Form factors and dispersive bounds: the low recoil region

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As in: A. Bharucha, T. Feldmann and M. Wick, “Theoretical and Phenomenological Constraints on Form Factors for Radiative and Semi-Leptonic B-Meson Decays,” JHEP **1009** (2010) 090 [arXiv:1004.3249 [hep-ph]]

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Rare b-Decays @ Low Recoil: DESY, Hamburg, June 17th, 2011

Introduction and Motivation

- Exclusive decays (we focus on $B \to K^{(*)}$, $B \to \rho$ and $B_2 \to \phi$) require form factors: non-perturbative quantities
- LCSR[1] at low q^2, Lattice[2] at high q^2 (for difficult channels, e.g. unstable final state, results still in progress, see talk by M. Wingate)
- Independent attempt to fit LCSR and Lattice using series expansion, coefficients satisfy dispersive bounds (new bound for tensor current)
- Investigate possibility to extrapolate LCSR to high q^2 where Lattice unavailable

[1] see e.g. P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232] and Phys. Rev. D **71** (2005) 014029 [arXiv:hep-ph/0412079]

[2] see e.g. A. Al-Haydari *et al.* [QCDSF Collaboration], Eur. Phys. J. A **43**, 107 (2010) [arXiv:0903.1664 [hep-lat]]

Outline

- Definition of helicity amplitudes
- Series expansion/ Simplified series expansion
- Brief derivation of Dispersive bounds
- Results of fits for $B \to K^{(*)}$

Definition of Helicity Amplitudes

B→P Form Factors:

Conventionally define $\mathbf{f_0(q^2)}$, $\mathbf{f_+(q^2)}$ and $\mathbf{f_T(q^2)}$ via

$$\langle P(k)|\bar{q}\gamma_\mu b|B(p)\rangle = \left(p_\mu + k_\mu - q_\mu \frac{m_B^2 - m_P^2}{q^2}\right) \mathbf{f_+(q^2)} + \frac{m_B^2 - m_P^2}{q^2} q_\mu \, \mathbf{f_0(q^2)} \,,$$

$$\langle P(k)|\bar{q}\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \frac{i}{m_B + m_P} \left(q^2(p+k)_\mu - (m_B^2 - m_P^2)\, q_\mu\right) \mathbf{f_T(q^2)} \,.$$

Note $q^2 = (p-k)^2$ and $f_+(0) = f_0(0)$

Definition of Helicity Amplitudes

Define HAs in order to simplify:

- implementation of dispersive bounds (will discuss later)
- choice of below-threshold resonance as have definite spin-parity
- relation to heavy quark/large energy limit
- expressions for observables

B→P Helicity Amplitudes:

$$\mathcal{A}_{V,\sigma}(q^2) = \sqrt{\frac{q^2}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle P(k)| \bar{q}\, \gamma_\mu\, b|\bar{B}(p)\rangle\,,$$

$$\mathcal{A}_{T,\sigma}(q^2) = (-i)\sqrt{\frac{1}{\lambda}}\, \varepsilon_\sigma^{*\mu}(q)\, \langle P(k)| \bar{q}\, \sigma_{\mu\nu} q^\nu\, b|\bar{B}(p)\rangle\,,$$

where $\varepsilon_\sigma^{*\mu}(q)$ are transverse, longitudinal or time-like polarization vectors for $\sigma = \pm, 0, t$

$$\mathcal{A}_{V,0}(q^2) = f_+(q^2)\,, \qquad \mathcal{A}_{V,t}(q^2) = \frac{m_B^2 - m_P^2}{\sqrt{\lambda}}\, f_0(q^2)\,,$$

$$\mathcal{A}_{T,0}(q^2) = \frac{\sqrt{q^2}}{m_B + m_P}\, f_T(q^2)\,.$$

Below threshold resonances

- An important factor are any low-lying resonances present with appropriate quantum numbers and $t_- < m_{\mathrm{R}}^2 < t_+$, where $t_\pm = m_B \pm m_L$
- Often parameterisations (i.e. Ball/Zwicky, Becirevic/Kaidalov) include such resonances via simple pole $P(t) = 1 - t/m_R^2$
- Series expansion approach includes such resonances via Blaschke factor, chosen to be $B(t) = z(t, m_{\mathrm{R}}^2)$ (see following slide)

