Hierarchies from landscape probability gradients and critical boundaries

Oleksii Matsedonskyi

arxiv:2311.10139

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Plan

- "Dynamical" solutions to hierarchy problems
- Probability gradients and critical boundaries
- Volume - weighed probabilities
- Local probabilities

Introduction

Gauge Hierarchy problem:

$$\delta m_h^2 \propto \Lambda^2 \ \leftarrow$$ any physics that Higgs interacts with

e.g. $$\frac{m_P^2}{m_h^2} \sim 10^{34}$$

Introduction

Single vacuum* approaches:

$$\delta m_h^2 = 0\,\Lambda^2 + \mathcal{O}(100GeV)$$

↑

supersymmetry

compositeness

extra dimensions

Introduction

Landscape/dynamical approaches:

$$m_h^2 \subset (-\Lambda^2, \Lambda^2)$$

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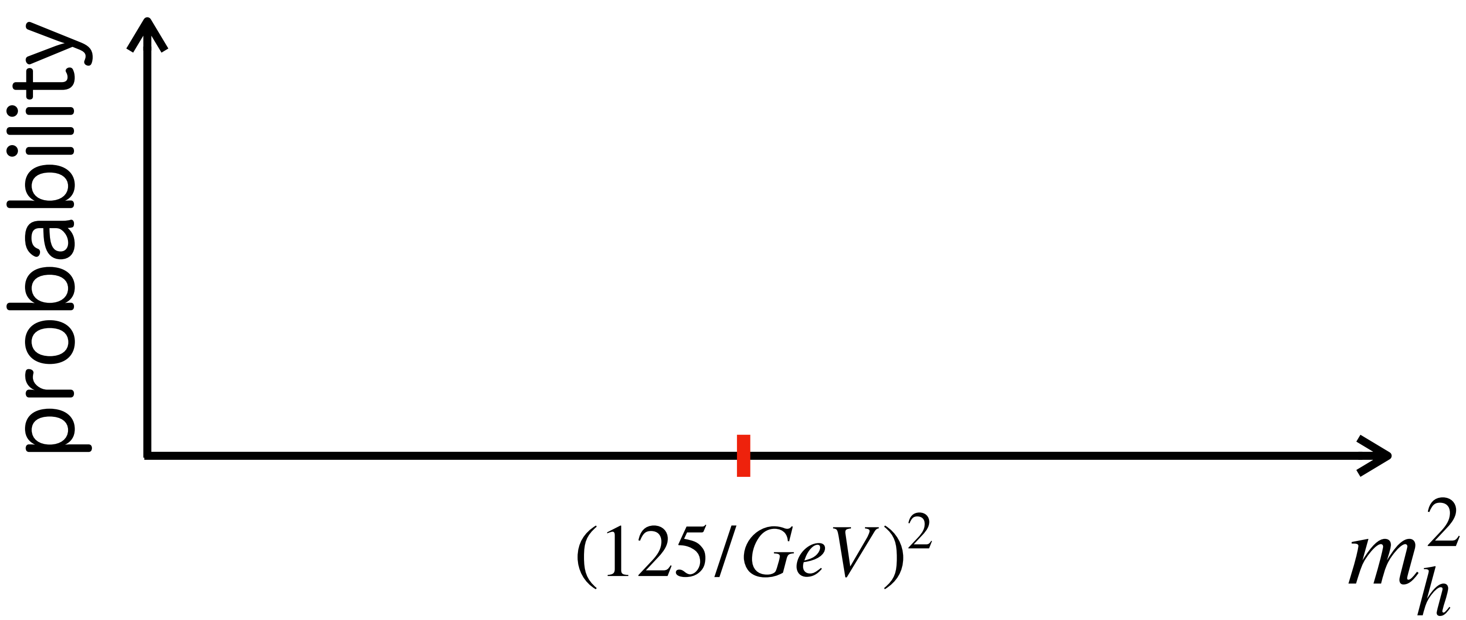

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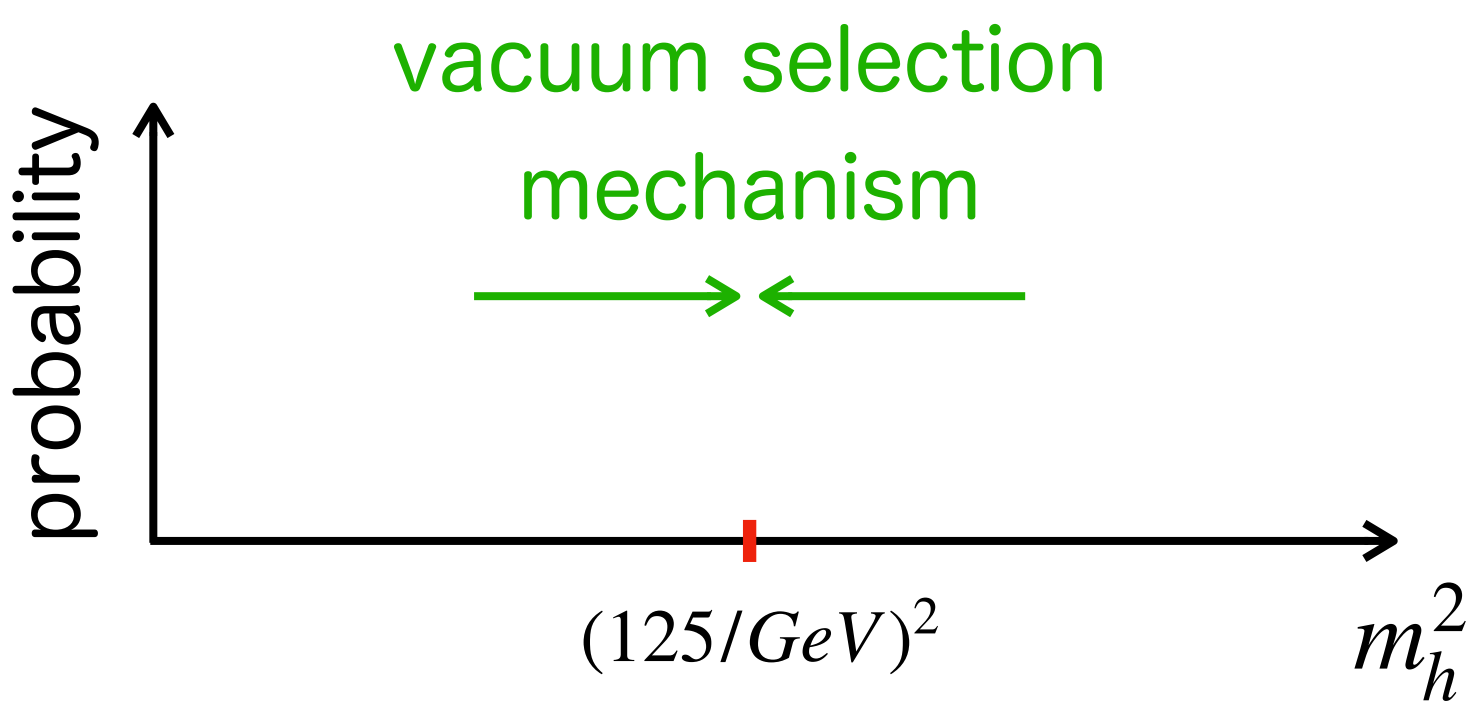

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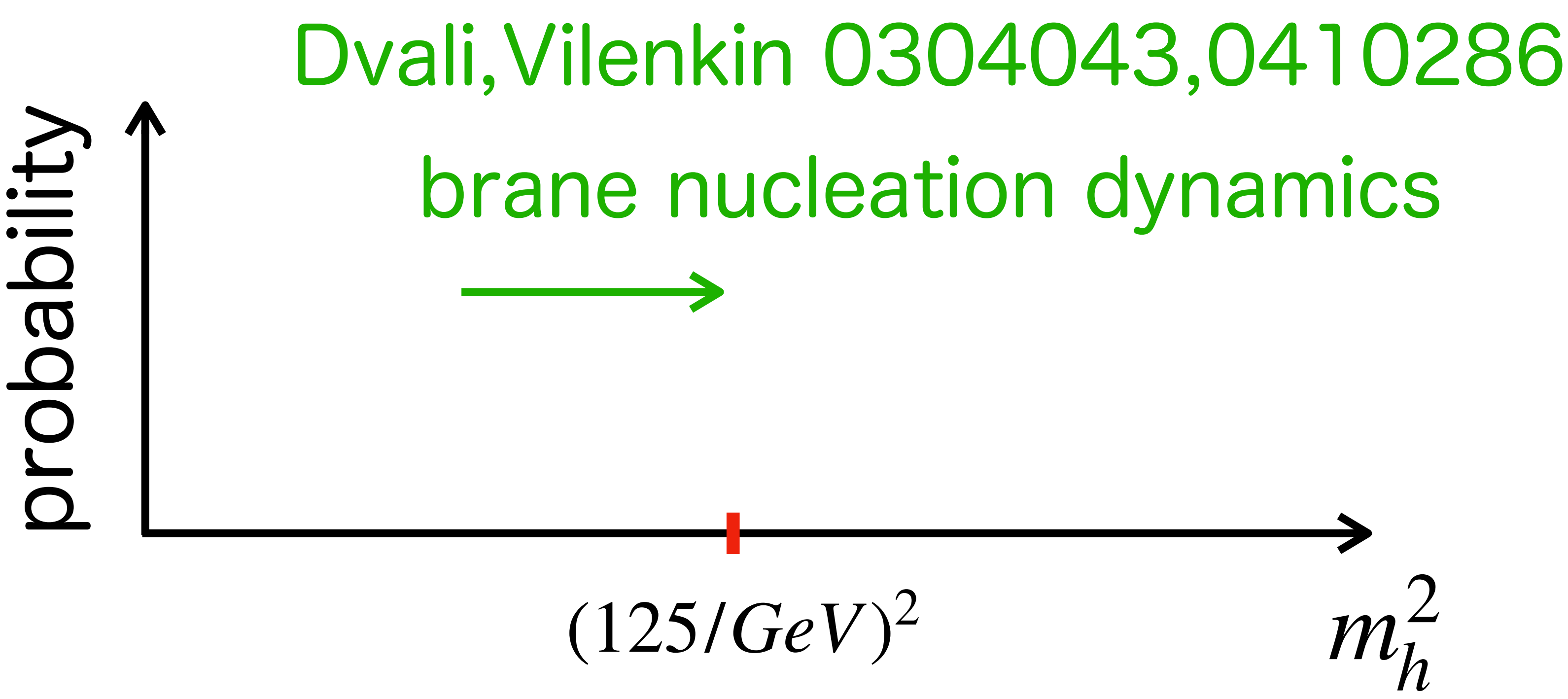

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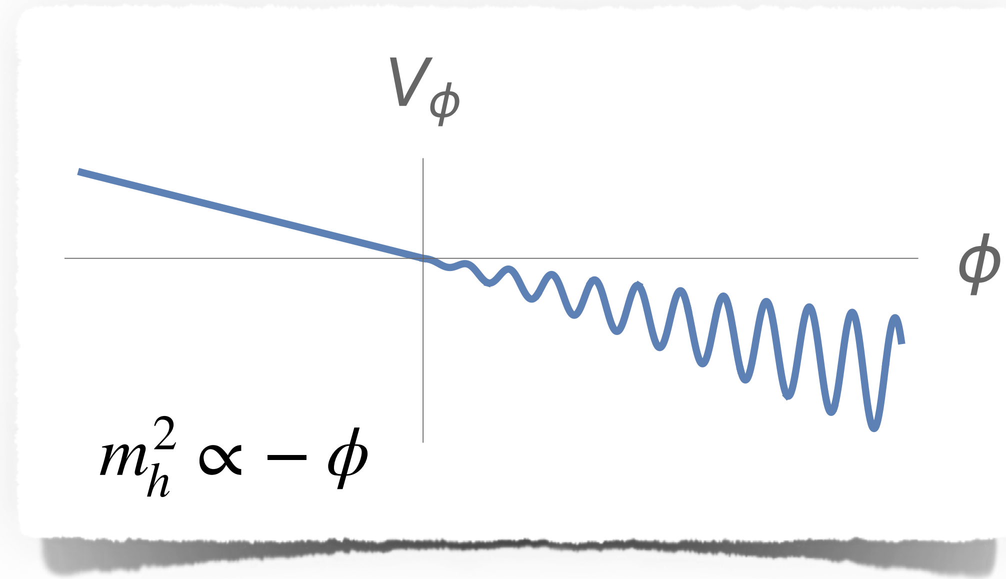
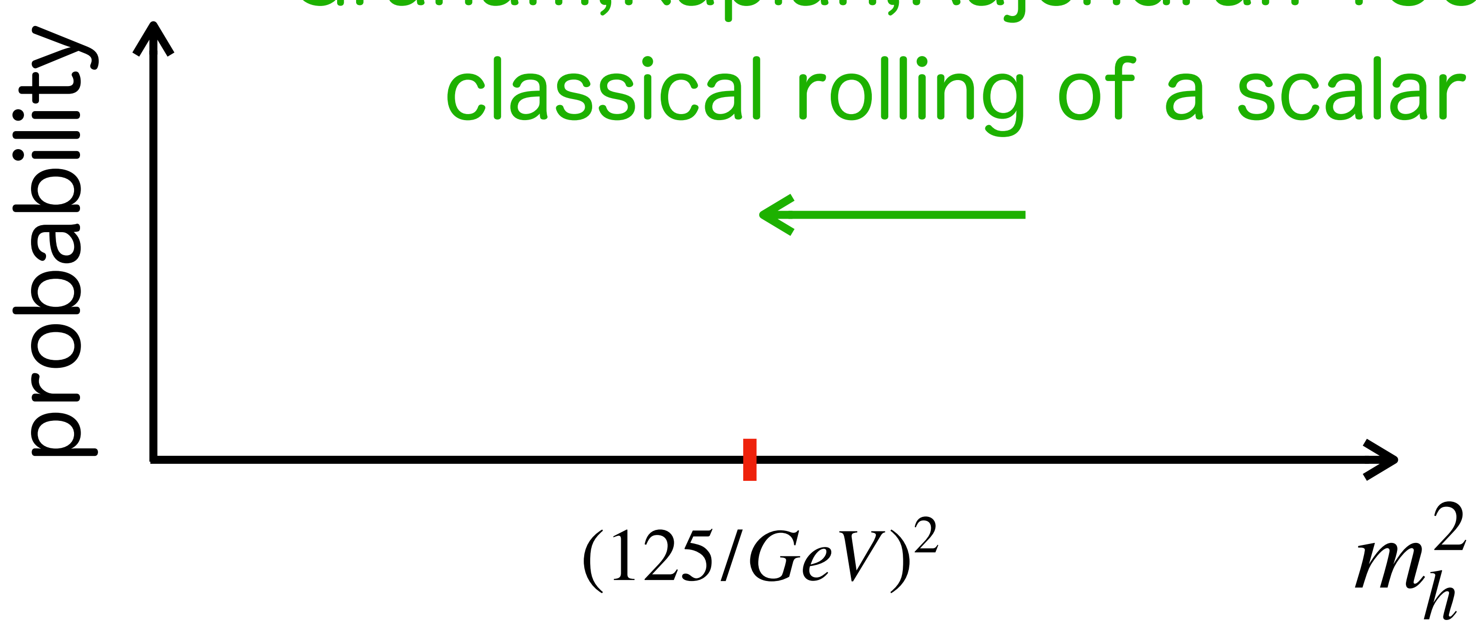


Graham,Kaplan,Rajendran 1504.07551
classical rolling of a scalar



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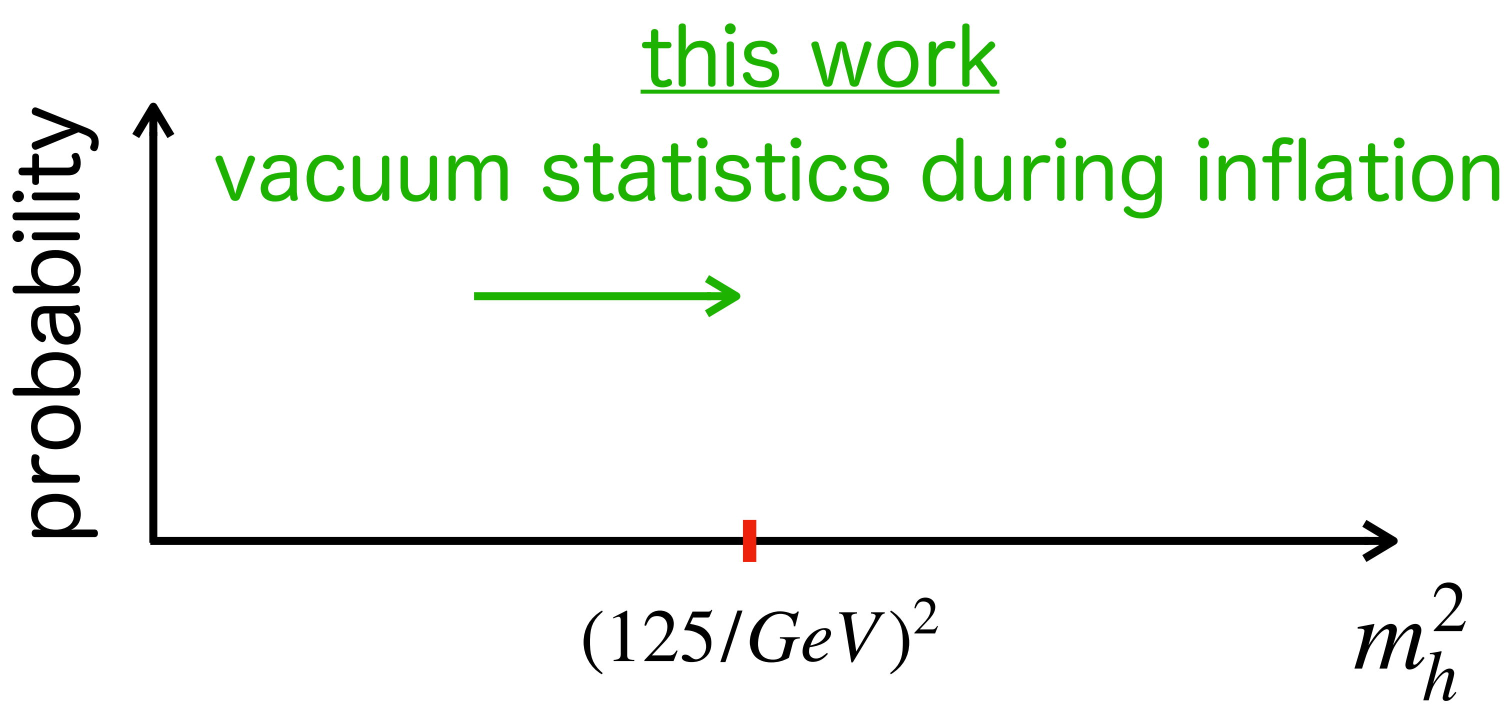

Introduction

Preview of the final mechanism

Scan both mH and CC. Why?

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Preview of the final mechanism

Scan both mH and CC. Why?

