The $B \rightarrow K(K^*)$ transition form factors in a relativistic constituent quark model

J. G. Körner (Mainz)

in collaboration of M. A. Ivanov, V. E. Lyubovitskij, P. Santorelli et al.

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Contents

Relativistic Quark Model of Hadrons

- **Formulate the constituent quark model in terms of Feynman loop integrals**
- **UV divergencies tempered through nonlocal (Gaussian) vertex functions with sufficient fall-off behaviour in the Euclidean domain. Introduces one size parameter Λ_H for each hadron.**
- **Interaction Lagrangian**

$$\mathcal{L}_{\rm int} = g_H \cdot H(x) \cdot J_H(x)$$

- **Quark currents**

$$J_M(x) = \int dx_1 \int dx_2 \, F_M(x; x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1) \, \Gamma_M \, q^a_{f_2}(x_2) \qquad \text{Meson}$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 \, F_B(x; x_1, x_2, x_3) \qquad \text{Baryon}$$

$$\times \, \Gamma_1 \, q^{a_1}_{f_1}(x_1) \left(q^{a_2}_{f_2}(x_2) C \, \Gamma_2 \, q^{a_3}_{f_3}(x_3) \right) \cdot \varepsilon^{a_1 a_2 a_3}$$

Compositeness condition is used to determine the hadron-quark coupling constant g_H.

Compositeness condition

- **Compositeness condition $Z_H = 0$**

Salam 1962; Weinberg 1963

"A hadron is either a hadron or a composite state, but not both"

$$\boxed{Z_H^{1/2} = < H_{\rm bare} | H_{\rm dressed} > = 0}$$

Wave function renormalization given by

$$Z_H = 1 - \tilde{\Pi}'(m_H^2) = 0$$

where $\tilde{\Pi}(p^2)'$ is the derivative of the hadron mass operator $\tilde{\Pi}(p^2)$.

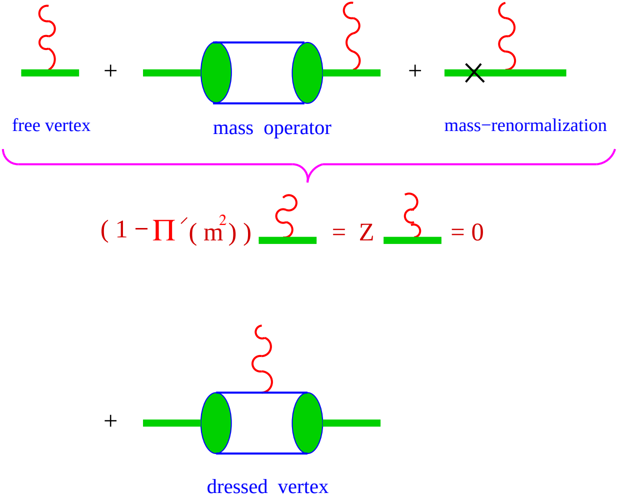
- For charged particles the compositeness provides for correct charge normalization, viz.

$$\frac{\partial}{\partial p^\mu} S(\ell + p) = S(\ell + p)\, \gamma_\mu S(\ell + p)$$

where the quark propagator $S(\ell + p)$ reads

$$S(\ell + p) = \frac{1}{m - \not\ell - \not p}$$

Z=0 : how it works



Mass operator contributions

- **Mesons** $\Pi_2(p^2)$ **two propagators; one loop**
- **Baryons** $\Pi_3(p^2)$ **three propagators; two loops**

The p^2–derivative adds one more propagator. Taking the derivative

$$\frac{\partial}{\partial p^2}\Pi_n(p^2 = \frac{1}{2p^2}\,p^\mu\,\frac{\partial}{\partial p^\mu}\Pi_n(p^2)$$

leads to

$$\frac{\partial}{\partial p^\mu}S(\ell+p) = S(\ell+p)\,\gamma_\mu S(\ell+p)$$

where the quark propagator $S(\ell+p)$ reads

$$S(\ell+p) = \frac{1}{m - \not\ell - \not p}$$

Need for efficient one- and two-loop integrations (loop-tensor integrals with different quark masses).

Need for confinement

- **Avoid the occurrence of free unconfined quarks/antiquarks in the loop integrals. For example, for $m_{u,d} = 0.235$ GeV; $m_s = 0.333$ GeV one has**

$$\mathbf{m_{u,d} + m_s < m_{K^*}}$$

There is a quark-antiquark branchpoint in the one-loop integral starting at $\mathbf{s = (m_u + m_s)^2}$.

