Rare exclusive B decays at low recoil

Dan Pirjol (IFIN)

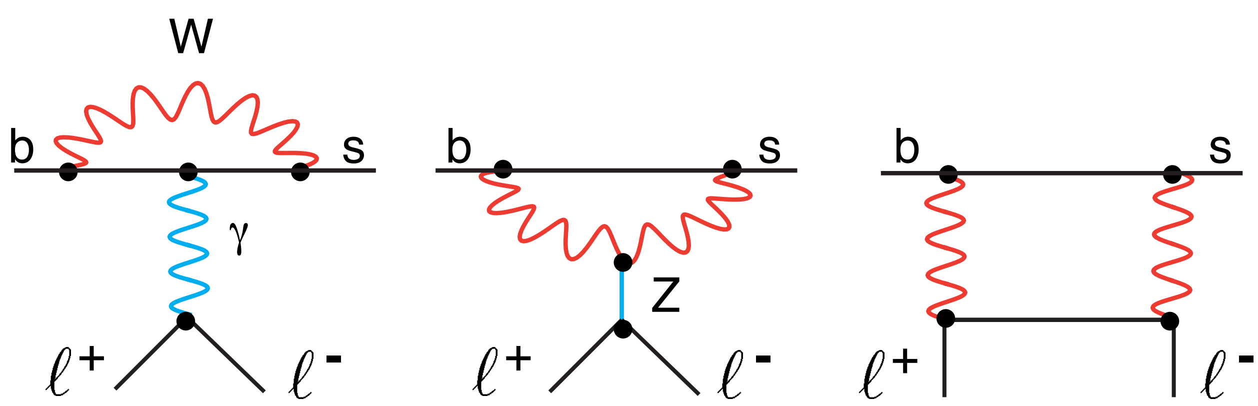
work in collaboration with Ben Grinstein

Workshop on B physics at low recoil DESY, 15-17 June 2011

Outline

- Motivation
- The theory of $b \to s\ell^+\ell^-$ at low recoil
  - Long-distance contributions - light quark loops
  - OPE for $b \to s\ell^+\ell^-$ at low recoil
- Improved form factor relations from heavy quark symmetry
- Determination of V(ub) from exclusive rare and semileptonic B decays
- Discussion

Rare radiative B decays



Mediated by photon/Z penguin and box diagrams

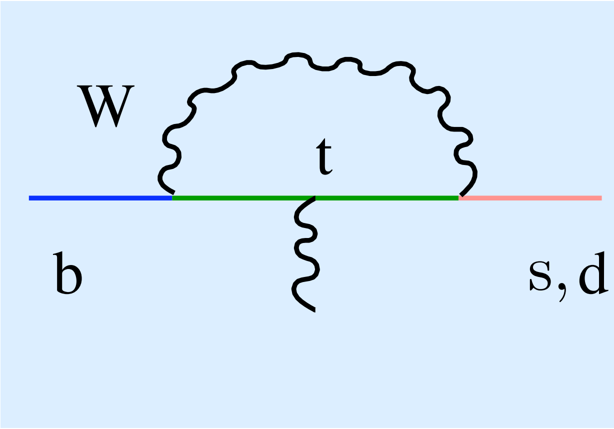
- Sensitive to CKM matrix elements
- Pure loop effect: sensitive to new particles running in the loop.
- Excellent probes of New Physics

V(ub) determination

Combine exclusive rare radiative and semileptonic decays

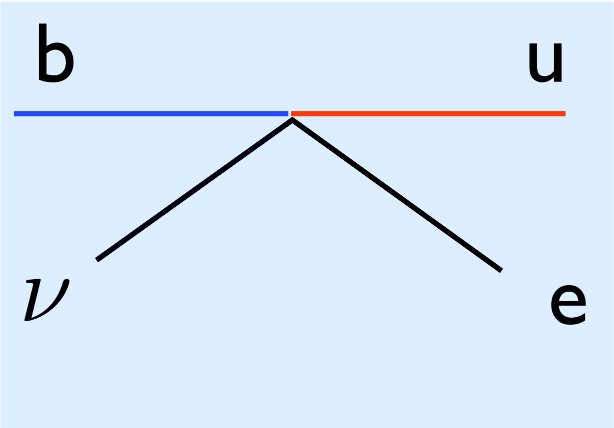
$B \to K^* \ell^- \ell^+$

$B \to \rho \ell^+ \ell^-$



vs.

$B \to \rho \ell^- \bar{\nu}_\ell$



CKM factors: $V_{tb} V_{ts}^*$

V_{ub}

Different form factors... but related in the heavy quark limit

Isgur, Wise, 1991

Tests for New Physics

- Many observables: decay rates, spectra, polarization, forward-backward asymmetries
- The goal is to determine the Wilson coefficients (and the complete operator basis) with minimal hadronic uncertainties
- Test the operator structure of the Standard model, and/or set constraints on the NP contributions

Kruger, Matias, hep-ph/0502060

Bobeth, Hiller, Piranishvili, arXiv:0709.4174, 0805.2525

Egede, Hurth, Matias, Ramon, Reece, arXiv:1005.0571, 0807.2589

Altmannshofer et al, arXiv:0811.1214

Bharucha, Reece, arXiv:1002.4310

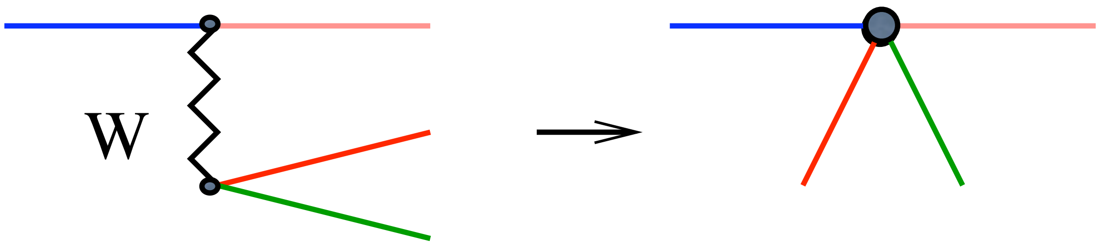
Bobeth, Hiller, van Dyk, arXiv:1006.5013, 11.05.0376

Descotes Genon et al, arXiv:1104.3342

Effective Hamiltonian for $b \to s\ell^+\ell^-$

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{QCD} \times \mathcal{H}_{QED} - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

