UNIVERSITÄT
BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

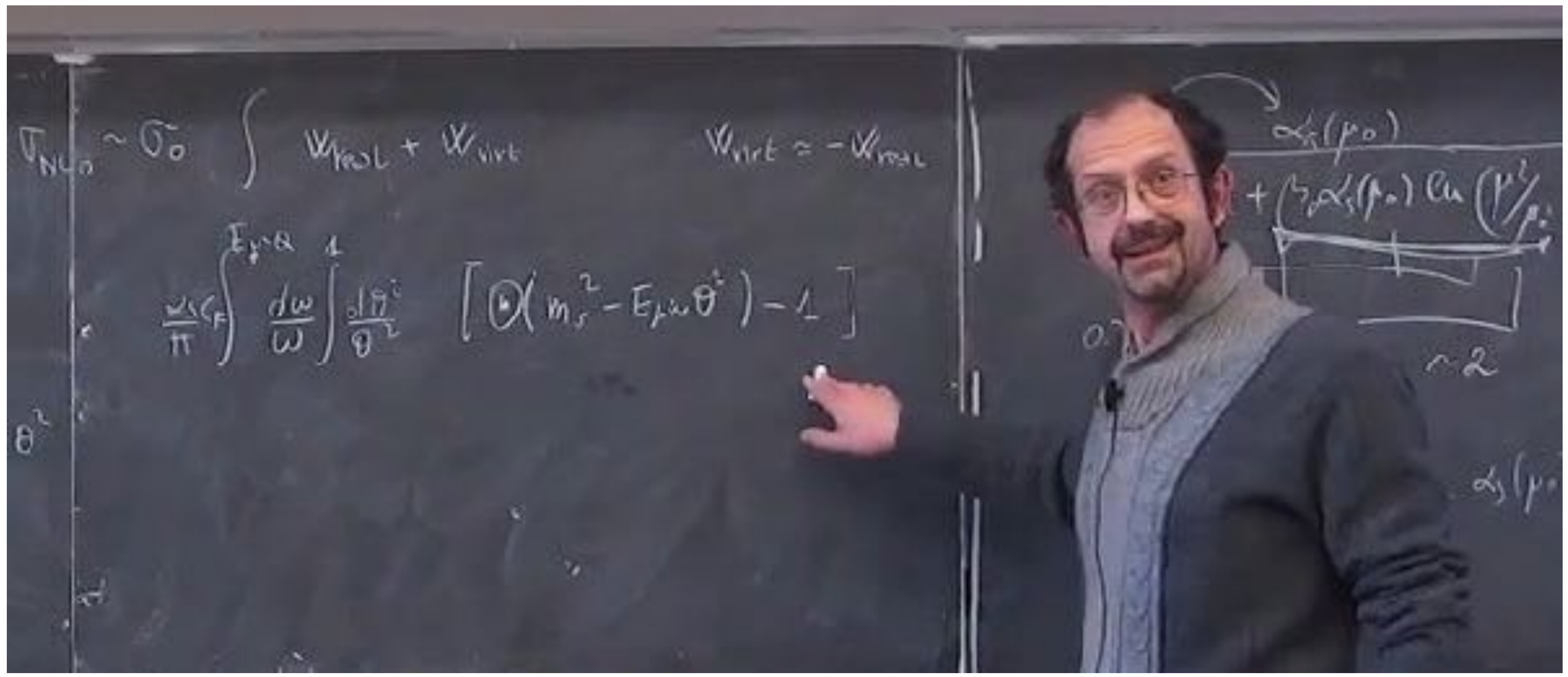
Resummation

Thomas Becher

University of Bern

Terascale Monte Carlo School 2024, Feb. 19-23, DESY

In memory of Stefano Catani



Pioneer of perturbative QCD and resummation, with many seminal contributions: soft-gluon resummation, jets (k_T algorithm), IR singularities (Catani formula), algorithms for fixed-order computations (Catani-Seymour, q_T subtraction) and merging (CKKW), small-x resummation (CCFM, CH)…

In processes involving disparate scales $Q \gg Q_0$, higher-order corrections are enhanced by large logarithms

$$\alpha_s^n \ln^m Q/Q_0$$

which can spoil perturbative expansion. Maximum power of logarithms depends on problem

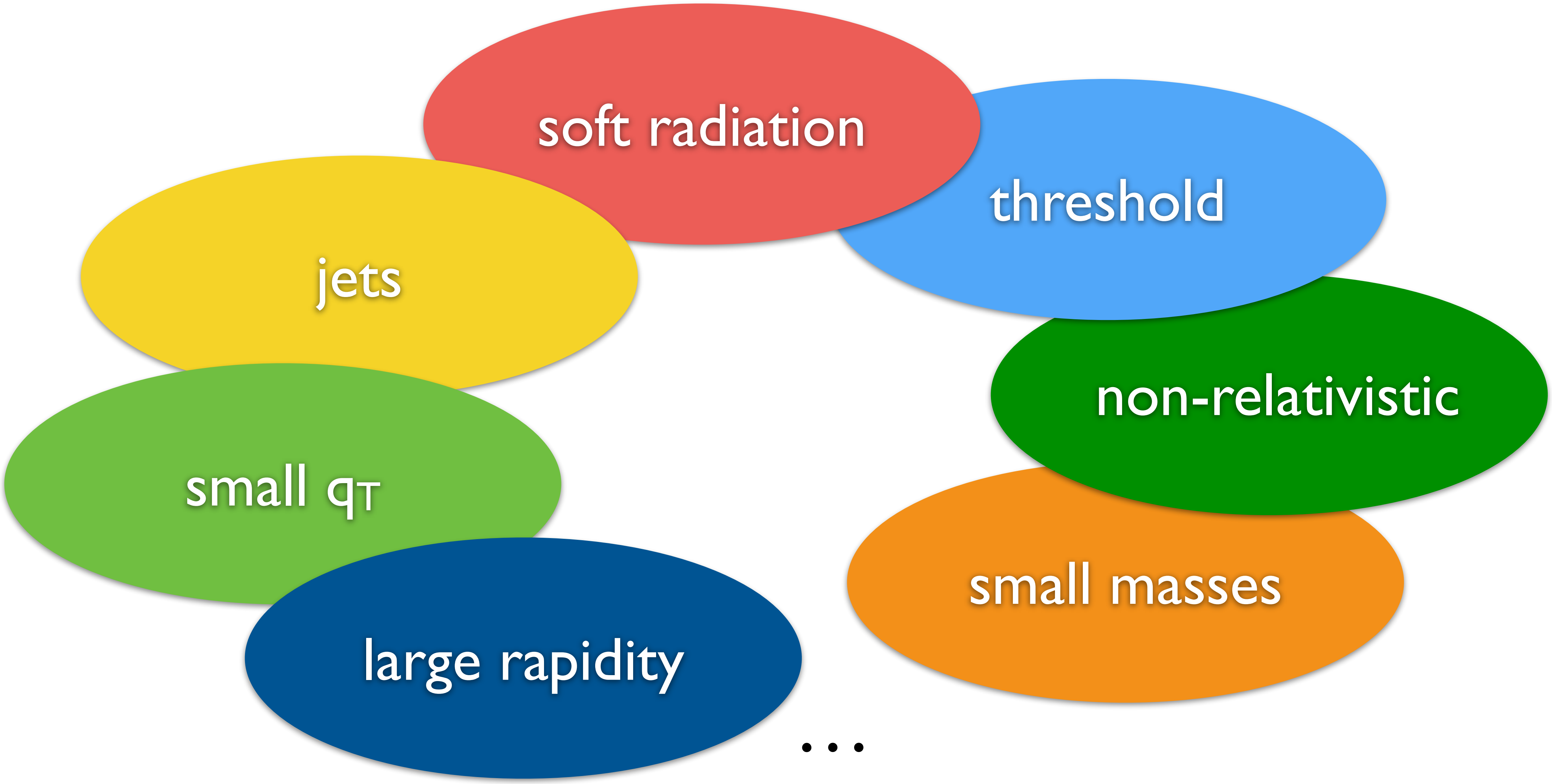
- Single logarithmic: $m \leqq n$
- Sudakov (soft + collinear): $m \leqq 2n$

Resum enhanced contributions to all orders.

