B form factors from lattice QCD

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in collaboration with

- ✤ Stefan Meinel (W&M) & Zhaofeng Liu (IHEP)
- ✤ Hart, Horgan, Müller
- ✤ HPQCD

Outline

- Sources of systematic errors in LQCD w/ *b*
- A couple features of lattice NRQCD for *b*
- A few plots showing current status
- Outline of our calculation
- Fitting correlation functions
- Some preliminary results
- Outlook

Lattice QCD

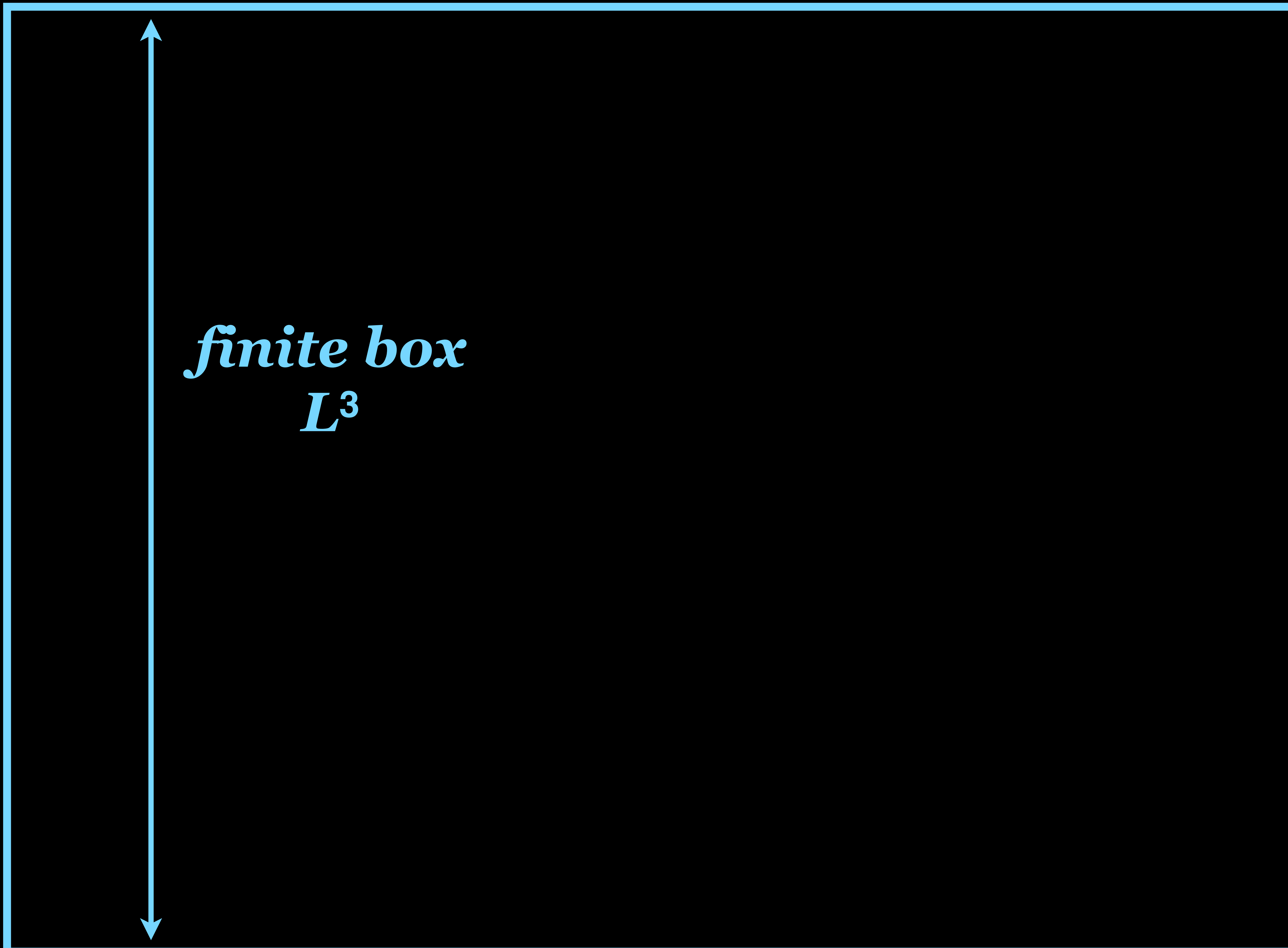
Lattice QCD

in 1 minute

Lattice QCD

in 1 minute

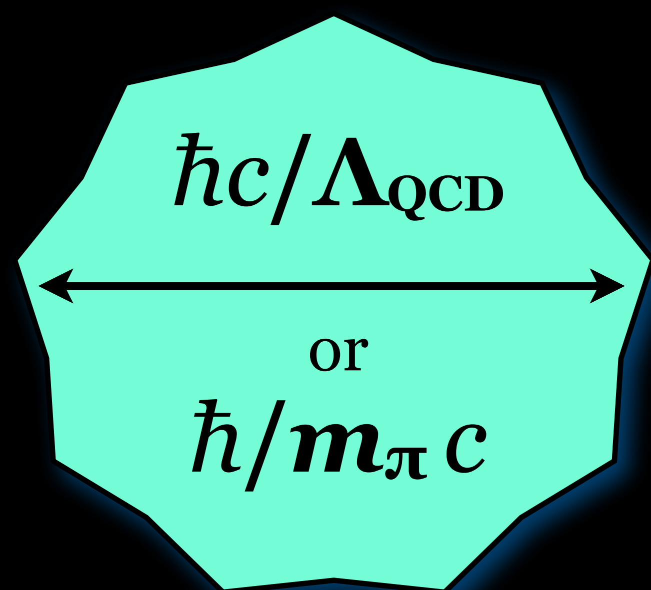
a silent film
by Matthew Wingate


finite box
L^3

finite imaginary time

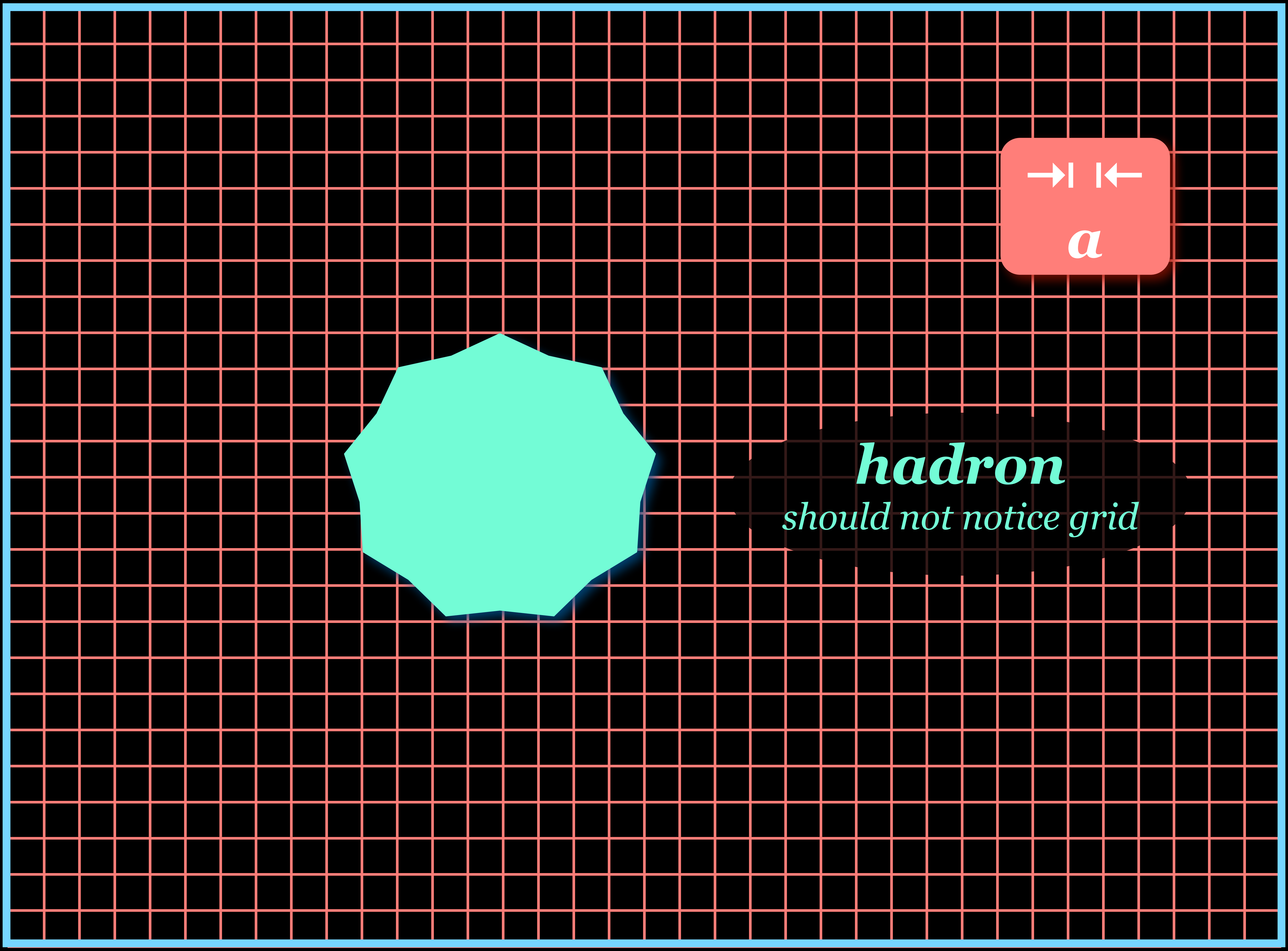
L_τ

$\therefore$ ***evolution operator:*** **$\exp(-\hat{H}\tau)$**

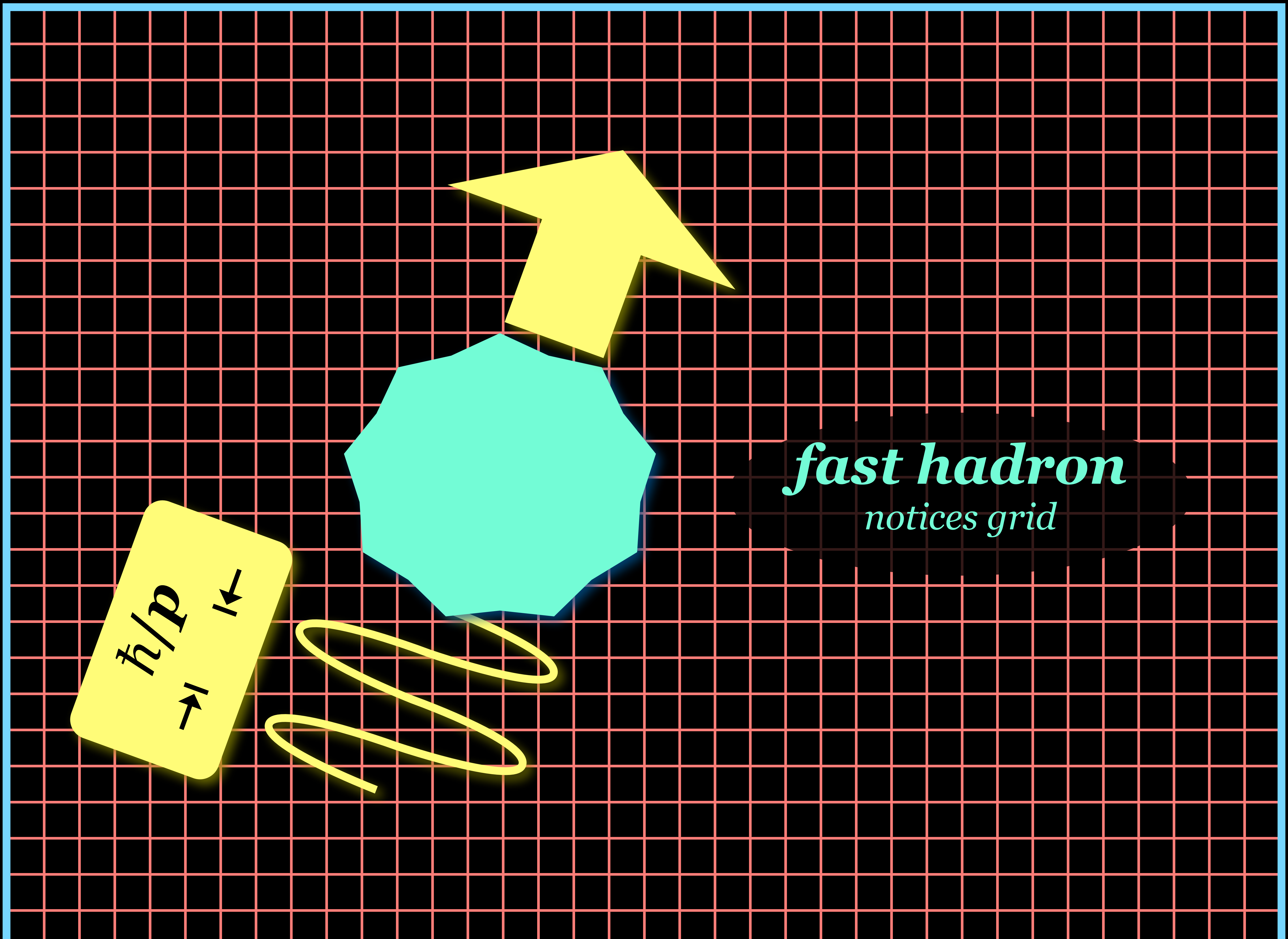


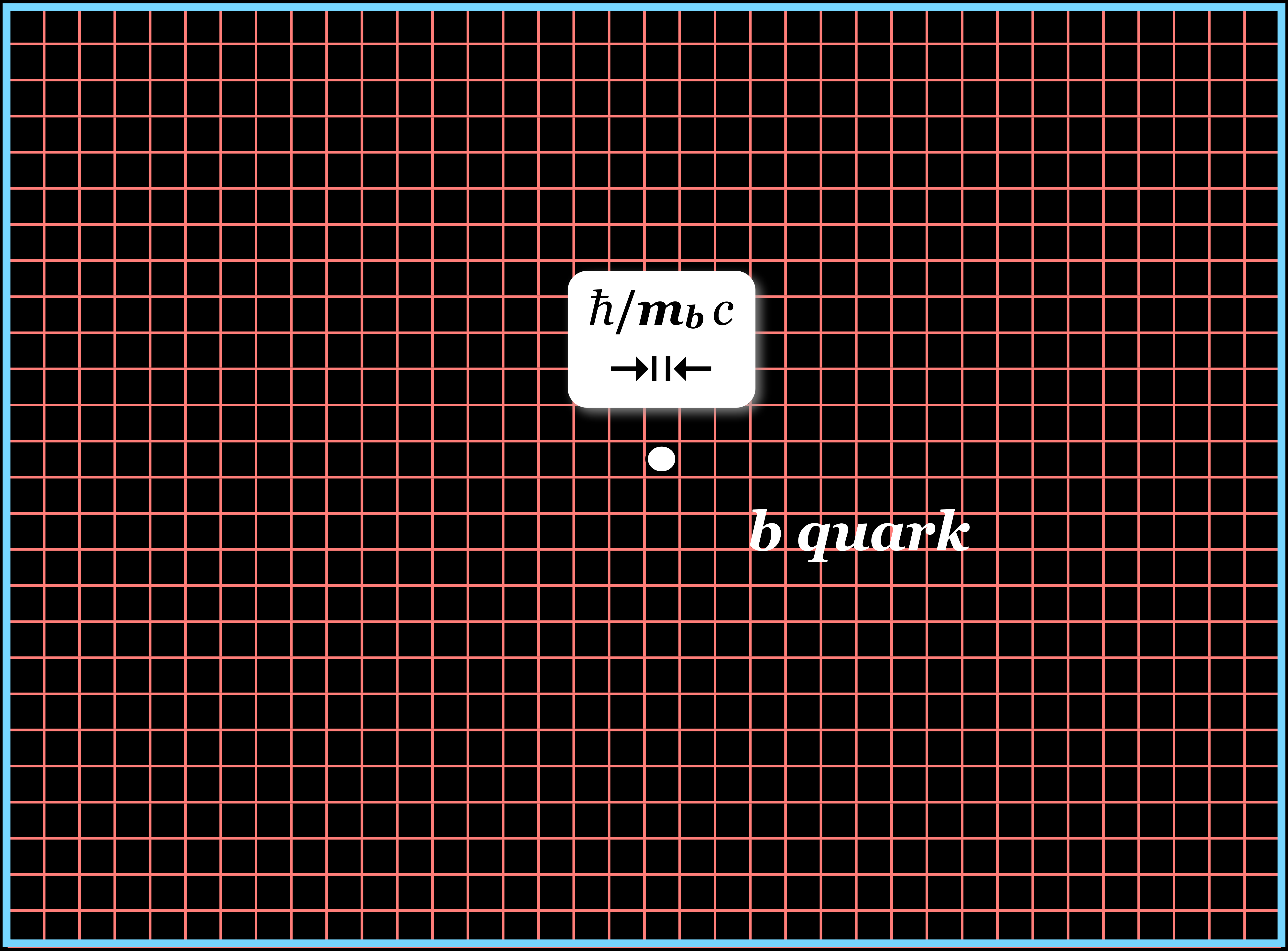
hadron

should be smaller than box

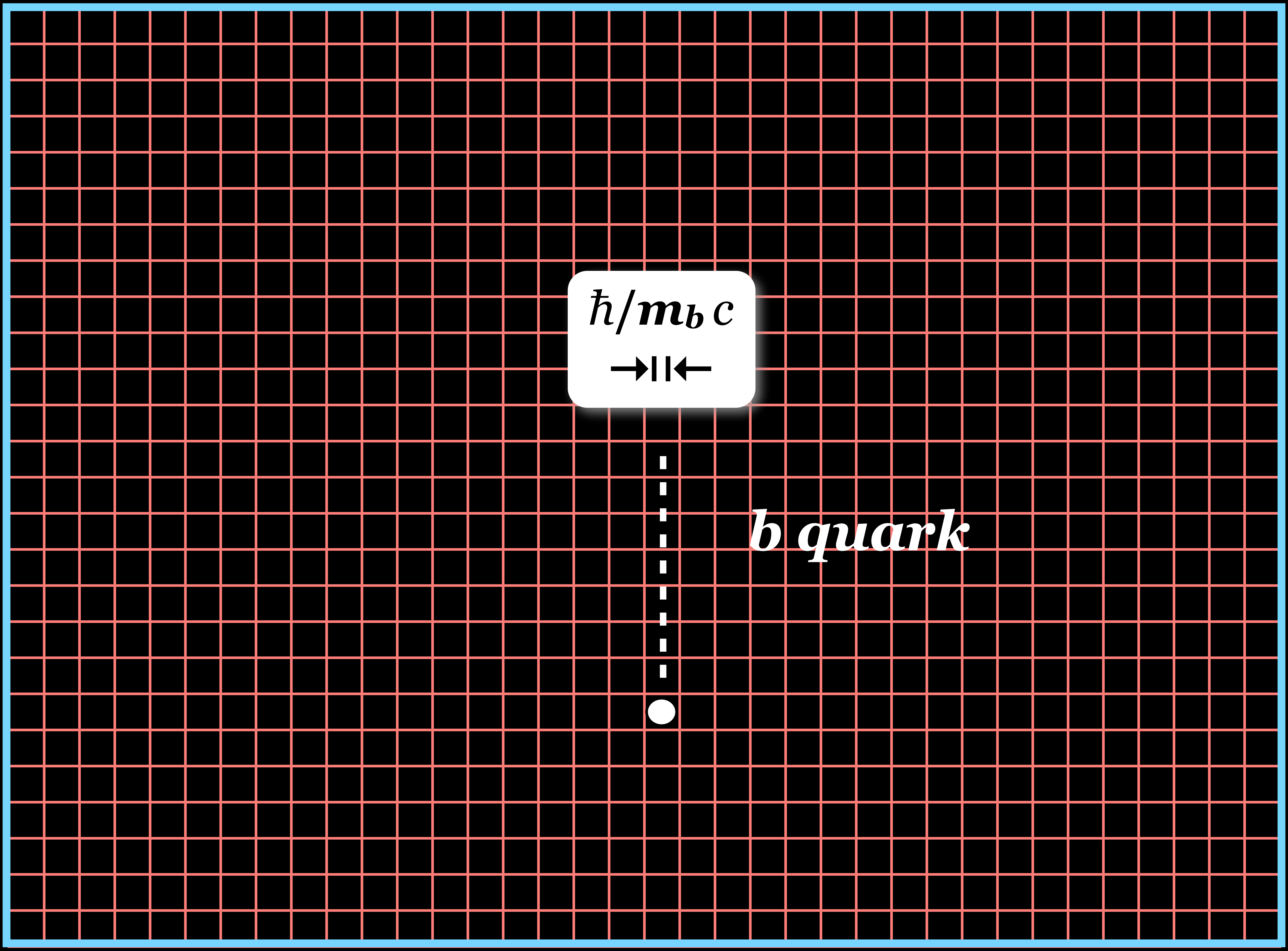


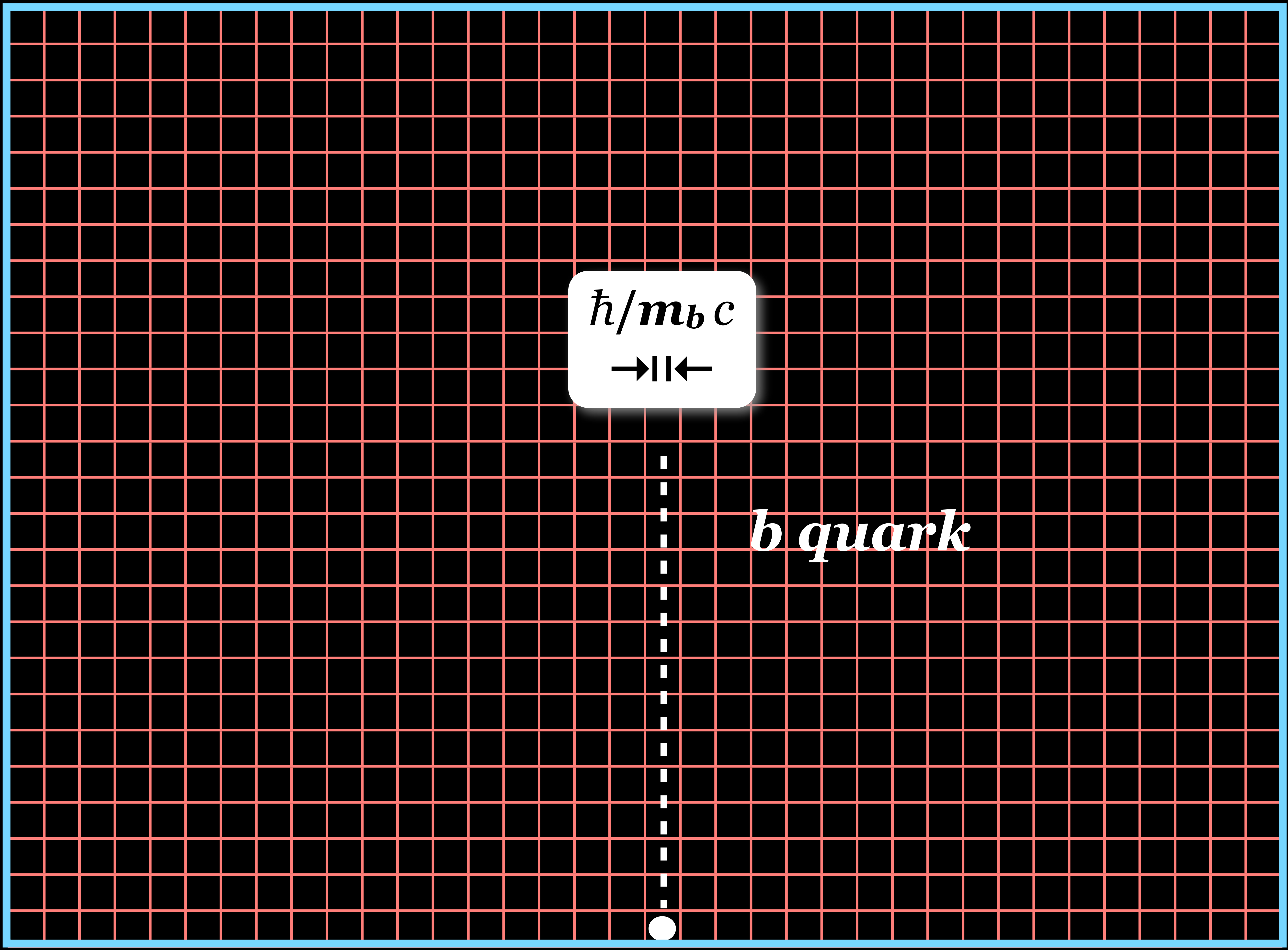
a
hadron
should not notice grid

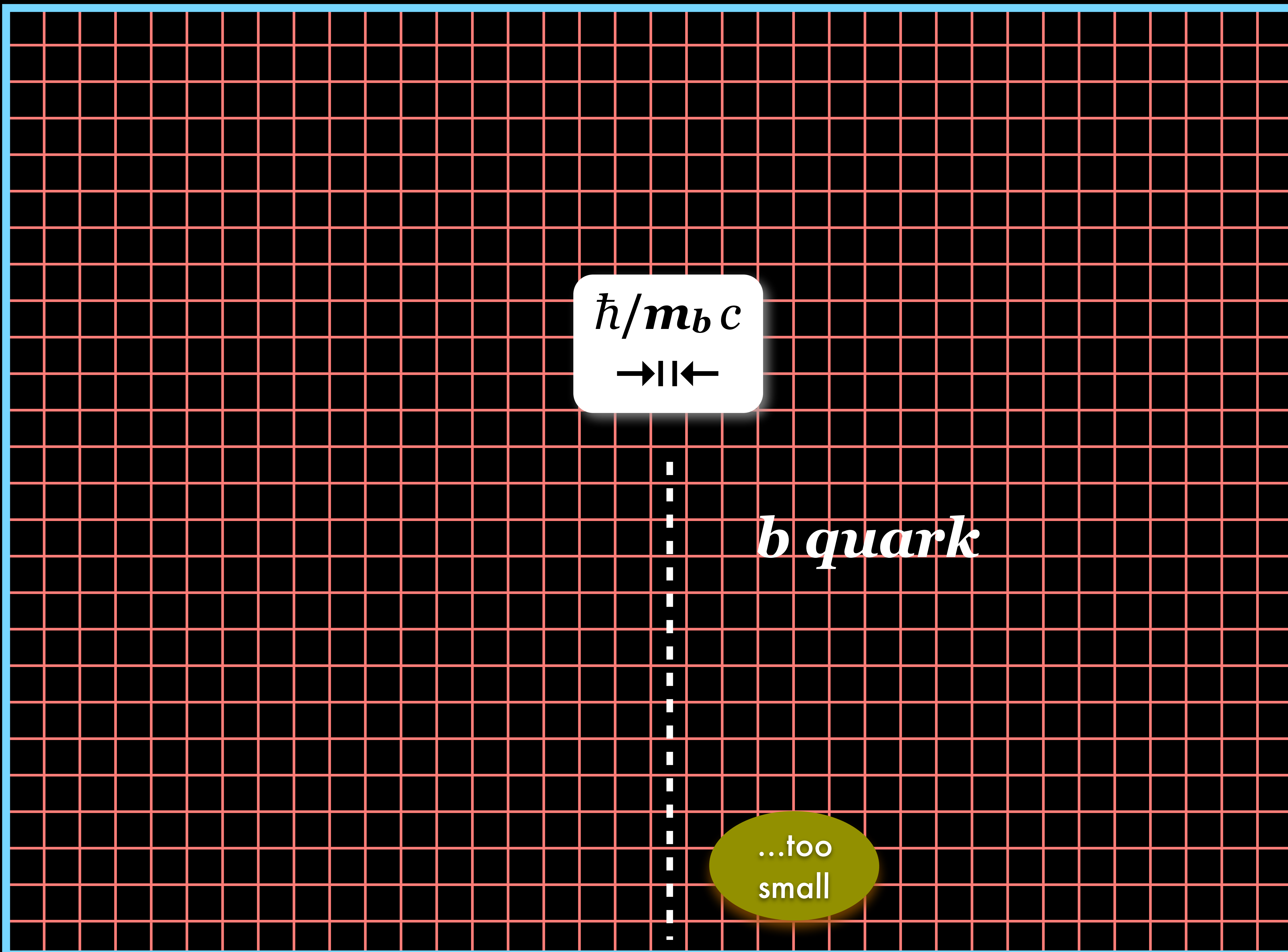

ħ/p
fast hadron
notices grid



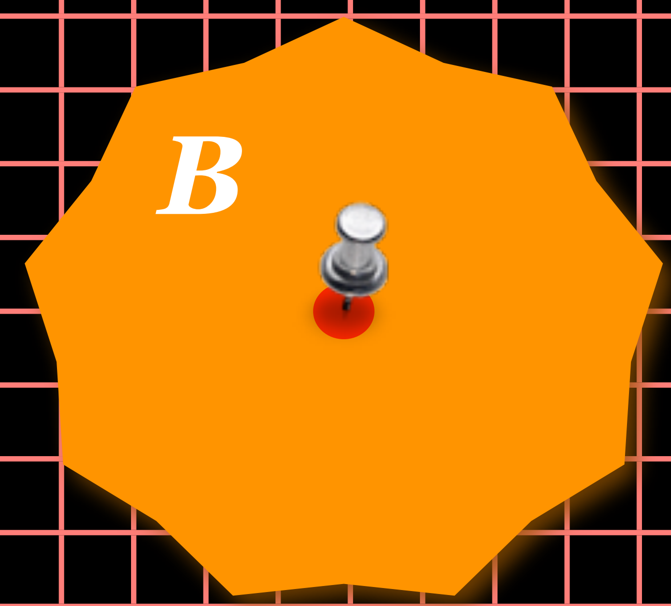
$\hbar/m_b c$
b quark


$\hbar/m_b c$
b quark


$\hbar/m_b c$
b quark


$\hbar/m_b c$
b quark
...too
small

HQET (NRQCD) *to the rescue!*



b as static color source

hadronic physics as “brown muck”

einde

Foldy-Wouthuyesen-Tani transformation

Generates operators which appear at tree-level

