Deriving constraints from the space of conformal line operators

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and work in progress with N. Drukker and P. Kravchuk

Self Introduction

- Education path

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- Hometown

- Hobbies: History, Literature...

Conformal Line Operator (Defect)

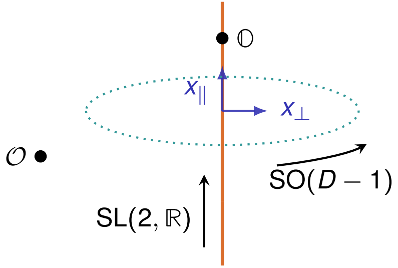
A conformal line operator is a 1d CFT.

As a defect inserted into a D dimensional bulk CFT, it breaks

- conformal symmetry: Translation, Lorentz, Dilatations, Special conformal transformations

$$SO(D+1,1) \to SL(2,\mathbb{R}) \times SO(D-1)$$

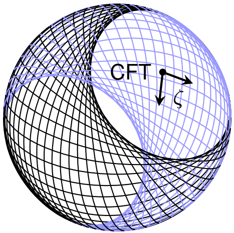
- **global-symmetry**: $G \to G'$



Defect Conformal Manifold

A defect conformal manifold $\mathcal{M}_{\mathsf{CFT}}$ is a family of defect CFT's parametrized by couplings $\{\zeta^i\}$. For each value of the couplings a different point on the manifold is a different CFT.

$$S \to S + \sum_i \zeta^i \int dx\, \mathbb{O}_i$$



$\mathbb{O}_i$ are defect exactly marginal operators.

This manifold admits a Riemannian structure:

- Zamolodchikov Metric
- Curvature tensor

Defect Exactly Marginal Operators

- Deforming a defect by **exactly marginal** operators $\mathbb{O}_i$ where $\Delta_{\mathbb{O}_i} = 1$, correlation functions of any operator ϕ

$$\langle\!\langle \phi_1 \cdots \phi_n \rangle\!\rangle \to \langle\!\langle e^{-\int \zeta^i \mathbb{O}_i dx} \phi_1 \cdots \phi_n \rangle\!\rangle$$

  - $\langle\!\langle \cdots \rangle\!\rangle$: defect correlation function normalized by the expectation value of the defect.

- If the defect breaks $G \to G'$, the conservation equation is

$$\partial_\mu \underset{\text{generators of } G}{J^{\mu a}} = \mathbb{O}_i(x_\parallel) \underset{\text{generators broken by the defect}}{\delta^{ia}} \delta^{D-1}(x_\perp)$$

When a defect breaks a global symmetry, the resulting defect conformal manifold is the symmetry breaking coset.

$$\mathcal{M} = G/G'$$

Geometric Structure

- Zamolodchikov Metric [Zamolodchikov]

$$g_{ij} = \langle\!\langle \mathbb{O}_i(\infty)\mathbb{O}_j(0) \rangle\!\rangle = C_{\mathbb{O}}\delta_{ij}, \quad \mathbb{O}_i(\infty) \equiv \lim_{x_{\|}\to\infty} |x_{\|}|^2 \mathbb{O}_i(x_{\|})$$

- Curvature tensor [Kutasov]

$$R_{ijkl} = \frac{1}{2}(\partial_j\partial_k g_{il} - \partial_i\partial_k g_{jl} - \partial_j\partial_l g_{ik} + \partial_i\partial_l g_{jk})$$

  - Take the first term for instance

$$\begin{aligned}\partial_j\partial_k g_{il} &= \left(\partial_j\partial_k \langle\!\langle e^{-\int \zeta^i \mathbb{O}_i dx} \mathbb{O}_i(\infty)\mathbb{O}_l(0) \rangle\!\rangle\right)\Big|_{\zeta^i=0} \\ &= \iint dx_1 dx_2 \langle\!\langle \mathbb{O}_j(x_1)\mathbb{O}_k(x_2)\mathbb{O}_i(\infty)\mathbb{O}_l(0) \rangle\!\rangle\end{aligned}$$

The curvature tensor a sum of double integrals of 4-pt functions $\Rightarrow$ integrals over cross-ratios. [Friedan, Konechny]

Maldacena-Wilson loop in $\mathcal{N} = 4$ SYM

The 1/2 BPS Wilson loop

$$W = \mathrm{Tr}\,\mathcal{P} e^{\int (iA_0 + \Phi_6) dt}$$

The breaking of global symmetry (R-symmetry) gives 5 exactly marginal defect operators Φ_i.

The defect conformal manifold is $S^5 = SO(6)/SO(5)$.