Integrate out the top and W

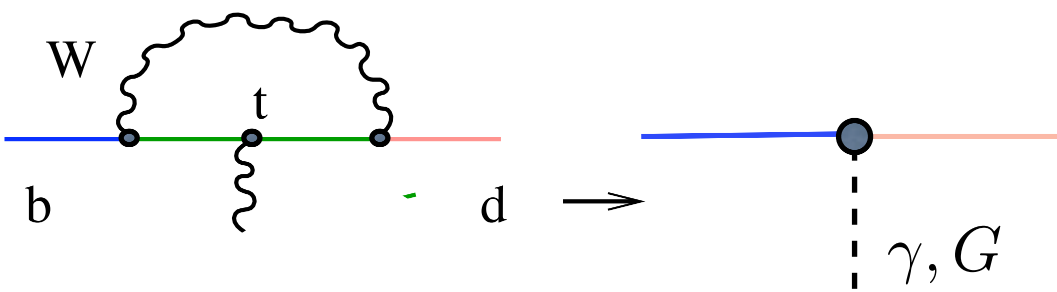


Tree operators

$$O_1^c = (\bar{c}_L \gamma_\mu b_L)(\bar{s}_L \gamma_\mu c_L)$$

$$O_2^c = (\bar{c}_L^i \gamma_\mu b_L^j)(\bar{s}_L^j \gamma_\mu c_L^i)$$

Penguin operators



$$O_{3-6} = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V\pm A}$$

$$O_7 = \frac{e m_b}{16\pi^2} (\bar{s}_L \sigma_{\mu\nu} F_{\mu\nu} b_R)$$

$$O_8 = \frac{g m_b}{16\pi^2} (\bar{s}_L \sigma_{\mu\nu} G_{\mu\nu} b_R)$$

Four-fermion operators

$$O_{9,10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma_\mu (\gamma_5) \ell)$$

+ new physics contributions...

O_i		
$(\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)$	i=1,2	$\lvert C_i(m_b)\rvert \sim 1$
$(\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma_i' q)$	i=3-6	$\lvert C_i(m_b)\rvert \leq 0.07$
$\frac{em_b}{16\pi^2}(\bar{s}_L\sigma^{\mu\nu}F_{\mu\nu}b_R)$	i=7	$C_7(m_b) \sim -0.3$
$\frac{gm_b}{16\pi^2}(\bar{s}_L\sigma^{\mu\nu}G_{\mu\nu}b_R)$	i=8	$C_8(m_b) \sim -0.15$
$\frac{e^2}{16\pi^2}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu(\gamma_5)\ell)$	i=9,10	$\lvert C_i(m_b)\rvert \sim 4$

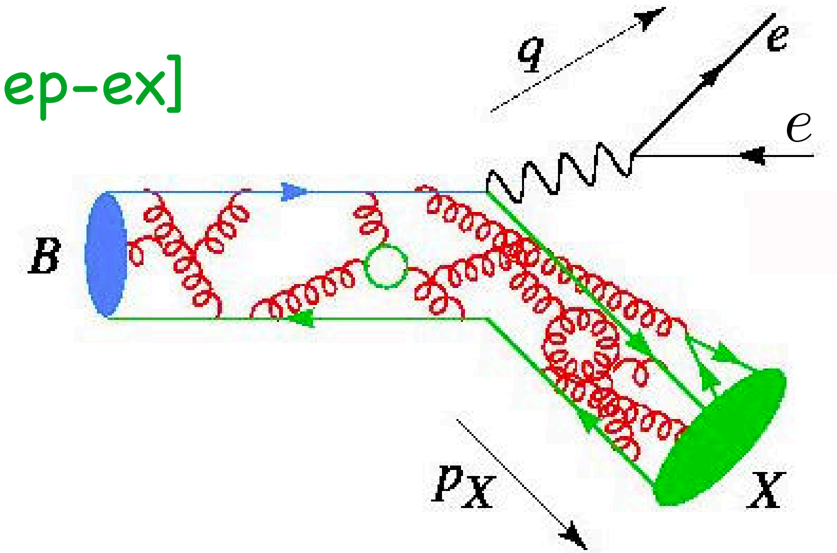
- Dominant contributions: O_7,O_9,O_{10}
- They give local contributions: local on the scale 1/M(W)
- Non-local contributions from $O_{1\text{-}6}$: contributions from intermediate states $b \to s(c\bar{c}) \to s\ell^+\ell^-$

Semileptonic FCNC B decays

- Exclusive rare B decays

$$B \to K^{(*)}\mu^+\mu^-$$

$$B_s \to \phi\mu^+\mu^-$$ CDF 1101.1028 [hep-ex]