$$\mathcal{L} \;=\; \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

Decouple quark and anti-quark degrees-of-freedom in $1/m$

$$\Psi \;=\; \exp\left(\frac{i\gamma^j D_j}{2m}\right)\Psi_{(1)}$$

$$\mathcal{L} \;=\; \bar{\Psi}_{(1)}(i\gamma^0 D_0 - m)\Psi_{(1)} \;+\; \sum_{n=1} \frac{1}{m^n}\bar{\Psi}_{(1)}O_{(1)n}\Psi_{(1)}$$

Further field redefinitions remove terms order-by-order which do not commute with γ^0

Decoupling

$$\Psi_{(k)} \;=\; e^{-imt\gamma^0}\begin{pmatrix}\psi\\ \xi\end{pmatrix} \qquad \bar{\Psi}_{(k)} \;=\; \left(\psi^\dagger, -\xi^\dagger\right) e^{imt\gamma^0}$$

$$\begin{aligned}\mathcal{L} \;=\; \psi^\dagger&\left[iD_0 \;+\; \frac{\vec{D}^2}{2m} \;+\; \frac{g}{2m}\vec{\sigma}\cdot\vec{B}\right]\psi \\ &+\xi^\dagger\left[iD_0 \;-\; \frac{\vec{D}^2}{2m} \;-\; \frac{g}{2m}\vec{\sigma}\cdot\vec{B}\right]\xi \;\; + \mathcal{O}(1/m^2)\end{aligned}$$

Note we implicitly worked in a frame where the heavy quark is slow

Lattice QCD in a Nutshell

Gluonic expectation values

$$\langle \Theta \rangle = \frac{1}{Z} \int [d\psi][d\bar{\psi}][dU]\, \Theta[U]\, e^{-S_g[U] - \bar{\psi} Q[U] \psi}$$
$$= \frac{1}{Z} \int [dU]\, \Theta[U]\, \det Q[U]\, e^{-S_g[U]}$$

Fermionic expectation values

Probability weight

$$\langle \bar{\psi} \Gamma \psi \rangle = \int [dU] \boxed{\frac{\delta}{\delta \bar{\zeta}} \Gamma \frac{\delta}{\delta \zeta}\, e^{-\bar{\zeta} Q^{-1}[U] \zeta}} \det Q[U] e^{-S_g[U]} \Big|_{\zeta, \bar{\zeta} \to 0}$$