Series expansion type parameterisations

- $z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$ excellent expansion parameter for the FFs, and allows us to impose dispersive bounds
- t_0 can be optimised to reduce $|z(t)|_{\max}$, $t_0\big|_{\rm opt.} = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$
- **Series Expansion (SE)**[3] corresponds to:

$$f(t) = \frac{1}{B(t)\,\phi_f(t)} \sum_k \alpha_k\, z^k(t)$$

- Alternative version is **Simplified series expansion (SSE)**[4]:

$$f(t) = \frac{1}{P(t)} \sum_k \tilde{\alpha}_k\, z^k(t, t_0)\,. \qquad (1)$$

[3] C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. **74**, 4603 (1995) [arXiv:hep-ph/9412324] and I. Caprini, L. Lellouch and M. Neubert, Nucl. Phys. B **530**, 153 (1998) [arXiv:hep-ph/9712417]

[4] C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D **79**, 013008 (2009) [arXiv:0807.2722 [hep-ph]]

What are dispersive bounds?

Consider the correlator: $\Pi^X_{\mu\nu}(q^2) = i \int d^4x\, e^{i\,q\cdot x}\, \langle 0|\, \mathrm{T}\, j^X_\mu(x)\, j^{\dagger\,X}_\nu(0)\,|0\rangle$

for currents e.g. $j^V_\mu = \bar{q}\gamma_\mu b\,, \qquad j^{V-A}_\mu = \bar{q}\gamma_\mu(1-\gamma^5)b$, etc.

Possible to calculate via Hadronic represenation or OPE

Use projectors $P^{\mu\nu}_L(q^2) = \frac{q^\mu q^\nu}{q^2}\,, \qquad P^{\mu\nu}_T(q^2) = \frac{1}{D-1}\left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)$ to rewrite $\Pi^X_{\mu\nu}(q^2)$ using Lorentz scalars

$$\Pi^X_I(q^2) \equiv P^{\mu\nu}_I(q^2)\,\Pi^X_{\mu\nu}(q^2)\,, \qquad (I = L, T).$$

which satisfy the n^{th} subtracted dispersion relation,

$$\chi^X_I(n) = \frac{1}{n!}\,\frac{d^n\Pi_X(q^2)}{dq^{2\,n}}\bigg|_{q^2=0} = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi^X_I(t)}{(t-q^2)^{n+1}}\bigg|_{q^2=0}\,,$$

Hadronic Representation

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma\,(2\pi)^4\,\delta^4(q-p_\Gamma)\,P_I^{\mu\nu}\ \langle 0|\,j_\mu^X\,|\Gamma\rangle\,\langle\Gamma|\,j_\nu^{\dagger X}\,|0\rangle$$

Note crossing symmetry relates **decay region** $0 < q^2 < t_-$ and **pair-production region** $q^2 > t_+$

$$\mathrm{Im}\,\Pi_{I,BL}^X(q^2) = \eta \int d\rho_{BL}\,P_I^{\mu\nu}\ \langle 0|\,j_\mu^X\,|BL\rangle\,\langle BL|\,j_\nu^{\dagger X}\,|0\rangle \le \mathrm{Im}\Pi_I^X(t)$$

$$\begin{aligned}
P_T^{\mu\nu}\,\langle P|j_\mu^V|B\rangle\langle B|j_\nu^{\dagger V}|P\rangle &= \frac{\lambda}{3q^2}\,|\mathcal{A}_{V,0}|^2\ , \\
P_L^{\mu\nu}\,\langle P|j_\mu^V|B\rangle\langle B|j_\nu^{\dagger V}|P\rangle &= \frac{\lambda}{q^2}\,|\mathcal{A}_{V,t}|^2\ , \\
P_T^{\mu\nu}\,\langle P|j_\mu^T|B\rangle\langle B|j_\nu^{\dagger T}|P\rangle &= \frac{\lambda}{3}\,|\mathcal{A}_{T,0}|^2\ ,
\end{aligned}$$

Operator product expansion

On the other hand, same correlator can be calculated via OPE:

$$\Pi^{X}_{I,\mathrm{OPE}}(q^2) = \sum_{k=1}^{\infty} C^{X}_{I,k}(q^2)\,\langle O_k \rangle$$

