MATCHING AND MERGING

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RESOURCES

- Black Book of Quantum Chromodynamics (Campbell, Huston, Krauss)
- QCD and Collider Physics (Ellis, Stirling, Webber)
- MCnet lectures (Gellersen, Krauss)
- Elements of QCD for hadron colliders (Salam)
- Next-to-leading-order event generators (Nason, Webber)
- Introduction to QCD (Skands)

COMPARING FIXED ORDER AND PARTON SHOWER

Parton shower	Fixed order
Correct only for soft/ collinear radiation	Hard radiation correctly described
High multiplicity final states possible	At most ~10 particles in final state
Realistic, hadronic final states	Only partonic final states
Hard to improve accuracy	Known how to systematically improve accuracy

COMBINING FIXED ORDER AND PARTON SHOWER

- Want to combine types of calculation to exploit best features of both. Two approaches to this problem:
- Merging combines samples with different multiplicities at FO and showers them without double counting
- Matching corrects first emissions of parton shower to be (N)NLO accurate and gives events with (N)NLO weight
- Final accuracy different in the two cases (matching includes more exact virtual corrections than merging)

MATCHING AT NLO

MATCHING NLO TO PARTON SHOWER

- Criteria for a successful combination of NLO+PS:
  - Total cross section inherited from NLO
  - Radiation pattern (first order) follows NLO real emission
  - Logarithmic accuracy of PS is maintained
- Recall NLO structure:

$$\sigma_N^{\mathrm{NLO}} = \int \mathrm{d}\Phi_{\mathscr{B}} \left[\mathscr{B}_N(\Phi_{\mathscr{B}}) + \mathscr{V}_N(\Phi_{\mathscr{B}}) + \mathscr{I}_N^{\mathscr{S}}(\Phi_{\mathscr{B}})\right] + \int \mathrm{d}\Phi_{\mathscr{R}} \left[\mathscr{R}_N(\Phi_{\mathscr{R}}) - \mathscr{S}_N(\Phi_{\mathscr{R}})\right]$$

IMPROVING THE PARTON SHOWER – MATRIX ELEMENT CORRECTIONS

- Parton shower good for soft/collinear, bad for hard emissions
- Can we correct it to get the hardest emission right?
- In many processes, parton shower is an overestimate of exact ME:

$$\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1) \leq \mathscr{B}_N(\Phi_{\mathscr{B}}) \otimes \mathscr{K}_N(\Phi_1)$$

- $\mathscr{K}_N$ is combined PS soft and collinear splitting kernel for emissions off an N body state – exact form depends on the shower

SUDAKOV FACTOR

- Recall that Sudakov form factor gives no-emission probability between two scales:

$$\Delta_N(Q_2, Q_1) = \exp\left[-\int_{Q_1}^{Q_2} \mathrm{d}\Phi_1 \mathscr{K}_N(\Phi_1)\right]$$

- Differentiating, can obtain probability of emission at a given 'time' t

$$\mathscr{P}_{\text{emission}}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\Delta_N(t, Q) = \mathscr{K}_N \Delta_N(t, Q)$$

MATRIX ELEMENT CORRECTIONS

- First emission pattern looks like:

$$d\sigma_N = d\Phi_{\mathcal{B}} \mathcal{B}_N(\Phi_{\mathcal{B}}) \left\{ \Delta_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\mathcal{K}_N(\Phi_1) \Delta_N(\mu_Q^2, t(\Phi_1)) \right] \right\}$$

No emission probability

Single emission probability at a given time t

- Terms in curly brackets integrate to 1 (shower is unitary)
- Let's modify the splitting kernel to make it look more like the real matrix element, at least for the first emission:

$$\tilde{\mathcal{K}}_N(\Phi_1) = \mathcal{R}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) / \mathcal{B}_N(\Phi_{\mathcal{B}})$$

MATRIX ELEMENT CORRECTIONS

- First emission pattern looks like:

$$d\sigma_N = d\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}})\left\{\Delta_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1\left[\mathcal{K}_N(\Phi_1)\Delta_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

No emission probability

Single emission probability at a given time t

- Terms in curly brackets integrate to 1 (shower is unitary)
- Define modified Sudakov factor as

$$\tilde{\Delta}_N(Q_2, Q_1) = \exp\left[-\int_{Q_1}^{Q_2} d\Phi_1 \frac{\mathcal{R}_N}{\mathcal{B}_N}\right]$$

MATRIX ELEMENT CORRECTIONS

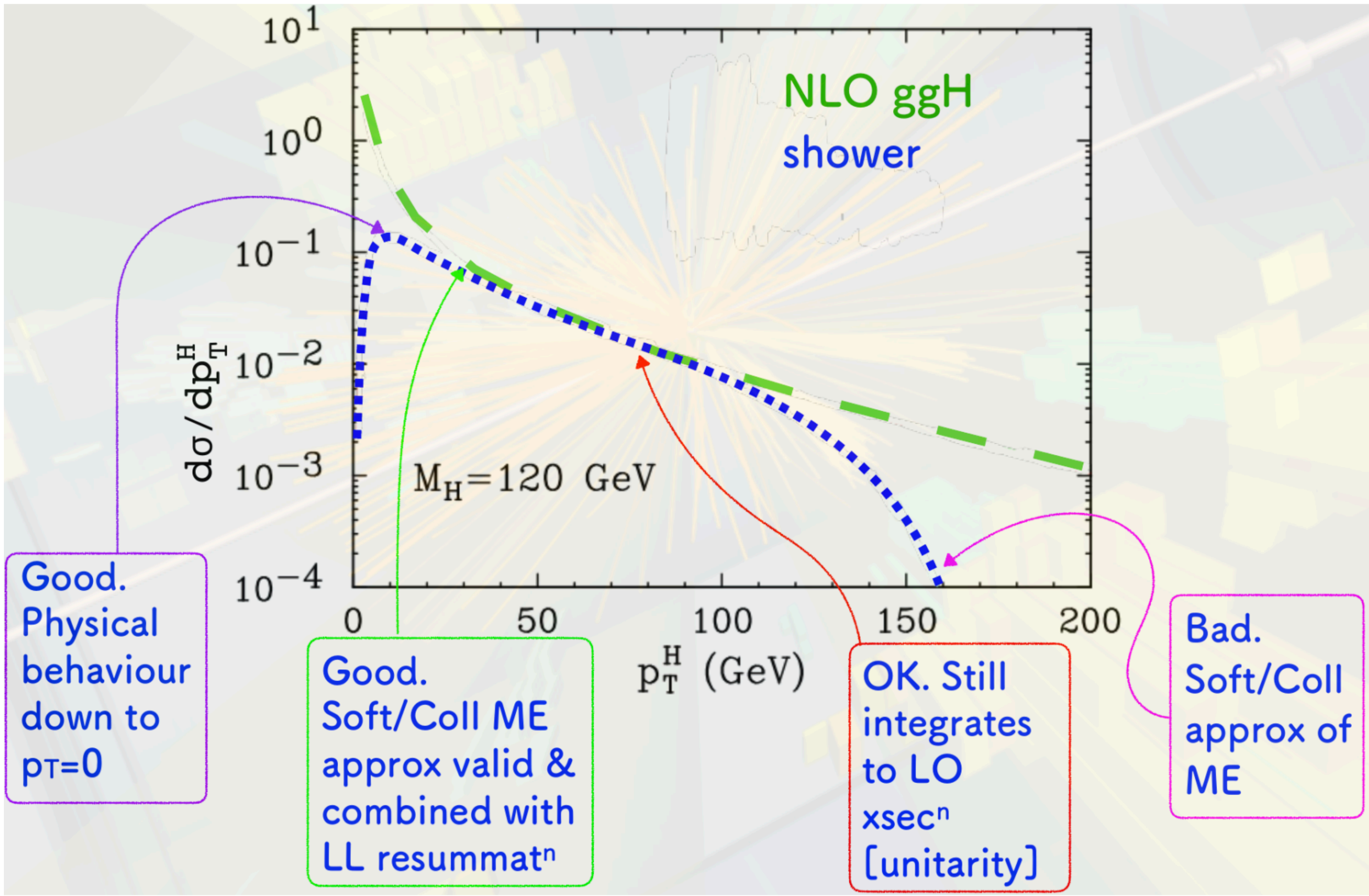
- First emission pattern modified to:

$$d\sigma_N = d\Phi_{\mathscr{B}}\mathscr{B}_N(\Phi_{\mathscr{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}_N(\Phi_{\mathscr{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

- Now first emission follows real matrix element!
- Practically, use normal shower kernels and simply accept/reject points with a probability

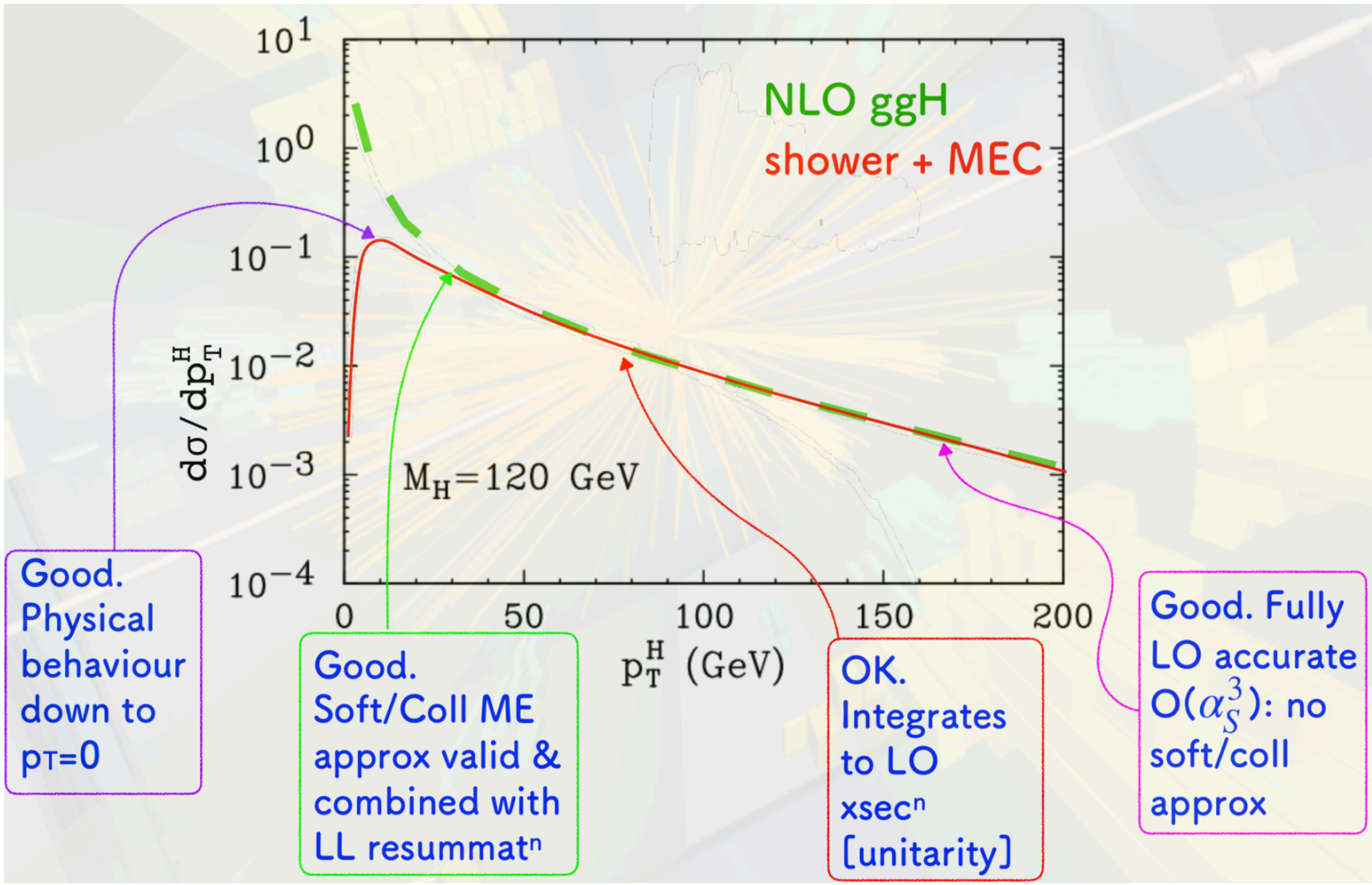
$$\mathscr{P}_{\mathrm{MEC}} = \frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}(\Phi_{\mathscr{B}}) \otimes \mathscr{K}_N(\Phi_1)}$$

MATRIX ELEMENT CORRECTIONS



Credit: Keith Hamilton

MATRIX ELEMENT CORRECTIONS



Credit: Keith Hamilton

NLO MATCHING – THE POWHEG METHOD

- Define Born-like configurations which give NLO-accurate cross section:

$$\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}}) = \mathscr{B}_N(\Phi_{\mathscr{B}}) + \overline{\mathscr{V}}_N(\Phi_{\mathscr{B}}) + \int \mathrm{d}\Phi_1 \left[\mathscr{R}_N(\Phi_\mathrm{B} \otimes \Phi_1) - \mathscr{S}_N(\Phi_\mathrm{B} \otimes \Phi_1)\right]$$

- IR-subtracted, UV-renormalised virtual piece is

$$\overline{\mathscr{V}}_N(\Phi_{\mathscr{B}}) = \mathscr{V}_N(\Phi_{\mathscr{B}}) + \mathscr{I}_N^{\mathscr{S}}(\Phi_{\mathscr{B}})$$

- Works if $\Phi_{\mathscr{R}} = \Phi_{\mathscr{B}} \otimes \Phi_1$. $\overline{\mathscr{B}}$ terms are fully differential cross sections of Born configurations with NLO weight.

NLO MATCHING – THE POWHEG METHOD

- Unitary PS cannot spoil NLO cross section
- Still need pattern of first emission to be correct up to $\mathcal{O}(\alpha_s)$
- Get this by applying matrix element corrections!
- POWHEG formula given by

$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_{\mathscr{B}}\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \left[\frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}_N(\Phi_{\mathscr{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

NLO MATCHING - THE POWHEG METHOD

- POWHEG formula given by

$$d\sigma_N = d\Phi_{\mathscr{B}}\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\frac{\mathscr{R}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}_N(\Phi_{\mathscr{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$

- Gets NLO cross section right (term in curly braces integrates to unity)
- Gets real radiation right at $\mathscr{O}(\alpha_s)$ - NLO terms in $\overline{\mathscr{B}}$ hitting $\mathscr{R}_N/\mathscr{B}_N$ are $\mathscr{O}(\alpha_s^2)$
- Subtleties in scale choices, starting scale of PS

SCALE CHOICES IN POWHEG

- Consider as an example $gg \to H$. What scale should I start the shower at?
- Arguments based on resummation suggest $\mu_Q \approx m_H$, which minimises the size of logs.
- This does not allow a description of the high p_T tail, since the phase space for hardest emissions is constrained to be below m_H. Lose accuracy over part of the phase space!
- Compromise between log and FO accuracy.

SCALE CHOICES IN POWHEG

- Assume we send $\mu_Q \to \sqrt{\hat{s}}$ so that the full phase space is opened up for the hardest emission.
- Local K-factor in $\overline{\mathscr{B}}_N$ is for inclusive production, after integrating out additional partons in $\mathscr{R}_N$. It is applied to all events, even when hardest emission is harder than m_H.
- Is this ok? Not necessarily the case that the K-factor for $gg \to H$ and $gg \to H + j$ are similar…

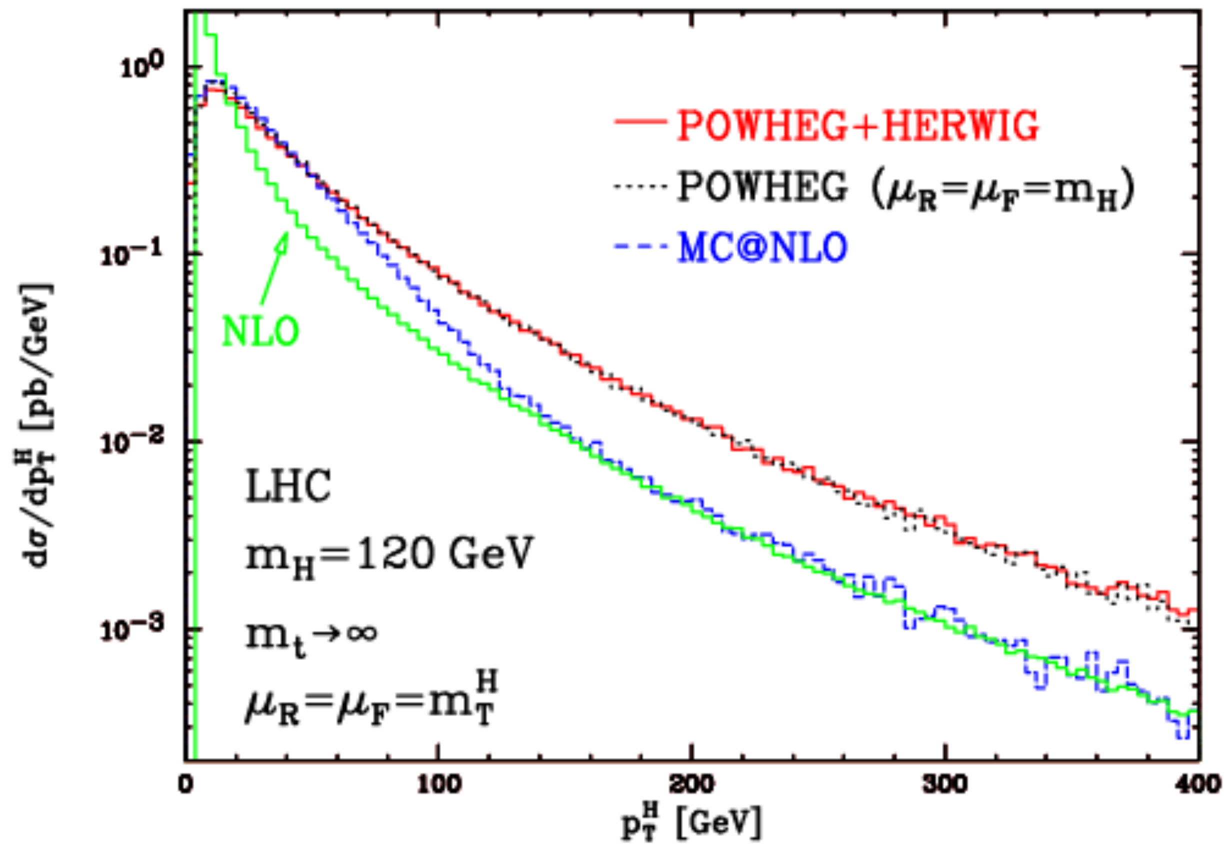
SCALE CHOICES IN POWHEG

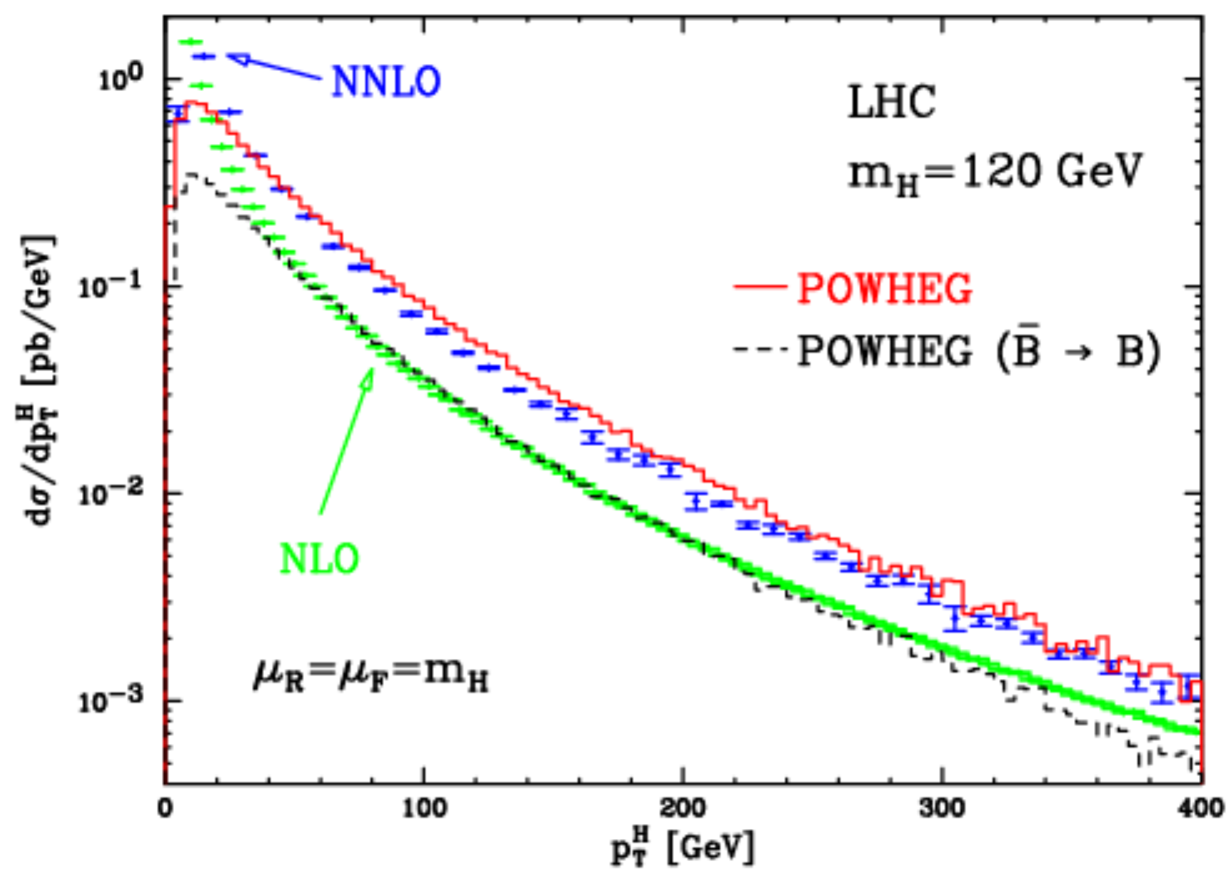
- POWHEG predictions differ from NLO result in high p_T region (upper plot). What is the cause?