Determinant in probability weight difficult

1) Requires nonlocal updating; 2) Matrix becomes singular

Partial quenching =

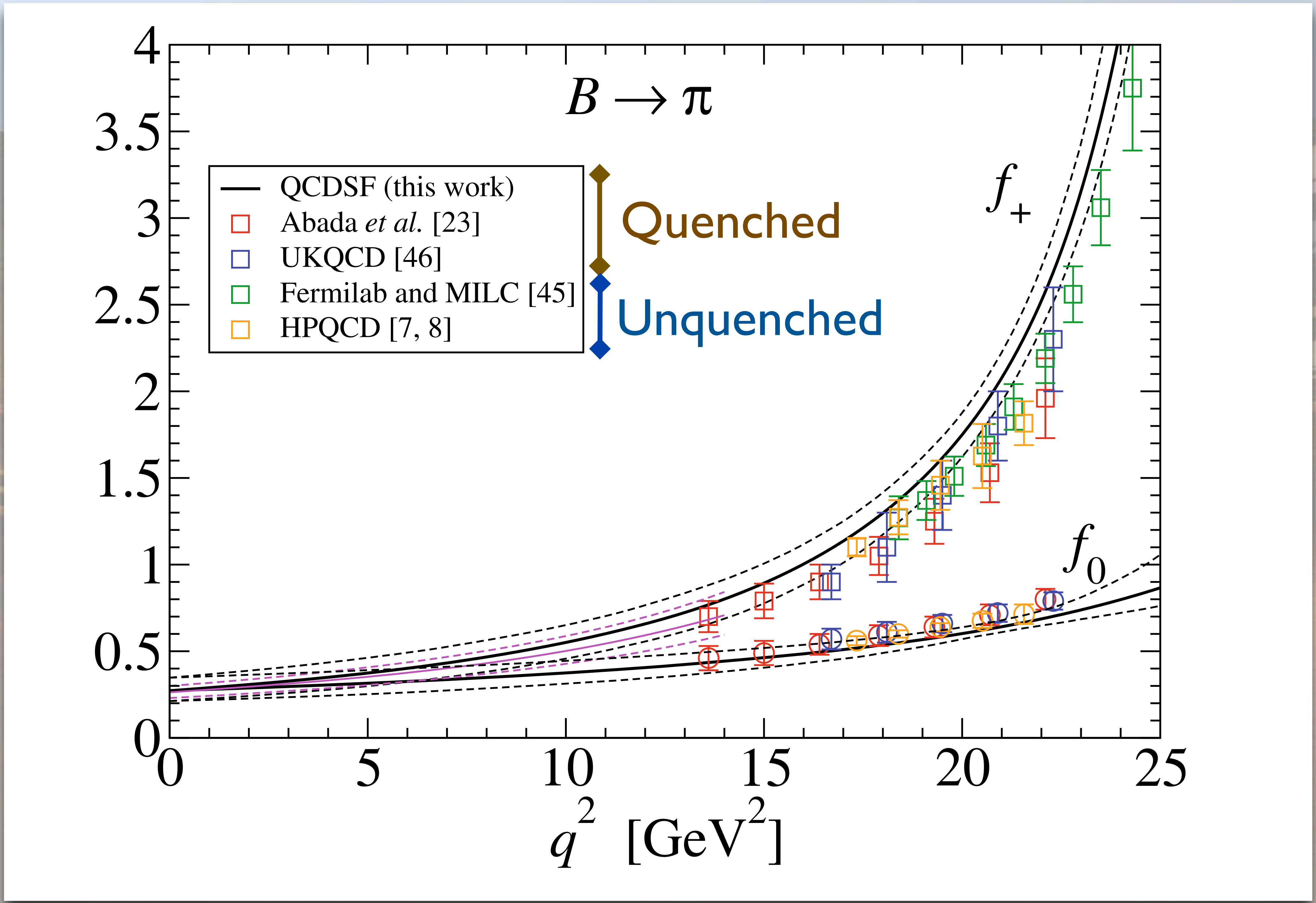
different mass for valence Q^{-1} than for sea $\det Q$

NRQCD Discretization

- Euclidean spacetime
- Replace derivatives by difference operators
- Lattice acts as regulator of the EFT
- Require $1/a \ll m_b$
- Errors are reducible, as in continuum EFTs
- Compare w/ LHQET: higher dimension operators not included in lattice action
- Can study multi-b systems
- NRQCD has better signal-to-noise vs. static action

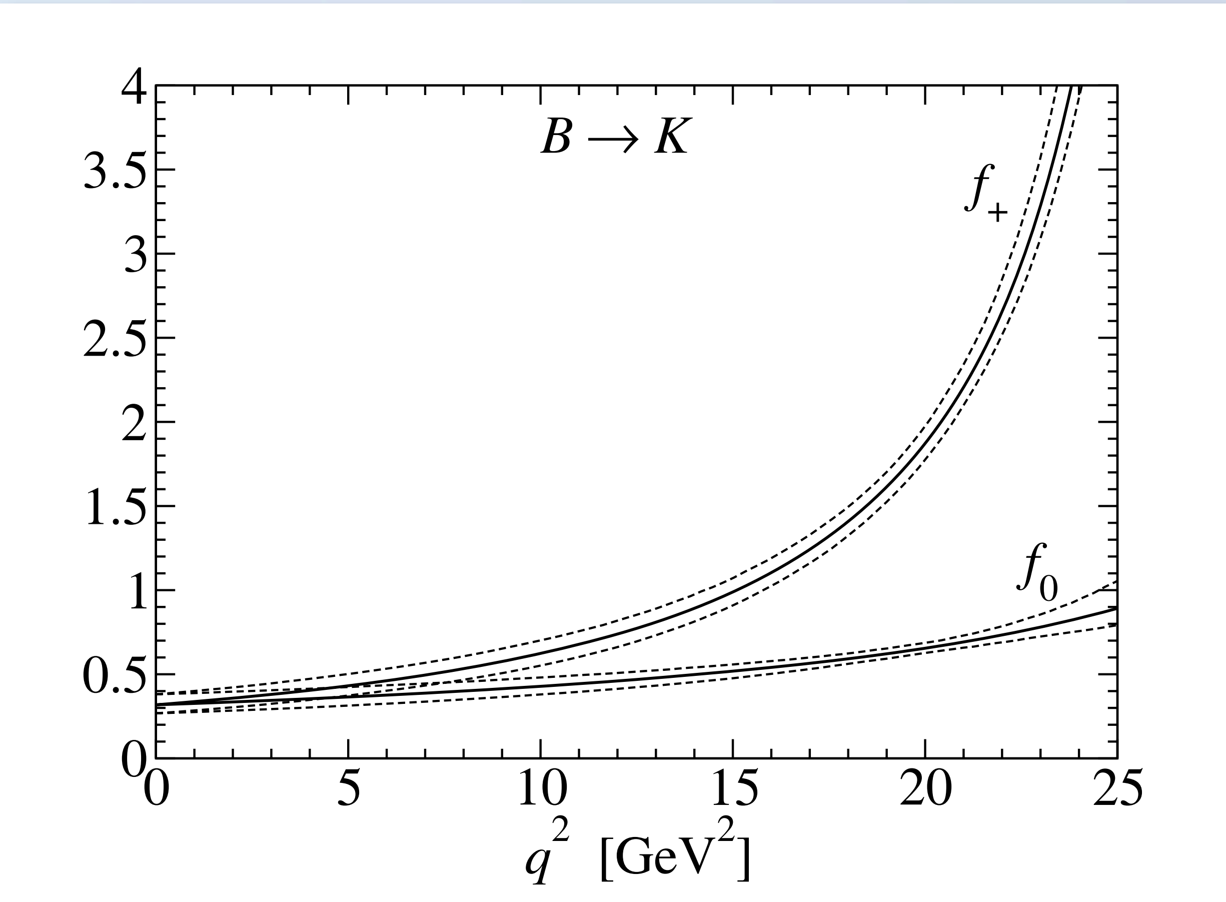
Recent summary

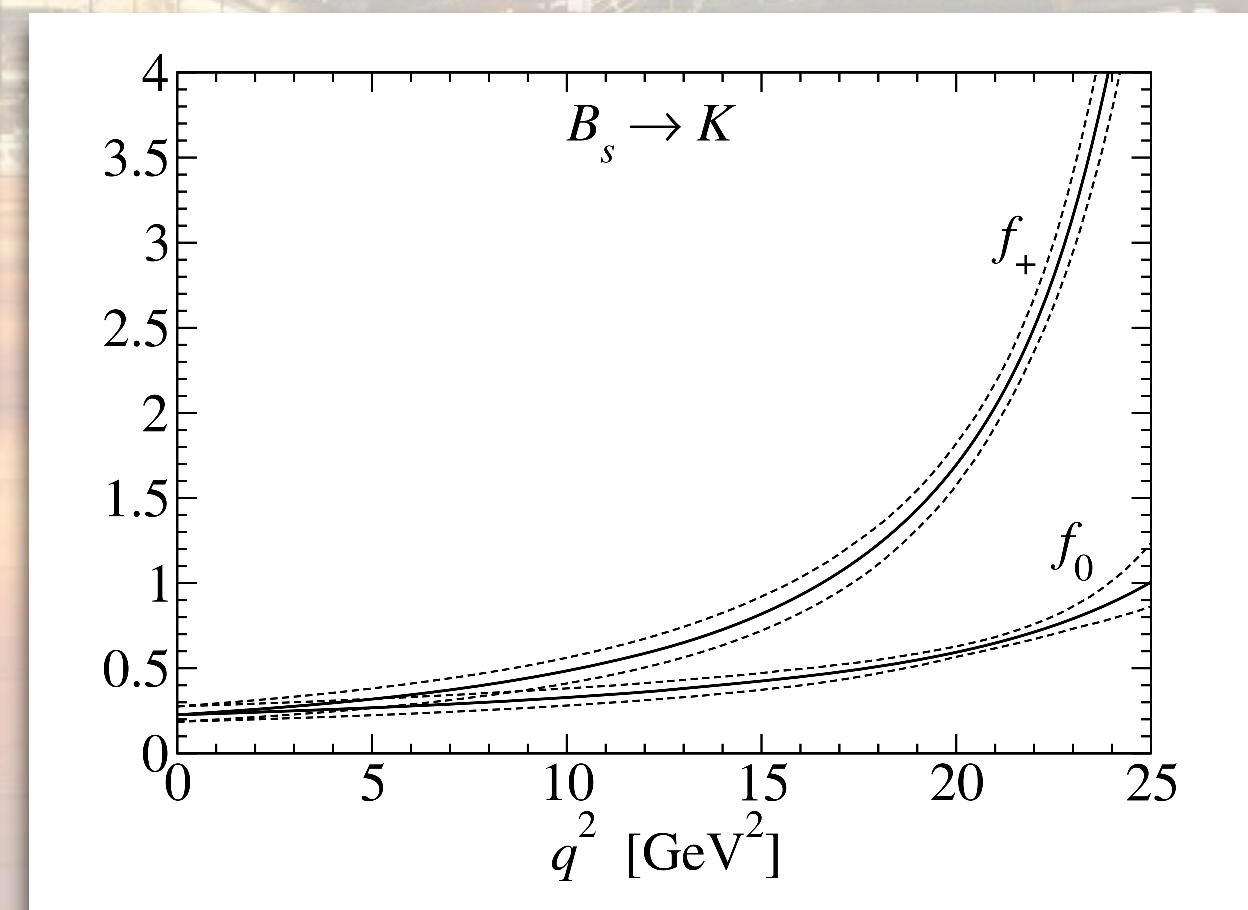
from Al-Haydari, *et al.* (QCDSF) EPJ A43 (2010)



QCDSF (quenched)

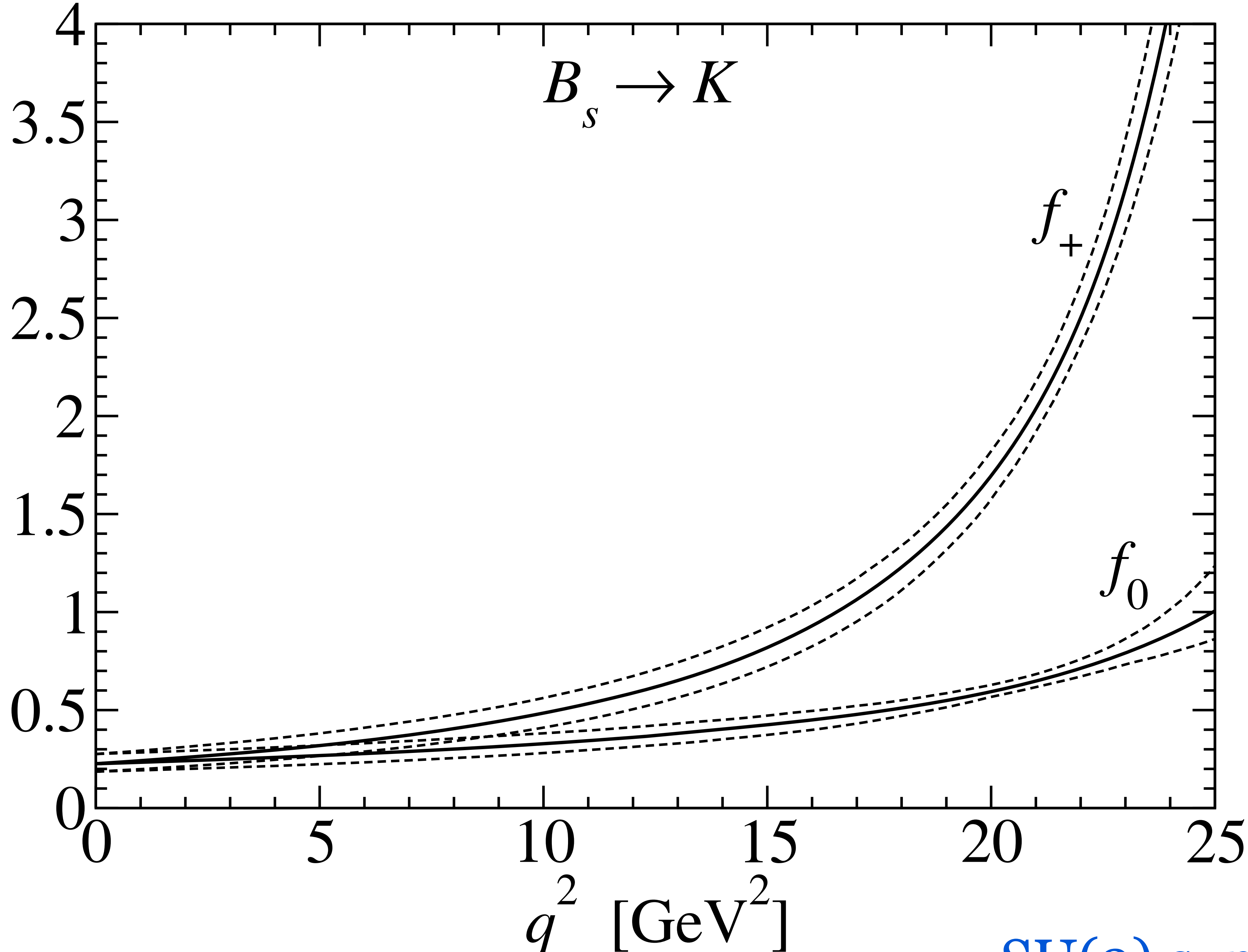
from Al-Haydari, *et al.* (QCDSF) EPJ A43 (2010)





QCDSF plots on top of each other

from Al-Haydari, *et al*. (QCDSF) EPJ A43 (2010)



SU(3) symmetry

QCDSF plots on top of each other

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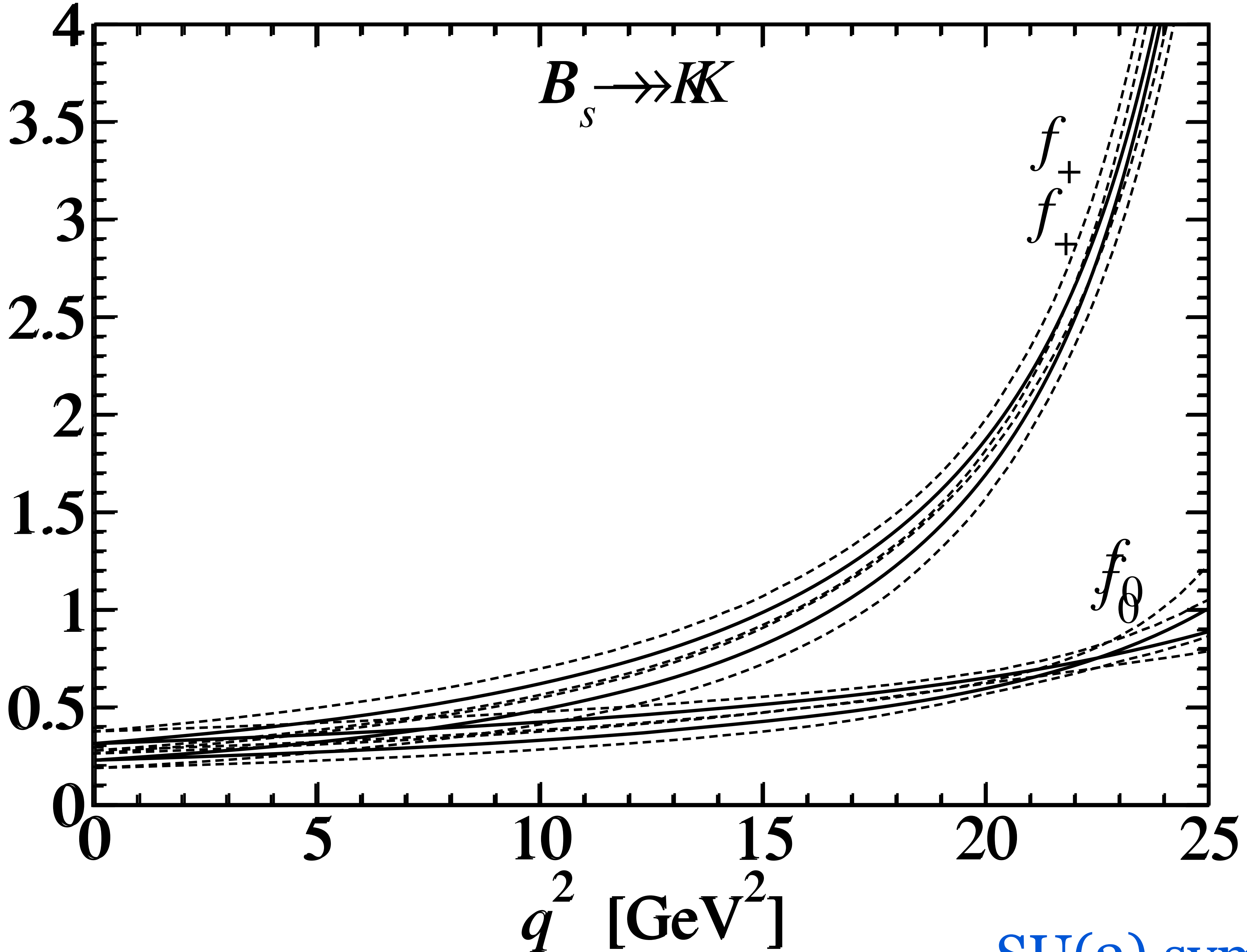