- Count $\ln(Q/Q_0) \sim 1/\alpha_s$
- Systematic expansion: LL, NLL, NNLL, …

Resummation techniques

- diagrammatic methods, factorization theorems, evolution equations (direct QCD)
- parton showers MCs
- Soft-Collinear Effective field Theory (SCET)
  - integrate out physics at high scale
  - renormalization group evolution to resum logarithms



Many types of scale hierarchies, many different types of resummations … and by now many different EFTs

Today's lecture

- Detailed discussion of the q_T spectrum in Drell-Yan production
  - motivation: M_W and α_s determination
  - basics
    - break down of fixed-order prediction at low q_T
    - counting of large logarithms
    - exponentiation and resummation
  - organization of the resummed result
    - SCET versus direct QCD
    - uncertainty estimate
    - switching off and matching
  - factorization theorem
  - $\mathrm{N}^n\mathrm{LL}$ results

Tomorrow's lecture

- QCD made simpler: the physics of soft and collinear emissions
    - factorization of soft and collinear emissions
- Jet physics and soft emissions
    - Non-global logarithms
    - Superleading logarithms

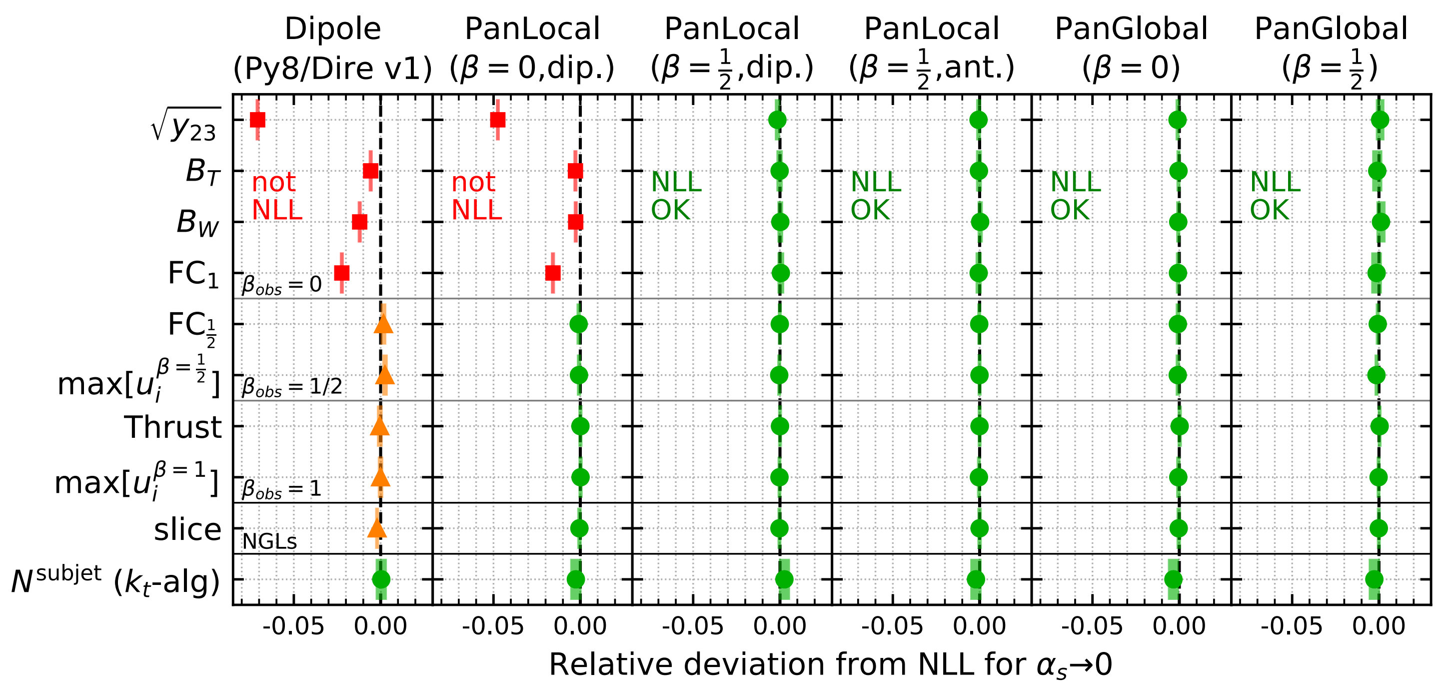

(a slide from MC school in 2015)
fixed order
Parton shower MC's
Higher-log resummation?

In the past, not too much cross talk between parton shower MCs and resummation

- analytical
- simple observables
- $\mathbf{N}^n\mathbf{LL}$ accuracy (by now up to n=4!)
- exact color
- non-perturbative matrix elements, fits

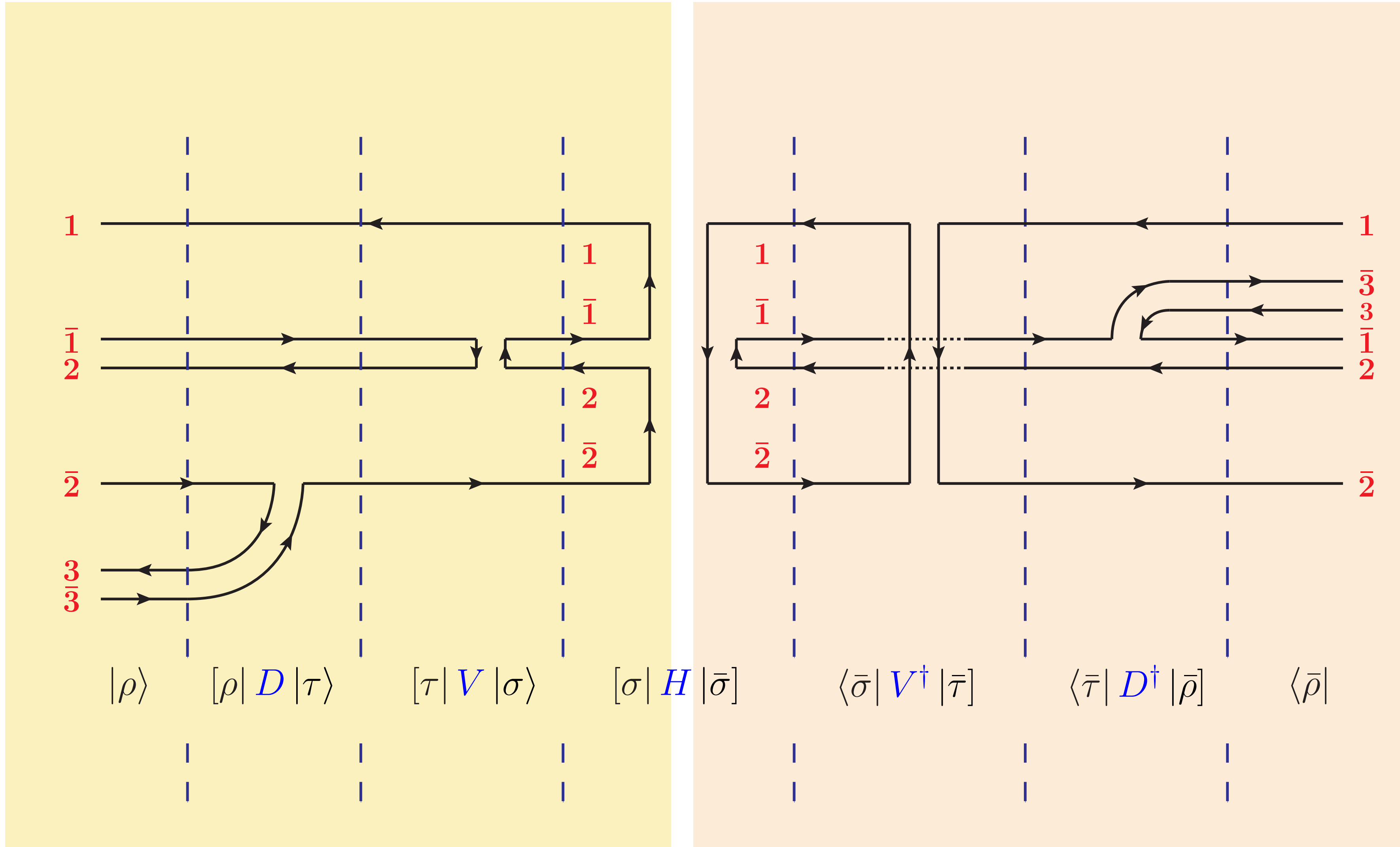
- numerical
- fully general
- $\mathbf{LL}$ + many subleading effects + tuning
- large-N_c limit + some
- hadronization models

The situation has changed!