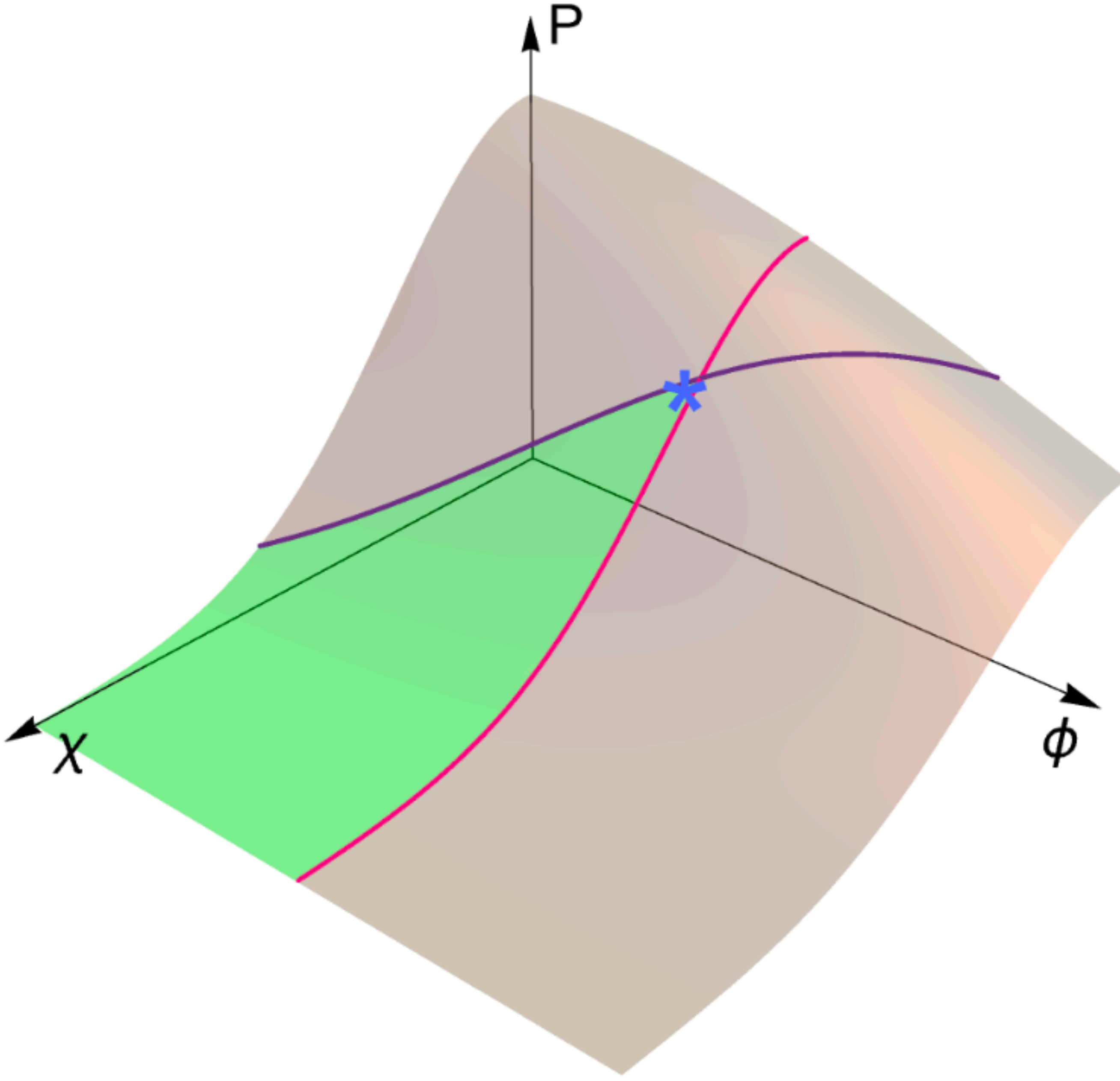
- $$\frac{m_P^4}{\Lambda_{cc}(obs)} \sim 10^{120}$$

- most straightforward approach to the smallness of CC is landscape + anthropics

- dynamics of the two landscapes generically interfere hence it is natural to consider them together

Introduction

Preview of the final mechanism



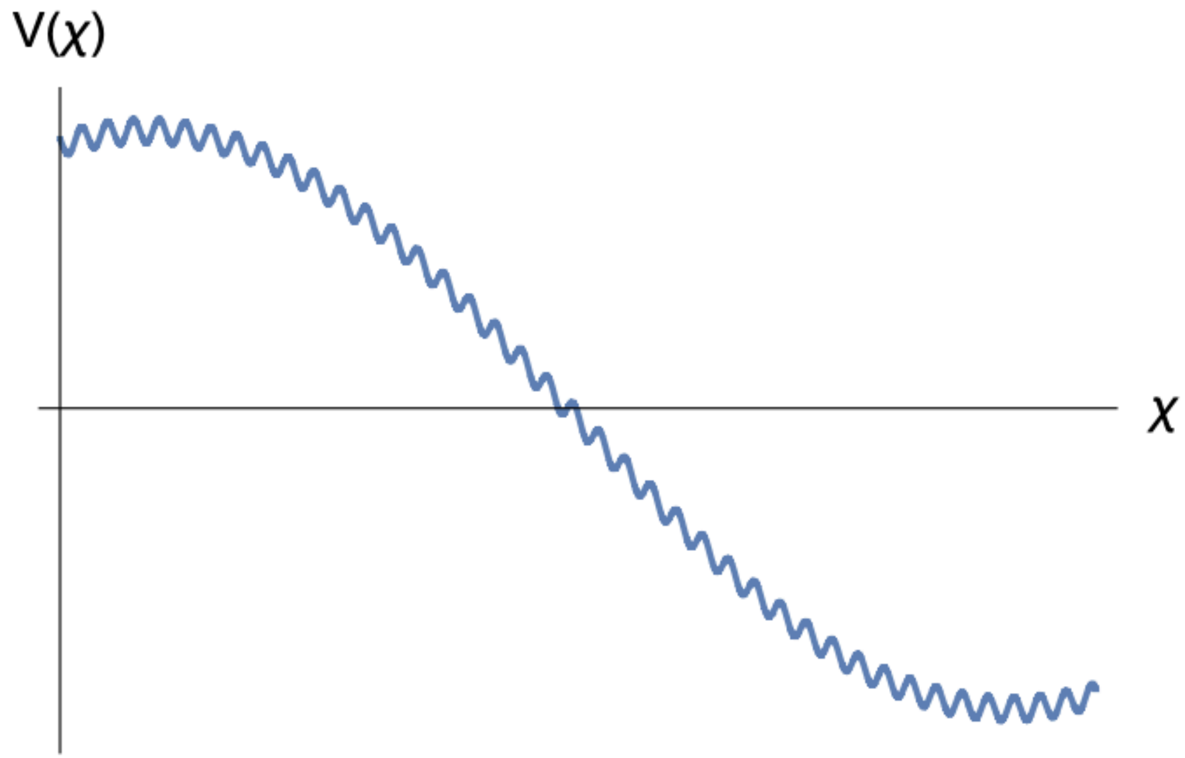
$$P \propto \exp[-\#\phi] \times \exp[-\#\chi]$$

$$m_h^2 \propto \phi$$

$$\Lambda_{cc} \propto \phi + \chi$$

Probability measures

What are the probabilities to observe different vacua?



$\chi \propto$ some fundamental parameter, e.g. mH

Probability measures

What are the probabilities to observe different vacua?

1. standard volume-weighted measure

A. D. Linde, Phys. Lett. B **175**, 395 (1986).
A. D. Linde, D. A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994), gr-qc/9306035.
A. D. Linde and A. Mezhlumian, Phys. Lett. B **307**, 25 (1993), gr-qc/9304015.

2. local measures

R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006), hep-th/0605263.
L. Susskind (2007), 0710.1129.
Y. Nomura, Astron. Rev. **7**, 36 (2012), 1205.2675.

Volume-weighted measures

Probability to observe some type of vacuum (labeled e.g. by the Higgs mass) $\propto$ overall volume of this vacuum at some proper time t

Volume-weighted measures

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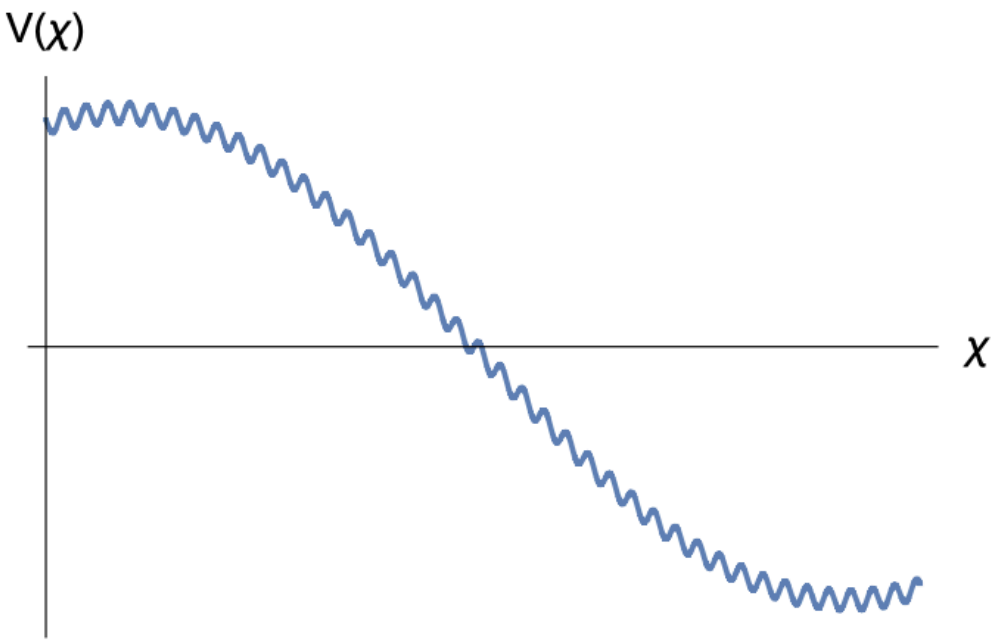
*Youngness paradox: assumed to be solved by the stationary measure prescription

Volume-weighted measures

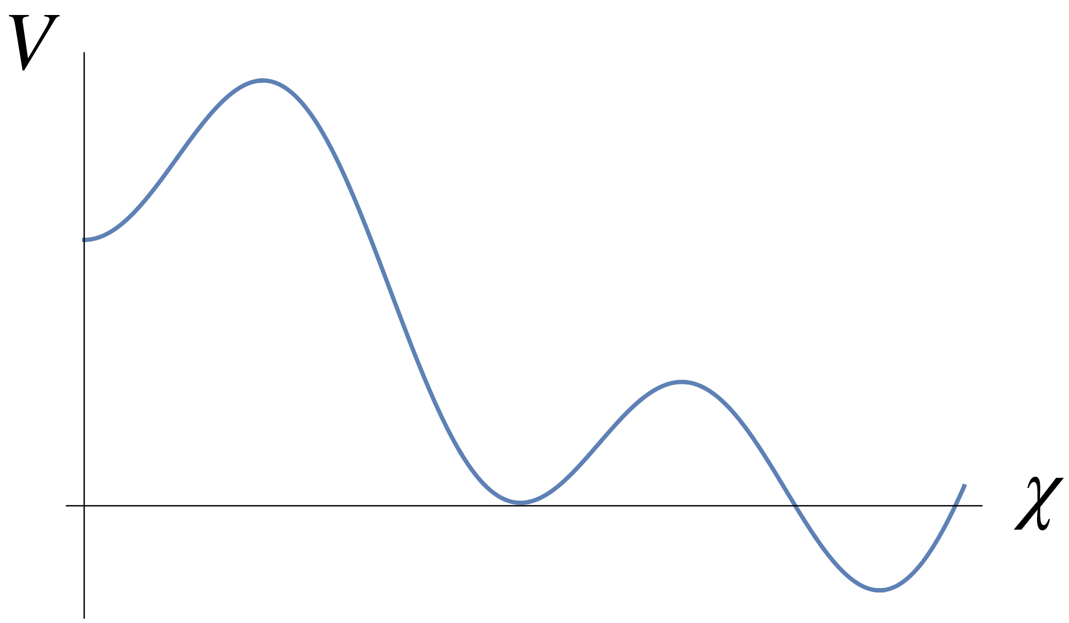
Probability gradients



$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i} + 3H_i P_i$$

Volume-weighted measures

Probability gradients



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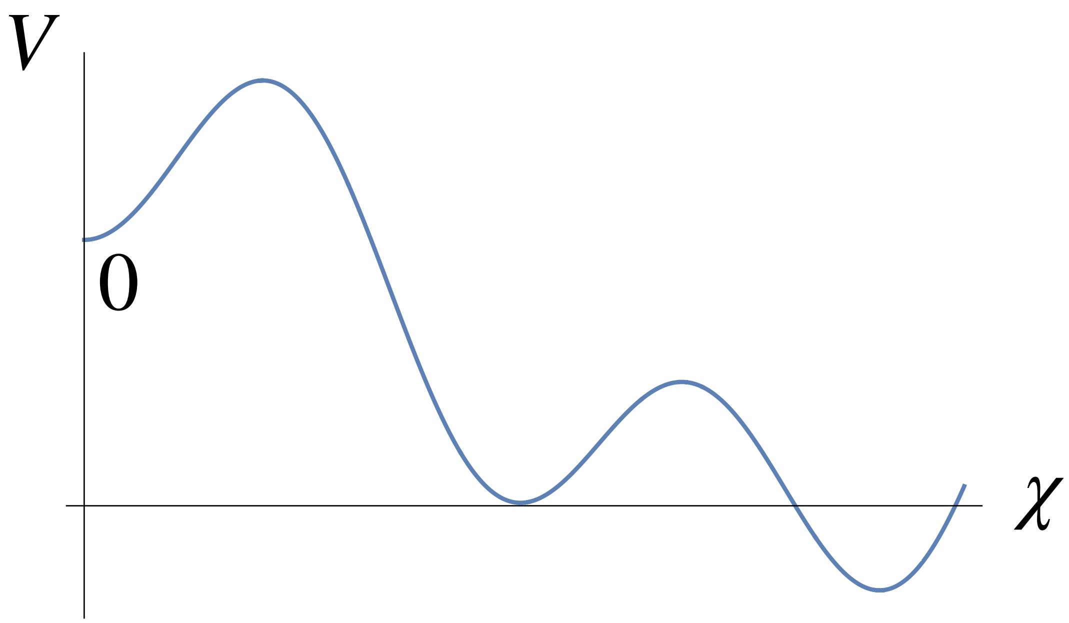


$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i} + 3H_i P_i$$

- Highest “parent” minimum

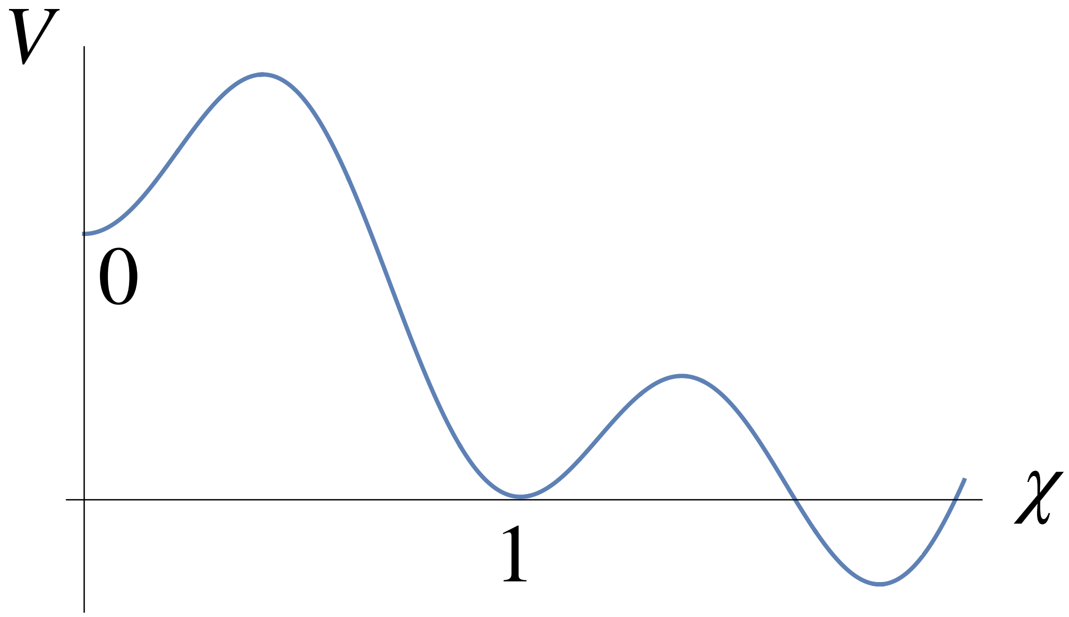
$$\dot{P}_0 \simeq 3H_0 P_0 \longrightarrow$$

stationary inflation:

$$P_0 = C_0\, e^{3H_0 t}$$

Volume-weighted measures

Probability gradients



$$\dot{P}_i = -P_i \sum_{j \neq i} \Gamma_{i \to j} + \sum_{j \neq i} P_j \Gamma_{j \to i} + 3H_i P_i$$

$j = 0$

- Lower vacuum:

$$\dot{P}_1 \simeq 3H_1 P_1 + P_0 \Gamma_{0 \to 1} \longrightarrow$$

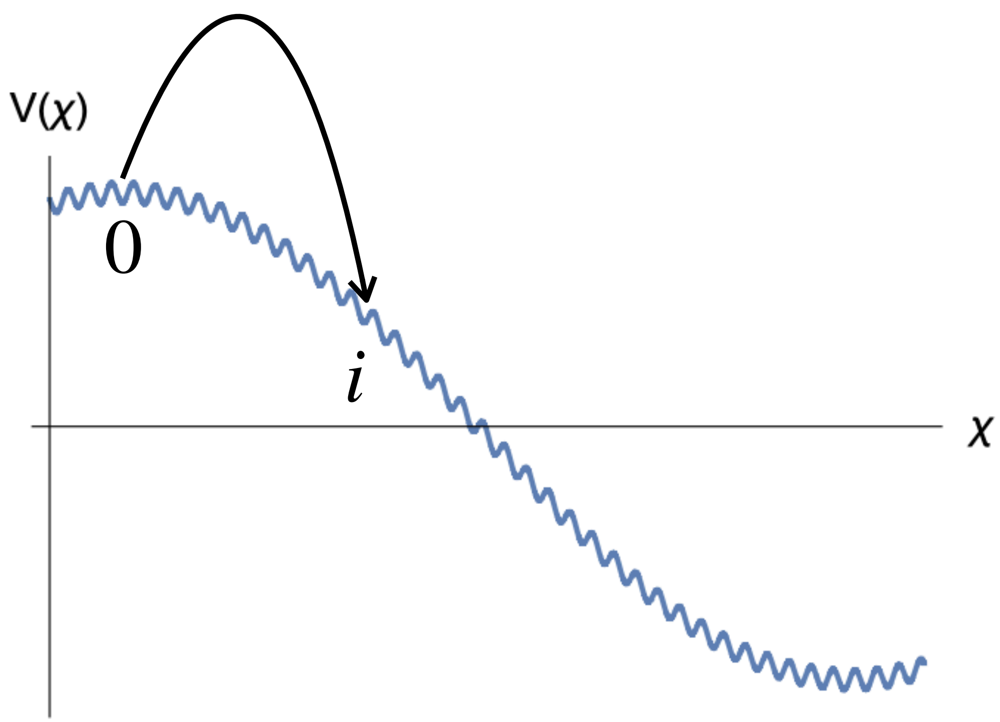
stationary inflation:

$$P_1 = C_1 \, e^{3H_0 t}$$

$$C_1 = \frac{\Gamma_{0 \to 1}}{3(H_0 - H_1)} C_0$$

Volume-weighted measures

Probability gradients



$$\dot{P}_i = -P_i \sum_{j\neq i} \Gamma_{i\to j} + \sum_{j\neq i} P_j \Gamma_{j\to i} + 3H_i P_i$$

- Chain rule leads to

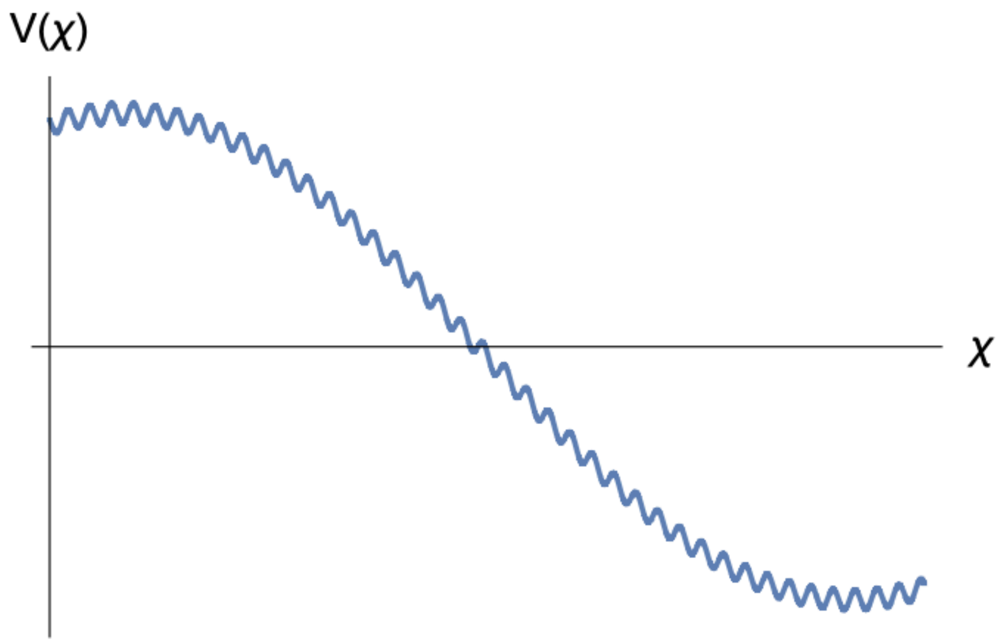
$$P_i = \left[\prod_{j=1}^{i} \frac{\Gamma_{\downarrow}}{3(H_0 - H_j)} \right] C_0 e^{3H_0 t}$$

HM tunneling (|m|<H):

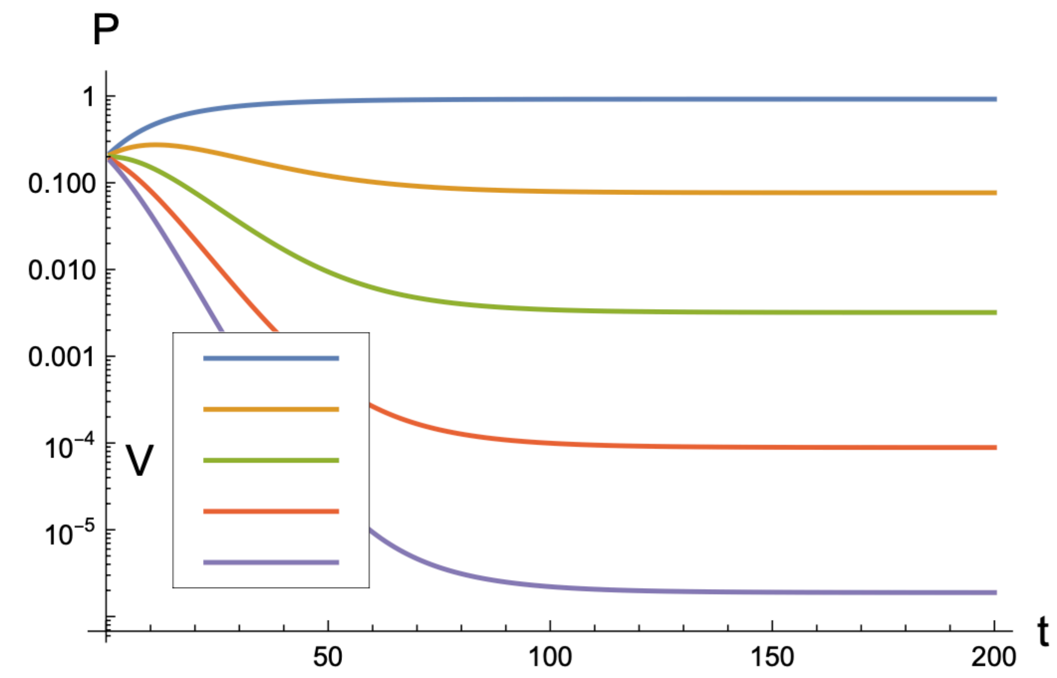
$$\Gamma_{j\to i} \sim H_j \exp\left[-\frac{8\pi^2}{3} \frac{\Delta V_B}{H_j^4} \right]$$

Volume-weighted measures

Probability gradients



numeric evol.