Cleaner experimentally

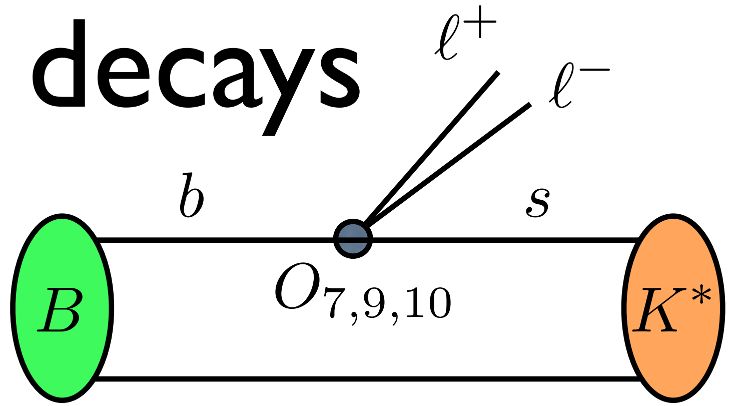
Babar, Belle, CDF, LHCb

- Inclusive decays

$$B \to X_s\mu^+\mu^-$$

Theoretically cleaner, can be treated systematically using an expansion in Λ/m_b

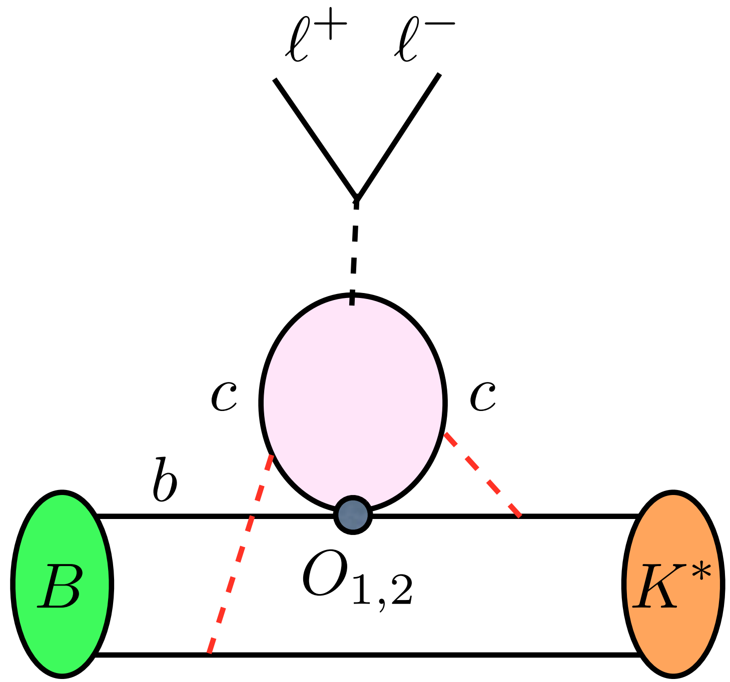
Exclusive rare B decays



Local contributions

Parameterized in terms of form factors

$$A_{\rm loc} = \sum_{i=7,9,10} C_i F_j^{BK^*} [\bar{u}(\ell^+)\Gamma_j u(\ell^-)]$$



Non-local contributions

Matrix element of a non-local operator

$$A_{\rm non-loc} = \sum_{i=1,2} C_i \langle K^* | T(O_i, j^\mu_{\rm e.m.}) | B\rangle [\bar{u}(\ell^+)\gamma_\mu u(\ell^-)]$$

Non-local contributions

Usually estimated using the factorization approximation and resonance saturation

Kruger, Sehgal

Lim, Morozumi, Sanda

$$\langle K^* | T(O_i, j^\mu_{\rm e.m.}) | B \rangle \simeq \sum_n \frac{\Gamma_{\psi_n} \Gamma(\psi_n \to \ell \bar{\ell})}{(q^2 - M^2_{\psi_n})/M_{\psi_n} + i\Gamma_{\psi_n}}$$

Sum over $J^{PC} = 1^{--}$ states

<u>Issues</u>

Renormalization scale dependence beyond leading log order

Difficult to account for non-factorizable corrections.

see also recent work

Khodjamirian et al.