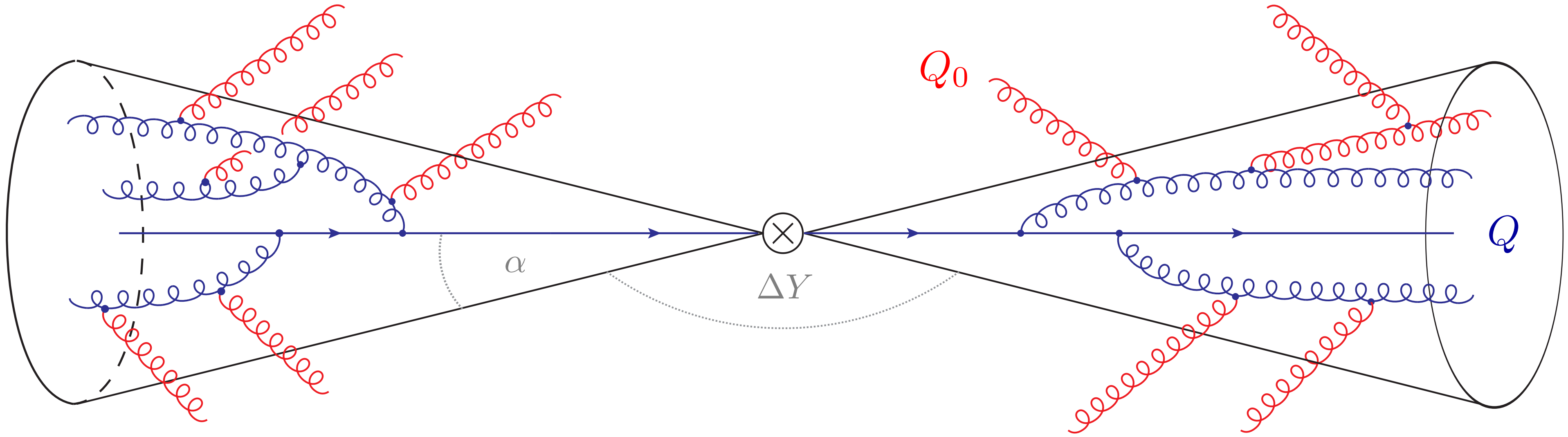
Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez '20

Development of parton showers which systematically include higher-log effects Deductor, PanScales, Alaric, … → Melissa's lecture



De Angelis, Forshaw, Plätzer ‘21

Amplitude evolution, development of full color showers. Deductor Nagy, Soper, CVolver Plätzer, Sjodahl, De Angelis, Forshaw, Holguin, …



Resummation of subleading soft logarithms in jet processes using MC method: `Gnole` Banfi, Dreyer, Monni '21 SCET framework TB, Schalch, Xu, '23 …

… and same result from PanScales MC Ferrario Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez '23.

Numerical agreement among the three approaches.

more in Friday's lecture

Anatomy of a resummed computation

Transverse momentum resummation in the Drell-Yan process

I first want to explain terminology such as

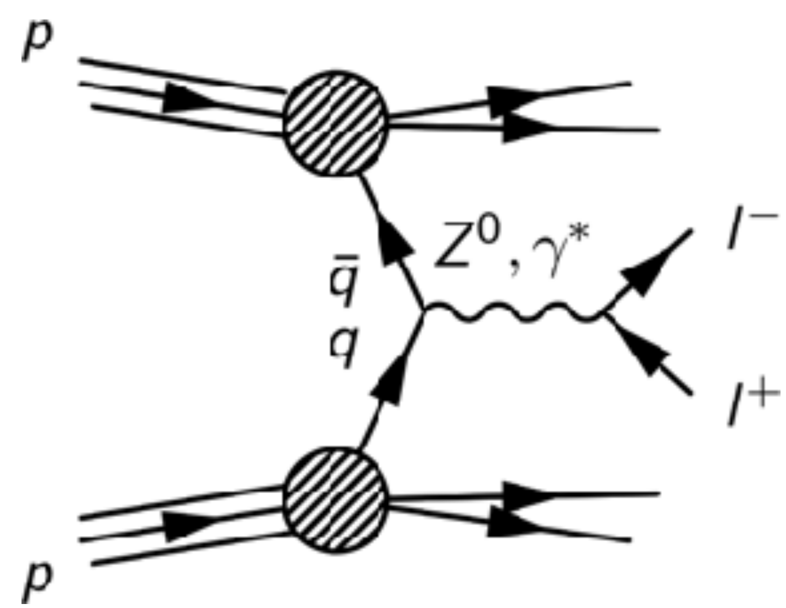
resummation

next-to-next-to-leading-logarithmic accuracy (NNLL)

To do so, I will use a classic example, the q_T spectrum in the Drell-Yan process

Will discuss the structure of resummed results and the associated theoretical uncertainties.

Drell-Yan Processes

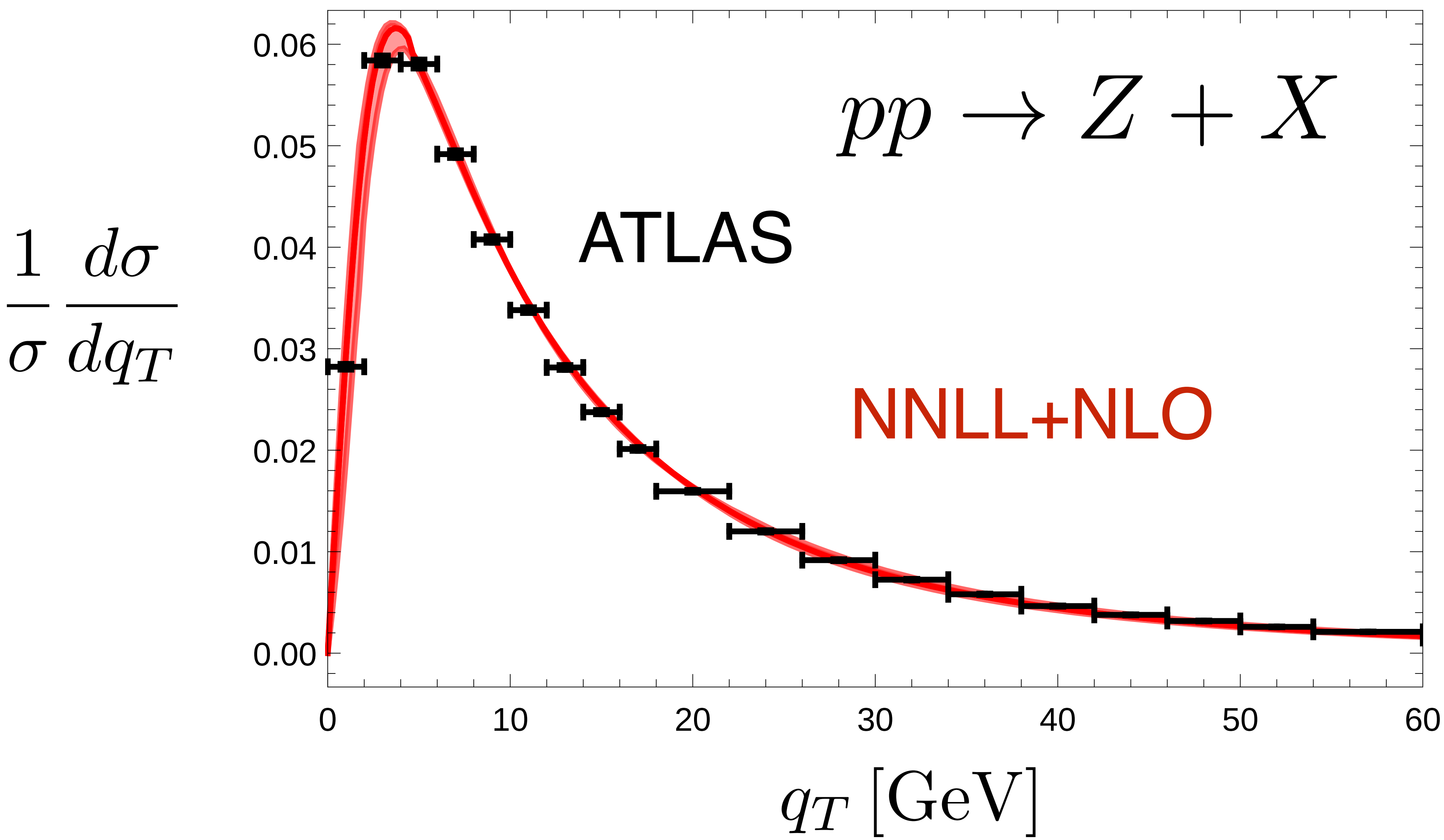


- Production of one or more electroweak bosons (*W*, *Z*, *γ* or H), together with arbitrary hadronic final state *X*.

- Leptonic decays of the weak bosons, only leptonic measurements.

