Applications of QCD Sum Rules to $B \to K^{(*)}\ell^+\ell^-$

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$B \to K^{(*)} \ell^+\ell^-$, the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \Big|_{\mu \sim m_b}$$

- "direct" $b \to s\ell\ell$, $b \to s\gamma$ operators:

$$O_{9(10)} = \frac{\alpha_{em}}{4\pi} [\bar{s}_L \gamma_\mu b_L] \ell\gamma^\mu(\gamma_5)\ell, \quad C_9(m_b) \simeq 4.4, \quad C_{10}(m_b) \simeq = -4.7,$$

$$O_{7\gamma} = -\frac{em_b}{8\pi^2} [\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b] F^{\mu\nu}, \quad C_7(m_b) \simeq -0.3$$

- quark-gluon operators, combined with quark e.m. current :

$$O_1^{(c)} = [\bar{s}_L \gamma_\rho c_L][\bar{c}_L \gamma^\rho b_L], \qquad C_1(m_b) \simeq 1.1$$
$$O_2^{(c)} = [\bar{c}_L \gamma_\rho c_L][\bar{s}_L \gamma^\rho b_L], \qquad C_2(m_b) \simeq -0.25$$

$$O_{8g} = -\frac{m_b}{8\pi^2} \bar{s}\sigma_{\mu\nu}(1+\gamma_5) b G^{\mu\nu}, \quad C_8(m_b) \simeq 0.2$$

O_{3-6} - quark-penguin operators , $\quad C_{3,4,5,6} < 0.03$

- the $\sim V_{ub} V_{us}^*$ part neglected

$B \to K^{(*)} \ell^+ \ell^-$ decay amplitude

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K^{(*)}\ell^+\ell^- \mid O_i \mid B \rangle$$

- hadronic matrix elements:

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Bigg[(\bar{\ell}\gamma^\rho\gamma_5\ell)\, C_{10} \langle K^{(*)}|\bar{s}\gamma_\rho(1-\gamma_5)b|B\rangle$$
$$+ (\bar{\ell}\gamma^\rho\ell)\Big(C_9 \langle K^{(*)}|\bar{s}\gamma_\rho b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K^{(*)}|\bar{s} i\sigma_{\nu\rho}(1+\gamma_5)b|B\rangle$$
$$+ \frac{8\pi^2}{q^2} \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^\rho \Big) \Bigg]$$

- nonlocal matrix elements

$$\boxed{\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)| i \int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle},$$

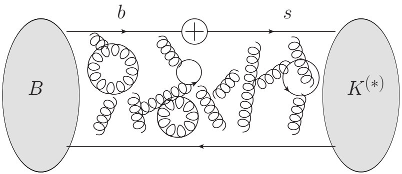
not all of them are reducible to form factors !

$B \to K^{(*)}$ form factors

momentum transfer q^2

$\Downarrow$

$\langle K^{(*)}(p)|\bar{s}\Gamma b|B(p+q)\rangle$



- the kinematical region in $B \to K^{(*)}\ell^+\ell^-$, $(m_\ell = 0)$:
  $0 < q^2 < (m_B - m_{K^{(*)}})^2 \sim 20\,\text{GeV}^2$
- in the form factors nonperturbative contributions dominate

$B \to K^{(*)}$ form factors

- definitions:

$$\langle K(p)|\bar{s}\gamma_\mu b|B(p+q)\rangle = f^+_{BK}(q^2)\Big[2p_\mu + \big(1 - \frac{m_B^2 - m_K^2}{q^2}\big)q_\mu\Big] + f^0_{BK}(q^2)\Big(\frac{m_B^2 - m_K^2}{q^2}\Big)q_\mu\,,$$

the $\sim q_\mu$ part heavily suppressed for $\ell = \mu, e$

$$\langle K(p)|\bar{s}\sigma_{\mu\nu}b|B(p+q)\rangle \Rightarrow f^T_{BK}(q^2)$$

$$\langle K^*(p)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle \Rightarrow V_{BK^*}(q^2), A^{1,2,3}_{BK^*}(q^2)$$

$$\langle K^*(p)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|B(p+q)\rangle \Rightarrow T^{1,2,3}_{BK^*}(q^2)$$

Use of flavour symmetries

- $SU(3)_{fl}$ symmetry ($m_s \simeq m_{u,d}$):
  $F_{B\to K}(q^2) \simeq F_{B\to\pi}(q^2)$, $F_{B\to K^*}(q^2) \simeq F_{B\to\rho}(q^2)$,
- heavy-quark symmetry ($m_b, m_c \to \infty$),
  relating $F_{B\to K^{(*)}}$ with $F_{D\to K^{(*)}}$
- both symmetries violated by finite quark masses,
  small q^2 (large recoil), typically at $\sim 20\%$
- low recoil region, double ratios ..