SU(3) symmetry

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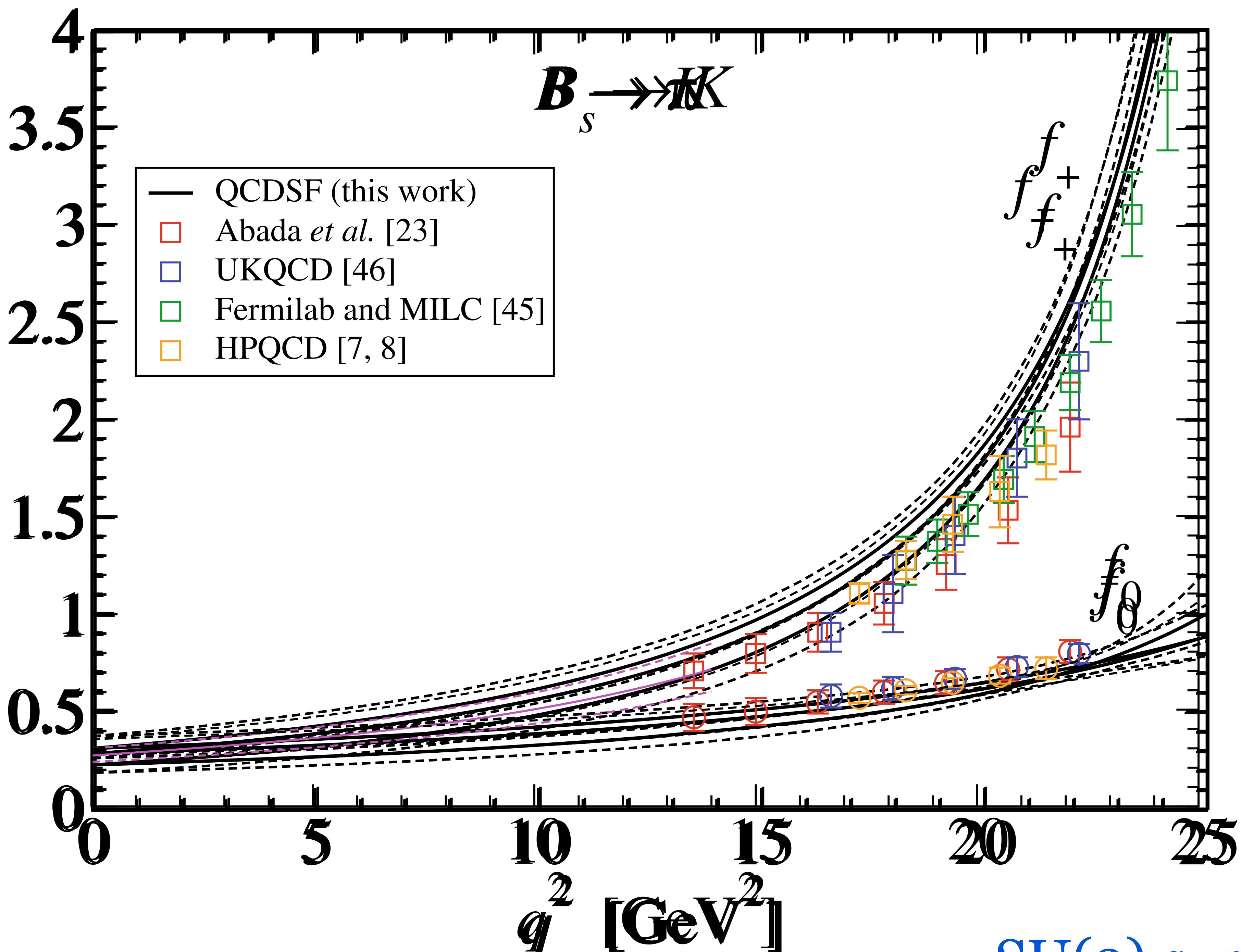


SU(3) symmetry

UKQCD quenched $B \to \rho l \nu$

Bowler, Gill, Maynard, Flynn, JHEP 05 (2004) 035

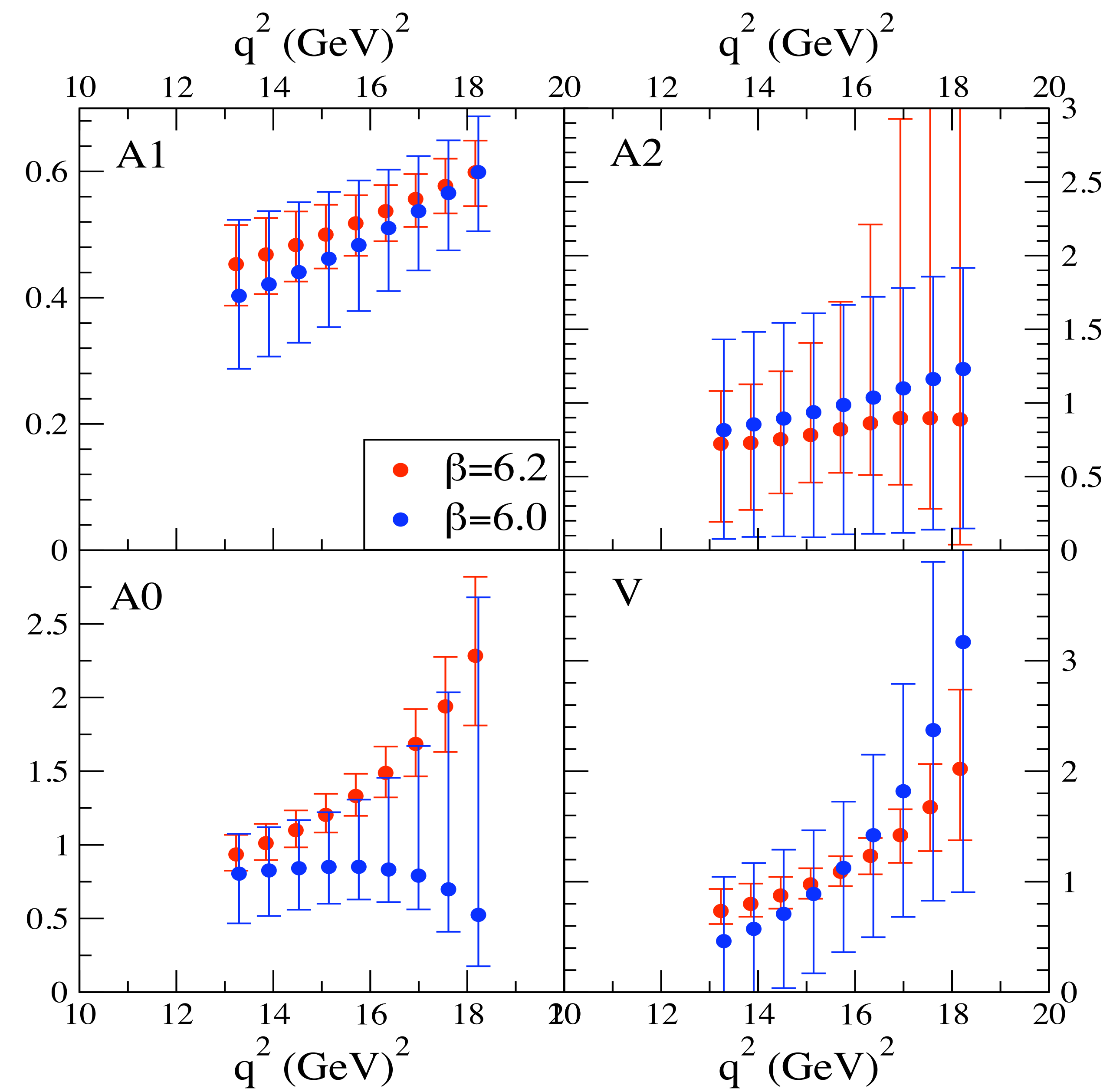
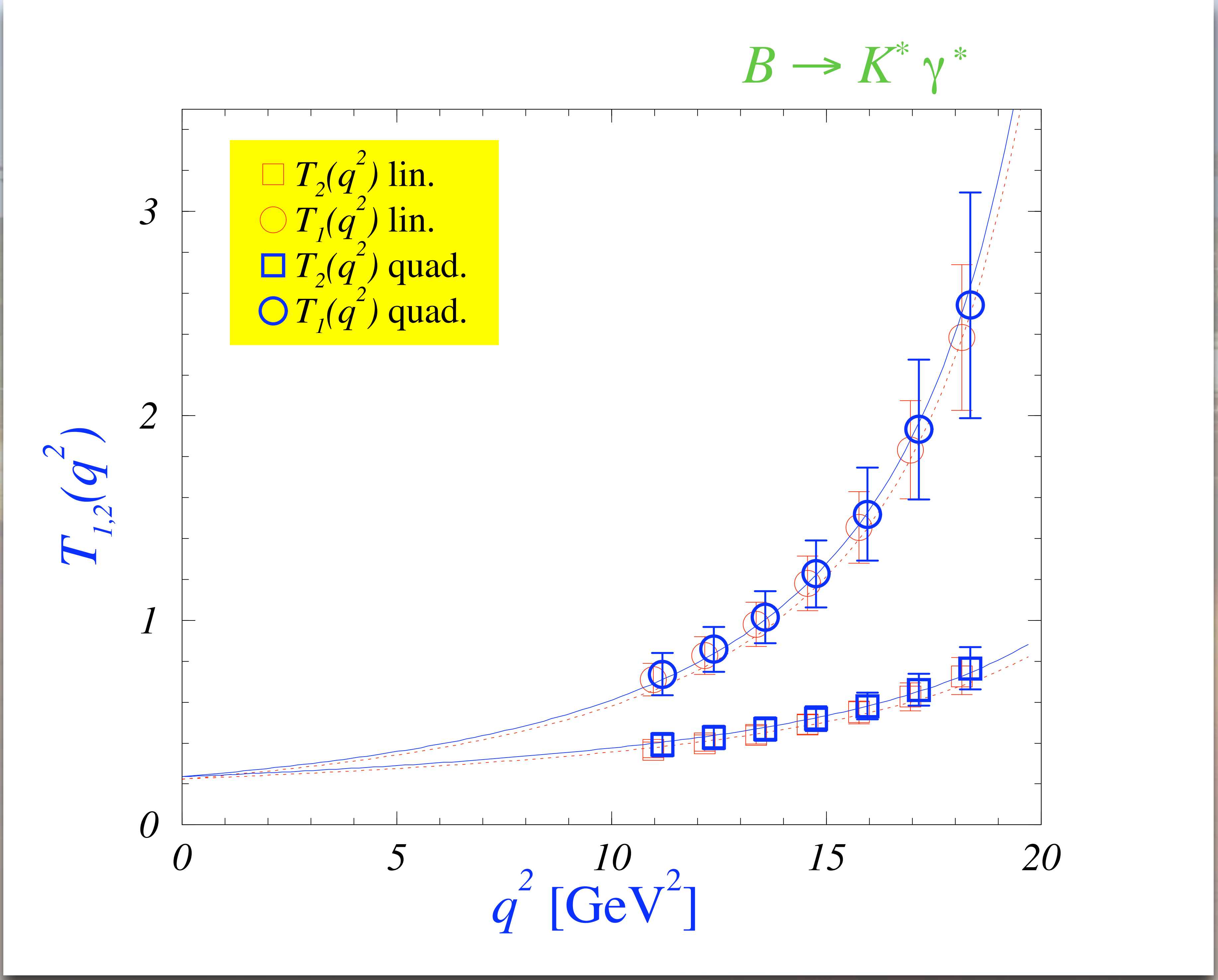


Figure 4: The form factors on both lattices. The vertical scale is different for each form factor.

BLM quenched $B \to K^*$

Bećirević-Lubicz-Mescia, Nucl. Phys. B769, 31 (2007)



Lattice data

High statistics

MILC lattices (2+1 asqtad staggered)

	a(fm)	am_{sea}	Volume	$N_{conf} \times N_{src}$	am_{val}	m_π (MeV)
coarse	$\sim$0.12	0.007/0.05	$20^3 \times 64$	2109×8	0.007/0.04	~300
		0.02/0.05	$20^3 \times 64$	2052×8	0.02/0.04	~460
fine	$\sim$0.09	0.0062/0.031	$28^3 \times 96$	1910×8	0.0062/0.031	~320

Light meson momenta (units of $2\pi/L$)

	$p^2/(2\pi/L)^2$
$(p_x, p_y, p_z) = (0,0,0)$.	0
$(\tilde{q}$,0,0), (0,$\tilde{q}$,0), (0,0,$\tilde{q}$), where $\tilde{q}$=1 or 2.	1 or 4
(1,1,0), (1,-1,0), (1,0,1), (1,0,-1), (0,1,1), (0,1,-1).	2
(1,1,1), (1,1,-1), (1,-1,1), (1,-1,-1).	3

Many Source/Sink separations (16 coarse, 22 fine)

So far, only v=0 NRQCD used (B at rest). Larger v (mNRQCD) next.

Leading order (HQET) current presently used.
$1/m_b$ current matrix elements computed, analysis in progress

pseudoscalar f.s.

$B \to \pi$

$B \to K$

$B_s \to K$

$B_s \to \pi_s$

vector f.s.

$B \to \rho(/\omega)$

$B \to K^*$

$B_s \to K^*$

$B_s \to \phi$

Stable for present lattice kinematics

Goal: Fit form factors for all bilinear operators
as functions of (q^2, m_{decay}, m_{spec}, a)
Generalize series expansion method
(as in $D \to K$ f.f. by HPQCD, 1008.4562)

Correlation functions

3-point function

$$C_{FJB}(\mathbf{p}',\, \mathbf{p},\, x_0,\, y_0,\, z_0) = \sum_{\mathbf{y}} \sum_{\mathbf{z}} \left\langle \Phi_F(x)\, J(y)\, \Phi_B^\dagger(z) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})} e^{-i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})}$$

2-point functions

$$C_{BB}(\mathbf{p},\, x_0,\, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_B(x)\, \Phi_B^\dagger(y) \right\rangle e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})},$$

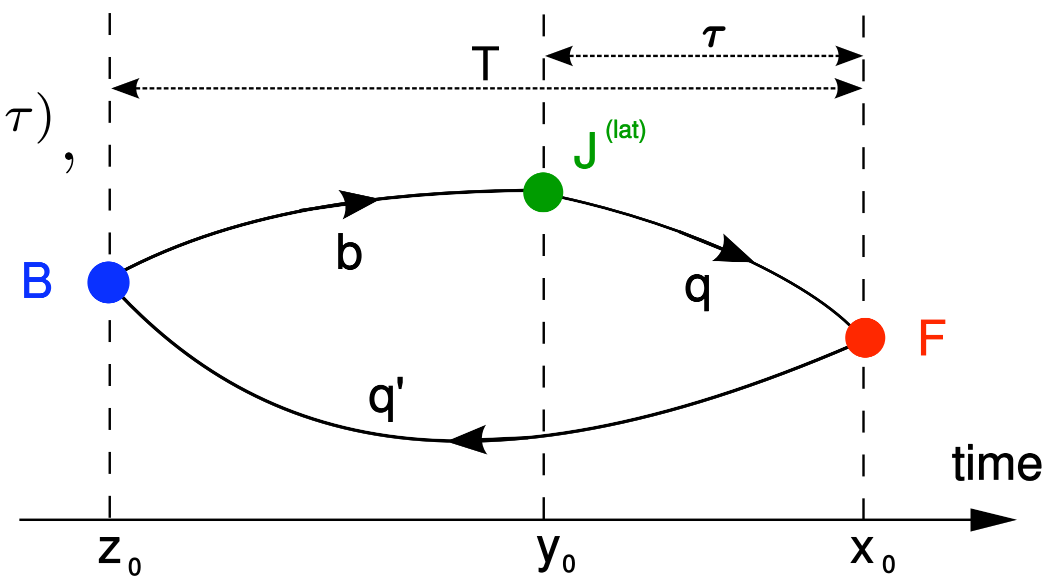
$$C_{FF}(\mathbf{p}',\, x_0,\, y_0) = \sum_{\mathbf{x}} \left\langle \Phi_F(x)\, \Phi_F^\dagger(y) \right\rangle e^{-i\mathbf{p}'\cdot(\mathbf{x}-\mathbf{y})}.$$