- $C^{X}_{I,n}(q)$ are Wilson coefficients for a given current X and projector I
- $\mathcal{O}_n$ are local gauge-invariant operators, ordered by increasing dimension k
- Consider **identity operator** and non-perturbative operators: **quark condensate** $\langle m_q\,\bar{q}q\rangle$, the **gluon condensate** $\langle \frac{\alpha_s}{\pi}\,G^2\rangle$, and the **mixed condensate** $\langle g_s\,\bar{q}\,(\sigma\cdot G)\,q\rangle$
- Eventually require coefficients describing $\chi^{X}_{I}(n)$ instead of $\Pi^{X}_{I,\mathrm{OPE}}(q^2)$ to impose bounds

Imposing the bounds

HAs diagonalise unitarity relations:

$$\frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi^X_{I,BL}(t)}{(t-q^2)^{n+1}}\bigg|_{q^2=0} = \frac{1}{\pi}\int_{t_+}^\infty dt\,\frac{\eta\,\lambda^{3/2}(t)}{48\pi\,t^{n+3}}\left|A^X_I(t)\right|^2 \leq \chi^{X,n}_{I,\mathrm{OPE}}\,,$$

$$\frac{1}{2\pi i}\oint\frac{dz}{z}\,|\phi^X_I\,A^X_I|^2(z)\leq 1 \qquad\qquad \Rightarrow \text{Define universal } \phi$$

HA parameterised via

$$A^X_I(t) = \frac{(\sqrt{-z(t,0)})^m(\sqrt{z(t,t_-)})^l}{B(t)\,\phi^X_I(t)}\sum_{k=0}^\infty \alpha_k\,z^k$$

so coefficients satisfy..... $\sum_{k=0}^\infty \alpha_k^2 < 1$

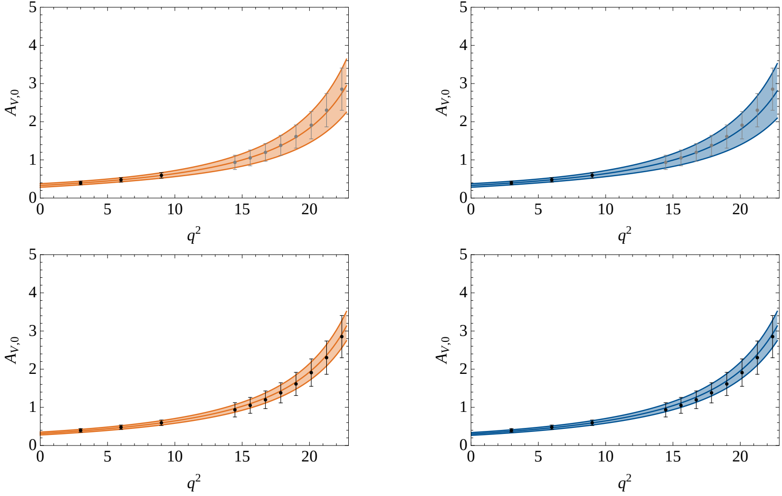
Summary of OPE results for the coefficients $\chi_I^X(n)$[5]

q	Correlator	Subtractions	LO	NLO	$\langle\bar{q}q\rangle$	$\langle\frac{\alpha}{\pi}G^2\rangle$	$\langle\bar{q}Gq\rangle$	Σ
	$100\times m_b^2\chi^S$	2	1.233	0.571	0.024	0.001	-0.003	1.83
	$100\times m_b^2\chi^P$	2	1.296	0.608	0.022	0.001	-0.003	1.93
	$100\times\chi_L^V$	1	1.172	0.229	0.023	0.000	-0.003	1.42
s	$100\times\chi_L^A$	1	1.361	0.187	0.023	0.002	-0.003	1.57
	$100\times m_b^2\chi_T^V$	2	0.980	0.237	-0.022	0.000	0.005	1.20
	$100\times m_b^2\chi_T^A$	2	0.916	0.238	-0.024	-0.002	0.006	1.13
	$100\times m_b^2\chi_T^T$	3	2.652	0.569	-0.023	0.001	0.006	3.21
	$100\times m_b^2\chi_T^{AT}$	3	2.404	0.603	-0.024	-0.002	0.007	2.99

$\mu = m_b = 4.2$ GeV, $m_d = 4.8$ MeV, $m_s = 104$ MeV, $\alpha_s = 0.2185$, $\langle\bar{d}d\rangle = (278\ \mathrm{MeV})^3$,
$\langle\bar{s}s\rangle = 0.8\langle\bar{d}d\rangle$, $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.038\ \mathrm{GeV}^4$, $\langle\bar{q}Gq\rangle = (1.4\ \mathrm{GeV})^2\langle\bar{q}q\rangle$

[5] Results consistent with C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. **74**, 4603 (1995) [arXiv:hep-ph/9412324], M. Jamin and M. Munz, Z. Phys. C **60**, 569 (1993) [arXiv:hep-ph/9208201]

Results: $\mathcal{A}_{V,0}$ for $B \to K$



Fit of SE (left) and SSE (right) to LCSR (top) and to LCSR and Lattice (bottom).
LCSR and Lattice data shown by black points.

