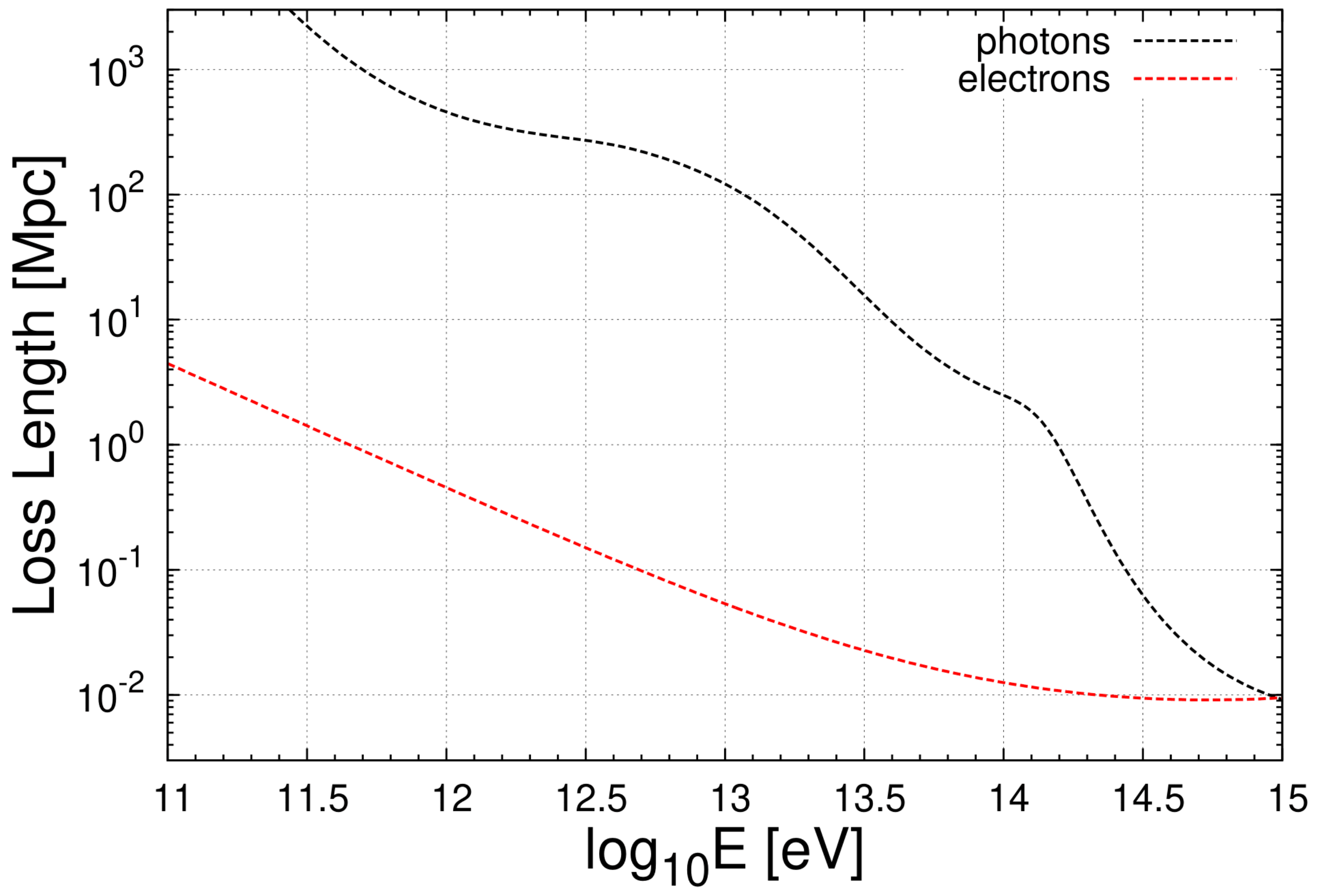
Ackermann et al.
The Astrophysical Journal Supplement Series, 237:32 (36pp), 2018 August
B [G]
λ [Mpc]
$\theta_{jet} = 6^\circ$
max. B in small angle approx.
$t_{max} = 10^1$ yrs
$t_{max} = 10^4$ yrs
$t_{max} = 10^7$ yrs

Ackermann et al.
The Astrophysical Journal Supplement Series, 237:32 (36pp), 2018 August

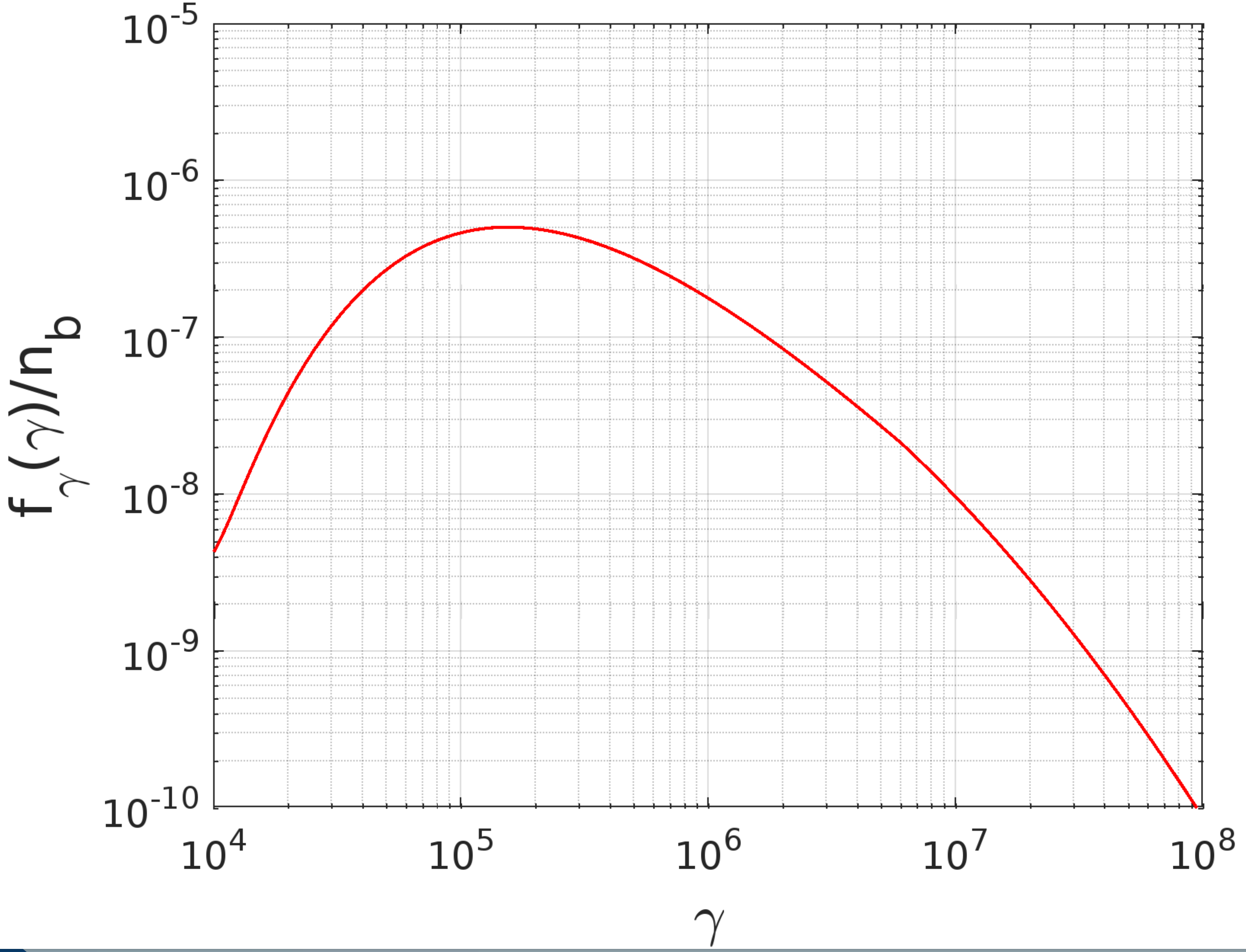


strongly on θ_{obs} and the orientation of the halo. The IGMF is modeled as cell-like; each cell has a side length of $\ell_B = 1\,\text{Mpc}$ and the B-field orientation changes randomly from one cell to the next. The templates are generated for each source and for seven values of the field strength, $B = 10^{-16}\,\text{G}, 10^{-15.5}\,\text{G}, \ldots, 10^{-13}\,\text{G}$. For higher values of B, the pairs are quickly isotropized and the cascade emission would appear as an additional component to the isotropic gamma-ray background in the LAT energy band. An example of the simulated energy-

Constraints on the IGMF with H.E.S.S. and Fermi LAT
ApJ Letters 2023, Volume 950, Number 2 950, L16

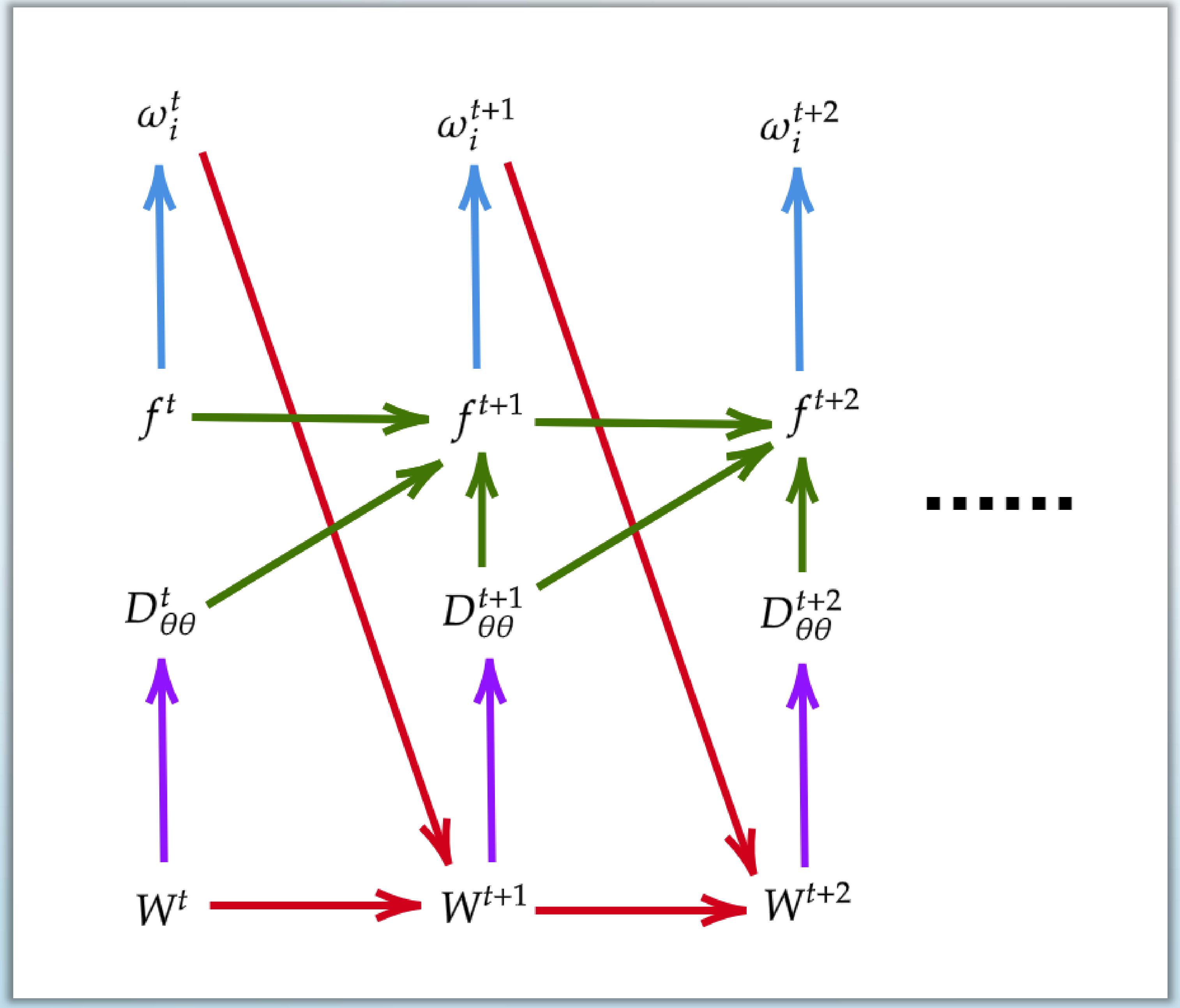


Andrew Taylor (private communication)



10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
f_γ(γ)/n_b
10^4
10^5
10^6
10^7
10^8
γ

Simulation steps



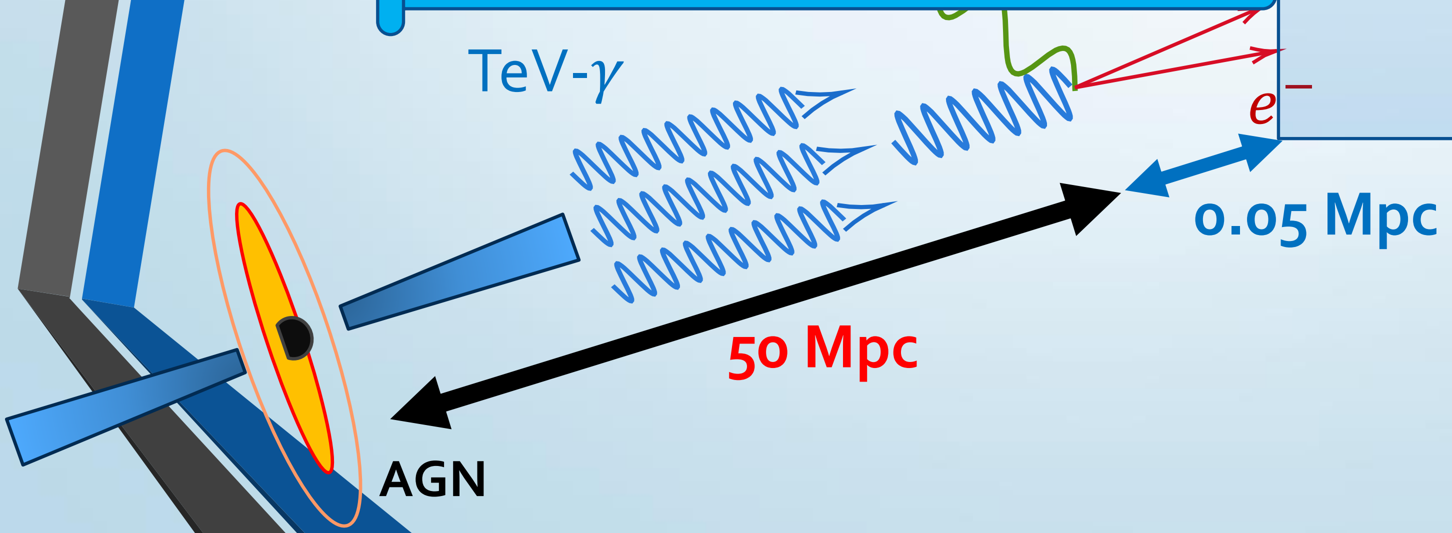
2D simulation of the widening feedback

Crank–Nicolson method

$$\frac{\partial f(p,\theta)}{\partial t} = \frac{1}{p^2\theta}\frac{\partial}{\partial\theta}\left(\theta D_{\theta\theta}\frac{\partial f}{\partial\theta}\right)$$

Forward time method

$$\frac{\partial W(\mathbf{k},t)}{\partial t} = 2\,(\omega_i(\mathbf{k}) + \omega_c)\,W(\mathbf{k},t)$$



For the wave spectrum, W, we use a logarithmic grid in the coordinates $(k_\perp, \theta^R)$ where $\theta^R = \left(\frac{ck_\parallel}{\omega_p} - 1\right) / \left(\frac{ck_\perp}{\omega_p}\right)$. We used 100 grid points for the perpendicular wave number, $k_\perp$, from $10^{-3}\ \frac{\omega_p}{c}$ to $10\ \frac{\omega_p}{c}$, we have verified a convergence of this by using 300 points. For the parameter, θ^R, we used 600 grid points for the interval 10^{-9} to 5×10^{-3} where we have tested this with 1500 grid points. For the beam distribution, f, we use a logarithmic grid in the coordinates (θ, γ) where γ is the beam particle Lorentz factor. We used 100 grid points for γ from 10^4 to 10^8 and verified a convergence of this with 300 grid points. Finally for the beam particle angle, θ, we used 600 grid points from 10^{-9} radian to 5×10^{-3} radians tested by using 1500 grid points.

More accurate collisional damping rate from Tigik et al. (2019)
20 times smaller

$$\omega_c(k) = -\omega_p \frac{g}{6\pi^{3/2}} \frac{1}{(1+3k^2\lambda_D^2)^3}. \qquad (8)$$

Here $g = (n_e \lambda_D^3)^{-1}$ is the plasma parameter, $\lambda_D = 6.9\ \mathrm{cm}\sqrt{\frac{T_e/K}{n_e/\mathrm{cm}^{-3}}}$ is the Debye length, $n_e = 10^{-7}(1+z)^3\mathrm{cm}^{-3}$ is the density of IGM electrons, and $T_e = 10^4 K$ is their temperature. We start integrating eq.(7) at the very low thermal fluctuations level.