- Zamolodchikov Metric: $g_{ij} = \langle\!\langle \Phi_i(0)\Phi_j(\infty) \rangle\!\rangle = C_\Phi \delta_{ij}$ with C_Φ twice the bremsstrahlung function [Drukker, Forini], [Correa, Henn, Maldacena, Sever], [Fiol, Garolera, Lewkowycz]

$$C_\Phi = \begin{cases} \frac{\lambda}{8\pi^2} - \frac{\lambda^2}{192\pi^2} + \frac{\lambda^3}{3072\pi^2} - \frac{\lambda^4}{46080\pi^2} + O(\lambda^5), & \lambda \ll 1 \\ \frac{\sqrt{\lambda}}{2\pi^2} - \frac{3}{4\pi^2} + \frac{3}{16\pi^2\sqrt{\lambda}} + \frac{3}{16\pi^2\lambda} + O(\lambda^{-3/2}), & \lambda \gg 1 \end{cases}$$

Curvature Tensor

Curvature Tensor: [Friedan, Konechny]

$$R_{ijkl} = -\mathrm{RV}\int_{-\infty}^{+\infty} d\eta \log|\eta| \Big[\langle\!\langle \Phi_i(1)\Phi_j(\eta)\Phi_k(\infty)\Phi_l(0) \rangle\!\rangle_c + \langle\!\langle \Phi_i(0)\Phi_j(1-\eta)\Phi_k(\infty)\Phi_l(1) \rangle\!\rangle_c \Big] \quad (*)$$

- 4-pt function: $\langle\!\langle \Phi_i(0)\Phi_j(\eta)\Phi_k(1)\Phi_l(\infty) \rangle\!\rangle$

$$= \frac{C_\Phi^2}{\eta^2}\left(\delta_{ik}\delta_{jl}h_2(\eta) + \delta_{il}\delta_{jk}h_1(\eta) + \delta_{ij}\delta_{kl}h_0(\eta)\right)$$

Plugging into $(*)$

$$R_{ijkl} = 2(g_{ik}g_{jl} - g_{il}g_{jk})\int_0^1 \frac{d\eta}{\eta^2}\log(\eta)\,(h_2 + h_1 - 2h_0)$$

Introduce $H = h_2 + h_1 - 2h_0$.

Integral Identity

Generally, for a sphere with radius r we have

$$R_{ijkl} = (g_{ik}g_{jl} - g_{il}g_{jk})/r^2$$

From the Zamolodchikov Metric, we expect $r = \sqrt{C_\Phi}$.

So the key point is to check

$$2\int_0^1 \frac{d\eta}{\eta^2} \log(\eta) H(\eta) = \frac{1}{C_\Phi}$$

- At strong coupling up to 3-loop order
  [Giombi, Roiban, Tseytlin], [Liendo, Meneghelli, Mitev], [Ferrero, Meneghelli]
- At weak coupling up to 2-loop order [Cavaglia, Gromov, Julius, Preti]

Other Examples

- Complex manifolds: 1/2 BPS Wilson loops in ABJM
$$\mathbb{CP}^3 = SU(4)/SU(3) \times U(1)$$
- Non-symmetric spaces: 1/3 BPS Wilson loops in ABJM
$$SU(4)/SU(2) \times U(1) \times U(1)$$
- Higher-dimensional defects:
  - 1/2 BPS surface operators in 6d $\mathcal{N} = (2,0)$ theory
$$S^4 = SO(5)/SO(4)$$
  - Non-supersymmetric theories: boundaries in free theories [Herzog, Schaub]
$$O(N)/\big(O(p) \times O(N-p)\big) \quad U(N)/\big(U(p) \times U(N-p)\big)$$

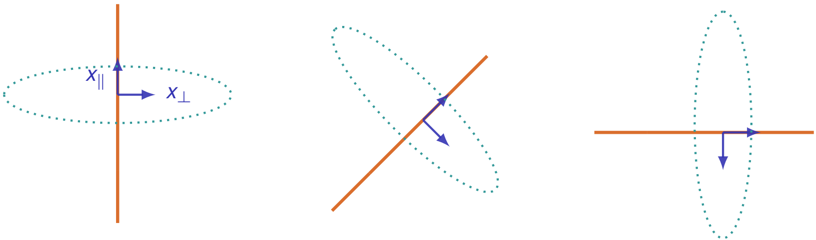
Defect conformal manifold arsing from broken global symmetry is the symmetry breaking coset

$$\mathcal{M}_{\mathrm{dCFT}} = G/G'$$

More Constraints from Spacetime

Another modified conservation equation

$$\partial_\mu T^{\mu i} = \mathbb{D}^i(x_\parallel)\delta^{D-1}(x_\perp)$$



$$\Rightarrow \sum \iint dx_1 dx_2 \, \langle\!\langle \mathbb{D}(x_1)\mathbb{D}(x_2)\mathbb{D}(0)\mathbb{D}(\infty) \rangle\!\rangle \propto \langle\!\langle \mathbb{D}(0)\mathbb{D}(\infty) \rangle\!\rangle$$

Outlook

1. Generalizations:
   - Defect marginal & non-marginal operators
   - Bulk & defect marginal operators
   - Bulk marginal operators
2. Similar constraints for higher point functions
3. $\cdots$

Thank you!