- At large p_T, POWHEG formula reduces to

$$d\sigma = \overline{\mathscr{B}}(\Phi_{\mathscr{B}})\frac{\mathscr{R}(\Phi_{\mathscr{R}})}{\mathscr{B}(\Phi_{\mathscr{B}})}d\Phi_1 d\Phi_{\mathscr{B}}$$

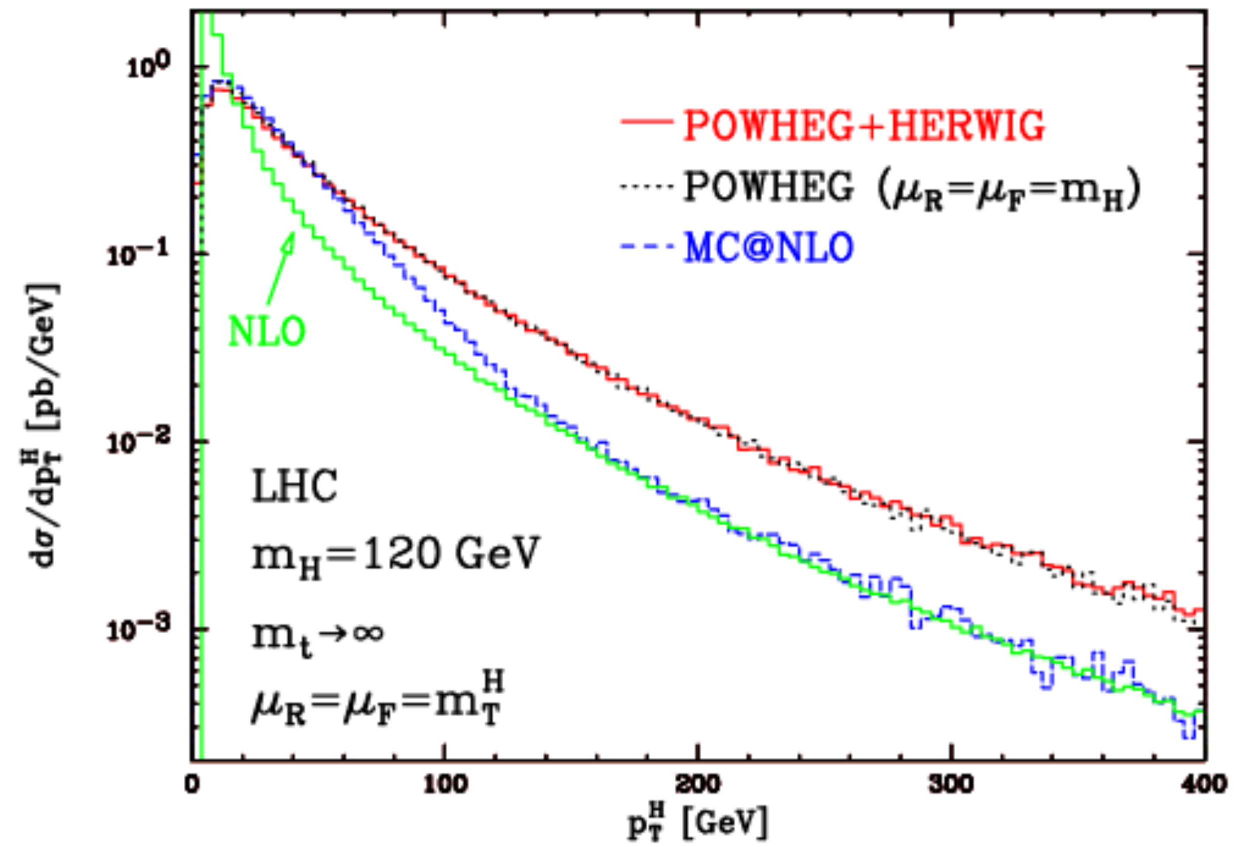
- $\dfrac{\overline{\mathscr{B}}(\Phi)}{\mathscr{B}(\Phi)} = 1 + \mathcal{O}(\alpha_s)$

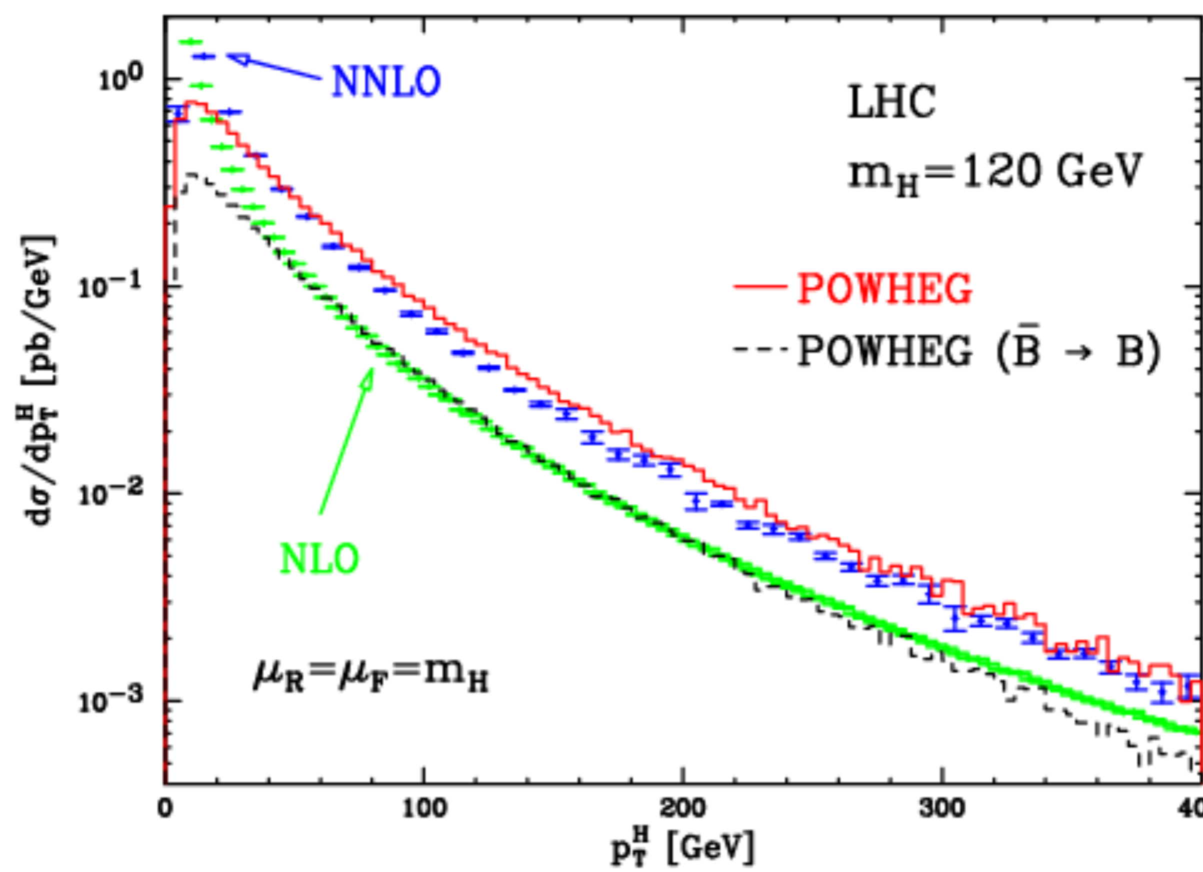




SCALE CHOICES IN POWHEG

- POWHEG predictions differ from NLO result in high p_T region (upper plot). What is the cause?
- Replacing $\mathscr{B}$ with $\overline{\mathscr{B}}$ in the Sudakov, the higher order terms are cancelled and the NLO result is recovered (lower plot).





IMPROVING THE POWHEG METHOD

- Possible to solve both of the above problems by splitting real emission phase space into soft and hard parts.

$$\mathscr{R}_N = \mathscr{R}_N \left(\frac{h^2}{p_T^2 + h^2} + \frac{p_T^2}{p_T^2 + h^2} \right) \equiv \mathscr{R}_N^{(S)} + \mathscr{R}_N^{(H)}$$

- $\mathscr{R}^{(S)}$ has divergences, $\mathscr{R}^{(H)}$ is finite. New parameter $h \approx m_H$ (can be tuned by comparison with dedicated resummed calculations).

- Use $\mathscr{R}^{(S)}$ for shower kernel and in definition of $\overline{\mathscr{B}}$, add extra term $\mathrm{d}\Phi_{\mathscr{R}}\mathscr{R}^{(H)}$ without K-factor.

IMPROVING THE POWHEG METHOD

- Improved POWHEG formula is given by

$$d\sigma_N = d\Phi_{\mathscr{B}}\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}})\left\{\tilde{\Delta}_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\frac{\mathscr{R}_N^{(S)}(\Phi_{\mathscr{B}} \otimes \Phi_1)}{\mathscr{B}_N(\Phi_{\mathscr{B}})}\tilde{\Delta}_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$
$$+ d\Phi_{\mathscr{R}}\mathscr{R}^{(H)}$$

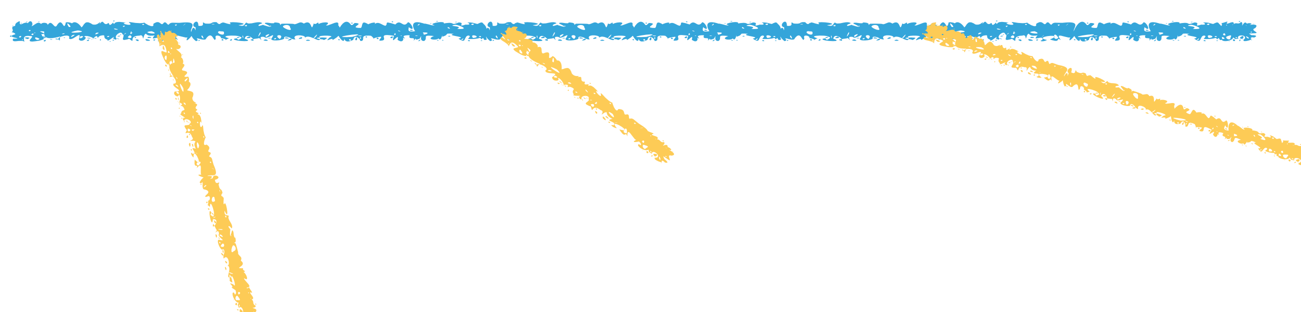
- Sudakov and $\overline{\mathscr{B}}_N$ now have only $\mathscr{R}_N^S$ inside
- Hard jet events in $\mathscr{R}_N^{(H)}$ no longer modified by an inappropriate K factor

POWHEG AND THE PARTON SHOWER INTERFACE

- POWHEG works by ordering emissions through 'hardness'.
- Typically this is something like transverse momentum, but many definitions possible and a given PS may use something different
- Easiest solution is to ensure the hardness scale in the FO part is identical to the shower evolution parameter.
- Alternatively, truncated showering can be used to account for mismatch between variables.

TRUNCATED SHOWERING

- POWHEG hardness is generally transverse momentum. What happens if we want to match to an angular ordered shower?
- Angular showers start with large angle soft emissions, later emissions can be hard (higher p_T)
- Truncated shower is needed to ensure POWHEG emission is the hardest



$$\theta_1 > \theta_2 > \theta_3$$

$$p_{T,1} > p_{T,3} > p_{T,2}$$

NLO MATCHING - THE MC@NLO METHOD

- MC@NLO was the first successful matching of NLO to parton shower. It splits the real term

$$\mathscr{R}_N(\Phi_{\mathscr{R}}) = \mathscr{R}_N^{(S)}(\Phi_{\mathscr{R}}) + \mathscr{R}_N^{(H)}(\Phi_{\mathscr{R}}) = \mathscr{S}_N(\Phi_{\mathscr{B}} \otimes \Phi_1) + \mathscr{H}_N(\Phi_{\mathscr{R}})$$

- The soft term is identified with the shower kernels

$$\mathscr{S}_N(\Phi_{\mathscr{B}} \otimes \Phi_1) = \mathscr{B}_N(\Phi_{\mathscr{B}}) \otimes \mathscr{K}(\Phi_1)$$

- Similar in spirit to a resummed computation matched to fixed order: soft term is like the resummed, hard term is like (FO - resummed expanded)

NLO MATCHING – THE MC@NLO METHOD

- MC@NLO formula is given by

$$\mathrm{d}\sigma_N = \mathrm{d}\Phi_{\mathscr{B}}\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}})\left\{\Delta_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2}\mathrm{d}\Phi_1\left[\mathscr{K}(\Phi_1)\Delta_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$
$$+\mathrm{d}\Phi_{\mathscr{R}}\mathscr{H}_N(\Phi_{\mathscr{R}})$$

- Modified Born term is

$$\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}}) = \mathscr{B}_N(\Phi_{\mathscr{B}}) + \overline{\mathscr{V}}_N(\Phi_{\mathscr{B}})$$

- Hard emission term corrects hardest PS emissions to follow real matrix element and also fills regions inaccessible by the PS

NLO MATCHING - THE MC@NLO METHOD

- MC@NLO formula is given by

$$d\sigma_N = d\Phi_{\mathscr{B}}\overline{\mathscr{B}}_N(\Phi_{\mathscr{B}})\left\{\Delta_N(\mu_Q^2, t_c) + \int_{t_c}^{\mu_Q^2} d\Phi_1 \left[\mathscr{K}(\Phi_1)\Delta_N(\mu_Q^2, t(\Phi_1))\right]\right\}$$
$$+ d\Phi_{\mathscr{R}}\mathscr{H}_N(\Phi_{\mathscr{R}})$$

- Disadvantage - negative weights can be present (counter-events), since $\mathscr{H}_N(\Phi_{\mathscr{R}}) = \mathscr{R}_N(\Phi_{\mathscr{R}}) - \mathscr{S}_N(\Phi_{\mathscr{B}} \otimes \Phi_1)$ is not guaranteed to be positive

- Several proposals in the literature to reduce the proportion of events with negative weight

TOOLS FOR NLO+PS MATCHING

- Main tools are aMC@NLO (automated) and POWHEG BOX (process-by-process). In addition,

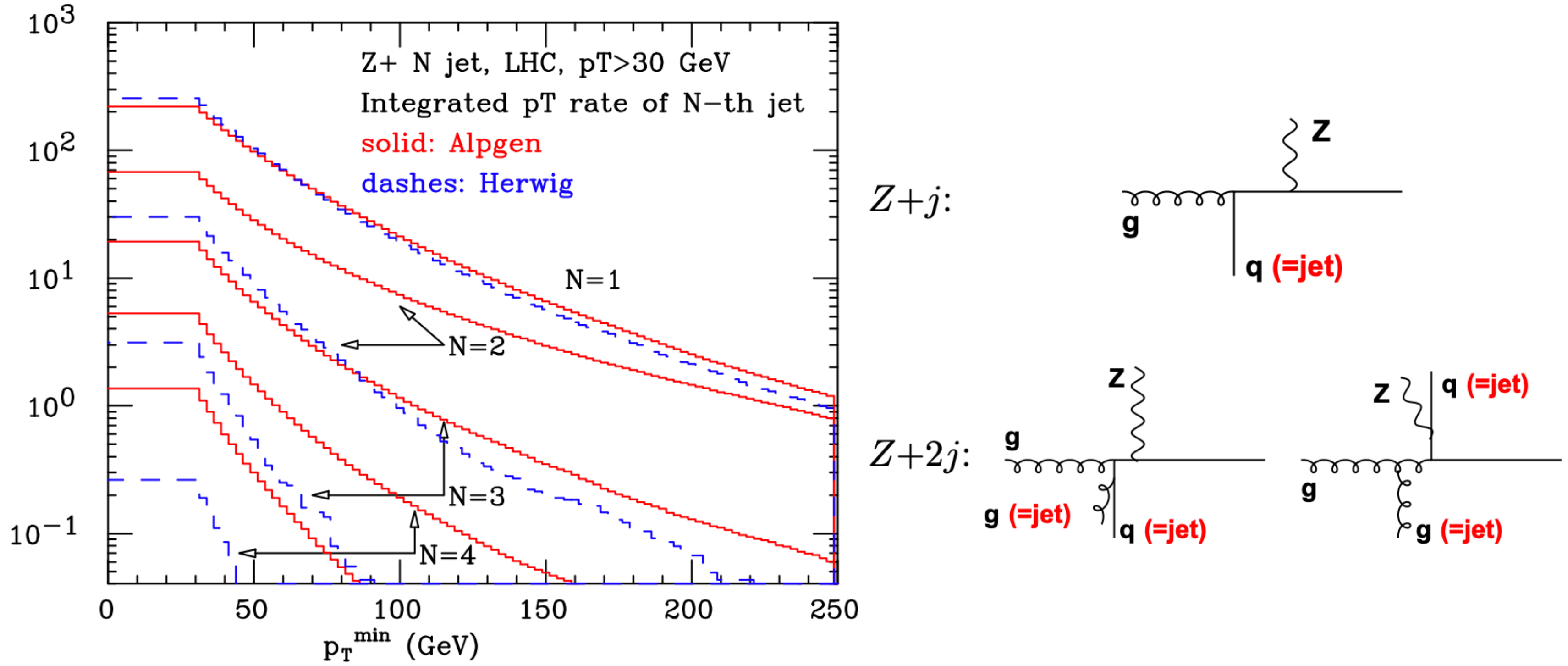
	POWHEG	MC@NLO
HERWIG7	✔	✔
PYTHIA8*	✘	✘
SHERPA	✔	✔
WHIZARD	✔	✘

* Interfaces to PYTHIA from aMC@NLO and POWHEG BOX are readily available, but PYTHIA itself does not perform the matching

MERGING AT LO

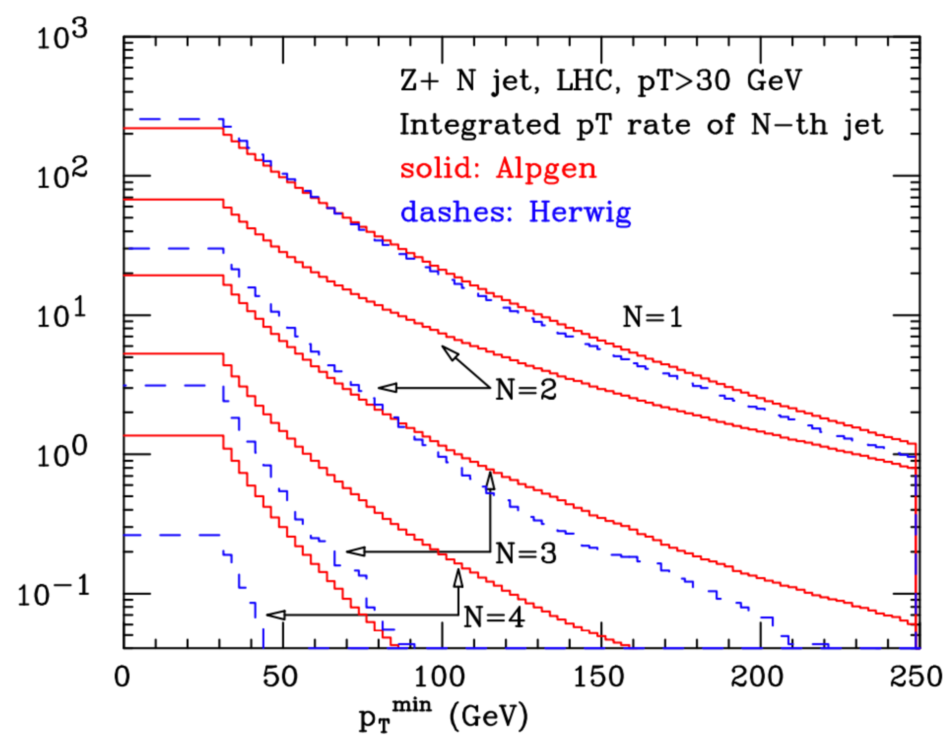
VECTOR BOSON + JET PRODUCTION

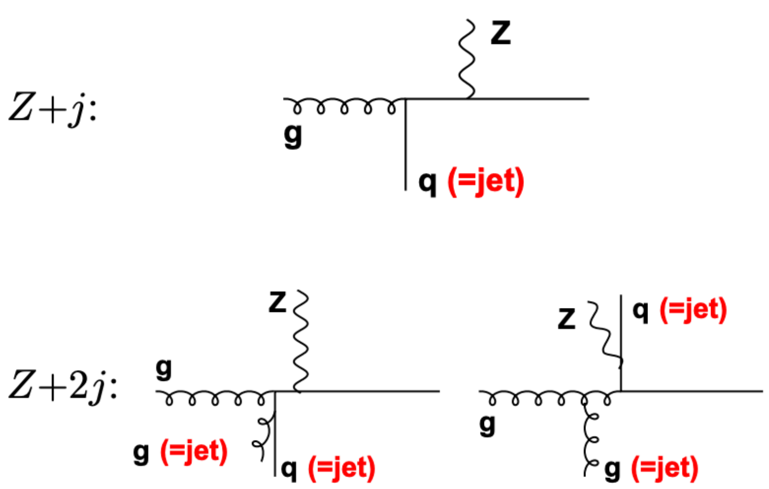
- Consider $Z + j$ production as the underlying hard process.
- Fig. shows cross section for N^{th} jet to have transverse energy above E_T
- PS and FO in agreement for 1st jet, but terrible for >2



VECTOR BOSON + JET PRODUCTION

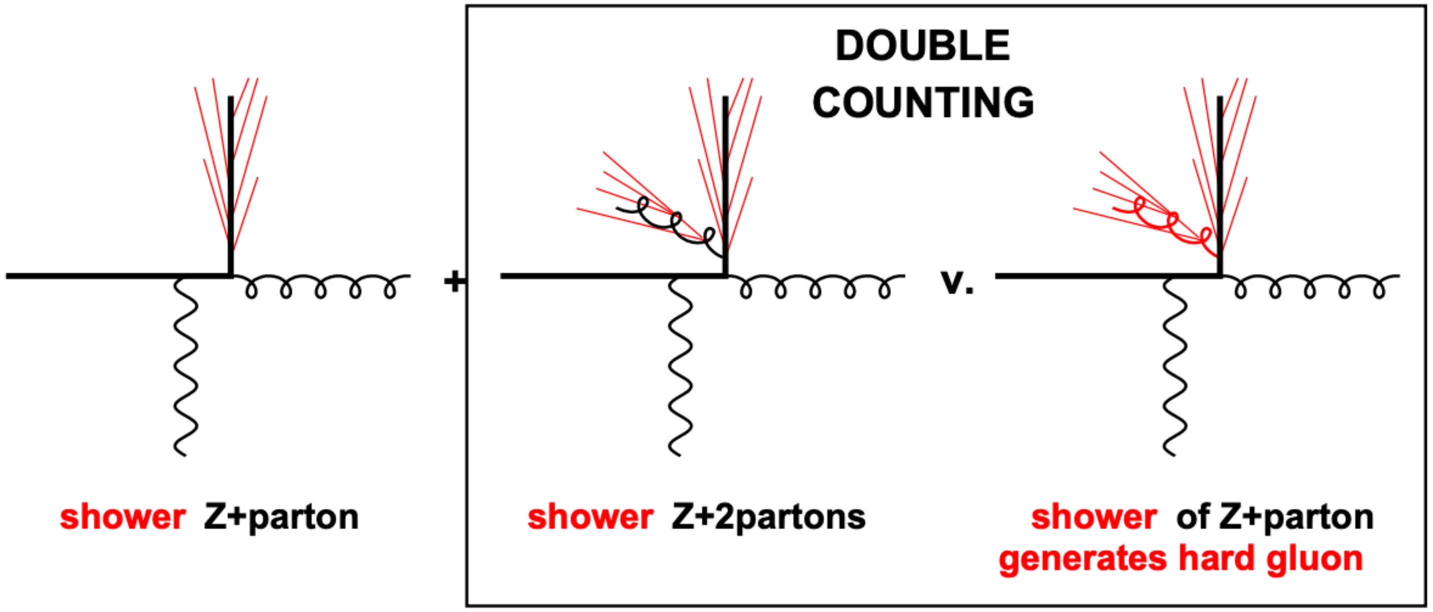
- Explanation: HERWIG generates hard $Z + j$ configs
- But: also soft/coll. enhanced events where Z is radiated off a dijet config, not captured by QCD shower alone





NAÏVE MULTIJET MERGING

- Want to combine LO calculations with different numbers of jets and then shower.
- Naïve solution: generate $Z+2$ with correct LO ME, then shower. Second emission now follows exact ME.
- Problem: double counting!



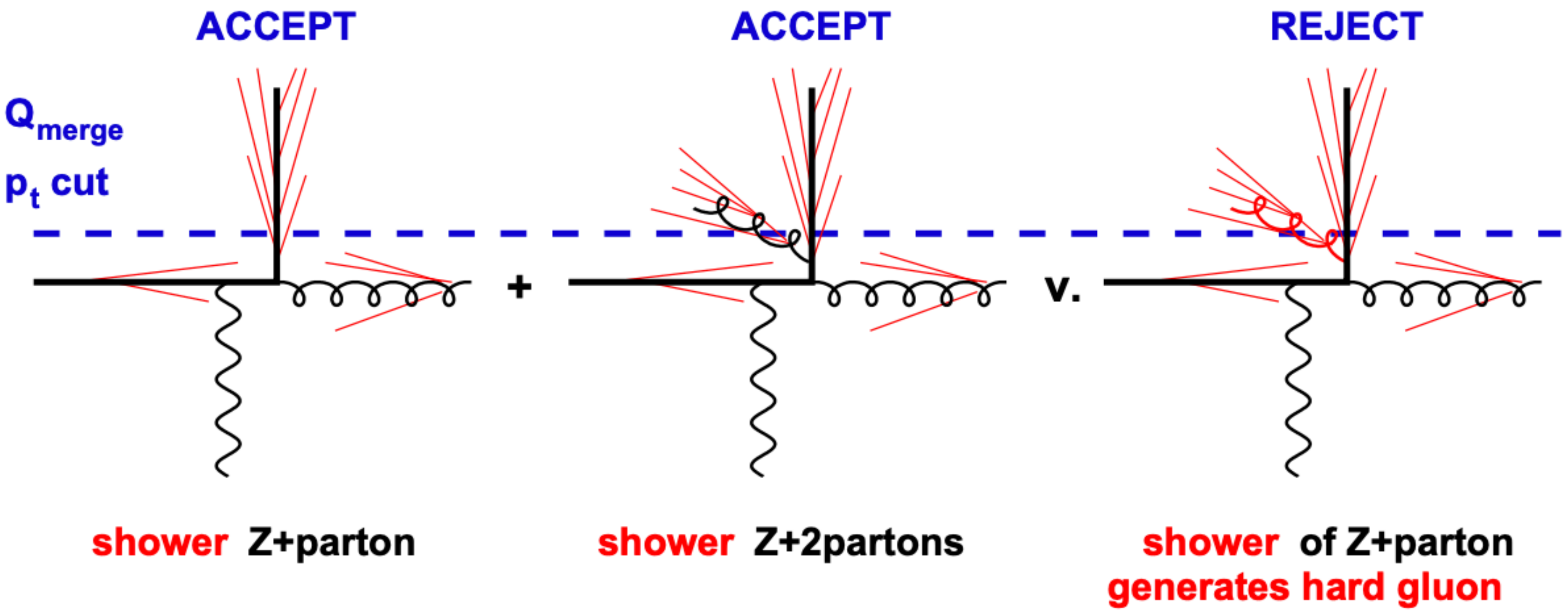
Credit: G. Salam

MULTIJET MERGING (COMME IL FAUT)

- Can solve double-counting issue by dividing phase space for each multiplicity into hard and soft regions, using a parameter ρ_{merge}
- Below ρ_{merge} shower, vetoing any new jets
- Above ρ_{merge}, use exact MEs and make exclusive by multiplying with Sudakov no-emission probabilities to mimic 'how shower got there' (virtual corrections)
- This ensures continuity across the merging scale (at NLL)

VETOING THE PARTON SHOWER

- Double-counting removed by rejection of hard radiation
- Hard jets come only from the matrix element



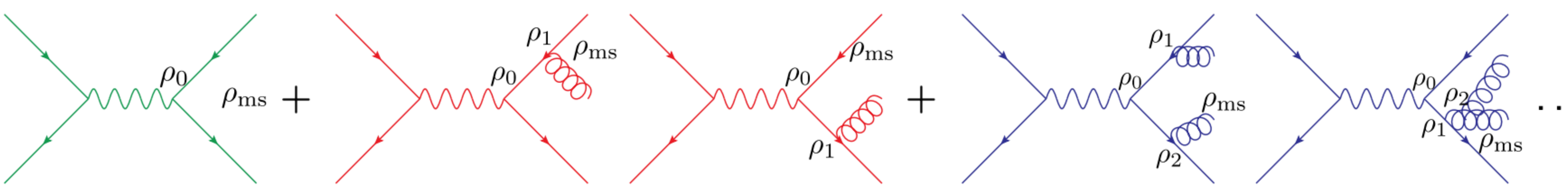
CALCULATING THE REWEIGHTING FACTORS

$$\langle \mathcal{O} \rangle = \int d\Phi_0 \left\{ \mathcal{O}_0 \mathcal{B}_0 w_0 + \int d\Phi_1 \mathcal{O}_1 \mathcal{B}_1 w_1 + \int d\Phi_1 \int d\Phi_2 \mathcal{O}_2 \mathcal{B}_2 w_2 \right\}$$

$$w_0 = \Delta_0(\rho_0, \rho_{\mathrm{merge}})$$

$$w_1 = \Delta_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Delta_1(\rho_1, \rho_{\mathrm{merge}})$$

$$w_2 = \Delta_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Delta_1(\rho_1, \rho_2) \frac{\alpha_s(\rho_2)}{\alpha_s(\mu_R)}$$

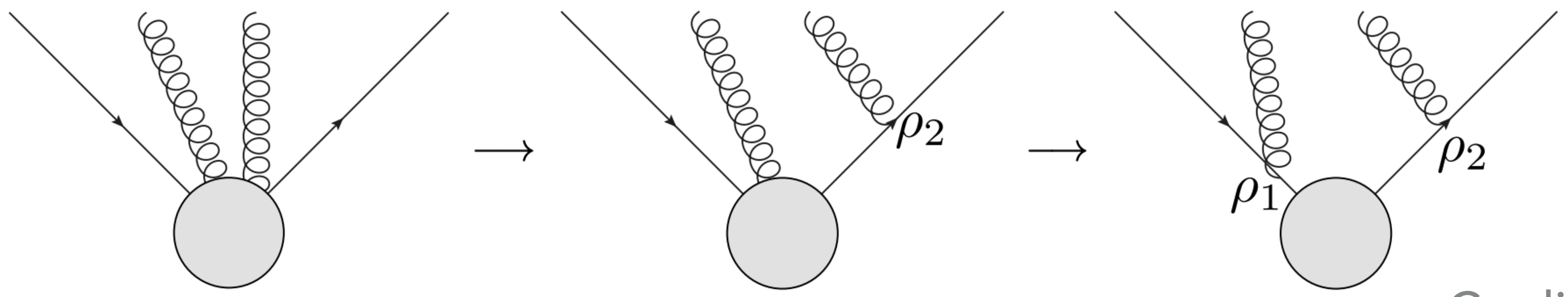


*For ISR case, need PDF factors as well

Credit: Leif Gellersen

WHERE DO THE EMISSIONS HAPPEN?