Tigik et al. (2019)

$$\omega_i(k_\perp, k_{||}) = \pi \omega_p \frac{n_b}{n_e} \left(\frac{\omega_p}{kc}\right)^3 \int_{p_{\min}}^{\infty} dp m_e c\, p \int_{\theta_1}^{\theta_2} d\theta \times \frac{-2f(p,\theta)\sin\theta + (\cos\theta - \frac{kv_b}{\omega_p}\cos\theta')\frac{\partial f(p,\theta)}{\partial\theta}}{[(\cos\theta_1 - \cos\theta)(\cos\theta - \cos\theta_2)]^{1/2}}, \tag{5.32}$$

where the boundaries are given by

$$\cos\theta_{1,2} = \frac{\omega_p}{kv_b}\left(\cos\theta' \pm \sin\theta' \sqrt{\left(\frac{kv_b}{\omega_p}\right)^2 - 1}\right), \tag{5.33}$$

and

$$p_{\min} = \sqrt{\frac{1 + \left(\frac{ck_\perp}{\omega_p}\right)^2 + 2\left(\frac{ck_{||}}{\omega_p} - 1\right)}{\left(\frac{ck_\perp}{\omega_p}\right)^2 + 2\left(\frac{ck_{||}}{\omega_p} - 1\right)}}. \tag{5.34}$$

$$\begin{Bmatrix} D_{pp} \\ D_{p\theta} \\ D_{\theta\theta} \end{Bmatrix} = \pi \frac{m_e \omega_p^2}{n_e} \int_{\omega_p/c}^{\infty} k^2 dk \int_{\cos\theta_1'}^{\cos\theta_2'} d\cos\theta' \frac{W(\mathbf{k})}{k v_b \sqrt{(\cos\theta' - \cos\theta_1')(\cos\theta_2' - \cos\theta')}} \begin{Bmatrix} 1 \\ \xi \\ \xi^2 \end{Bmatrix}, \tag{5.16}$$

where

$$\xi = \frac{\cos\theta \frac{\omega_p}{k v_b} - \cos\theta'}{\sin\theta}. \tag{5.17}$$

and the boundaries of $\cos\theta'$ are given by

$$\cos\theta_{1,2}' = \frac{\omega_p}{k v_b}\left[\cos\theta \pm \sin\theta \sqrt{\left(\frac{k v_b}{\omega_p}\right)^2 - 1}\right]. \tag{5.18}$$

$$\theta^R = \left(\frac{ck_{||}}{\omega_p} - 1\right) / \left(\frac{ck_\perp}{\omega_p}\right)$$

$$\left\{\begin{array}{c} D_{pp} \\ D_{p\theta} \\ D_{\theta\theta} \end{array}\right\} = \pi \frac{m_e \omega_p^2}{n_e c \theta} \int_{R(\theta,\gamma)} dk_\perp k_\perp \int_{R(\theta,\gamma)} d\theta^R$$

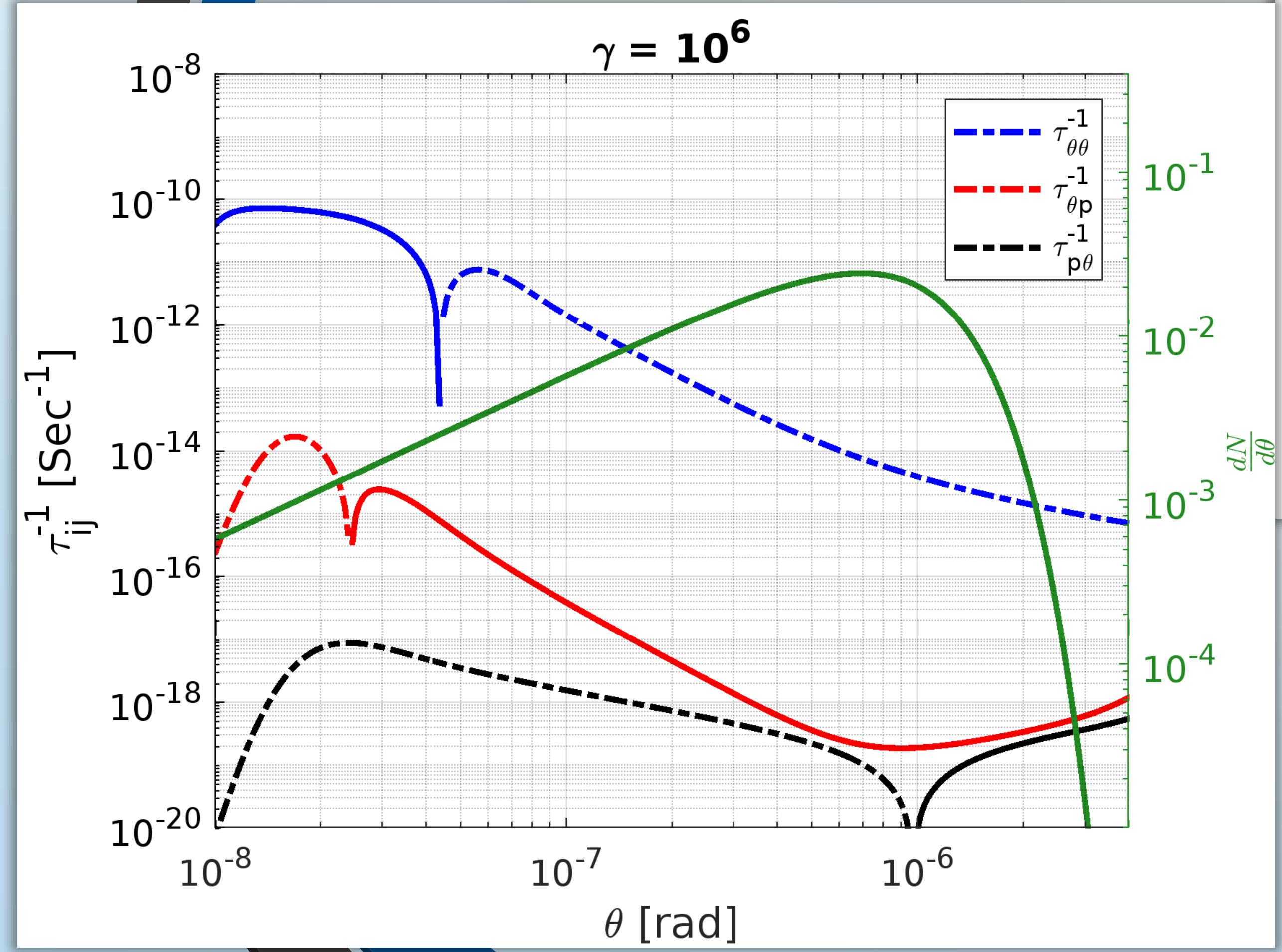
$$\times \frac{W(k_\perp, \theta^R)}{\sqrt{1 - \left(\frac{\theta^R}{\theta}\right)^2 + \frac{\theta^R}{ck_\perp/\omega_p}\left[1 + \left(\frac{1}{\gamma\theta}\right)^2\right] - \left(\frac{\omega_p}{ck_\perp}\right)^2 \left[\frac{1}{2\gamma^2\theta} + \frac{\theta}{2}\right]^2}} \left\{\begin{array}{c} 1 \\ \xi \\ \xi^2 \end{array}\right\}, \tag{B.34}$$

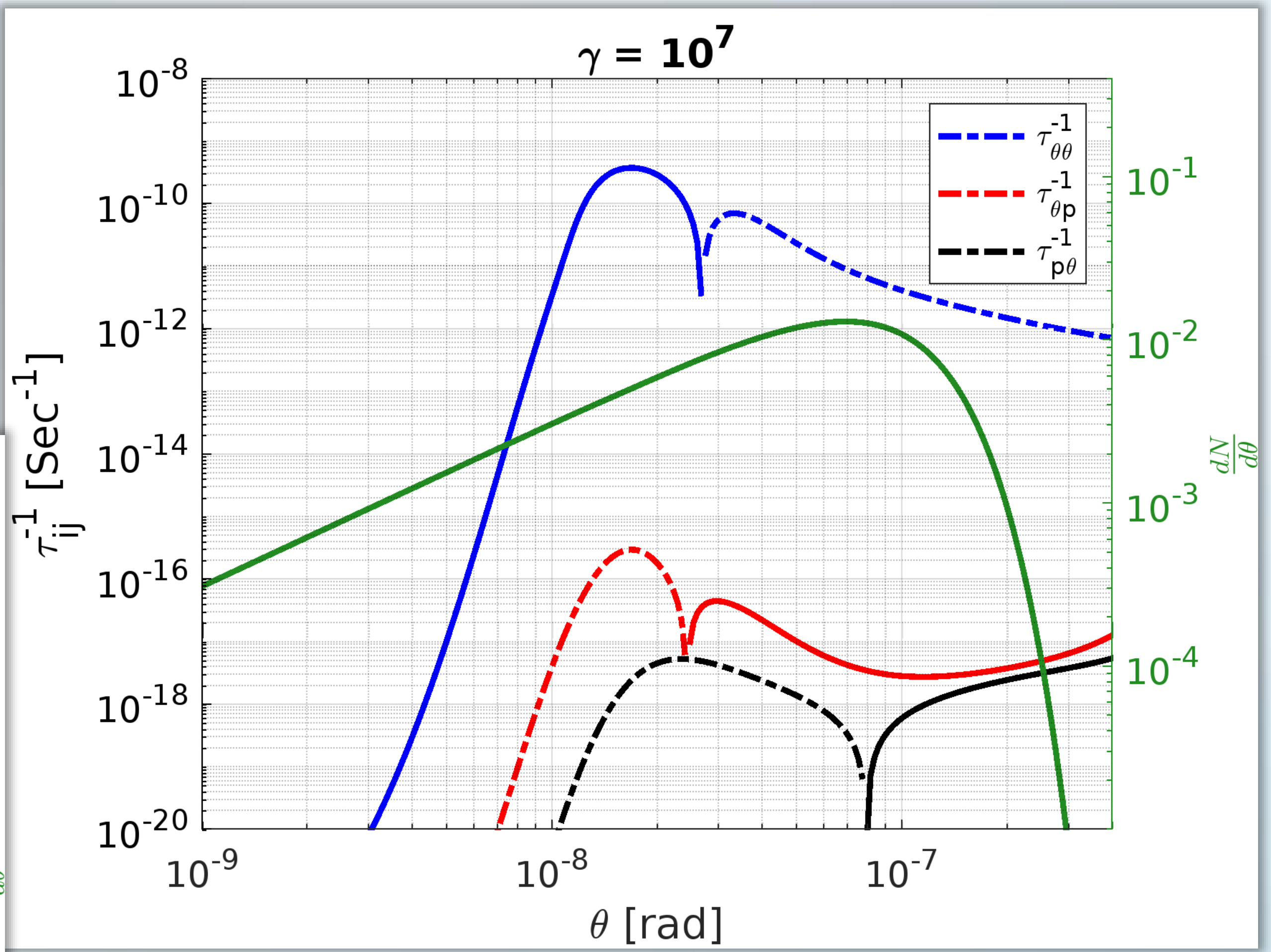
where

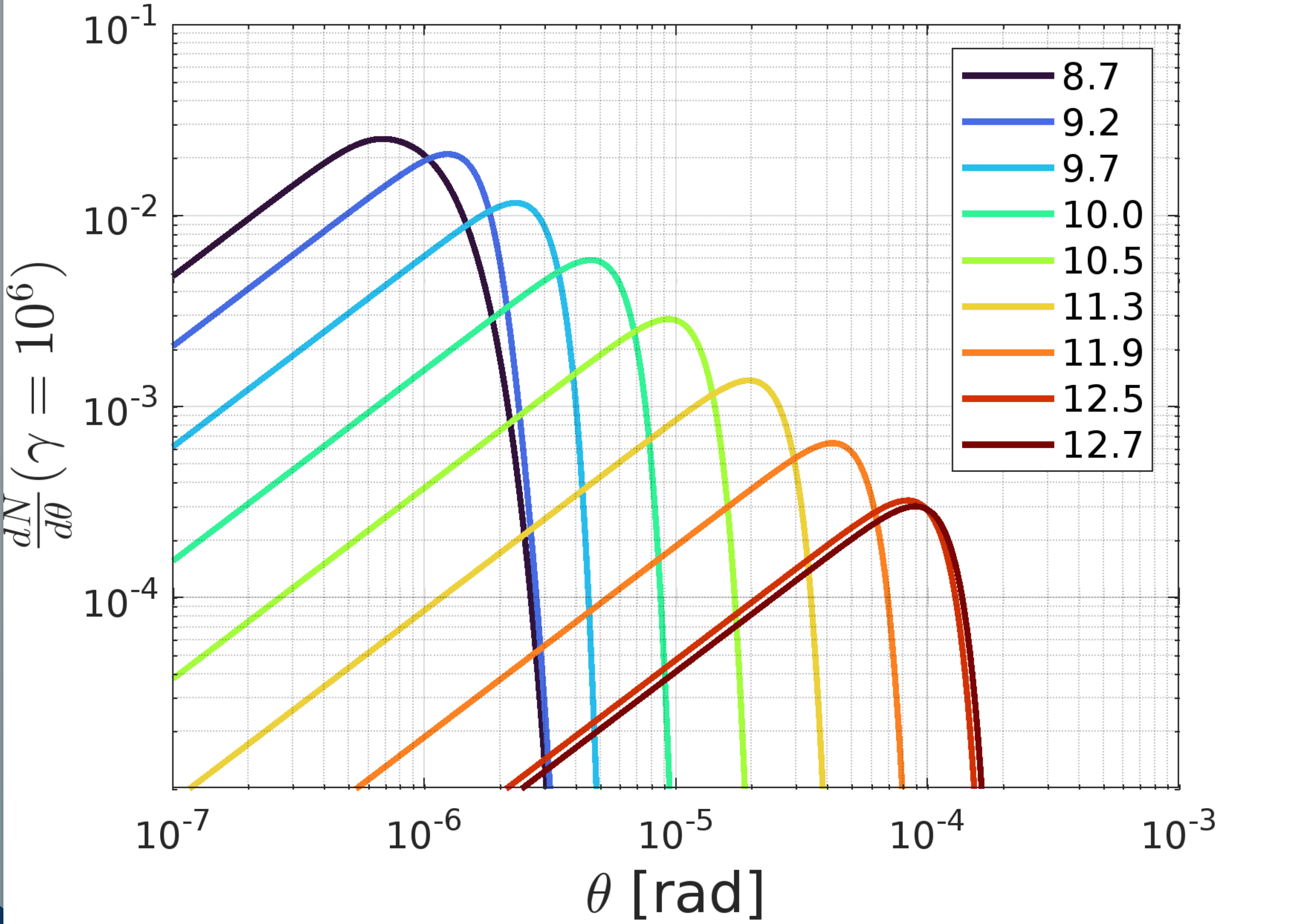
$$\xi = -\frac{1}{\sqrt{1 + 2\theta^R (ck_\perp/\omega_p) + (ck_\perp/\omega_p)^2 (1 + \theta^{R^2})}} \left[\frac{\theta^R}{\theta}\frac{ck_\perp}{\omega_p} + \frac{\theta}{2} - \frac{1}{2\theta\gamma^2}\right], \tag{B.35}$$

and the resonance region $R(\theta, \gamma)$ is defined by the following condition

$$\left(\frac{ck_\perp}{\omega_p}\right)^2 \left(\theta^2 - \theta^{R^2}\right) + \frac{ck_\perp}{\omega_p}\theta^R \left[\theta^2 + \frac{1}{\gamma^2}\right] - \left[\frac{1}{2\gamma^2} + \frac{\theta^2}{2}\right]^2 \geq 0. \tag{B.36}$$


γ = 10^6
τ_θθ^-1
τ_θp^-1
τ_pθ^-1
τ_ij^-1 [Sec^-1]
θ [rad]
dN/dθ
10^-8
10^-10
10^-12
10^-14
10^-16
10^-18
10^-20
10^-1
10^-2
10^-3
10^-4
10^-7
10^-6

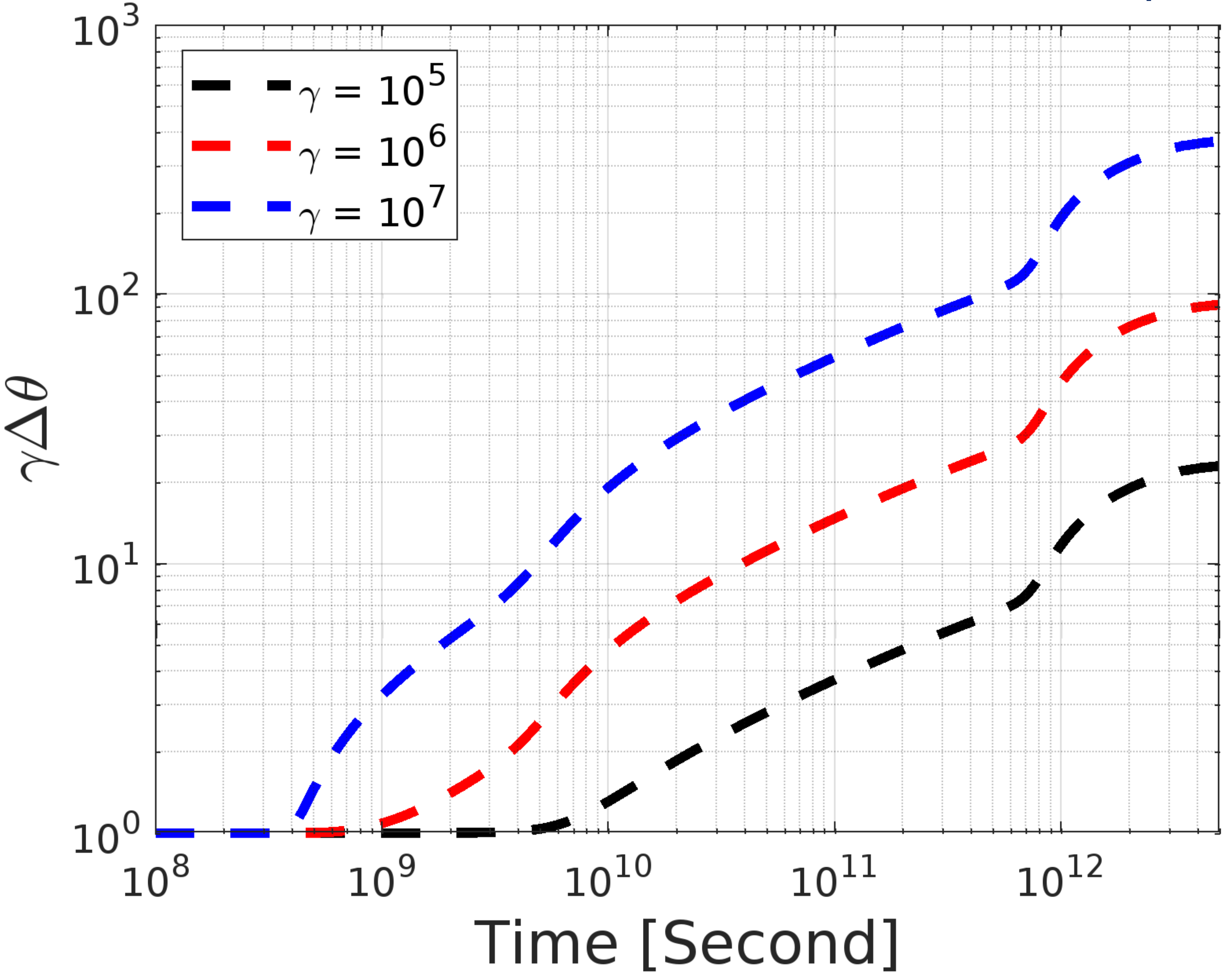

γ = 10^7
τ_θθ^-1
τ_θp^-1
τ_pθ^-1
τ_ij^-1 [Sec^-1]
θ [rad]
dN/dθ
10^-8
10^-10
10^-12
10^-14
10^-16
10^-18
10^-20
10^-1
10^-2
10^-3
10^-4
10^-9
10^-8
10^-7



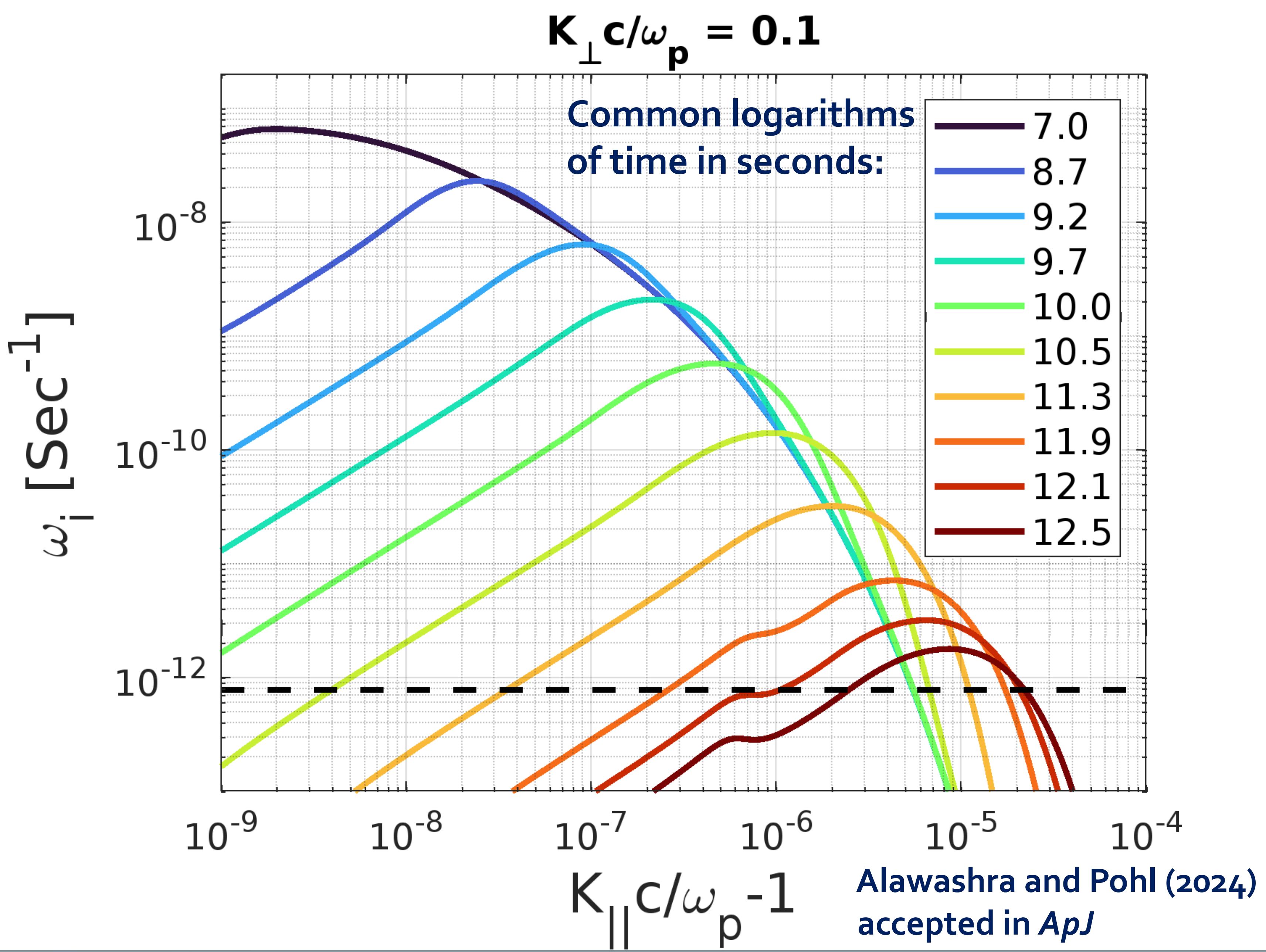
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\frac{dN}{d\theta}(\gamma = 10^6)$
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
θ [rad]
8.7
9.2
9.7
10.0
10.5
11.3
11.9
12.5
12.7