QCD calculation of $B \to K^{(*)}$ form factors

- a nonperturbative QCD method
- with finite quark masses

- lattice QCD with growing accuracy
  (currently, $B \to \pi$ at large q^2 , $n_f = 3$)

- non-lattice methods ?

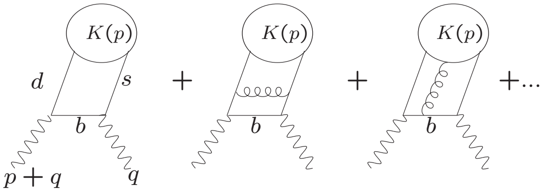
Light-Cone Sum Rules for $B \to K^{(*)}$ form factors

- correlation function in QCD = hadronic sum
  $\Rightarrow$ ground state contribution
- a specific correlation function

$$F_\mu(q,p) = i \int d^4x\, e^{iqx} \langle K(p) \,|\, T\{\bar{s}(x)\gamma_\mu b(x), \bar{b}(0) i\gamma_5 d(0)\} \,|\, 0\rangle$$

$q^2, (p+q)^2 \ll m_b^2$,

b-quark
highly virtual
$\Longrightarrow\ x^2 \sim 0$



- an intermediate scale $\Lambda_{QCD} \ll \chi \ll m_b$

the method: [I.Balitsky,V.Braun et al (1989); V.Chernyak, I.Zhitnitsky (1989)]

$B \to K$ form factor [V.Belyaev, A.K., R.Rückl (1993)],

$B \to K^*$ [A.Ali, V.M.Braun, H. Simma (1994)]

QCD calculation

- the result

$$F_\mu(q,p) = \sum_{t=2,3,4,..} \int du\, T_\mu^{(t)}(q^2,(p+q)^2,m_b^2,\alpha_s,u)\, \varphi_K^{(t)}(u,\mu)$$

hard scattering ampl. $\otimes$ kaon light-cone DA

- nonperturbative objects: distribution amplitudes (DA's): vacuum-kaon hadronic matrix elements,

$$\langle K(q)|\bar{s}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -iq_\mu f_K \int_0^1 du\, e^{iuqx} \varphi_K(u)\,.$$

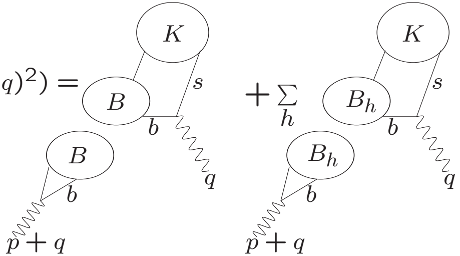
- DA's, defined originally for the pion,
  determines $\gamma\gamma^* \to \pi^0$ (CLEO,BABAR)
  $SU(3)_{fl}$ breaking in f_K/f_π and in DA's (Gegenbauer expansion)

- $t = 3,4$ contributions (soft gluon) power suppressed,
  nontrivial factorization in $O(\alpha_s)$

Derivation of LCSR

- Hadronic dispersion relation in the variable $(p+q)^2$:

$F(q^2,(p+q)^2) =$



$$f_B\ f^+_{BK}(q^2) \qquad\qquad \sum_{B_h} \rightarrow duality\ (s_0^B)$$

(fixed $q^2 \ll m_b^2$)

$$[F((p+q)^2,q^2)]_{QCD} = \frac{m_B^2 f_B f^+_{BK}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s,q^2)]_{QCD}}{s-(p+q)^2}$$

quark-hadron duality approximation, f_B - from two-point QCD sum rule

Status and accuracy of LCSR calculations

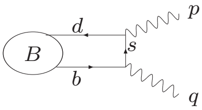
- $q^2 \leq 12 - 15~\text{GeV}^2$ accessible,
  complementing the lattice FF's
- $B \to \pi$ recent major update ($\overline{MS}$ b-quark mass)):
  [G.Duplancic, AK, Th.Mannel, B.Melic, N.Offen (2008)] ,

  taken as an input for $|V_{ub}|$ by BABAR
  [A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]
- $B \to K$
  [G.Duplancic, B.Melic (2008)], *[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]*
- the same method/input for $D \to \pi, K$
  [A.K.,Ch.Klein, Th.Mannel, N.Offen (2009)]
- estimated uncertainties for $B \to \pi, K$ $\pm(12 - 15)\%$
- $B_{(s)} \to \rho, \omega, K^*, \phi$ form factors,
  [P.Ball, R.Zwicky (2005)]
  DA's of K^* and ρ in "quenched" approximation,
  the width of ρ, K^* neglected