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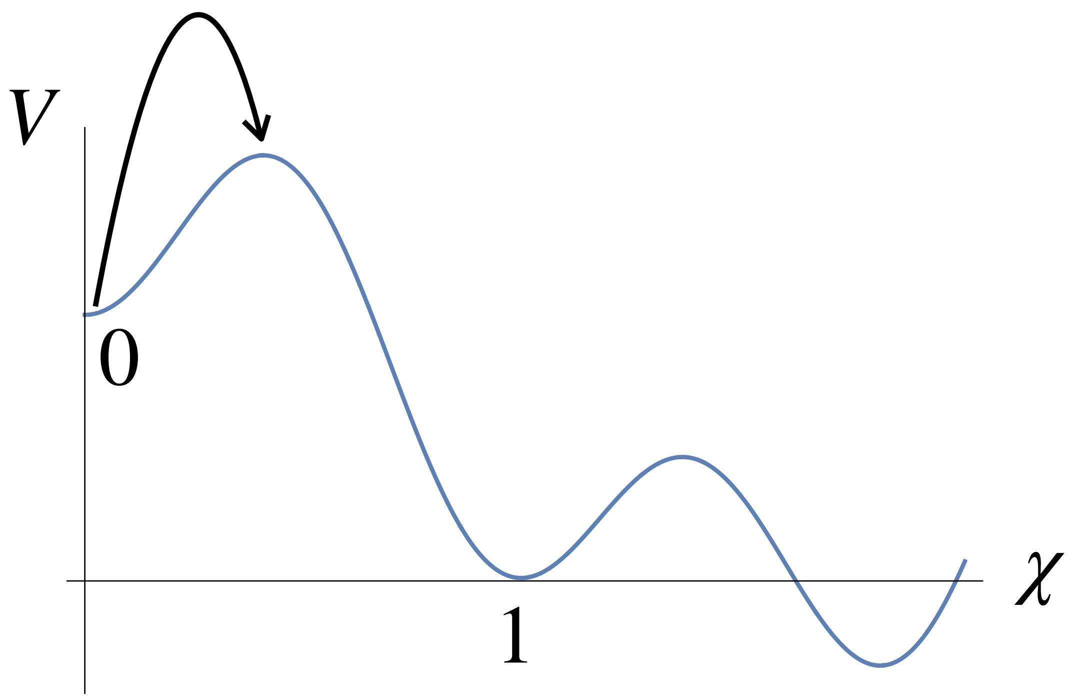
Volume-weighted measures

Stochastic approach



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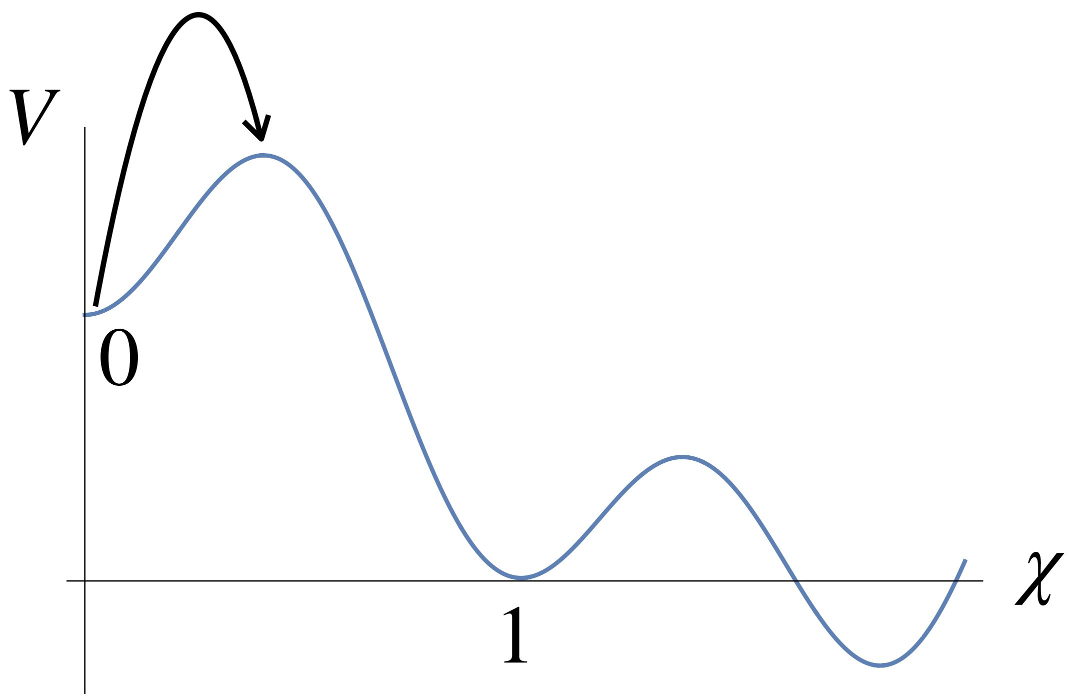
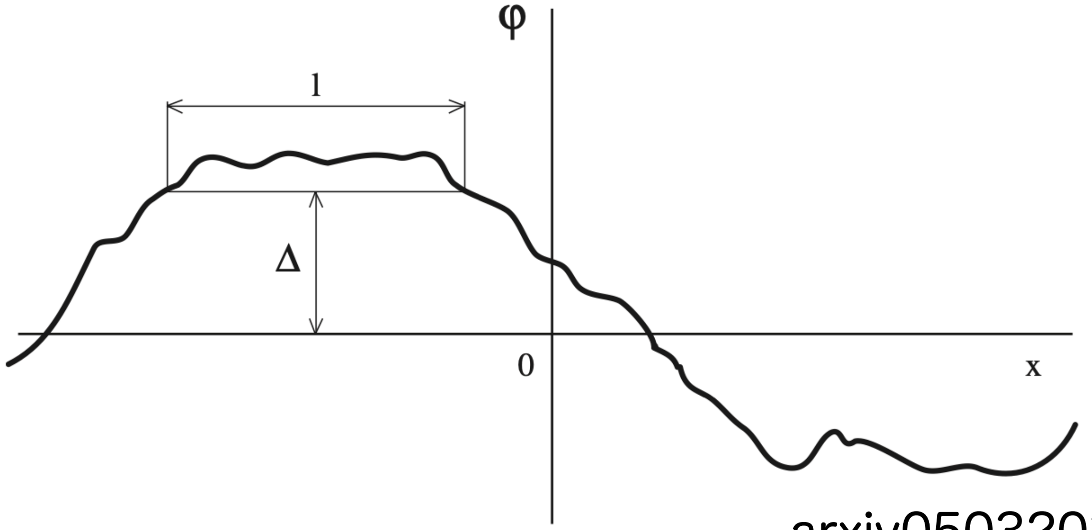


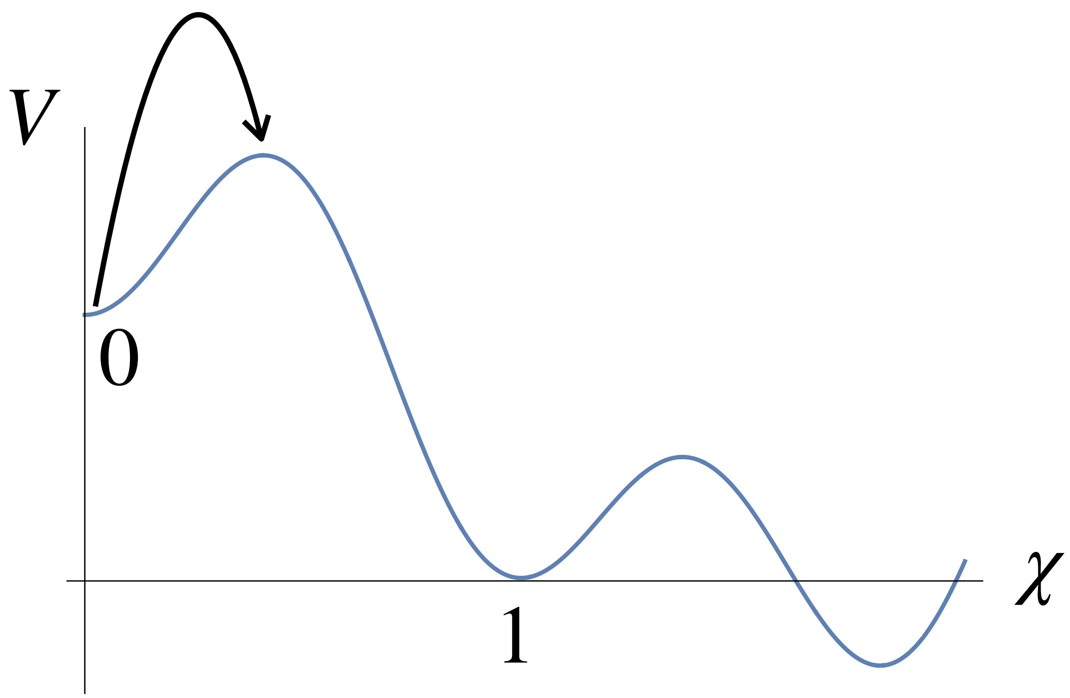
“quantum spread”



arxiv0503203

Volume-weighted measures

Stochastic approach



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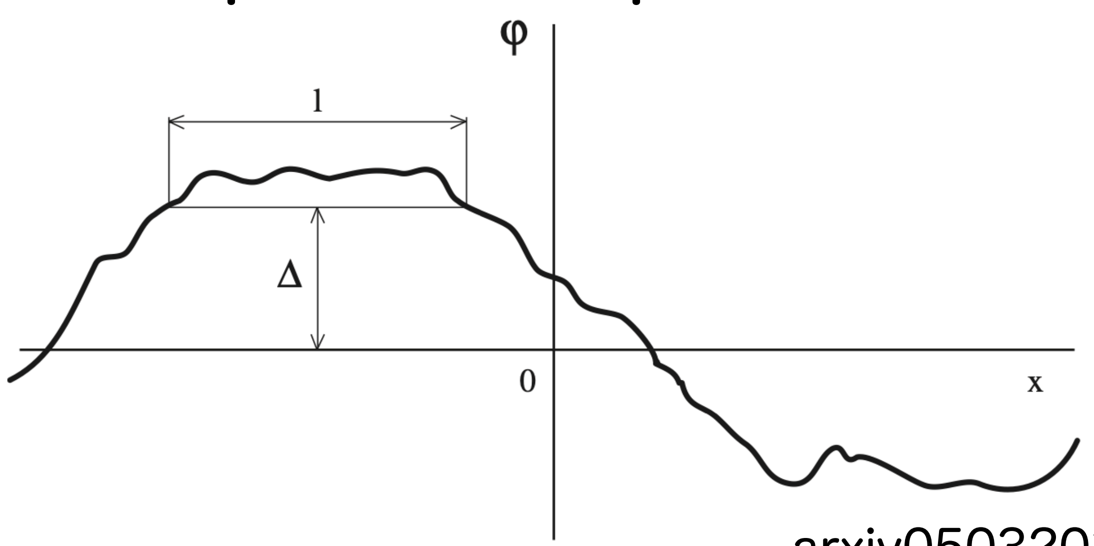


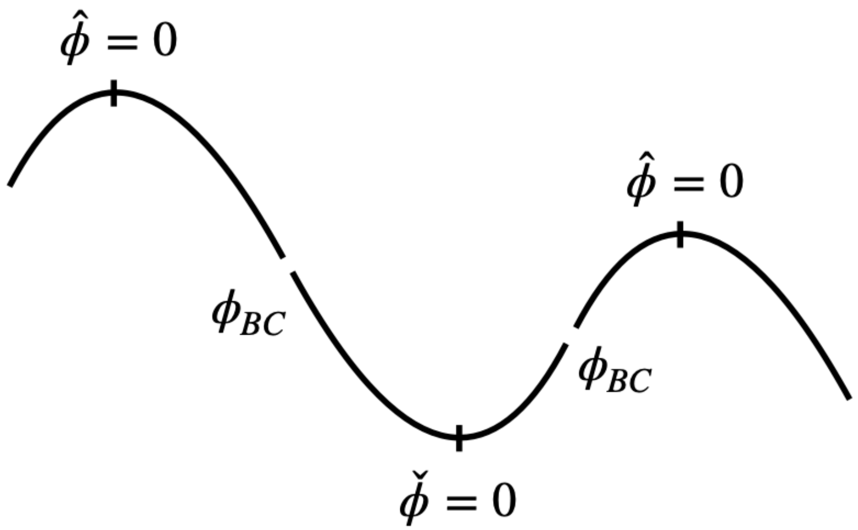
arxiv0503203

$$P(\chi_i) \to P(\chi)$$

$$\dot{P} = \frac{\partial}{\partial \phi}\left(\frac{H^{3(1-\beta)}}{8\pi^2}\frac{\partial}{\partial \phi}(H^{3\beta}P)\right) + \frac{\partial}{\partial \phi}\left(\frac{V'}{3H}P\right) + 3HP$$

Volume-weighted measures

Stochastic approach



$$V = \Lambda + \frac{1}{2}m^2\phi^2$$

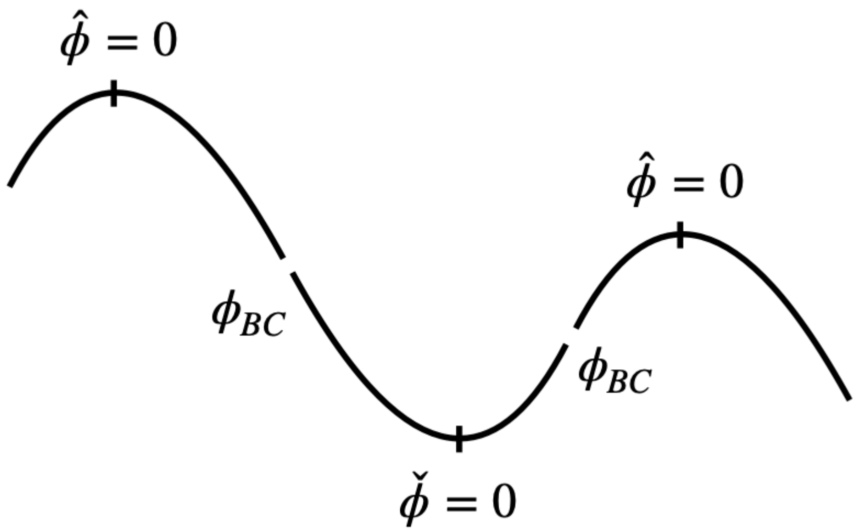
general solution:

$$P_\nu = \exp\left[-A\phi^2\right]\left\{\mathbf{c}_+ D_\nu\left[B\phi\right] + \mathbf{c}_- D_\nu\left[-B\phi\right]\right\}$$

eigenmodes of $\nu \propto -H_s^2 + \ldots$

Volume-weighted measures

Matching



Volume-weighted measures

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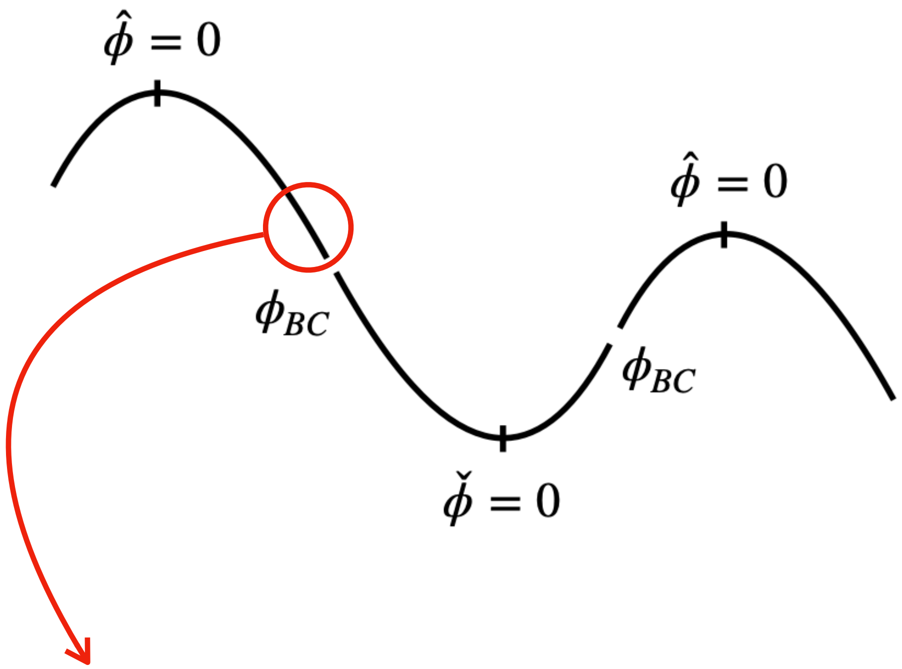

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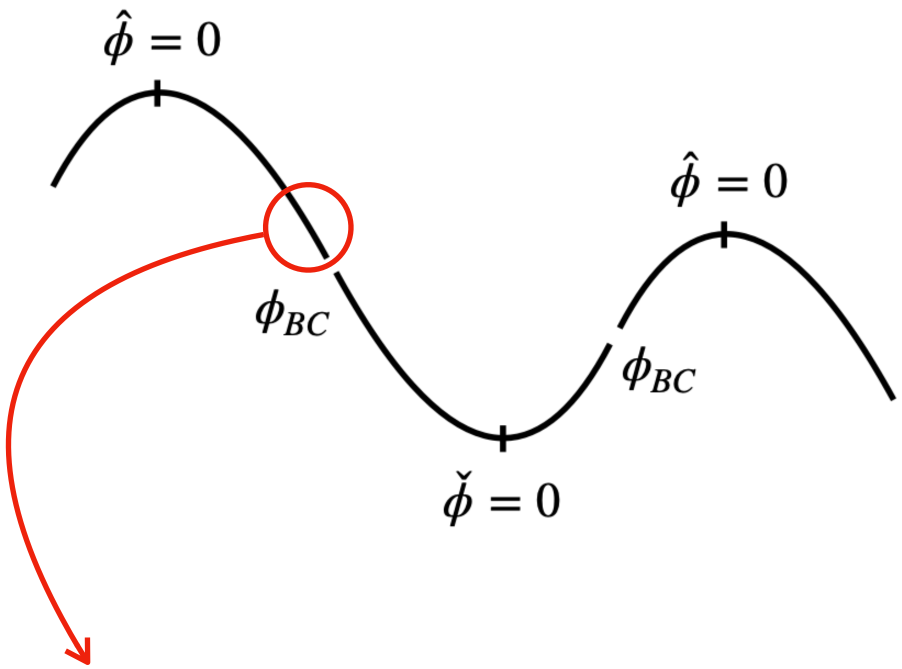


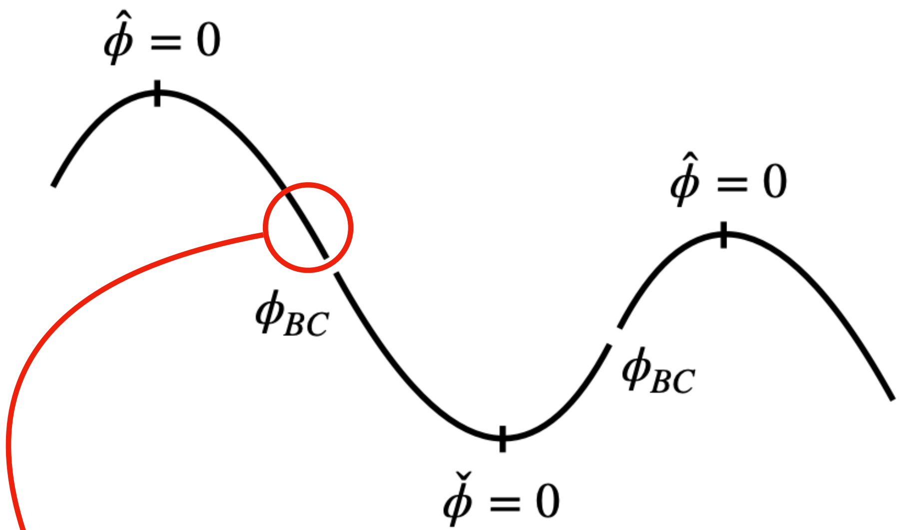
$$P_\nu = \exp\left[-A\phi^2\right]\left\{\mathbf{c}_+ D_\nu[B\phi] + \mathbf{c}_- D_\nu\left[-B\phi\right]\right\}$$