- How do we determine the jet resolution scales ρ_i?
- Approach 1: Find a unique splitting history by reclustering emissions with a sequential 2→1 jet algorithm
- Approach 2: Find all possible splitting histories by reclustering 3→2, choose one with probability ∝ product of splitting probs.

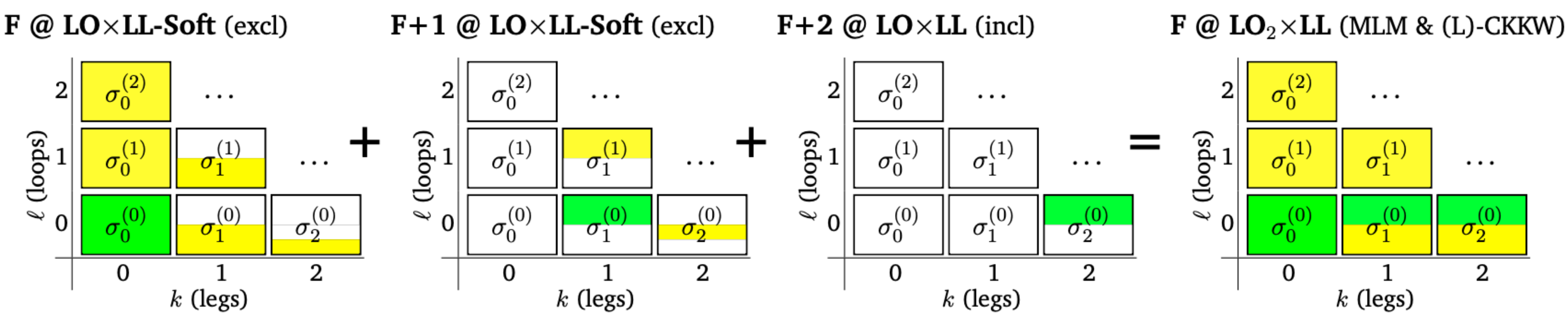


Credit: Leif Gellersen

SUMMARY OF MERGING PROCEDURE

- Calculate inclusive cross sections for $X + n$ partons, from $n = 0$ to N. Cut off singularities in MEs at a scale ρ_{merge}.
- Find scales at which emissions happened by jet algorithm reclustering or reconstructing PS history.
- Multiply by merging weight (Sudakovs, α_s/PDF factors).
- For $n < N$, multiply by no-emission probability up to ρ_{merge}.
- Shower all samples. For $n < N$, veto extra jets above ρ_{merge}.

SUMMARY OF MERGING PROCEDURE



Credit: Peter Skands

THE MERGING SCALE

- What value do I choose for ρ_{merge}? Naïvely, want to push to as small a value as possible.
- Problem 1: higher multiplicity MEs are singular in this limit and become numerically unstable.
- Problem 2 (related) : large logarithms of Q/ρ_{merge} are introduced. These can invalidate the convergence of perturbation theory.
- Choose ρ_{merge} no smaller than $\sim Q/10.$

CKKW MERGING

- Clustering method: k_T jet algorithm
- Analytic NLL-accurate Sudakov factors give no-emission probabilities
- Need a truncated shower, since shower evolution variable not exactly the same as merging scale cut ρ_{merge}.
- Implemented in SHERPA (1.1)

CKKW-L MERGING

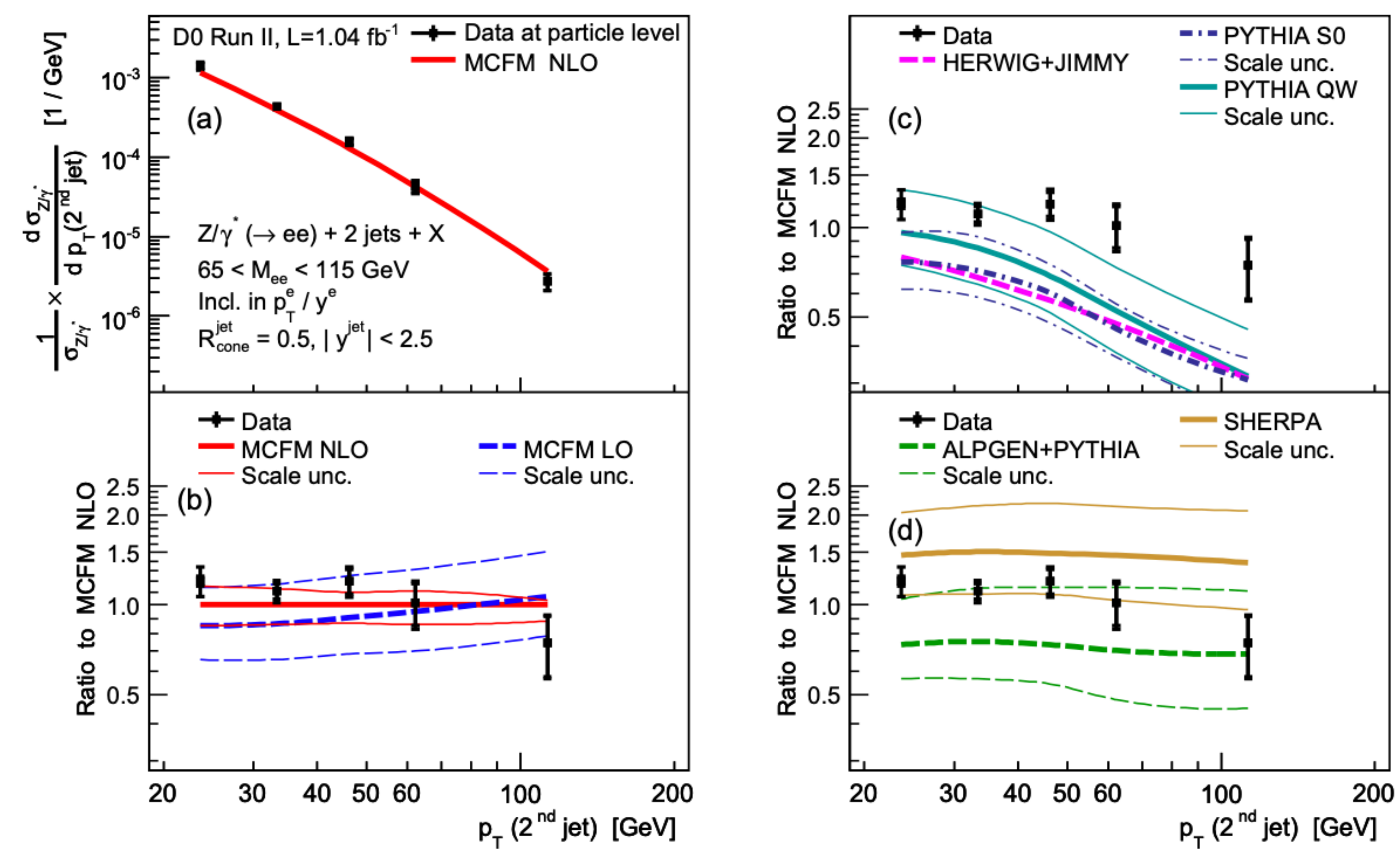
- Clustering method: splitting probabilities in parton shower
- No-emission probabilities generated directly using parton shower
- Shower step-by-step, starting from clustering scale and vetoing event if emission occurs at value larger than next clustering scale
- Weaker merging scale dependence, since Sudakov and shower match by construction
- Implemented in SHERPA (>1.1) , PYTHIA8, HERWIG7

MLM MERGING

- Run shower on ME starting from ρ_0
- Perform jet clustering, and reject if PS emits any jets harder than original partons or partons that are not clustered to hard partons
- Gives a simple estimate of Sudakov suppression - Sudakov factor corresponds to final partons only, not accounting for intermediate states
- Simplest scheme and can be used generally, but Sudakov suppression not exact

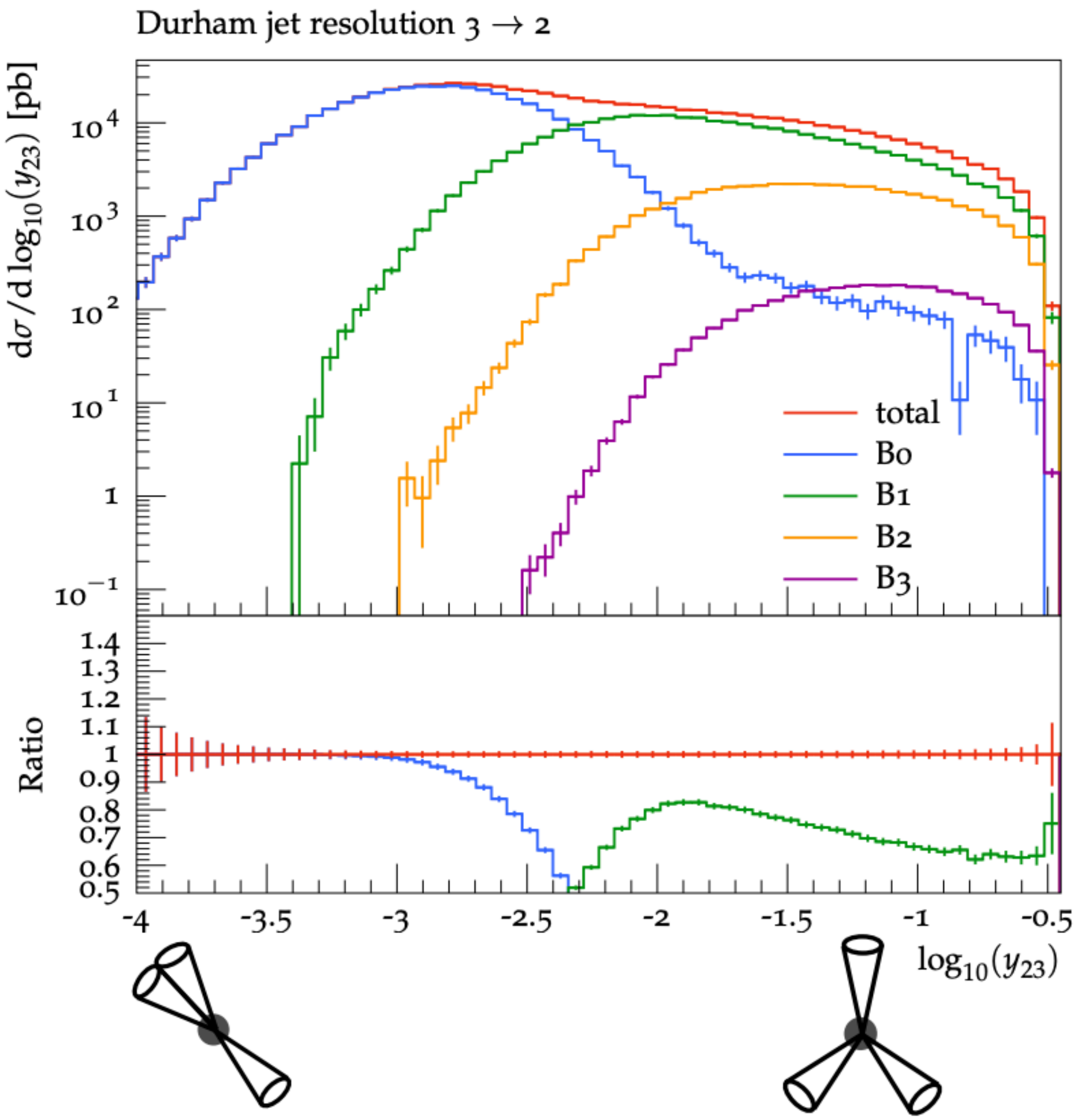
MLM VS FIXED ORDER AND PARTON SHOWER

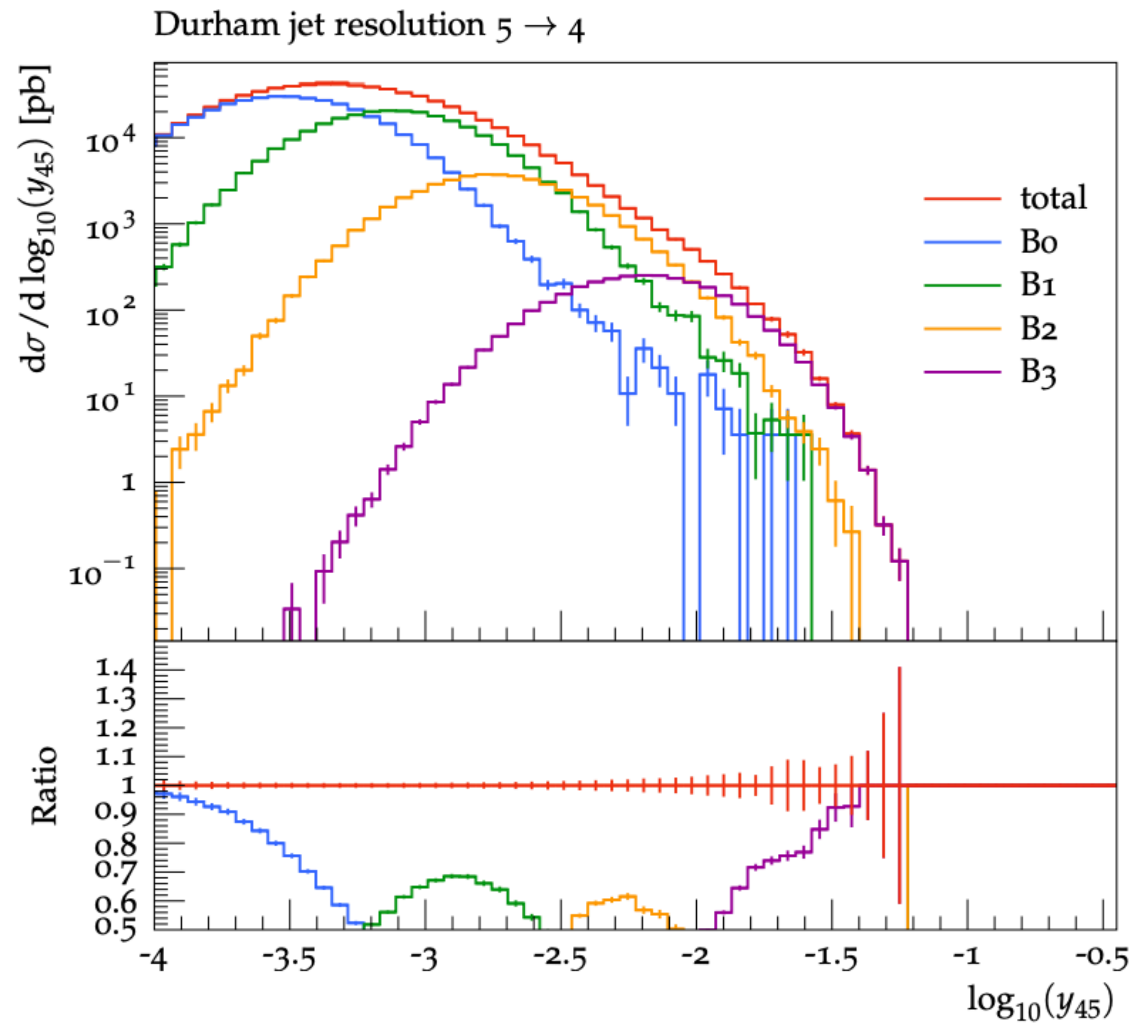
- MLM (green) gets shape right
- Large scale uncertainty and normalisation wrong, much worse than NLO (red)



- Second problem fixed by matching methods, e.g. POWHEG, MC@NLO

JET PRODUCTION IN e^+e^-

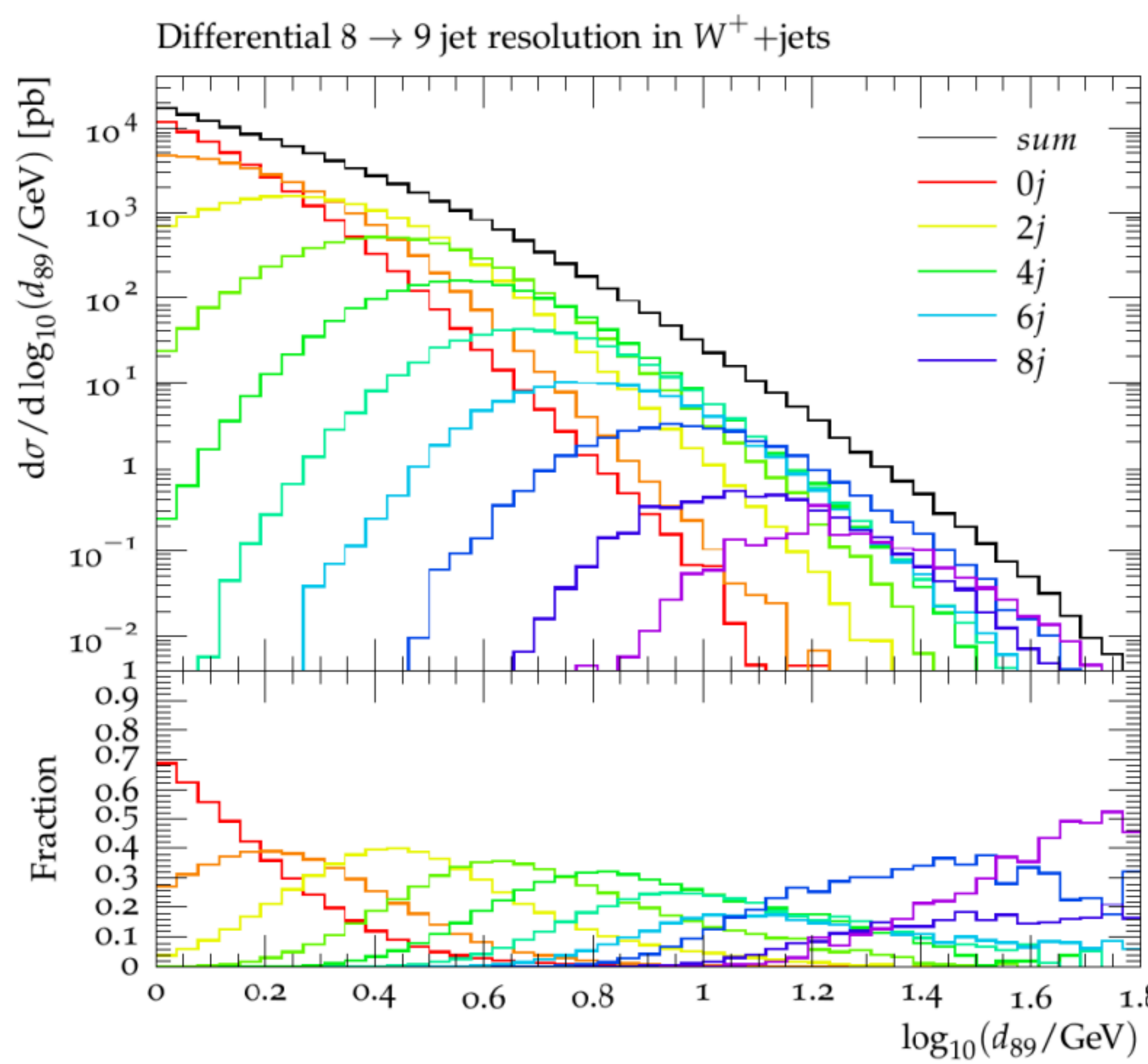




Credit: Leif Gellersen

HIGH MULTIPLICITIES

- Many jet final states are challenging
- Factorial growth in shower history reconstruction makes merging difficult for $N \geq 5$
- Approaches include winner-take-all clustering, sector showers



MATCHING AND MERGING AT HIGHER ACCURACIES

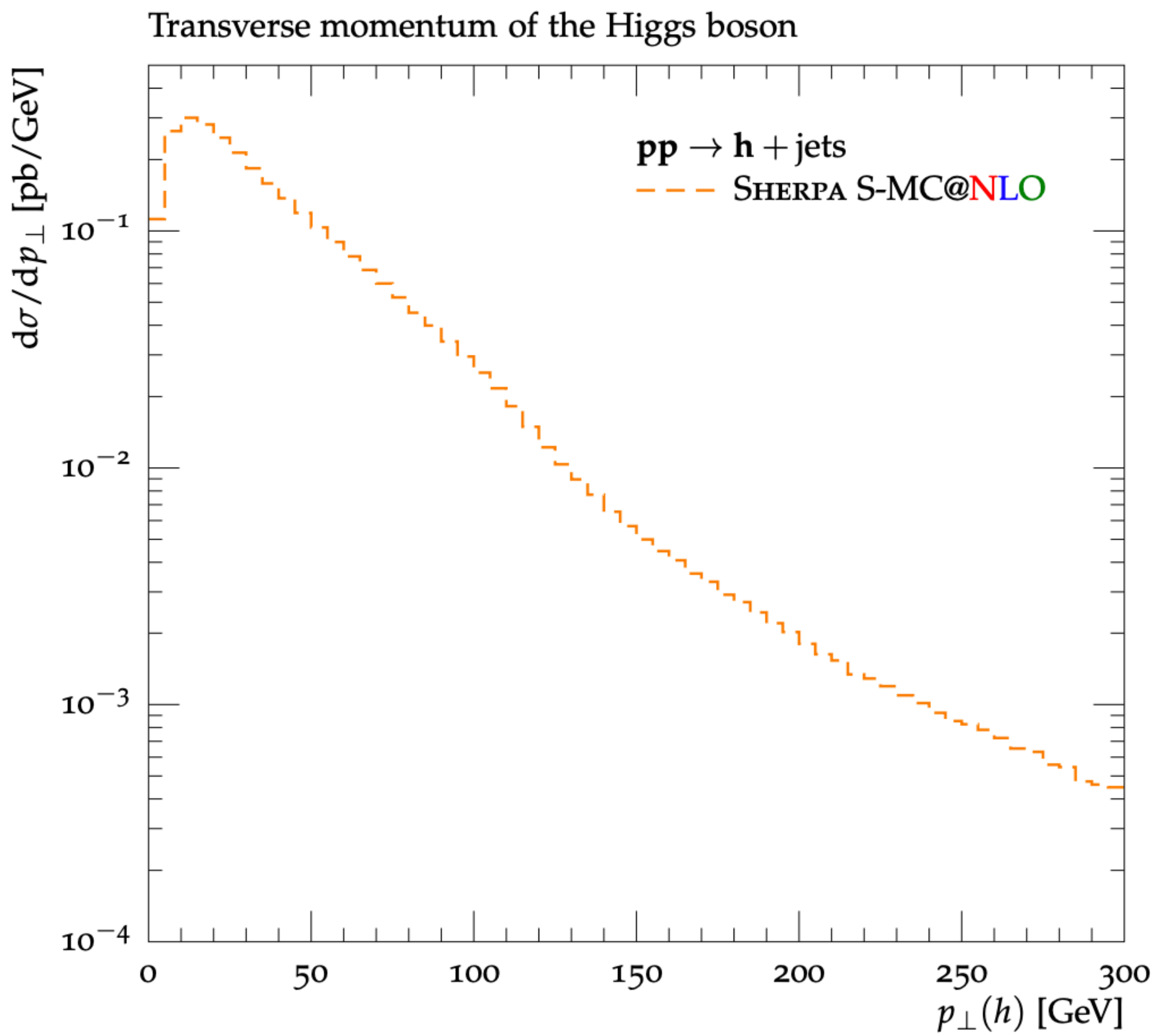
MERGING AT NLO

- Merged strategies are LO+LL accurate for exclusive quantities involving jets (Born + 1,2,…)
- Matched strategies are NLO+LL accurate for inclusive quantities.
- NLO matching gives more accurate normalisation, reduced theoretical uncertainties, but it is only LO+LL accurate for Born + 1 jet exclusive quantities (just LL for > 1 jet).
- Is there a way to combine the advantages of matching and merging?