Effective Implementation of Confinement

I. Schwinger vs. Feynman parametrization of loop integrals

Feynman :
$$\frac{1}{AB} = \int_0^1 d\alpha \frac{1}{\Big(\alpha A + (1-\alpha)B\Big)^2}$$

Schwinger :
$$\frac{1}{A} = \int_0^\infty d\alpha e^{-\alpha A} \quad A > 0$$
$$\frac{1}{AB} = \int_0^\infty \int_0^\infty d\alpha_1 d\alpha_2 e^{-\alpha_1 A - \alpha_2 B}$$

Making use of the identity $1 = \int_0^\infty dt\, \delta(t - \alpha_1 - \alpha_2)$ **one finds**

$$\frac{1}{AB} = \int_0^1 \int_0^1 d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2) \int_0^\infty dt\, t\, e^{-t(\alpha_1 A + \alpha_2 B)}$$

Finally, do the t–integration to recover Feynman parametrization

$$\frac{1}{AB} = \int_0^1 d\alpha \frac{1}{\Big(\alpha A + (1-\alpha)B\Big)^2}$$

There are two advantages of using Schwinger parameters in our method:

- **tensor loop integrals are easily done by converting loop momenta to derivatives, viz. (ℓ^μ is loop momentum)**

$$\frac{m + \not\ell + \not p}{m^2 - (\ell + p)^2} = (m + \not\ell + \not p) \int_0^\infty d\alpha e^{2\ell\alpha p} e^{\alpha(\ell^2 + p^2 - m^2)}$$

. One can then convert the loop momentum ℓ^μ to a derivative via $(\alpha p^\mu = r^\mu)$

$$\ell^\mu e^{2\ell r} = \frac{1}{2} \frac{\partial}{\partial r_\mu} e^{2\ell r} .$$

After loop momentum integration one finds $\frac{\partial}{\partial r_\mu} \to p_\mu$ where p_μ is an outer momentum. When there are higher loop momentum tensors $\ell^{\mu_1} \ell^{\mu_2} \dots$ the procedure is iterative and can be easily programmed analytically.

- **When changing the upper (infrared) limit on the t–integration**

$$\int_0^\infty dt \quad \to \quad \int_0^{1/\lambda^2} dt$$

one removes all quark thresholds, i.e. one has effective confinement. We choose a universal infrared parameter λ for all processes.

Examples of effective confinement

1. Single quark propagator (omit numerator)

$$\begin{aligned}\frac{1}{m^2 - p^2} &= \int_0^\infty d\alpha e^{-\alpha(m^2-p^2)} \\ &\rightarrow \int_0^{1/\lambda^2} d\alpha e^{-\alpha(m^2-p^2)} \\ &\qquad = \frac{1 - e^{-(m^2-p^2)/\lambda^2}}{m^2 - p^2}\end{aligned}$$

Pole has been removed!

One-loop two-point function

- Consider the case of a scalar one–loop two–point function:

$$\Pi_2(p^2) = \int \frac{d^4 k_E}{\pi^2} \frac{e^{-s\,k_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

  where the numerator factor $e^{-s\,k_E^2}$ comes from the product of nonlocal vertex form factors of Gaussian form. k_E, p_E are Euclidean momenta ($p_E^2 = -\,p^2$).

- Doing the loop integration one obtains

$$\Pi_2(p^2) = \int_0^\infty dt \frac{t}{(s+t)^2} \int_0^1 d\alpha \; \exp\left\{-t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2\right\}$$

  One has a branch point at $p^2 = 4m^2$

Infrared confinement

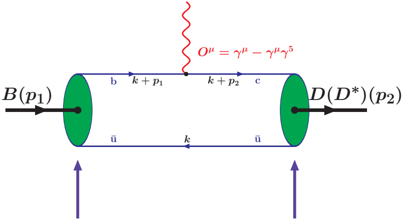
- By introducing a cut-off in the t–integration one obtains

$$\Pi_2^c(p^2) = \int\limits_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int\limits_0^1 d\alpha \, \exp\left\{-t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\left(\alpha - \frac{1}{2}\right)^2 p^2\right\}$$

where the one–loop two–point function $\Pi_2^c(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