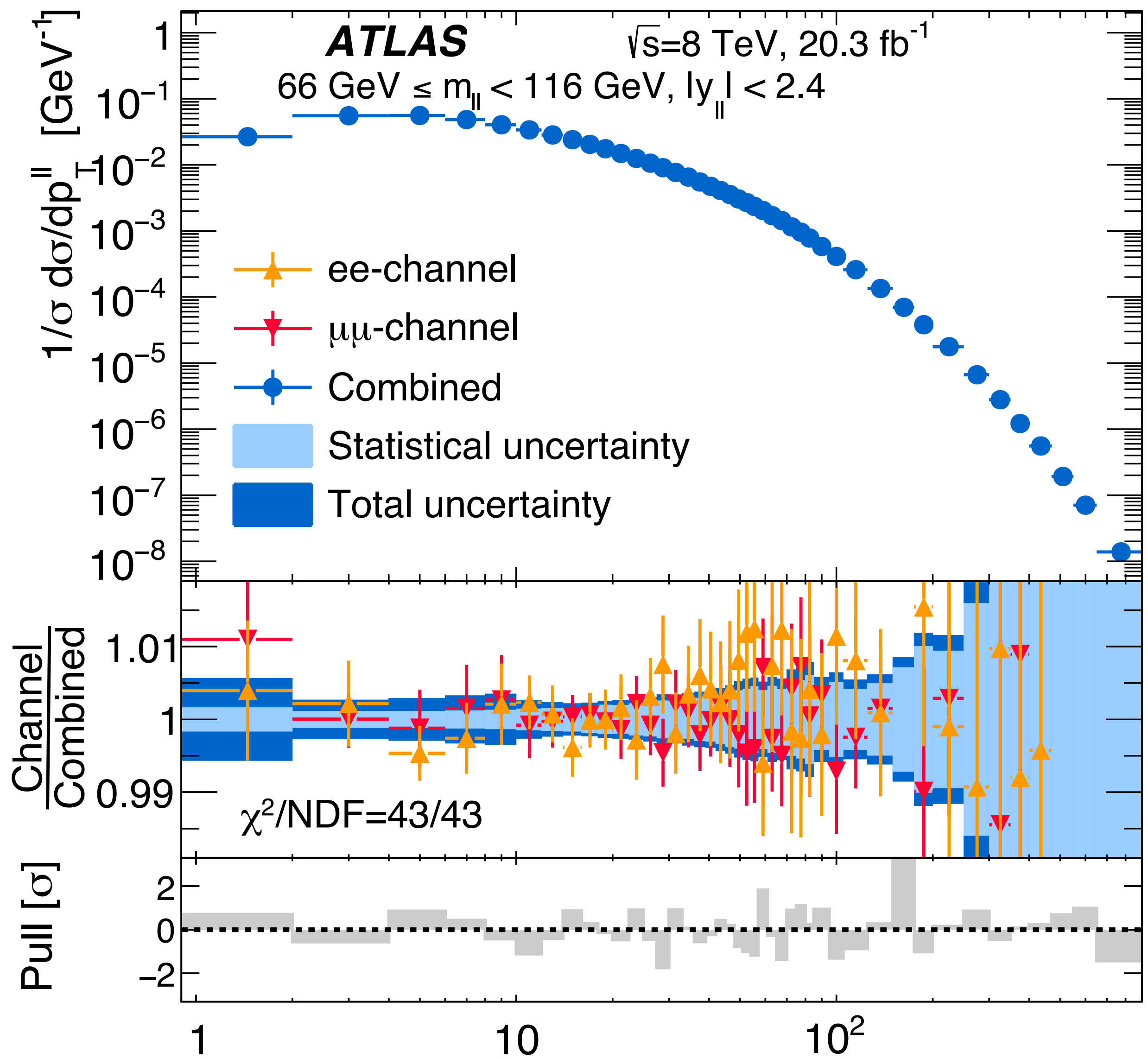
- Simplest hard process at hadron colliders. Precision results from the LHC, even for multi-boson final states.

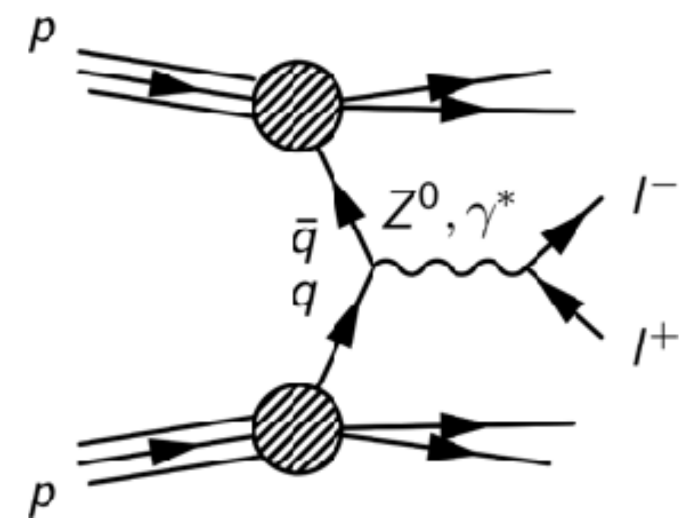
q_T spectrum of Z-bosons



- q_T is the transverse momentum of the lepton pair!
- Experimental uncertainties invisible on this plot!

A precision measurement at the LHC



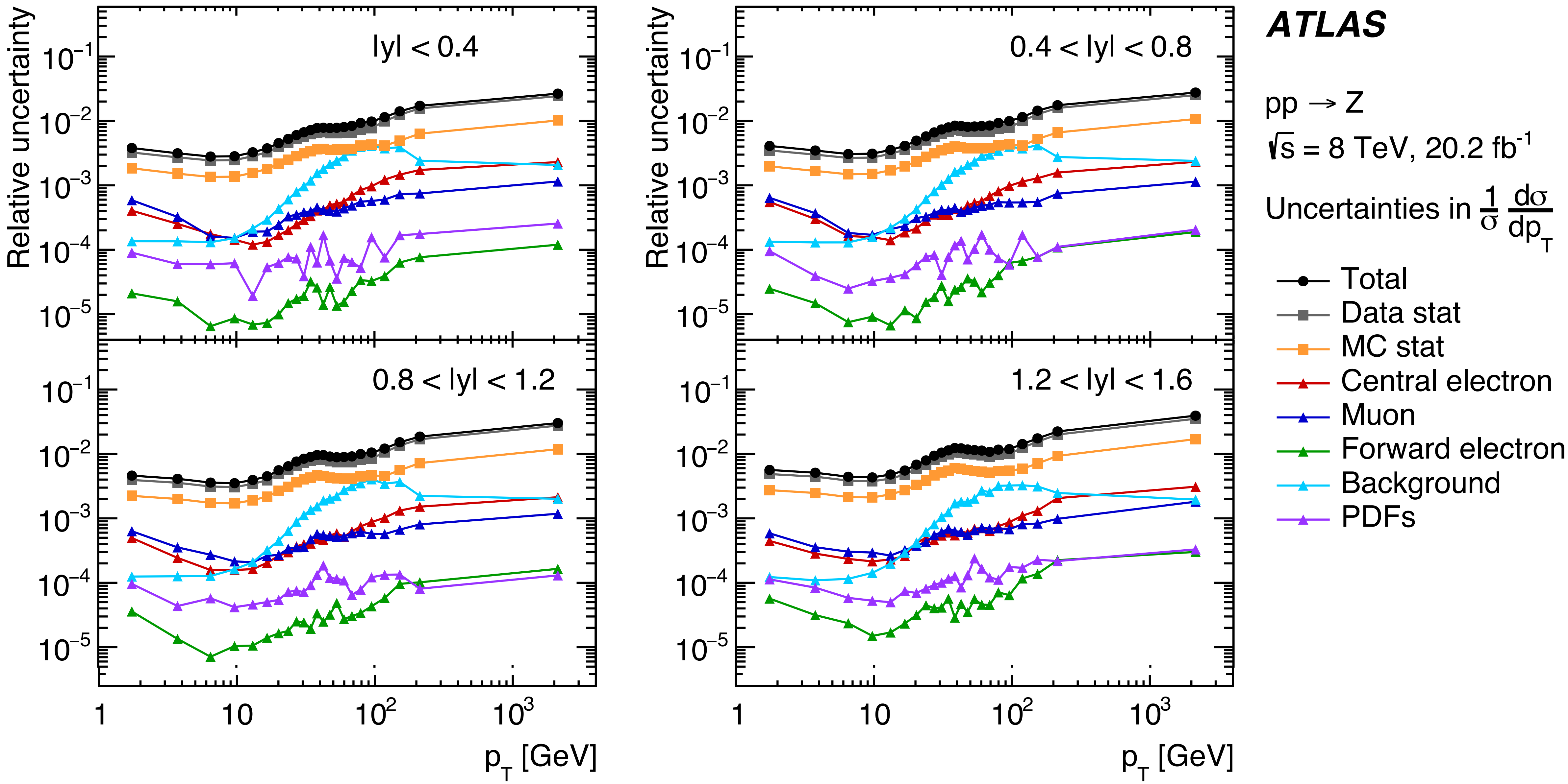


Sub-percent accuracy
over large range of energies and
many orders of cross section!