Large Euclidean-time behavior

$$C_{FJB}(\mathbf{p}',\, \mathbf{p},\, \tau,\, T) \rightarrow A^{(FJB)} e^{-E_F \tau}\, e^{-E_B(T-\tau)},$$

$$C_{FF}(\mathbf{p},\, \tau) \rightarrow A^{(FF)}\, e^{-E_F \tau},$$

$$C_{BB}(\mathbf{p},\, \tau) \rightarrow A^{(BB)}\, e^{-E_B \tau},$$



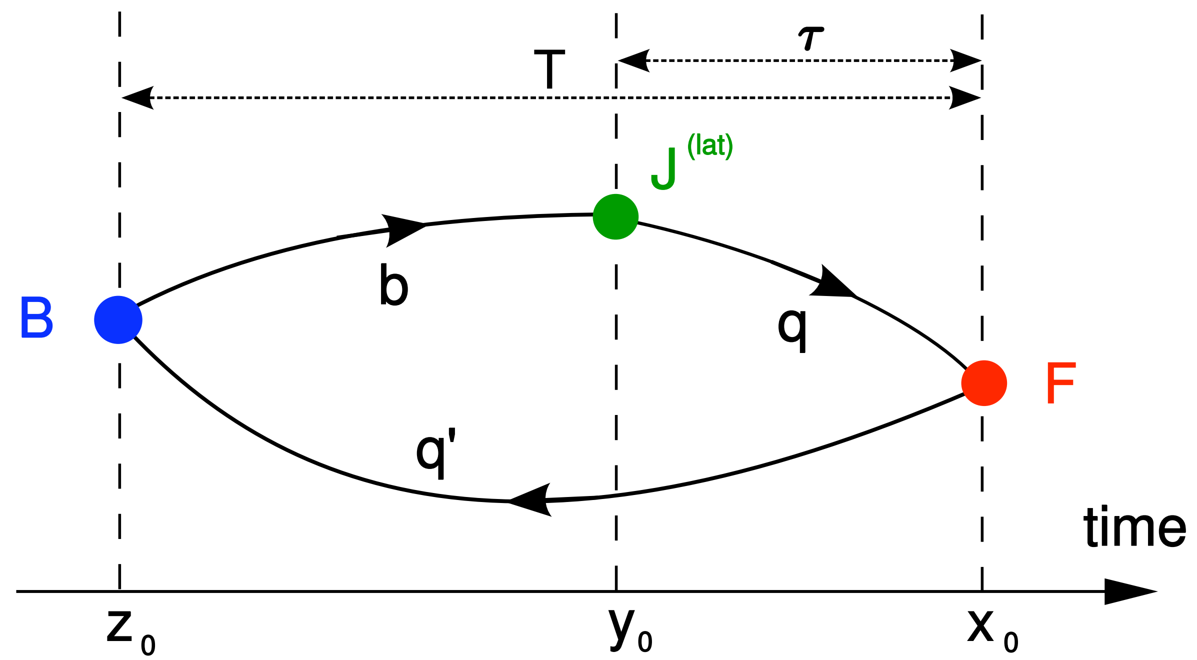
Correlation functions

Matrix element from amplitudes

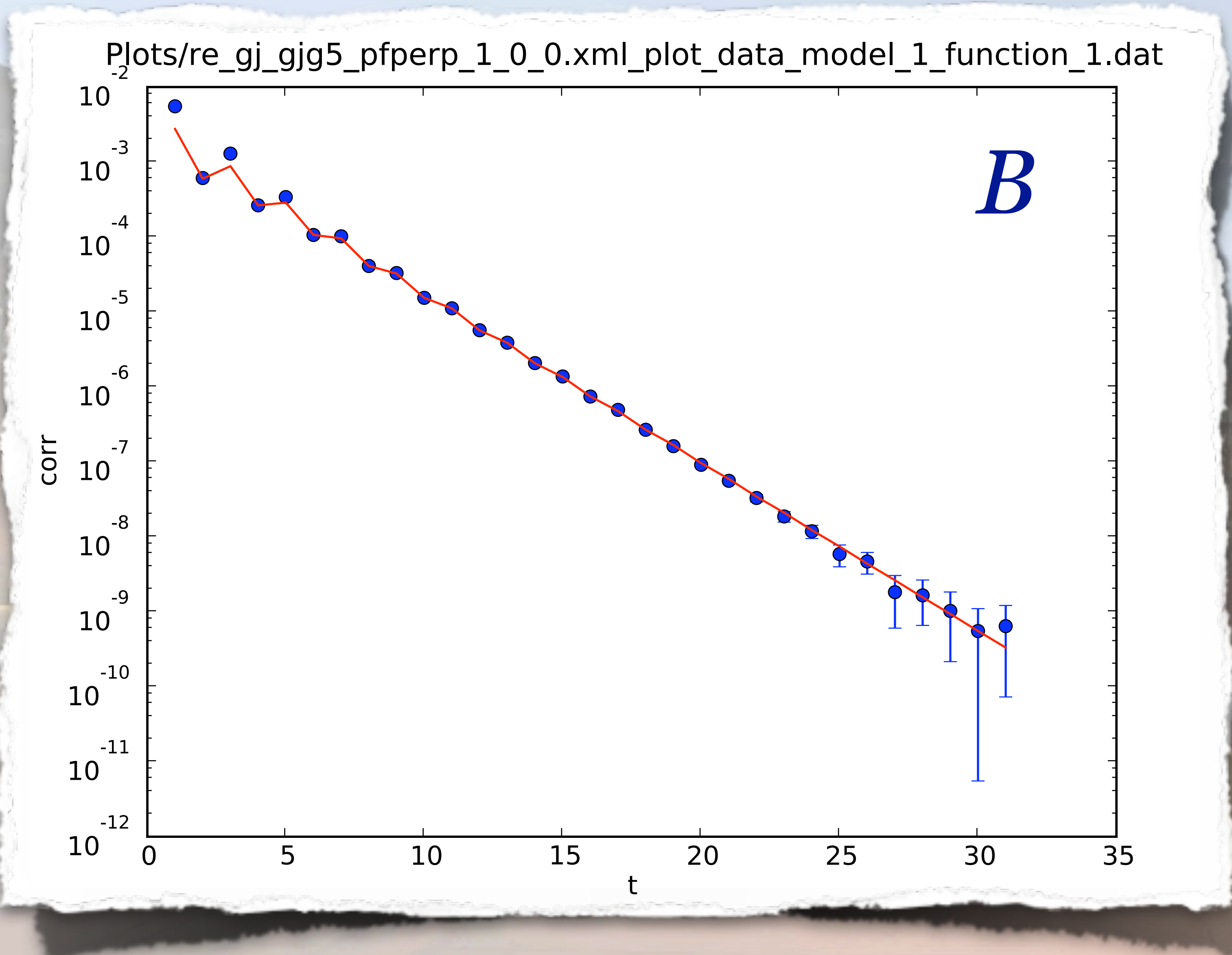
$$A^{(FJB)} = \frac{\sqrt{Z_V}}{2E_V}\frac{\sqrt{Z_B}}{2E_B}\sum_s \varepsilon_j(p',s)\,\langle V\left(p',\varepsilon(p',s)\right)|\,J\,|B(p)\rangle,$$

$$A^{(BB)} = \frac{Z_B}{2E_B},$$

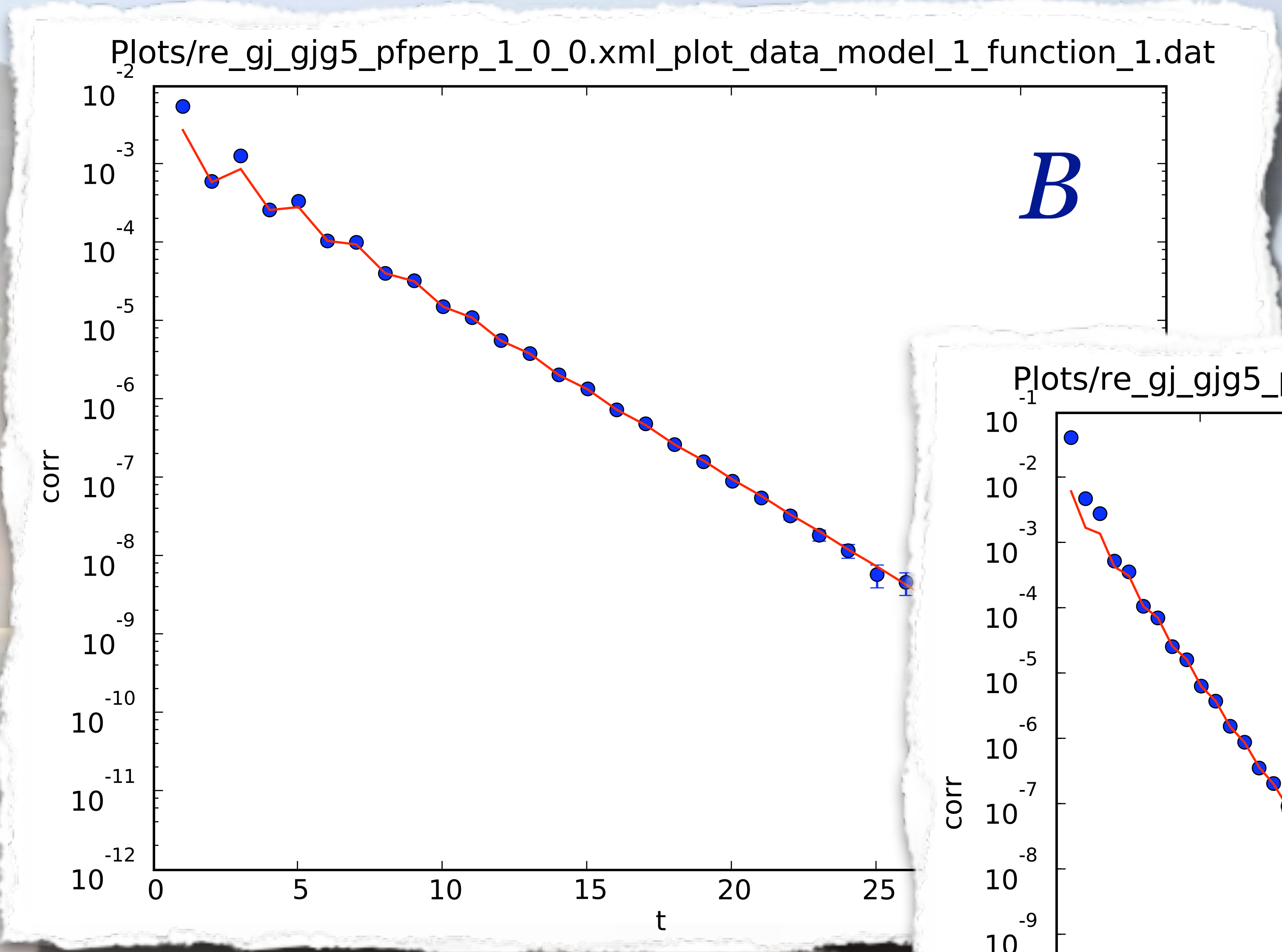
$$A^{(FF)} = \sum_s \frac{Z_V}{2E_V}\,\varepsilon_j^*(p',s)\varepsilon_j(p',s)$$