LCSR with B-meson distribution amplitudes

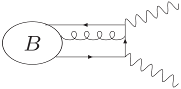
[A.K., Th. Mannel, N.Offen (2005)]

- In the correlation function:
  B-meson ⇒ an on-shell state,
  light meson ⇒ current,

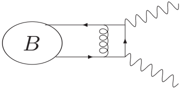


(a)

- *B*-meson DA's,
  defined in HQET,
  light mesons
  from quark-hadron duality



(b)



(c)

- so far only tree-level calculations, 2,3-particle DA's
- all $B \to \pi, K^{(*)}, \rho$ form factors calculated at $q^2 \leq 10$ GeV^2
  [A.K., Th.Mannel, N.Offen (2007)]
- LCSR in SCET [F. De Fazio, Th. Feldmann T.Hurth (2006)]

B-meson DA's

- defined in HQET:

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not{v})\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not{x}\right\}\gamma_5\right]_{\beta\alpha}$$

- key input parameter: the inverse moment of ϕ_+^B

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV

  [V.Braun, D.Ivanov, G.Korchemsky,2004]

- QCD sum rule based model for 3-particle DA's

[A.K., T.Mannel, N.Offen (2007)]

Extrapolation to large q^2

- Series parameterization of form factors based on conformal mapping:

  ...., [Boyd,Grinstein,Lebed(1995)],... [Bourrely,Caprini, Lellouch(2008)]

$$z(q^2,\tau_0) = \frac{\sqrt{\tau_+ - q^2} - \sqrt{\tau_+ - \tau_0}}{\sqrt{\tau_+ - q^2} + \sqrt{\tau_+ - \tau_0}}$$

$$\tau_+ = (m_B + m_{K^{(*)}})^2\,, \qquad \tau_- = (m_B - m_{K^{(*)}})^2 \quad \tau_0 = \tau_+ - \sqrt{\tau_+ - \tau_-}\sqrt{\tau_+}\,.$$

- we use BCL parameterization

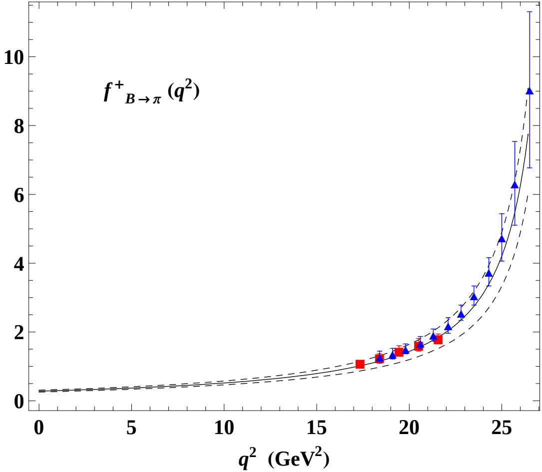
$$F(q^2) = \frac{F(0)}{1 - q^2/m^2_{B_s(J^P)}}\Big\{1 + b_1\Big(z(q^2,t_0) - z(0,t_0) + \frac{1}{2}\left[z(q^2,t_0)^2 - z(0,t_0)^2\right]\Big)\Big\}\,,$$

- allows to extra(inter)polate the LCSR and lattice QCD FF's

  $B \to K^{(*)}$ [A.Bharucha, Th.Feldmann, M.Wick, 1004.3249[hep-ph].