MERGING AT NLO

- Combine MC@NLO simulations for Born, Born+1 jet, Born+2 jets…
- Naïve combination results in double counting, since Sudakov form factors (LL/NLL accurate) also encode some of the NLO corrections
- Subtracting double-counted term results in consistent combination of NLO samples
- Minor differences in implementation details (MEPS@NLO, FxFx, UNLOPS)

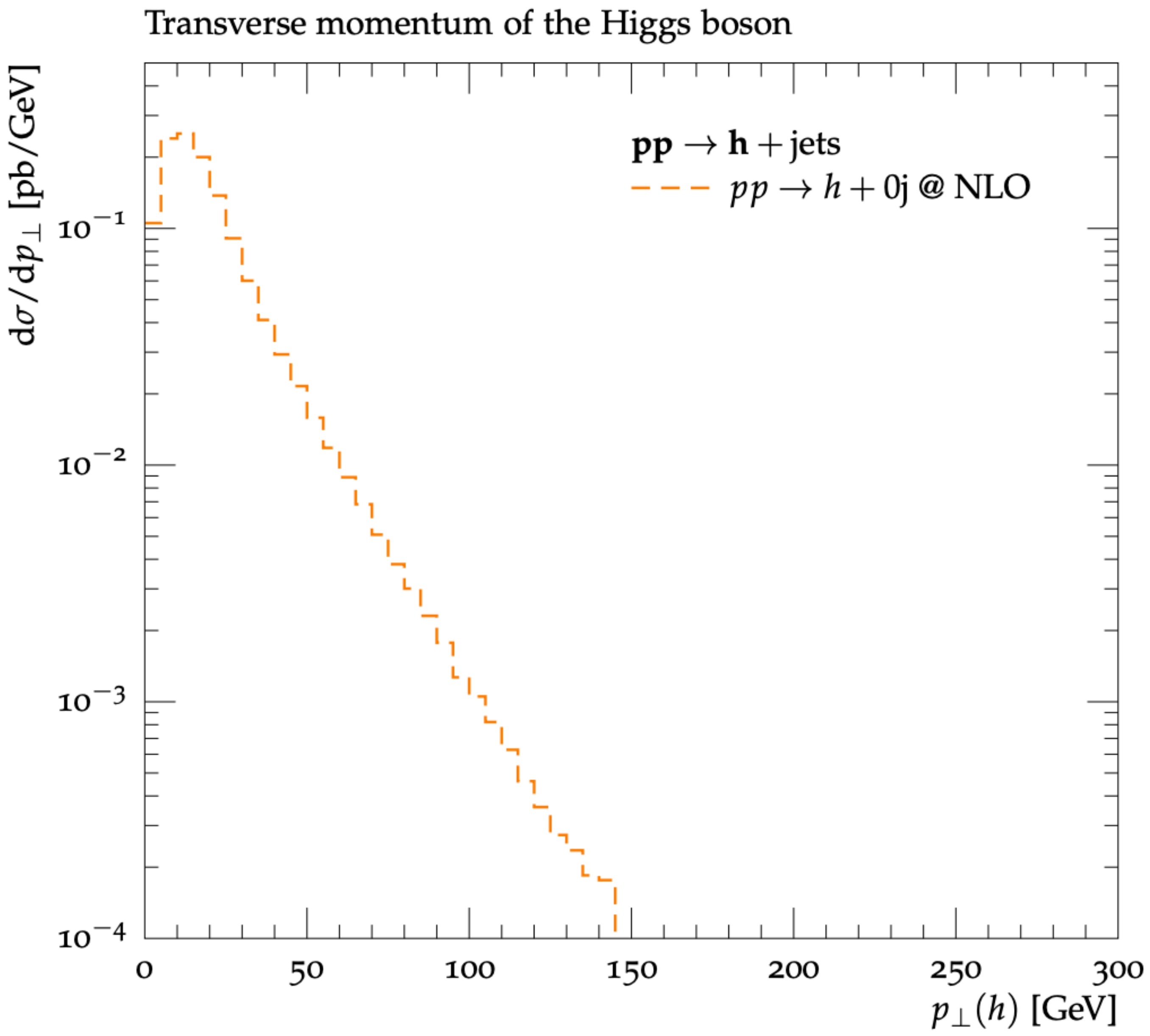
EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



Credit: F. Krauss

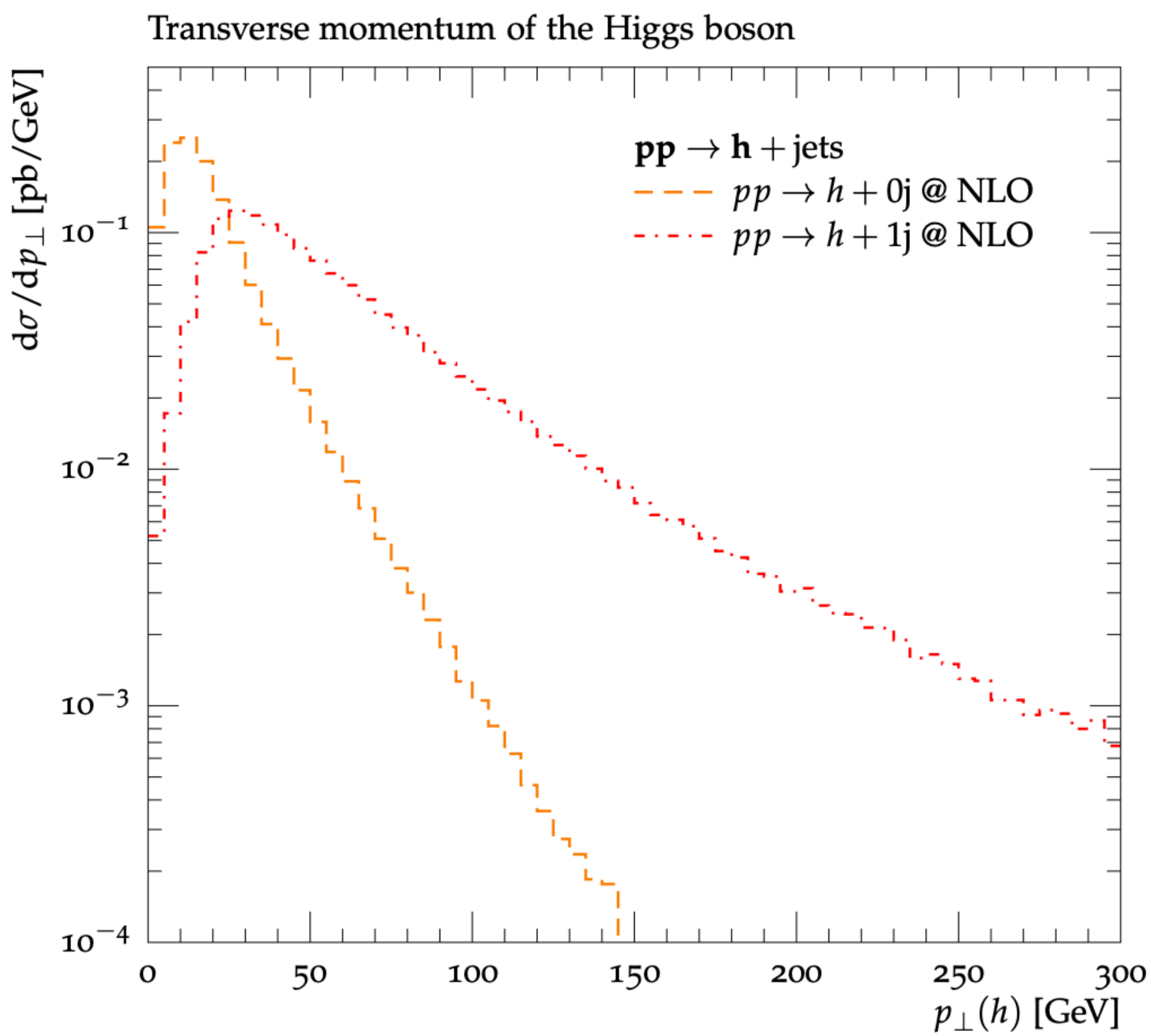
- First emission by MC@NLO

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



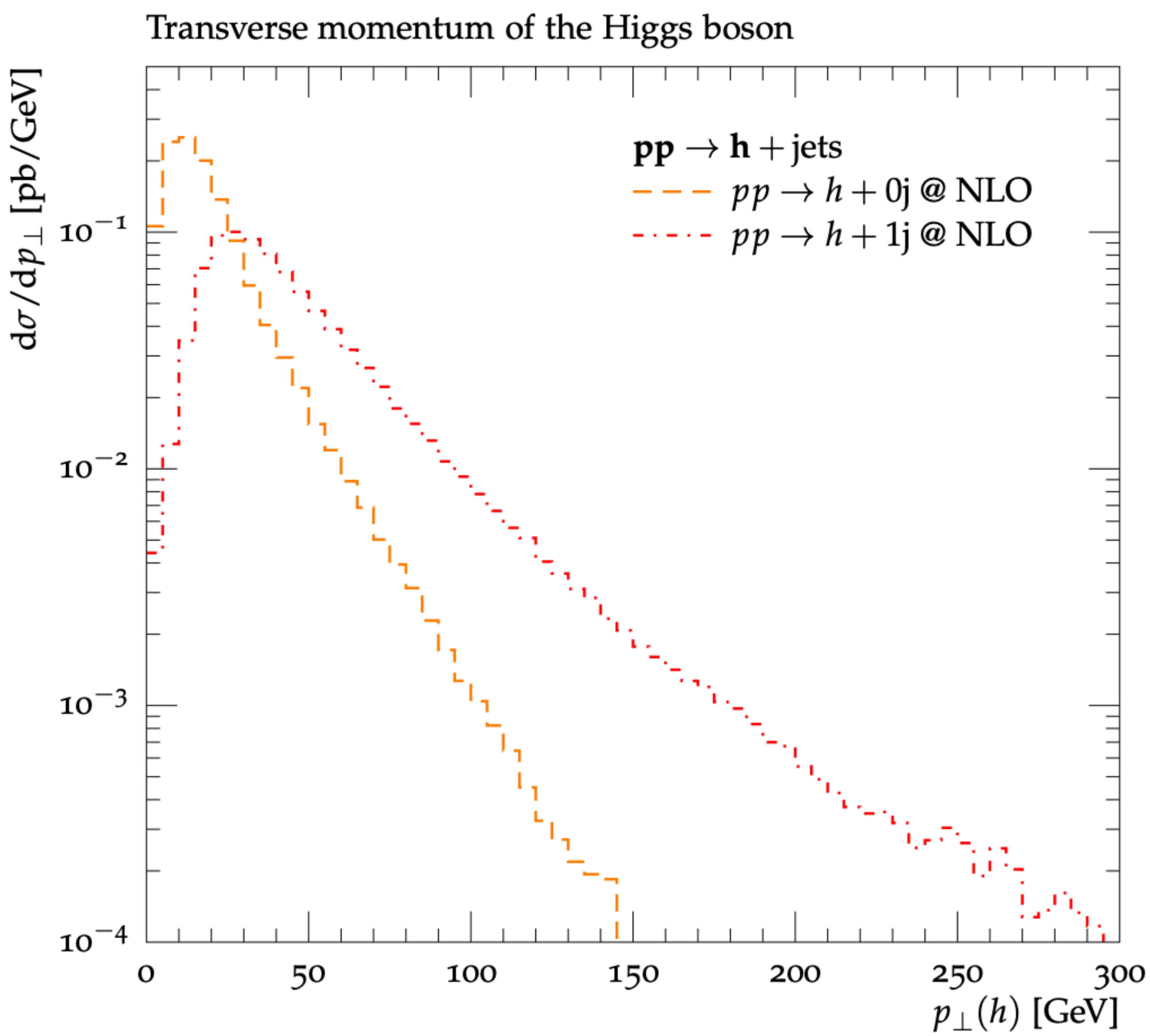
- First emission by MC@NLO, restricted to $Q_{N+1} < Q_{\mathrm{cut}}$

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



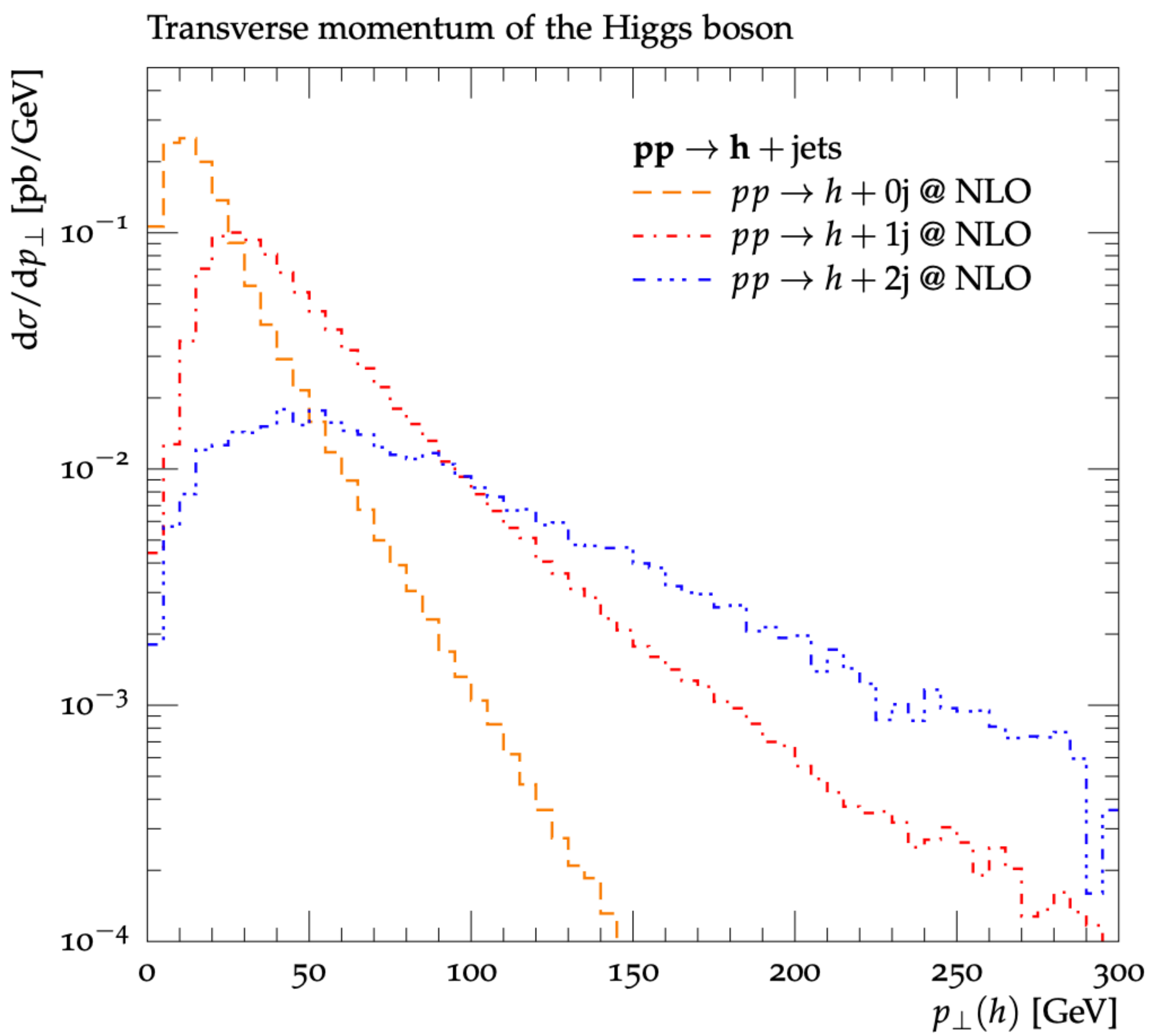
- First emission by MC@NLO, restricted to $Q_{N+1} < Q_{\rm cut}$
- MC@NLO for H+jet, $Q_{N+1} > Q_{\rm cut}$

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



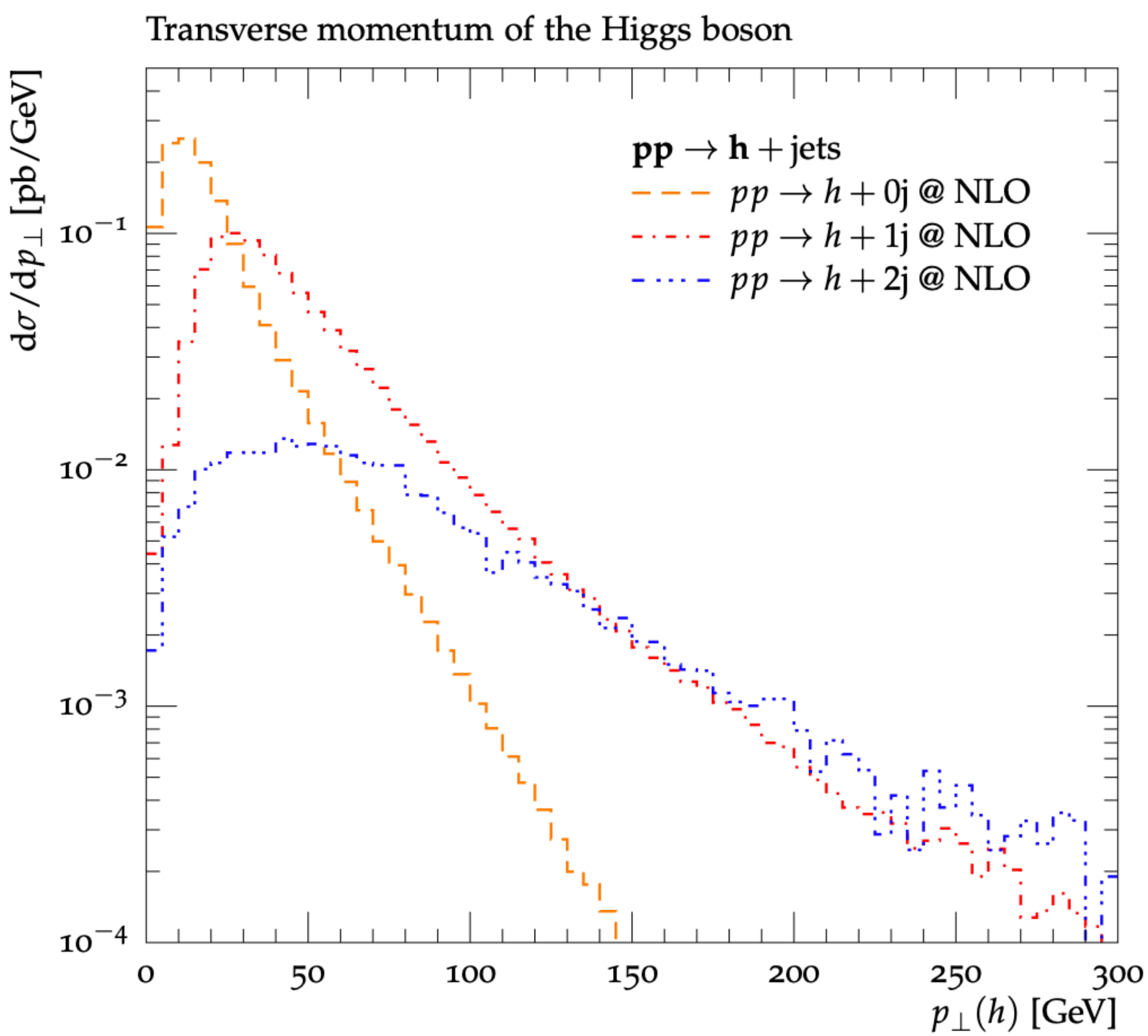
- First emission by MC@NLO, restricted to $Q_{N+1} < Q_{\mathrm{cut}}$
- MC@NLO for H+jet, $Q_{N+1} > Q_{\mathrm{cut}}$, restricted to $Q_{N+2} < Q_{\mathrm{cut}}$

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



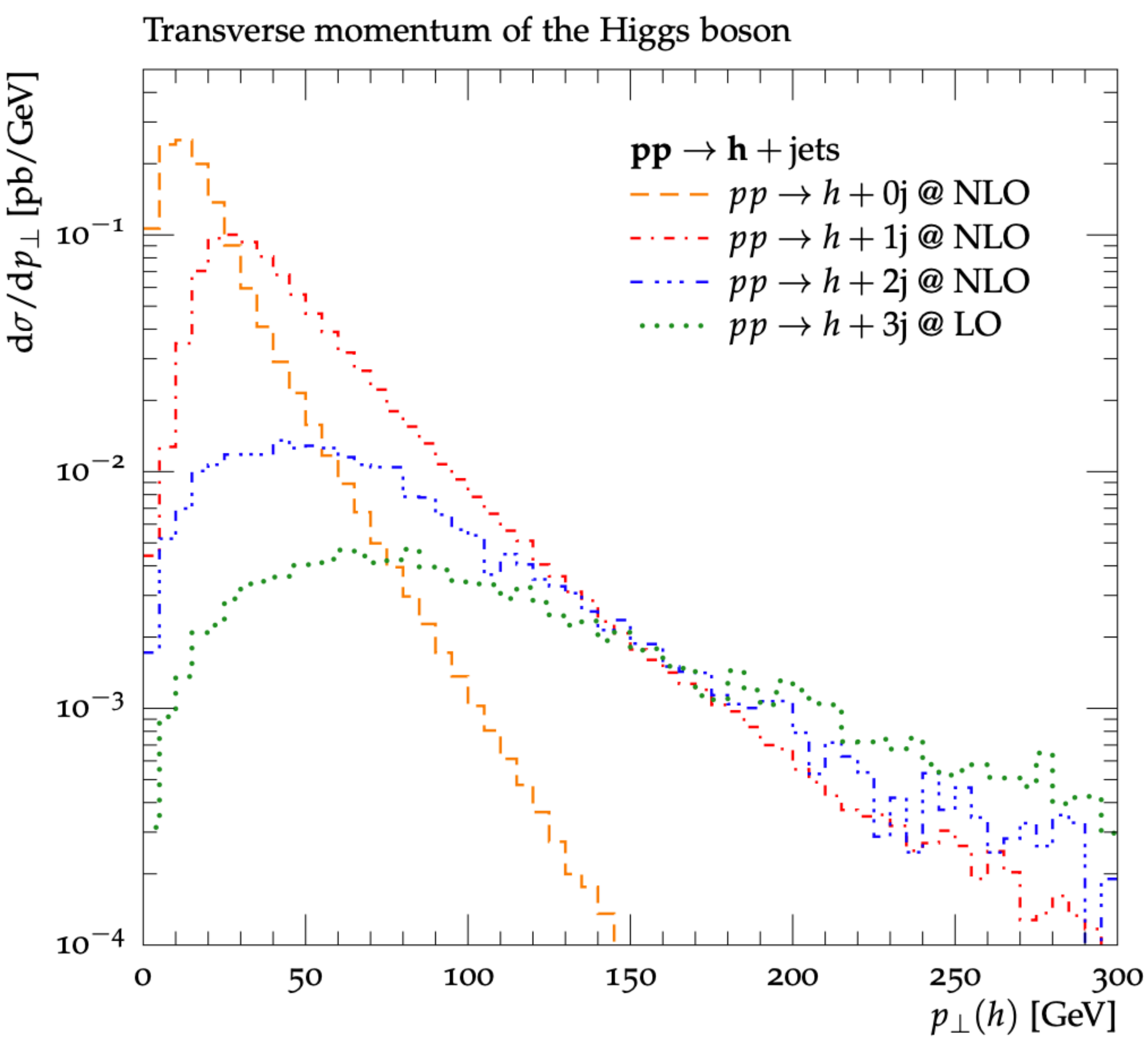
- First emission by MC@NLO, restricted to $Q_0 < Q_{\text{cut}}$
- MC@NLO for H+jet, $Q_0 > Q_{\text{cut}}$, restricted to $Q_1 < Q_{\text{cut}}$
- Iterate

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



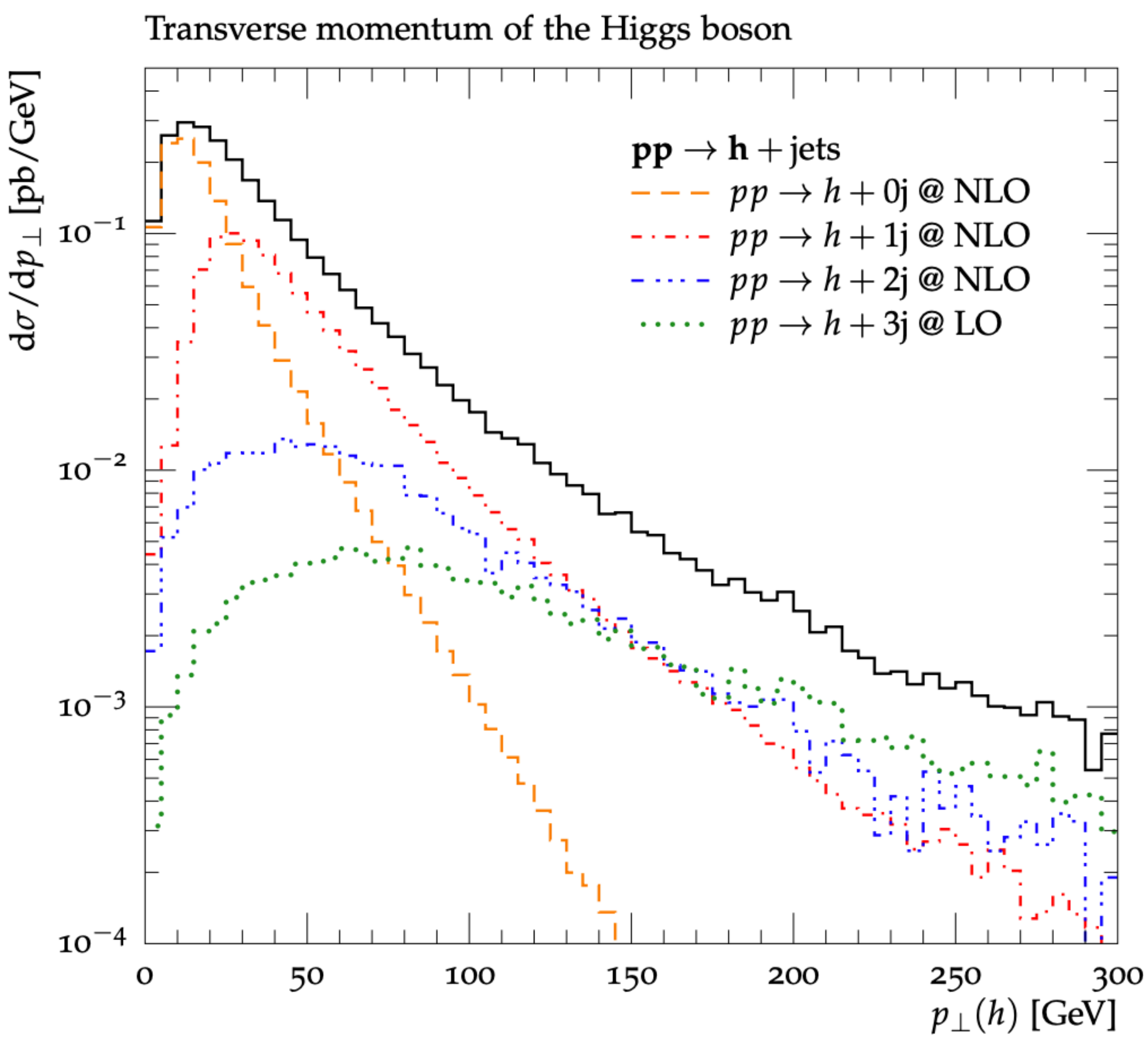
- First emission by MC@NLO, restricted to $Q_0 < Q_{\mathrm{cut}}$
- MC@NLO for H+jet, $Q_0 > Q_{\mathrm{cut}}$, restricted to $Q_1 < Q_{\mathrm{cut}}$
- Iterate