p_T^{ll} [GeV] $= q_T$, the transverse momentum of the lepton pair

ATLAS, Eur. Phys. J. C 76 (2016) 291

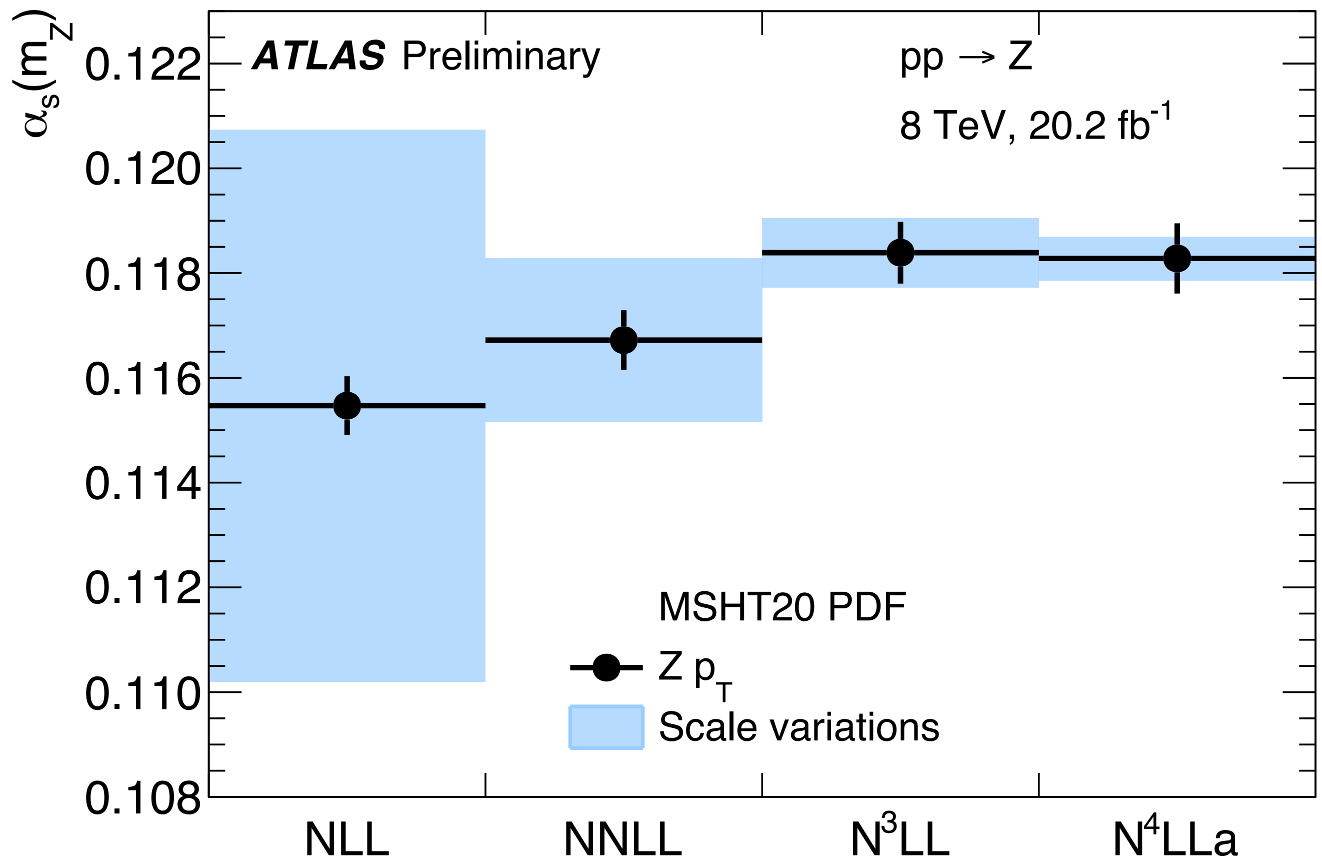
Potential for precision determinations of SM parameters but huge challenge for theory!



2309.09318

Tiny experimental uncertainties!

Strong coupling constant from q_T spectrum



2309.12986

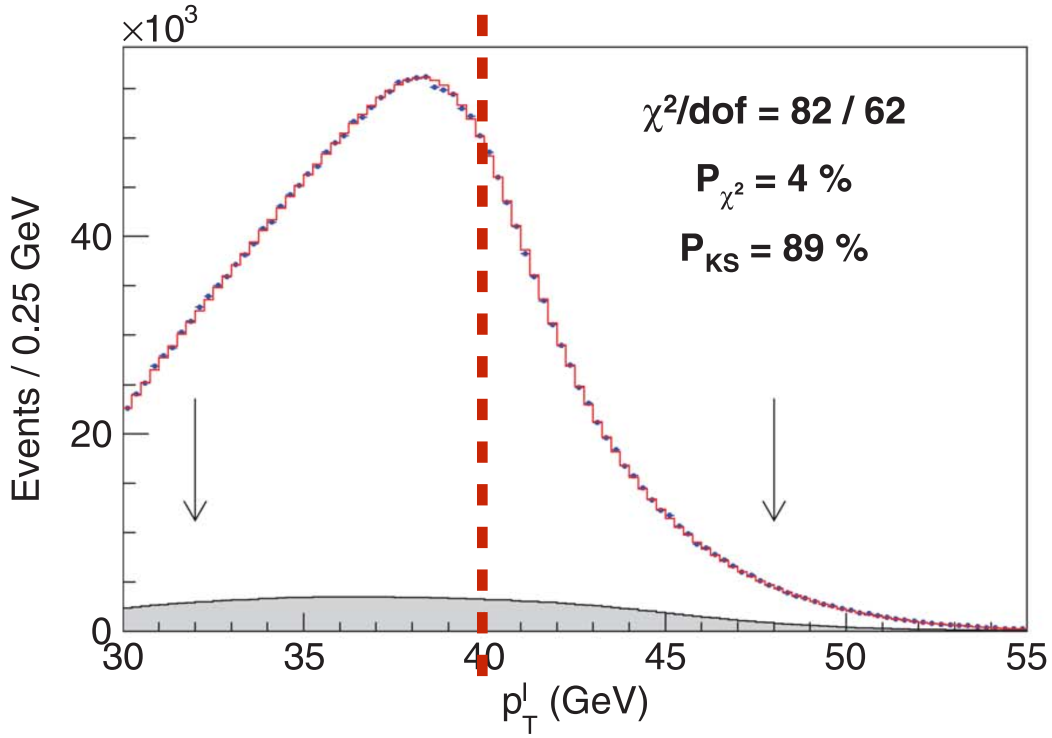
higher precision of resummation

Precise determination of strong coupling constant:

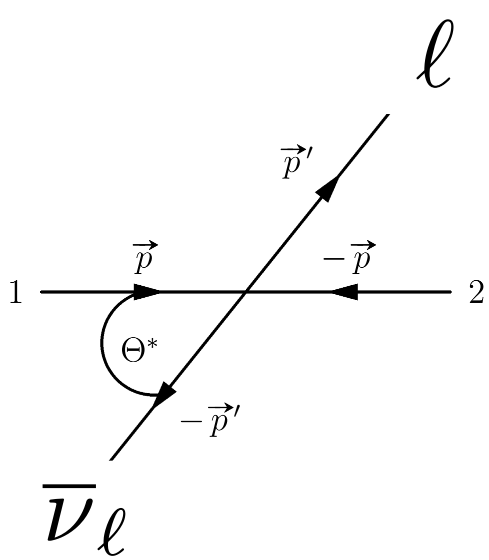
$$\alpha_s(m_Z) = 0.1183 \pm 0.0009$$

m_W from W-production

CDF et al., Science 376, 170–176 (2022)