$B\phi \to \infty$

$$(B\phi)^2 \propto \frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}$$

Volume-weighted measures

Matching



P and P' not tunable unless

$$|c_-/c_+| \sim e^{-\frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}}$$

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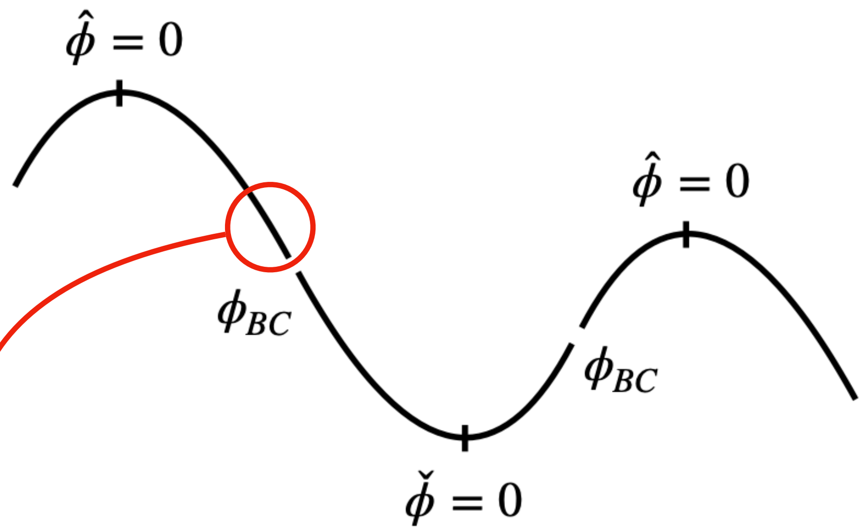
$\ll 1$

Volume-weighted measures

Matching



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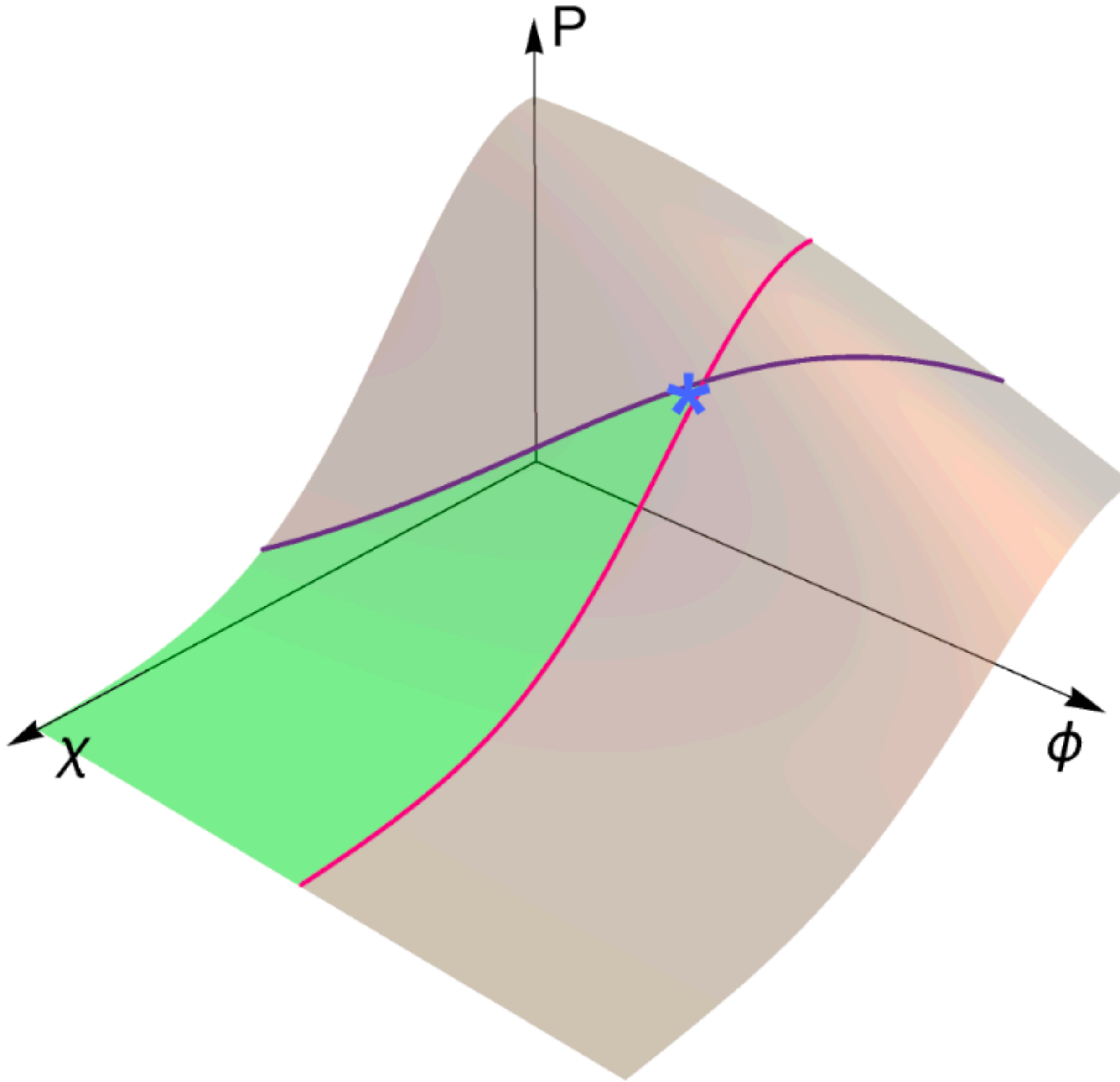
$$P_\nu = \exp\left[-A\phi^2\right]\left\{\mathbf{c}_+ D_\nu\left[B\phi\right] + \mathbf{c}_- D_\nu\left[-B\phi\right]\right\}$$

$$\frac{\check{P}_{i+1}(0)}{\check{P}_i(0)} \simeq \frac{\Gamma[-\hat{\nu}_i]\Gamma[-\check{\nu}_{i+1}]}{2\pi}|B\phi_{\rm BC}|^{2(\check{\nu}_i+\hat{\nu}_i+1)} e^{-\frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}} + \mathcal{O}(\epsilon^2)$$

$$\left(\epsilon \sim \frac{H^4}{m_p^2 m^2}\right)$$

Volume-weighted measures

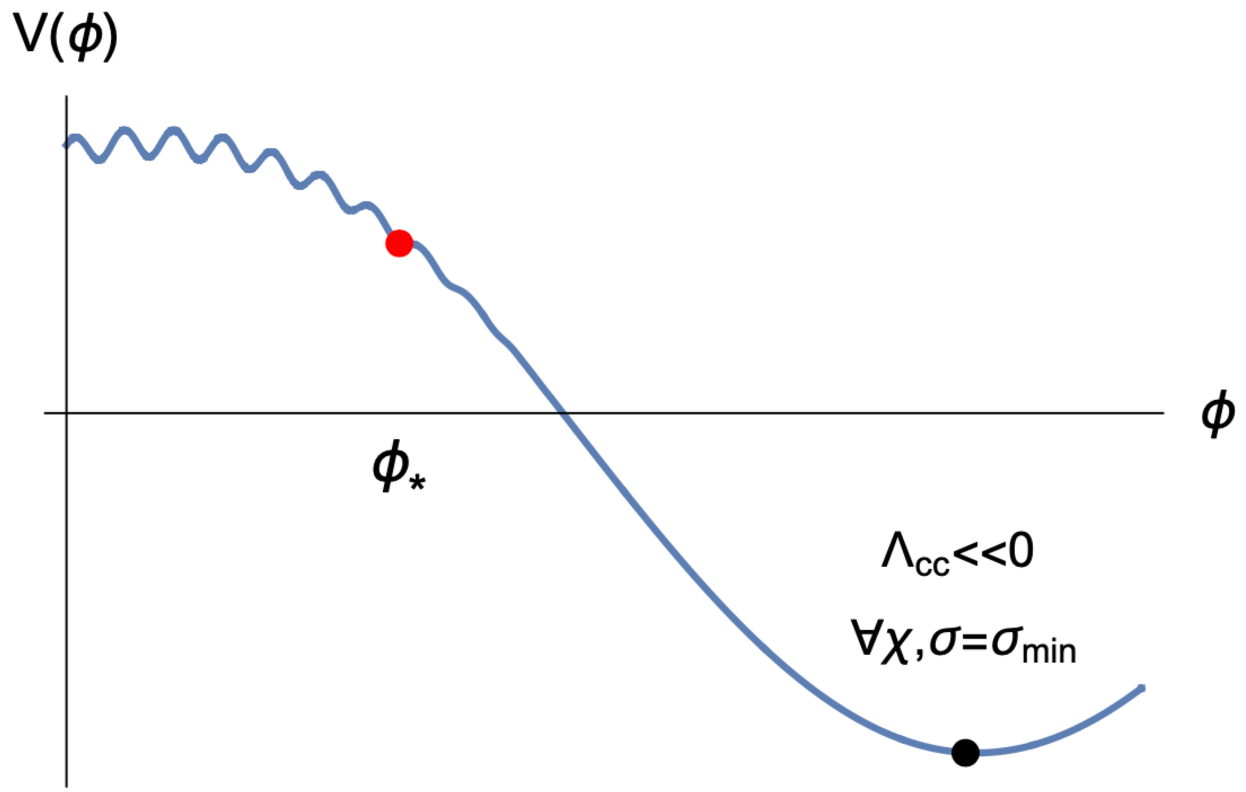
We got the gradients



We need to scan mH and introduce the boundaries

mH and CC from gradients & boundaries

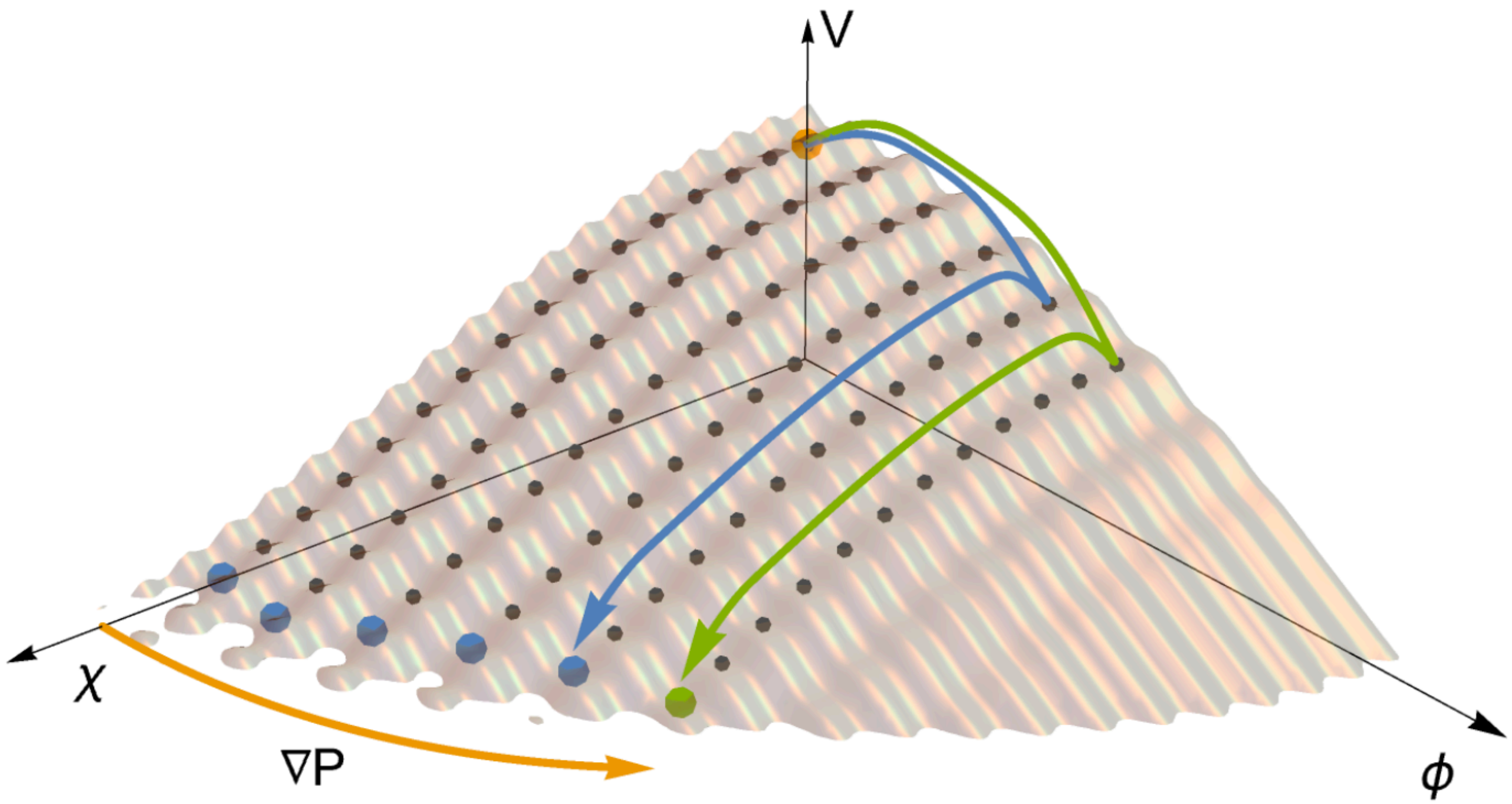
Higgs-VEV dependent critical boundary



$$V(\phi, h) = M_\phi^4 \cos\phi/F_\phi - c_1 M_\phi^2 h^2 \cos\phi/F_\phi - c_2 M_\phi^2 h^2 + \frac{1}{4}\lambda_h h^4 + \mu_\phi^2 h^2 \cos\phi/f_\phi$$

$$m_h^2 \simeq -2c_2 M_\phi^2 - 2c_1 M_\phi^2 \cos\phi/F_\phi$$

mH and CC from gradients & boundaries

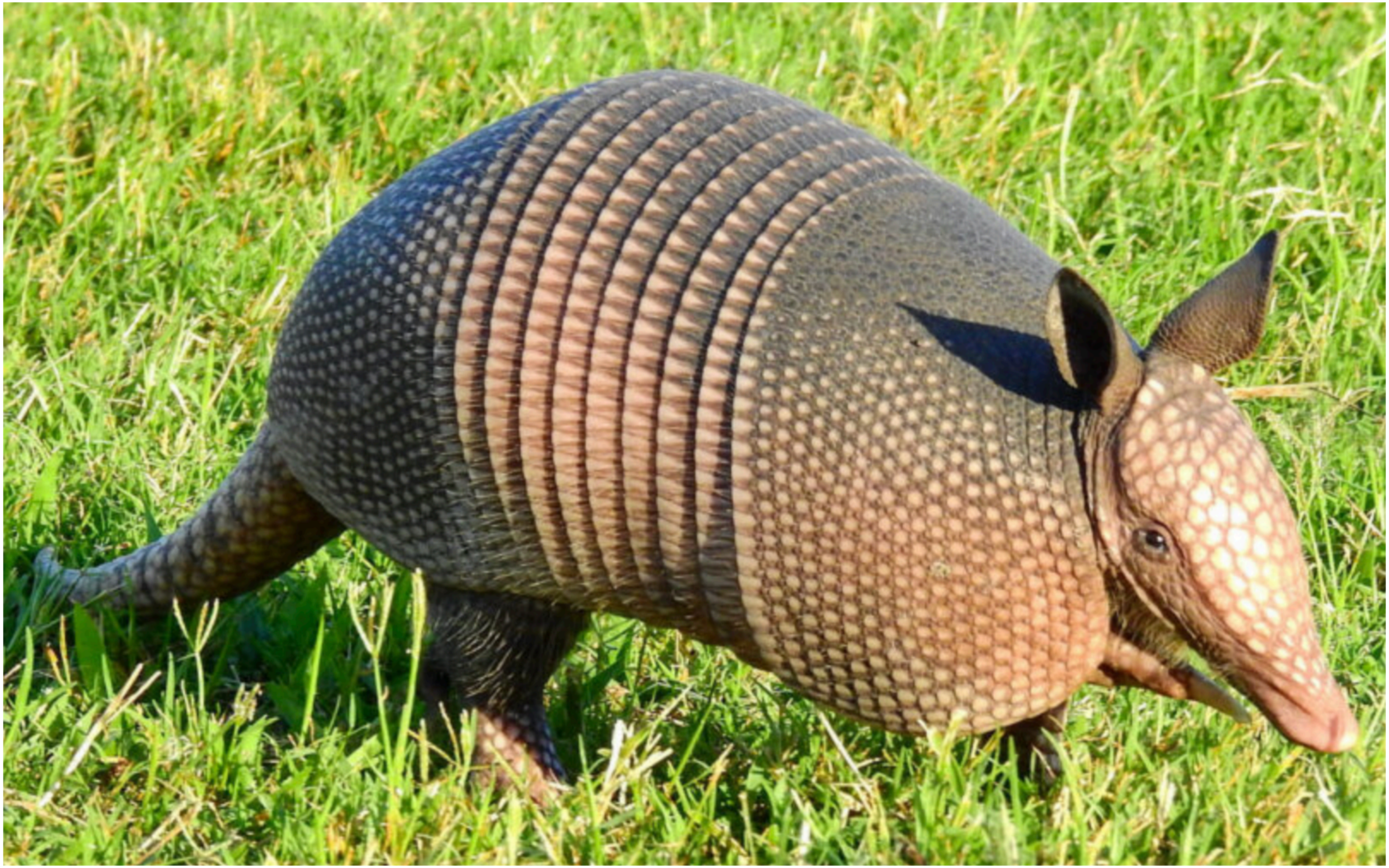


factorization:

$$P(\phi,\chi) \simeq P(\phi)\,P(\chi)$$

$$\rightarrow \qquad \frac{P_{\color{green}\bullet}}{P_{\color{cyan}\bullet}} \sim \frac{\Gamma_\phi}{\Gamma_\chi} \gg 1$$