Significant widening of the beam

Alawashra and Pohl (2024) accepted in *ApJ*

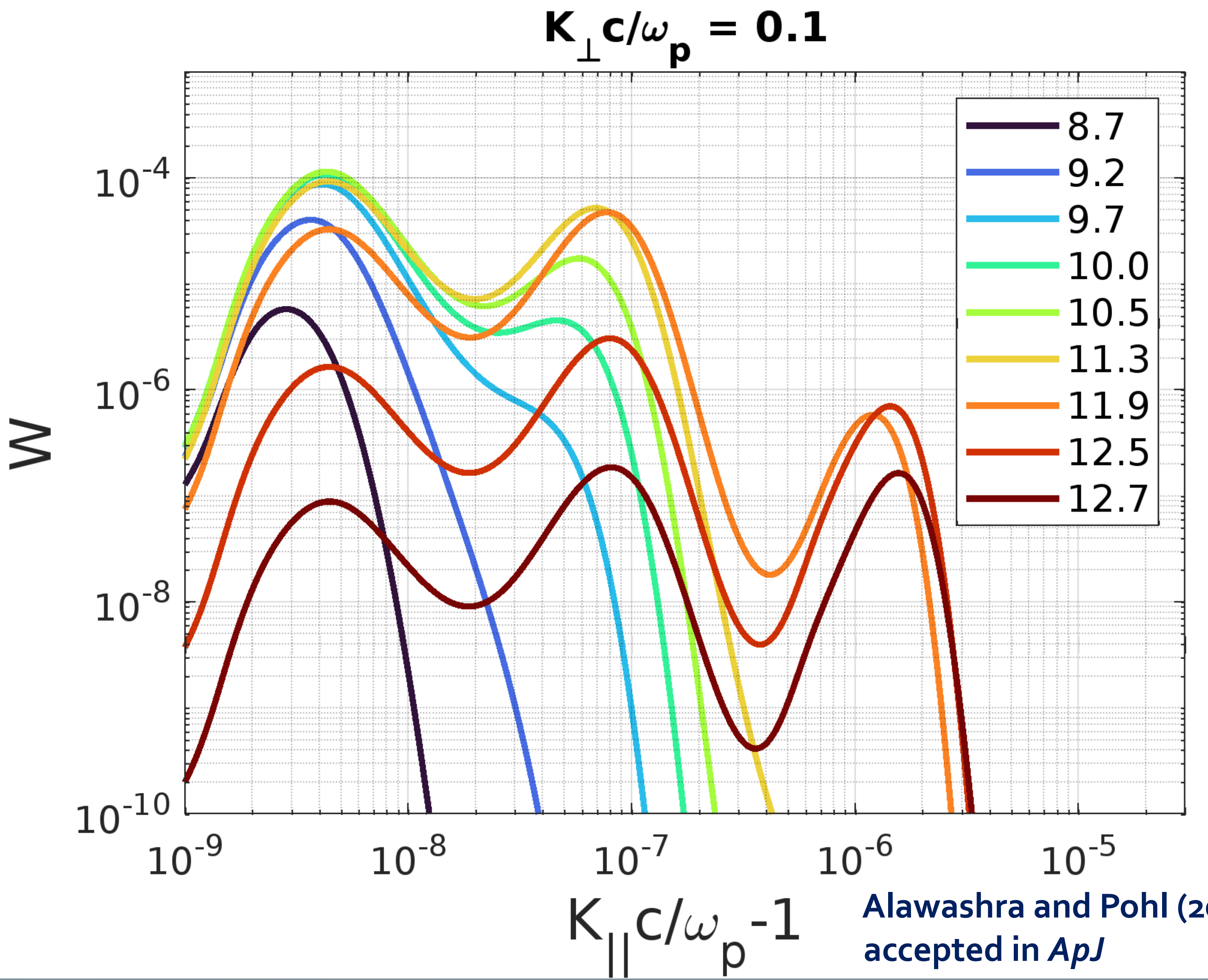


The instability is suppressed by the widening

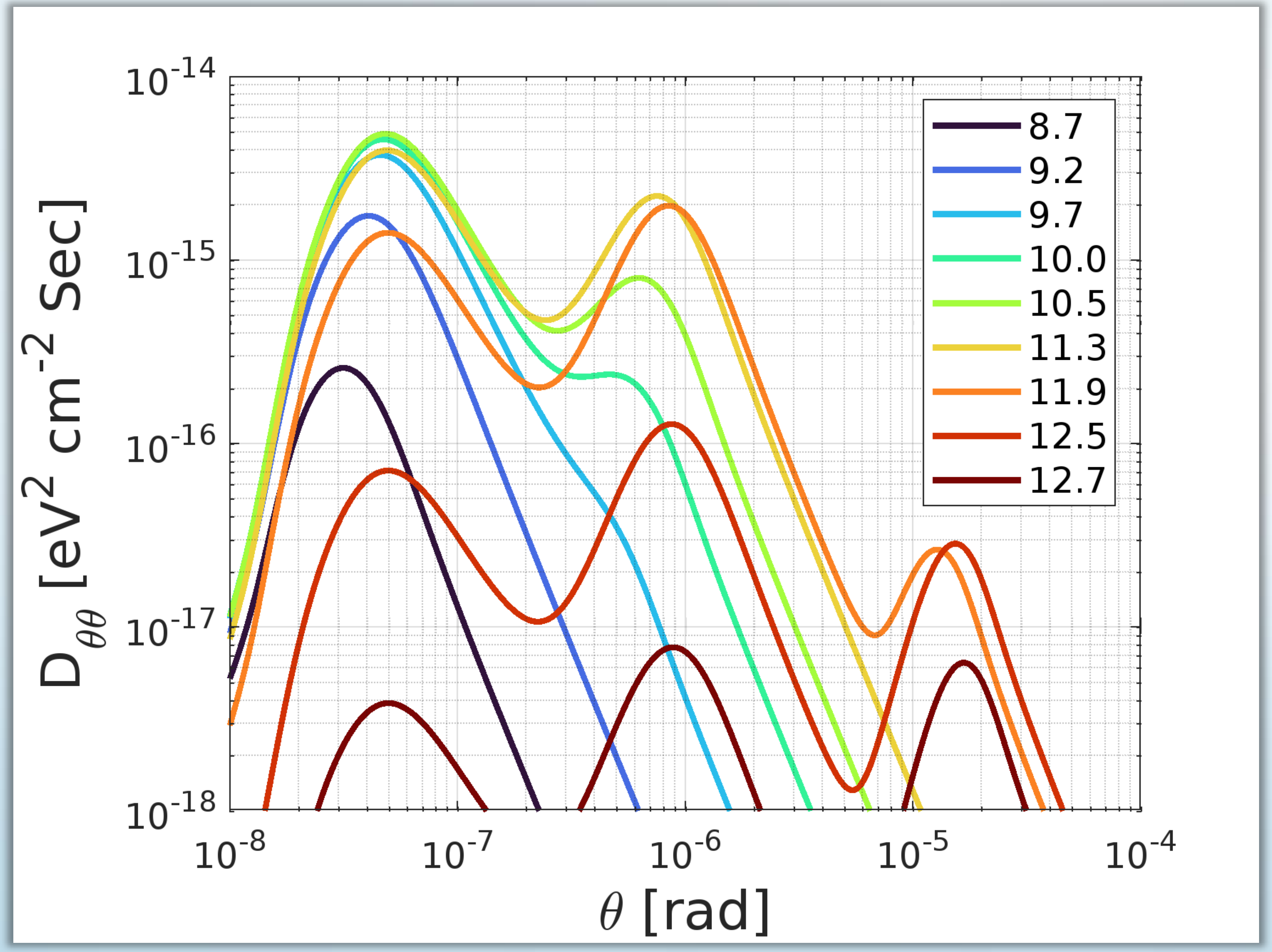


Alawashra and Pohl (2024) accepted in *ApJ*

Unstable wave spectrum evolution



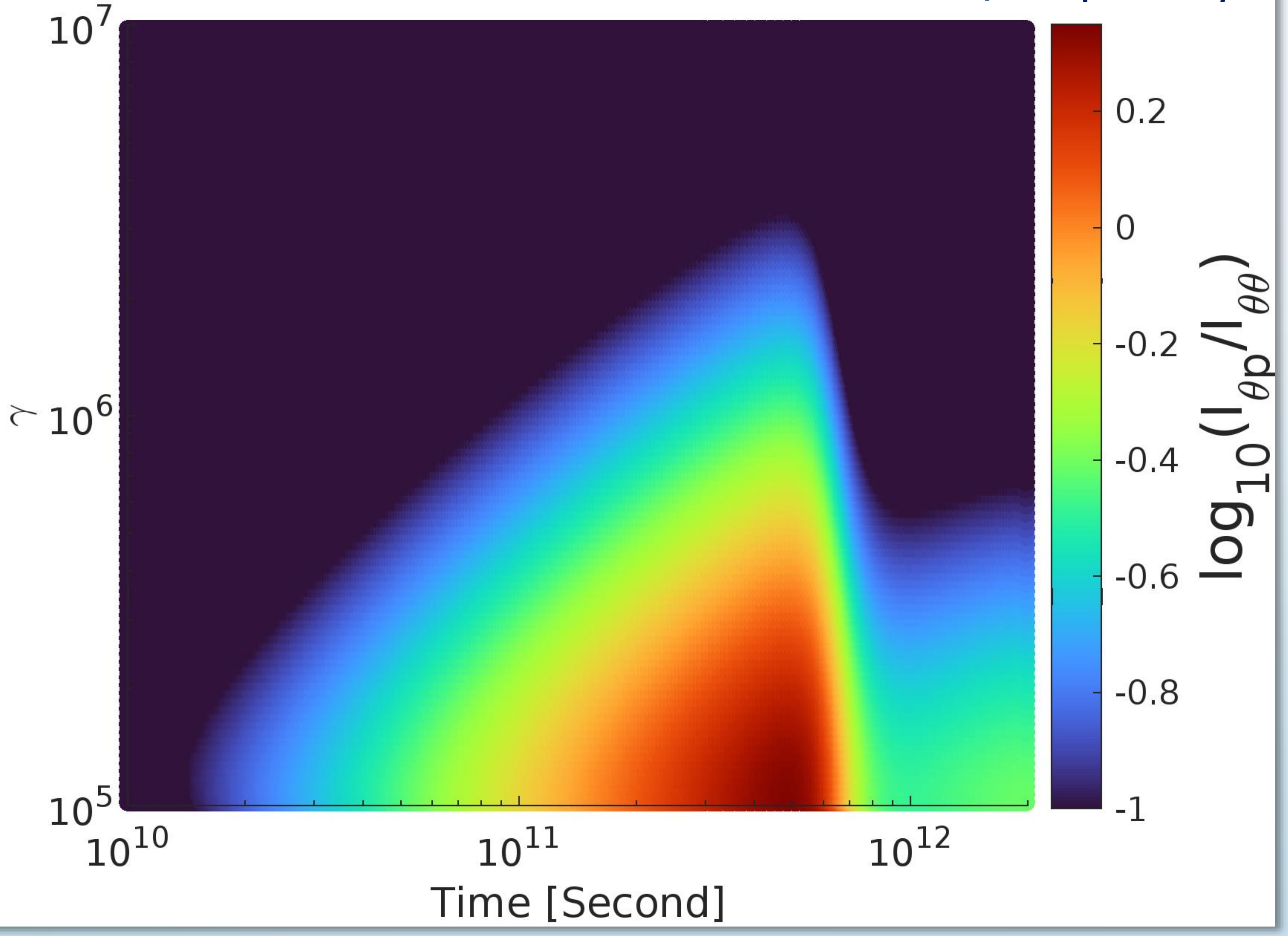
Alawashra and Pohl (2024) accepted in *ApJ*

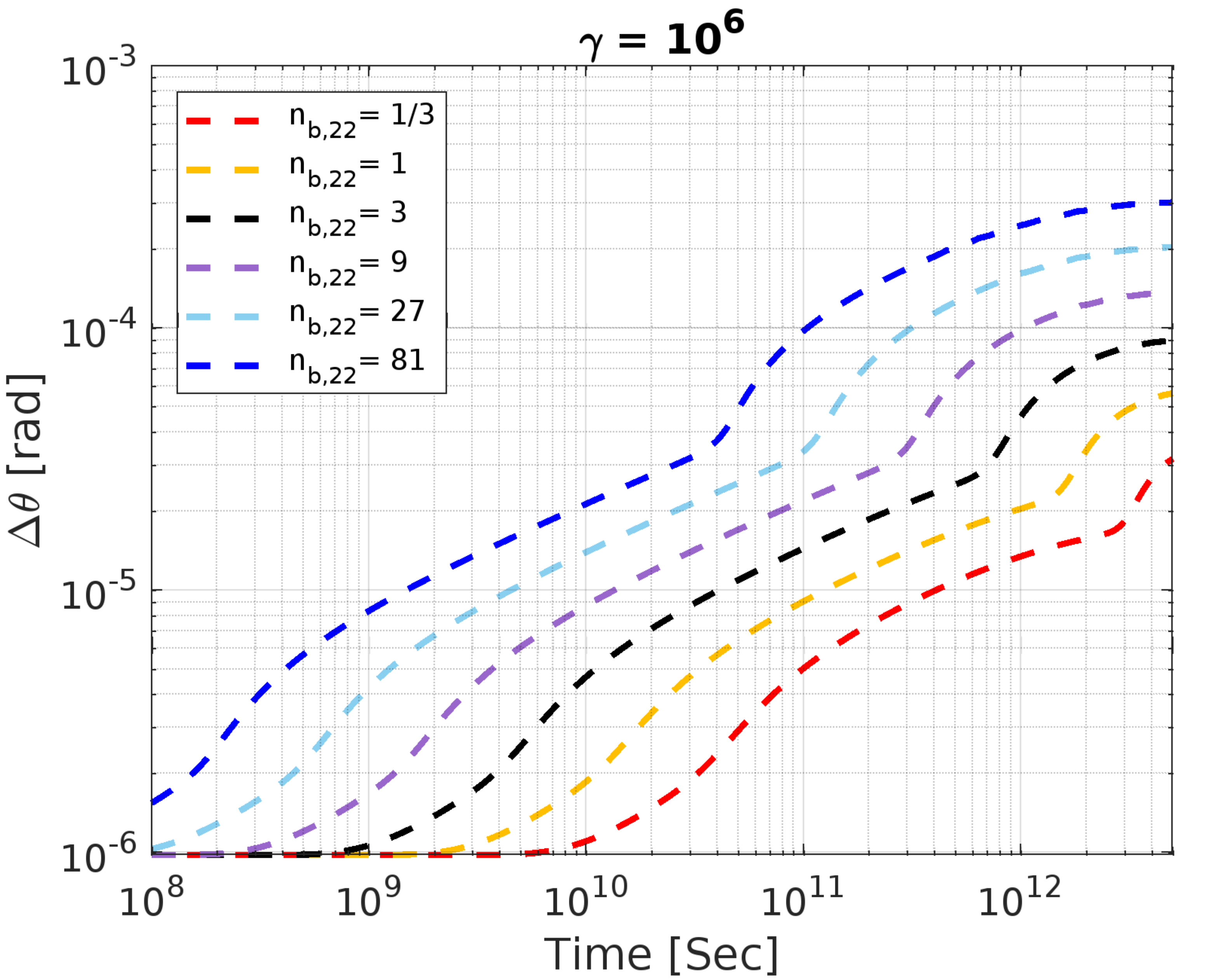


8.7
9.2
9.7
10.0
10.5
11.3
11.9
12.5
12.7
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
$D_{\theta\theta}$ [eV^2 cm^{-2} Sec]
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
θ [rad]