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO



- First emission by MC@NLO, restricted to $Q_0 < Q_{\text{cut}}$
- MC@NLO for H+jet, $Q_0 > Q_{\text{cut}}$, restricted to $Q_1 < Q_{\text{cut}}$
- Iterate

EXAMPLE: HIGGS PRODUCTION USING MEPS@NLO

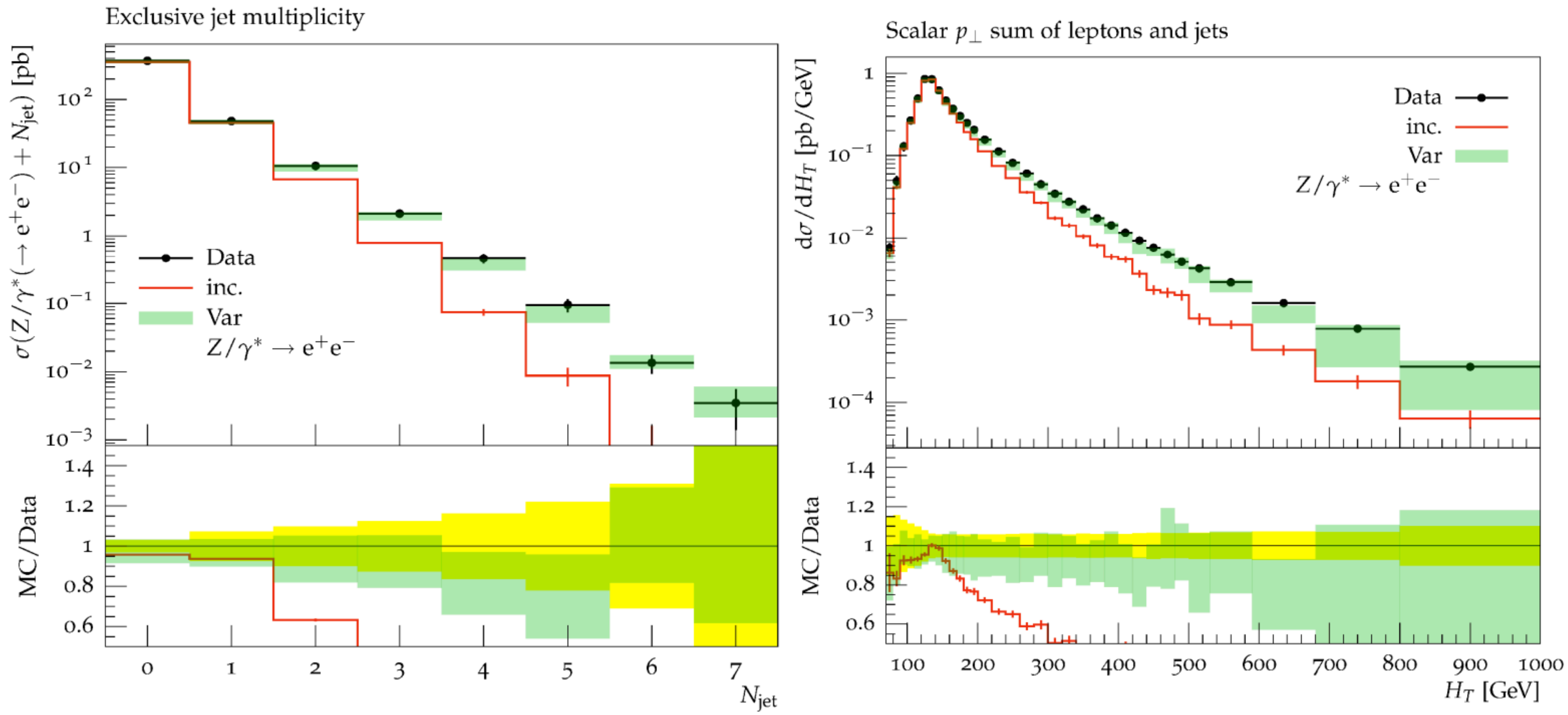


- First emission by MC@NLO, restricted to $Q_0 < Q_{cut}$
- MC@NLO for H+jet, $Q_0 > Q_{cut}$, restricted to $Q_1 < Q_{cut}$
- Iterate
- Sum contributions

EXAMPLE: DRELL-YAN USING FXFX

(Data from ATLAS, 1304.7098, aMC@NLO_MADGRAPH with HERWIG++)

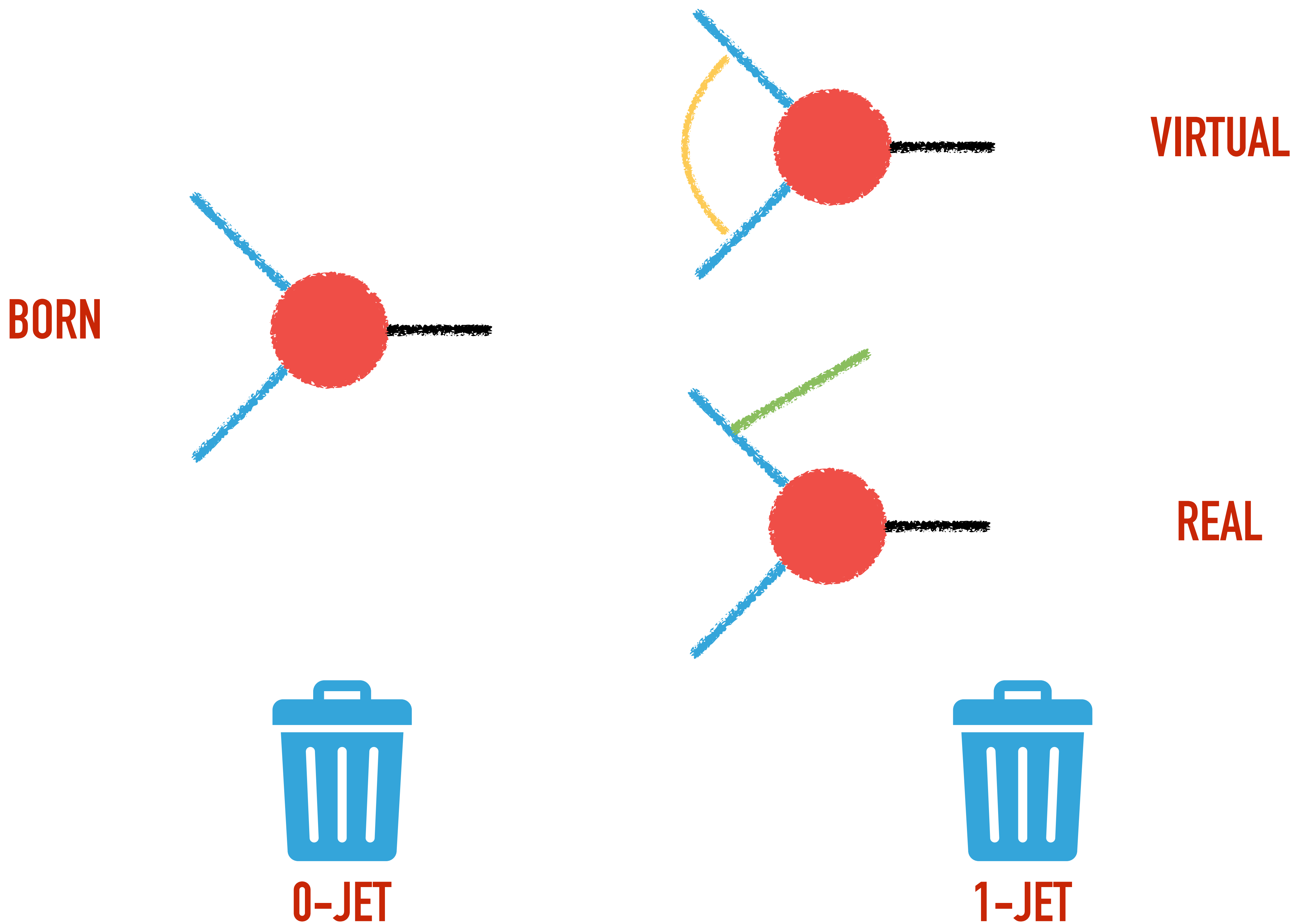
(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



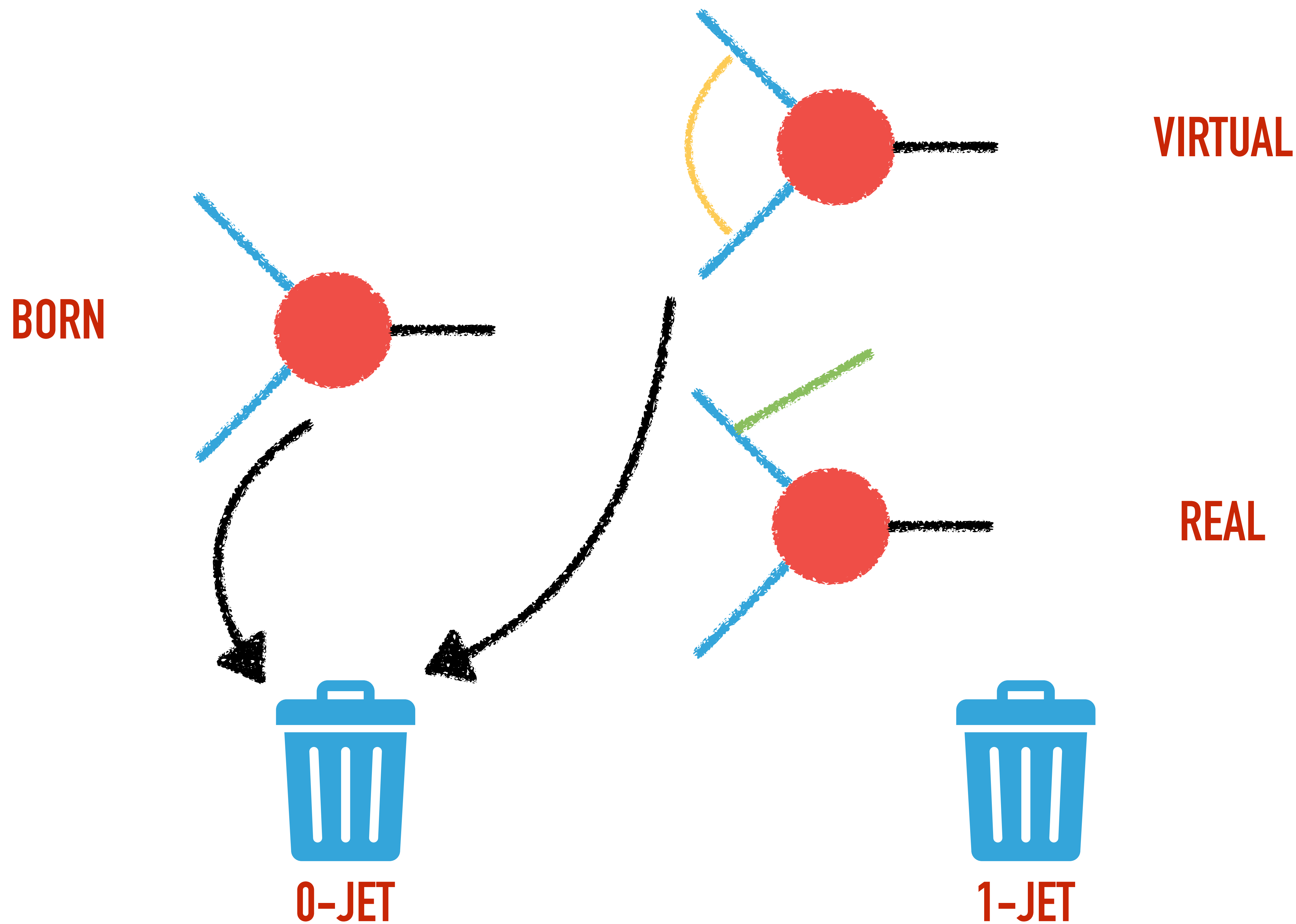
NNLO MATCHING

- Just as we matched NLO calculations to parton shower, can we match NNLO?
- Aim to get NNLO normalisation for inclusive quantities, NLO+LL for 1-jet quantities and LO+LL for 2-jet
- Learn from NLO merging, introducing resolution cuts to divide phase space
- Sending merging cuts to small values requires exquisite control of large logarithms