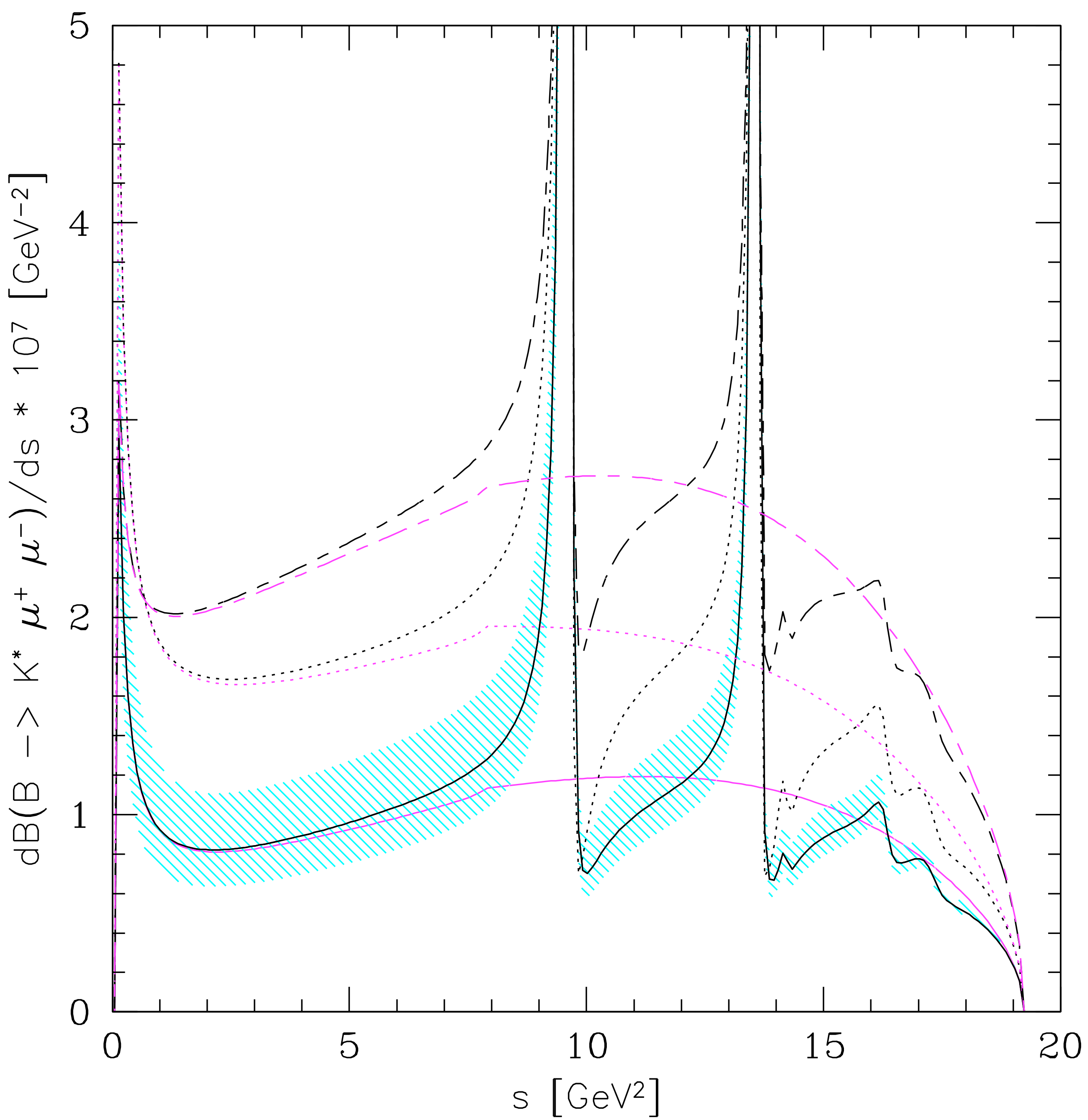
1006.4945

Example

Estimated effect ~ 10-15% in the rates

LD effect more pronounced at large q2

Difficult to extend beyond Leading Log order



Ali, Ball, Handoko, Hiller - hep-ph/9910221

Operator-product expansion for the low recoil region in $b \to s\ell^+\ell^-$

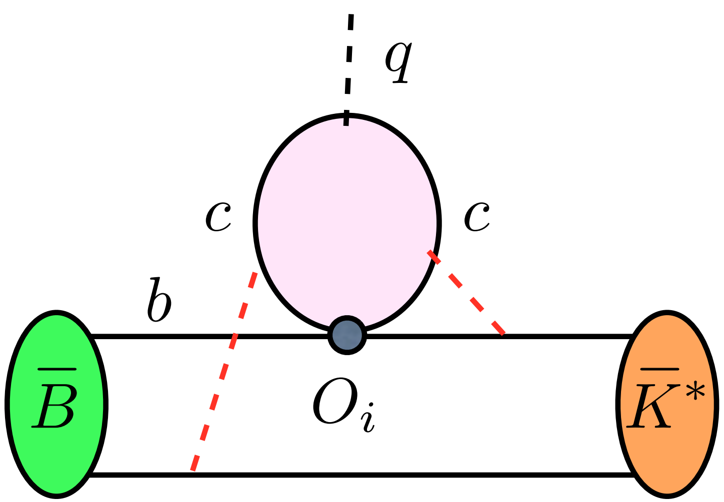
Closer look at the nonlocal contribution

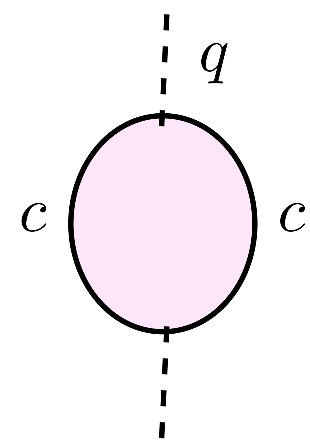
Study the structure of the non-local amplitude

$$T_i^{\mu}(q^2, v.q) = i \int d^4x e^{iq.x} \langle K^*(p)|TO_i(0)(\bar{c}\gamma^{\mu}c)(x)|\bar{B}(v)\rangle$$

Similar to the T-product appearing in $e^+e^- \rightarrow \text{hadrons}$

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq.x} \langle 0|Tj_{\mu}(0)j_{\nu}(x)|0\rangle$$





OPE for $e^+e^- \to \mathrm{hadrons}$

Consider the correlator of two equal-mass vector currents of massive quarks

$$\Pi\mu\nu(q) = \Pi(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

One-loop order result, expanded in $m^2/Q^2 \ll 1$

$$\Pi(Q^2) = \frac{N_c}{(4\pi)^2}\Big(-\frac{4}{3}\log\frac{Q^2}{\mu^2} + \frac{20}{9} - 8\frac{m^2}{Q^2} - 8\frac{m^4}{Q^4}\Big[\log\frac{m^2}{Q^2} - \frac{1}{2}\Big] + \cdots\Big)$$

Known to 3-loop order in QCD, up to $O(\frac{m^4}{Q^4})$ Chetyrkin, Kuhn

Contains also nonperturbative power corrections, proportional to vevs of higher dimensional operators

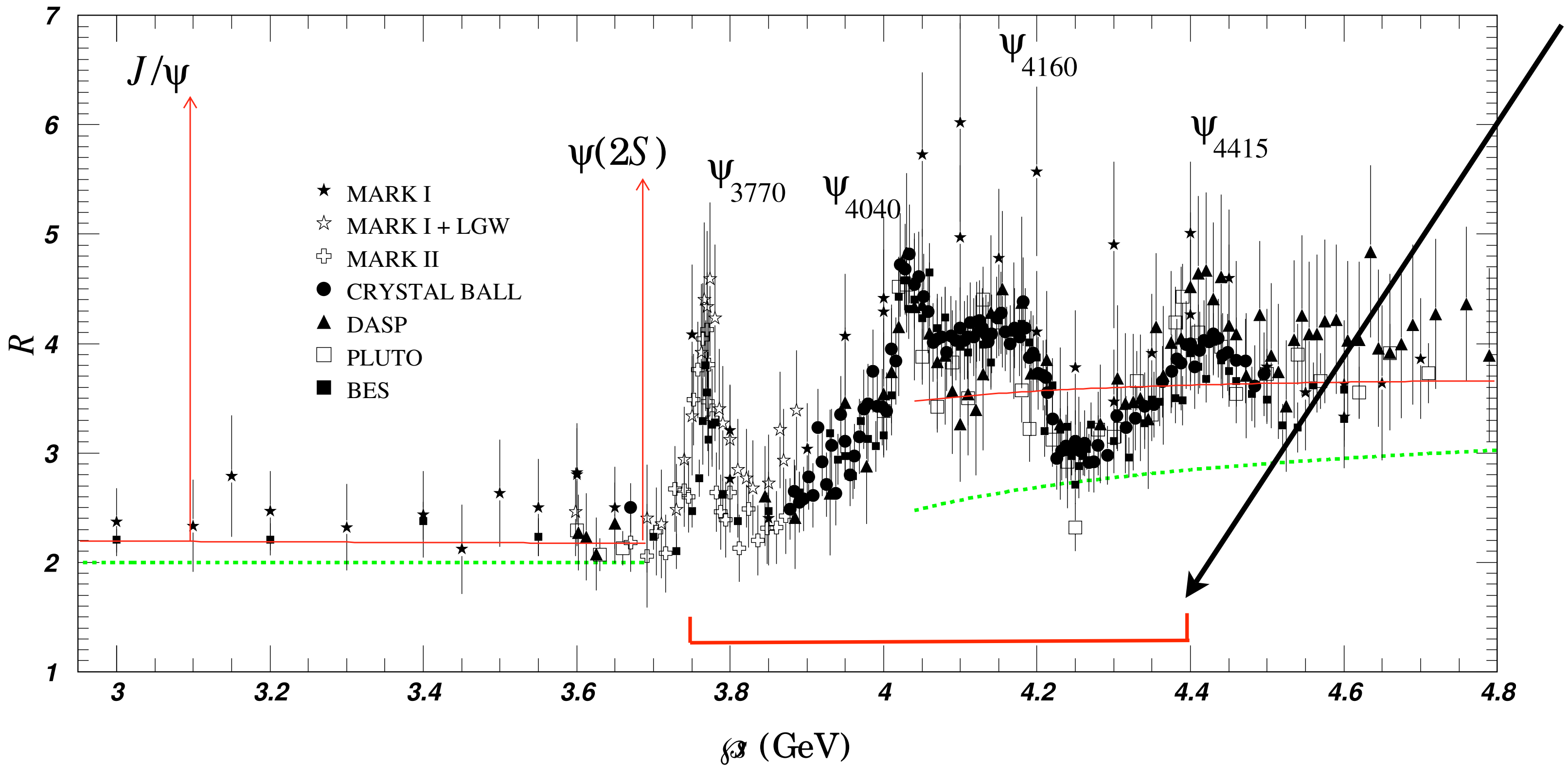
$$\Pi_{\mu\nu}(q) \to (q_\mu q_\nu - q^2 g_{\mu\nu})\Big\{\Pi(Q^2) + \frac{1}{Q^4}\langle\frac{\alpha_s}{\pi}G^2\rangle + \frac{1}{Q^4}m_q\langle\bar{q}q\rangle + \cdots\Big\}$$