$B \to \pi$ form factor: LCSR vs lattice QCD

[A.K, Th.Mannel, N.Offen, Y-M. Wang (2011)]



$q^2 \leq 12$ GeV^2 -LCSR, $q^2 > 12$ GeV^2 - [HPQCD, FNAL/MILC]

$B \to K, K^{(*)}$ form factors from LCSR

[A.K, Th.Mannel, A.Pivovarov, Y-M. Wang (2010)]

form factor	$F^i_{BK^{(*)}}(0)$	b^i_1	$B_s(J^P)$	input at $q^2 < 12$ GeV^2
f^+_{BK}	$0.34^{+0.05}_{-0.02}$	$-2.1^{+0.9}_{-1.6}$	$B^*_s(1^-)$	
f^0_{BK}	$0.34^{+0.05}_{-0.02}$	$-4.3^{+0.8}_{-0.9}$	no pole	LCSR
f^T_{BK}	$0.39^{+0.05}_{-0.03}$	$-2.2^{+1.0}_{-2.00}$	$B^*_s(1^-)$	with K DA's
V^{BK^*}	$0.36^{+0.23}_{-0.12}$	$-4.8^{+0.8}_{-0.4}$	$B^*_s(1^-)$	
$A^{BK^*}_1$	$0.25^{+0.16}_{-0.10}$	$0.34^{+0.86}_{-0.80}$	$B_s(1^+)$	
$A^{BK^*}_2$	$0.23^{+0.19}_{-0.10}$	$-0.85^{+2.88}_{-1.35}$	$B_s(1^+)$	LCSR
$A^{BK^*}_0$	$0.29^{+0.10}_{-0.07}$	$-18.2^{+1.3}_{-3.0}$	$B_s(0^-)$	with B DA's
$T^{BK^*}_1$	$0.31^{+0.18}_{-0.10}$	$-4.6^{+0.81}_{-0.41}$	$B^*_s(1^-)$	
$T^{BK^*}_2$	$0.31^{+0.18}_{-0.10}$	$-3.2^{+2.1}_{-2.2}$	$B_s(1^+)$	
$T^{BK^*}_3$	$0.22^{+0.17}_{-0.10}$	$-10.3^{+2.5}_{-3.1}$	$B_s(1^+)$	

Summary on $B \to K^{(*)}$ FF's

- $B \to K^{(*)}$, accessible at low q^2, two independent LCSR techniques available,
- LCSR with light meson DA's K, K^* , finite m_b, m_s
  zero width approximation for K^*
- LCSR with B DA's (SCET LCSR) have a more "universal" input , allow to account for the finite width of K^* ,
  need $1/m_b$ corrections
- z-parameterization $\Rightarrow$ large q^2
- complements lattice QCD
- LCSR remain an essentially approximate method, the accuracy of FF's is limited at $\sim 10-15\%$
- we have to learn to calculate $B \to K\pi$ FF's, with $K\pi$ in both $J^P = 1^-$ and $J^P = 0^+, 2^+$ states including the resonances in these channels

Nonlocal hadronic matrix elements in $B \to K^{(*)}\ell^+\ell^-$

- generic expression:

  $\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)| i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle$

  $j^\rho_{em} = \sum\limits_{q=u,d,s,c,b} Q_q \bar{q}\gamma^\rho q$, the hierarchy $O_i = O_{1,2}^{(c)},\ O_{8g}, O^{(q)}_{3,4,5,6}, O^{(u)}_{1,2}$

- in LO only two contributions:

  1) quark loop with photon emission,
  $O_{1,2}^{(c)}$ dominate → "charm-loop effect"

  2) "weak annihilation"

- adding (perturbative) gluons:

  O_{8g} enters, diagrams/ topologies for O_i proliferate,
  "nonspectator" and "spectator contributions

- $\mathcal{H}_i$'s should be calculated one by one and included in
  $A(B \to K^{(*)}\ell^+\ell^-)$, as corrections to C_9

Use of OPE and effective theories

- at low $q^2 \ll 4m_c^2$ (large recoil of $K^{(*)}$):

- $\mathcal{H}_i$ including $O(\alpha_s)$ “catalogized”, estimated from QCD factorization ($m_b \to \infty, E_{K^{(*)}} \sim m_b$)
- "nonspectator" contributions $\Rightarrow B \to K^*$ FF’s, "spectator " contributions factorized; nonpert. inputs $f_B, f_K^{(*)}, \lambda_B$

  [M.Beneke, Th.Feldmann, D.Seidel (2001)], ...