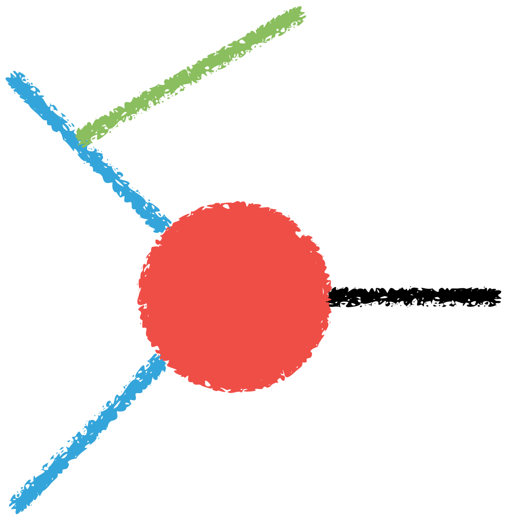
DEFINING IR-FINITE EVENTS



DEFINING IR-FINITE EVENTS



DEFINING IR-FINITE EVENTS



REAL

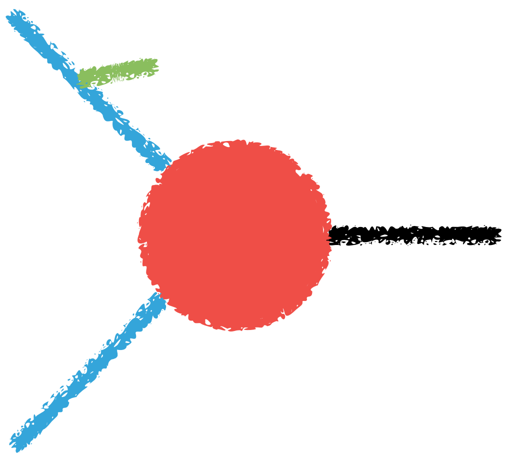
0-JET

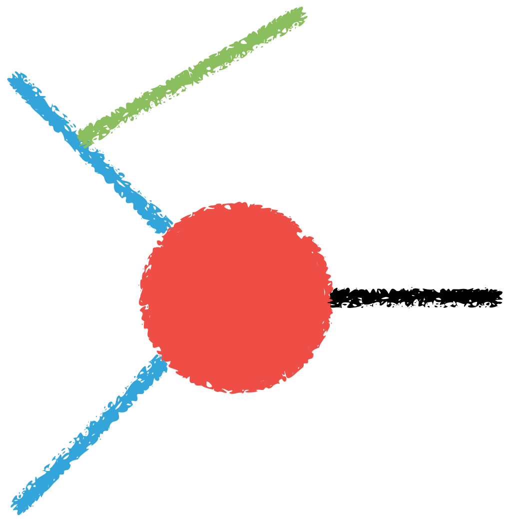
1-JET

DEFINING IR-FINITE EVENTS

SOFT/COLL. REAL

$r_0 < r_0^{\mathrm{cut}}$





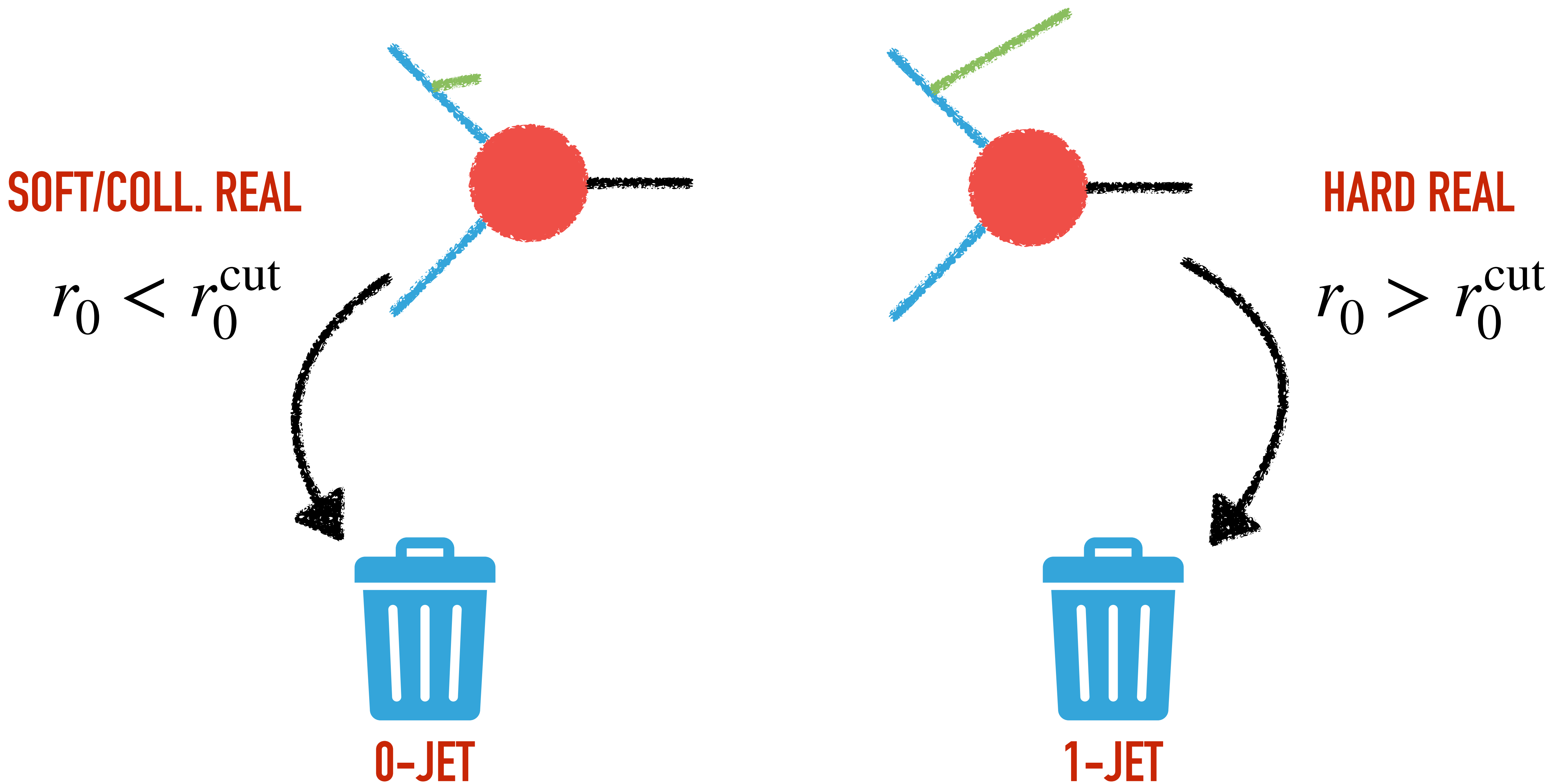
HARD REAL

$r_0 > r_0^{\mathrm{cut}}$

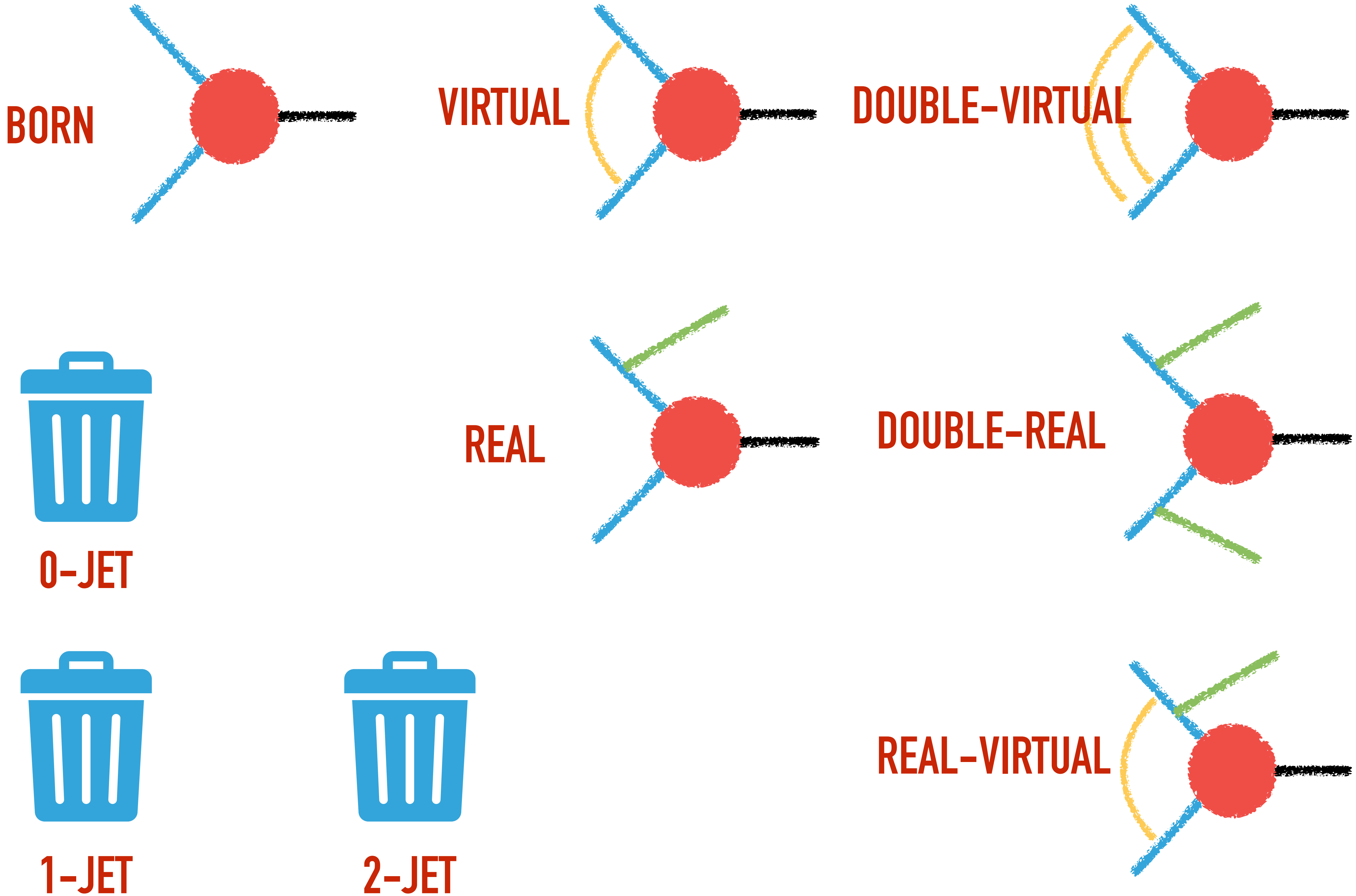
0-JET

1-JET

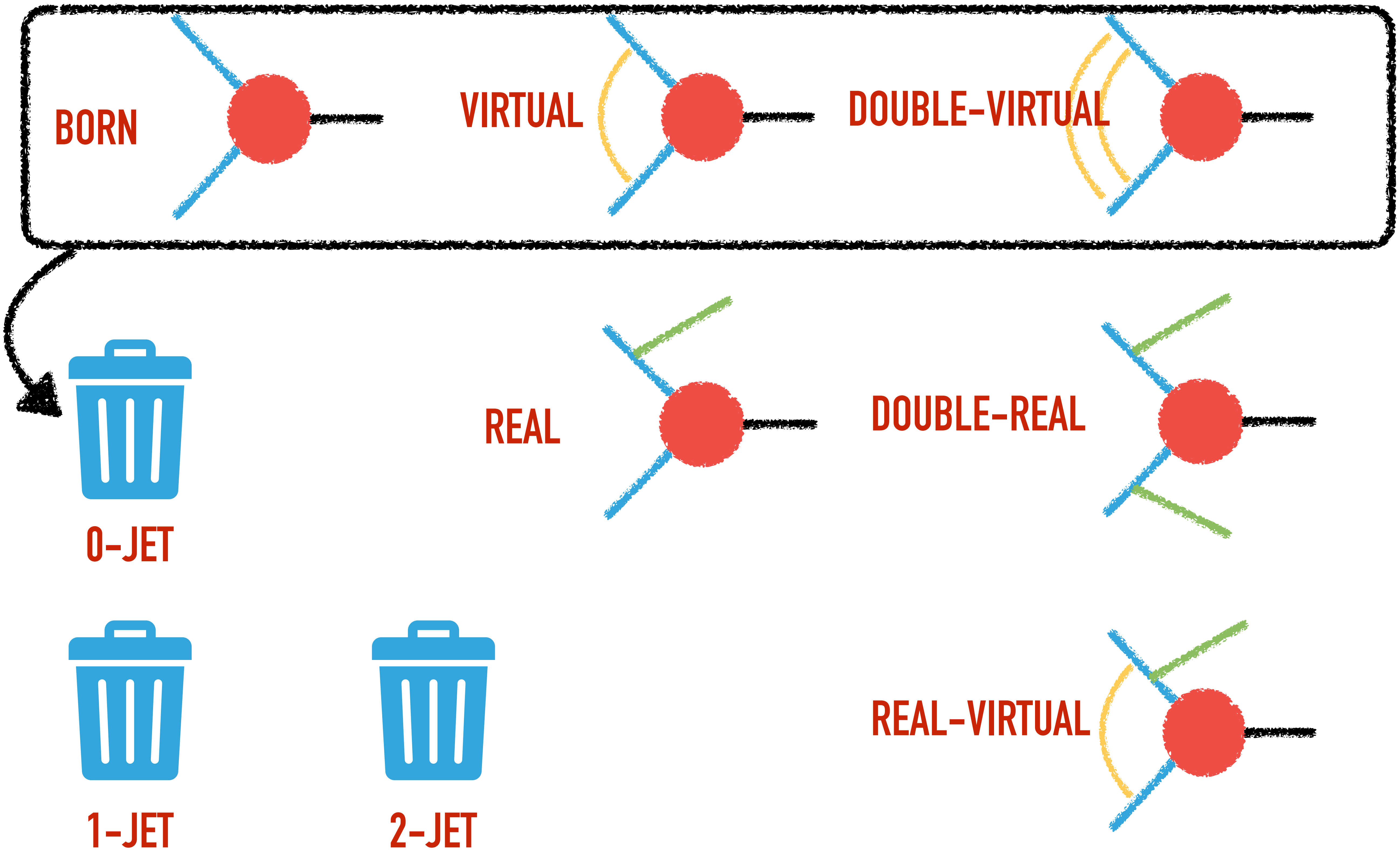
DEFINING IR-FINITE EVENTS



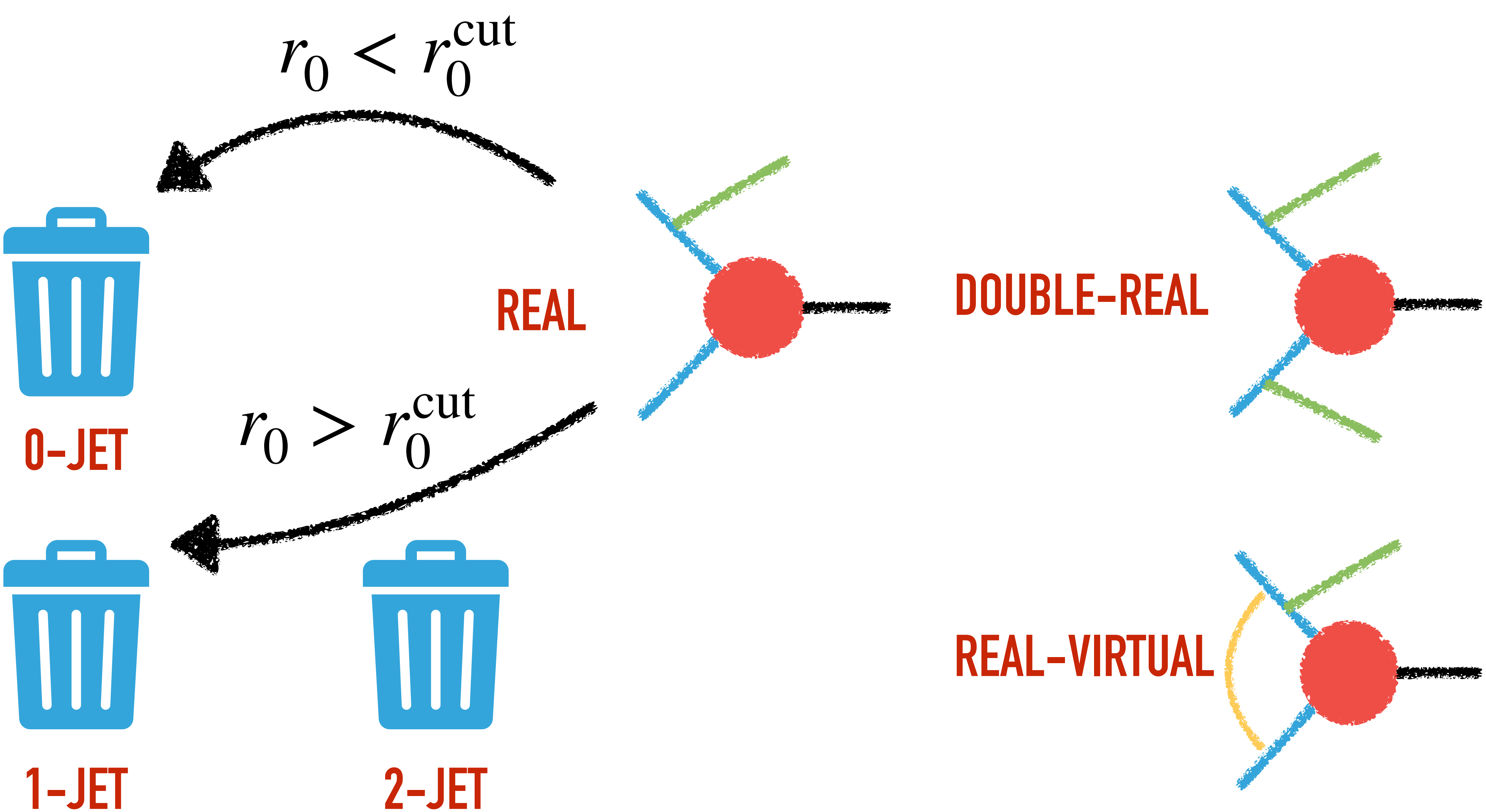
DEFINING IR-FINITE EVENTS



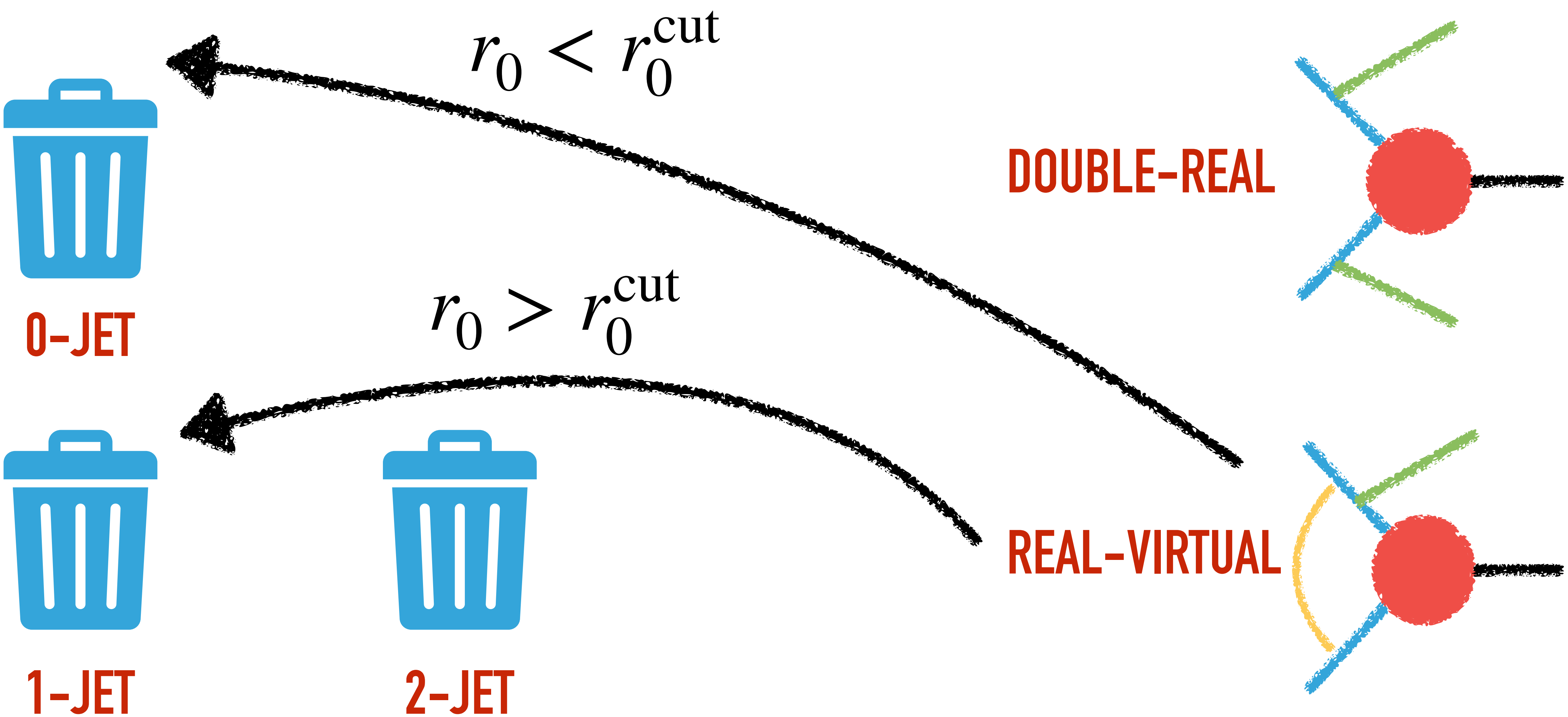
DEFINING IR-FINITE EVENTS



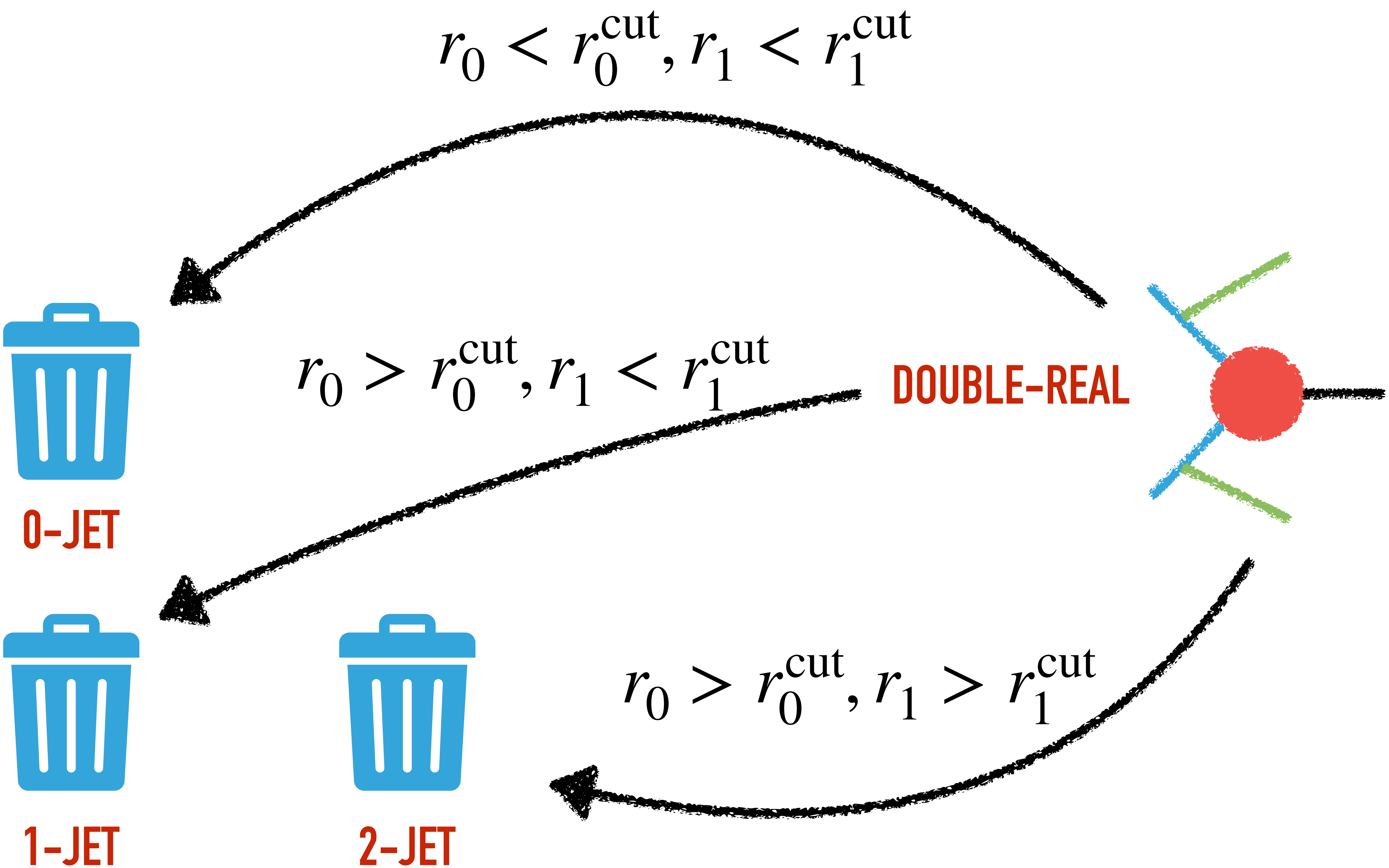
DEFINING IR-FINITE EVENTS



DEFINING IR-FINITE EVENTS



DEFINING IR-FINITE EVENTS



DEFINING IR-FINITE EVENTS

- Defining events this way introduced a projection from a higher multiplicity to a lower multiplicity phase space - want to set merging scale as small as possible

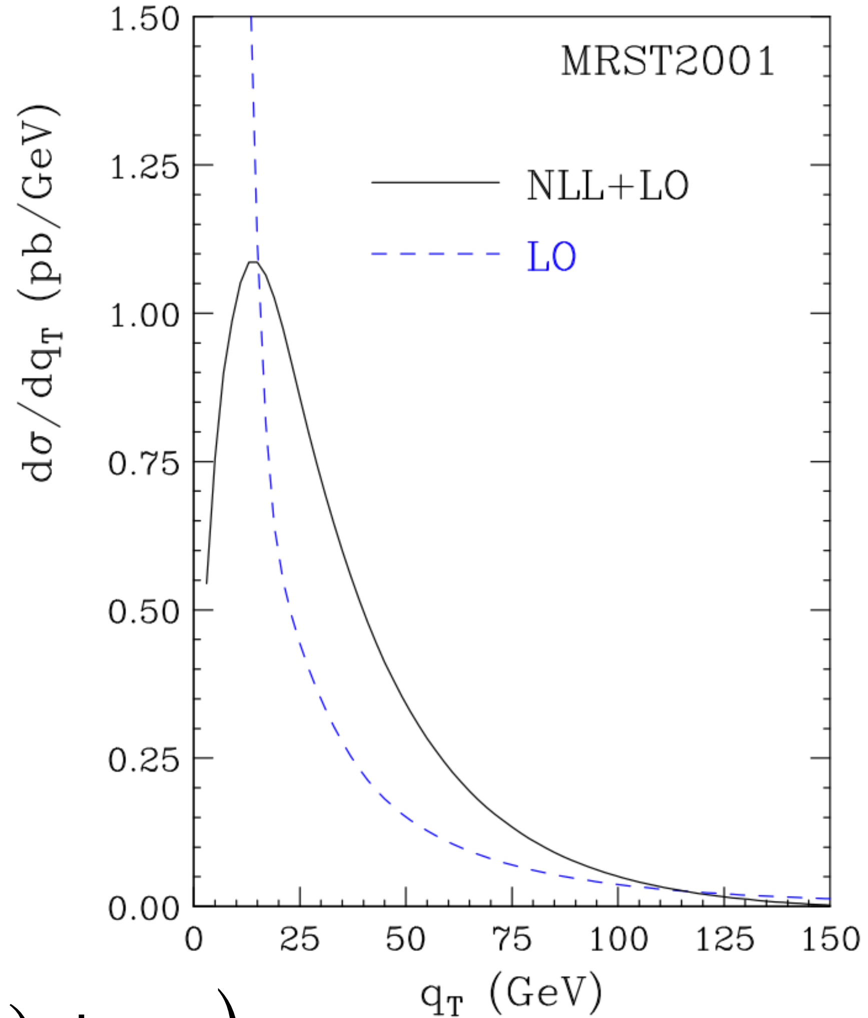
- Results are only (N)NLO accurate up to power corrections in r_0^{cut} - as $r_0^{\mathrm{cut}} \to 0$, exact fixed order result is recovered

- Causes large logarithms to appear which spoil perturbative convergence!

$$L = \log(Q/r_0^{\mathrm{cut}})$$ becomes large…

RESUMMATION – THE CURE FOR LARGE LOGS

- Large logs signal the breakdown of the perturbative series in the coupling, leading term $\alpha L^2 \sim 1 \Rightarrow \alpha L \ll 1$
- Reordering the series to expand in a genuinely small parameter cures behaviour



$$d\sigma = C(\alpha_s)\, \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

- Different formalisms available to achieve this

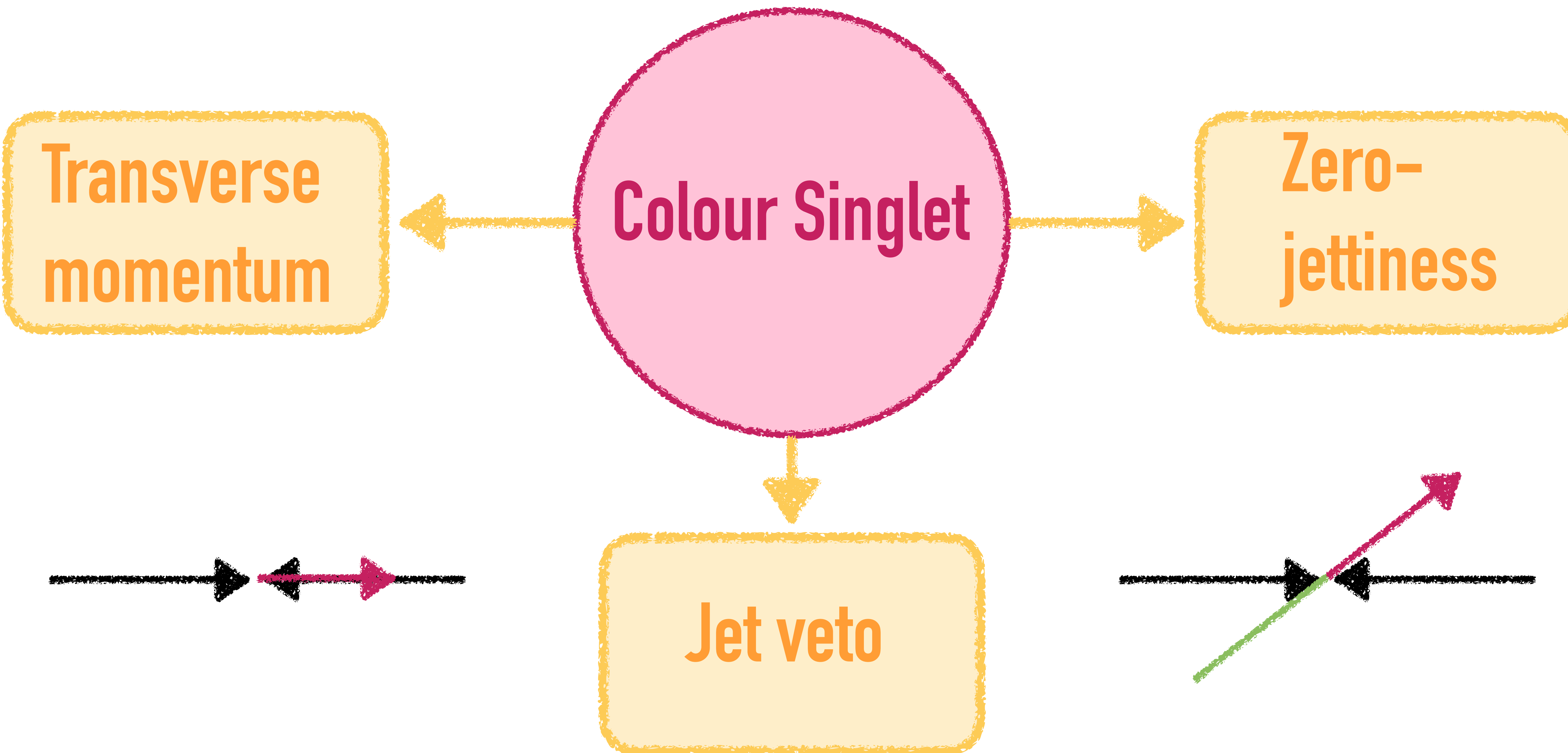
NNLO+PS MATCHING IN GENEVA

- Replace low-accuracy Sudakov resummation (LL/NLL) with higher-accuracy analytic resummed formula (SCET)
- Combine resummed calculation with fixed order, subtracting double counting
- Pass IR-finite events to shower

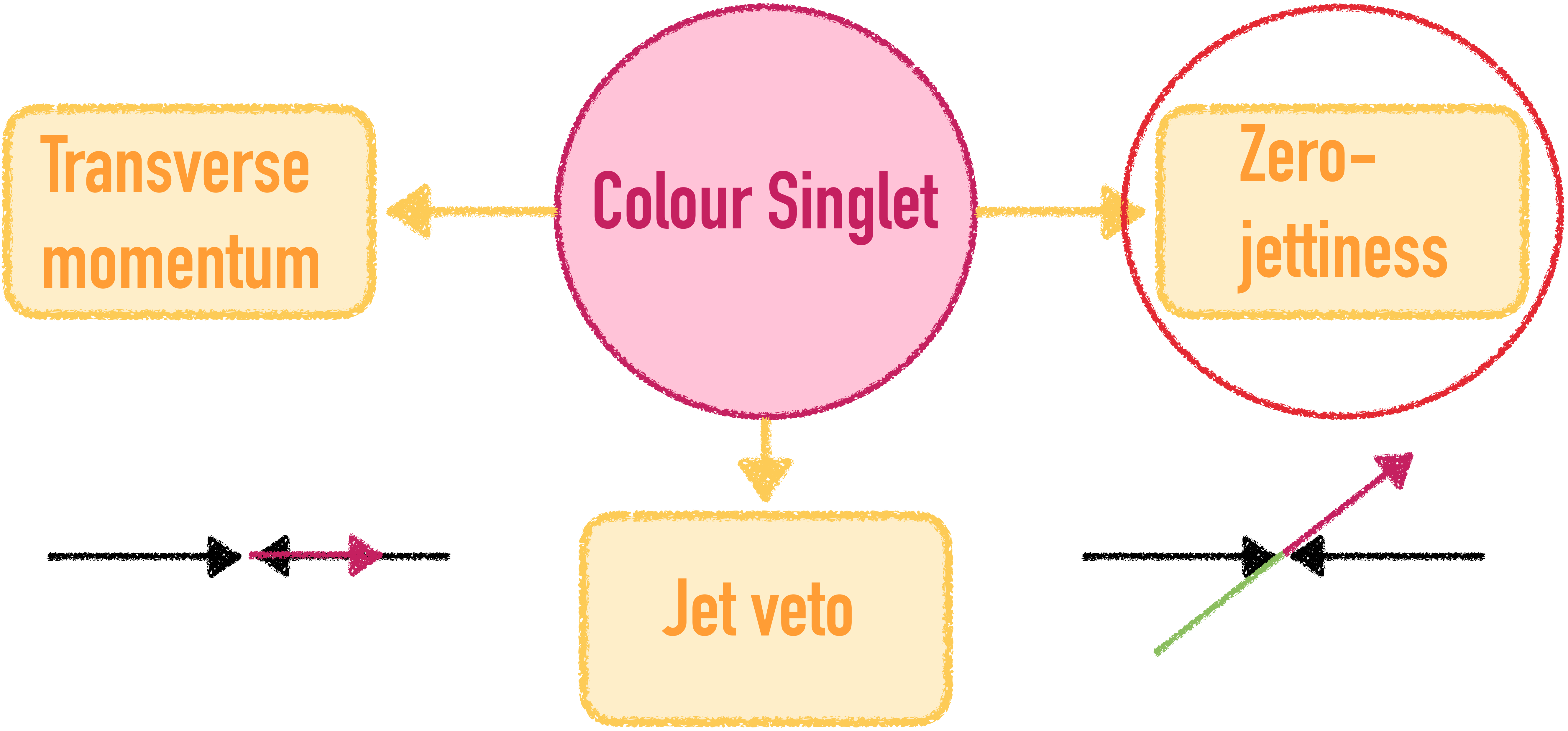
$$\frac{d\sigma}{d\Phi_{N+1}} = \frac{d\sigma^{\mathrm{NNLL}'}}{dr\, d\Phi_N}\mathscr{P}(\Phi_{N+1}) + \frac{d\sigma^{\mathrm{NLO}_1}}{d\Phi_{N+1}} - \left[\frac{d\sigma^{\mathrm{NNLL}'}}{dr\, d\Phi_N}\mathscr{P}(\Phi_{N+1})\right]_{\mathrm{NLO}_1}$$