 Shifman, Vainshtein, Zakharov

- How well does it work?
- Can we use a similar approach for the non-local contribution in $b \to s\ell^+\ell^-$?

zero recoil

$B \to K^*\ell^+\ell^-$



The ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in the kinematical region accessible in $B \to K^*e^+e^-$

$$E_{K^*} - m_{K^*} \leq \Lambda \to 3.8\text{GeV} \leq \sqrt{q^2} \leq m_B - m_{K^*} = 4.4 \text{ GeV}$$

Energy scales in $b \to s\ell^+\ell^-$

Hierarchy of scales at low recoil

$$m_b \sim Q > m_c > \Lambda$$

Two possible strategies:

1. Integrate out only $m_b \sim Q$ and leave charm as a dynamical field

The non-local amplitude can be expanded in $\frac{m_c^2}{Q^2} \sim 10\%, \frac{\Lambda}{Q} \sim 10\%$

Pros: • control $\log(m_c^2/Q^2)$ terms using RGE methods Grinstein, DP

• no potentially large $O(\Lambda^2/m_c^2)$ power corrections

Cons: • many operators in the OPE

2. Integrate out all three scales m_b, Q, m_c simultaneously

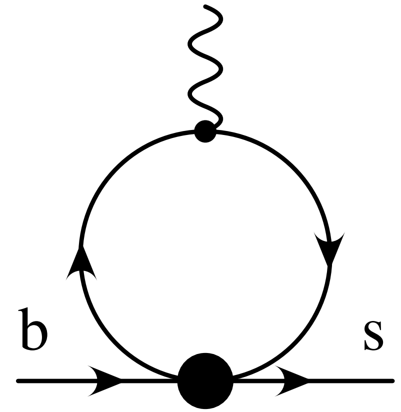
Beylich, Buchalla, Feldmann

Pro: • simpler structure of the OPE operators

Con: • $O(\Lambda^2/m_c^2)$ power corrections could be present

$$\mathcal{T}_\mu(q^2) = i\int d^4x e^{iq.x}\langle \bar{K}^*|T\mathcal{H}_W(x)\,,j_\mu^{\rm ew}|\bar{B}\rangle$$

Integrate out the large scale $m_b \sim Q$



Expand $\mathcal{T}_\mu(q^2)$ into an OPE including all possible HQET operators satisfying the Ward identity

$$\mathcal{T}_\mu(q^2)q^\mu = 0$$

$$\mathcal{T}_\mu(q^2) \to C_1^{(0)}(\mu)\bar{s}_L q^2\gamma^\mu b_v + C_2^{(0)}(\mu)\bar{s}_L i m_b\sigma^{\mu\nu}q_\nu b_v \quad O(1)$$

$$+C_1^{(2)}(\mu)m_c^2\bar{s}_L\gamma^\mu b_v + C_2^{(2)}(\mu)\frac{m_b m_c^2}{q^2}\bar{s}_L i\sigma^{\mu\nu}q_\nu b_v \quad O(\frac{m_c^2}{Q^2})$$

$$+\sum_{i=1}^{5} C_i^{(1)}(\mu)m_b\bar{s}_L\Gamma_i^{\mu\nu}(iD_\nu)b_v + \cdots \quad O(\frac{\Lambda}{Q})$$

Operator basis

Leading terms $O(Q^2)$

$$\boxed{\begin{array}{l} \bar{s}_L(q^2\gamma_\mu - \not{q} q_\mu)h_{vL} \\ i m_b[\bar{s}_L \sigma_{\mu\nu} q^\nu h_{vR}] \end{array}}$$

$O(\Lambda Q)$

$$\boxed{\begin{array}{l} m_b \bar{s}_L[iD_\mu - v_\mu(v.iD)]h_{vR} \\ m_b(v.i\partial)\bar{s}_L[\gamma_\mu - v_\mu \not{v}]h_{vL} \\ m_b(i\partial_\mu - v_\mu(v.i\partial))(\bar{s}_L h_{vR}) \\ m_b(i\partial_\nu)(\bar{s}_L[\gamma_\mu - v_\mu \not{v}]\gamma^\nu h_{vR}) \\ m_b m_s \bar{s}_R(\gamma_\mu - v_\mu \not{v})h_{vR} \end{array}}$$

$O(m_c^2)$

$$\boxed{\begin{array}{l} m_c^2 \bar{s}_L(\gamma_\mu - \not{q} q_\mu/q^2)h_{vL} \\ i m_b \dfrac{m_c^2}{q^2}[\bar{s}_L \sigma_{\mu\nu} q^\nu h_{vR}] \end{array}}$$