The Heavy Quark Limit

Semileptonic $\mathbf{B - D(D^*)}$ transition



$$\phi_B\left(-\left(k + w_u\, p_1\right)^2\right) \qquad \phi_{D(D^*)}\left(-\left(k + w_u'\, p_2\right)^2\right)$$

$$w_u = \frac{m_u}{m_u + m_b} \qquad\qquad w_u' = \frac{m_u}{m_u + m_c}$$

Heavy quark limit:

Take $m_H = m_Q + \Lambda_Q$ **and** $m_Q \to \infty$

$$\frac{1}{m_i - \not{k} - \not{p}_i} \quad \to \quad -\frac{1 + \not{v}_i}{2} \cdot \frac{1}{k v_i + \Lambda_Q}, \qquad v_i = \frac{p_i}{m_i}$$

With the vertex factors $\binom{\gamma_5}{\gamma_\mu}$ **one recovers the HQET wave functions** $\binom{(1+\not{v})\gamma_5}{(1+\not{v})\not{\epsilon}}$ **which factor from the loop integral. The remaining loop integral with the above HQET propagators determines the Isgur-Wise function** $\xi(\omega)$**. The correct zero recoil normalization** $\xi(\omega = 1) = 1$ **follows from the compositeness condition** $Z_H = 0$**.**

Spin-Kinematical Considerations

I LS analysis of two-particle decays

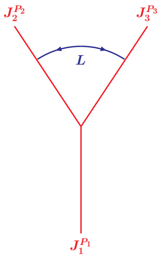
Consider the two-particle decay

$$H_1(J_1^{P_1}) \longrightarrow H_2(J_2^{P_2}) + H_3(J_3^{P_3})$$

It is convenient to introduce LS-amplitudes (LS), where L is the orbital momentum of the final state and S is the total spin of the final state $S = J_2 \otimes J_3$.

The advantages are

- **amplitude counting**
- **importance sampling $(LS) \sim |\vec{p}|^L$**
- **parity nature of transition**



Mesons $m_\ell = 0$

Apply the *LS* analysis to the decay $H_1(J_1^{P_1}) \to H_2(J_2^{P_2}) + W_{off-shell}(0^-, 1^-)$.

Remember that one has $|\vec{p}| = M_2\sqrt{(\omega+1)(\omega-1)}$.

<u>Case A</u> Mesons $m_\ell = 0$ $W_{off-shell}$ **has** $J^P = 1^-$

- $P \to P$ **transitions:** $0^- \to 0^- + 1^- \rightsquigarrow L = 1; S = 1$

$$LS\,\textbf{amplitude}: \quad (11) \sim (\omega - 1)^{1/2} \quad p.c.$$

- $P \to V$ **transitions:** $0^- \to 1^- + 1^- \rightsquigarrow L = 0, 1, 2;\ S = 0, 1, 2$

$$\begin{array}{lcll} LS\,\textbf{amplitudes}: (00) & \sim & (\omega-1)^0 & p.v. \\ (11) & \sim & (\omega-1)^{1/2} & p.c. \\ (22) & \sim & (\omega-1)^1 & p.v. \end{array}$$

$B \to D$ is suppressed @ low recoil

Mesons $m_\ell \neq 0$

Case B **Mesons** $m_\ell \neq 0$ $W_{off-shell}$ **has** $J^P = 0^+ + 1^-$**. There are additional amplitudes from** $W_{off-shell}(0^+)$**.**

- $P \to P$ **transitions:** $0^- \to 0^- + 0^+ \rightsquigarrow L = 0; S = 0$

$$LS\,\textbf{amplitude}: \quad (00) \sim (\omega - 1)^0 \quad p.c.$$

- $P \to V$ **transitions:** $0^- \to 1^- + 0^+ \rightsquigarrow L = 1; S = 1$

$$LS\,\textbf{amplitudes}: (11) \sim (\omega - 1)^{1/2} \qquad p.v.$$

$B \to D$ **no longer suppressed @ low recoil**

Baryons

Case C **Baryons** $1/2^+ \to 1/2^+ + W_{off-shell}(1^-)$ **with** $m_\ell = 0$.