Splitting function adds dependence in two extra variables

RESOLUTION VARIABLES



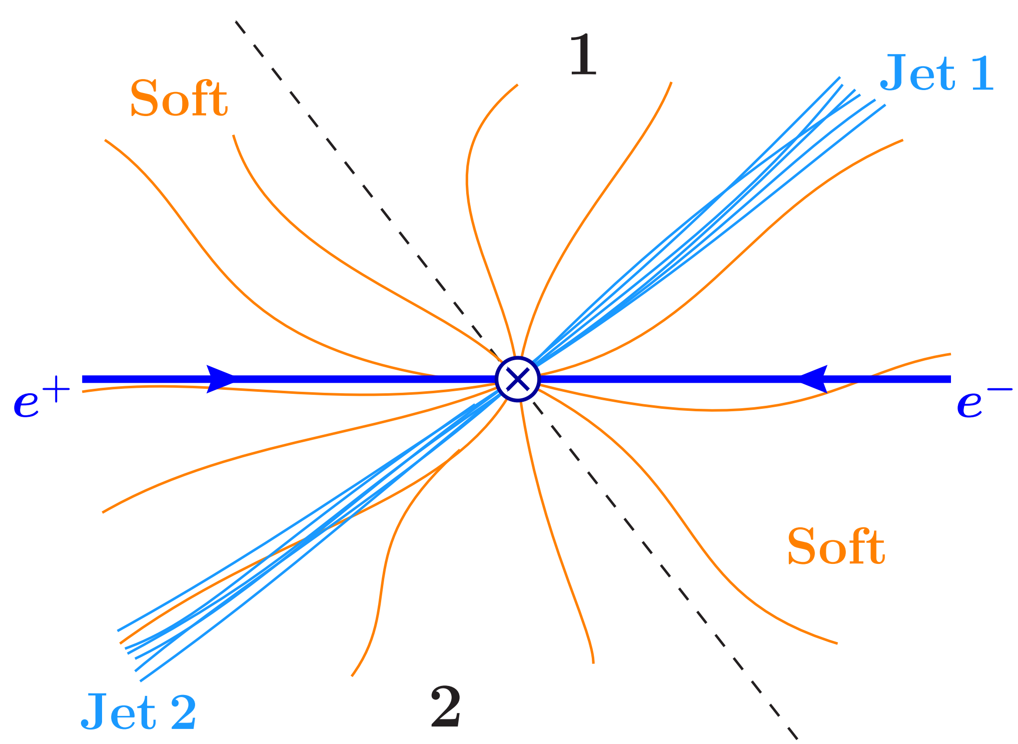
RESOLUTION VARIABLES

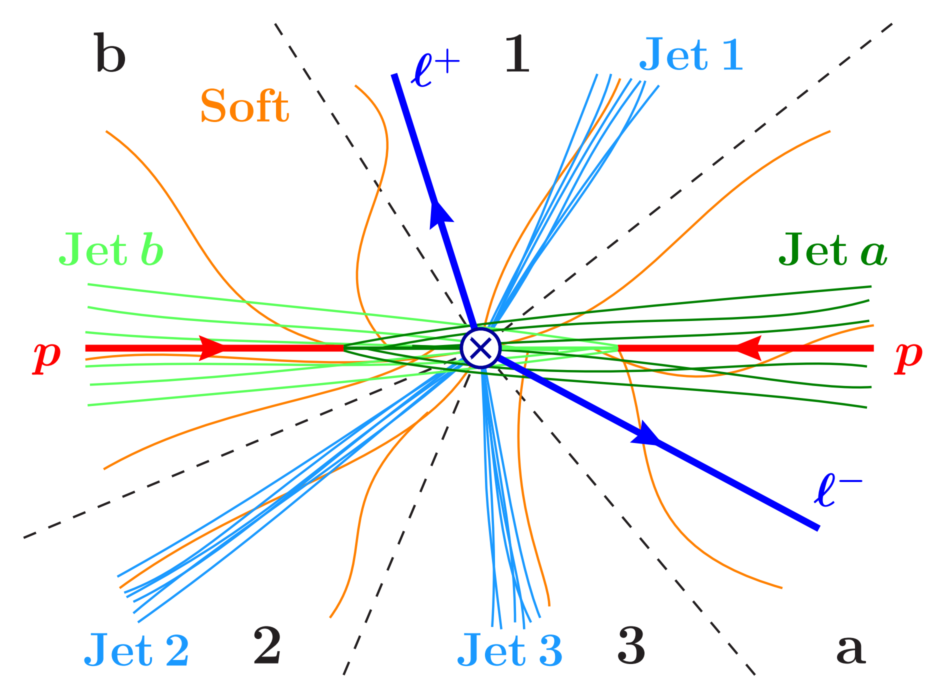


THE N-JETTINESS OBSERVABLE

- $\mathcal{T}_N = 0$ implies there are exactly N pencil-like jets
- Large $\mathcal{T}_N$ implies a spherical distribution of radiation

$$\mathcal{T}_N = \frac{2}{Q} \sum_k \min\{q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k\}$$





ZERO-JETTINESS RESUMMATION FOR COLOUR SINGLET

SCET allows us to write a factorisation formula as

$$\frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \int dt_a dt_b \, B_i(t_a, x_a, \mu_B)\, B_j(t_b, x_b, \mu_B)\, H_{ij}(\Phi_0, \mu_H)\, S\!\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu_S\right)$$

Beams

Hard

Soft

All single-scale objects!

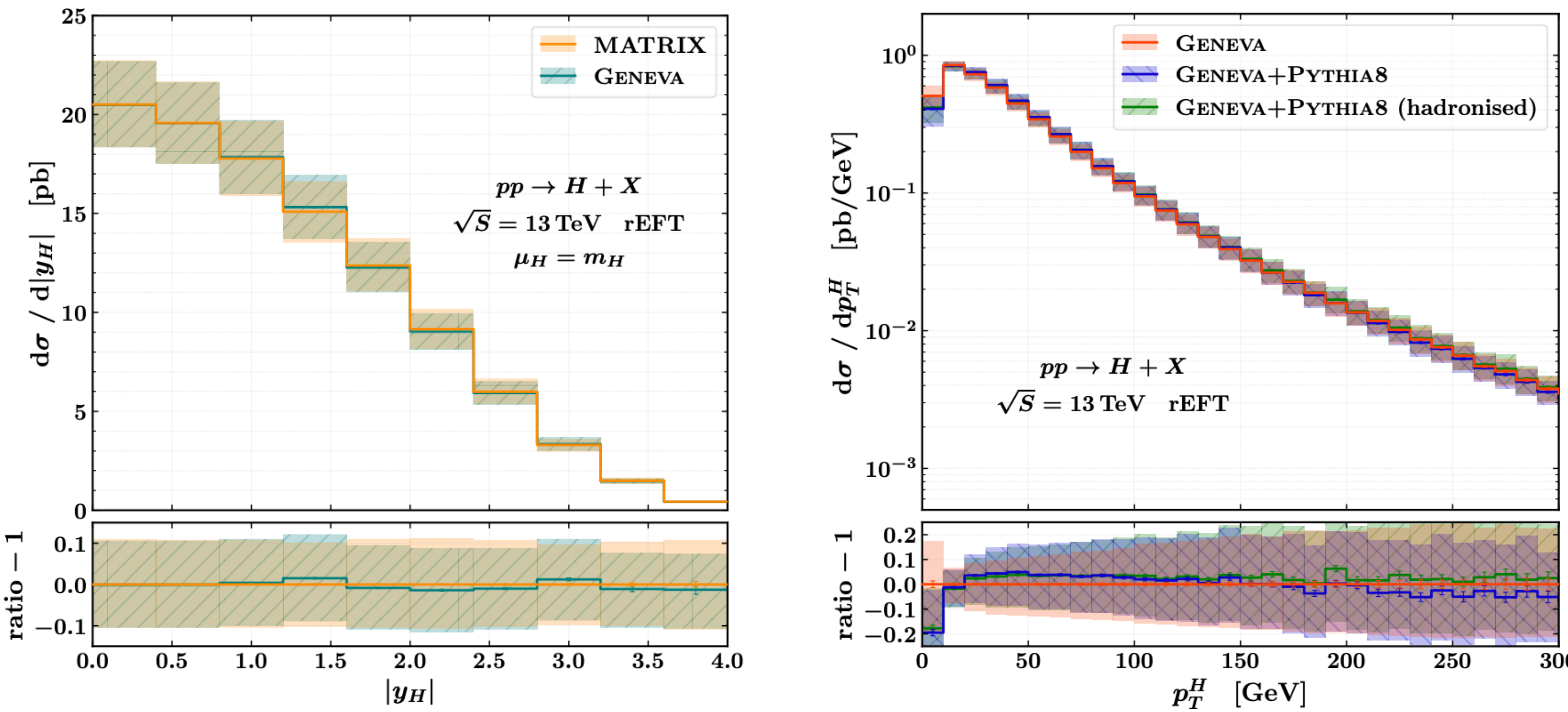
Resummation via RGE running to common scale:

$$B_i(t_a, x_a, \mu) = B_i(t_a, x_a, \mu_B) \otimes U_B(\mu, \mu_B)$$

Resums logs of μ/μ_B

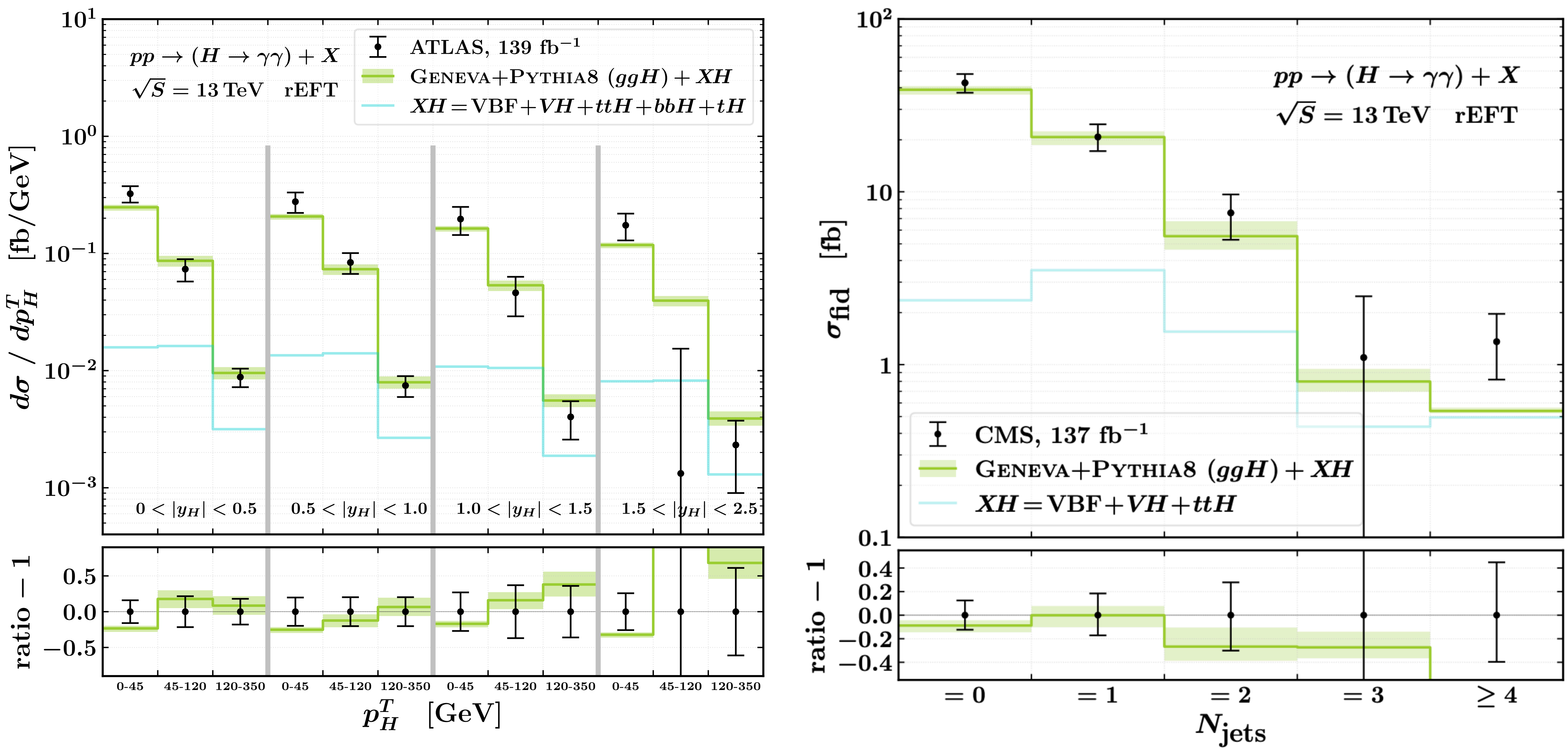
0910.0467, I. Stewart, F. Tackmann, W. Waalewijn

ZERO-JETTINESS RESUMMATION IN GENEVA



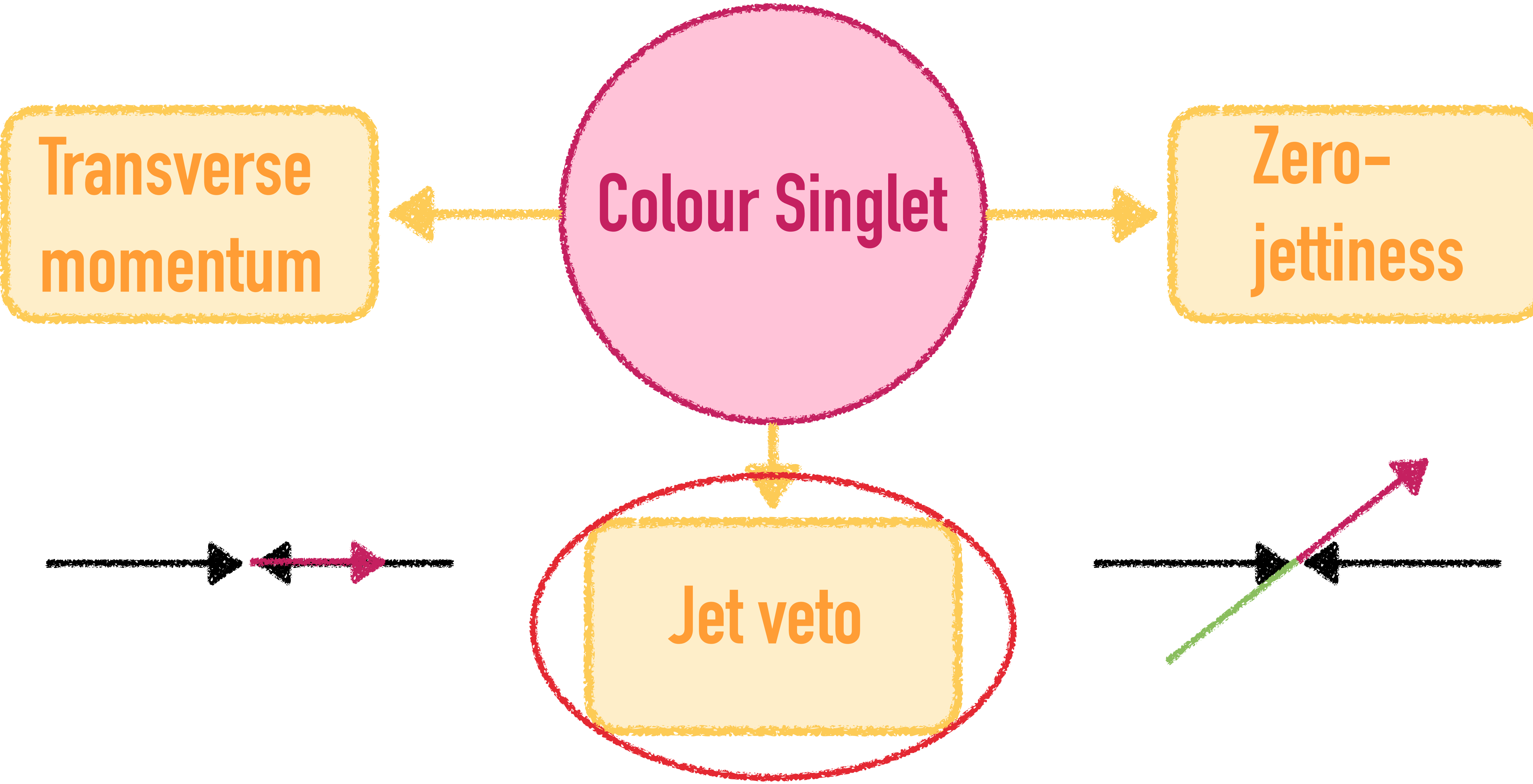
2301.11875, S. Alioli, G. Billis, A. Broggio, A. Gavardi, S. Kallweit, MAL, G. Marinelli, R. Nagar, D. Napoletano

ZERO-JETTINESS RESUMMATION IN GENEVA



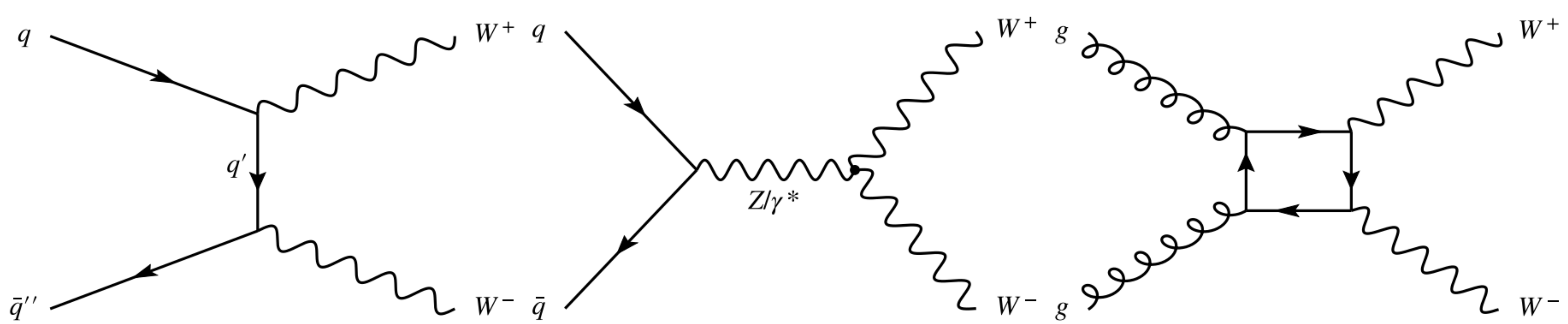
2301.11875, S. Alioli, G. Billis, A. Broggio, A. Gavardi, S. Kallweit, MAL, G. Marinelli, R. Nagar, D. Napoletano

RESOLUTION VARIABLES



GENEVA USING JET VETO RESUMMATION

- W^+W^- production an interesting case study - jet vetoes used in analyses to reject $t\bar{t}$ background
- Aim to improve description of jet-vetoed cross section within an NNLO+PS event generator
- Combine NNLL′ resummation for WW + 0 jets with NLL′ resummation for WW + 1 jet to define events at NNLO



FACTORISATION WITH A JET VETO FOR COLOUR SINGLET

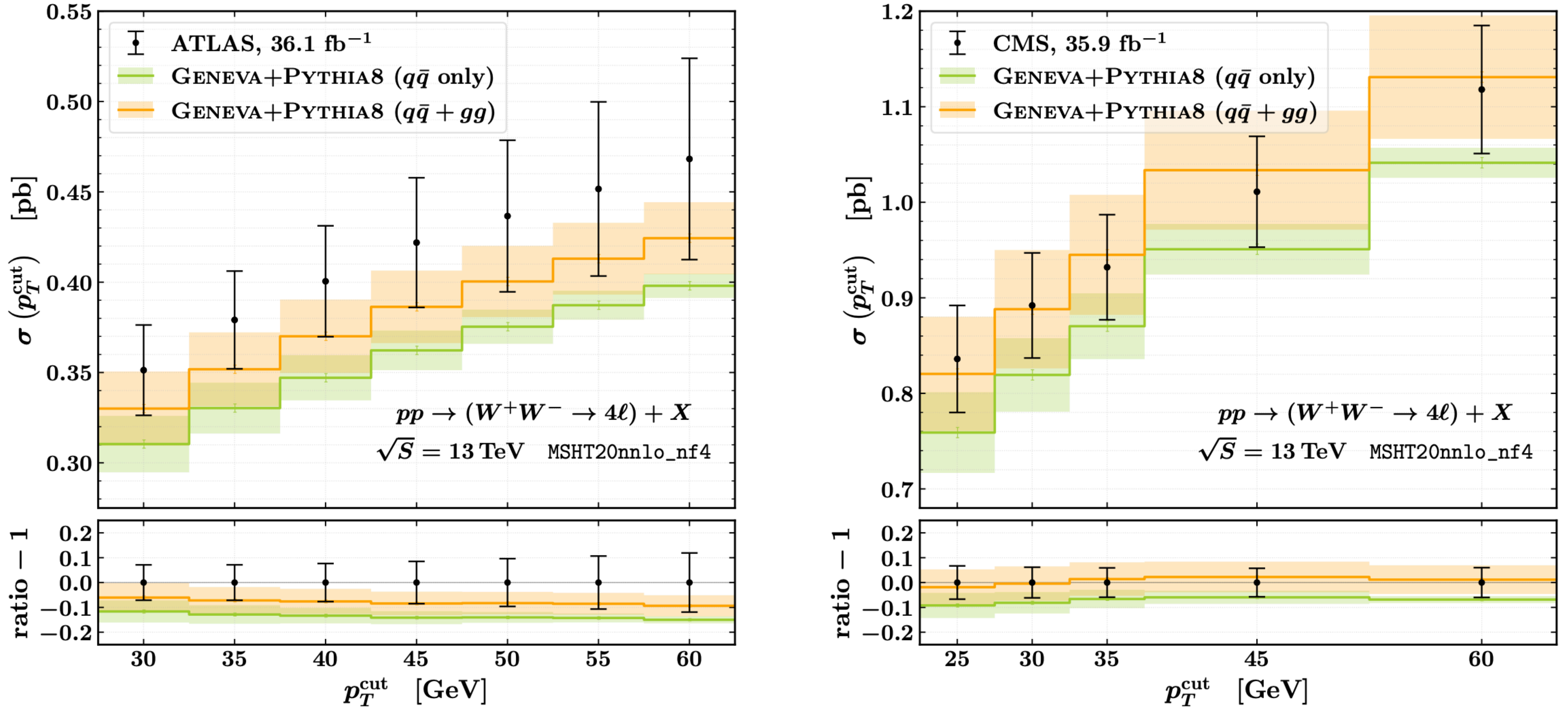
- Consider colour singlet production, vetoing all jets with $p_T > p_T^{\rm veto}$. Resummation has been studied in both QCD and SCET. T. Becher, M. Neubert, 1205.3806, F. Tackmann, J. Walsh, S. Zuberi, 1206.4312, A. Banfi, G. Salam, G. Zanderighi, 1203.5773, I. Stewart, F. Tackmann, J. Walsh, S. Zuberi, 1307.1808, T. Becher, M. Neubert, L. Rothen, 1307.0025
- Factorisation into hard, beam and soft functions

$$\frac{\mathrm{d}\sigma(p_T^{\rm veto})}{\mathrm{d}\Phi_0} = H(\Phi_0,\mu)\;[B_a \times B_b](p_T^{\rm veto},R,x_a,x_b,\mu,\nu)\;S_{ab}(p_T^{\rm veto},R,\mu,\nu)$$

- Radius of vetoed jets R
- Additional scale ν necessary to separate soft/collinear modes (SCET II)

COMPARISON TO ATLAS/CMS

- Vetoed cross section measurements



2308.11577 A. Gavardi, MAL, S. Alioli, F. Tackmann

SUMMARY

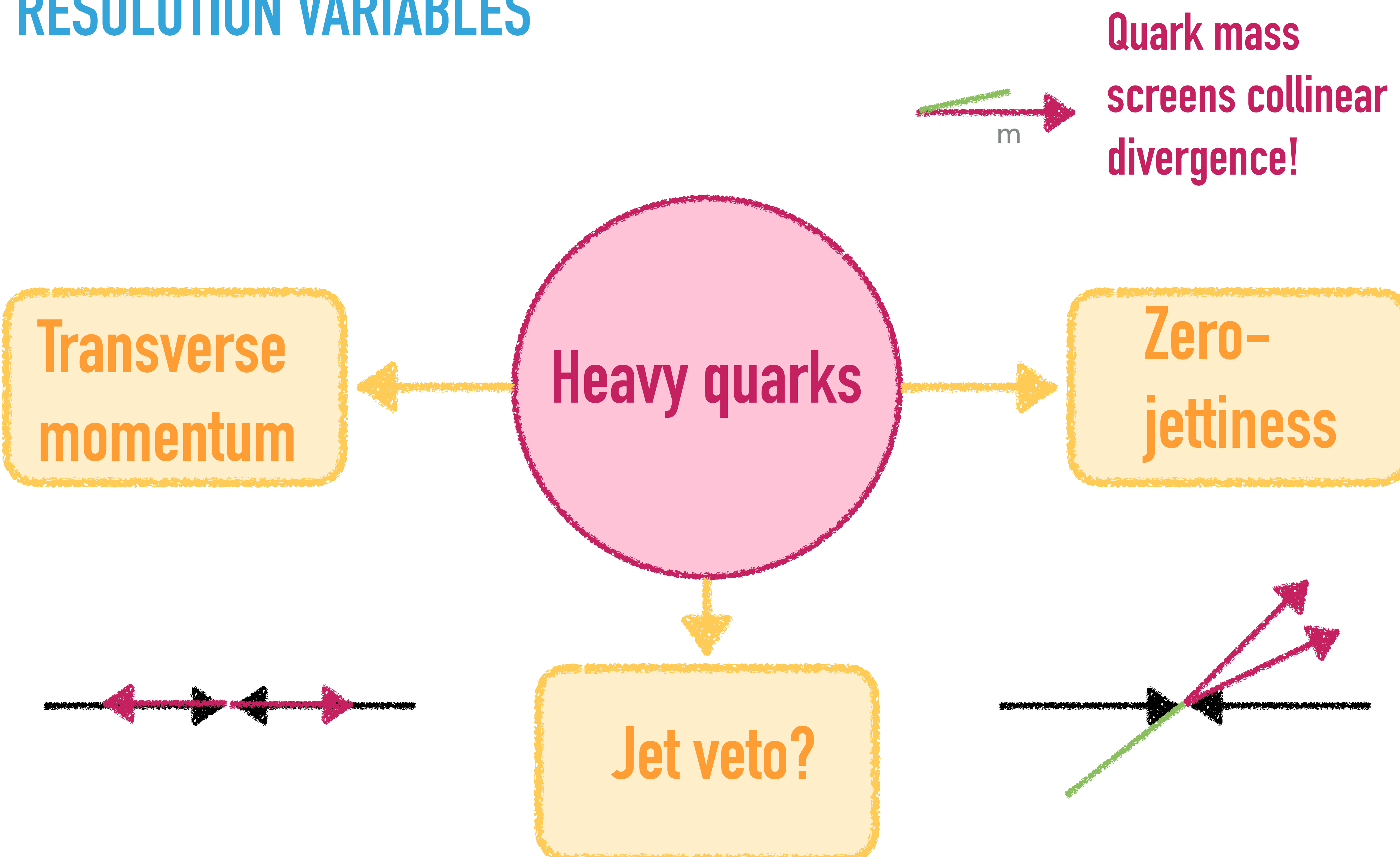
- Fixed order and parton shower calculations have different advantages - important to be able to combine them to achieve best theoretical description
- Merging combines samples with different multiplicities at FO and showers them without double counting
- Matching corrects first emissions of parton shower to be (N)NLO accurate and gives events with (N)NLO weight