same operators as $O(Q^2)$

can be included without introducing any new hadronic uncertainty

$O(\Lambda^2)$

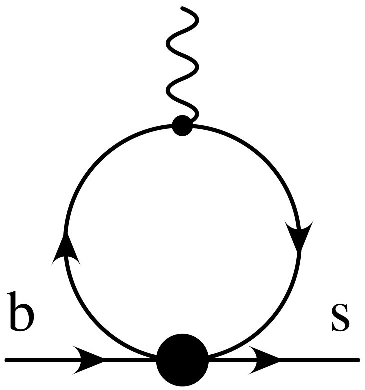
$$\bar{s}\Gamma(iD_\mu)(iD_\nu)h_v$$

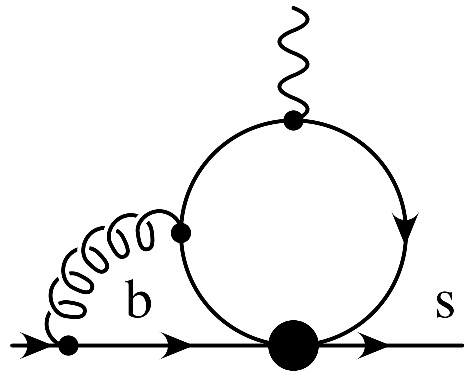
$$\bar{s}\Gamma g G_{\mu\nu} h_v$$

$O(m_c^4/Q^2)$

$$\frac{1}{Q^2}(\bar{s}\Gamma h_v)(\bar{c}\Gamma_c iD_\mu c)$$

Matching



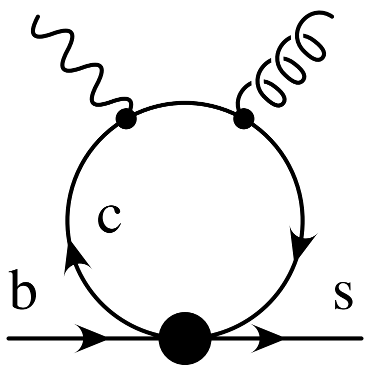


$$+\cdots \to C_i^{(-2)}(\mu)\,\bar{s}\Gamma_i b_v$$

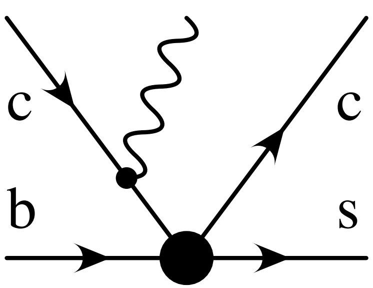
known to $O(\alpha_s(Q))$

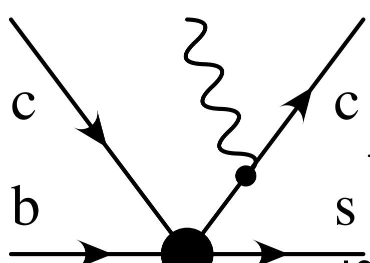
2-loop result (mc=0)

Seidel, hep-ph/0403185



$$+\cdots \to C_i^{(0)}(\mu)\,\frac{1}{Q^2}\,\bar{s}\Gamma_i G^{\mu\nu} b_v$$





$$+\cdots \to C_i^{(2)}(\mu)\,\frac{1}{Q^2}\,(\bar{s}\Gamma_i b_v)(\bar{c}\Gamma_j(iD)c)$$

Summary

- Summary of the OPE for the long-distance amplitude $\longrightarrow$

$$\mathcal{T}_\mu(q) \to \left(C_i^{(-1)} + \frac{m_c^2}{Q^2} C_i^{(0)}\right) \langle \bar{s}_L \Gamma_i b_v \rangle + \frac{1}{Q} C_i^{(-1)} \langle \bar{s}_L \Gamma_i (iD) b_v \rangle + \frac{1}{Q^2} C_j^{(0)} \langle \bar{s}_L \Gamma_j g G^{\mu\nu} b_v \rangle + \cdots$$

- The O(1) and O(m_c^2/Q^2) terms in the OPE produce the same quark bilinears as $Q_{7,9}$ in the short-distance amplitude: their effects can be absorbed into 'effective' Wilson coefficients

$$C_9^{\text{eff}} = C_9 + \sum_{i=1}^{6} C_i(\mu)[C_{1,i}^{(0)}(\mu) + \frac{m_c^2}{Q^2} C_{1,i}^{(2)}(\mu)]$$

- The O(Λ/Q) power corrections come from dimension-4 operators : new form factors. Their Wilson coeffs start at 1-loop, so they are further suppressed.

The OPE expansion for the long-distance contribution produces a RG-invariant (to the given order) amplitude

$$A_\mu^{(V)} = C_9^{\rm eff}\langle \bar{s}_L \gamma_\mu b_L\rangle - C_7^{\rm eff}(\mu)\frac{2m_b}{q^2}\langle \bar{s}_L i\sigma_{\mu\nu} q^\nu b_R\rangle + \frac{1}{q^2}\sum_{i=1}^{5} B_i(\mu)\langle O_i^{(-1)}\rangle + \cdots$$

Including C_i at NNL, and OPE matching at 1-loop (only terms ~ $C_{1,2}$)

$$\mu\frac{d}{d\mu}C_9^{\rm eff}(\mu) = O(\alpha_s^2 C_{1,2}, \alpha_s C_{3-6})$$

Bobeth,Gambino,Gorbahn, Haisch, hep-ph/0312090

The scale dependence decreases from LL to NLL

	$({\rm Re}C_9^{\rm eff}, {\rm Im}C_9^{\rm eff})$
LL:	2.4% , 113%
NLL:	2.1% , 27%

$$\mu = 4.8\ {\rm GeV}(\pm 50\%)$$

Power corrections

The leading power corrections are of $O\Big(\alpha_s(Q)\dfrac{\Lambda}{Q}\Big)$

They are proportional to the form factors of dimension-4 HQET quark bilinears

$$\langle V(k,\eta)|\bar{q}iD_\mu b_v|\bar{B}(v)\rangle = d^{(0)}(q^2)i\varepsilon_{\mu\nu\lambda\sigma}\eta^{*\nu}(p+k)^\lambda(p-k)^\sigma$$