Relevant for pairs with Lorentz factors less than 10^6

Alawashra and Pohl (2024) accepted in *ApJ*

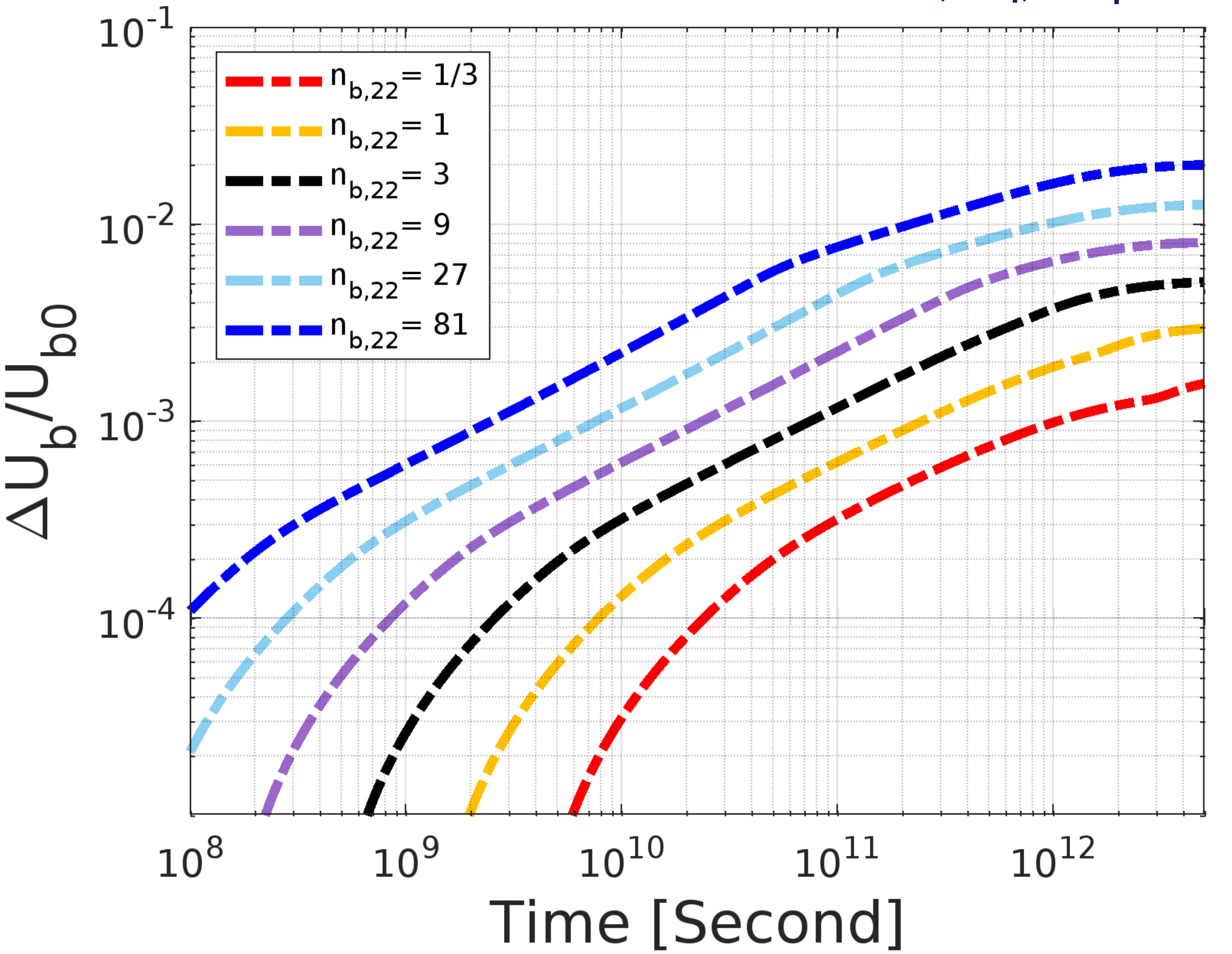




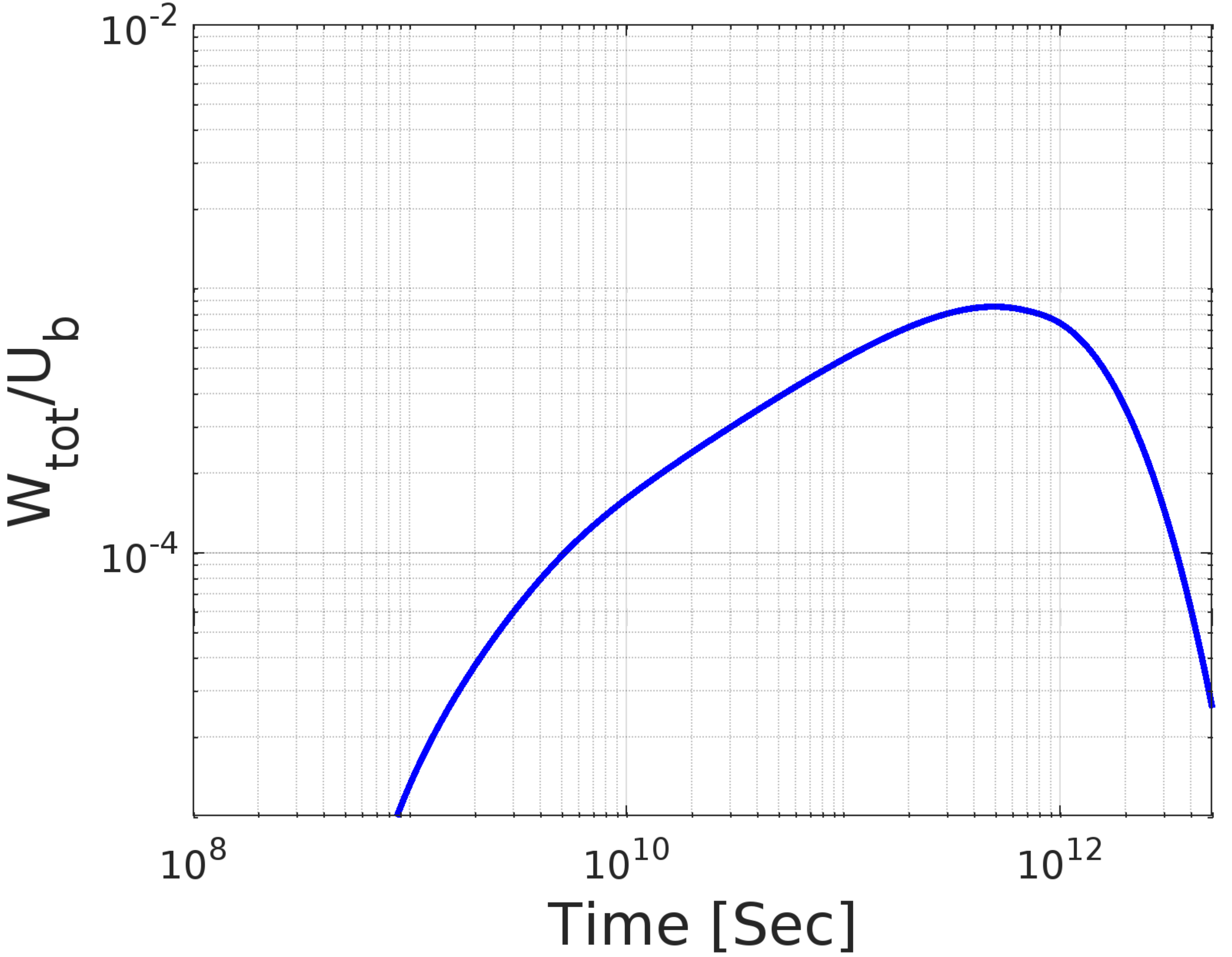
γ = 10^6
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Delta\theta$ [rad]
10^8
10^9
10^{10}
10^{11}
10^{12}
Time [Sec]
$n_{b,22}$= 1/3
$n_{b,22}$= 1
$n_{b,22}$= 3
$n_{b,22}$= 9
$n_{b,22}$= 27
$n_{b,22}$= 81

Small energy loss even for higher densities

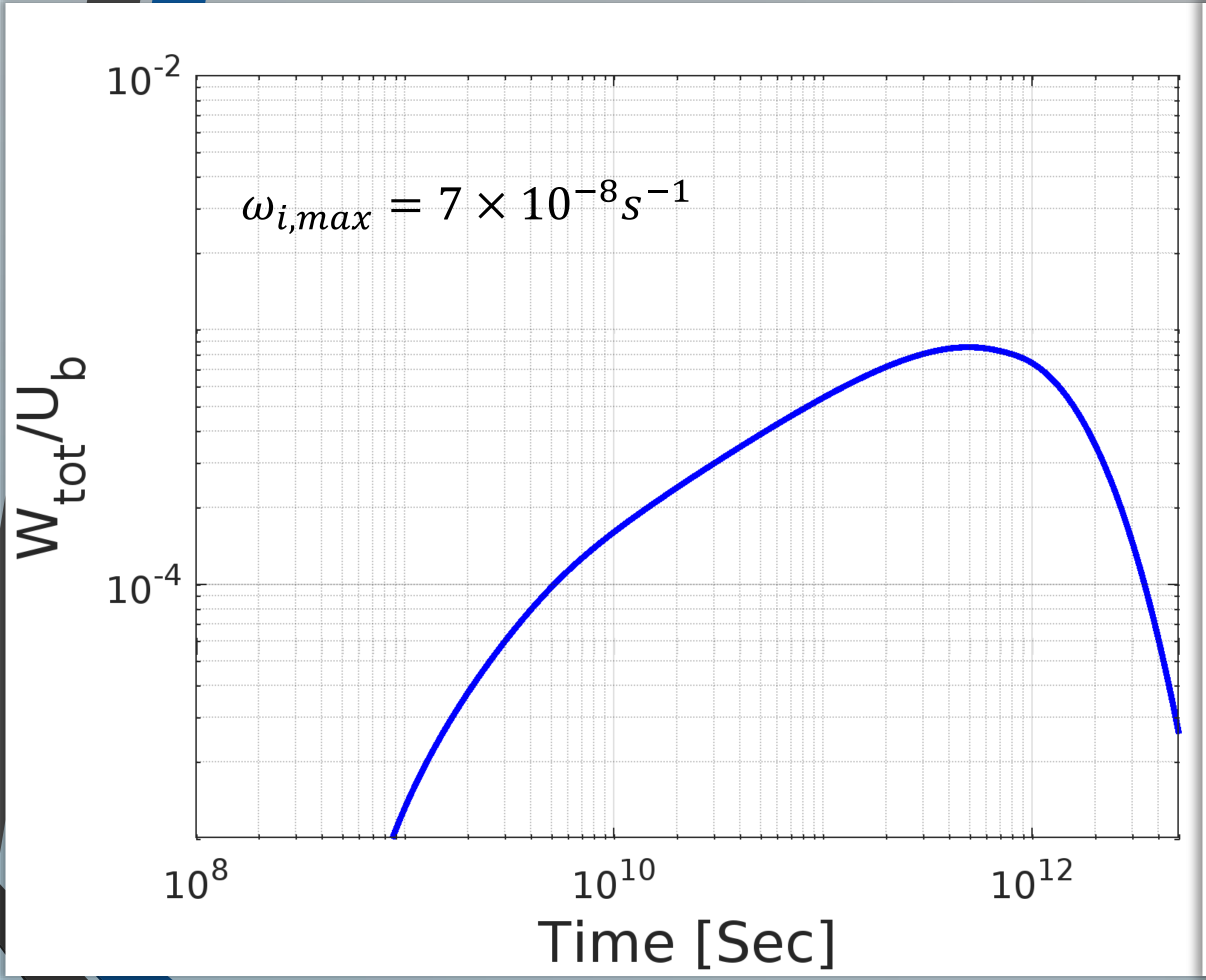
Alawashra and Pohl (2024) accepted in *ApJ*

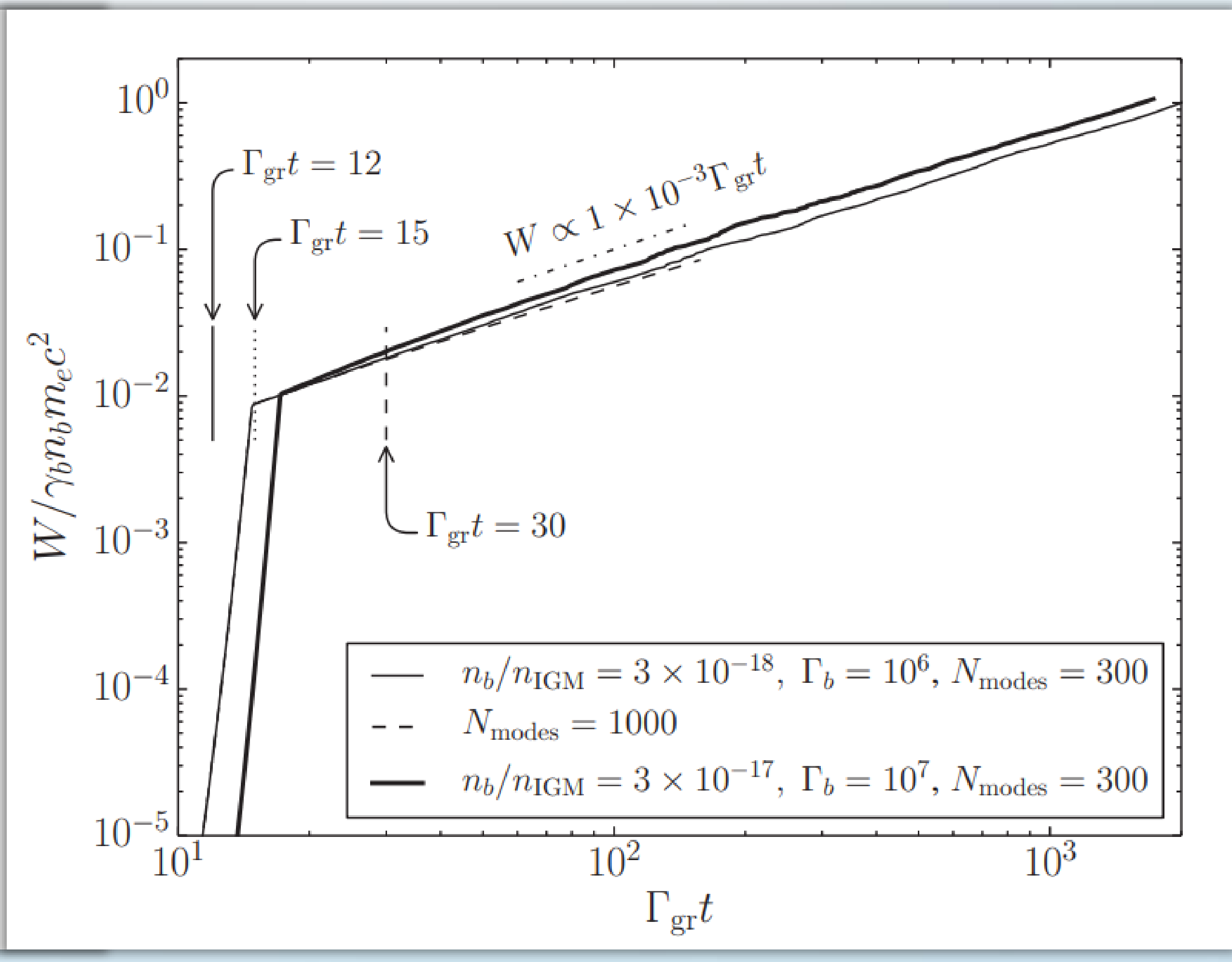


$$\begin{aligned}\frac{dU_b}{dt}(t) &= -2\frac{dW_{\text{tot}}}{dt}(t) \\ &= -8\pi \int dk_\perp k_\perp \int dk_{||} W(k_\perp, k_{||}, t)\,\omega_i(k_\perp, k_{||}, t),\end{aligned} \tag{5.38}$$

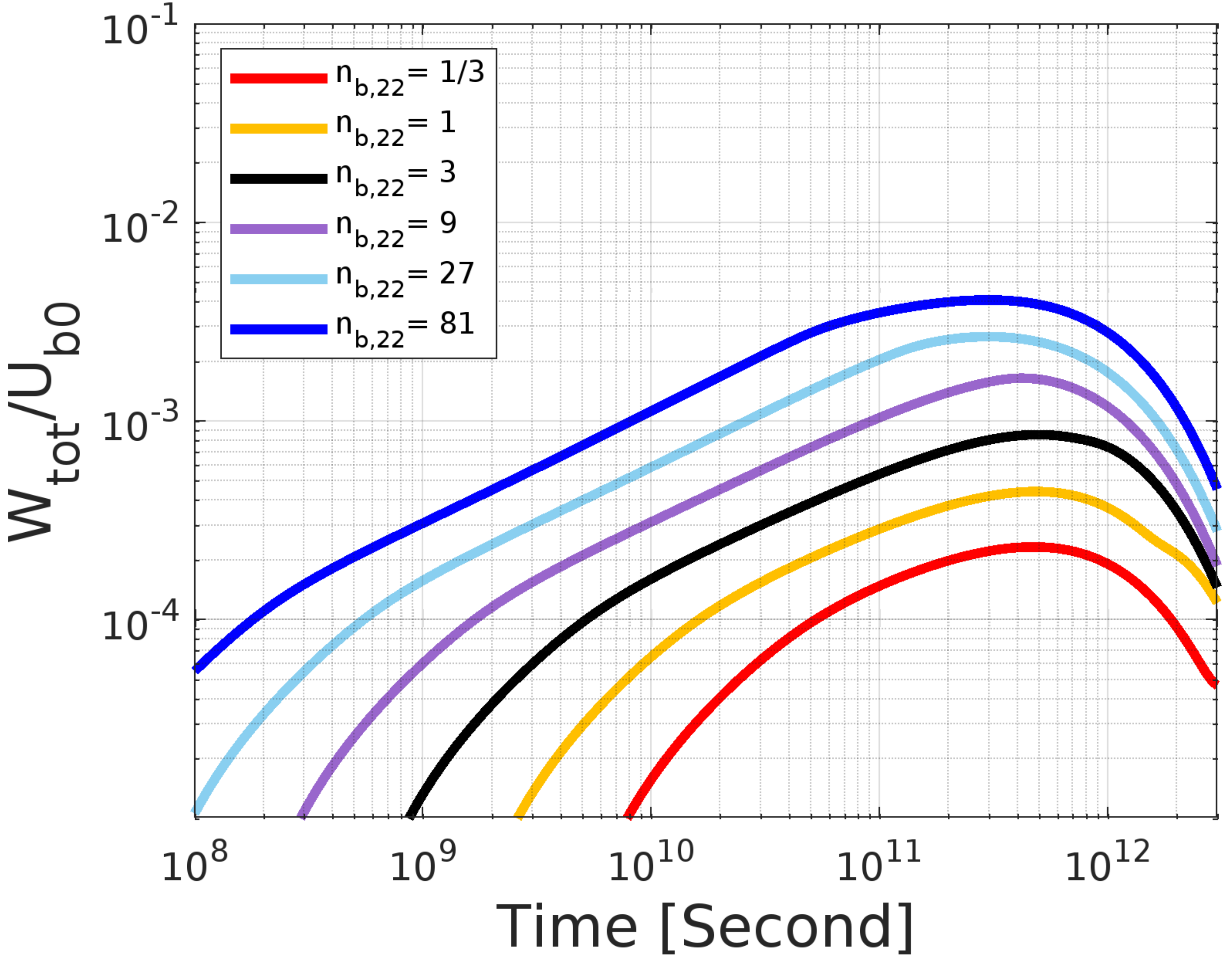


10^{-2}
10^{-4}
W_{tot}/U_b
10^{8}
10^{10}
10^{12}
Time [Sec]