Armadillo



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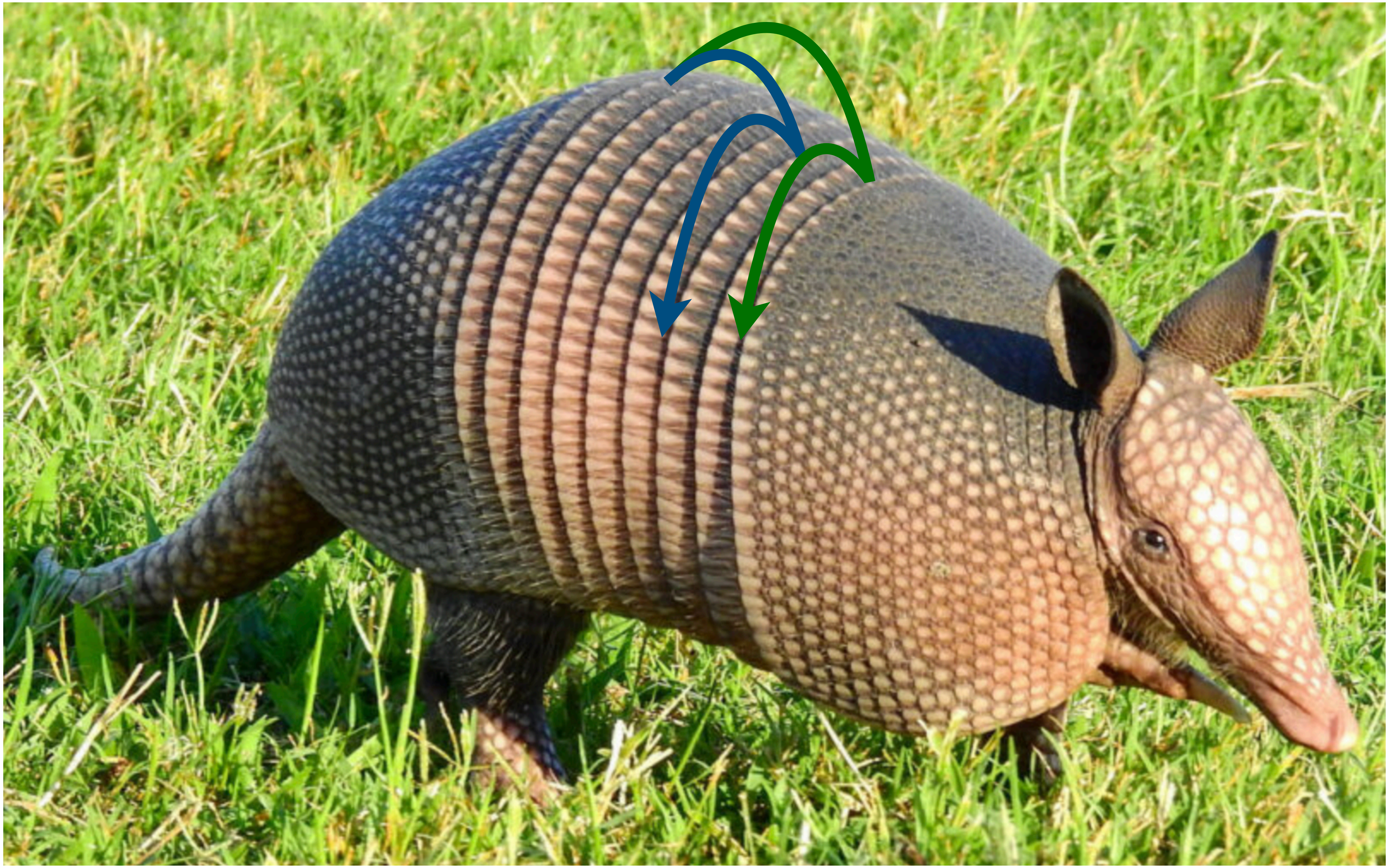
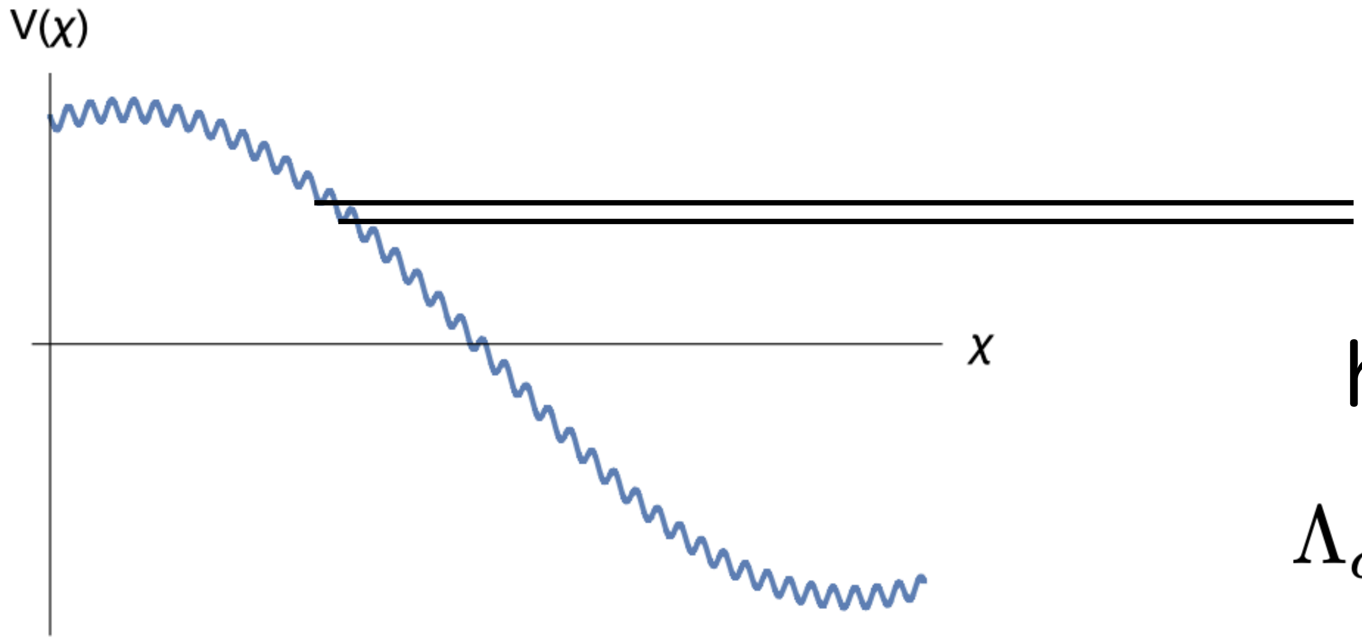


mH and CC from gradients & boundaries

CC solution?



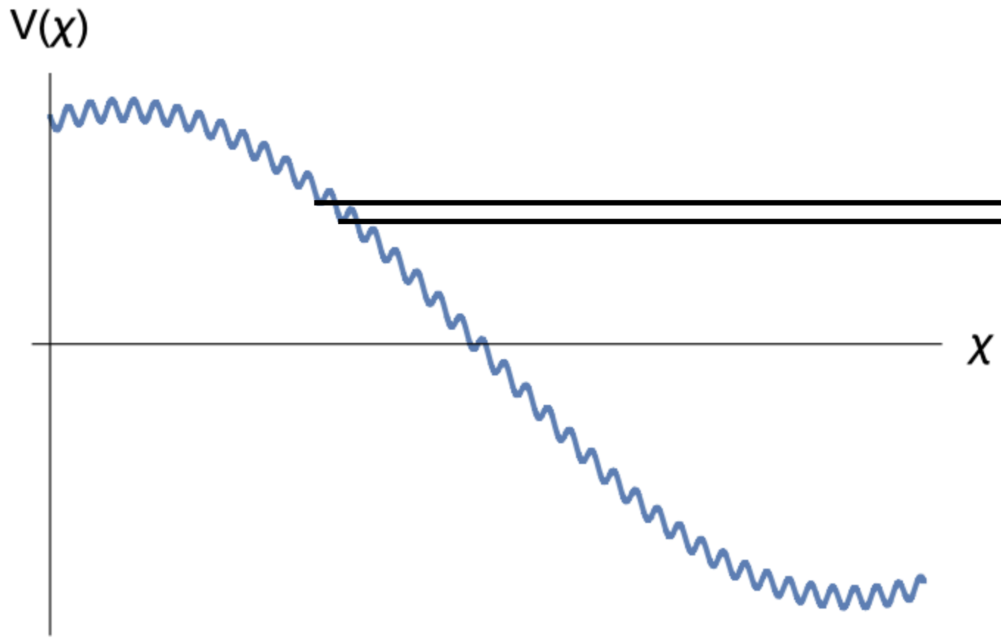
$$\Delta\Lambda_{cc\,\chi} \simeq M_\chi^4/N_\chi$$

has to be within

$$\Lambda_{cc(obs.)} \simeq 10^{-47}\mathrm{GeV}^4 \quad (1)$$

mH and CC from gradients & boundaries

CC solution?



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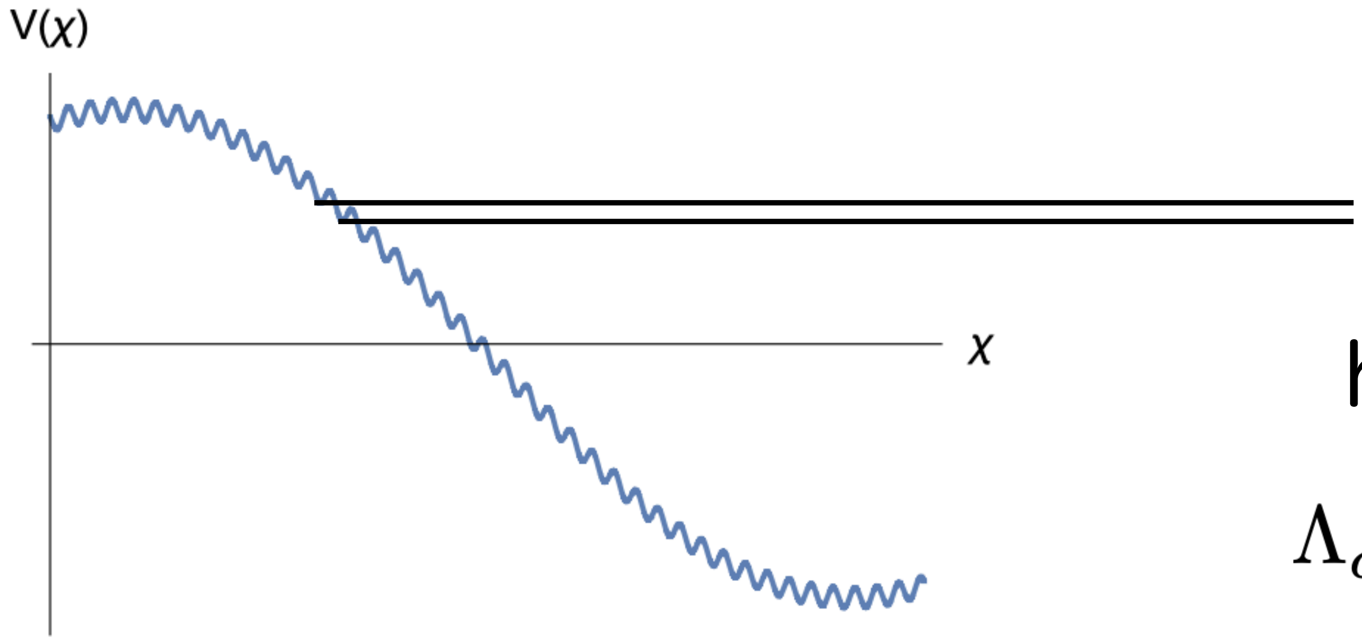
In addition, $P(\chi)$ prefers less tunnelings, hence higher Λ, close to the upper anthropic bound $\sim\ 10^3\Lambda_{cc(obs.)}$

$\Rightarrow$ one needs a sufficiently mild grad $P(\chi)$ (2)

mH and CC from gradients & boundaries

CC solution?



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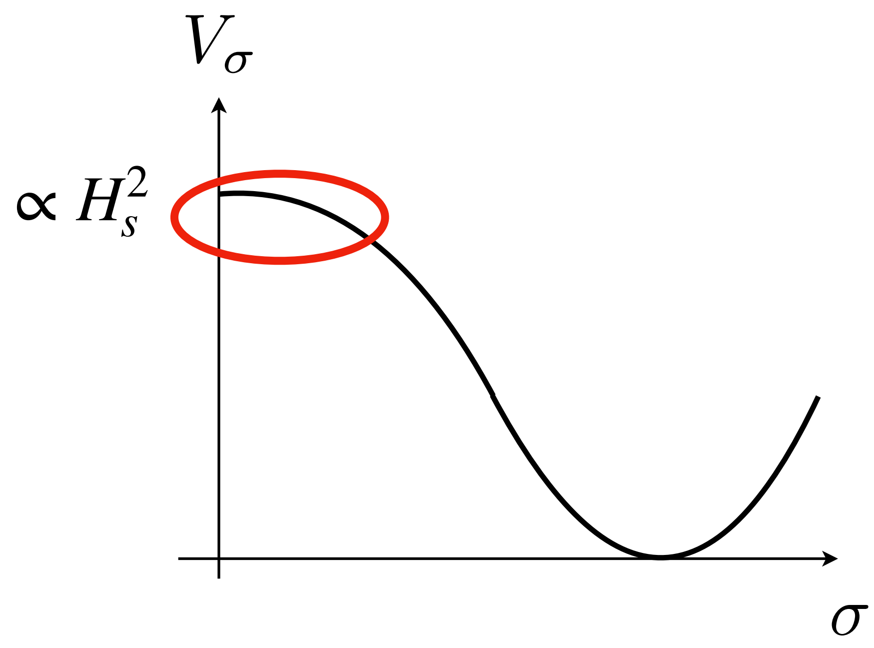
We evade (1), (2) by assuming some additional fine-scanning sector.

mH and CC from gradients & boundaries

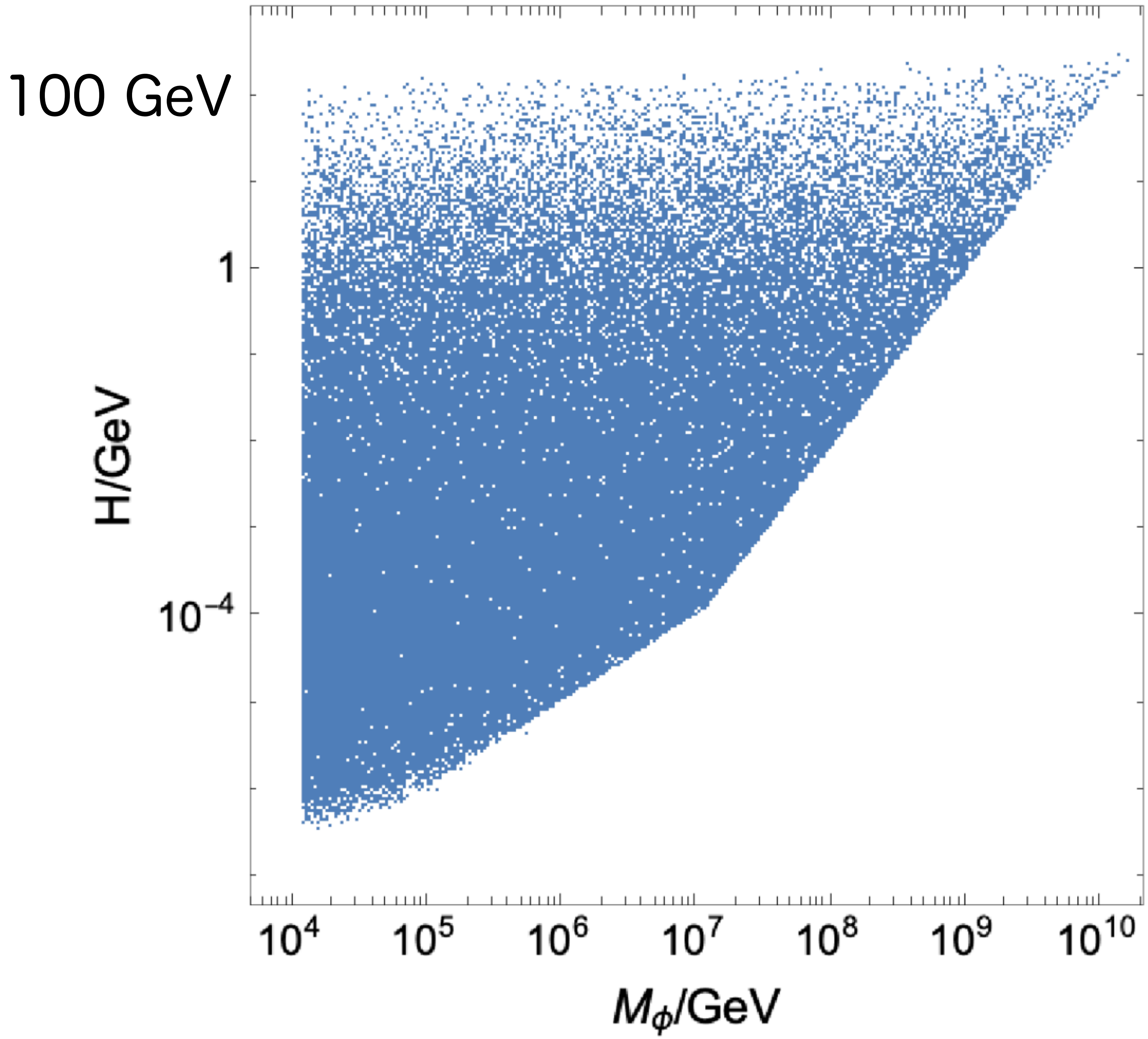
Slow-roll inflation



We assume some slow-roll inflation in the background, responsible for eternal inflation at a scale H_s

mH and CC from gradients & boundaries

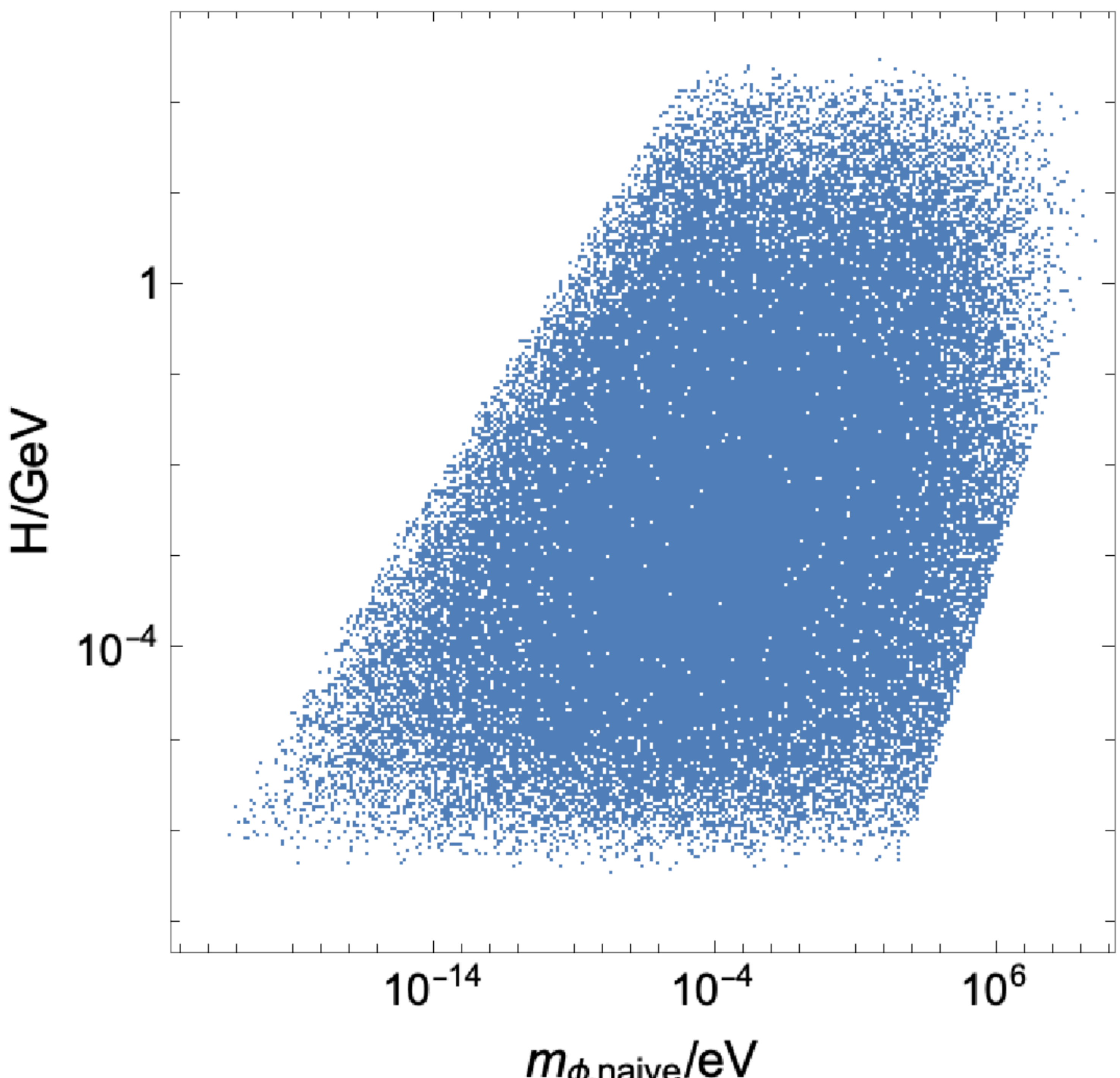
Parameter space



where EW physics is

mH and CC from gradients & boundaries

Parameter space



mH and CC from gradients & boundaries

Parameter space

- Hierarchical suppression over ϕ landscape requires

$$\Gamma_\phi \sim \exp\left[-\frac{8\pi^2}{3}\frac{\Delta V_B}{H^4}\right] \ll 1 \quad \Rightarrow \quad H \lesssim \Delta V_B{}^{1/4} \sim \sqrt{\mu_\phi v_{\rm SM}} \lesssim \mathrm{v_{sm}}$$

I'm too restrictive here!

- Landscape energy contribution is subdominant in H_s

$$M_\phi \lesssim \sqrt{m_P H} \lesssim \sqrt{m_P \mathrm{v}_{sm}}$$

Local measures

Motivation

Extrapolation of black hole complementarity to inflationary space.

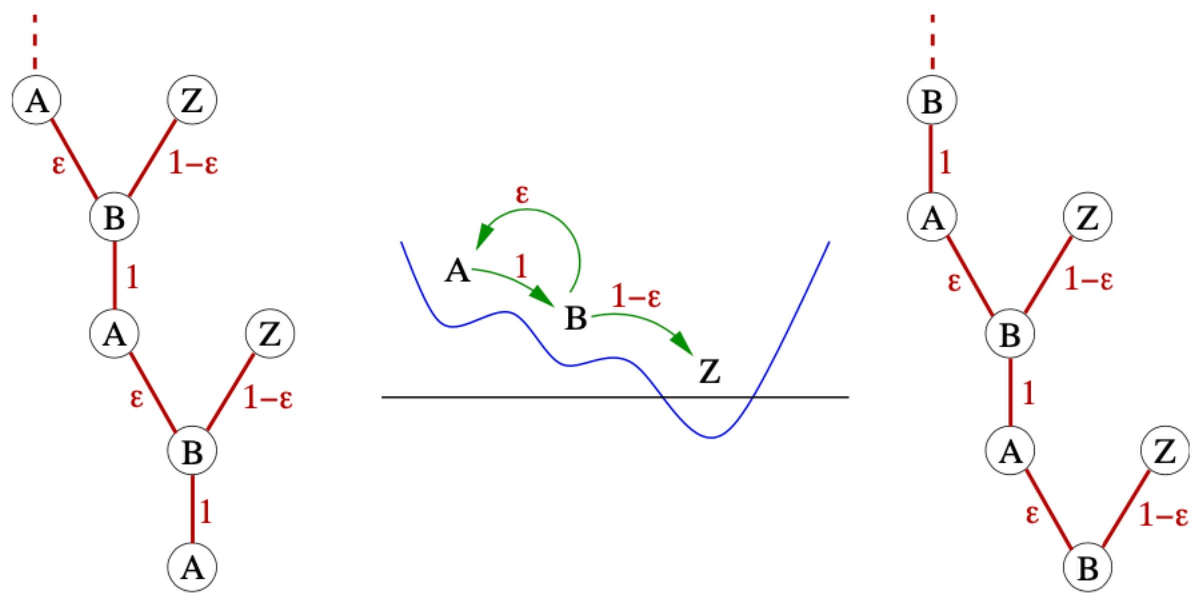
The physically meaningful description of the universe should be confined to a region of space accessible to some hypothetical observer.