- $B_b \to B_s$ **transitions:** $1/2^- \to 1/2^- + 1^- \rightsquigarrow L = 0, 1, 2; S = 1/2, 3/2$

$$\begin{array}{llll} LS\ \textbf{amplitudes}: (0\,\tfrac{1}{2}) & \sim & (\omega-1)^0 & p.v. \\ (1\,\tfrac{1}{2}) & \sim & (\omega-1)^{1/2} & p.c. \\ (1\,\tfrac{3}{2}) & \sim & (\omega-1)^{1/2} & p.c. \\ (2\,\tfrac{3}{2}) & \sim & (\omega-1)^1 & p.v. \end{array}$$

No suppression @ low recoil

Model parameters

- **Model parameters:**
  - **the constituent quark masses m_q,**
  - **the scale parameter λ characterizing the infrared confinement,**
  - **the size parameters Λ_H.**
- **They were determined by using a least square fit to leptonic weak/radiative constants, $\pi^0 \to 2\gamma$, *etc*.**
- **Fit values of the constituent quark masses and the confinement scale:**

$m_{u/d}$	m_s	m_c	m_b	λ	
0.235	0.333	1.67	5.06	0.181	GeV

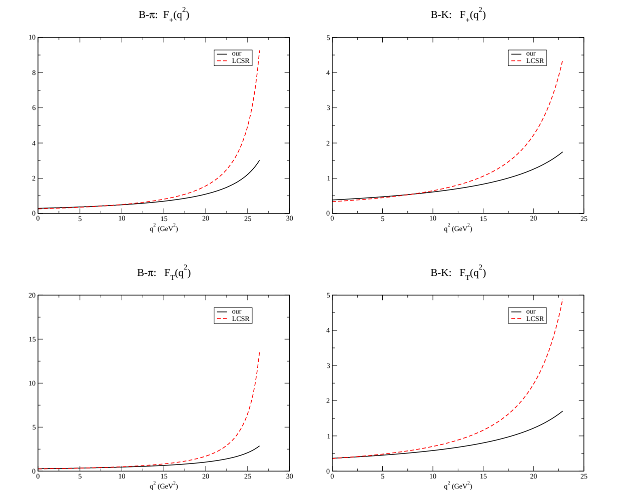
- **Values of some of the size parameters:**

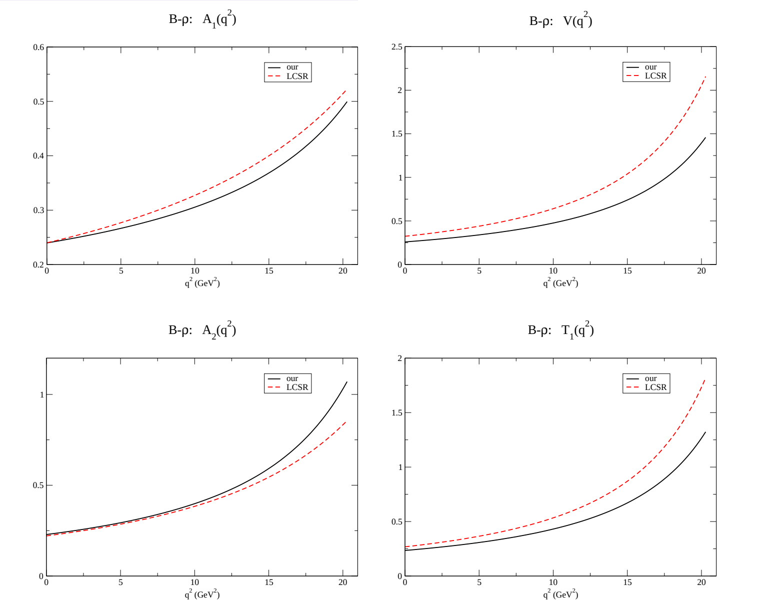
Λ_π	Λ_K	Λ_ρ	Λ_{K^*}	Λ_B	
0.90	1.15	0.61	0.52	2.2	GeV

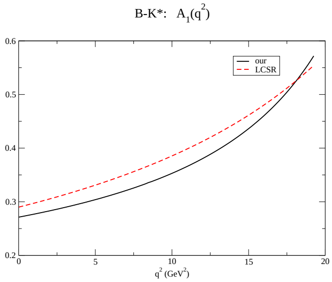
Form factors at $q^2 = 0$.