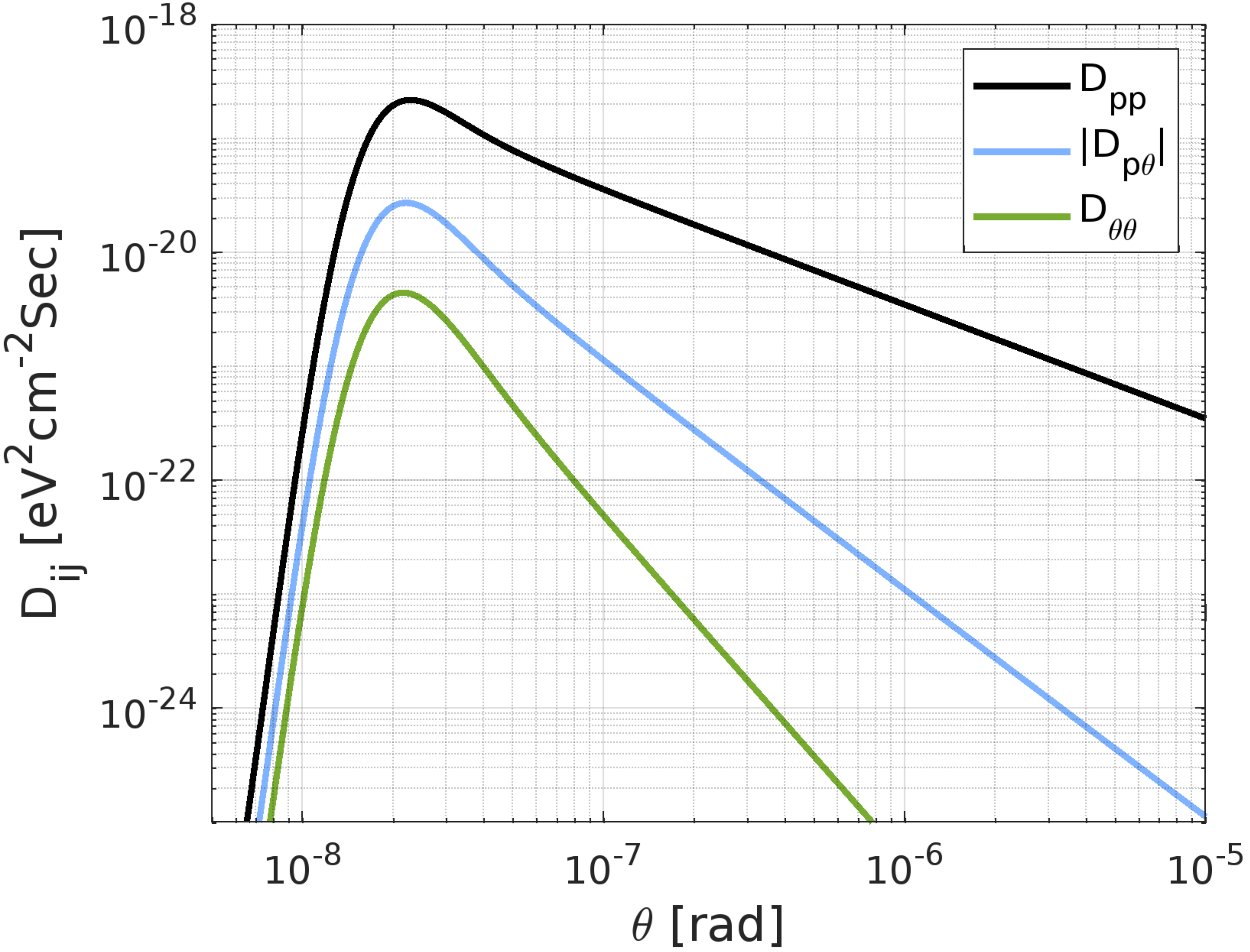




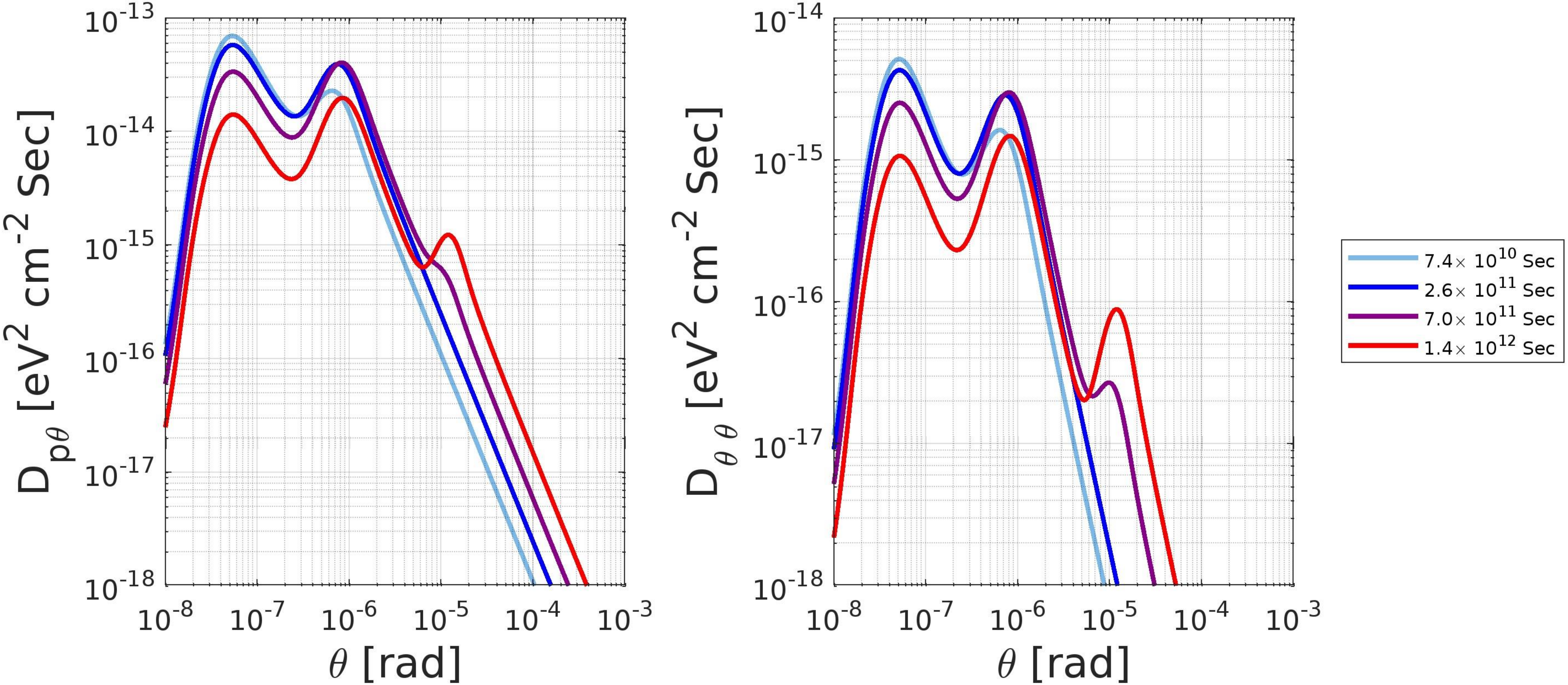
Chang et al.
The Astrophysical Journal, 797:110 (6pp), 2014 December 20



$n_{b,22}$= 1/3
$n_{b,22}$= 1
$n_{b,22}$= 3
$n_{b,22}$= 9
$n_{b,22}$= 27
$n_{b,22}$= 81
W_{tot}/U_{b0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
Time [Second]

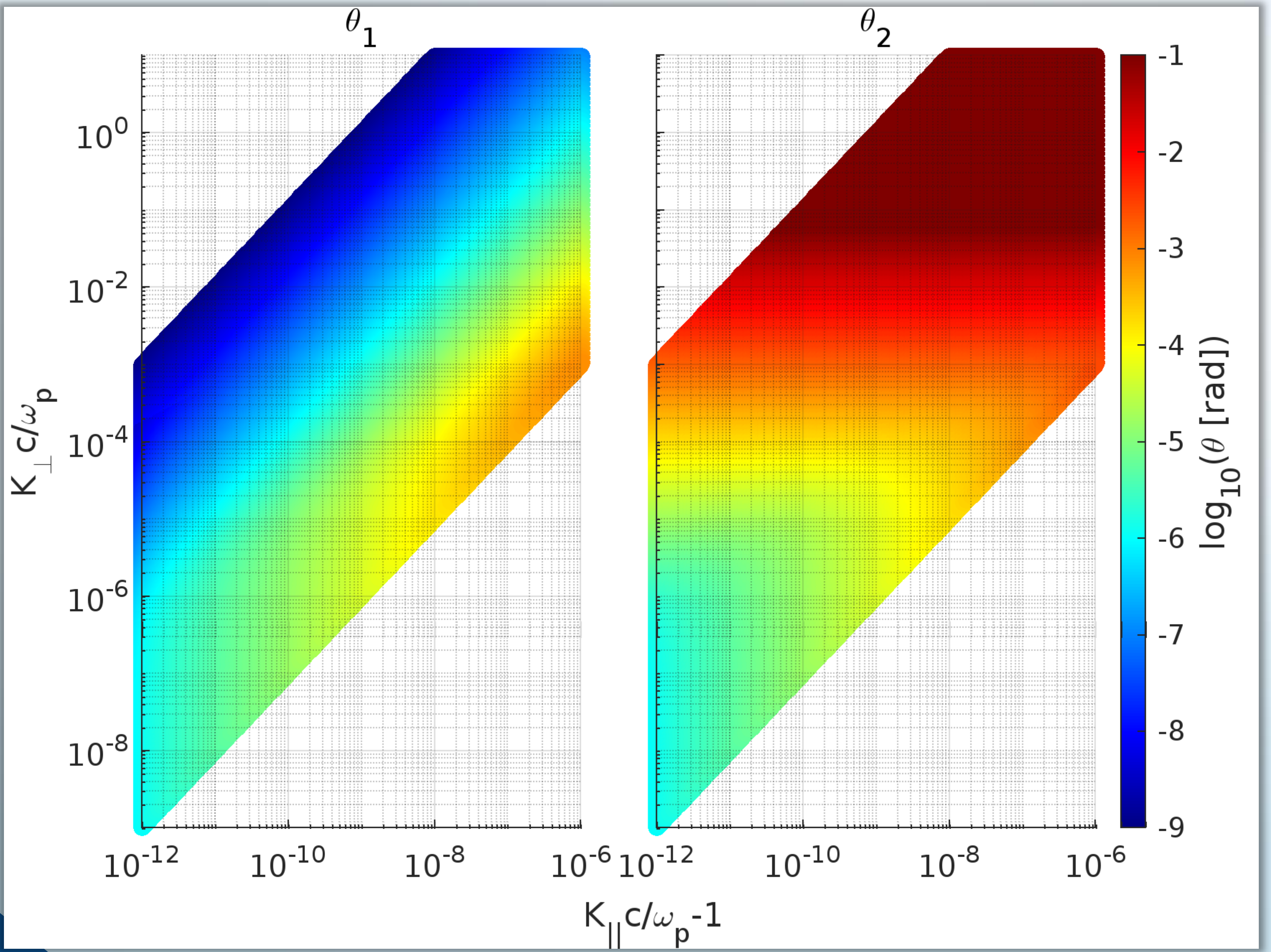


D_{pp}
$|D_{p\theta}|$
$D_{\theta\theta}$
10^{-18}
10^{-20}
10^{-22}
10^{-24}
D_{ij} [$eV^2cm^{-2}Sec$]
10^{-8}
10^{-7}
10^{-6}
10^{-5}
θ [rad]

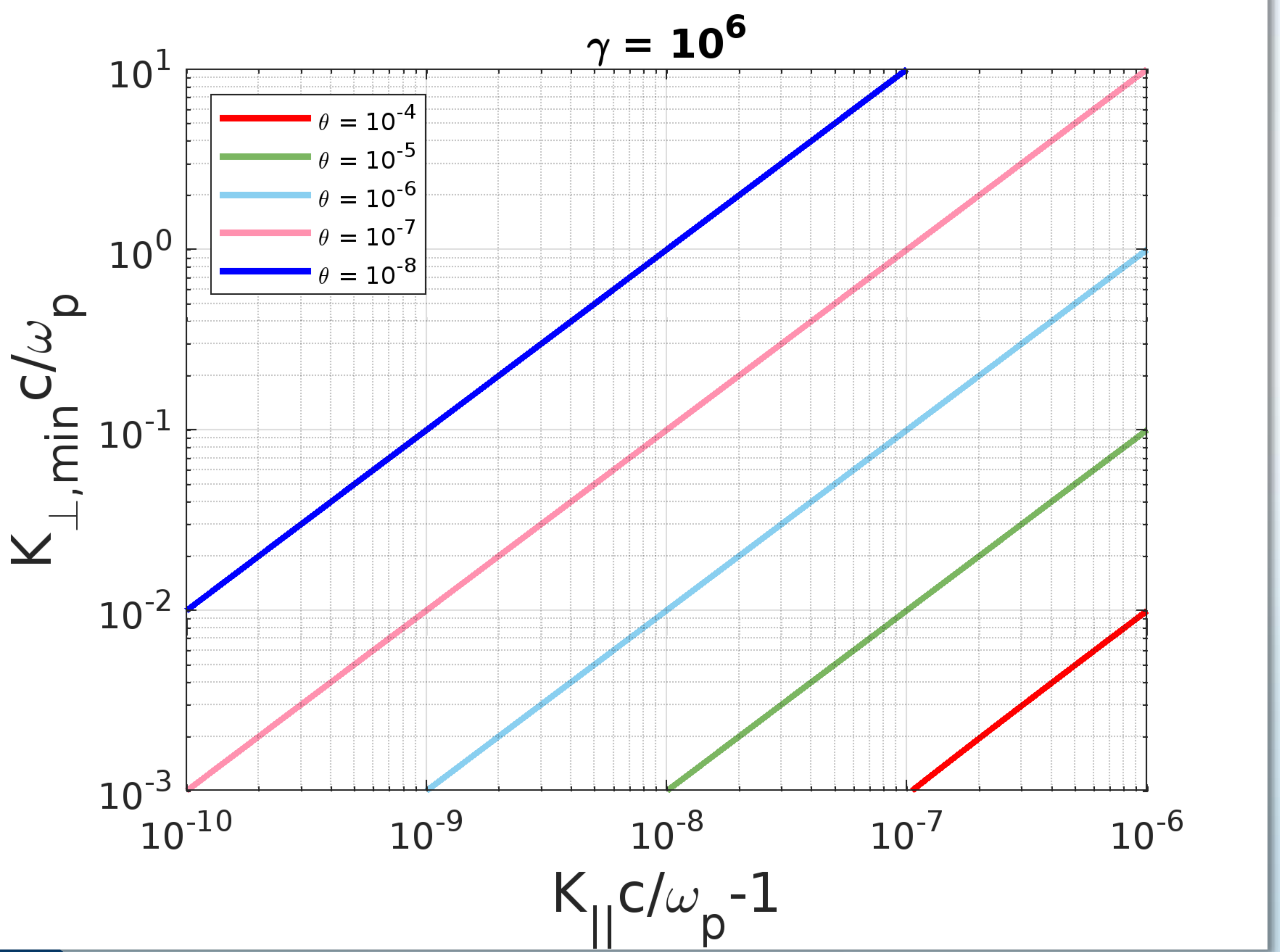


$D_{p\theta}$ [eV^2 cm^{-2} Sec]
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3}
θ [rad]
$D_{\theta\theta}$ [eV^2 cm^{-2} Sec]
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3}
θ [rad]
7.4×10^{10} Sec
2.6×10^{11} Sec
7.0×10^{11} Sec
1.4×10^{12} Sec

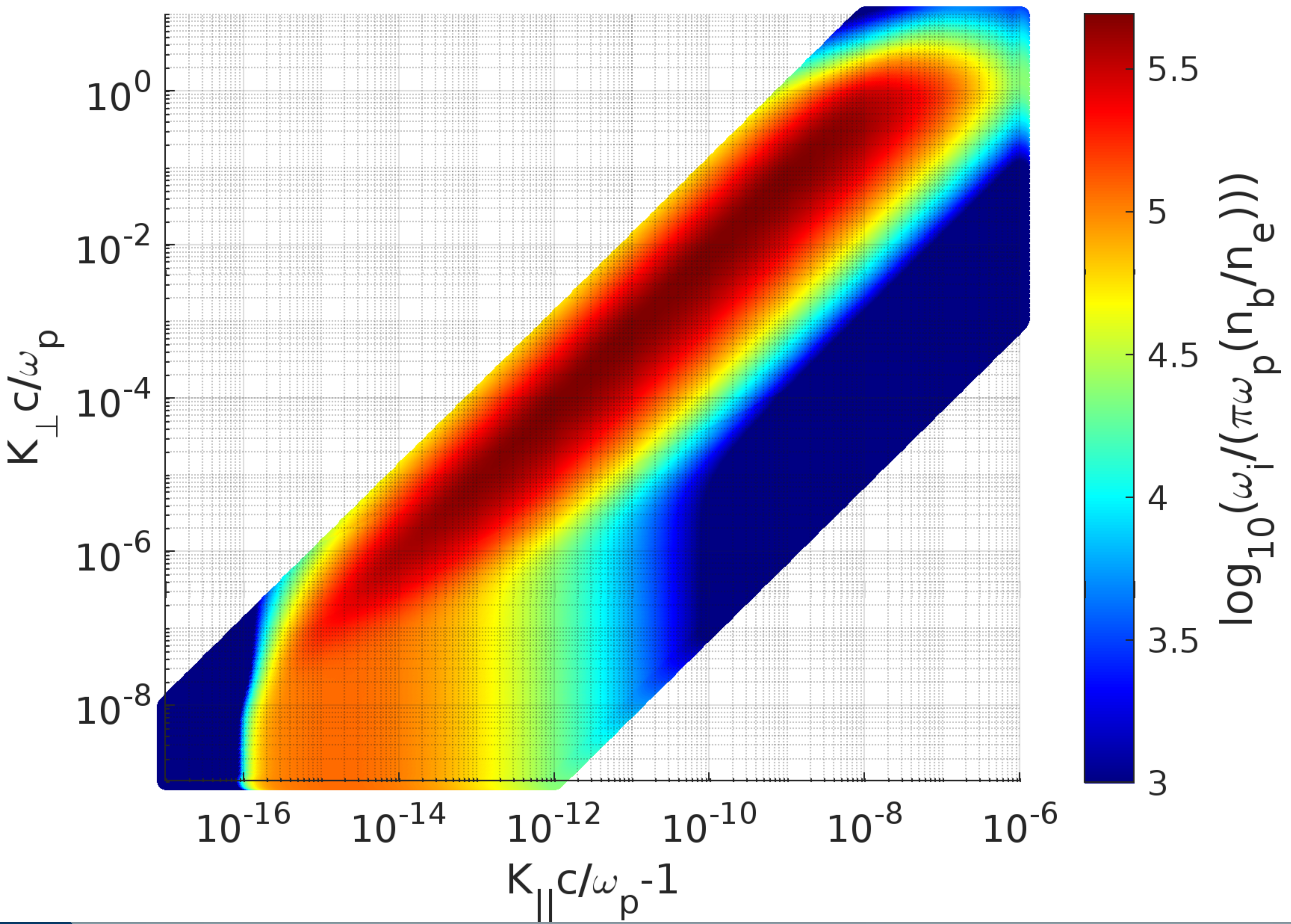
Resonance



θ_1
θ_2
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$K_\perp c/\omega_p$
10^{-12}
10^{-10}
10^{-8}
10^{-6}
$K_{||} c/\omega_p$-1
-1
-2
-3
-4
-5
-6
-7
-8
-9
$\log_{10}(\theta$ [rad])

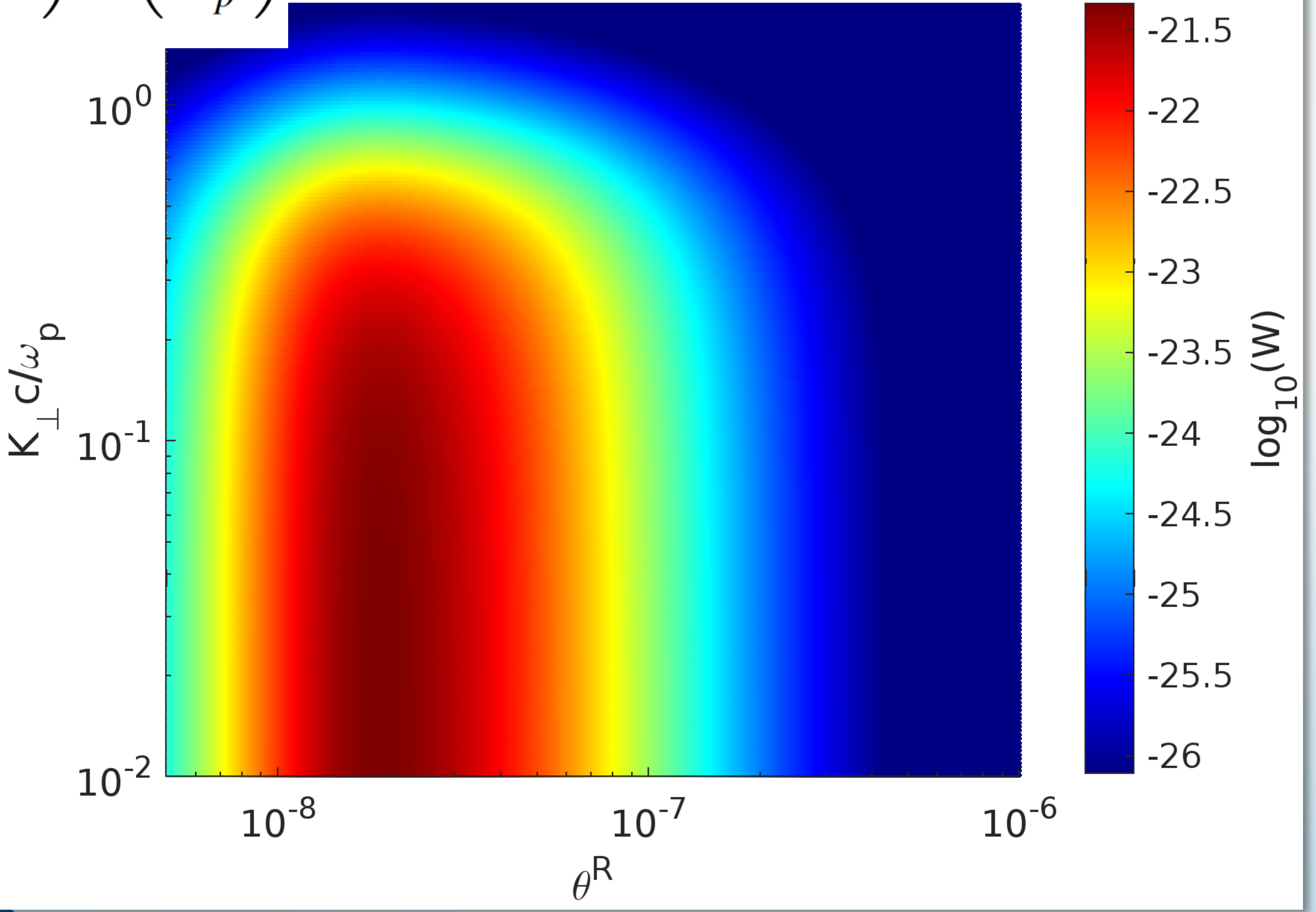


γ = 10^6
θ = 10^-4
θ = 10^-5
θ = 10^-6
θ = 10^-7
θ = 10^-8
K⊥,min c/ωp
K|| c/ωp -1
10^1
10^0
10^-1
10^-2
10^-3
10^-10
10^-9
10^-8
10^-7
10^-6