R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006), hep-th/0605263.
L. Susskind (2007), 0710.1129.
Y. Nomura, Astron. Rev. **7**, 36 (2012), 1205.2675.

Local measures

What is P(vac)?

Time that a worldline spends (or number of times it enters) in a given vacuum on its way to AdS



Bousso, 0605263

Local measures

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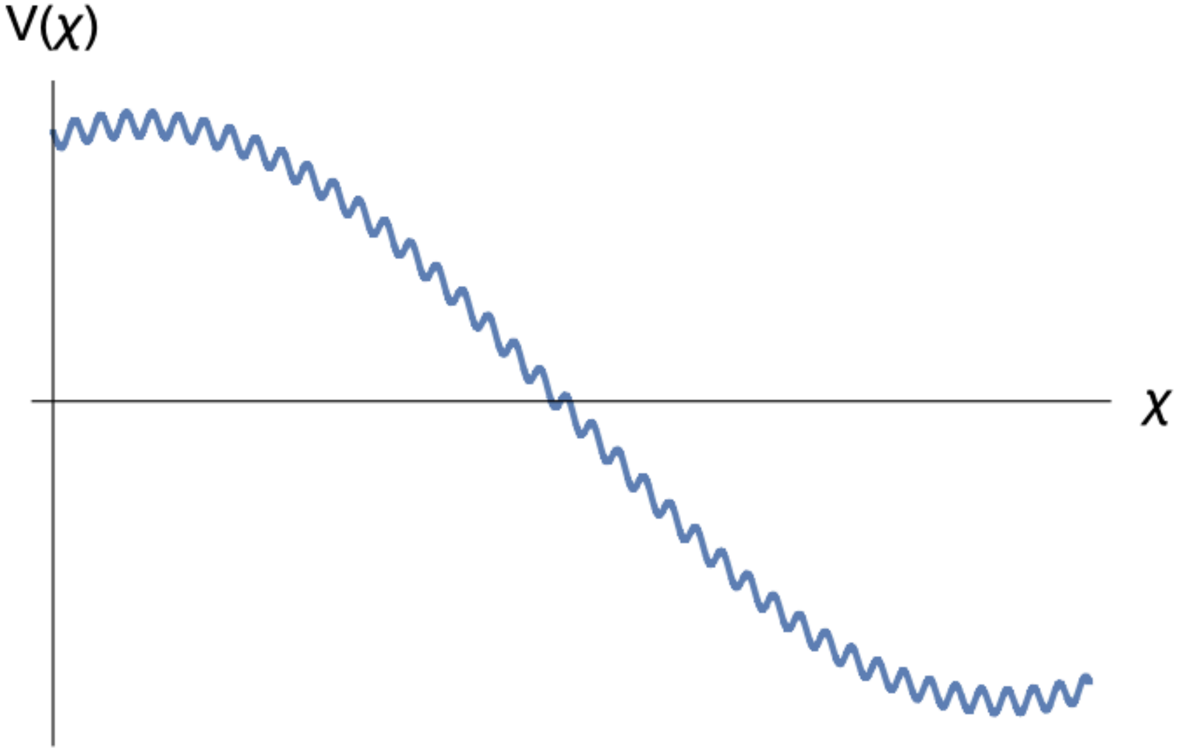
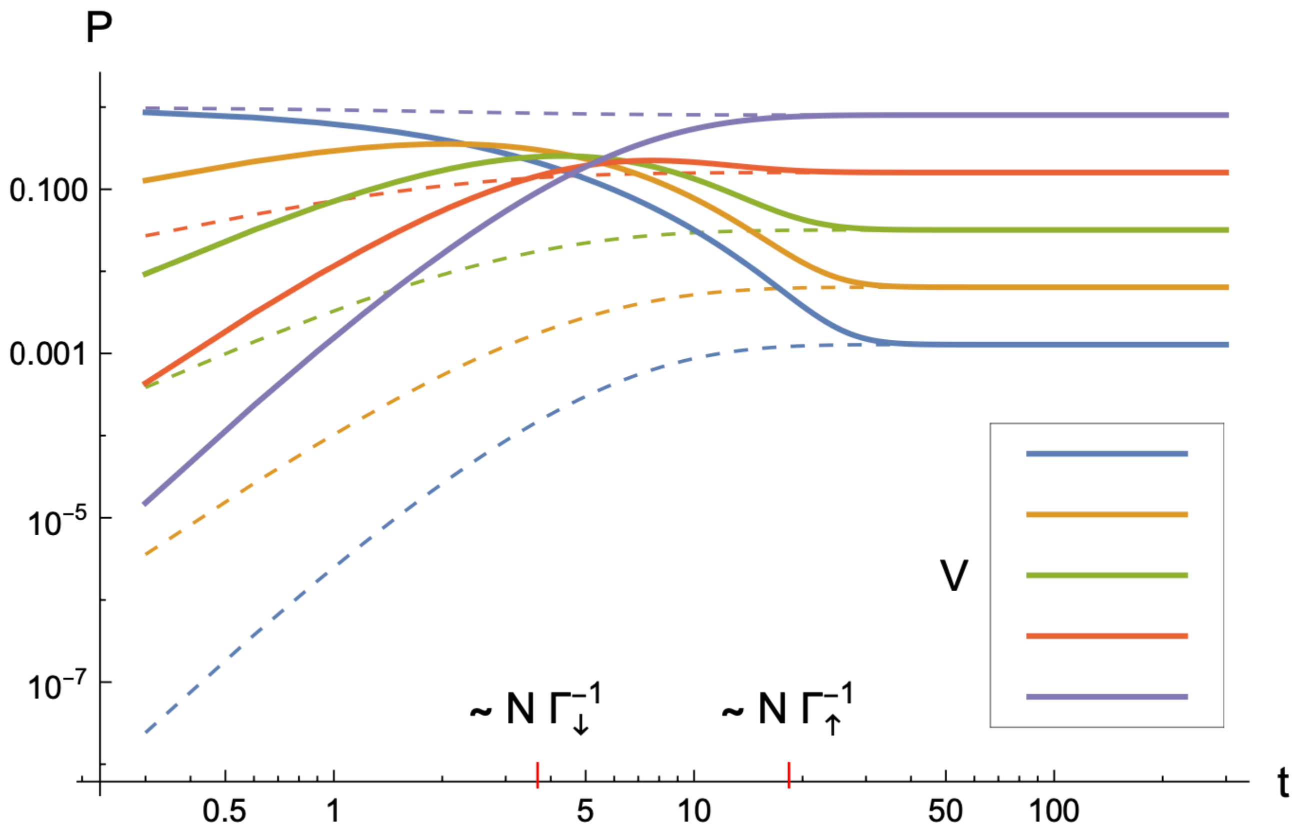
Local measures

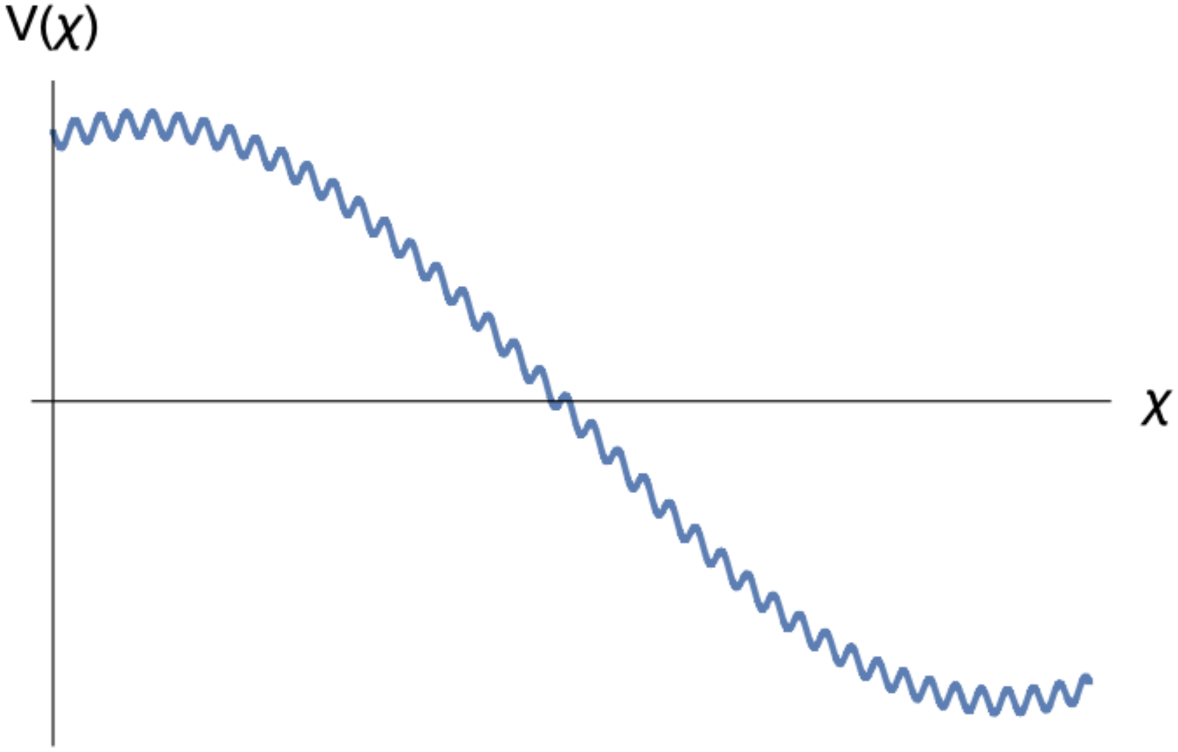
Probability gradients

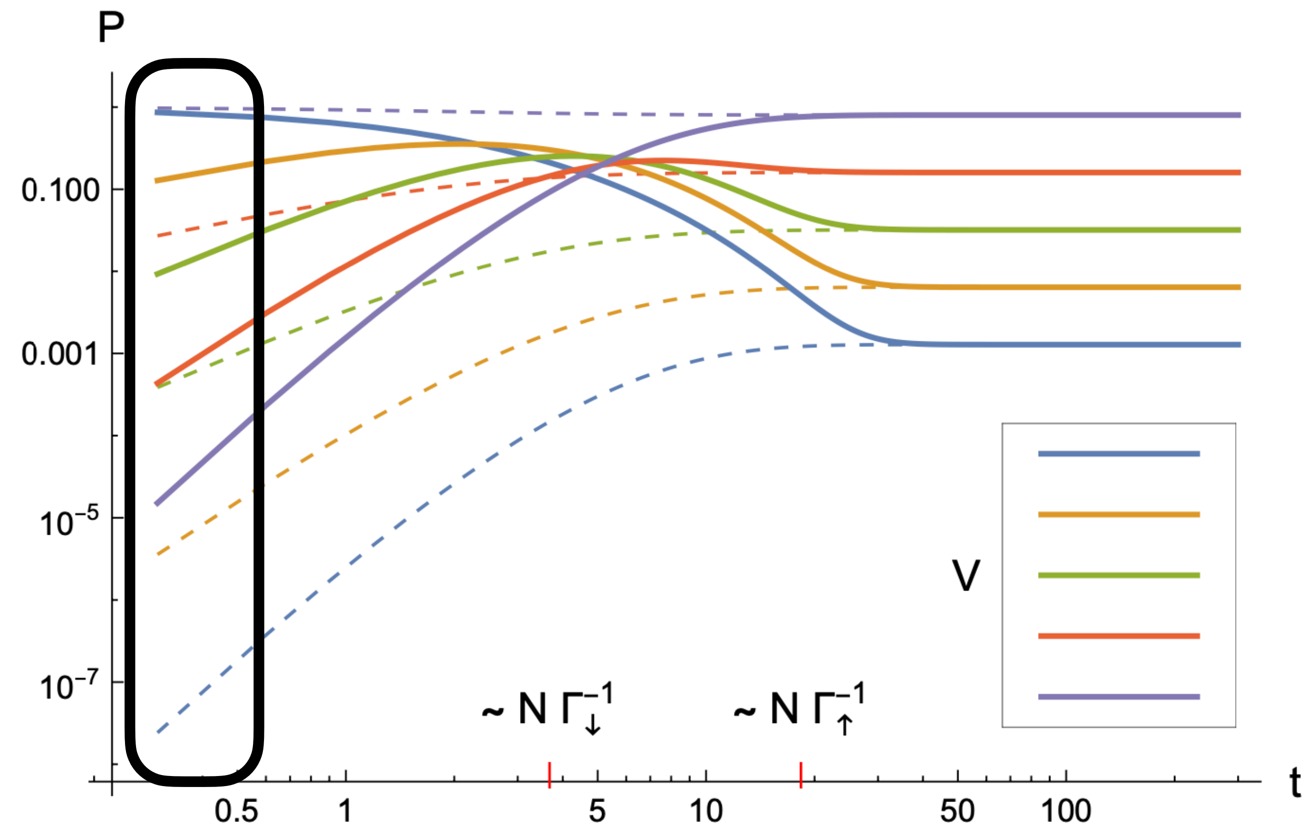




Local measures

Probability gradients





1. Dominated by initial conditions

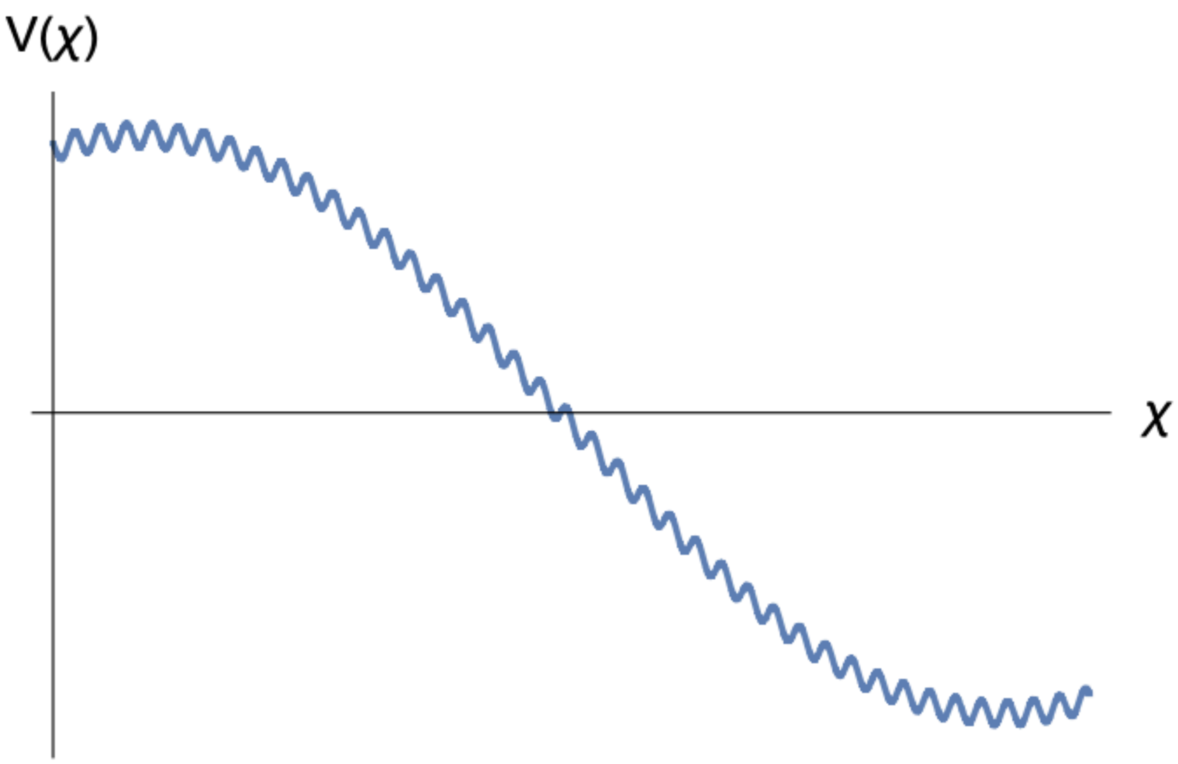
e.g. "quantum creation of the universe"

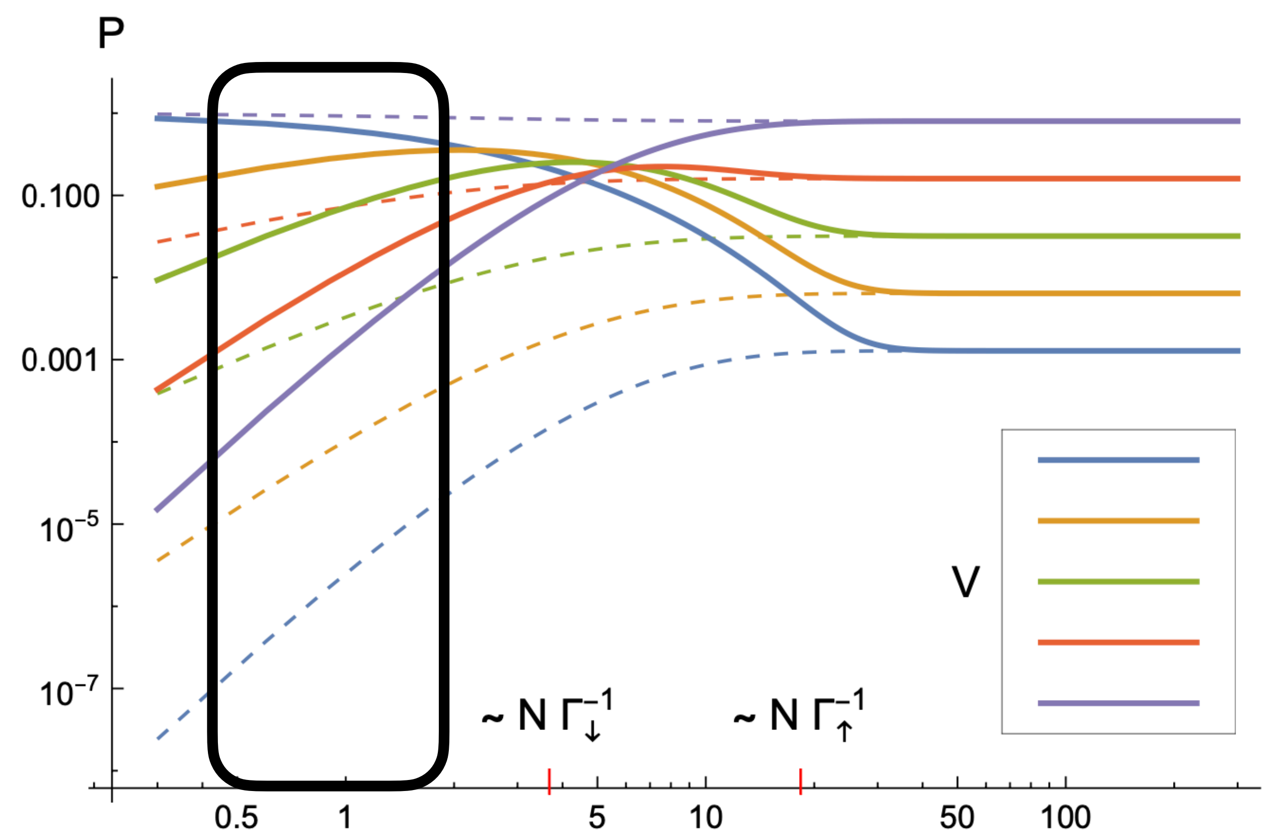
$$P(t=0) \propto \exp\left[-\frac{3}{8}\frac{m_P^4}{V(\chi)}\right] \propto \exp\left[\frac{8\pi^2}{3}\frac{V(\chi)}{H^4}\right]$$