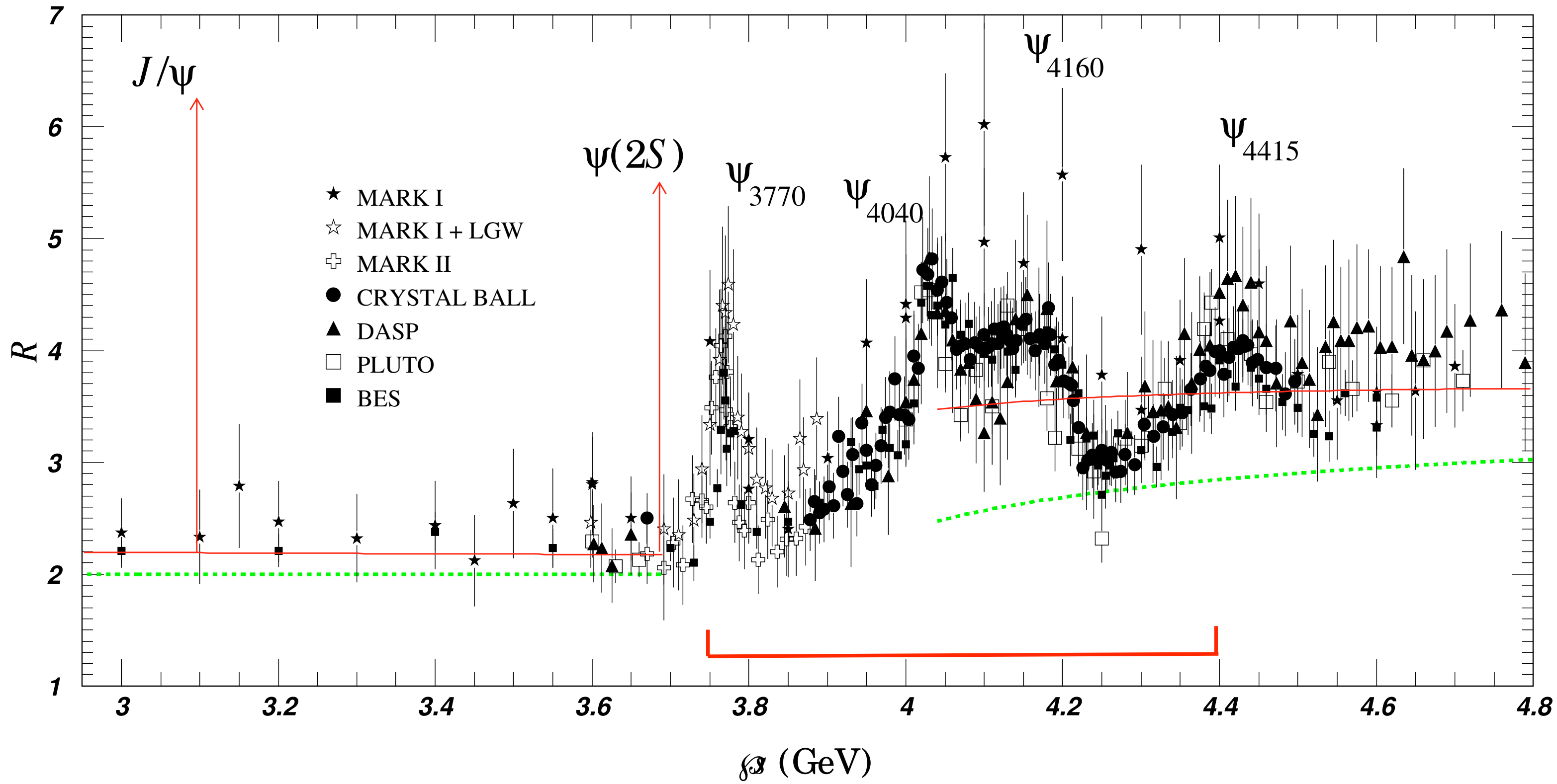
$$\langle V(k,\eta)|\bar{q}iD_\mu\gamma_5 b_v|\bar{B}(v)\rangle = d_1^{(0)}(q^2)\eta^*_\mu + d_-^{(0)}(q^2)(\eta^*.p)(p+k)_\mu + d_+^{(0)}(q^2)(\eta^*.p)(p-k)_\mu$$

We need these form factors in the low recoil region - within the applicability region of lattice QCD

Technical detail:

Although the OPE was performed in terms of operators built out of the HQET field b_v, the leading order operators can be expressed only in terms of the QCD field b.

Duality violation



The ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in the kinematical region accessible in $B \to K^* e^+ e^-$

$$E_{K^*} - m_{K^*} \leq \Lambda \to 3.8\text{GeV} \leq \sqrt{q^2} \leq m_B - m_{K^*} = 4.4 \text{ GeV}$$

Suggests duality violation in $[C_9^{\rm eff}]_{LD}$ of about 20%

5% in $\mathrm{Re}(C_9^{\rm eff})$

20% in $\mathrm{Im}(C_9^{\rm eff})$

Improved symmetry relations for heavy-light form factors

Heavy-light form factors

The hadronic matrix elements of the quark bilinears are parameterized by form factors

Example: $B \to V$ transitions

$$\langle V(k,\eta)|\bar{q}\gamma_\mu b|\bar{B}(p)\rangle = g(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\eta^{*\nu}(p+k)^\lambda(p-k)^\sigma$$

$$\begin{aligned}\langle V(k,\eta)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle = f(q^2)\eta^{*\mu} + a_+(q^2)(\eta^*\cdot p)(p+k)_\mu \\ +a_-(q^2)(\eta^*\cdot p)(p-k)_\mu\end{aligned}$$

$$\begin{aligned}\langle V(k,\eta)|\bar{q}i\sigma_{\mu\nu} b|\bar{B}(p)\rangle = g_+(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\eta^{*\lambda}(p+k)^\sigma + g_-(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\eta^{*\lambda}(p-k)^\sigma \\ +h(q^2)(\eta^*\cdot p) i\varepsilon_{\mu\nu\lambda\sigma}(p+k)^\lambda(p-k)^\sigma\end{aligned}$$

How does heavy quark symmetry help?

Two types of predictions

1. Flavor symmetry: can relate e.g. $B \to \rho\ell\nu$ and $D \to \rho\ell\nu$ form factors

$$f^{B\to K^*}(y) = \sqrt{\frac{m_B}{m_D}} f^{D\to K^*}(y) + O(\Lambda/m_c)$$

2. Spin symmetry: can relate tensor and vector-axial form factors

e.g. $$g_+(y) - g_-(y) + 2m_B g(y) = O(m_b^{-1/2})$$

The relations hold between form factors at the same value of the light meson energy $E = m_V y$

Heavy quark symmetry gives relations among B -> M heavy-light form factors at low recoil

Example: tensor vs. vector form factors

$$\underbrace{g_+(q^2,\mu) = -\kappa(\mu) m_b g(q^2)}_{\text{leading order relation}} - 2d(q^2)$$

power correction

$$\kappa(\mu) = 1 + O(\alpha_s(m_b))$$

$$\langle V(k,\eta)|\bar{q} i D_\mu b_v|\bar{B}(p)\rangle = d(q^2) i\varepsilon_{\mu\nu\lambda\sigma}\eta^{*\nu}(p+k)^\lambda (p-k)^\sigma$$

Extends the Isgur-Wise relations by including also hard loops radiative and power corrections

Including the radiative corrections

Match the dim.-4 quark bilinears onto HQET

$$\bar{q} i D_\mu b = D_0^{(v)}(\mu) m_b \bar{q}\gamma_\mu b_v + D_1^{(v)}(\mu) m_b \bar{q} v_\mu b_v + \bar{q} i D_\mu b_v + \cdots$$

Note the enhanced O(1) terms!