10^0
10^-2
10^-4
10^-6
10^-8
K⊥ c/ωp
10^-16
10^-14
10^-12
10^-10
10^-8
10^-6
K|| c/ωp-1
5.5
5
4.5
4
3.5
3
log10(ωi/(πωp(nb/ne)))

$$\theta^R = \left(\frac{ck_{||}}{\omega_p} - 1\right) / \left(\frac{ck_{\perp}}{\omega_p}\right)$$



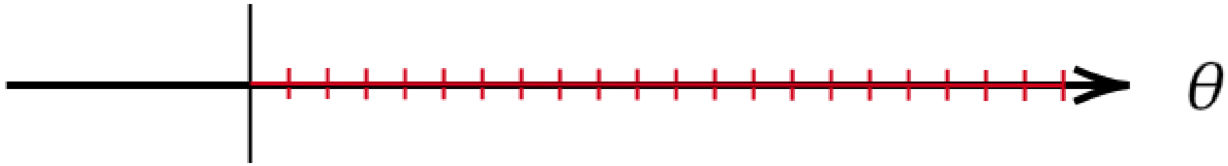
For $\frac{ck_\perp}{\omega_p} > 10^{-2}$, $\gamma > 10^3$ and $\theta < 10^{-3}$

The resonance condion: $\frac{ck_{||}}{\omega_p} - 1 = \frac{ck_\perp}{\omega_p}\theta$

Let's look at the case of fixed $\frac{ck_\perp}{\omega_p}$

For plasma waves with $\frac{ck_{||}}{\omega_p} - 1$

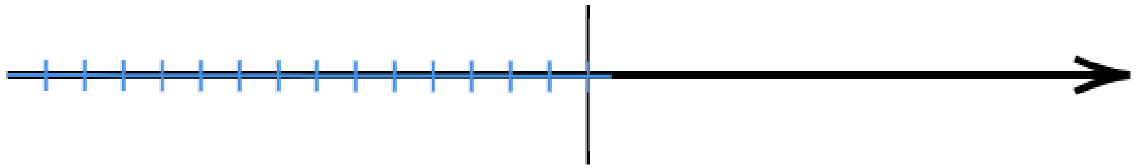
The resonance of the beam is



$$\theta_{R,min} = \frac{\frac{ck_{||}}{\omega_p} - 1}{\frac{ck_{\perp}}{\omega_p}}$$

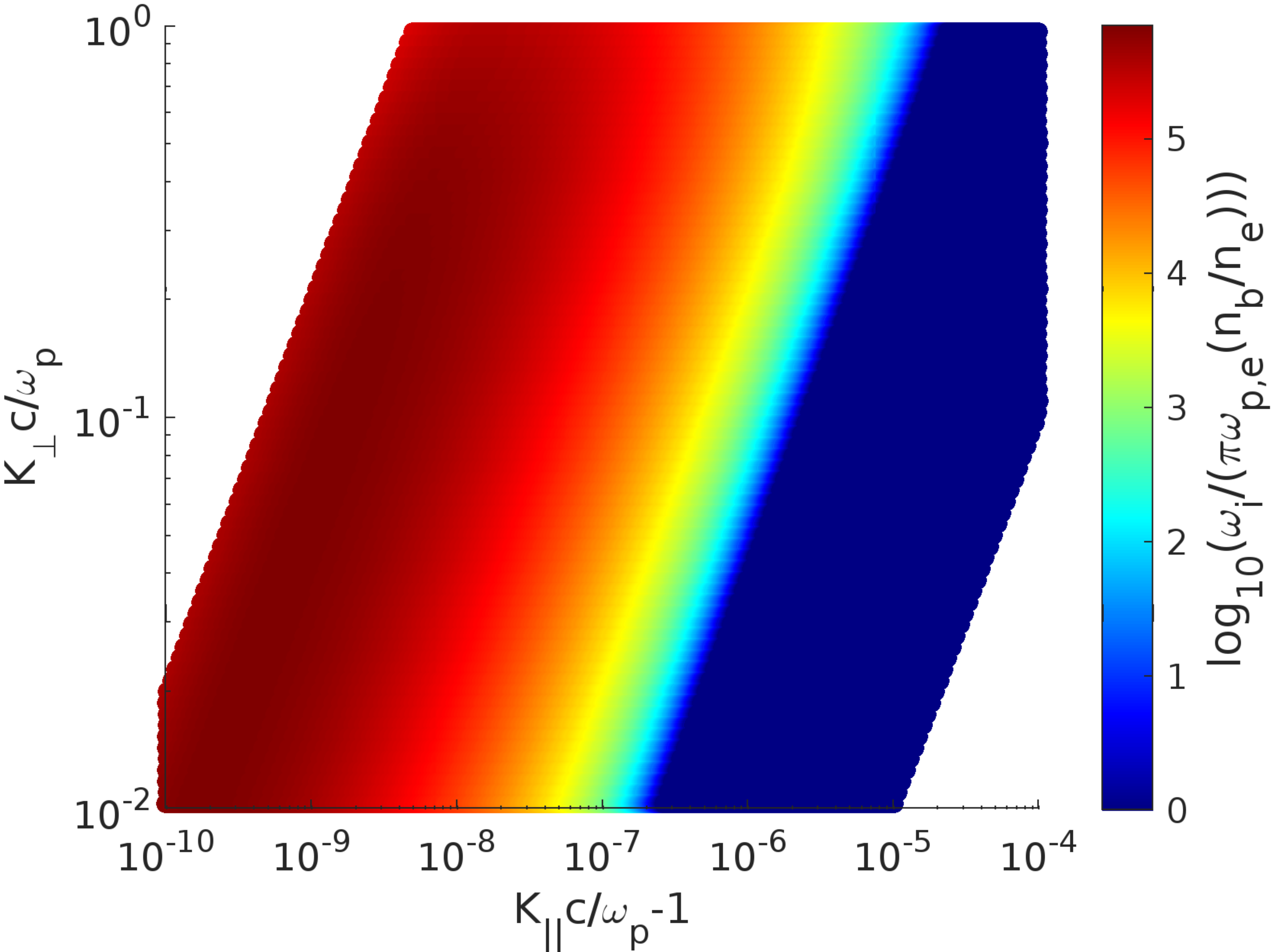
For beam angles with θ

The resonance of the waves is



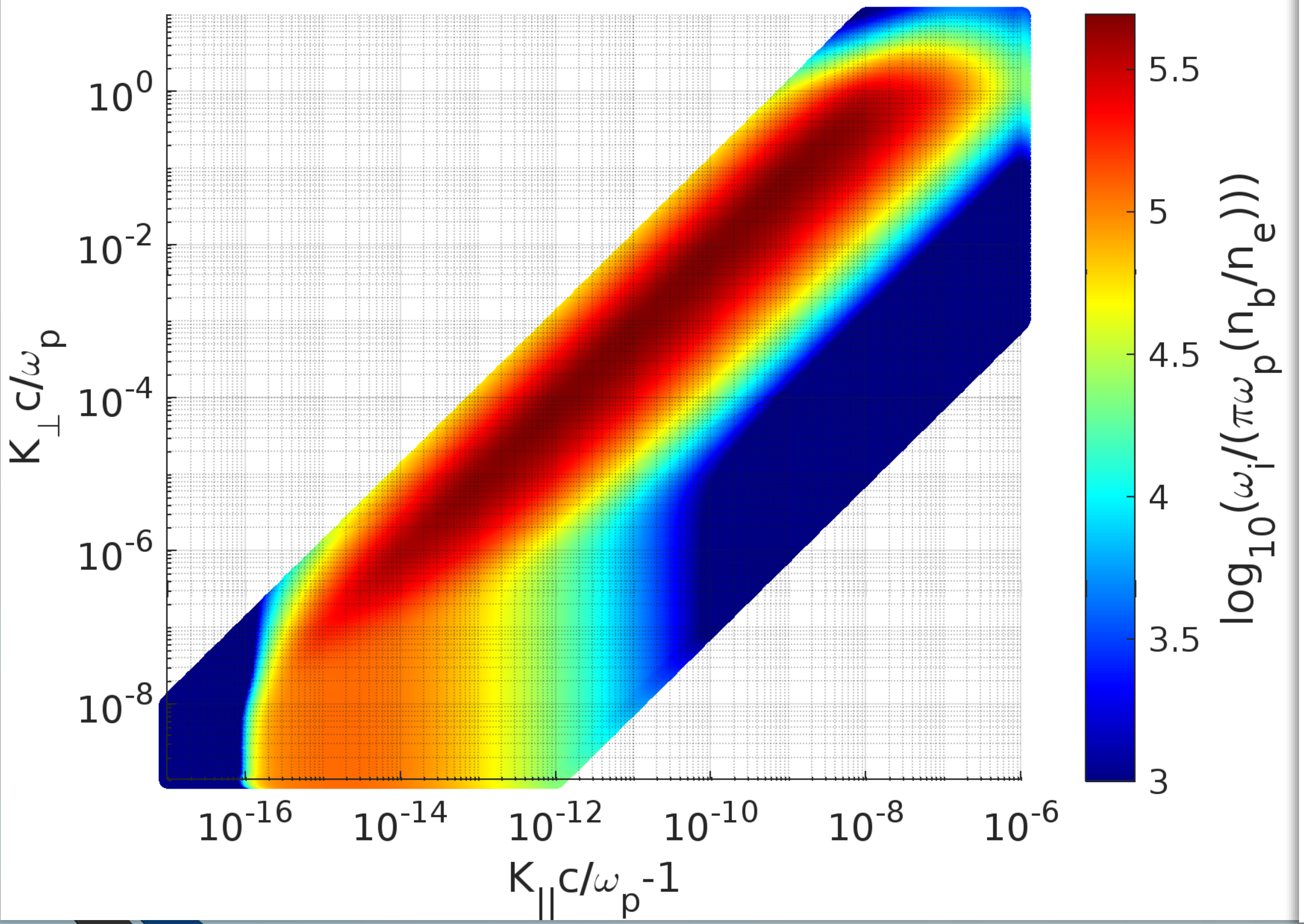
$$\frac{ck_{||}}{\omega_p} - 1$$

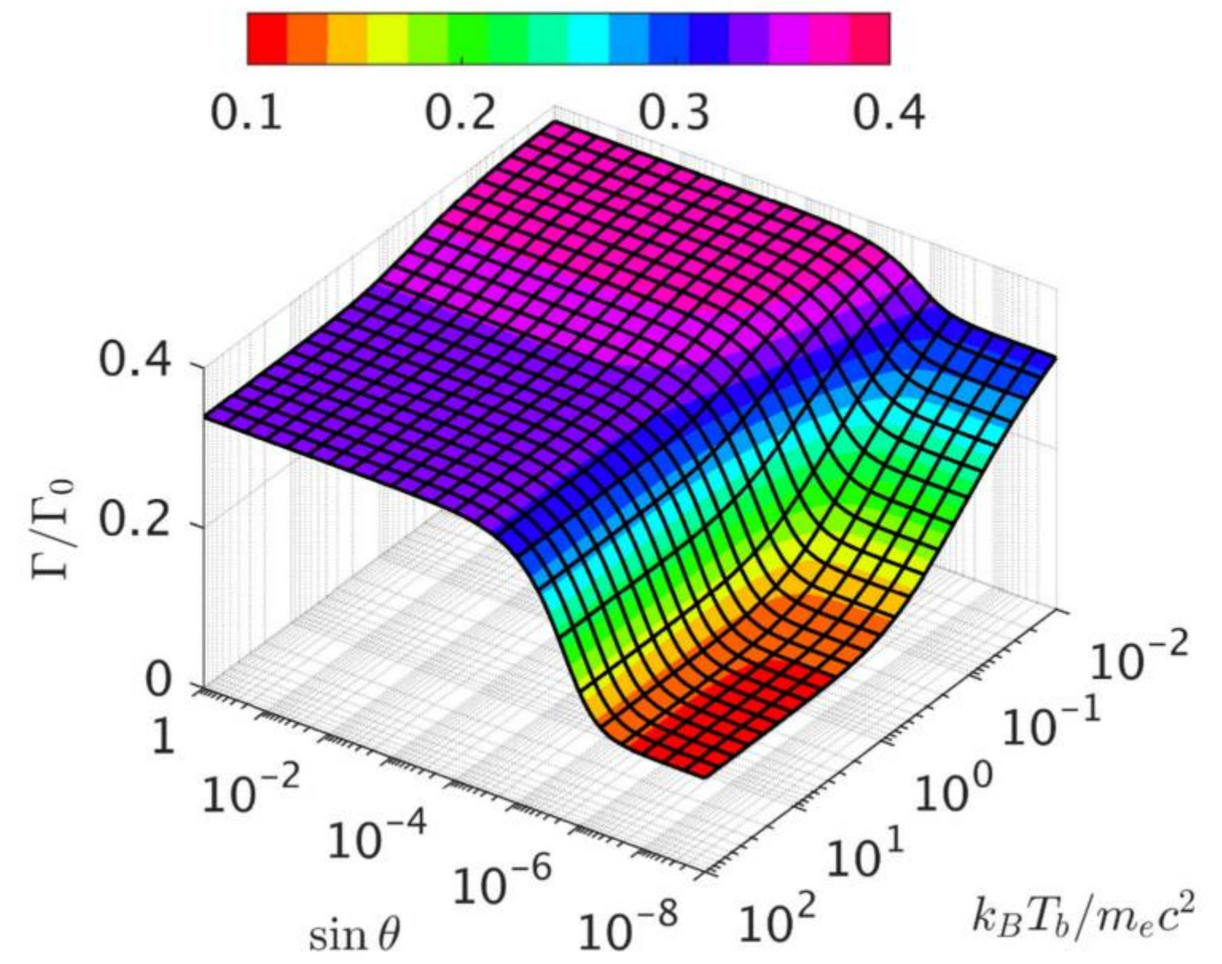
$$\left(\frac{ck_{||}}{\omega_p} - 1 \right)_{R,max} = \theta \, \frac{ck_{\perp}}{\omega_p}$$



10^0
10^-1
10^-2
K⊥c/ωp
10^-10
10^-9
10^-8
10^-7
10^-6
10^-5
10^-4
K||c/ωp-1
0
1
2
3
4
5
log10(ωi/(πωp,e(nb/ne)))

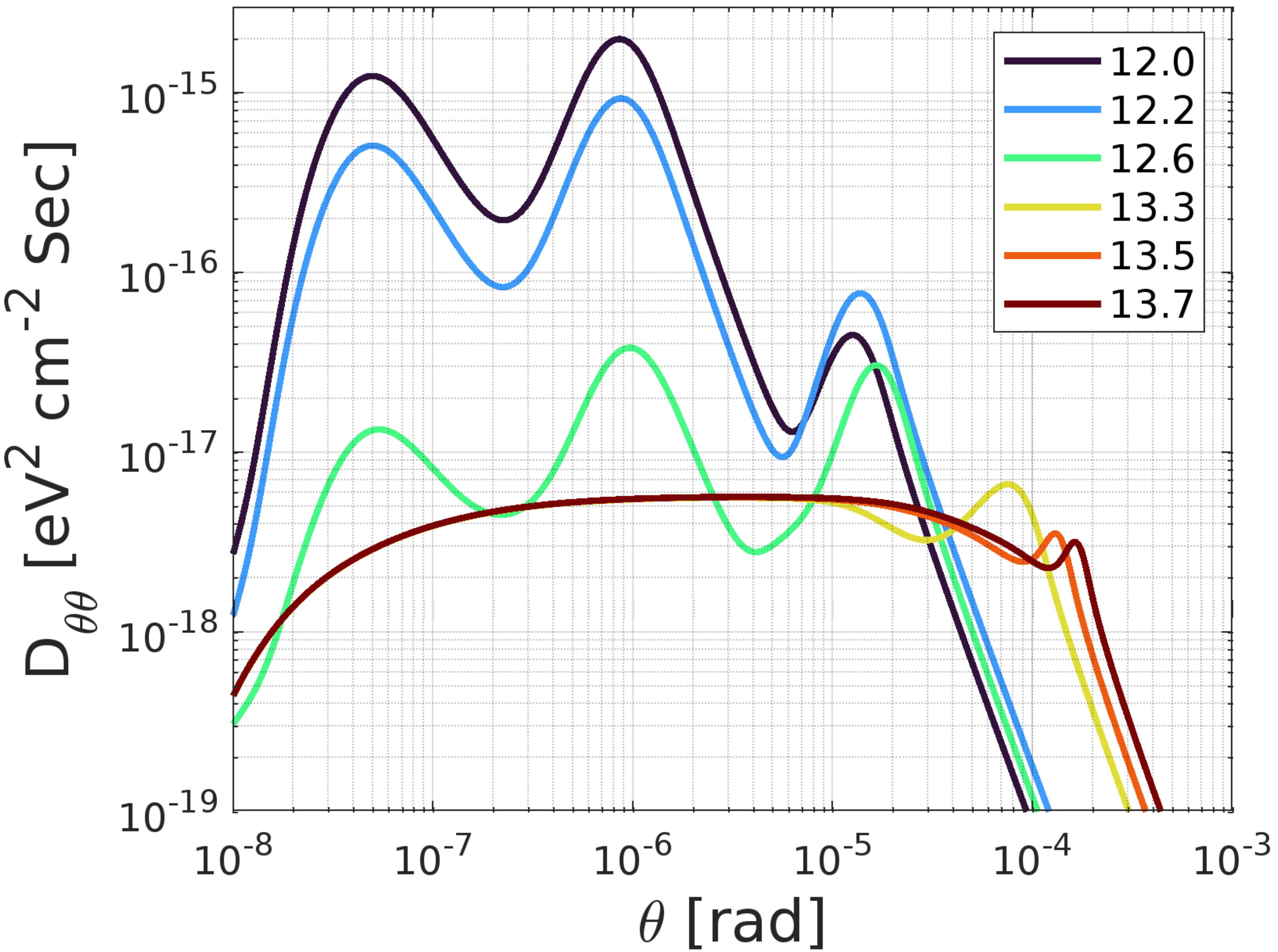
$$\Gamma_0 \equiv \omega_p \gamma_b \frac{n_b}{n_t} \frac{m_e v_b^2}{k_B T_b}.$$