A. D. Linde, Lett. Nuovo Cim. **39**, 401 (1984).
A. Vilenkin, Phys. Rev. D **30**, 509 (1984).

Local measures

Probability gradients



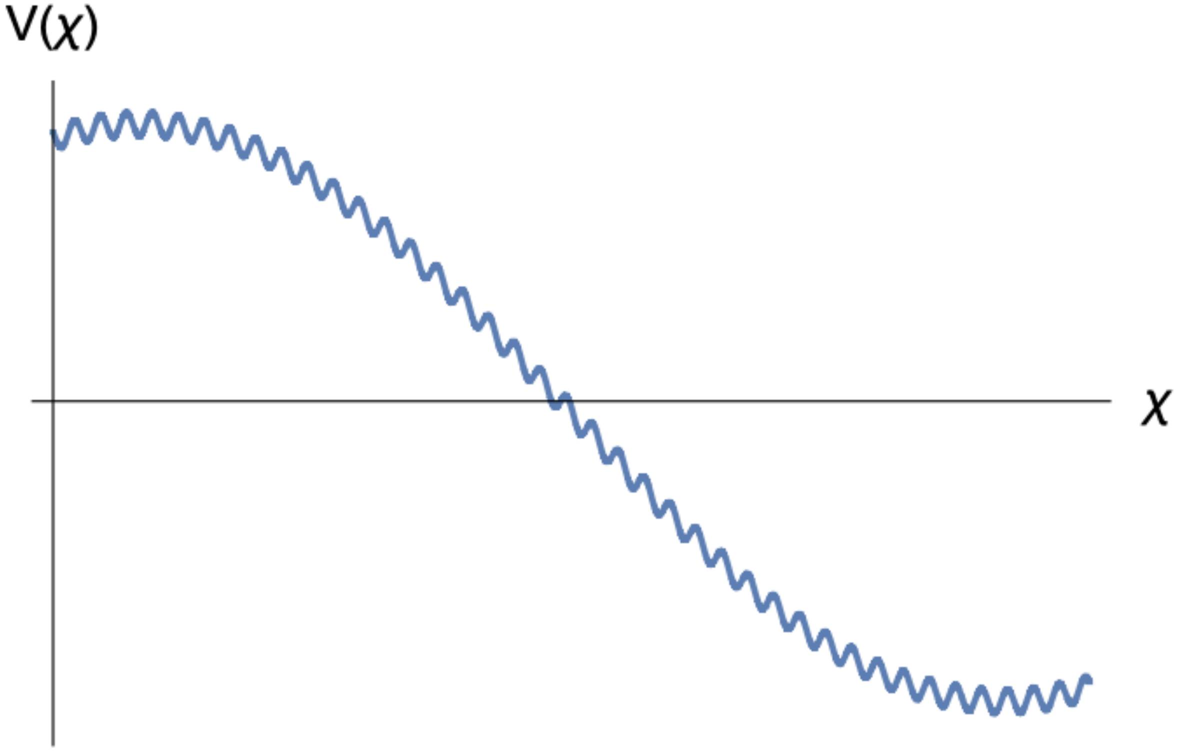


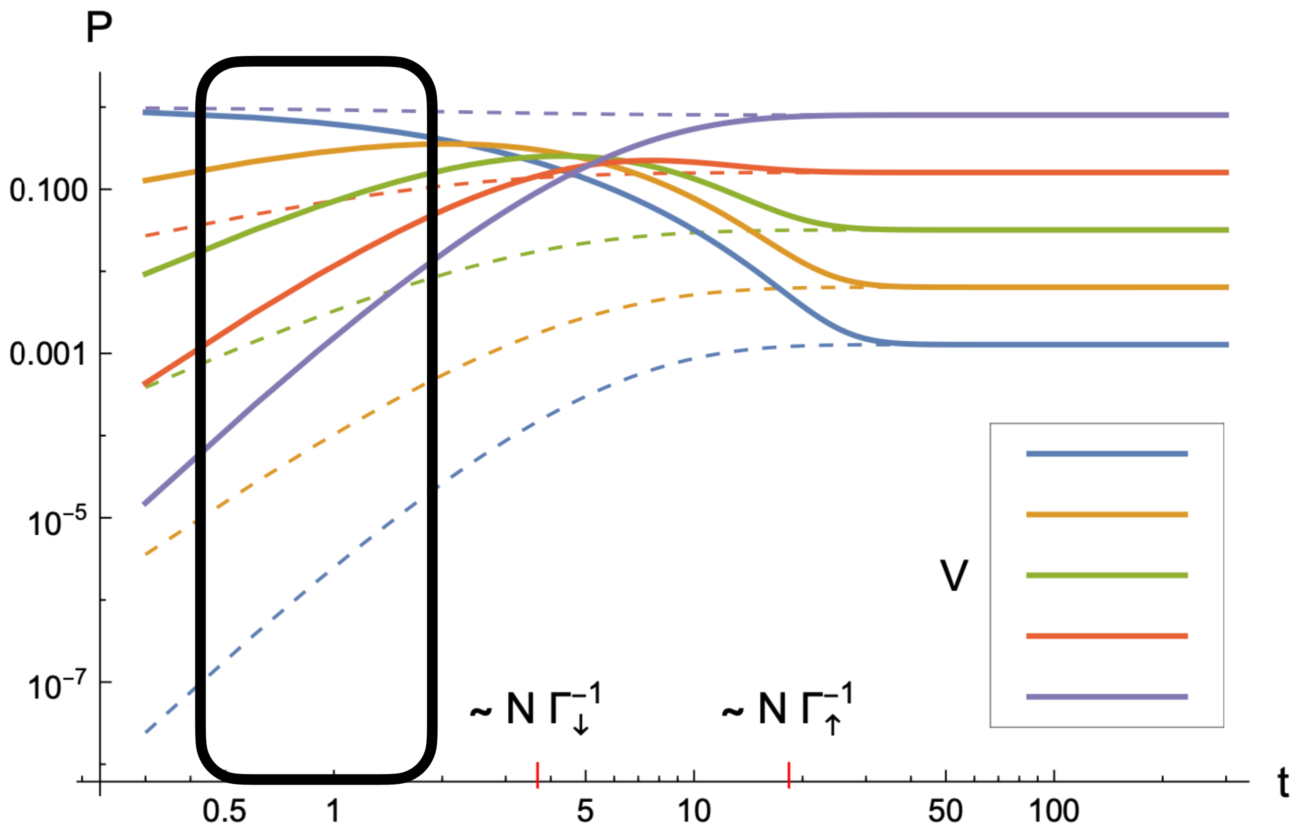
2. I.C. + Dynamics

$$P = \exp[\kappa t] P_{t=0}, \text{with } \kappa_{ij} = \Gamma_{j\to i} - \delta_{ij} \sum_k \Gamma_{j\to k}$$

Local measures

Probability gradients



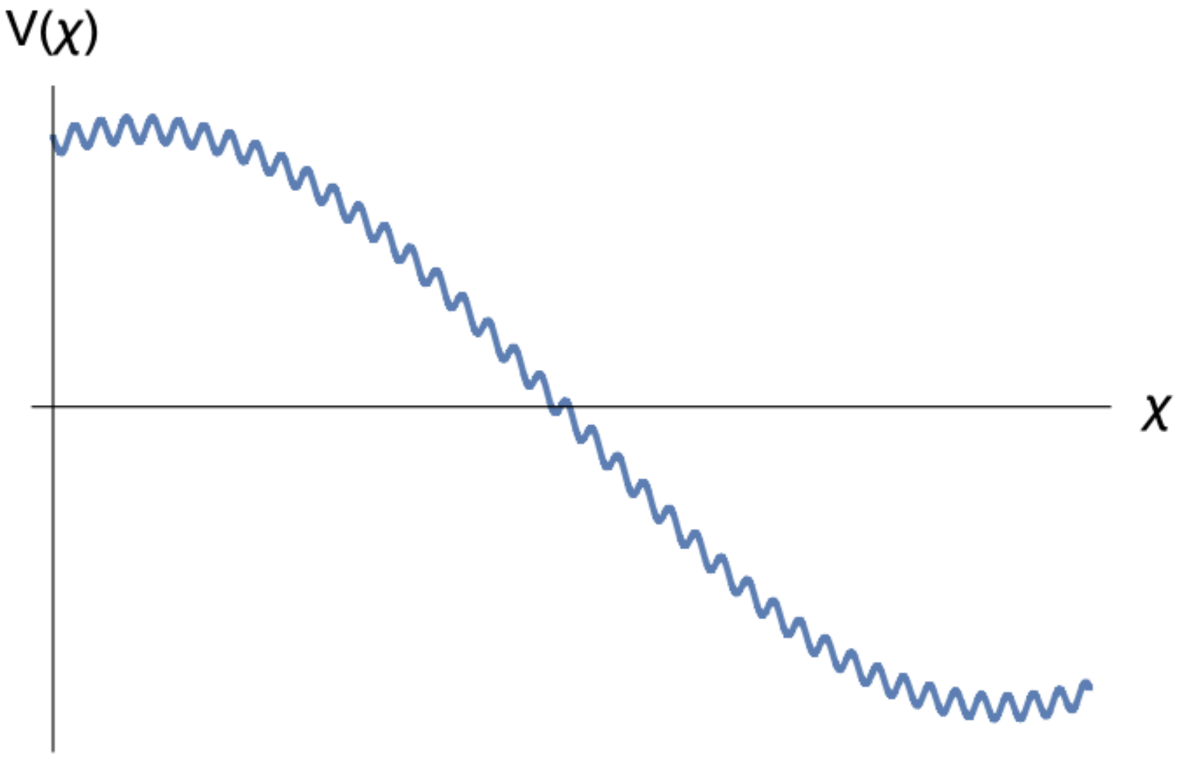


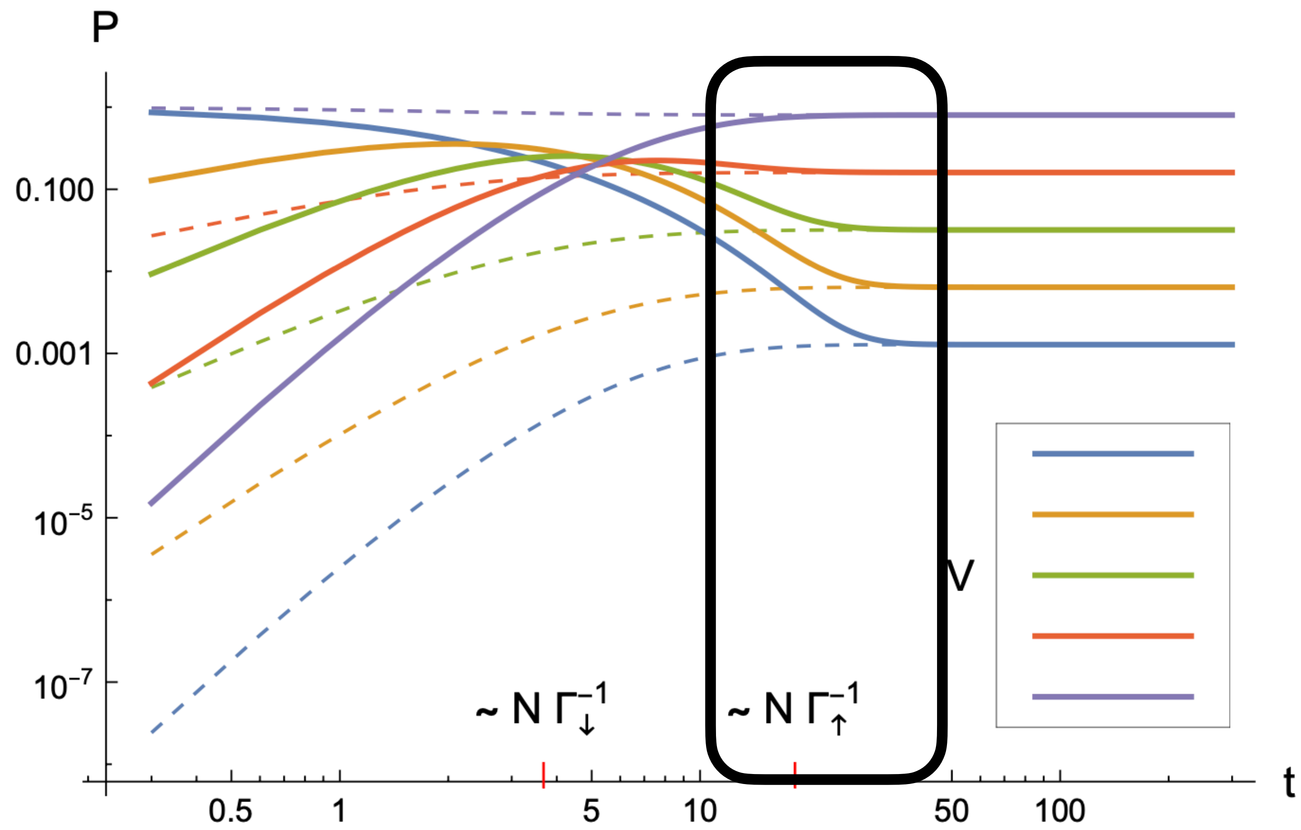
2. I.C. + Dynamics

$$P_i \simeq \frac{1}{i!}(\kappa t)^i P_{t=0} \simeq \frac{1}{i!}(\Gamma_\downarrow t)^i$$

Local measures

Probability gradients





3. Equilibrium independent of I.C.

(if no sinks)

$$P_i \propto \exp\left[\frac{3}{8}\frac{m_P^4}{V(\chi_i)}\right] \propto \exp\left[-\frac{8\pi^2}{3}\frac{V(\chi_i)}{H^4}\right]$$

Local measures

Probability gradients

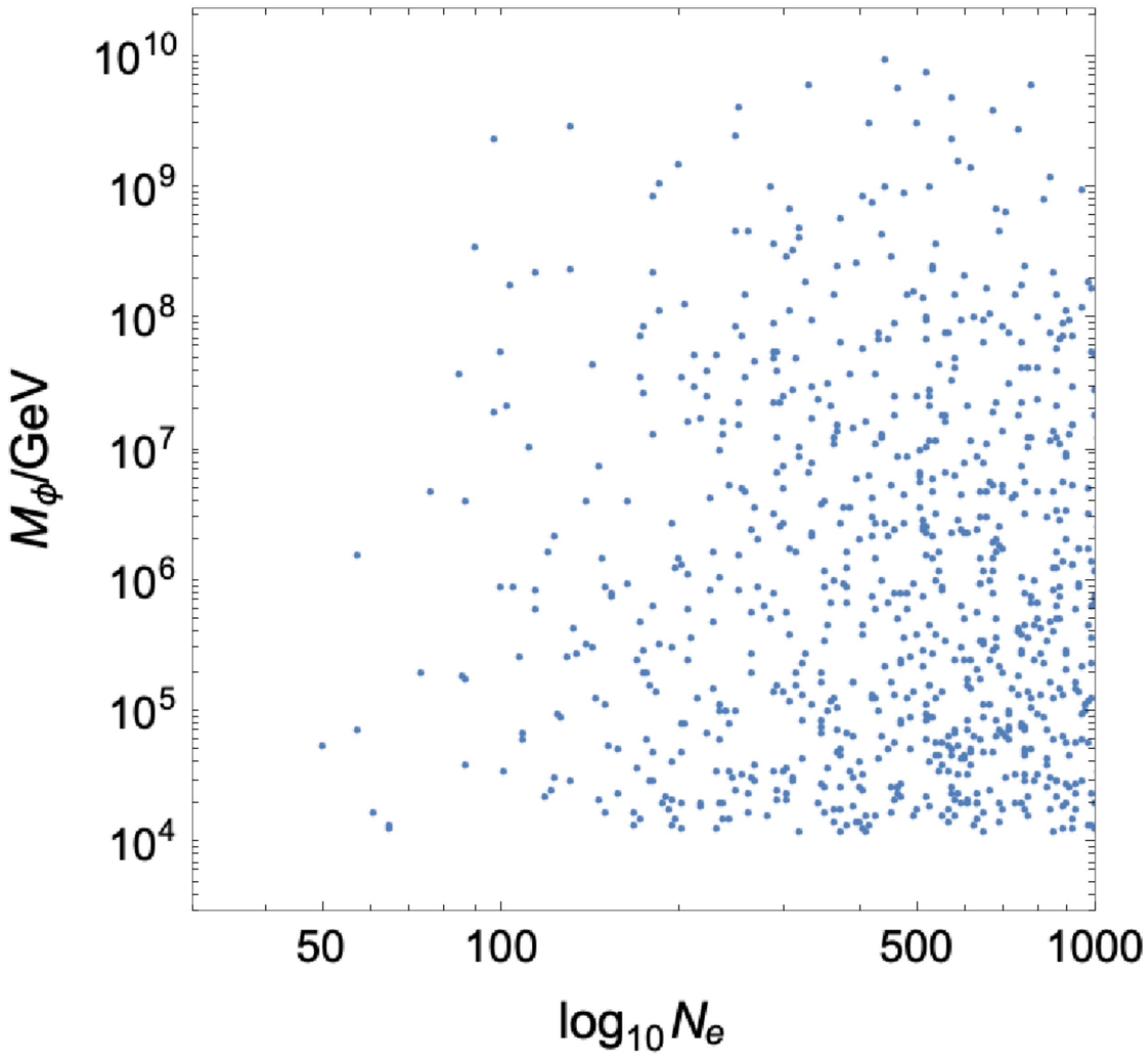
3 regimes, end of slow-roll picks the time of sampling.

Regime 2 has no probability-vacuum energy degeneracy.

* Although the degeneracy can be broken e.g. by changing slope after inflation.

Local measures

Parameter space:



(Other params similar to volume-weighted)

Local measures

Parameter space:

Main bounds on Ne:

1) domain walls

2) requirement to erase V-dependent initial conditions

$$\frac{1}{n_\phi!}\left[\Gamma_{\phi\downarrow}t_R\right]^{n_\phi} > \exp\left[-\frac{8\pi^2}{3}\frac{V(0)-V(n_\phi)}{H^4}\right]$$

Experimental tests

All the pheno associated with the relaxion.

(although param. space is somewhat different)

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape.

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>
> A.Guth, *0002188*

Predictions are uncertain, which doesn't mean that they are not physically significant.

Conclusions

Dynamical solution for the Higgs mass in the presence of the CC landscape.

> When I told Rocky Kolb that I was going to be talking about eternal inflation, he said, "That's OK, we can talk about physics later." So that's the point I'd like to address here.
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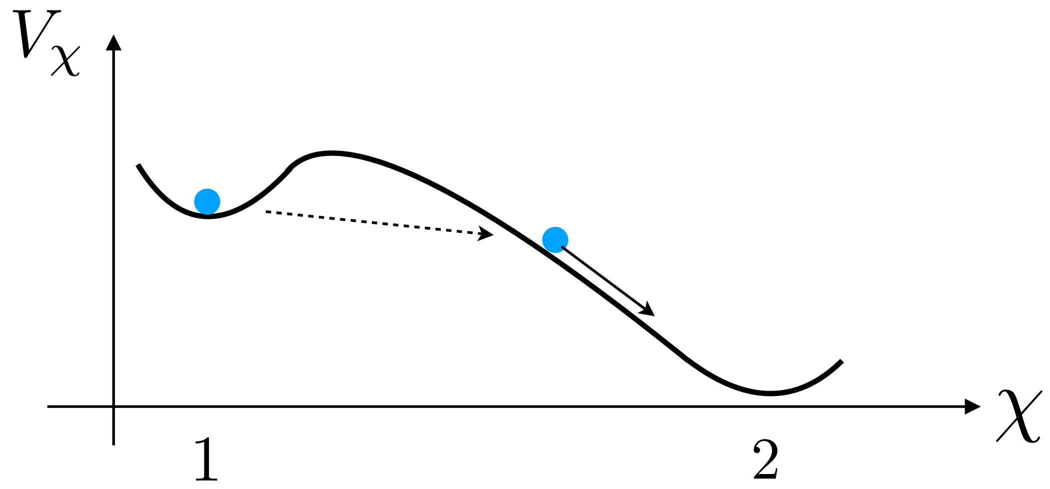
Predictions are uncertain, which doesn't mean that they are not physically significant.