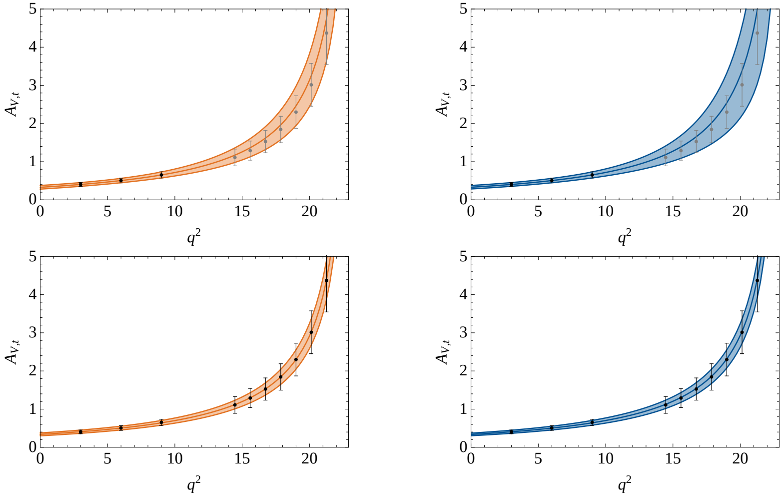
Dependence on below-threshold resonances

Transition	J^P	Mass (GeV)	J^P	Mass (GeV)	Ref.
$b \to d$	0^-	5.28	1^-	5.33	PDG
	0^+	5.63	1^+	5.68	BEH
	1^+	5.72	2^+	5.75	PDG
$b \to s$	0^-	5.37	1^-	5.42	PDG
	0^+	5.72	1^+	5.77	BEH
	1^+	5.83	2^+	5.84	PDG

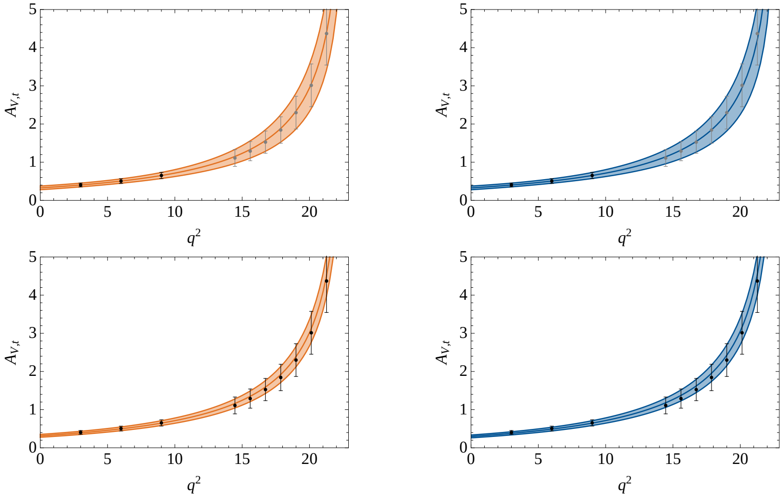
Masses of resonances from PDG and/or theoretical estimates from heavy-quark/chiral symmetry by Bardeen, Eichten and Hill, 2003 (BEH)[6]

[6] Masses for $(0^+, 1^+)$ not yet confirmed experimentally. PDG quotes "effective" resonances $B_J^*(5698)$ and $B_{sJ}^*(5853)$ with undetermined spin/parity.