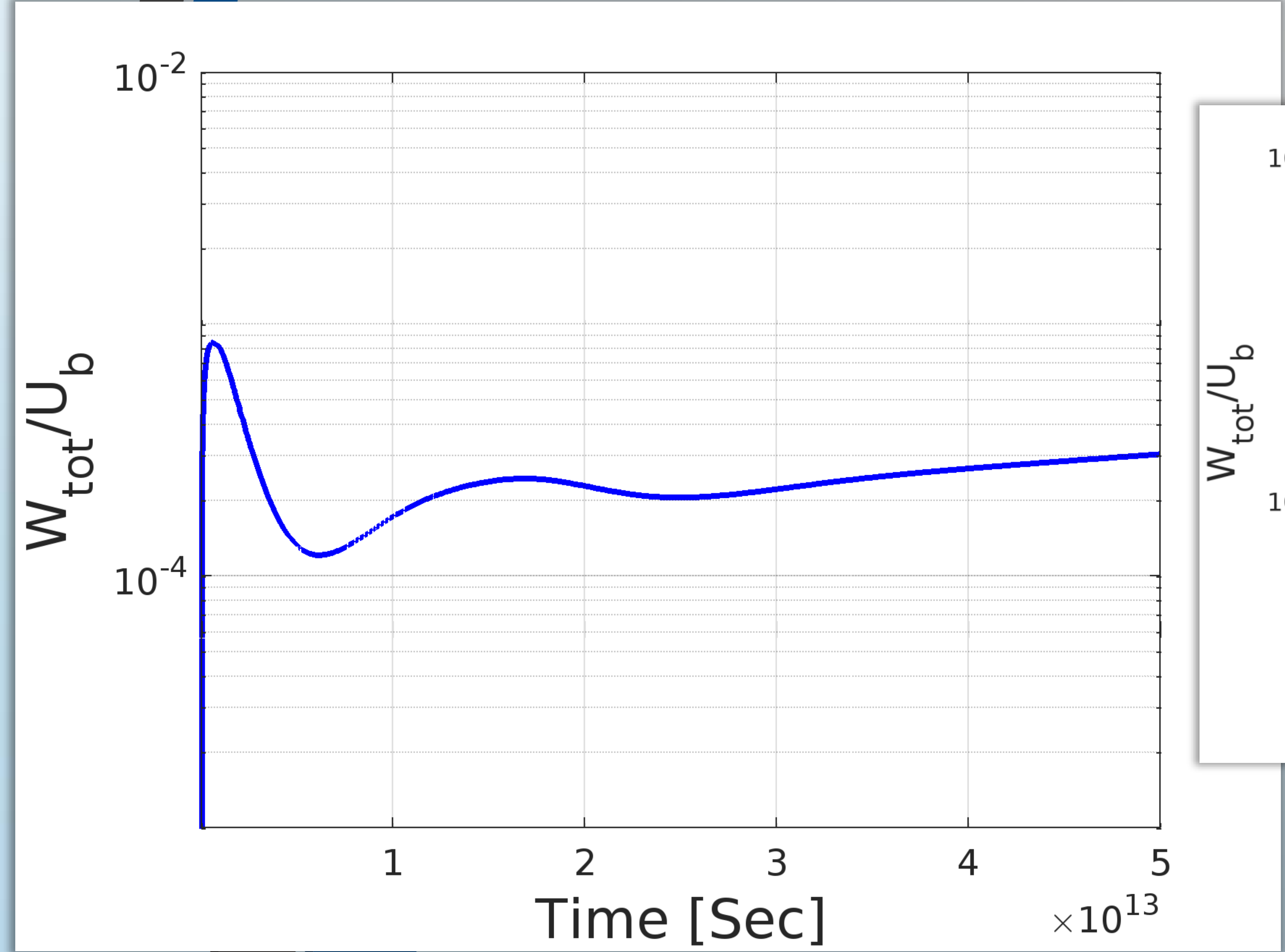
Chang et al.
The Astrophysical Journal, 833:118 (12pp), 2016 December 10

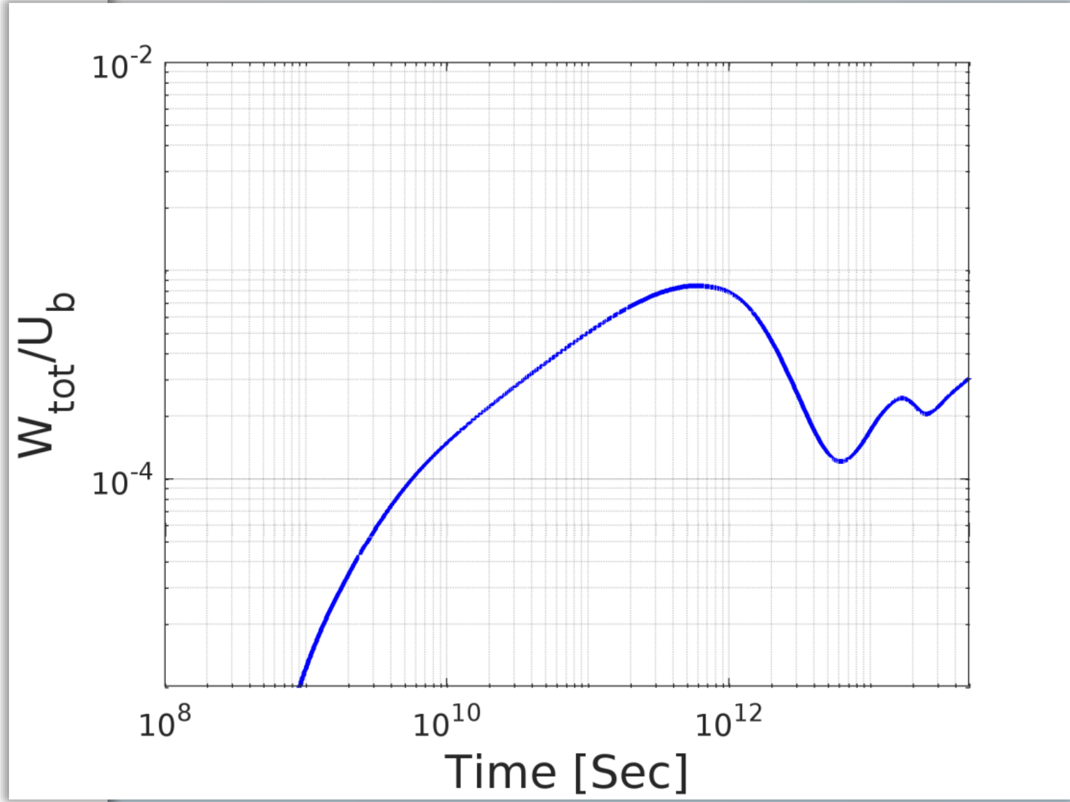
Injection simulation



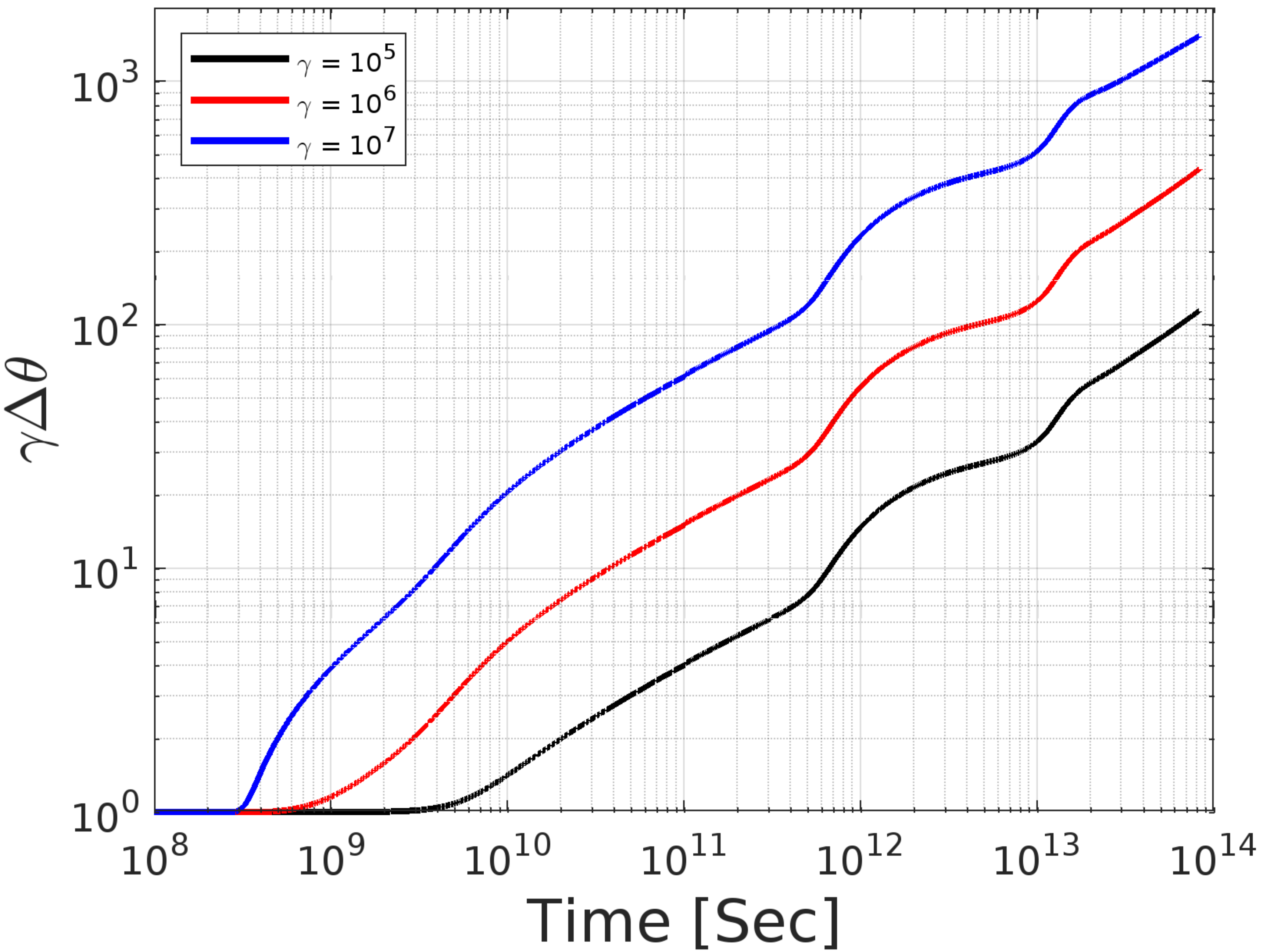
12.0
12.2
12.6
13.3
13.5
13.7
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
$D_{\theta\theta}$ [eV^2 cm^{-2} Sec]
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
θ [rad]