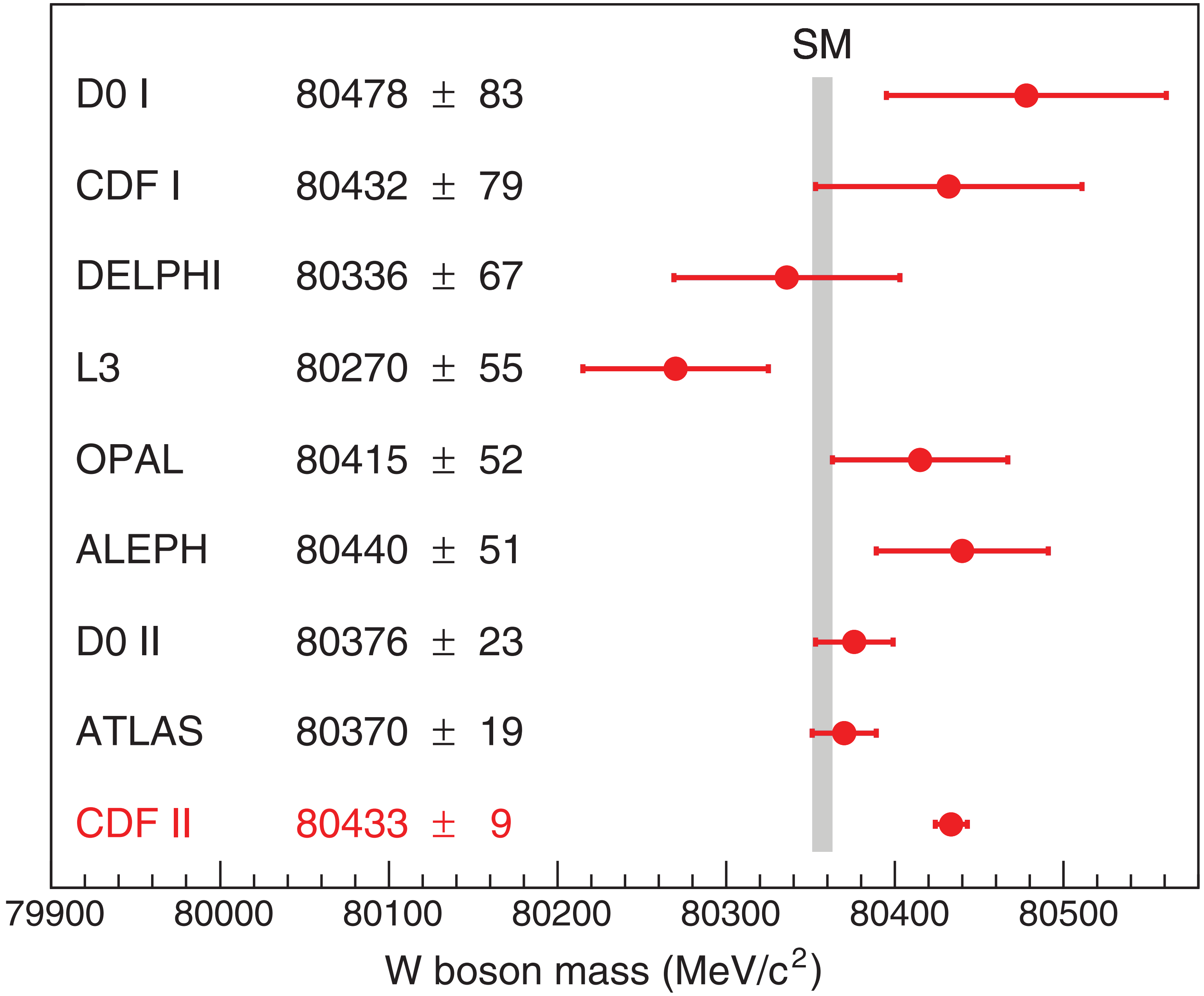
LO: $p_T{}^{\ell} < M_W/2$



- Neutrino transverse momentum indirectly through

$$\vec{p}_T^{\,\nu} = -\vec{p}_T^{\,\ell} - \vec{p}_T^{\,X}$$

- m_W from template fits to $p_T{}^{\ell}$, $p_T{}^{\nu}$, and m_T.



CDF et al., Science 376, 170–176 (2022)

$$M_W = 80{,}433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}} = 80{,}433.5 \pm 9.4\ \text{MeV}/c^2$$

7 σ difference of CDF value to the world average!

Discussion of theoretical systematics in LHC precision measurements

26 Feb 2024, 09:00 → 27 Feb 2024, 18:45 Europe/Zurich

4/3-006 - TH Conference Room (CERN)

Description The Workshop will review the theory challenges opened by recent high-precision measurements at the LHC of m_W and α_S, where theoretical systematics plays a critical role in setting the ultimate precision that the experiments can achieve.

Videoconference LHC precision measurements discussion — Join

Registration Registration form — Register

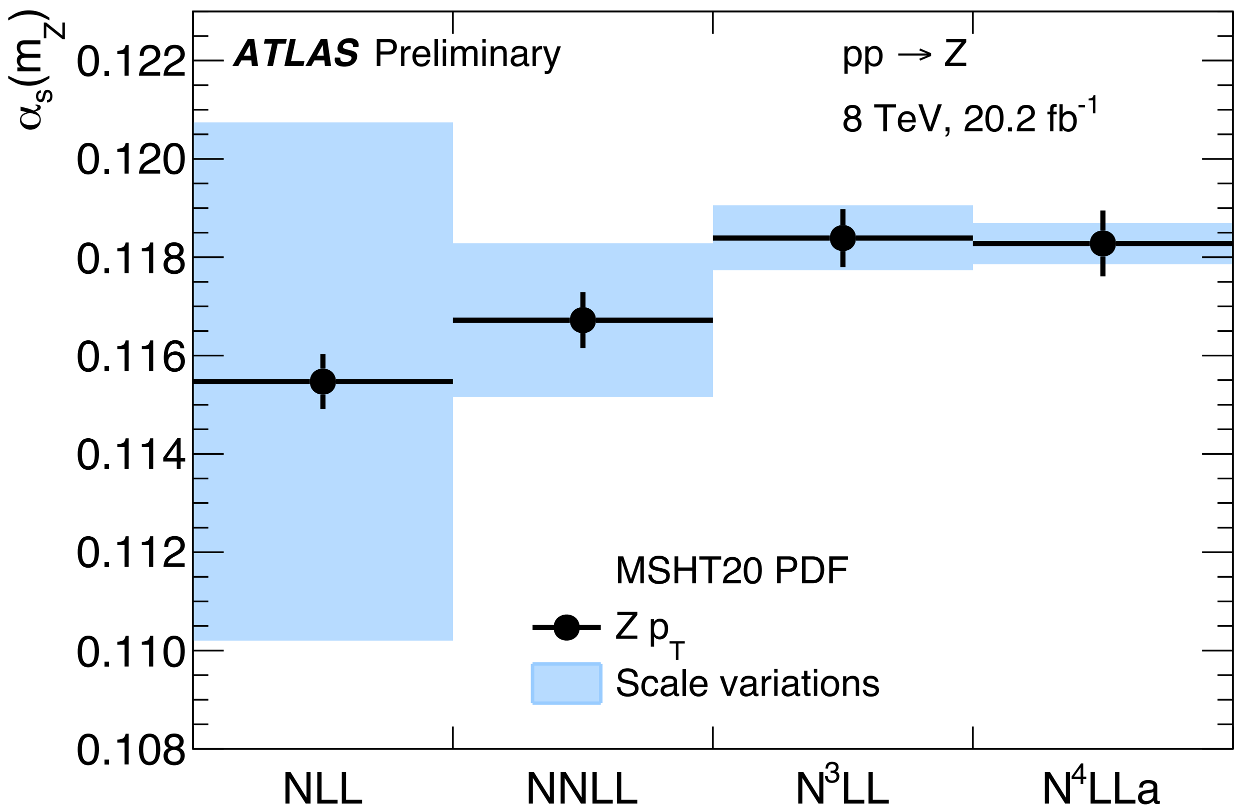
Participants Alessandro Bacchetta, Alessandro Tricoli, Alexander Yohei Huss, Aman Desai, Andrea Piccinelli, Bogdan Malaescu, Costas Vellidis, Daniel Britzger, Daniel Froidevaux, David d'Enterria

Contact thworkshops.secretariat@cern.ch

Monday, 26 February

09:00 → 09:05	**Introduction**		5m
09:05 → 09:35	**Overview of LHC EW WG1 studies in the context of precision measurements**		30m
09:35 → 10:15	**Fixed-order predictions for leptonic observables (QCD, EW, PDFs).**	**Speaker:** Dr Xuan Chen (Shandong University)	40m
10:15 → 10:55	**Resummation and PDF uncertainties for leptonic observables**	**Speaker:** Tobias Neumann (Illinois Tech / Fermi National Accelerator Laboratory)	40m
10:55 → 11:20	**Coffee break**		25m

11:20 → 11:50	**Non-perturbative aspects**	Speakers: Prof. Alessandro Bacchetta, Giuseppe Bozzi (University of Cagliari and INFN, Cagliari), Dr Valerio Bertone (C.E.A. Paris-Saclay)	30m
11:50 → 12:20	**Propagation of scale uncertainties to ptlep templates after tuning to ptZ**	Speaker: Paolo Torrielli (Universita e INFN Torino (IT))	30m
12:20 → 13:00	**A perspective on TH uncertainties**	Speaker: Frank Tackmann	40m
13:00 → 14:00	**Lunch break**		1h
14:00 → 14:30	**Statistical interpretation of TH uncertainties**	Speaker: Alexander Yohei Huss (CERN)	30m
14:30 → 15:10	**Theory uncertainties in the LHCb MW measurement**	Speaker: Mika Anton Vesterinen (University of Warwick (GB))	40m
15:10 → 15:50	**Theory uncertainties in the alphaS measurement from ptZ**	Speaker: Stefano Camarda (CERN)	40m
15:50 → 16:20	**Coffee break**		30m
16:20 → 17:00	**Propagation of TH uncertainties in data driven approaches**	Speaker: Maarten Boonekamp (Université Paris-Saclay (FR))	40m
17:00 → 17:30	**Profiling of PDF uncertainties**	Speaker: Simone Amoroso (Deutsches Elektronen-Synchrotron (DE))	30m



Important to have precise theoretical control over the transverse momentum spectra.