- at large $\sqrt{q^2} \sim m_b$ (low recoil of $K^{(*)}$):

- local OPE in $1/\sqrt{q^2} \sim 1/m_b$,
- quark-hadron duality:

  $\mathcal{H}_i(q^2 \ll 0) \Rightarrow \mathcal{H}_i(q^2 \gg m_{res}^2)$

  [Grinstein, Pirjol (2004)], [Beylich,Buchalla,Feldmann(2011)],

  the talks at this workshop

Questions to address

1. hadronic inputs for the OPE estimates of $\mathcal{H}_i$:
   FF's, decay constants and light-cone DA's of B, K, K^*,

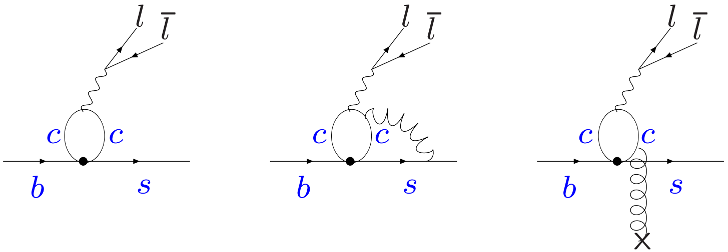
   are there additional contributions to $\mathcal{H}_i$
   due to soft gluons ?
2. can we estimate these contributions using LCSR?
3. how they influence the duality relation between large $q^2 < 0$ and large $q^2 > 0$?

   In what follows: 1.- 3. are addressed for the dominant nonlocal contribution: the charm-loop

Charm-loops in $B \to K^{(*)}\ell^+\ell^-$

[A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, 1006.4945 [hep-ph]]

- charm-loop effect: a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$
- low q^2, contract the virtual c-quark fields



Isolating the charm-loop in the decay amplitude

- hereafter $B \to K\ell^+\ell^-$,

$$A(B \to K\ell^+\ell^-)^{(O_{1,2})} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\rho\ell}{q^2}\mathcal{H}_\rho^{(B\to K)}$$

- the hadronic matrix element:

$$\mathcal{H}_\rho^{(B\to K)}(q,p) = i\langle K(p)|\int d^4x\, e^{iq\cdot x}\, T\Big\{\bar{c}(x)\gamma_\rho c(x)\,,$$
$$\Big[C_1[\bar{s}_L(0)\gamma_\mu c_L(0)\bar{c}_L(0)\gamma^\mu b_L(0)] + C_2...\Big]\Big\}|B(p+q)\rangle$$

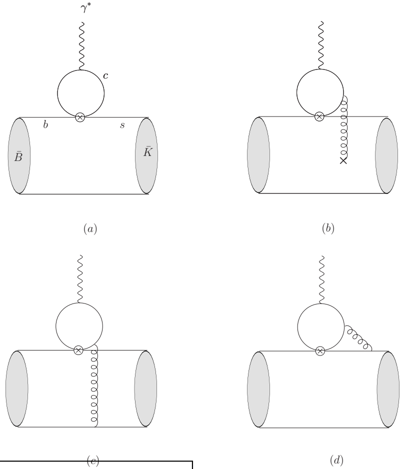
- the invariant amplitude:

$$\mathcal{H}_\rho^{(B\to K)}(q,p) = \big[(p\cdot q)q_\rho - q^2 p_\rho\big]\mathcal{H}^{(B\to K)}(q^2)$$

Charm-loop in $B \to K^{(*)}\ell^+\ell^-$ at $q^2 \ll 4m_c^2$

- factorizable c-quark loop
  $C_9 \to C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$
- perturbative gluons $\to$ (nonfactorizable) corrections being factorized in $O(\alpha_s)$ and added to C_9

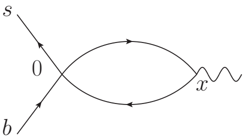
[M. Beneke,T. Feldmann,D. Seidel (2001)]



How important are the soft gluons
(low-virtuality, nonvanishing momenta)
emitted from the c-quark loop ?

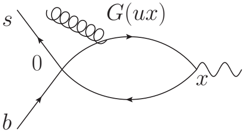
Expansion near the light-cone

- at $q^2 \ll 4m_c^2$, the dominant region: $\langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$
- T- product of $\bar{c}c$-operators can be expanded near the light-cone $x^2 \sim 0$, diagrammatically:



the simple loop

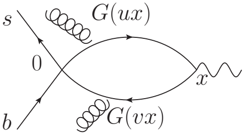
(unit operator)



one-gluon emission

(non-local, $x \neq 0$)

$(\bar{s}(0)G(x)b(0))$



two-gluon emission

.....