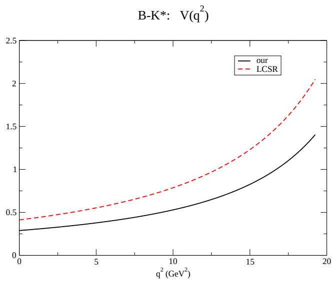
	Our	LCSR-1	LCSR-2
$F_+^{B\pi}(0)$	0.29	0.258±0.031	0.25±0.05
$F_+^{BK}(0)$	0.38	0.335±0.042	0.31±0.04
$F_T^{B\pi}(0)$	0.27	0.253±0.028	0.21±0.04
$F_T^{BK}(0)$	0.37	0.359±0.038	0.27±0.04
$V^{B\rho}(0)$	0.26	0.324±0.029	0.32±0.10
$V^{BK^*}(0)$	0.29	0.412±0.045	0.39±0.11
$A_1^{B\rho}(0)$	0.24	0.240±0.024	0.24±0.08
$A_1^{BK^*}(0)$	0.27	0.290±0.036	0.30±0.08
$A_2^{B\rho}(0)$	0.23	0.221±0.023	0.21±0.09
$A_2^{BK^*}(0)$	0.26	0.258±0.035	0.26±0.08
$T_1^{B\rho}(0)$	0.23	0.268±0.021	0.28±0.09
$T_1^{BK^*}(0)$	0.26	0.332±0.037	0.33±0.10

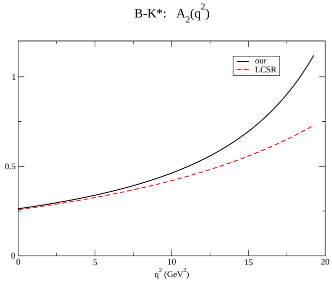
[1] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005); D 71, 014029 (2005)
[2] A. Khodjamirian, T. Mannel, N. Offen, Phys. Rev. D 75, 054013 (2007).

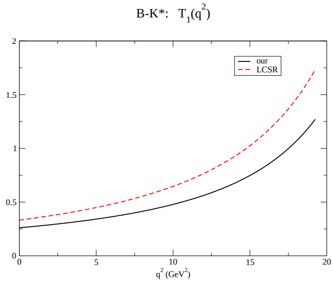

B-π: F+(q2)
our
LCSR
q2 (GeV2)
B-K: F+(q2)
our
LCSR
q2 (GeV2)
B-π: FT(q2)
our
LCSR
q2 (GeV2)
B-K: FT(q2)
our
LCSR
q2 (GeV2)


B-ρ: A1(q2)
B-ρ: V(q2)
B-ρ: A2(q2)
B-ρ: T1(q2)
our
LCSR
q2 (GeV2)


B-K*: A1(q2)
our
LCSR
q2 (GeV2)

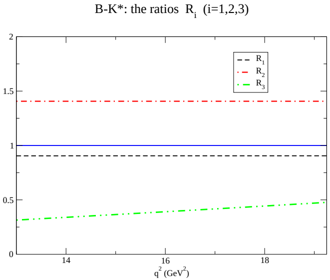

B-K*: V(q2)
our
LCSR
q2 (GeV2)

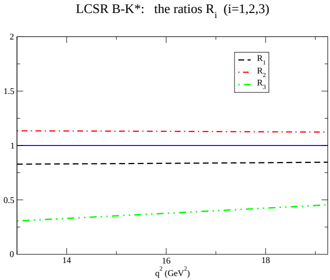

B-K*: A2(q2)
our
LCSR
q2 (GeV2)


B-K*: T1(q2)
our
LCSR
q2 (GeV2)

Ratios of the form factors

$$\mathbf{R_1 = \frac{T_1(q^2)}{V(q^2)}, \qquad R_2 = \frac{T_2(q^2)}{A_1(q^2)}, \qquad R_3 = \frac{q^2}{m_B^2}\frac{T_3(q^2)}{A_2(q^2)}.}$$





Concluding Remarks

- **The relativistic quark model can easily be applied to other rare decays such as**

$$(b \to s) - \textbf{transitions} : \qquad \begin{array}{lcl} B_s & \to & \phi + \ell^+\ell^- \\ B_c & \to & D_s(D_s^*) + \ell^+\ell^- \\ \Lambda_b & \to & \Lambda + \ell^+\ell^- \end{array}$$

$$(b \to d) - \textbf{transitions} : \qquad \begin{array}{lcl} B_c & \to & D(D^*) + \ell^+\ell^- \\ \Lambda_b & \to & n + \ell^+\ell^- \end{array}$$

- **The computer codes were double-checked by independent implementations of M. A. Ivanov and P. Santorelli. Analytical calculations were done in FORM and Mathematica. Numerical calculations were done in Fortran within the NAG library. Every q^2–point in the form factors took only a tiny fraction of a second.**