Optimistically: one could infer the correct measure from probing the landscape structure.

back-up slides

Volume-weighted measures

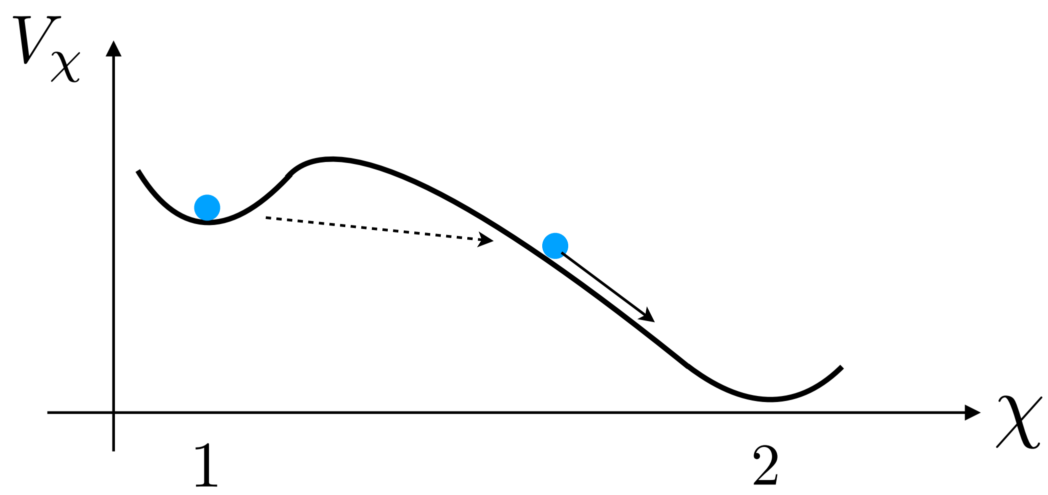
“Youngness paradox”



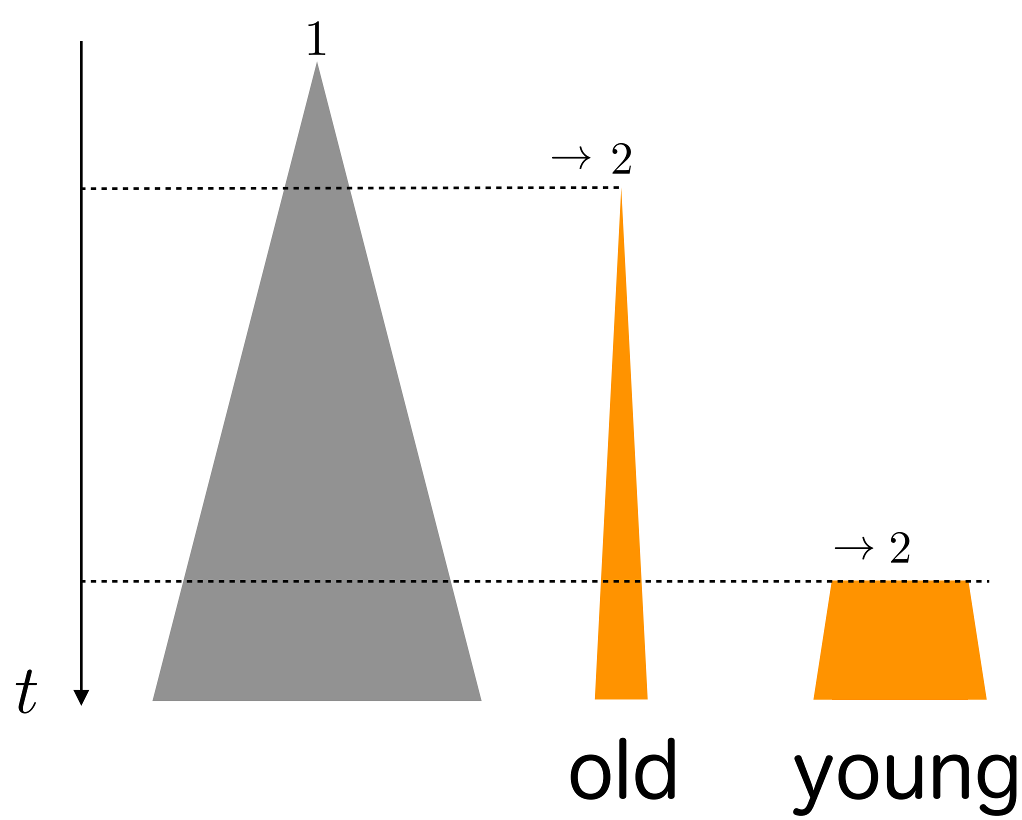
$\Rightarrow$ eternal inflation driven by vacuum 1

Volume-weighted measures

"Youngness paradox"



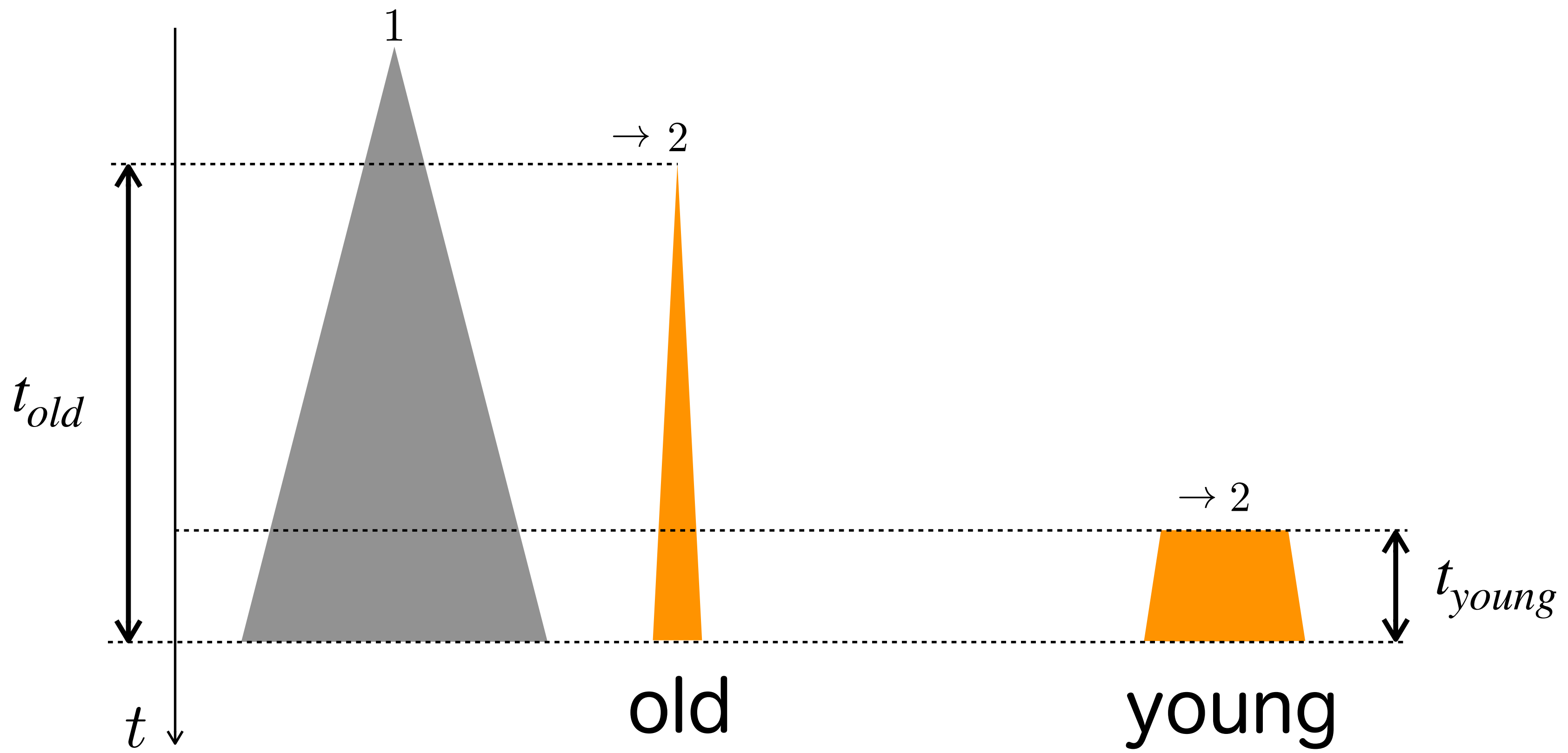
$\Rightarrow$ eternal inflation driven by vacuum 1



$\Rightarrow$ exponentially more young universes

Volume-weighted measures

“Stationary measure”



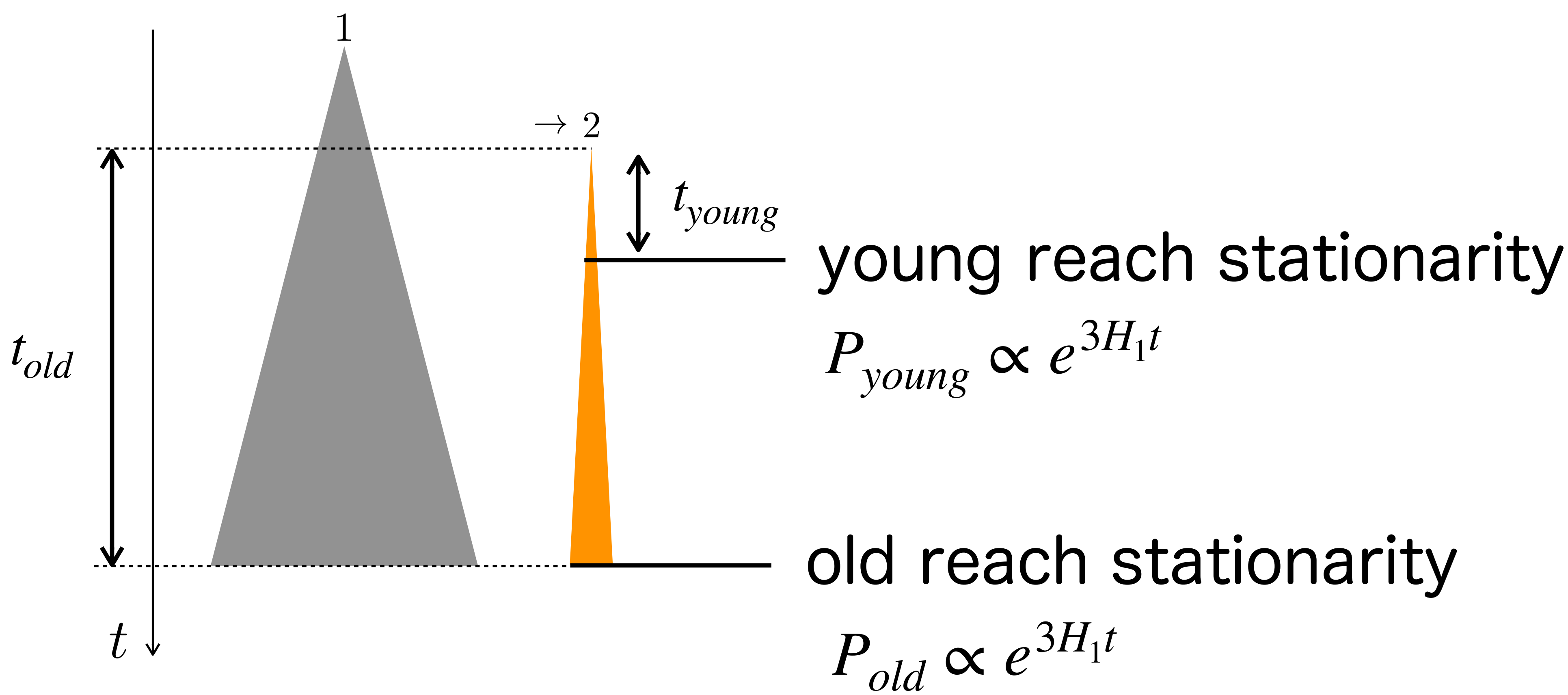
A. D. Linde, JCAP **06**, 017 (2007), 0705.1160

A. D. Linde, V. Vanchurin, and S. Winitzki, JCAP **01**, 031 (2009), 0812.0005

Volume-weighted measures

"Stationary measure"

gist: P are compared at the time of reaching stationarity



A. D. Linde, JCAP **06**, 017 (2007), 0705.1160

A. D. Linde, V. Vanchurin, and S. Winitzki, JCAP **01**, 031 (2009), 0812.0005

Similar approaches

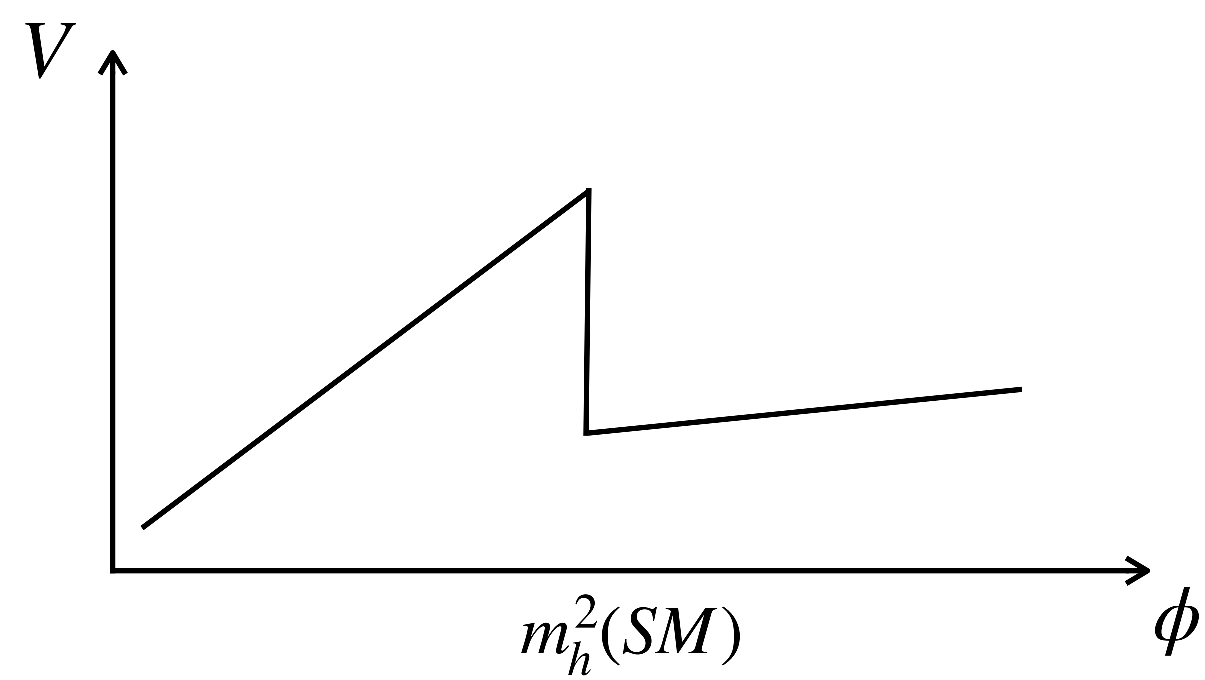
V-weighted

assuming non-eternal

M. Geller, Y. Hochberg, and E. Kuflik, Phys. Rev. Lett. **122**, 191802 (2019), 1809.07338.

C. Cheung and P. Saraswat (2018), 1811.12390.

G. F. Giudice, M. McCullough, and T. You, JHEP **10**, 093 (2021), 2105.08617.



Volume-weighted measures

Stochastic approach

$$V = \Lambda + \frac{1}{2} m^2 \phi^2 \qquad \Rightarrow \qquad P_\nu = \exp\left[-A\phi^2\right] \left\{ \mathbf{c}_+ D_\nu\left[B\phi\right] + \mathbf{c}_- D_\nu\left[-B\phi\right] \right\}$$

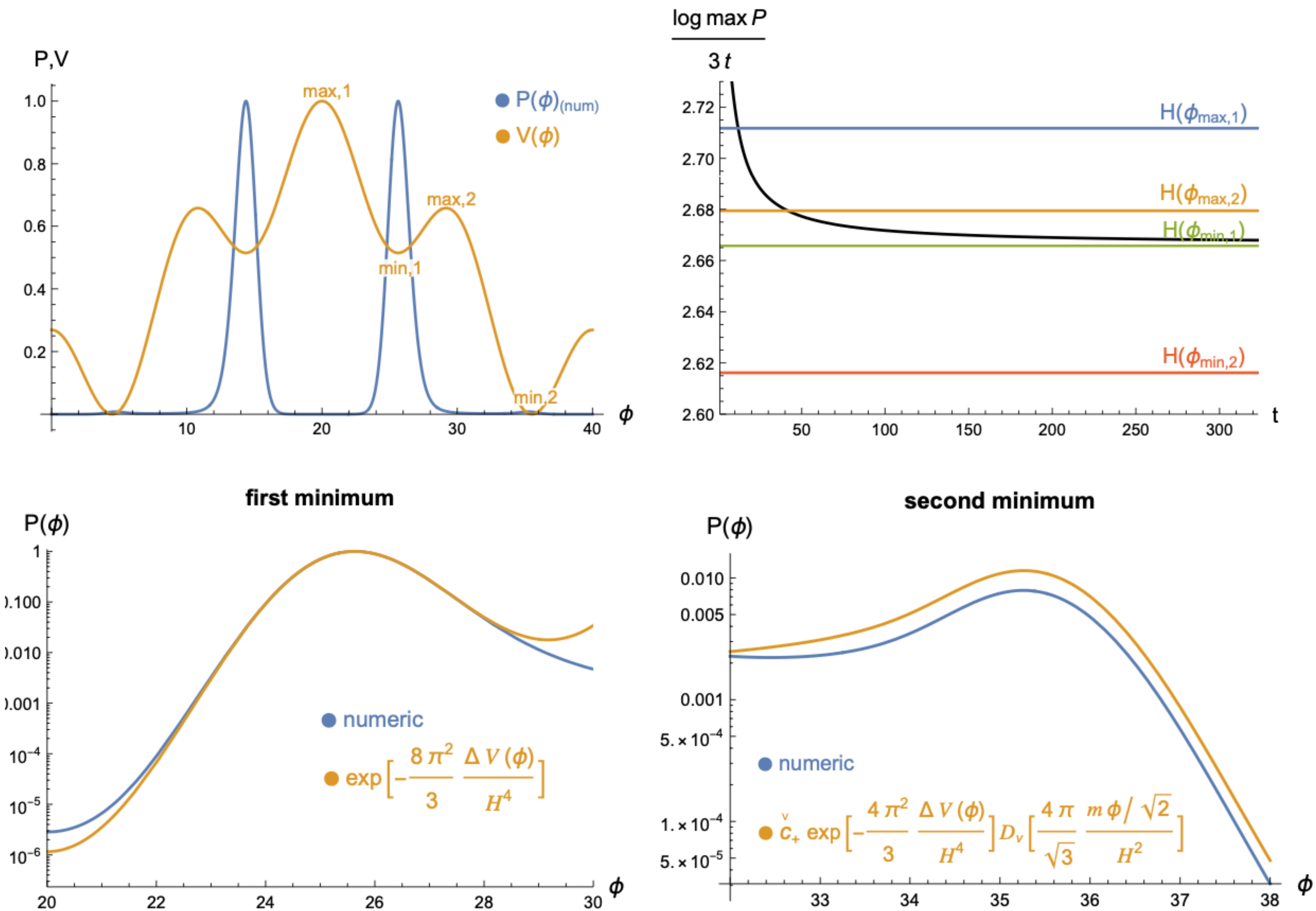
$$\begin{aligned} A\phi^2 &= \frac{4\pi^2}{3} \frac{V(\phi) - V(0)}{H(0)^4}, \\ B\phi &= \left\{ 4 \frac{4\pi^2}{3} \frac{|V(\phi) - V(0)|}{H(0)^4} \sqrt{1 - \frac{9}{\pi} \frac{H(0)^4}{m^2 m_P^2}} \right\}^{1/2} \text{sign}[\phi] \\ \nu &= \frac{9(H(0)^2 - H_s^2) + m^2}{2|m^2| \sqrt{1 - \frac{9}{\pi} \frac{H(0)^4}{m^2 m_P^2}}} - \frac{1}{2}. \end{aligned}$$

asymptote: $D_\nu(x) \underset{x \to \infty}{\longrightarrow} |x|^\nu e^{-x^2/4}$

$$\underset{x \to -\infty}{\longrightarrow} (-1)^\nu |x|^\nu e^{-x^2/4} + \frac{\sqrt{2\pi}}{\Gamma[-\nu]} |x|^{-\nu-1} e^{x^2/4}$$

Volume-weighted measures

Stochastic approach



mH and CC from gradients & boundaries

FPV: $$\dot{P}_{n_\phi,n_\chi} = \Gamma_{\downarrow\phi} P_{n_\phi-1,n_\chi} + \Gamma_{\downarrow\chi} P_{n_\phi,n_\chi-1} + 3H_{n_\phi,n_\chi} P_{n_\phi,n_\chi}$$

factorization: $$P_{n_\phi,n_\chi} = \left[\prod_{i=1}^{n_\phi} \frac{\Gamma_{\downarrow\phi}}{3i\Delta H_\phi}\right]\left[\prod_{j=1}^{n_\chi} \frac{\Gamma_{\downarrow\chi}}{3j\Delta H_\chi}\right] C_0 e^{3H_s t}$$

anthropic line:

$$n_\chi|_{V^{(\text{today})}=0} = \frac{1}{2\pi}\frac{F_\chi}{f_\chi} \arccos\left(const - (M_\phi/M_\chi)^4 \cos(2\pi n_\phi f_\phi/F_\phi)\right) \simeq -\kappa\, n_\phi + const$$

P on anthropic line:

$$P(\phi,\chi)|_{V=0} \propto \left(\frac{\Gamma_\phi}{3(n_\phi/e)\Delta H_\phi}\right)^{n_\phi}\left(\frac{\Gamma_\chi}{3(n_\chi/e)\Delta H_\chi}\right)^{n_\chi} \propto \left(\frac{e\Gamma_\phi}{3n_\phi\Delta H_\phi}\left(\frac{3n_\chi\Delta H_\chi}{e\Gamma_\chi}\right)^\kappa\right)^{n_\phi}$$

peaked at correct mh if:

$$\frac{e\Gamma_\phi}{3n_\phi\Delta H_\phi}\left(\frac{3n_\chi\Delta H_\chi}{e\Gamma_\chi}\right)^\kappa > 1$$ where $$\kappa = \frac{N_\chi}{N_\phi}\frac{M_\phi^4}{M_\chi^4}\frac{\sin\phi_0}{\sin\chi_0}, \quad N_\phi = \frac{F_\phi}{f_\phi}, \quad N_\chi = \frac{F_\chi}{f_\chi}$$