The resulting effective operators

- LO reduced to simple $\bar{c}c$ -loop,
  no difference between local and LC,

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho}) \frac{9}{32\pi^2}\, g(m_c^2, q^2)\bar{s}_L \gamma^\rho b_L \,.$$

- gluon emission: use c-quark propagator near the light-cone in the external gluon field [I. Balitsky, V. Braun (1999)]
- define LC kinematics ($n_\pm$) in the rest-frame of B,
  $q \simeq (m_b/2) n_+$
- one-gluon emission yields a new nonlocal operator:

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega)\bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta} b_L \ ,$$

The hierarchy of contributions in LC OPE

after integrating over x and taking hadronic matrix element

- each extra gluon brings one power of $\sim \frac{\Lambda_{QCD}^2}{4m_c^2 - q^2}$ suppression
- perturbative gluon corrections are α_s suppressed
- reexpanding the one-gluon nonlocal operator near $x = 0$ in derivatives of $G_{\mu\nu}(0)$:

$$\text{term with } k\text{-th derivative} \Rightarrow \sum_{k=0}^{\infty} \frac{(q\Lambda_{QCD})^k}{(4m_c^2 - q^2)^{k+1}}$$

$q \sim m_b/2$ and $m_b \Lambda_{QCD} \sim m_c^2$.

the OPE near the light-cone works, but not the local OPE

The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}^{(0)}_{\mu}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^{\rho}\widetilde{G}_{\alpha\beta}b_L \ ,$$

$$I_{\mu\rho\alpha\beta}(q,m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha}g_{\rho\beta}) \times \frac{1}{16\pi^2}\int_0^1 dt\, \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
*[Z. Ligeti, L. Randall and M.B. Wise,(1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage,(1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]*

Hadronic matrix elements for the charm-loop effect in $B \to K\ell^+\ell^-$

- the LO: factorized $\bar{c}c$ loop

$$\left[\mathcal{H}_\mu^{(B\to K)}(p,q)\right]_{fact} = \left(\frac{C_1}{3} + C_2\right) \langle K(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle \,,$$

  reduced to $B \to K^{(*)}$ form factors

- The gluon emission yields:

$$\left[\mathcal{H}_\mu^{(B\to K)}(p,q)\right]_{nonfact} = 2C_1 \langle K(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle \,.$$

- new hadronic matrix element

$$\langle K(p)|\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L|B(p+q)\rangle \,,$$

- the result for nonlocal matrix element:

$$\mathcal{H}^{(B\to K)} = \mathcal{H}_{fact}^{(B\to K)} + \mathcal{H}_{nonfact}^{(B\to K)} = \left(\frac{C_1}{3} + C_2\right) A(q^2) + 2C_1\widetilde{A}(q^2) \quad (2)$$

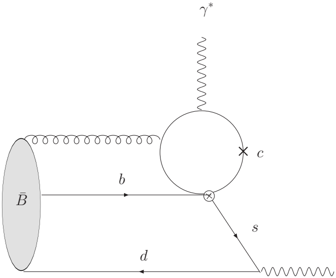
Charm-loop effect in $B \to K\ell^+\ell^-$

- the factorizable part $A(q^2) = \frac{9}{32\pi^2}\, g(m_c^2, q^2) f_{BK}^+(q^2)$
- Wilson coefficients enhance the nonfact. part $C_1/3 + C_2 \ll C_1$
- need nonperturbative QCD methods to calculate the form factor $f_{BK}^+(q^2)$ and the nonfactorizable amplitude $\widetilde{A}(q^2)$
- use one and the same LCSR approach for $A(q^2)$ and $\widetilde{A}(q^2)$

LCSR for the soft-gluon hadronic matrix element

- the correlation function:

  $\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}$
  $\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + \int_{s_h}^{\infty} ds\, \frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s-p^2}$$

Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
  a process-dependent correction to be added:

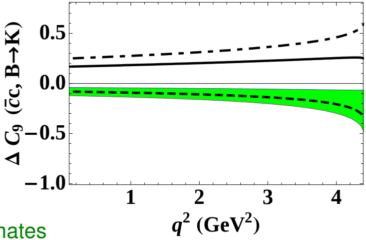
$$\Delta C_9^{(\bar{c}c, B\to K)}(q^2) = \frac{32\pi^2}{3} \frac{\mathcal{H}^{(B\to K)}(q^2)}{f_{BK}^+(q^2)}$$

$$= (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3} \frac{\tilde{A}(q^2)}{f_{BK}^+(q^2)}$$