With Injection

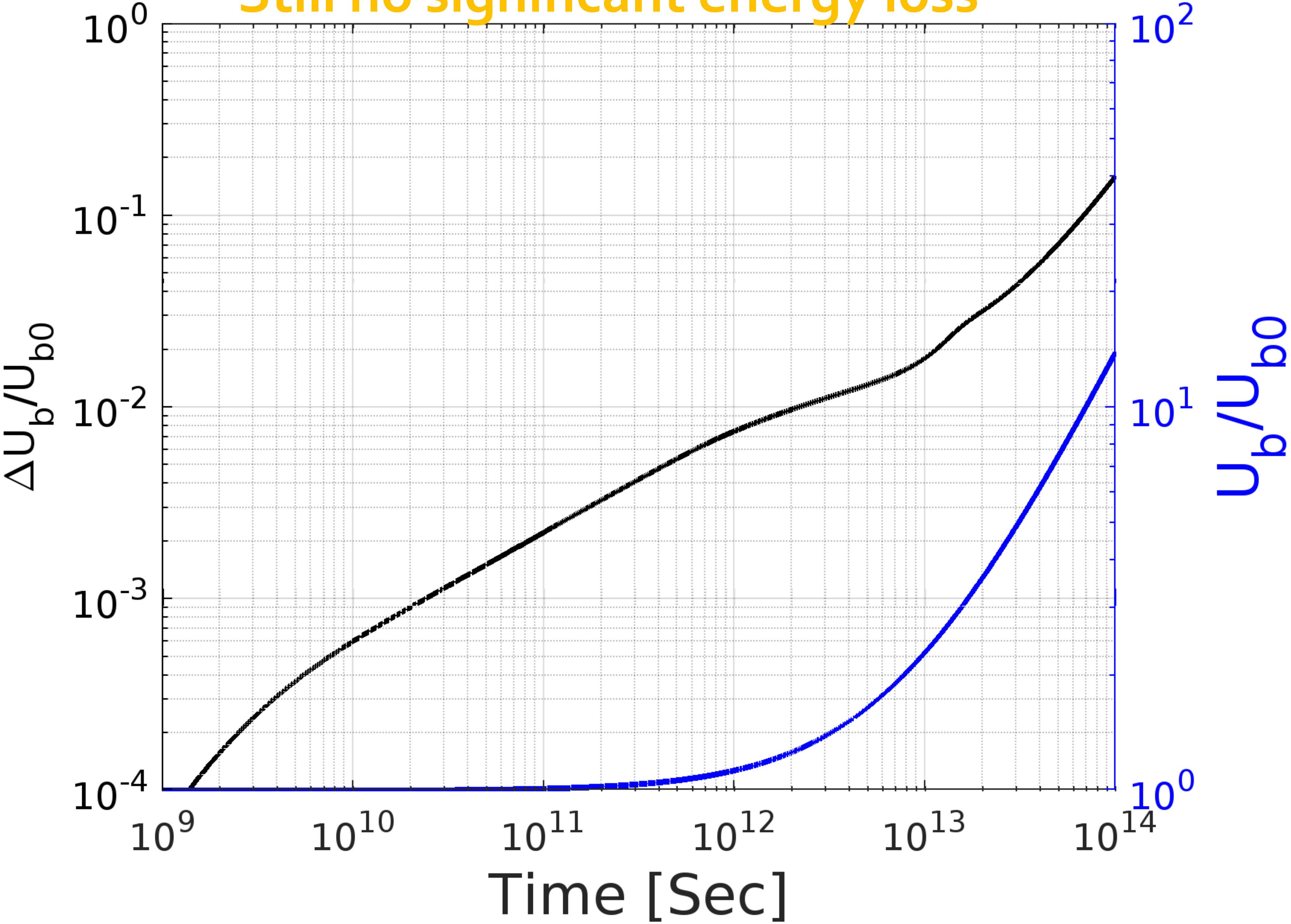




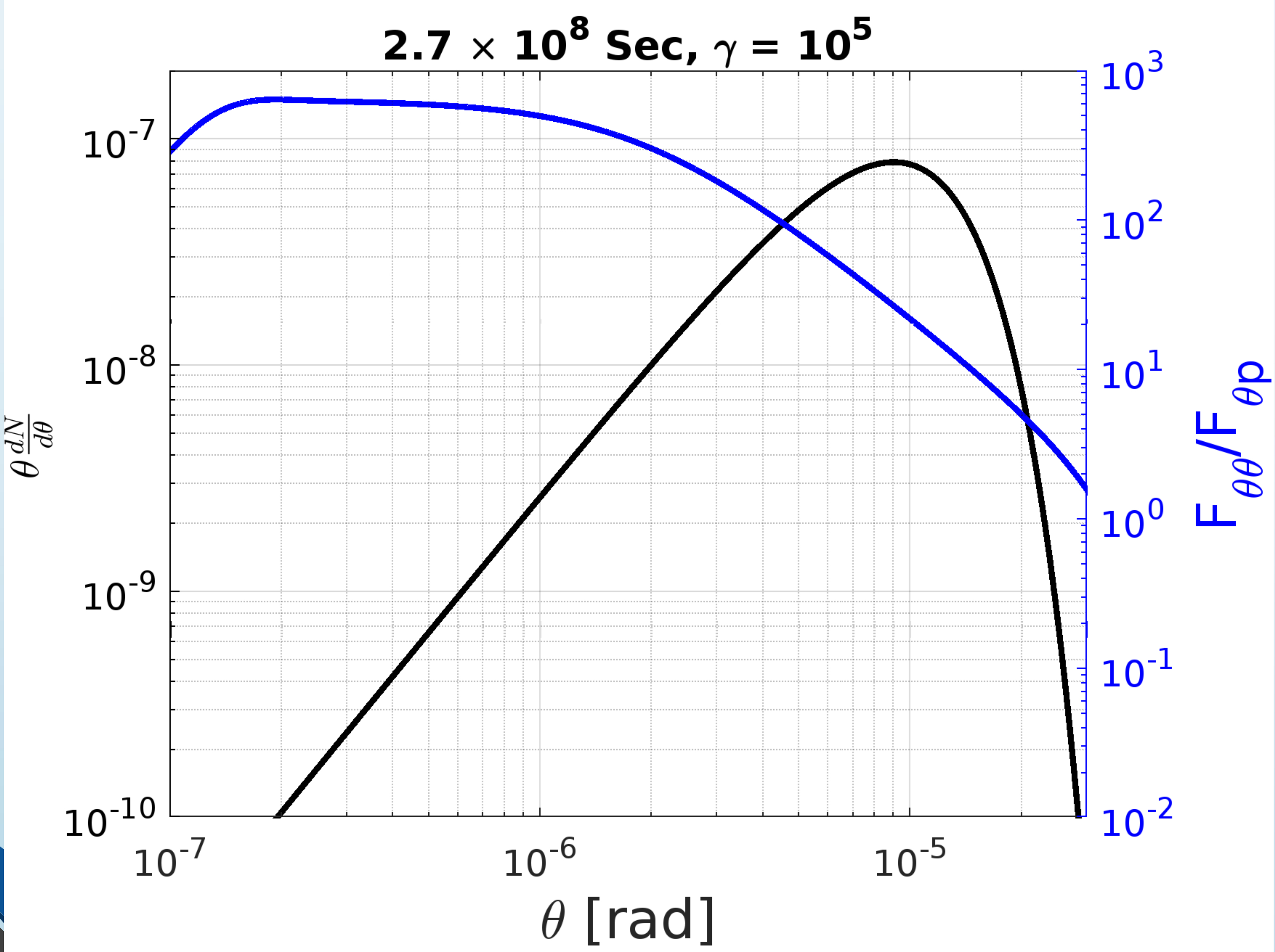
The beam keep widens



Still no significant energy loss

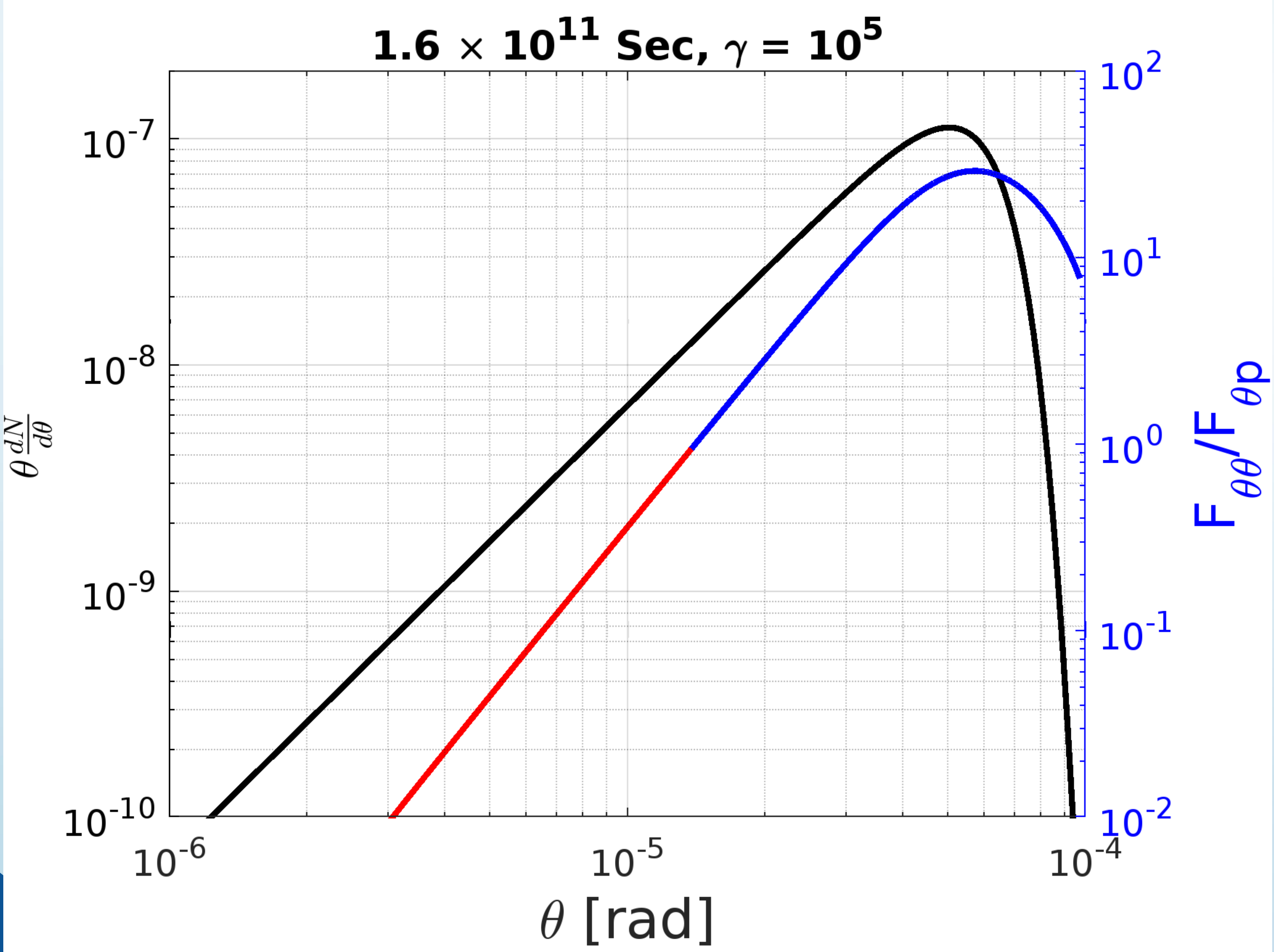


2D analysis of diffusion equation

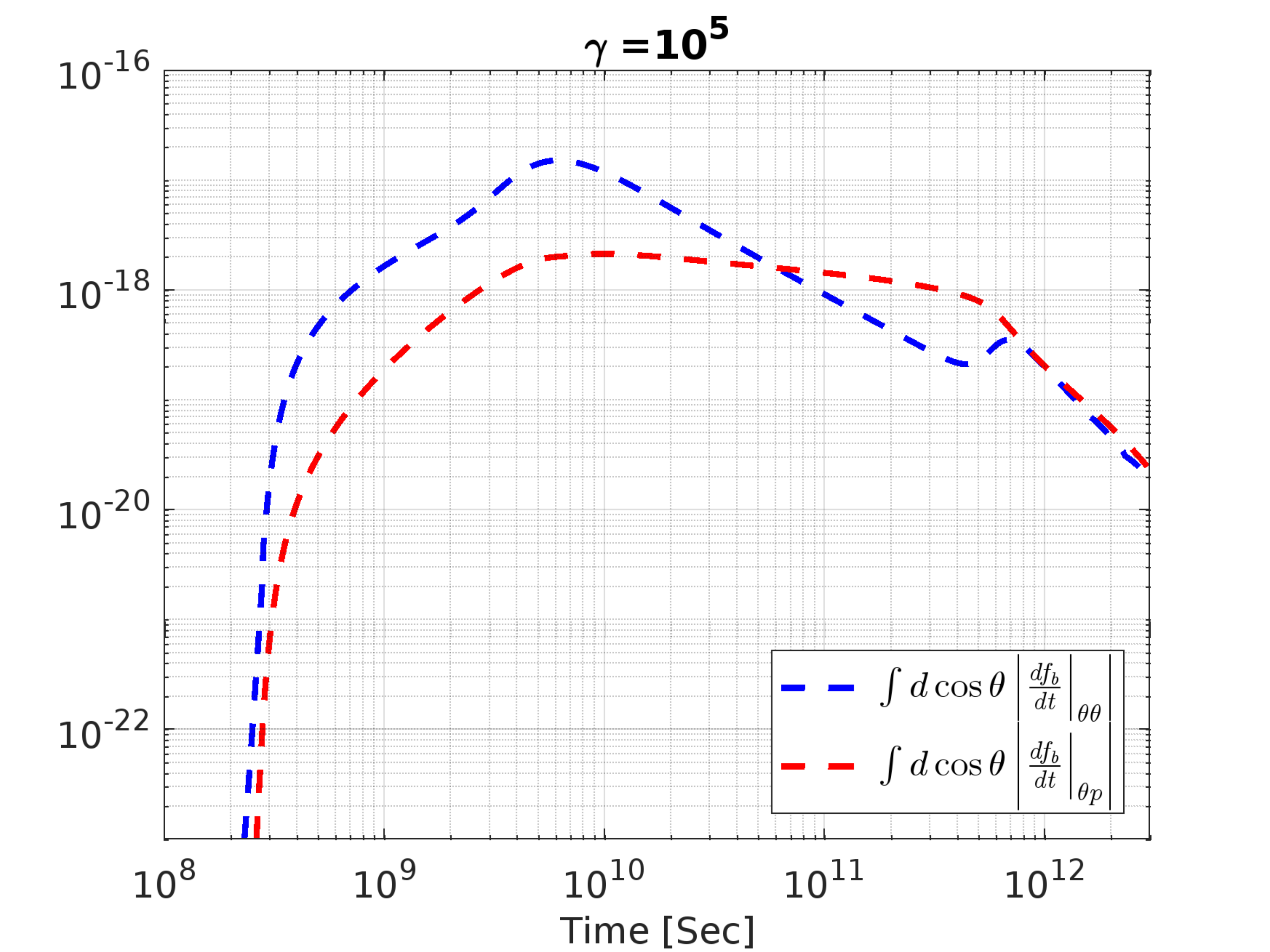


2.7 × 10^8 Sec, γ = 10^5
θ dN/dθ
θ [rad]
F_θθ/F_θp
10^-7
10^-8
10^-9
10^-10
10^-6
10^-5
10^3
10^2
10^1
10^0
10^-1
10^-2

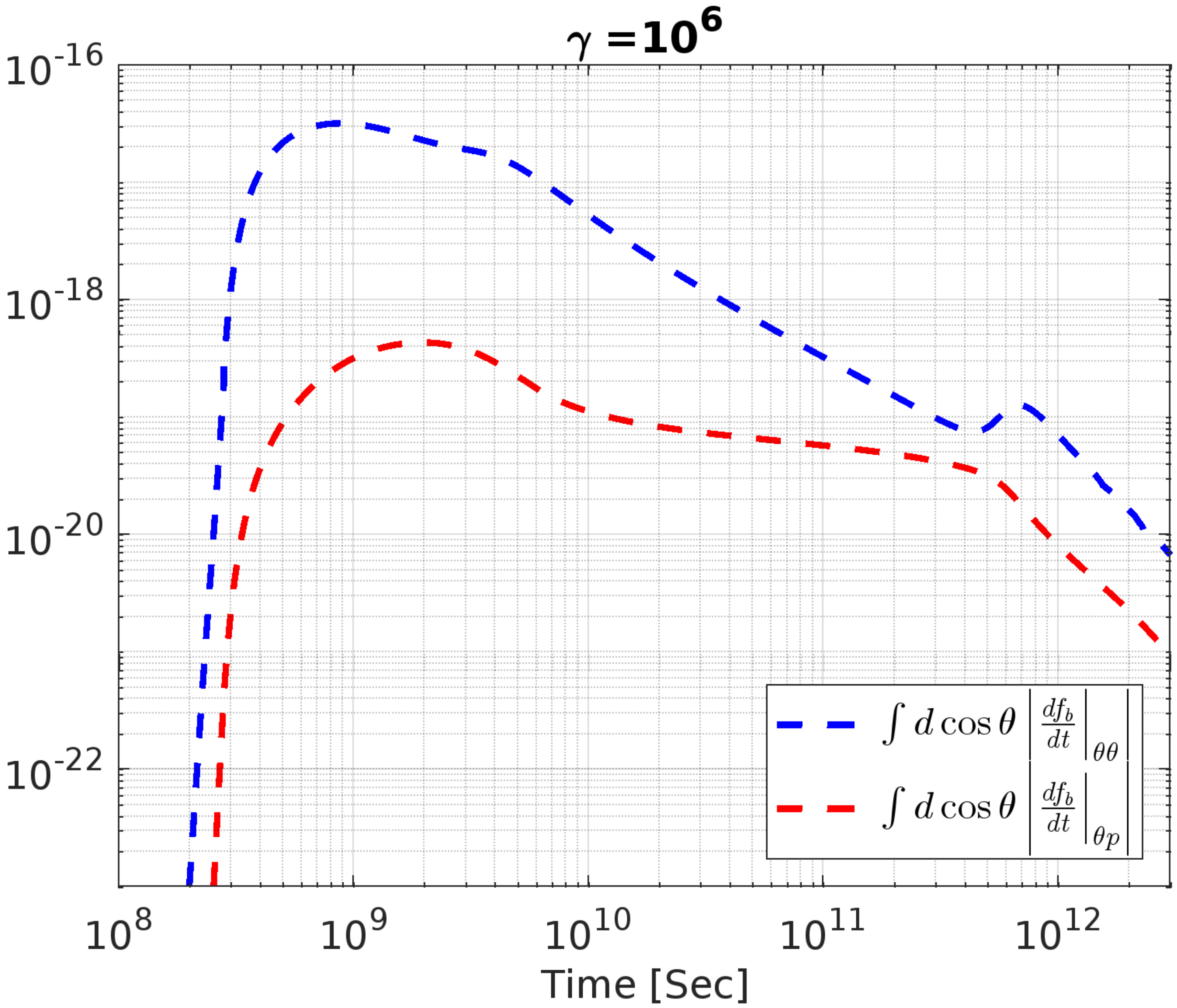
Impact of widening at small angles



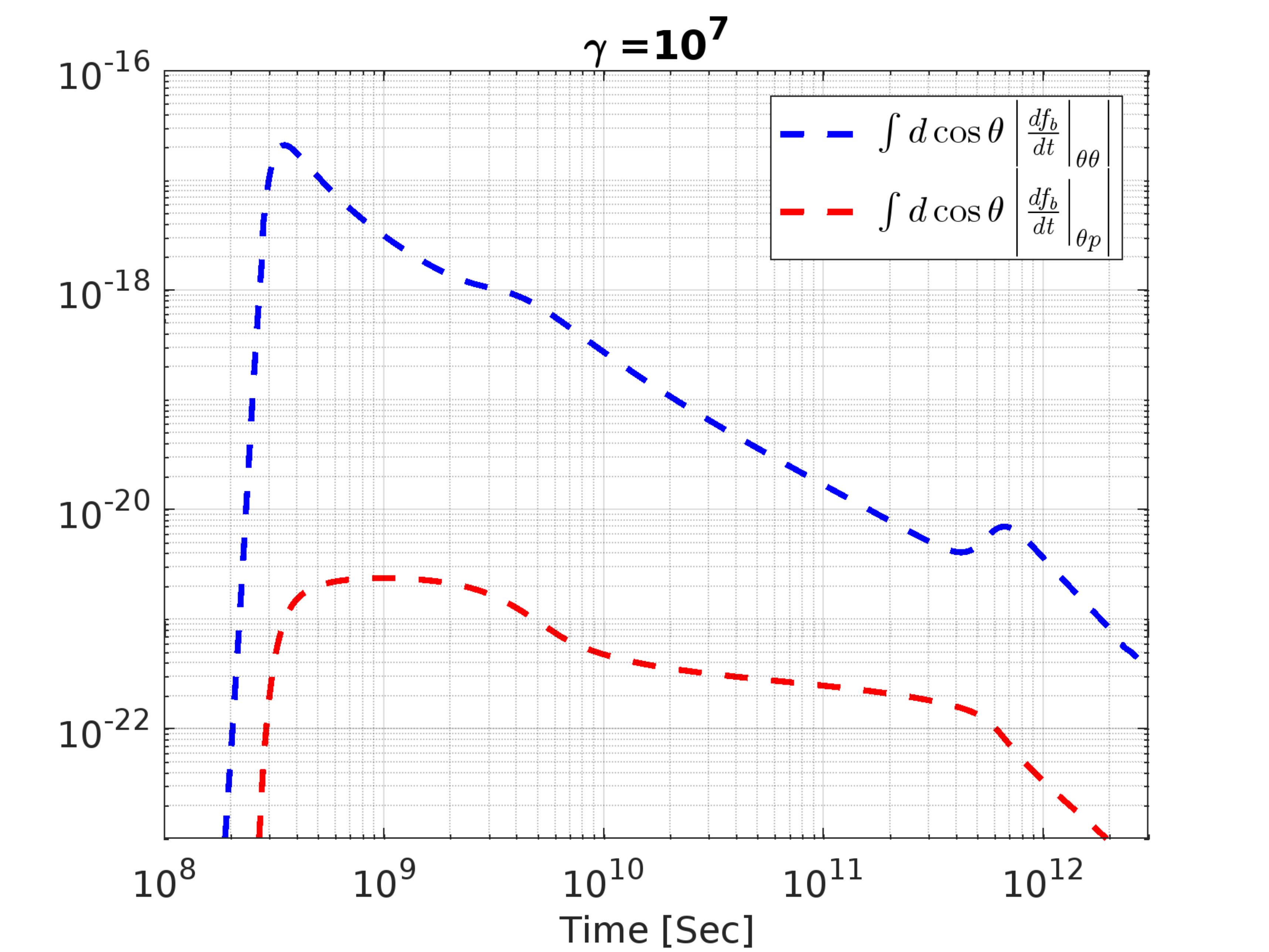
The other term is relevant here



Not the same case for higher Lorentz factors

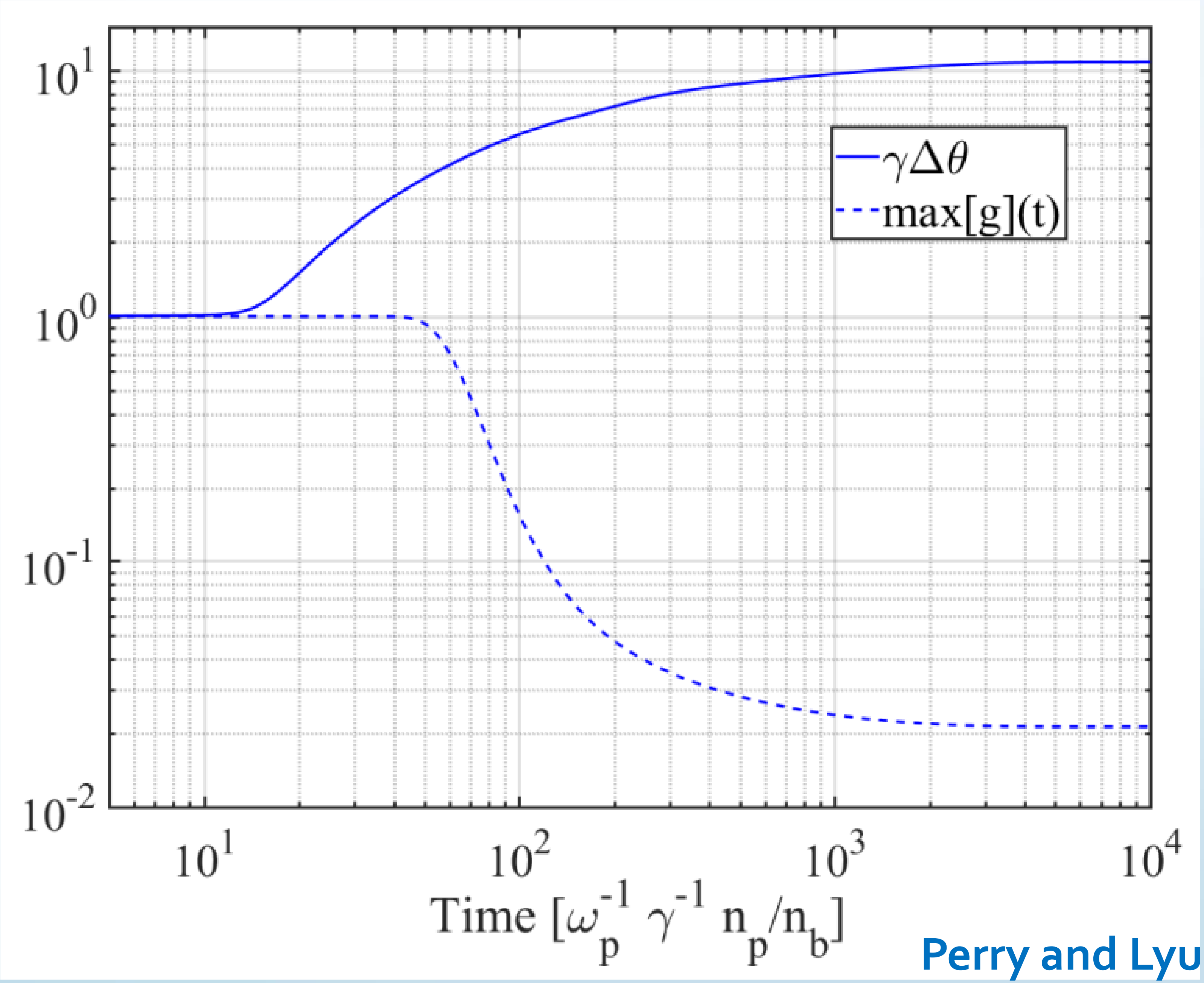


Angular widening dominate for larger Lorentz factors



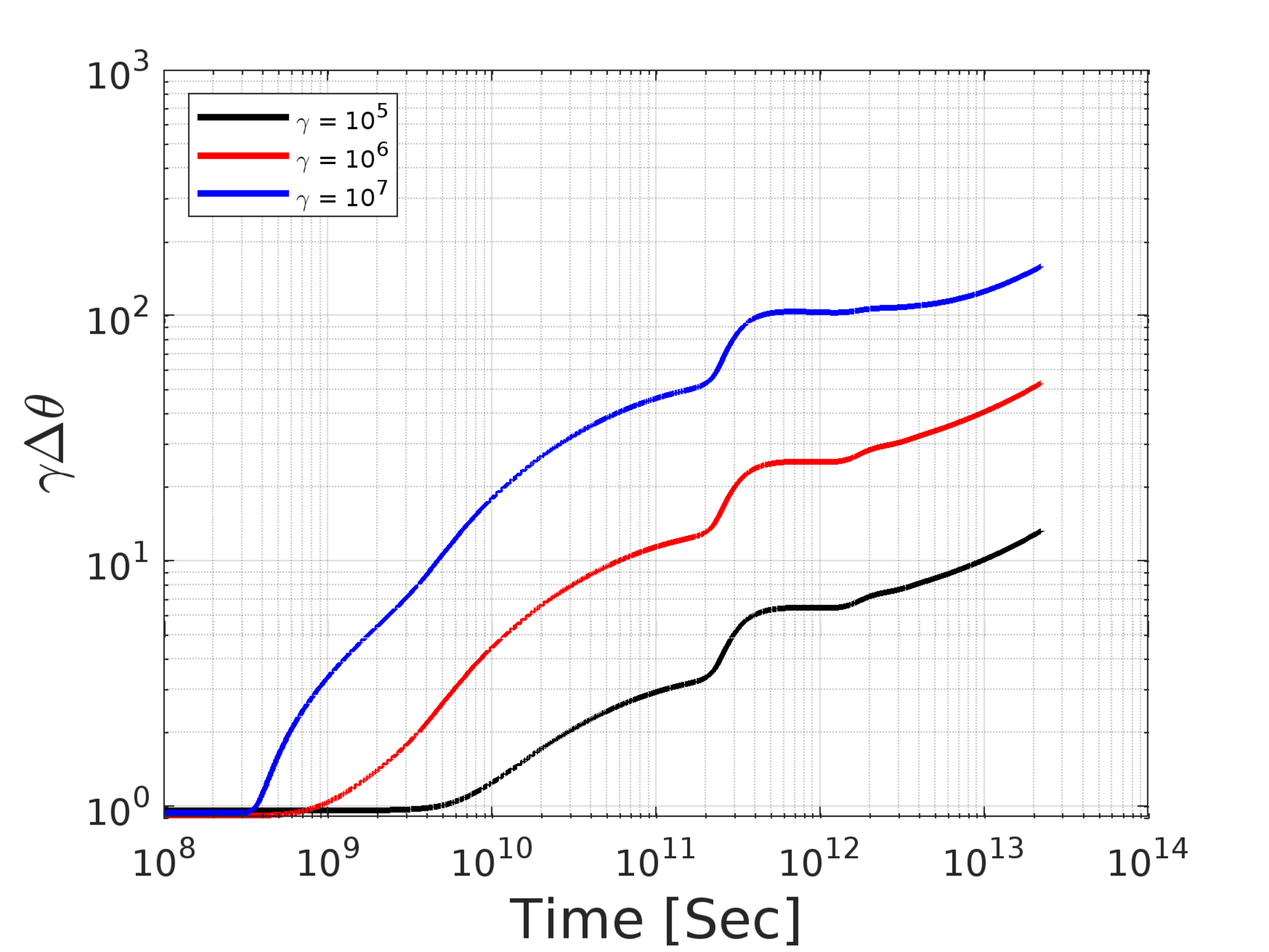
Perry and Lyubarsky (2021)

Significant beam broadening yields instability suppression



Perry and Lyubarsky (2021)

Old Collisional damping with Injection



Plasma waves energy density evolution

$$\frac{W_{\mathrm{tot}}}{n_e T_e} > 3(k\lambda_D)^2.$$