Let’s compute them!

from: QCD and Collider Physics, Ellis, Sterling, Webber

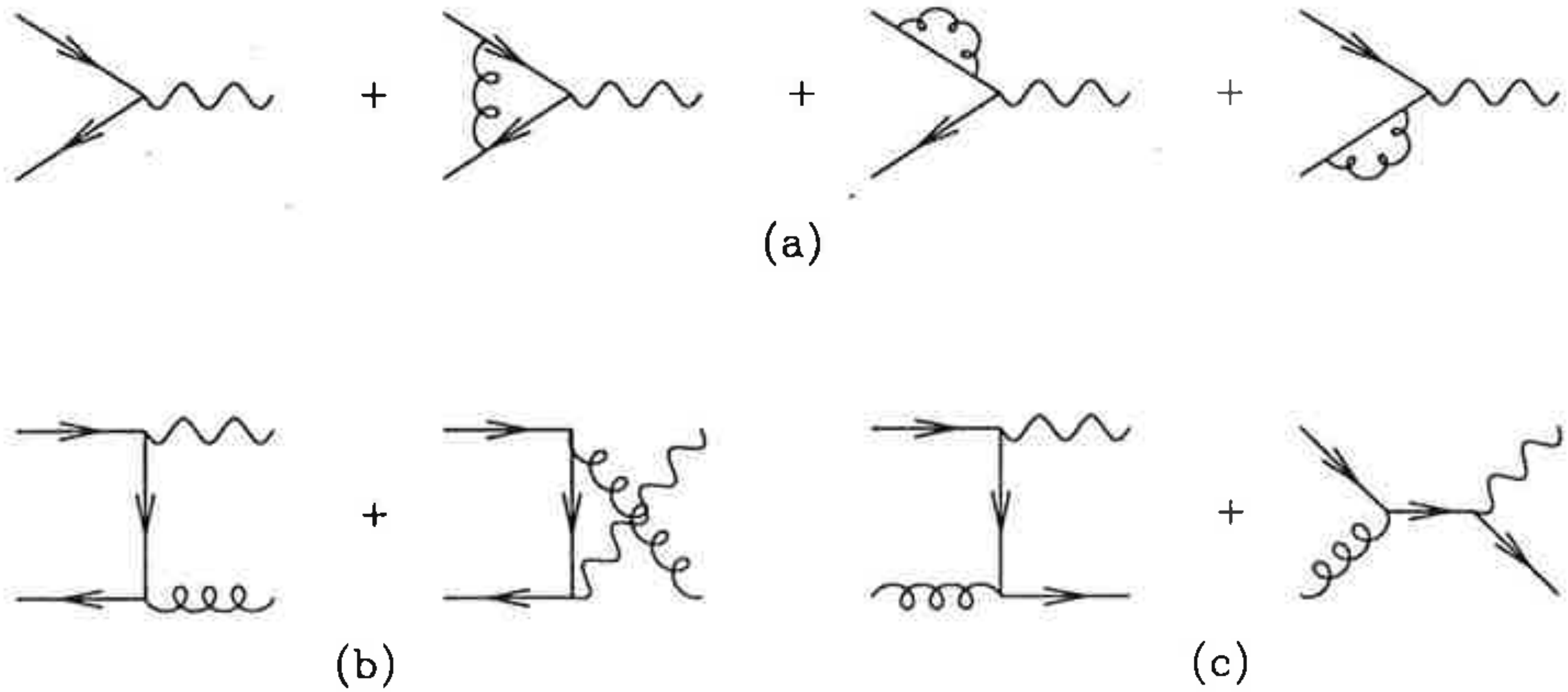
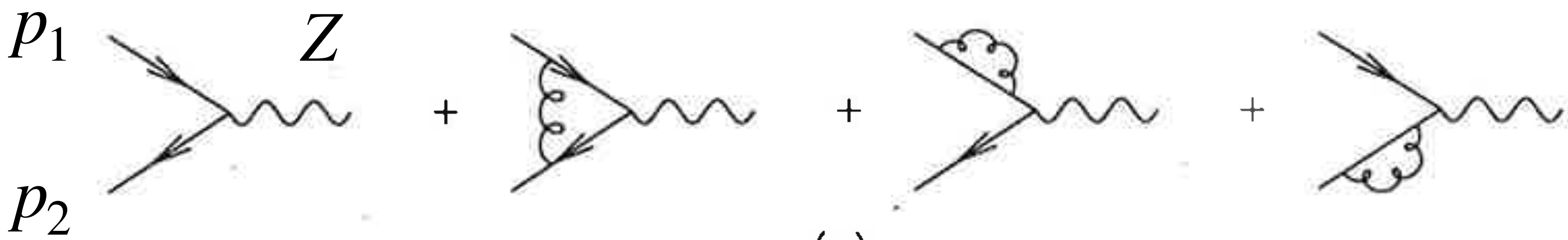


Fig. 9.3. The leading- and next-to-leading-order diagrams for the Drell-Yan process.

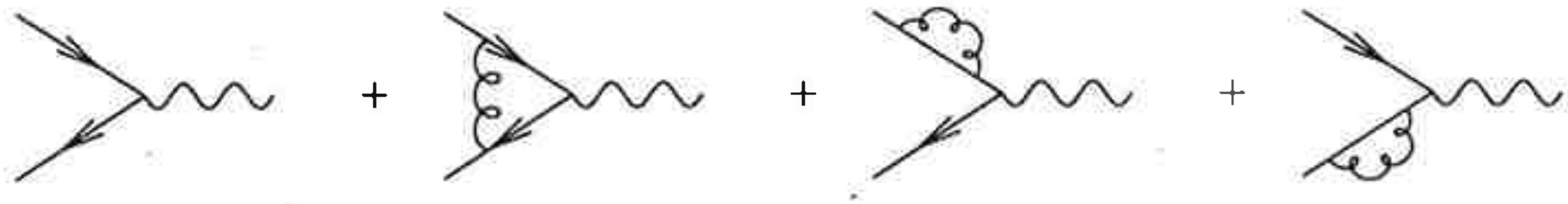
9.2 Perturbative QCD corrections

In this section we calculate the $O(\alpha_S)$ corrections to the parton model Drell-Yan cross section. The calculation is similar in many respects to that for the corresponding correction to the deep inelastic structure function F_2, described in Chapter 4. We begin by considering the parton-level Drell-Yan cross section for the leading-order process $q(p_1)+\bar{q}(p_2) \to l^+l^-$:



(a)