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b)$

loop (dash-dotted),

soft-gluon (dotted), total (solid)



- $O(\alpha_s)$ effects to be included separately,
  can be taken from QCD factorization estimates

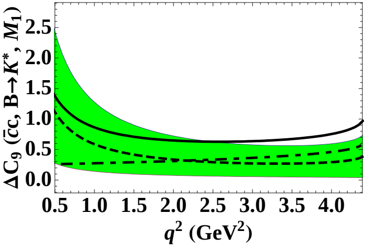
Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2\, V^{BK^*}(q^2)} ,$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\text{GeV}^2) = 0.7^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\text{GeV}^2) = 0.8^{+0.6}_{-0.4}$
$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\text{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Can we access the large q^2 region ?

- analyticity of the hadronic matrix element in q^2,
  $\oplus$ unitarity $\Rightarrow$ hadronic dispersion relation:

$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2 \Big[\sum_{\psi=J/\psi,\psi(2S),..} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)} \Big]$$

- the residues $|A_{B\psi K}|$ and $|f_\psi|$ determined by $BR(B\to\psi K)$, $BR(\psi\to\ell^+\ell^-)$
- complex FSI phase in each $A(B\to\psi K)$, (Im part in $(p+q)^2$)
  destructive interferences between different ψ terms possible !
- we only control $\mathcal{H}^{(B\to K)}$ at small q^2 with LC OPE
  use data for J/ψ and $\psi(2S)$ and fit an eff.pole ansatz for the rest

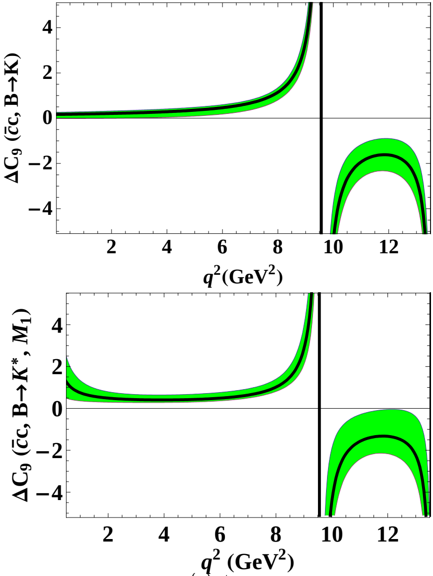
Can we control the $B \to \psi K^{(*)}$ amplitudes?

- the naive factorization fails in $B \to \psi K^{(*)}$,
  $(\psi = J/\psi, \psi(2S), \psi(3770))$,
  $A(B \to \psi K) \sim \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1 + c_2/3) f_\psi f_{BK}^+ (q^2 = m_\psi^2)$
  predicts $\Gamma(B \to \psi K) \ll$ *exp.*
- indicating large nonfactorizable contribuitons
- in accordance with QCD factorization expectations
- *previous uses of dispersion relation:*
  factorizable and positive residues, universal a_2
  [F. Krüger, L. Sehgal (1997),...]

Charm-loop effect at large q^2

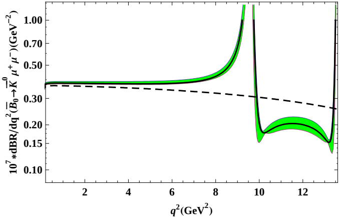
solid- central input,
green-shaded - uncertainties

- the dispersion relation ansatz coinsides with OPE result at $q^2 < 4.0\ \text{GeV}^2$ and is valid up to $s = 4m_D^2$ (at $q^2 < m_{J/\psi}^2$ largely independent of higher-states ansatz)



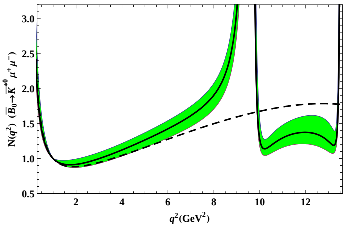
Influence on the observables for $B \to K\ell^+\ell^-$

- adding $\delta C_9(q^2)$ to the decay amplitude
- differential distribution in q^2 with (solid) and without (dashed) charn-loop effect