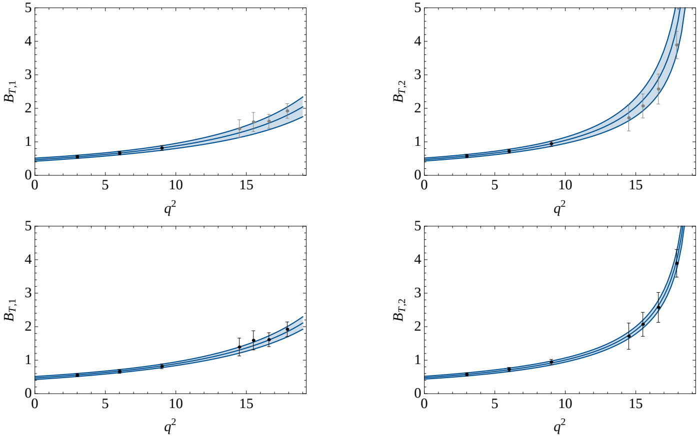
Results: $\mathcal{A}_{V,t}$ for $B \to K$



Fit of SE (left) and SSE (right) to LCSR (top) and to LCSR and Lattice (bottom)



Without using the scalar B_s resonance in the fit ansatz.



Fit of SSE(left) to LCSR(top) and to LCSR and Lattice[7](bottom)

[7] Z. Liu, S. Meinel, A. Hart, R. R. Horgan, E. H. Muller and M. Wingate, arXiv:1101.2726 [hep-ph]

Summary and Outlook

Summary

- 2 coefficient expansion provides a good description of form factors over entire range in q^2
- Define convenient helicity amplitudes
- Calculate dispersive bounds for the tensor current at NLO
- Good agreement with/without lattice: reliable fits

Outlook

- Include latest Lattice results for $B \to K^*$ etc.
- Re-normalization of LCSR at $q^2 = 0$ using $B \to K^*\gamma$ experimental results

Fitting procedure[8]

- $\chi^2(\vec{\theta}) = \Big(F_i - F(t_i, \vec{\theta})\Big) \left[V^{-1}\right]_{ij} \Big(F_j - F(t_j, \vec{\theta})\Big)$,
- **For SE:** $\vec{\theta} = \{\alpha_0, \alpha_1\}$, satisfying $\sum \alpha_i^2 \overset{!}{<} 1$
- **For SSE:** $\vec{\theta} = \{\tilde{\alpha}_0, \tilde{\alpha}_1\}$, satisfying $\sum_{i,j=0}^{1} C_{ij}\, \tilde{\alpha}_i \tilde{\alpha}_j \overset{!}{<} 1$
- Combine LCSR and Lattice in Block Diagonal form:
  $\chi^2 = \chi^2_{\mathrm{LCSR}} + \chi^2_{\mathrm{Lat}}$

[8] We closely follow the procedure outlined in C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D **79**, 013008 (2009)[arXiv:0807.2722 [hep-ph]]

Fit to SE for $B \to K$

A_X	m_R	α_0	α_1	Fit to	$\chi^2_{\rm fit}$	$X\sum_i \alpha_i^2$
$\mathcal{A}_{V,0}$	5.41	-2.4×10^{-2}	6.2×10^{-2}	LCSR and Lattice	5.07×10^{-3}	4.43×10^{-3}
$\mathcal{A}_{V,t}$	-	-6.8×10^{-2}	0.20	LCSR and Lattice	0.200	0.129
$\mathcal{A}_{V,t}$	5.72	-4.8×10^{-2}	0.11	LCSR and Lattice	1.54×10^{-4}	4.34×10^{-2}
$\mathcal{A}_{V,0}$	5.41	-2.8×10^{-2}	6.0×10^{-2}	LCSR	3.94×10^{-3}	4.40×10^{-3}
$\mathcal{A}_{V,t}$	-	-6.7×10^{-2}	0.18	LCSR	1.44×10^{-3}	0.111
$\mathcal{A}_{V,t}$	5.72	-2.5×10^{-2}	7.2×10^{-2}	LCSR	0.329	5.77×10^{-3}
$\mathcal{A}_{T,0}$	5.41	-4.5×10^{-2}	8.9×10^{-2}	LCSR	0.234	2.99×10^{-2}

$B \to K$: Fit of SE parameterisation to LCSR or LCSR/Lattice results, for $\mathcal{A}_{V,0}$ $(X=1)$, $\mathcal{A}_{V,t}$ $(X=3)$ and $\mathcal{A}_{T,0}$ $(X=1)$