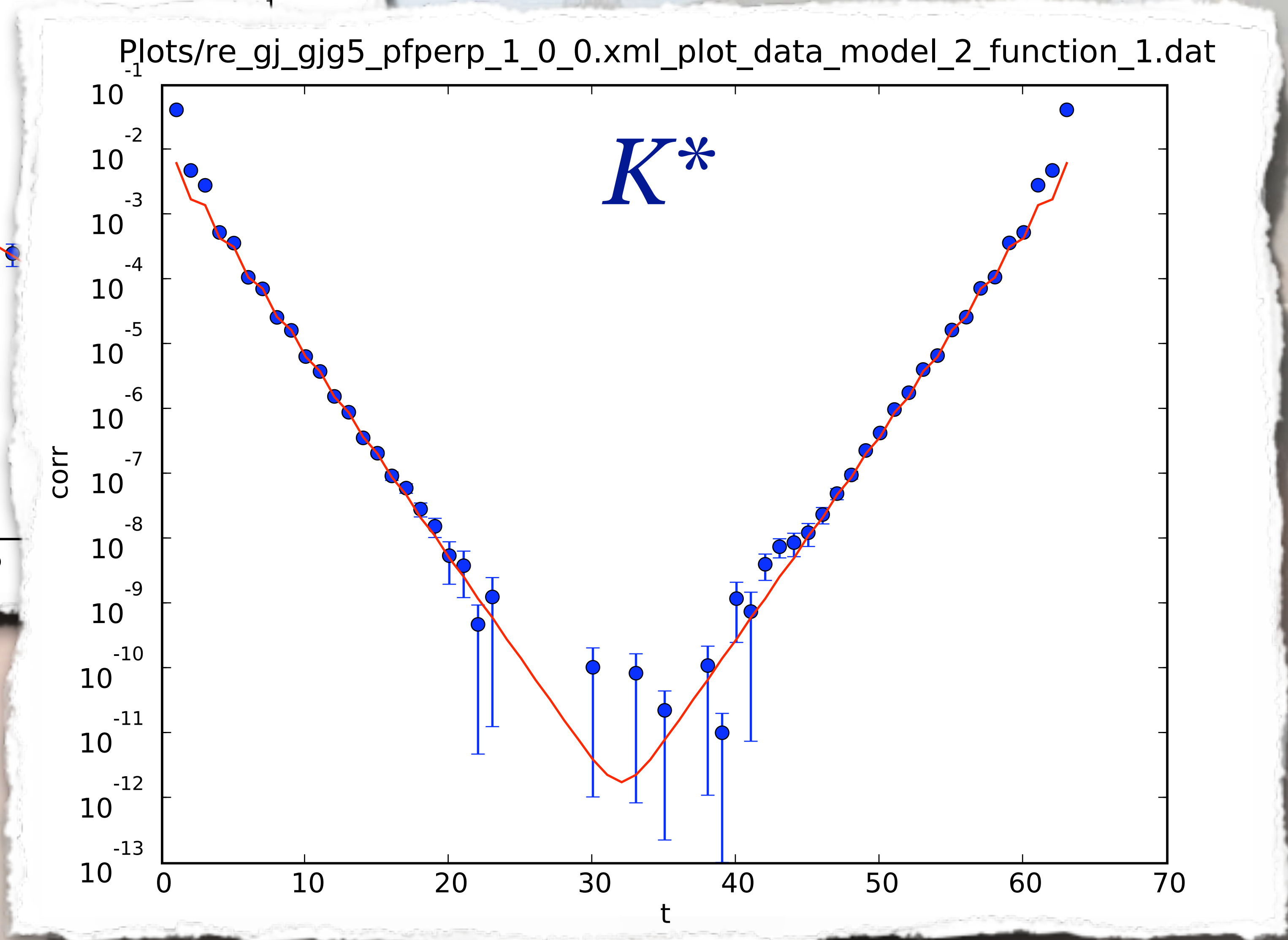


Example plots



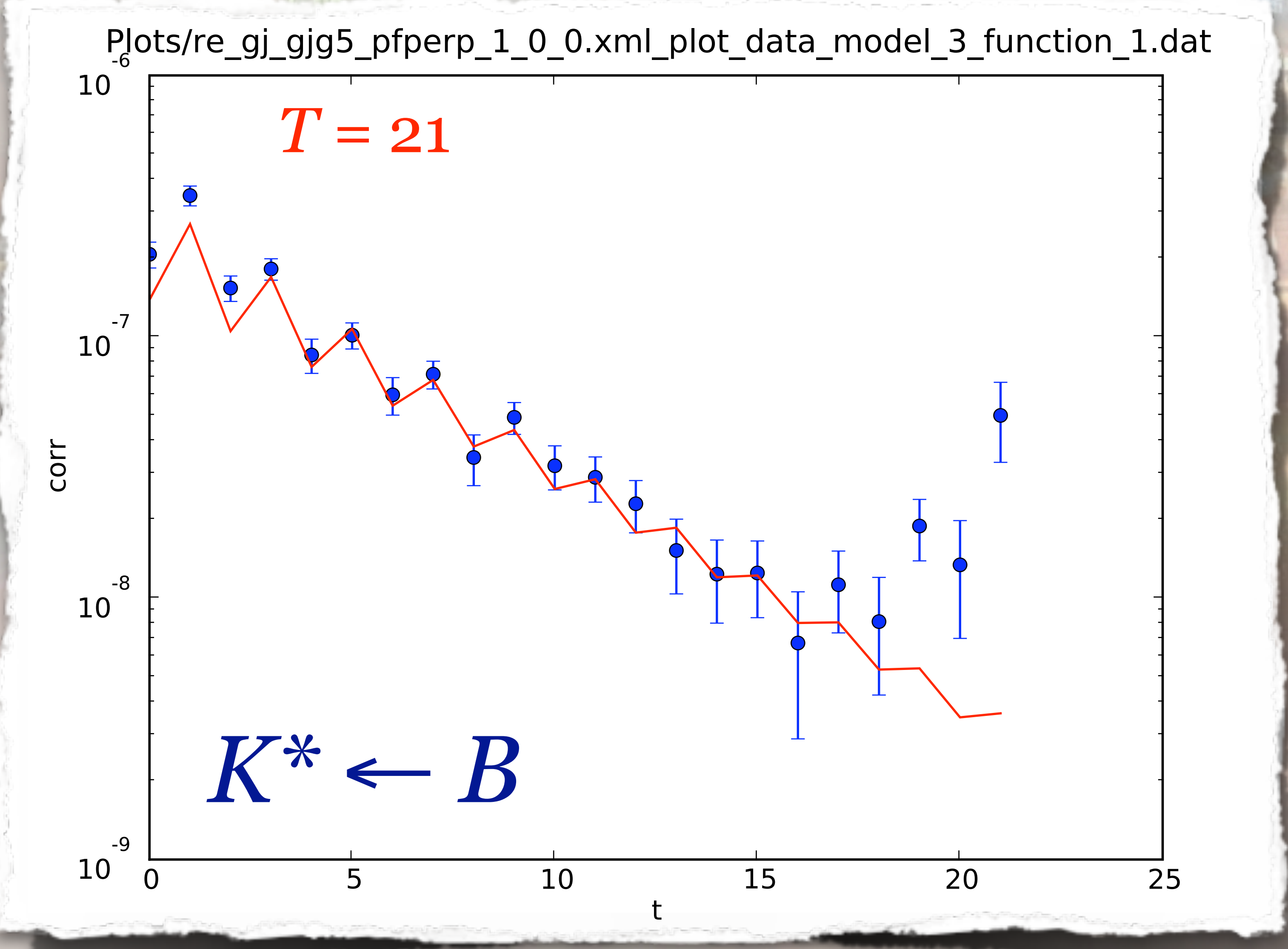
Example plots



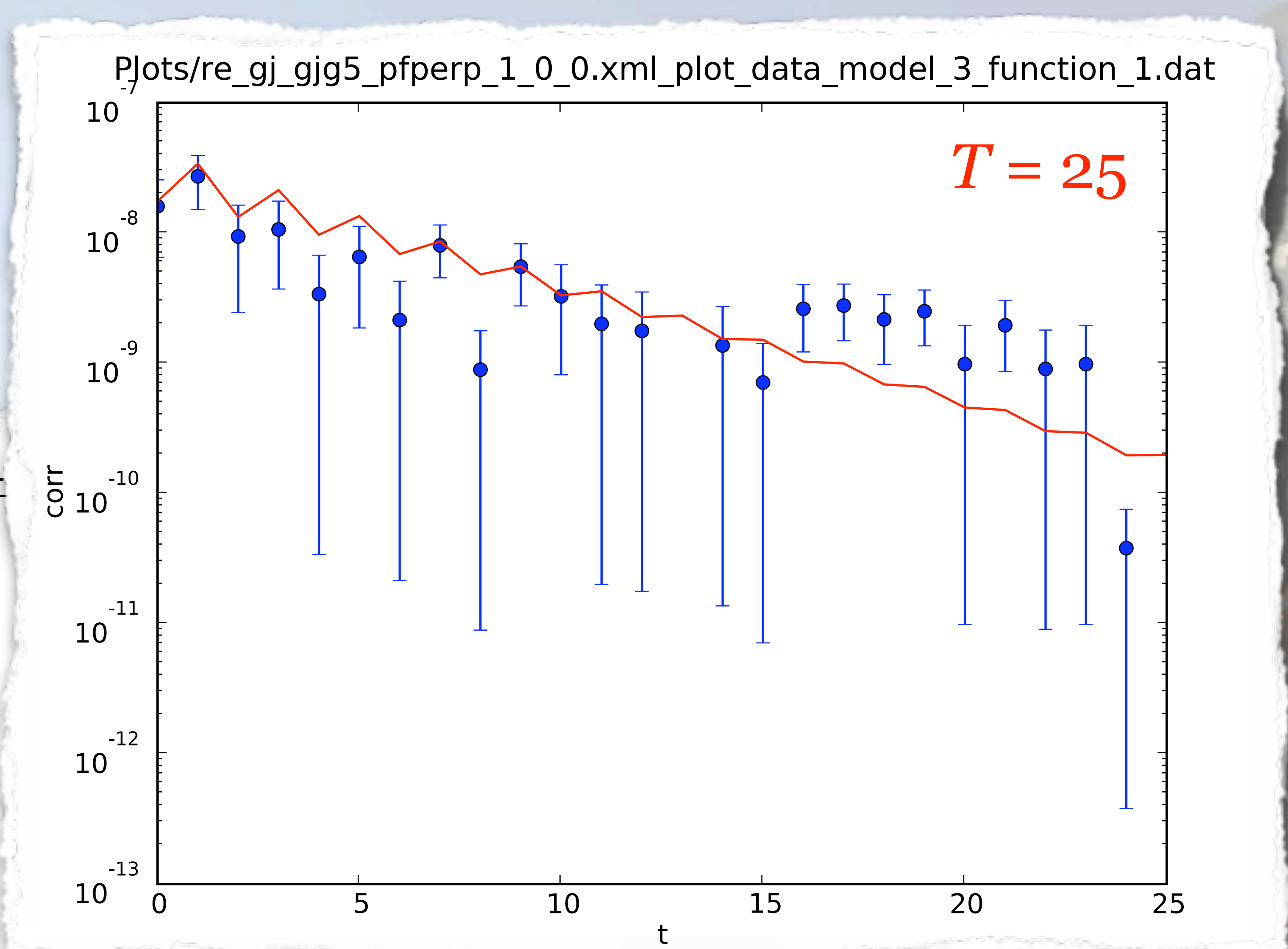


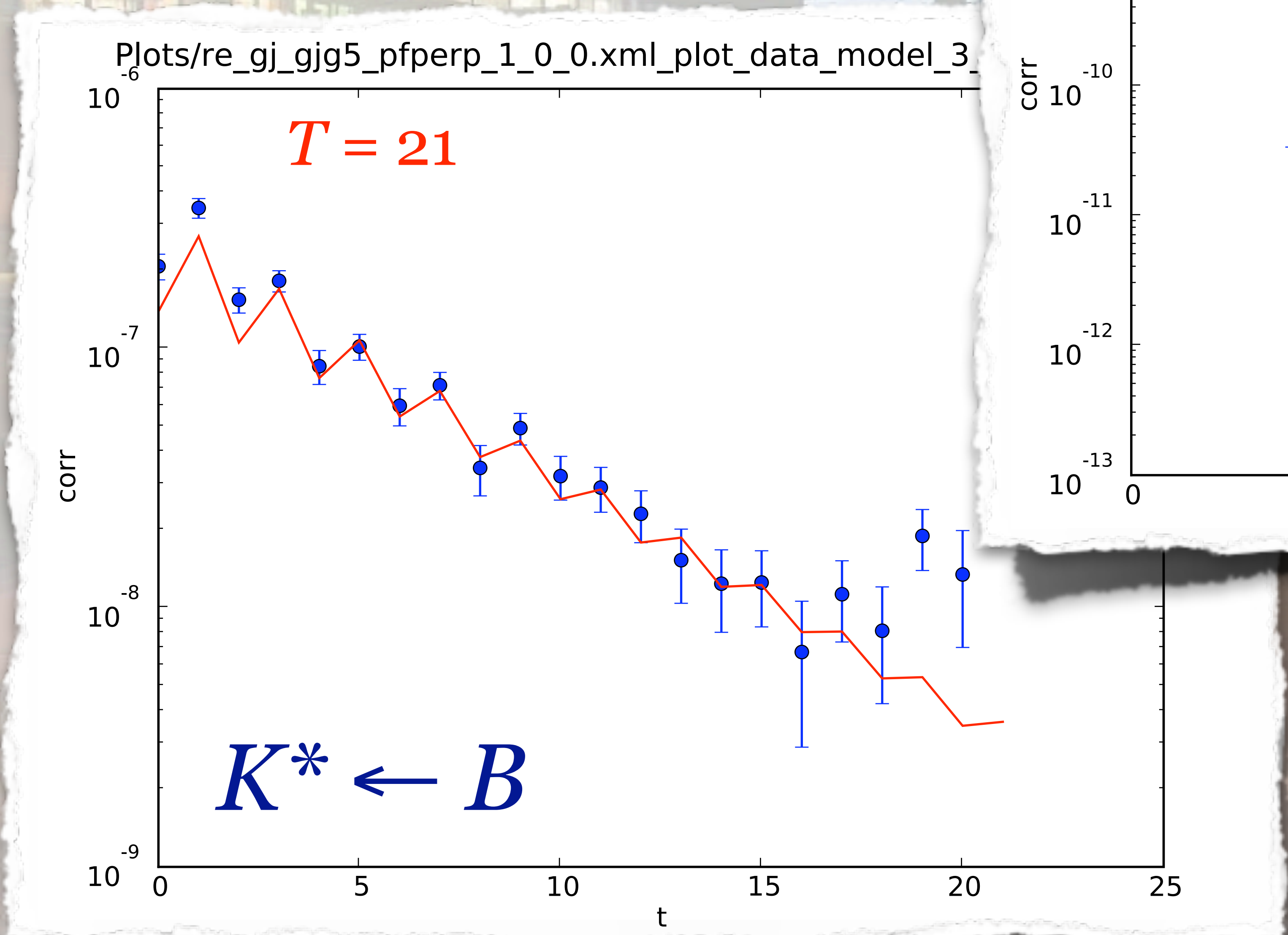
Example plots

Example plots

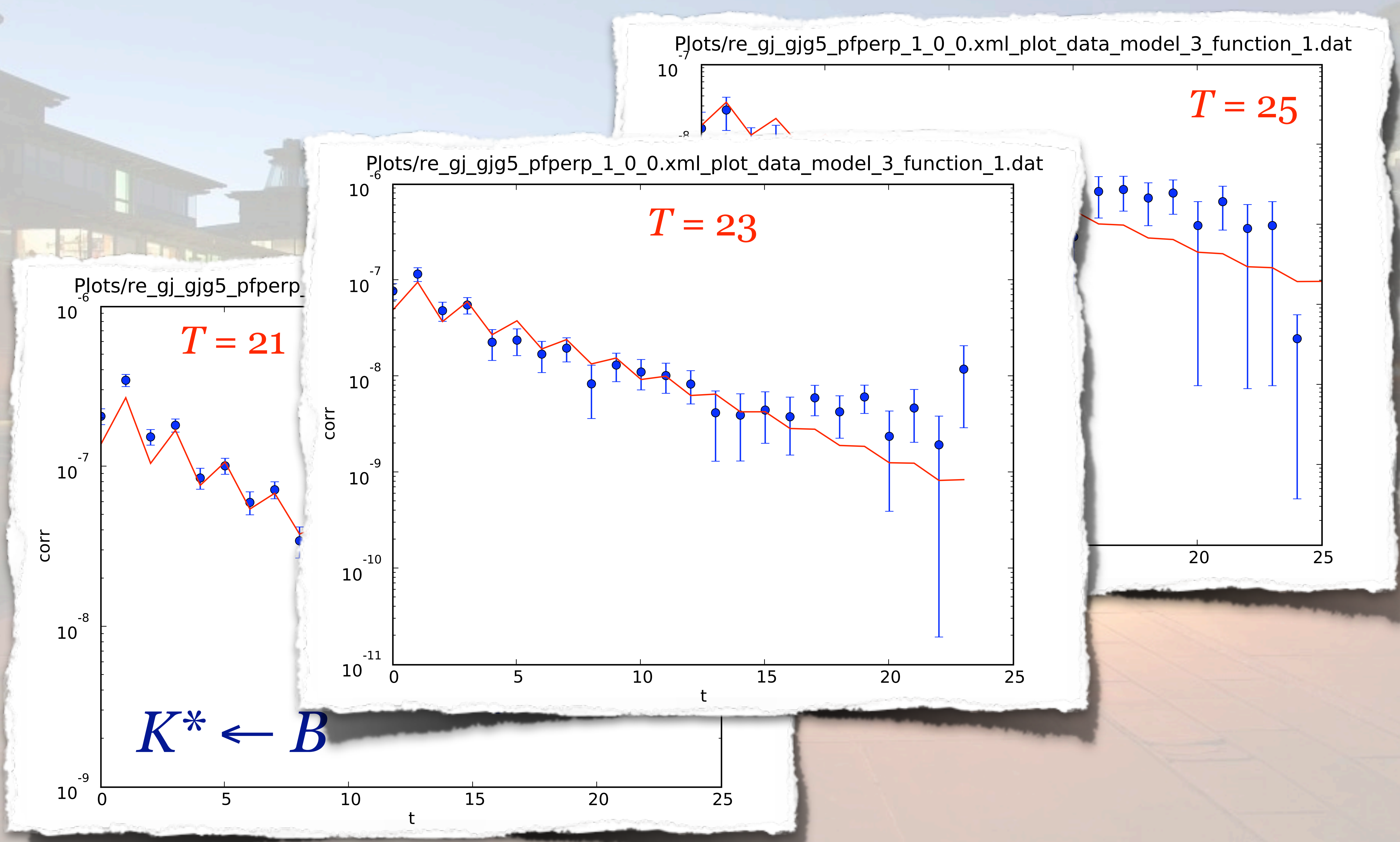


Example plots





Example plots



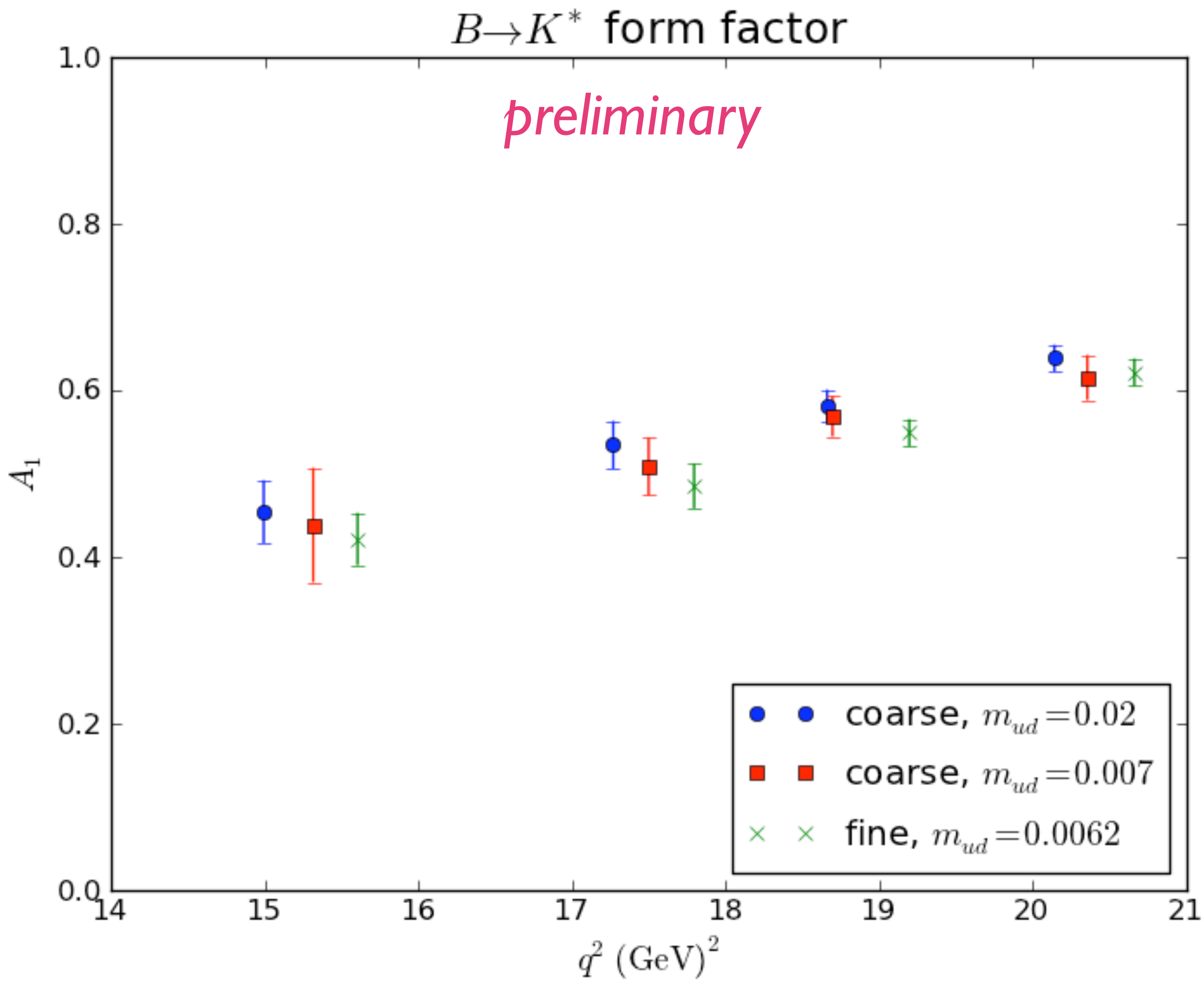
Excited state contamination

- Bayesian fits to multi-exponential models
  - Difficult to fit all data (esp. all T values)
- Frequentist fits with "cuts"
  - Fit with 500 choices for t_{cut}'s, find top few fits
  - Include uncertainty by varying t_{cut}'s bootstrap-by-bootstrap
- Useful to have separate analyses