Their Wilson coefficients can be computed exactly in terms of the HQET coefficients of the LO operators

$$\bar{q}\gamma_\mu b = C_0^{(v)}(\mu)\bar{q}\gamma_\mu b_v + C_1^{(v)}(\mu)\bar{q} v_\mu b_v + \cdots$$

$$\bar{q} i\sigma_{\mu\nu} q^\nu b = C_0^{(t)}(\mu)\bar{q} i\sigma_{\mu\nu} q^\nu b_v + C_1^{(t)}(\mu)\bar{q}[(v.q)\gamma_\mu - \not{q} v_\mu] b_v + \cdots$$

as

$$D_0^{(v)}(\mu) = -\frac{1}{2} C_0^{(v)}(\mu) + \frac{1}{2}\frac{m_b^{pole}}{m_b(\mu)}[C_0^{(t)}(\mu) - C_1^{(t)}(\mu)]$$

all-order relation - checked by explicit 1-loop matching

Heavy quark symmetry form factor relation, including also the radiative correction

Expresses one of the B->V tensor form factors in terms of the vector form factor

$$g_+(q^2) = -\left(1 + 2\frac{D_0^{(v)}}{C_0^{(v)}}\right) m_b g(q^2) - 2d^{(0)}(q^2)$$

The radiative correction factor depends only on known QCD -> HQET heavy-to-light Wilson coefficients

$$\kappa_1(\mu) = \left(1 + 2\frac{D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right)\frac{m_b(\mu)}{m_B} = \frac{C_0^{(t)}(\mu) - C_1^{(t)}(\mu)}{C_0^{(v)}(\mu)}$$

This factor is essential in order to reproduce the correct RG scaling of the total amplitude

Application to $B \to K^* \ell^+ \ell^-$

The heavy quark symmetry relations connect the matrix elements of the penguin O_7 operator to those of O_9

$$A(B \to K^* \ell^+ \ell^-) \propto -C_7(\mu) \frac{2m_b}{q^2} \langle \bar{s} i \sigma^{\mu\nu} q_\nu P_R b \rangle + C_9(\mu) \langle \bar{s} \gamma^\mu P_L b \rangle$$

The $B \to K^* \ell^+ \ell^-$ helicity amplitudes can be expressed in terms of those for the semileptonic decay $B \to \rho \ell \bar{\nu}$

$$H_\lambda^{B \to K^*} = C_9 \Big(1 + \frac{2m_b^2}{q^2} \frac{C_7}{C_9} + r_\lambda + O(\Lambda^2/m_b^2) \Big) H_\lambda^{B \to \rho}$$

with r_λ non-universal but calculable power corrections

Precise V(ub) determination from exclusive B decays

Determine V(ub) from the ratio of decay rates

$$\frac{d\Gamma(B \to \rho e \nu)}{d\Gamma(B \to K^* e^+ e^-)} = \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2} \frac{8\pi^2}{\alpha^2} \frac{1}{N_{\text{eff}}(q^2)} \frac{\sum_\lambda |H_\lambda^{B\to\rho}(q^2)|^2}{\sum_\lambda |H_\lambda^{B\to K^*}(q^2)|^2}$$

1. Compute the RG invariant factor N_{eff} using the OPE

2. Use double ratios and D data to determine the SU(3) breaking factor

$$R_{B\to V}(y) \equiv \frac{\sum_\lambda |H_\lambda^{B\to\rho}(y)|^2}{\sum_\lambda |H_\lambda^{B\to K^*}(y)|^2} \qquad R_{D\to V}(y) \equiv \frac{\sum_\lambda |H_\lambda^{D\to\rho}(y)|^2}{\sum_\lambda |H_\lambda^{D\to K^*}(y)|^2}$$

as $$R_{B\to V}(y) = R_{D\to V}(y)\left(1 + O\left(\frac{m_s}{m_b} - \frac{m_s}{m_c}\right)\right)$$

$$\boxed{|V_{ub}|}$$

Theory gives the factor

$$N_{\text{eff}}(q^2) = \left| C_9^{eff} \left(1 + \frac{2m_b^2}{q^2} \frac{C_7^{eff}}{C_9^{eff}} \right) \right|^2 + |C_{10}|^2$$

Ingredients

- 2-loop matching at $\mu \sim M_W$
- 3-loop running for $C_{1,2}$ and 2-loop running for C_{3-6}
- 2-loop matching at $\mu \sim Q$

$N_{\text{eff}}(q^2)$ should be RG invariant up to NNL order in perturbation theory

Results

$$\frac{d\Gamma(B\to\rho e\nu)}{d\Gamma(B\to K^* e^+ e^-)} = \frac{|V_{ub}|^2}{|V_{tb}V_{ts}^*|^2}\,\frac{8\pi^2}{\alpha^2}\,\frac{1}{N_{\rm eff}(q^2)}\,\frac{\sum_\lambda |H_\lambda^{B\to\rho}(q^2)|^2}{\sum_\lambda |H_\lambda^{B\to K^*}(q^2)|^2}$$

	μ_b (GeV)	$N_{\rm eff}(y=1)$	$N_{\rm eff}(y=1.5)$
LL	2.4	30.80	28.96
	4.8	33.37	32.34
	9.6	35.81	35.38
NLL	2.4	32.75	30.83
	4.8	32.76	31.11
	9.6	33.46	32.10

Improved scale dependence 15% (LL) to 2% (NLL)

Small dependence on kinematics (y)

Theory errors

Sources of theoretical uncertainty in $N_{\rm eff}(q^2)$ (hence in $|V_{ub}|^2$)

scale dependence	15% (LL) 2% (NLL)
power corrections	1%
duality violation	5%
total	8-10%

Experimental data is not yet at this precision level

Ideal application for a super-B factory

Summary and outlook

- Progress in the theory of exclusive radiative b->se+e- decays
  - OPE for long-distance charm effects in exclusive B decays at low recoil
  - Derived improved heavy quark symmetry relations among form factors at low recoil
- Model-independent theory of the exclusive rare radiative decays at low recoil
- Application of the formalism: model-independent determination of V(ub) from exclusive B decays, with a theory precision better than 10%
- Many other applications possible: e.g. tests for NP effects, polarization effects, etc.