Fit to SSE for $B \to K$

A_X	m_R	$\tilde{\alpha}_0$	$\tilde{\alpha}_1$	Fit to	χ^2_{fit}	$X \sum_{i,j} C_{i,j}\tilde{\alpha}_i\tilde{\alpha}_j$
$\mathcal{A}_{V,0}$	5.41	0.48	-1.0	LCSR and Lattice	5.15×10^{-3}	4.04×10^{-3}
$\mathcal{A}_{V,t}$	-	0.54	-1.7	LCSR and Lattice	0.904	0.142
$\mathcal{A}_{V,t}$	5.72	0.30	0.20	LCSR and Lattice	7.17×10^{-5}	5.32×10^{-2}
$\mathcal{A}_{V,0}$	5.41	0.48	-1.1	LCSR	8.15×10^{-3}	3.06×10^{-3}
$\mathcal{A}_{V,t}$	-	0.52	-1.4	LCSR	2.27×10^{-3}	9.55×10^{-2}
$\mathcal{A}_{V,t}$	5.72	0.50	-1.4	LCSR	0.940	6.51×10^{-3}
$\mathcal{A}_{T,0}$	5.41	0.28	0.35	LCSR	0.128	3.15×10^{-2}

$B \to K$: Fit of SSE parameterisation to LCSR or LCSR/Lattice results, for $\mathcal{A}_{V,0}$ $(X = 1)$, $\mathcal{A}_{V,t}$ $(X = 3)$ and $\mathcal{A}_{T,0}$ $(X = 1)$

Covariance Matrices: SE

The fit of $B \to K$ FFs to LCSR data alone gives the covariances matrices:

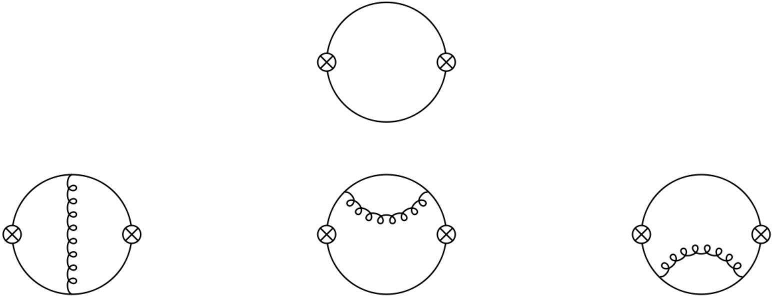
	SE	SSE
$\mathcal{A}_{V,0}$	$\begin{pmatrix} 1.56 \times 10^{-5} & -1.04 \times 10^{-4} \\ -1.04 \times 10^{-4} & 9.59 \times 10^{-4} \end{pmatrix}$	$\begin{pmatrix} 4.39 \times 10^{-3} & -2.91 \times 10^{-2} \\ -2.91 \times 10^{-2} & 0.266 \end{pmatrix}$
$\mathcal{A}_{V,t}^{\text{no res.}}$	$\begin{pmatrix} 1.19 \times 10^{-4} & -7.87 \times 10^{-4} \\ -7.87 \times 10^{-4} & 6.98 \times 10^{-3} \end{pmatrix}$	$\begin{pmatrix} 7.17 \times 10^{-3} & -4.75 \times 10^{-2} \\ -4.75 \times 10^{-2} & 0.423 \end{pmatrix}$
$\mathcal{A}_{V,t}$	$\begin{pmatrix} 6.27 \times 10^{-6} & -2.72 \times 10^{-5} \\ -2.72 \times 10^{-5} & 2.19 \times 10^{-4} \end{pmatrix}$	$\begin{pmatrix} 2.61 \times 10^{-3} & -1.08 \times 10^{-2} \\ -1.08 \times 10^{-2} & 8.86 \times 10^{-2} \end{pmatrix}$
$\mathcal{A}_{T,0}$	$\begin{pmatrix} 2.1 \times 10^{-5} & -6.55 \times 10^{-5} \\ -6.55 \times 10^{-5} & 5.37 \times 10^{-4} \end{pmatrix}$	$\begin{pmatrix} 7.63 \times 10^{-4} & 6.3 \times 10^{-4} \\ 6.3 \times 10^{-4} & 8.32 \times 10^{-3} \end{pmatrix}$

Covariance Matrices: SSE

For the fit of scalar/vector $B \to K$ FFs to LCSR and Lattice data, we obtain the covariance matrices:

	SE	SSE
$\mathcal{A}_{V,0}$	$\begin{pmatrix} 1.48 \times 10^{-5} & -9.81 \times 10^{-5} \\ -9.81 \times 10^{-5} & 8.76 \times 10^{-4} \end{pmatrix}$	$\begin{pmatrix} 6.26 \times 10^{-3} & -4.15 \times 10^{-2} \\ -4.15 \times 10^{-2} & 0.382 \end{pmatrix}$
$\mathcal{A}_{V,t}^{\text{no res.}}$	$\begin{pmatrix} 4.82 \times 10^{-5} & -2.03 \times 10^{-4} \\ -2.03 \times 10^{-4} & 1.6 \times 10^{-3} \end{pmatrix}$	$\begin{pmatrix} 3.08 \times 10^{-3} & -1.39 \times 10^{-2} \\ -1.39 \times 10^{-2} & 0.11 \end{pmatrix}$
$\mathcal{A}_{V,t}$	$\begin{pmatrix} 6.21 \times 10^{-5} & -4.11 \times 10^{-4} \\ -4.11 \times 10^{-4} & 3.75 \times 10^{-3} \end{pmatrix}$	$\begin{pmatrix} 3.45 \times 10^{-3} & -2.37 \times 10^{-2} \\ -2.37 \times 10^{-2} & 0.261 \end{pmatrix}$

Calculation of Wilson coefficients



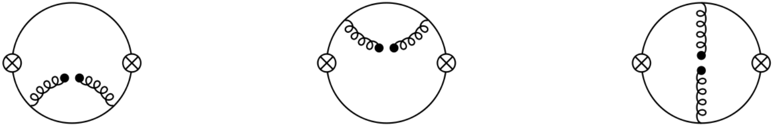
- LO and NLO diagrams contributing to correlation function (crossed circle indicates insertion of appropriate current)
- Expansion about $q^2 \to 0$ simplifies calculation
- Use tensor reduction and recurrence relations to express loop integrals in terms of master integrals

Calculation of Wilson coefficients

- Diagrams involving quark condensate, indicated by two solid dots
- Calculation follows Jamin and Münz[9]

[9] M. Jamin and M. Munz, Z. Phys. C **60**, 569 (1993) [arXiv:hep-ph/9208201]

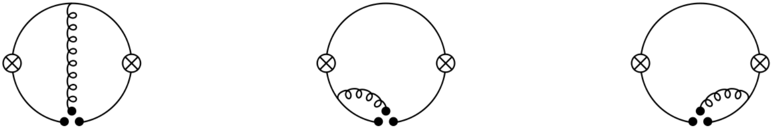
Calculation of Wilson coefficients



- Diagrams involving gluon condensate, indicated by two solid dots
- Calculation uses fixed-point gauge technique[9]

[9] see L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127**, 1 (1985)

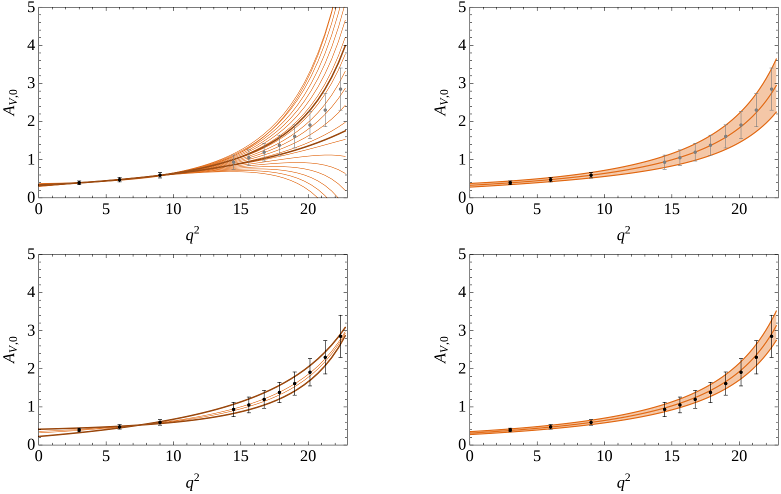
Calculation of Wilson coefficients



- Diagrams involving mixed condensate, indicated by three solid dots
- Calculation follows Jamin and Münz[9]

[9] M. Jamin and M. Munz, Z. Phys. C **60**, 569 (1993) [arXiv:hep-ph/9208201]

Results: $\mathcal{A}_{V,0}$ for $B \to K$, effect of a_2



Fit of SE to LCSR or LCSR/Lattice, for a_2 in range $[-0.9, +0.9]$ (thin lines). Thick, dark lines show $a_2 = \pm 0.25$. On right we show again results for truncated fit after a_1.