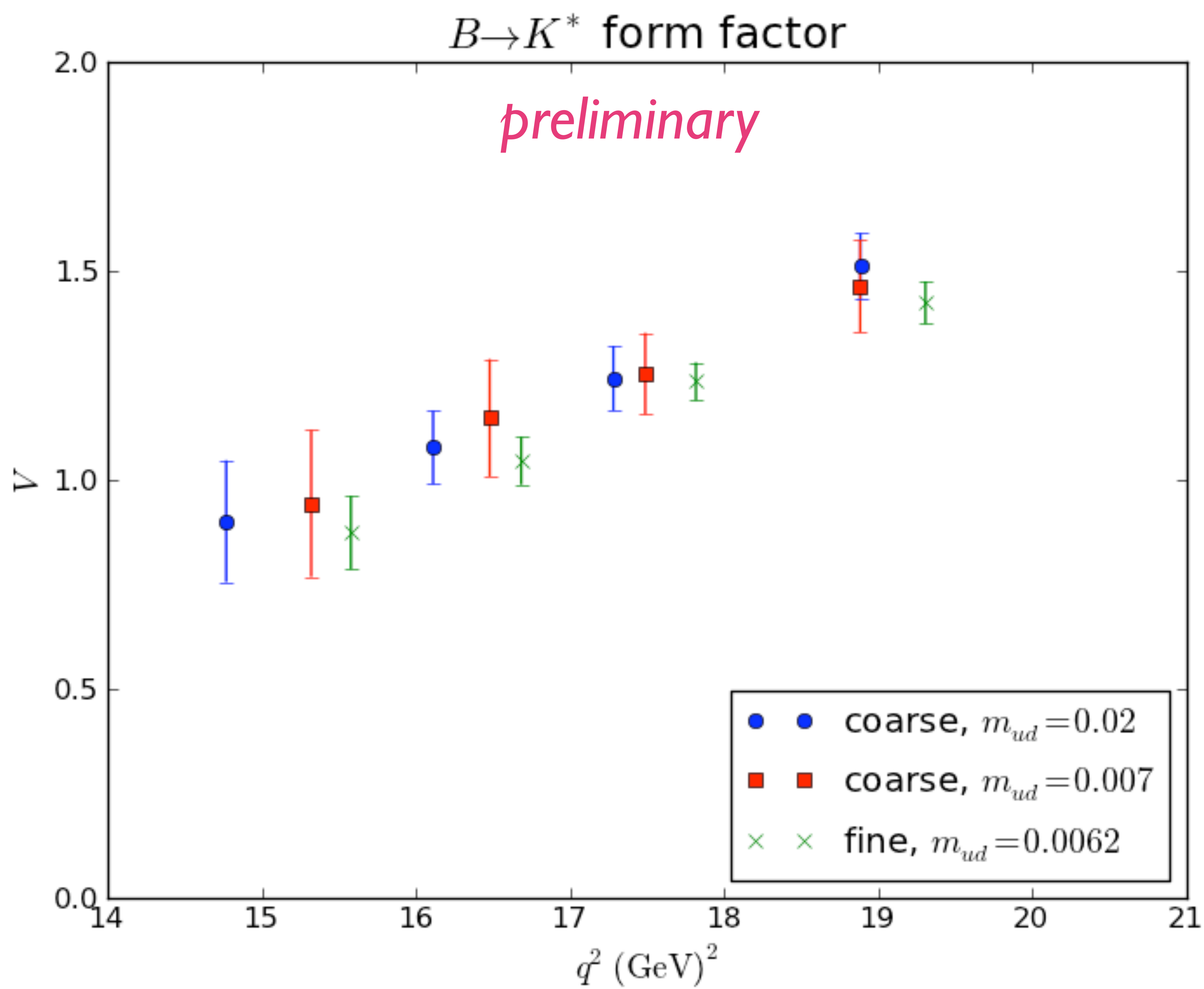
Form factors of V, A matrix elements

$$
\begin{aligned}
\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle &= \frac{2iV(q^2)}{M_B+M_V}\,\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu\, p'_\rho p_\sigma\,, \\
\langle V(p',\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}_5 b|B(p)\rangle &= 2M_V A_0(q^2)\,\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu \\
&\quad +(M_B+M_V)\,A_1(q^2)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu\right] \\
&\quad -A_2(q^2)\,\frac{\varepsilon^*\cdot q}{M_B+M_V}\left[p^\mu+p'^\mu-\frac{M_B^2-M_V^2}{q^2}\,q^\mu\right]
\end{aligned}
$$

- So far, we use **p**=0
- V straightforward to extract
- A_1 straightforward to extract when 1 component of **p’** is 0
- Linear combinations then yield A_0 and A_2

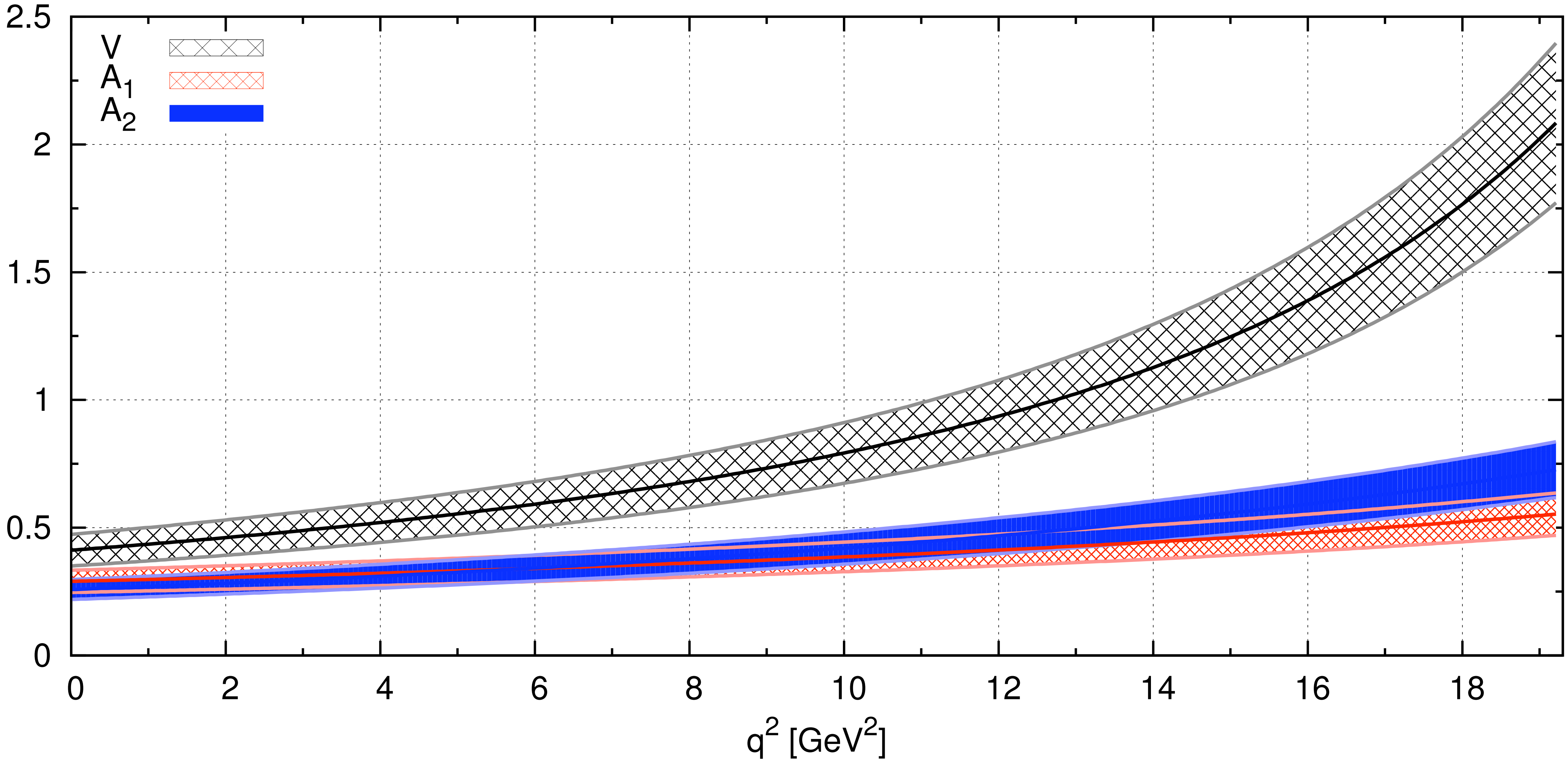


$B \to K^*$ form factor
preliminary
1.0
0.8
0.6
0.4
0.2
0.0
14
15
16
17
18
19
20
21
A_1
q^2 (GeV)^2
coarse, $m_{ud}=0.02$
coarse, $m_{ud}=0.007$
fine, $m_{ud}=0.0062$

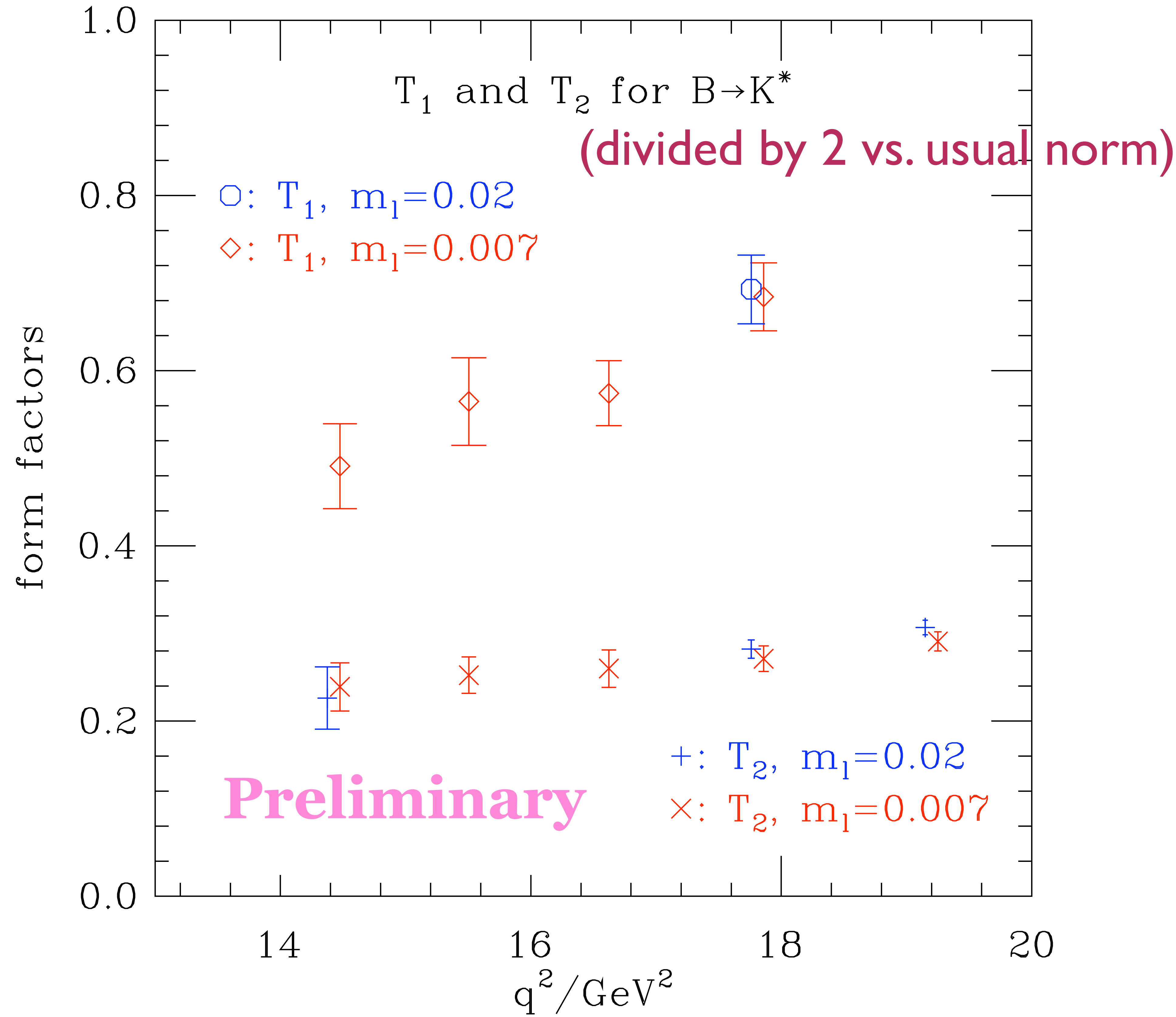


$B \to K^*$ form factor
preliminary
2.0
1.5
1.0
0.5
0.0
V
14
15
16
17
18
19
20
21
q^2 (GeV)^2
coarse, $m_{ud} = 0.02$
coarse, $m_{ud} = 0.007$
fine, $m_{ud} = 0.0062$

Extrapolation of sum rule f.f.



Bobeth, Hiller, van Dyk, extrapolating from Ball & Zwicky



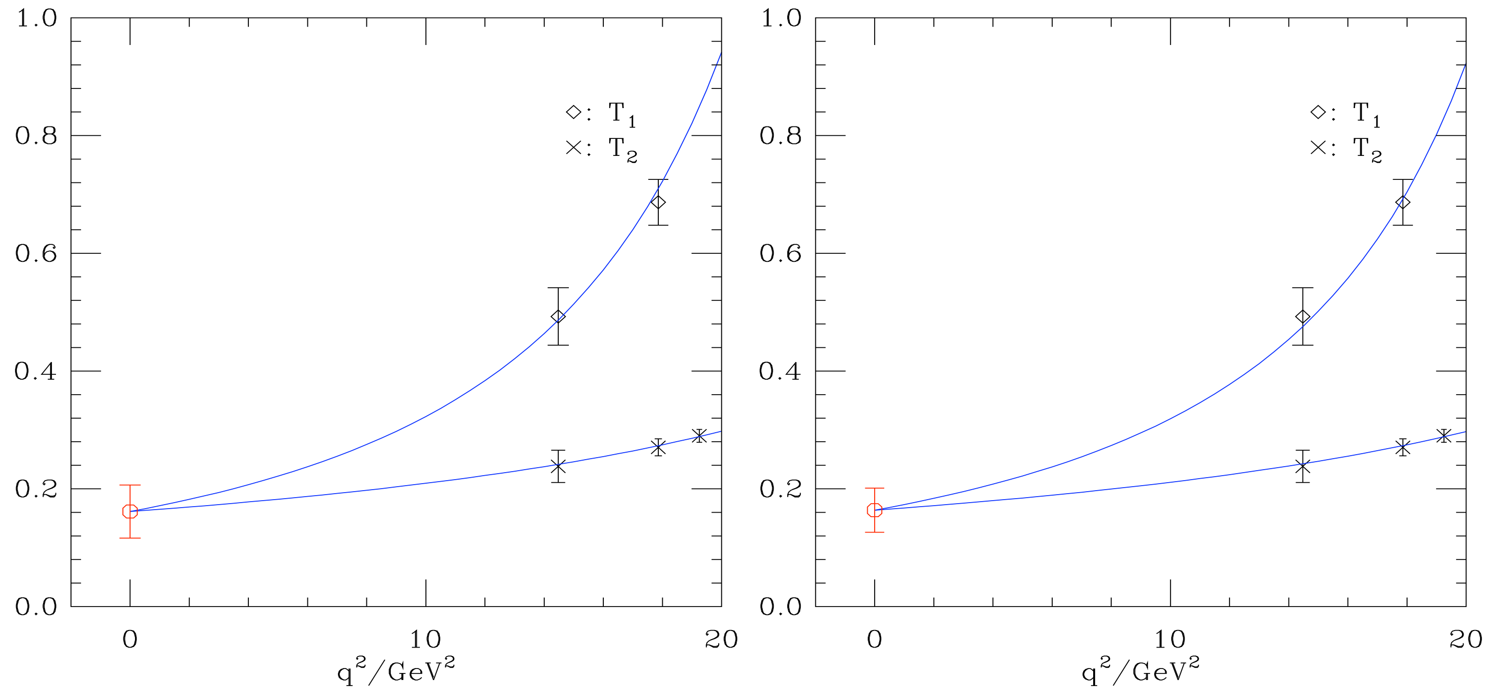
T_1 and T_2 for $B \to K^*$
(divided by 2 vs. usual norm)
○: T_1, m_l=0.02
◇: T_1, m_l=0.007
+: T_2, m_l=0.02
×: T_2, m_l=0.007
Preliminary
form factors
q^2/GeV^2
1.0
0.8
0.6
0.4
0.2
0.0
14
16
18
20

Preliminary results

Extrapolation of T_1 and T_2 to $q^2 = 0$

Pole dominance [Becirevic & Kaidalov (2000), Ball & Zwicky (2005), Becirevic et al. (2007)]

$$T_1(q^2) = \frac{T(0)}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \quad T_2(q^2) = \frac{T(0)}{1-\tilde{q}^2/\beta}, \quad \tilde{q}^2 = q^2/M^2_{B_s^*}.$$



$T(0) = 0.161(45)$ if $M_{B_s^*}$ is a free parameter (left graph).
$T(0) = 0.164(38)$ if $M_{B_s^*} = 5.4158$ GeV is fixed from PDG2010.