SUMMARY

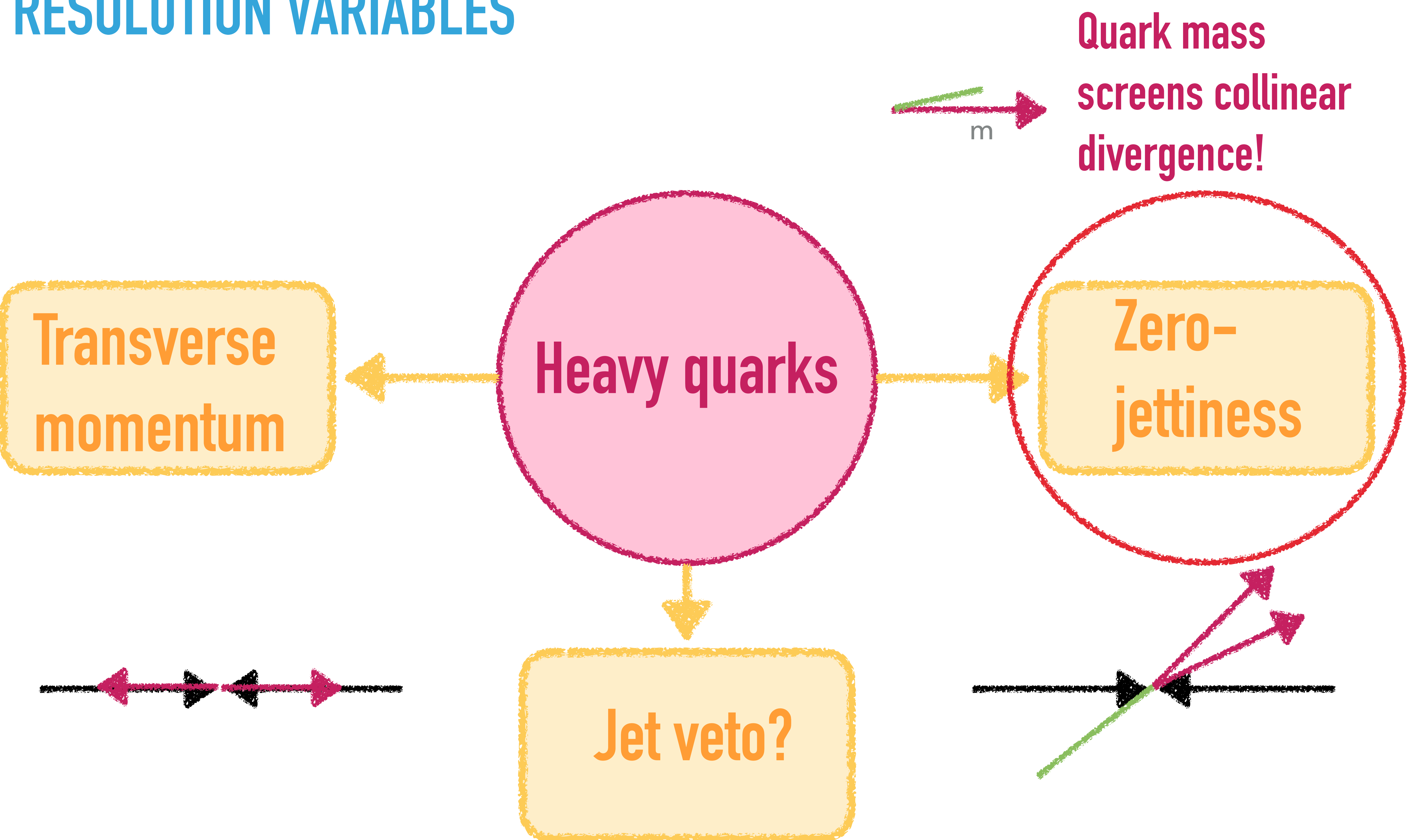
- Important not to overestimate accuracy of matched and merged samples!
- (N)NLO matching is (N)NLO for inclusive quantities - cannot get e.g. 5th jet multiplicity correct, which is only provided by parton shower
- (N)LO merged strategies are better at higher multiplicities, but must be cautious about merging scale dependence/normalisation

BACKUP SLIDES

RESOLUTION VARIABLES



RESOLUTION VARIABLES



ZERO-JETTINESS RESUMMATION FOR HEAVY QUARK PAIRS

SCET allows us to write a factorisation formula as

$$\frac{d\sigma^{\text{resum}}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \int dt_a dt_b \, B_i(t_a, x_a, \mu_B)\, B_j(t_b, x_b, \mu_B) \, \text{Tr}\left\{ \mathbf{H}_{ij}(\Phi_0, \mu_H)\, \mathbf{S}\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \Phi_0, \mu_S\right)\right\}$$

Same as before

Matrices in colour space!

Arises from exchange of soft gluons from heavy quark lines. Evolution equations more complicated:

$$\mathbf{H}(\Phi_0, \mu) = \mathbf{U}(\Phi_0, \mu, \mu_H)\mathbf{H}(\Phi_0, \mu_H)\mathbf{U}^\dagger(\Phi_0, \mu, \mu_H)$$

2111.03632, S. Alioli, A. Broggio, MAL

ZERO-JETTINESS RESUMMATION FOR HEAVY QUARK PAIRS

Derived for the first time! Ingredients partially unknown.

$$\frac{d\sigma^{\mathrm{resum}}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \int dt_a dt_b \, B_i(t_a, x_a, \mu_B) \, B_j(t_b, x_b, \mu_B) \, \mathrm{Tr} \left\{ \mathbf{H}_{ij}(\Phi_0, \mu_H) \, \mathbf{S} \left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \Phi_0, \mu_S \right) \right\}$$

Known up to 3-loops

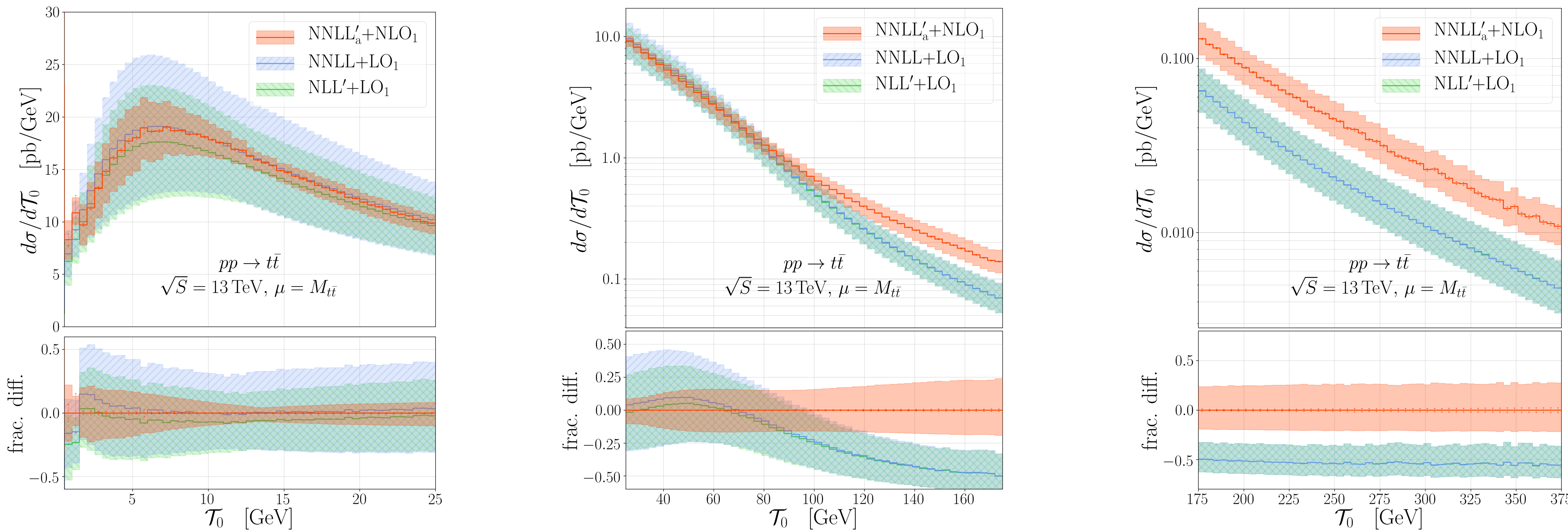
Known up to 2-loops (in principle)

Unknown!

We computed the soft function up to 1-loop. Some 2-loop terms can be obtained via RGE.

2111.03632, S. Alioli, A. Broggio, MAL

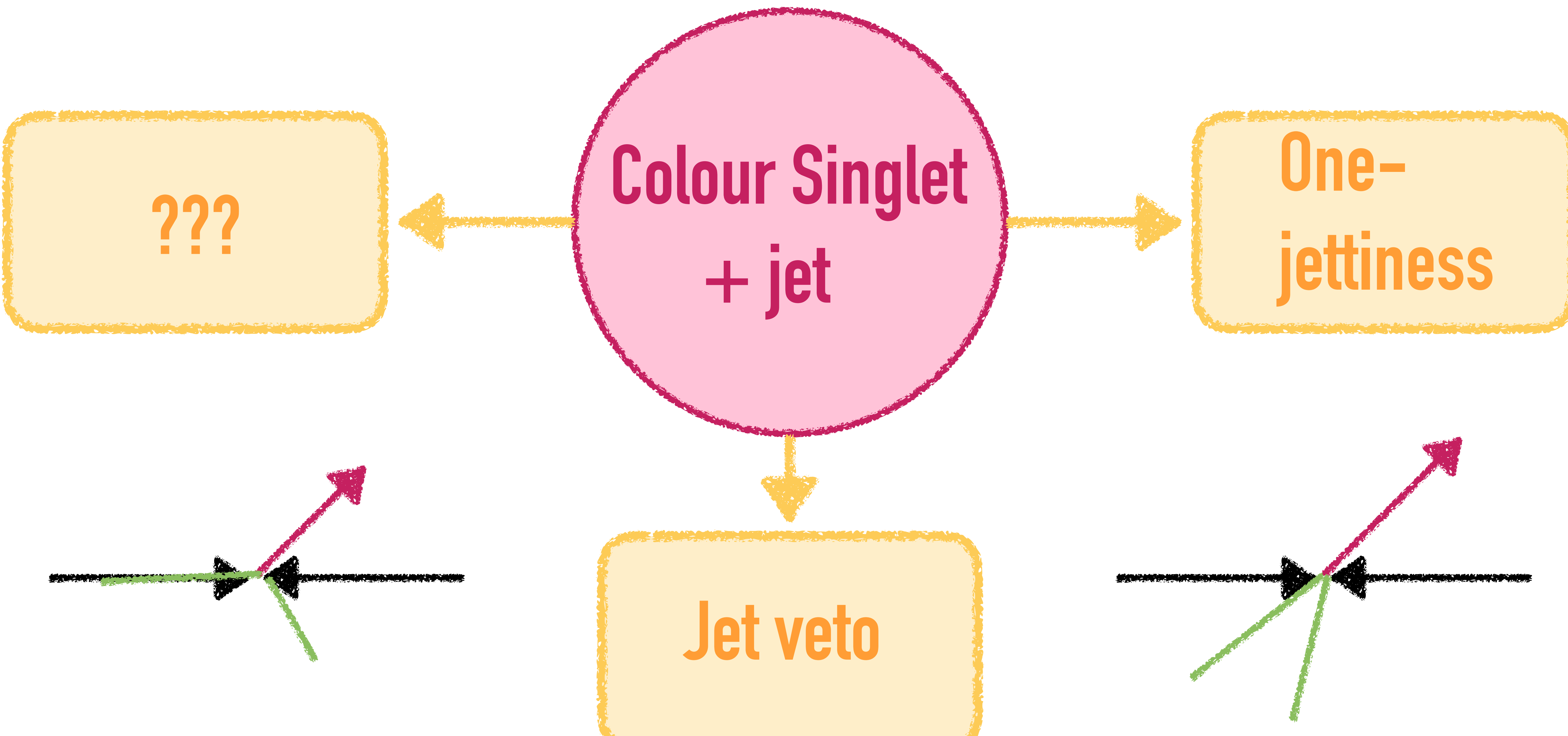
ZERO-JETTINESS RESUMMATION FOR TOP-QUARK PAIRS



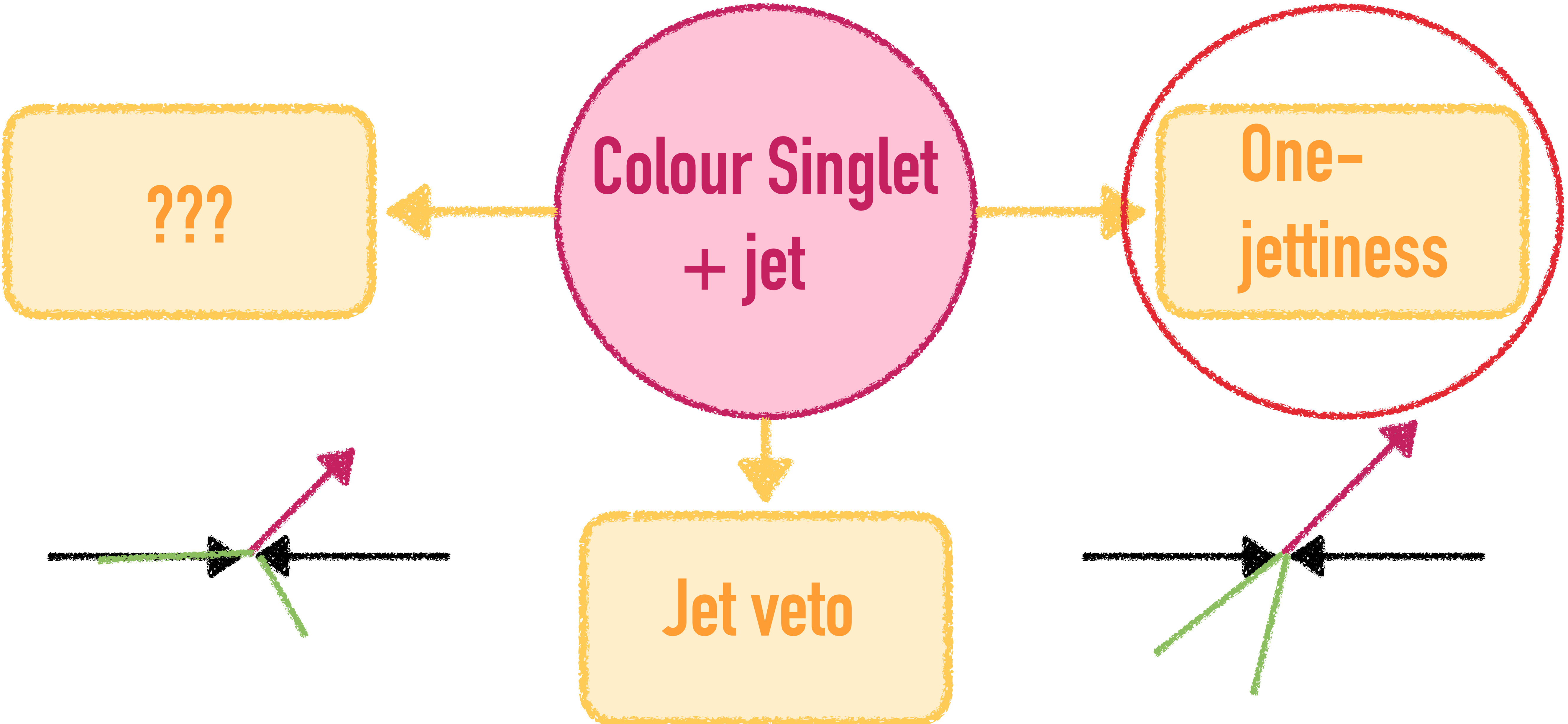
- Still missing - two-loop hard (not included here) and one piece of the two-loop soft.
- Allows approximate NNLL' accuracy.

2111.03632, S. Alioli, A. Broggio, MAL

RESOLUTION VARIABLES



RESOLUTION VARIABLES



ONE-JETTINESS RESUMMATION FOR COLOUR SINGLET + JET

Similar factorisation to zero-jet case:

$$\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{ijk} \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J \;\; B_i(t_a, x_a, \mu_B)\, B_j(t_b, x_b, \mu_B)\, J_k(s_J, \mu_J)\, \mathrm{Tr}\left\{ \mathbf{H}_{ij}(\Phi_1, \mu_H)\;\; \mathbf{S}\left(\mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}, \Phi_1, \mu_S \right) \right\}$$

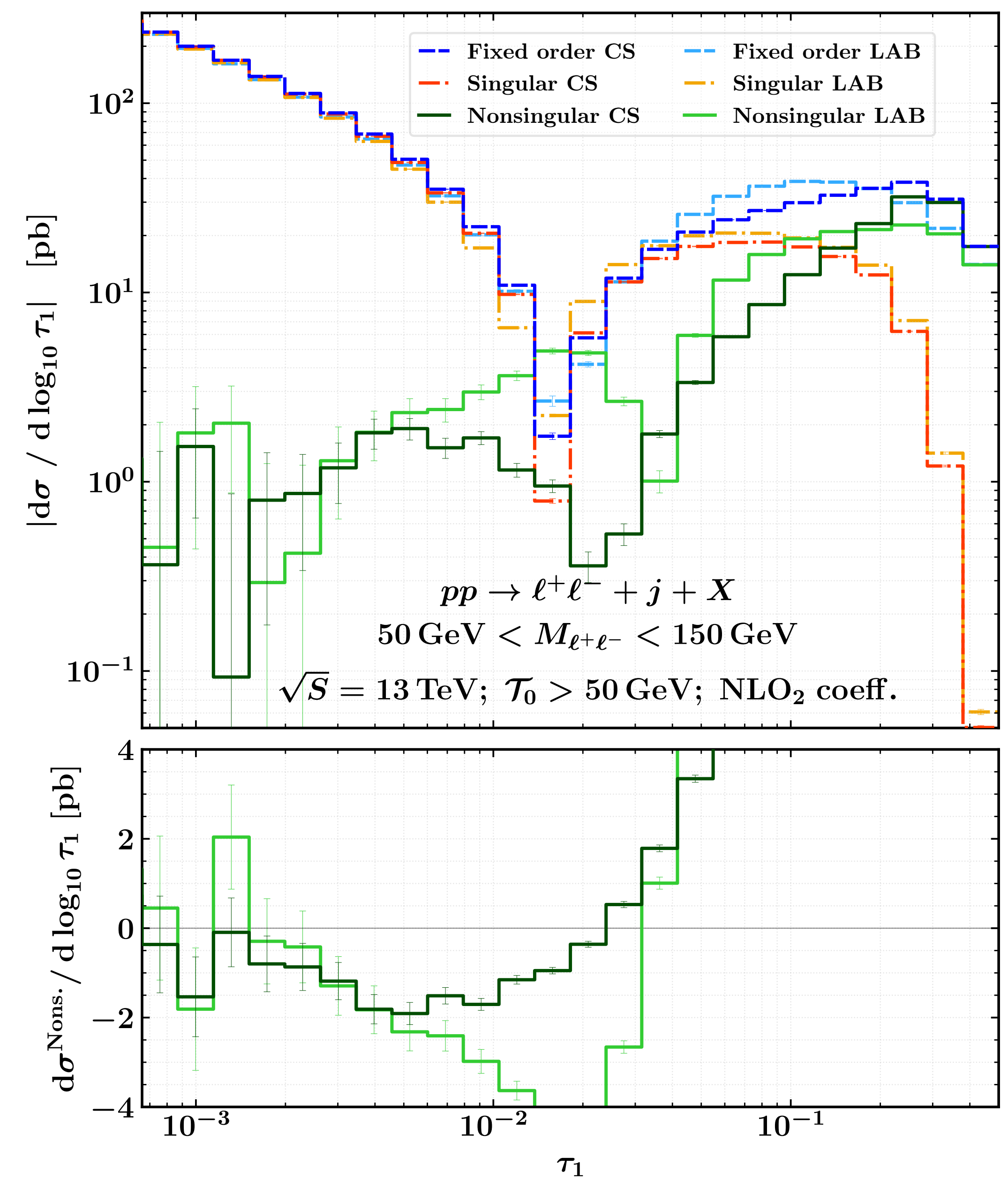
New jet function

- Only three coloured legs - colour algebra is diagonal
- Ingredients for N^{3}LL all known, we use new numerical of two-loop soft function from SoftSERVE
- One-jettiness definition requires choice of frame - can evaluate energies in lab or in CS centre-of-mass

0910.0467, 1302.0846, T. Jouttenus, I. Stewart, F. Tackmann, W. Waalewijn

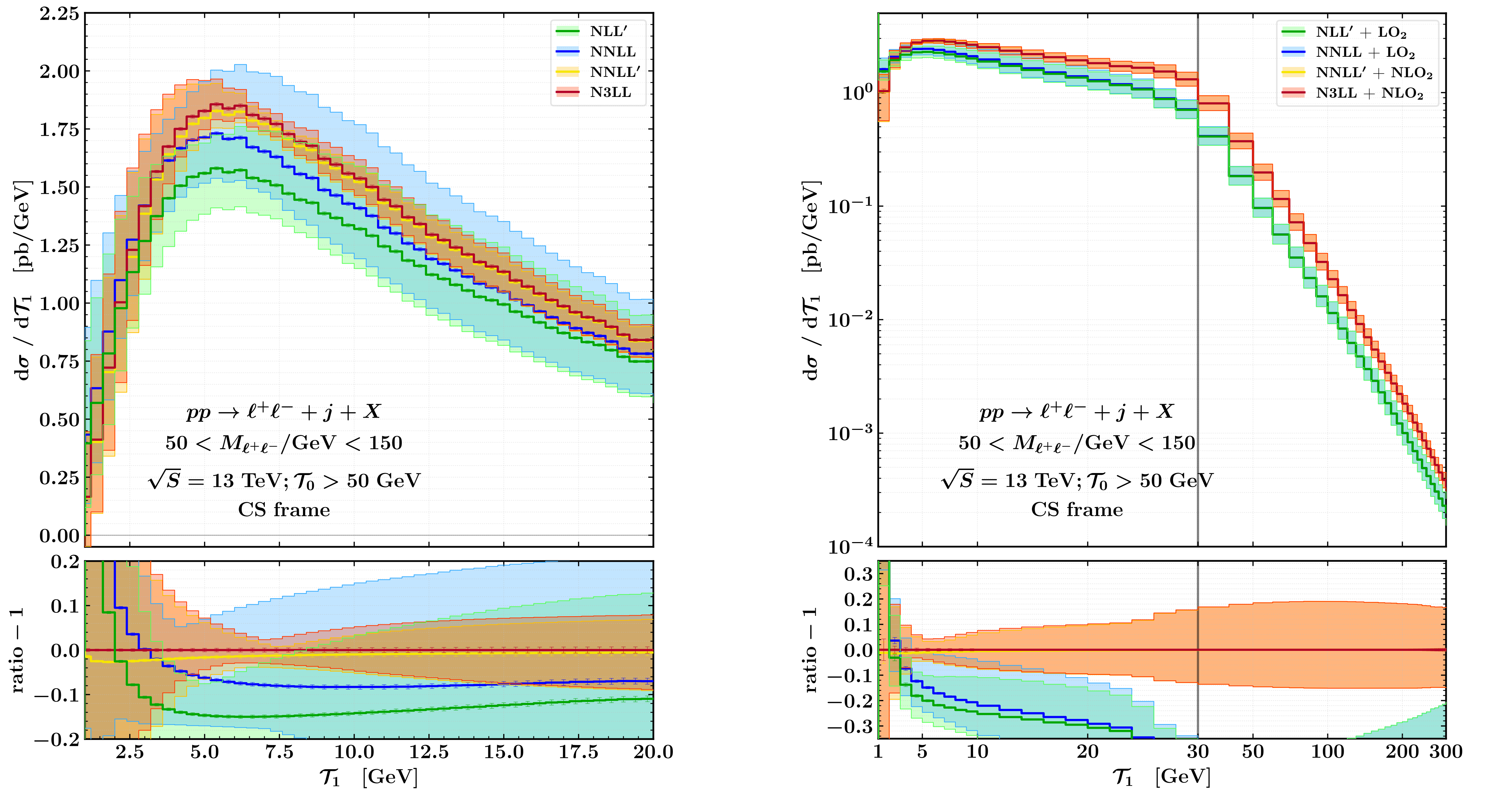
FIXED-ORDER VALIDATION OF ONE-JETTINESS FACTORISATION

- Factorisation theorem must reproduce result of fixed order in the small $\tau_1 = \mathscr{T}_1/Q$ limit
- Size of nonsingular difference has implications for numerical accuracy of slicing calculations



2312.06496, S. Alioli, G. Bell, G. Billis, A. Broggio, B. Dehnadi, MAL, G. Marinelli, R. Nagar, D. Napoletano, R. Rahn

RESUMMED AND MATCHED ONE-JETTINESS SPECTRA



2312.06496, S. Alioli, G. Bell, G. Billis, A. Broggio, B. Dehnadi, MAL, G. Marinelli, R. Nagar, D. Napoletano, R. Rahn