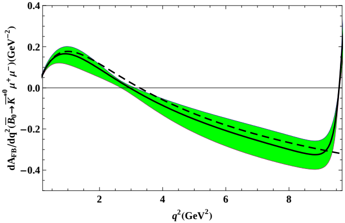
Observables for $B \to K^* \ell^+ \ell^-$

- differential distribution in q^2 with (solid) and without (dashed) charm-loop effect



- forward-backward asymmetry : $q_0^2 = 2.9^{+0.2}_{-0.3} \mathrm{GeV}^2$

$\sim$ 10% larger without nonfactorizable correction



OPE at large timelike q^2

- taking large $q^2 < 0$, $|q^2| \sim m_b^2 \gg 4m_c^2$,
  one indeed recovers the local OPE:

$$\frac{q\Lambda_{QCD}}{4m_c^2 - q^2} \to 1/m_b$$

- is a duality transition to large timelike q^2 possible ?

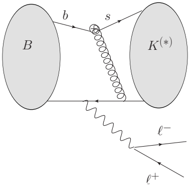
  dispersion relation with interfering $\psi' s$:
  the situation different from $R(e^+e^- \to hadrons)$!

- a possible solution for the low recoil region
  $q^2 = 15 - 20$ GeV^2:
  use dispersion relation and refine the resonance model
  including all charmonium levels
- more data on $B \to \psi K^{(*)}$ and $B \to \bar{D}DK^{(*)}$ needed
- local OPE $\oplus$ duality at $q^2 > (m_B - m_{K^*})^2$ can be used to additionally constrain the dispersion relation

Summary on nonlocal contributions in $B \to K^{(*)}\ell^+\ell^-$

- contribution of four-quark operators with c-quarks in $B \to K^{(*)}\ell^+\ell^-$, obtained at $q^2 \ll 4m_c^2$ from light-cone OPE
- soft-gluon emission - a nonlocal operator, effective resummmation of local operators, $\sim 1/(4m_c^2 - q^2)$-suppression
- LCSR with B meson DA's used to calculate the emerging hadronic matrix element
- hard-gluon nonfactorizable charm-loop effects also accessible with LCSR but technically very difficult, we can use QCD factorization estimates
- charm loop with soft-gluon contribution yields an important correction to C_9 , especially for $B \to K^*\ell^+\ell^-$, also ΔC_7 for $B \to K^*\gamma$

- analytical continuation using dispersion relation and data on $B \to \psi K$ allows to access $q^2 \leq 4m_D^2$
- accuracy can be improved by including $O(\alpha_s)$ effects and CKM/Wilson coeff. suppressed loop corrections
- other nonlocal effects, e.g. from O_{8g}, is there a soft contribution?



- the large recoil (small $q^2 \leq 5-6$ GeV^2) region is under theory control
- the low recoil (large q^2) region has a nontrivial strong dynamics and remains a phenomenological challenge

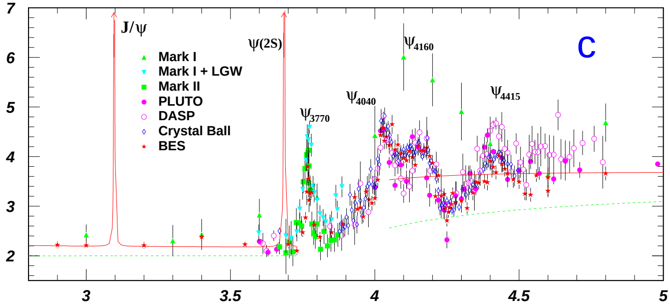
"Best accuracy is with weapons that have low recoil"

from an advertisement of a store selling firearmes in the US

BACKUP SLIDES

Charm loop turns charmonium

- at $q^2 \to m^2_{J/\psi}$, $\bar{c}c$ loop becomes a hadronic state:
  $B \to K^{(*)}\ell^+\ell-$ = { $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$}
- heavier ψ-levels (charmonia with $J^P = 1^-$) at $q^2 = m^2_\psi$,
  $\bar{c}c$ states with the masses up to $m_B - m_K^{(*)} \simeq 4.8\text{GeV}(\simeq 4.4\text{GeV})$
- spectrum of ψ states as seen in $e^+e^- \to$ *hadrons*



[PDG, V.V. Ezhela et al. hep-ph/0312114]

Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_{1,2}\,,$$

$$\left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
$$[\Delta C_7^{(\bar{c}c, B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (3)$$

  [P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]