Preliminary results

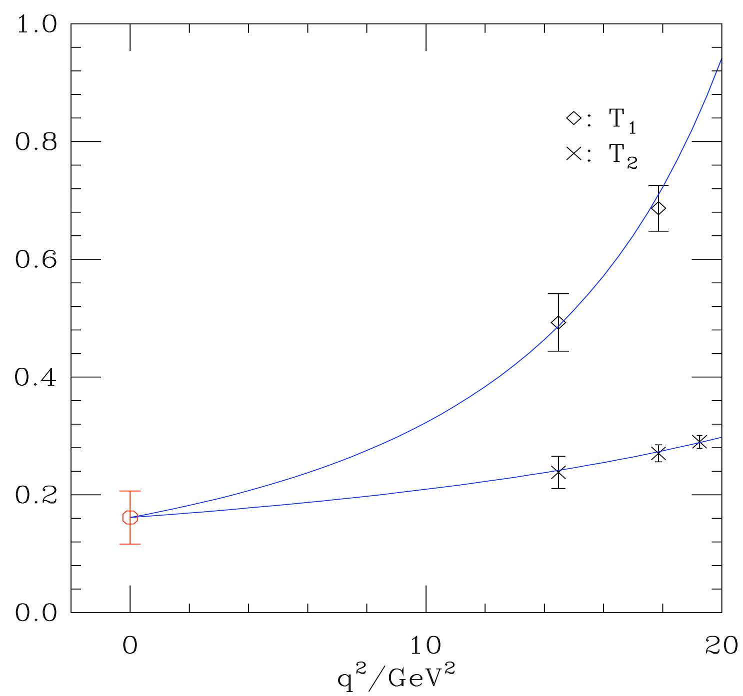
Multiply by 2 to compare Becirevic, et al; Ball, et al.

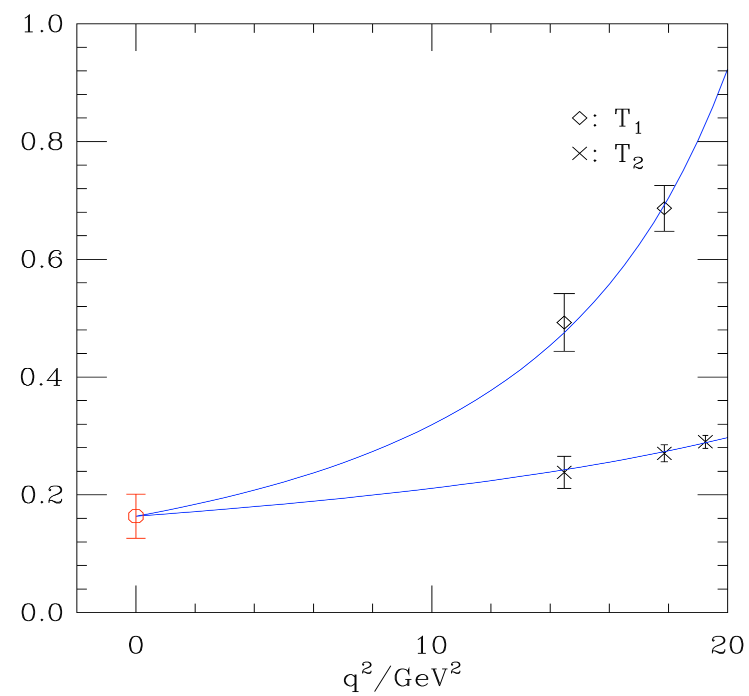
Z. Liu, CKM2010

Extrapolation of T_1 and T_2 to $q^2 = 0$

Pole dominance [Becirevic & Kaidalov (2000), B... Becirevic et al. (2007)]

$$T_1(q^2) = \frac{T(0)}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)}, \quad T_2(q^2) = \frac{T \ldots}{1-\tilde{q} \ldots} \ldots \quad \tilde{q}^2 = q^2/M^2_{B_s^*}.$$

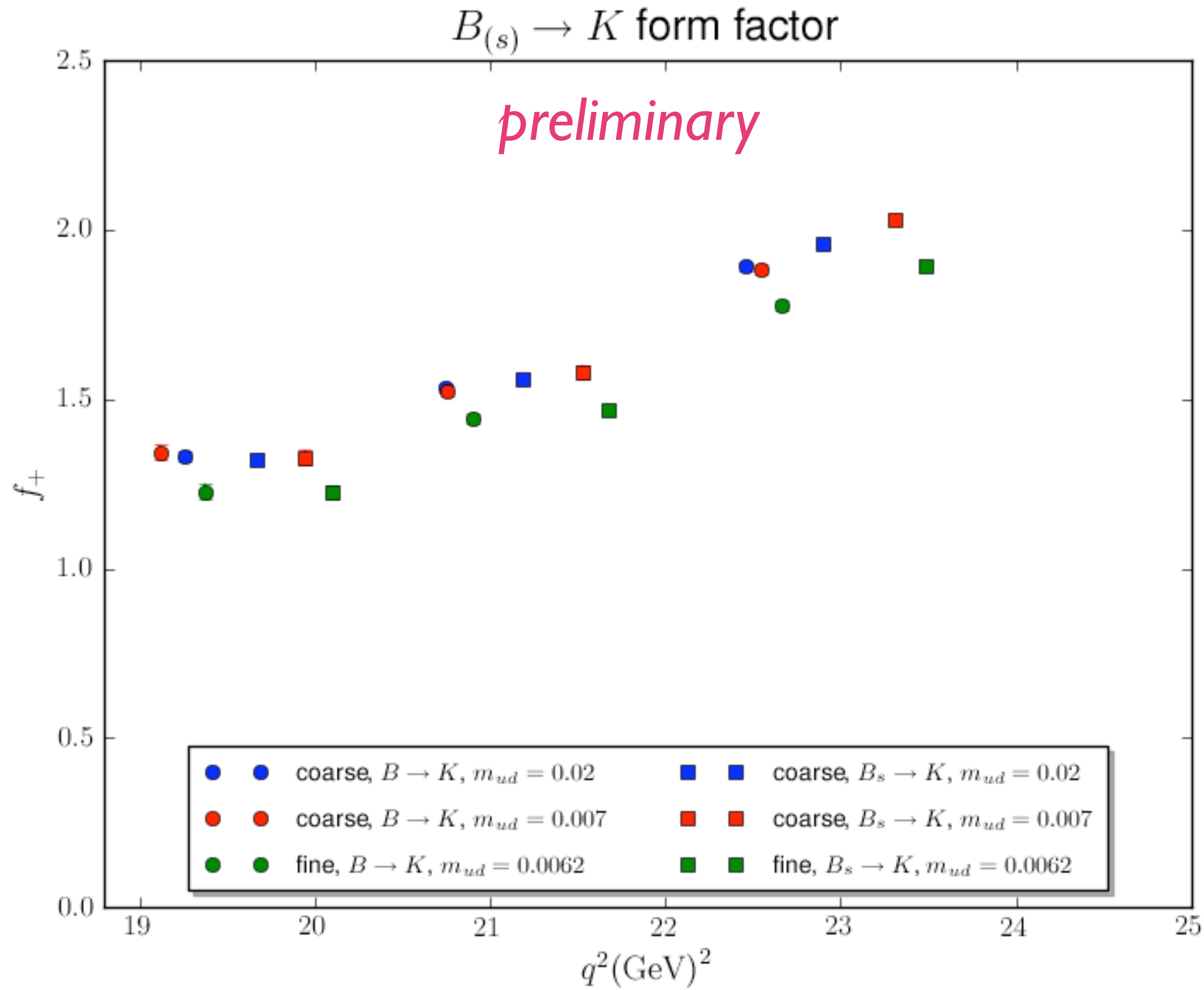




$T(0) = 0.161(45)$ if $M_{B_s^*}$ is a free parameter (left graph).
$T(0) = 0.164(38)$ if $M_{B_s^*} = 5.4158$ GeV is fixed from PDG2010.

B → V plan

- Check factors of 2, kinematic factors
- Look for SU(3) breaking effects (should be smaller than errors)
- Series expansion fits
- How to estimate errors due to finite width?


$B_{(s)} \to K$ form factor
preliminary
f_+
$q^2(\mathrm{GeV})^2$
2.5
2.0
1.5
1.0
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coarse, $B \to K$, $m_{ud} = 0.02$
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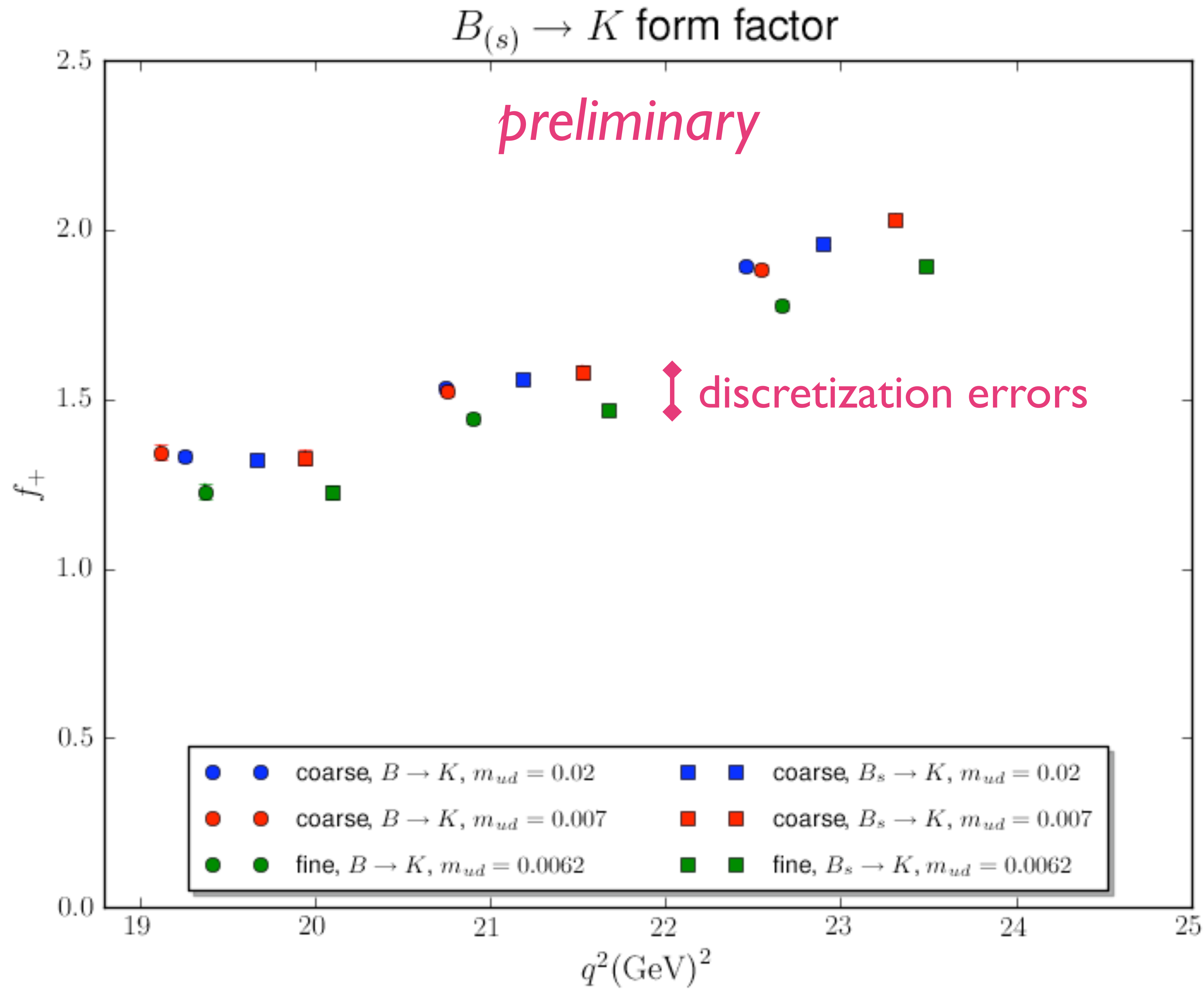

$B_{(s)} \to K$ form factor
preliminary
discretization errors
f_+
$q^2 (\mathrm{GeV})^2$
2.5
2.0
1.5
1.0
0.5
0.0
19
20
21
22
23
24
25
coarse, $B \to K$, $m_{ud} = 0.02$
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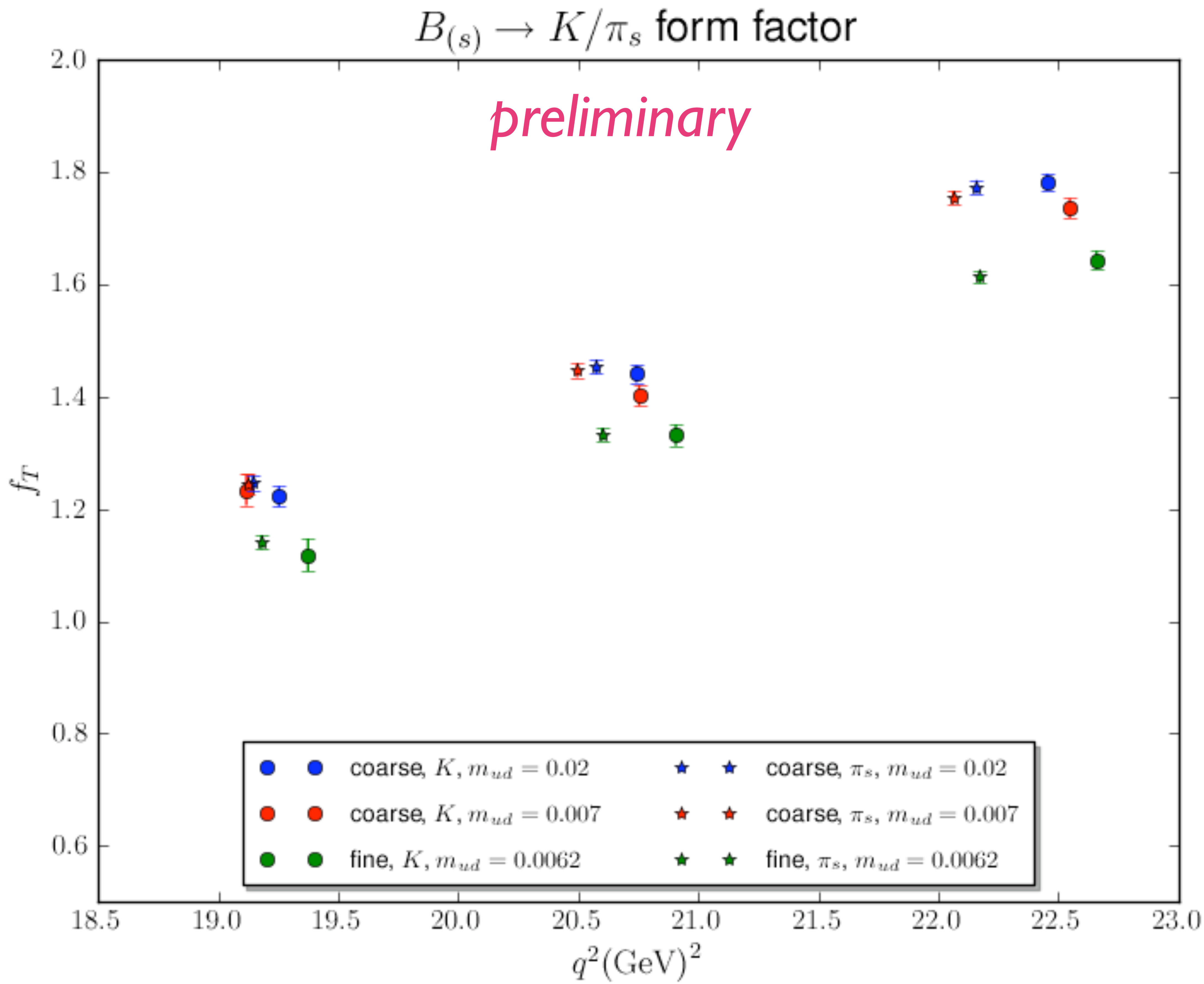
$B_{(s)} \to K/\pi_s$ form factor
preliminary
f_T
$q^2(\mathrm{GeV})^2$
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
18.5
19.0
19.5
20.0
20.5
21.0
21.5
22.0
22.5
23.0
coarse, $K, m_{ud} = 0.02$
coarse, $K, m_{ud} = 0.007$
fine, $K, m_{ud} = 0.0062$
coarse, $\pi_s, m_{ud} = 0.02$
coarse, $\pi_s, m_{ud} = 0.007$
fine, $\pi_s, m_{ud} = 0.0062$

Isgur-Wise relation

$$f_+ - \frac{m_B}{m_B + m_P} f_T = O\left(\frac{\Lambda_{\rm QCD}}{m_B}\right)^{1/2}$$

We see roughly

$$f_+ \approx 1.05 f_T$$

$B \to P$ plan

- Series expansion fits (generalized to include varying masses and finite a)
- Find ratios which isolate SU(3) breaking
- Quantify f_{+}, f_{T} deviation from Isgur-Wise
- Look to better actions to reduce a dependence

Summary

- Unquenched calculation of $B \to V$ form factors
- Enhanced statistical precision exposing discretization errors
- Appears that $f_T < f_+$. $1/m$ effects computable
  - Action accurate through $1/m^2$
  - Still need to analyze m.e. of $1/m$ operators (small)
- Difficult to extract $SU(3)_F$ breaking effects

Forecast

- New lattices with smaller discretization errors (AsqTad → HISQ)
- Beyond NRQCD, reduction in matching uncertainty which is dominant

ECT* Workshop, Trento, April 2012

- ***"Beautiful Mesons and Baryons on the Lattice"***
- Organizers: M.W., W. Detmold, C.-J. D. Lin
- Bring together phenomenology & lattice, etc
  - Rare decays, *b* baryons: lattice, pheno, expt
  - Status of LQCD results, reducing errors, new opportunities
  - χPT for *b* hadrons, lattice methods for b
- Talk to each other Mon-Wed
- Try to keep technical talks to Thurs, Fri