$$\frac{1}{\sigma}\frac{d\sigma}{dq_T} = \,?$$



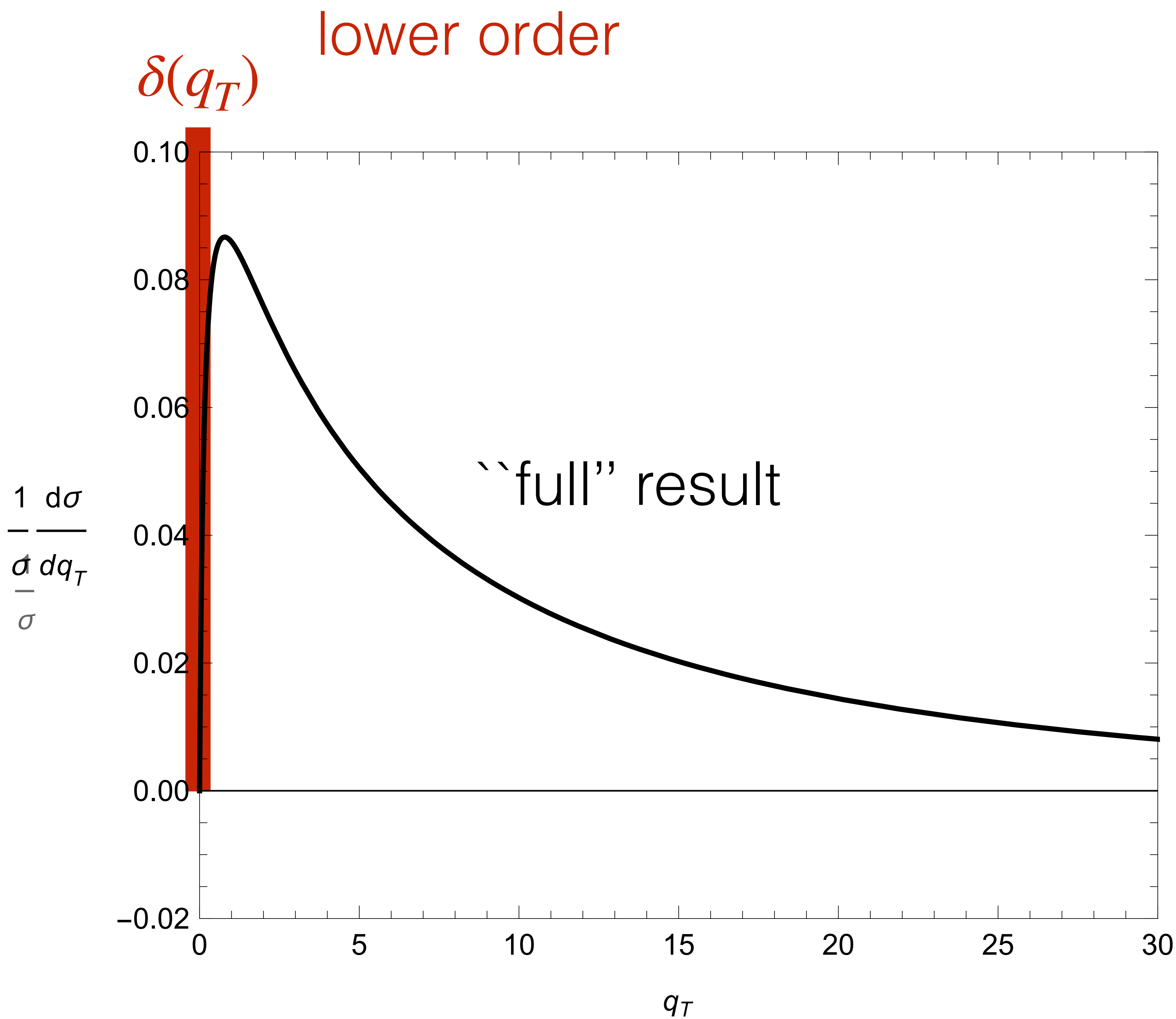
(a)

- Lowest order $q\bar{q} \to Z$ process has no hadronic state X, therefore $q_T = 0$:

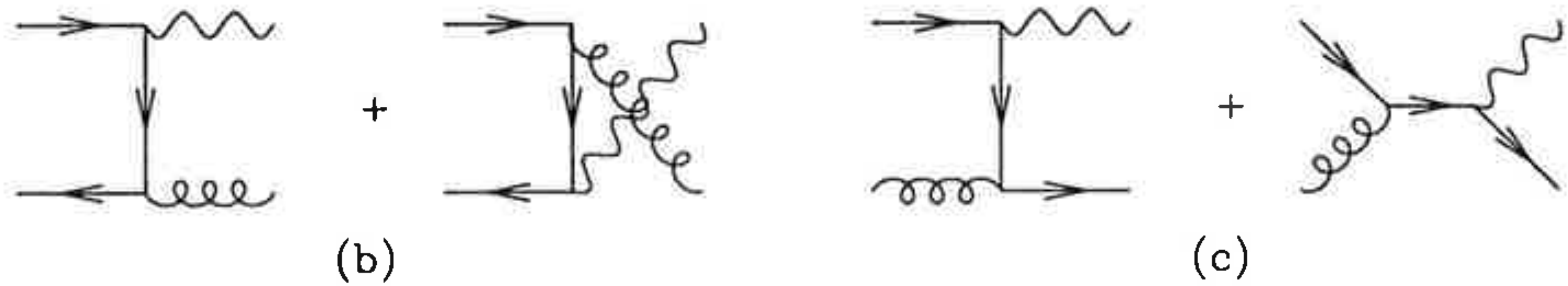
$$\boxed{\frac{1}{\sigma}\frac{d\sigma}{dq_T} = \delta(q_T) + \mathcal{O}(\alpha_s)}$$

- Note: cross section is a distribution in q_T.

- PDF convolution drops out in spectrum.



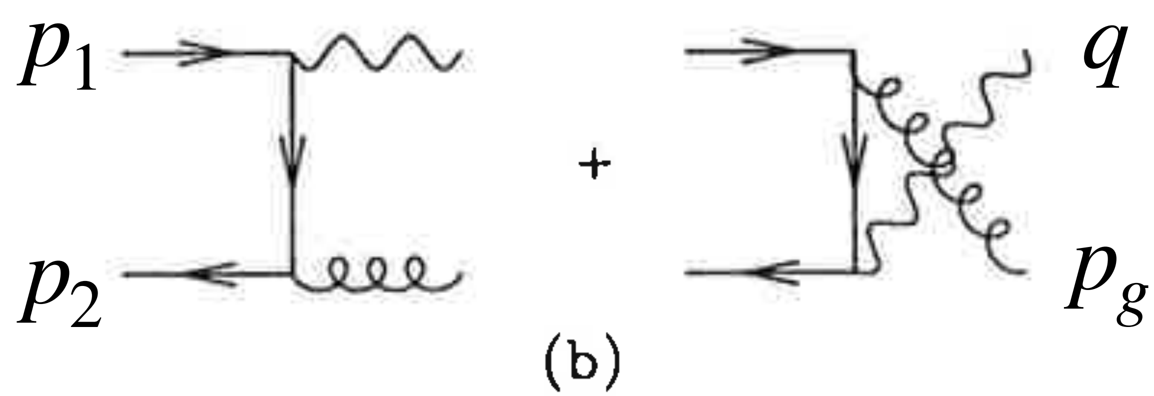
lower order
$\delta(q_T)$
``full'' result
$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
q_T
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
0
5
10
15
20
25
30



- At the next order Z-boson recoils against the gluon (b) or quark (c):
- At low q_T the cross section is enhanced:

$$\frac{1}{\sigma}\frac{d\sigma}{dq_T} \sim \alpha_s \frac{\ln(q_T/Q)}{q_T} + \ldots$$

- Distribution: integral over q_T is defined and gives 1.



Squared amplitudes (b)

$$\sum_{\text{spins}} |\mathscr{M}_{q\bar{q}\to Z+g}|^2 = \sigma_0 \frac{2C_F g^2\left[\left(p_1\cdot q\right)^2 + \left(p_2\cdot q\right)^2\right]}{q^2\, p_1\cdot p_g\, p_2\cdot p_g}$$

diverges when

$p_g \parallel p_1$ $\qquad$ $p_g \parallel p_2$ $\qquad$ $p_g \to 0$

collinear limits $\qquad$ soft limit

One can use dimensional regularization $d = 4 - 2\epsilon$ for phase space (as well as loop integrals). Gives expressions such as

$$\int_0^{Q_T} dq_T \, q_T^{-1-2\epsilon} = -\frac{1}{2\epsilon} Q_T^{-2\epsilon} = -\frac{1}{2\epsilon} + \ln(Q_T) - \epsilon \ln^2(Q_T) + \ldots$$

or differentially

$$q_T^{-1-2\epsilon} = -\frac{1}{2\epsilon}\,\delta(q_T) + \left(\frac{1}{q_T}\right)_* + \ldots$$

Soft and collinear $1/\epsilon$ divergences cancel in cross section (real against virtual!) up to collinear terms which are absorbed into the PDFs, but logarithms remain.

Full partonic cross section in $d = 4 - 2\epsilon$

$$\frac{d\hat{\sigma}_{\bar{q}q\to Z+g}}{dz\, d\cos\theta} = \sigma_0 \frac{C_F\,\alpha_s}{4\pi} \frac{e^{\gamma\epsilon}}{\Gamma(1-\epsilon)} 2^{1+2\epsilon}(1-z)^{-1-2\epsilon} z^{\epsilon} \left(1-\cos^2\theta\right)^{-1-\epsilon}$$
$$\times \left[4 + (1-z)\big((1-z)\left(\cos^2\theta + 1 - 2\epsilon\right) - 4\big)\right]$$

θ : gluon scattering angle in partonic CMS

$z = M^2/\hat{s}$: energy variable.

Soft limit is $z \to 1$

$$\frac{d\hat{\sigma}}{dz\, d\cos\theta} \to \sigma_0 \frac{C_F\,\alpha_s}{4\pi} \frac{e^{\gamma\epsilon}}{\Gamma(1-\epsilon)} 2^{3+2\epsilon} \boxed{(1-z)^{-1-2\epsilon}} \boxed{\left(1-\cos^2\theta\right)^{-1-\epsilon}}$$

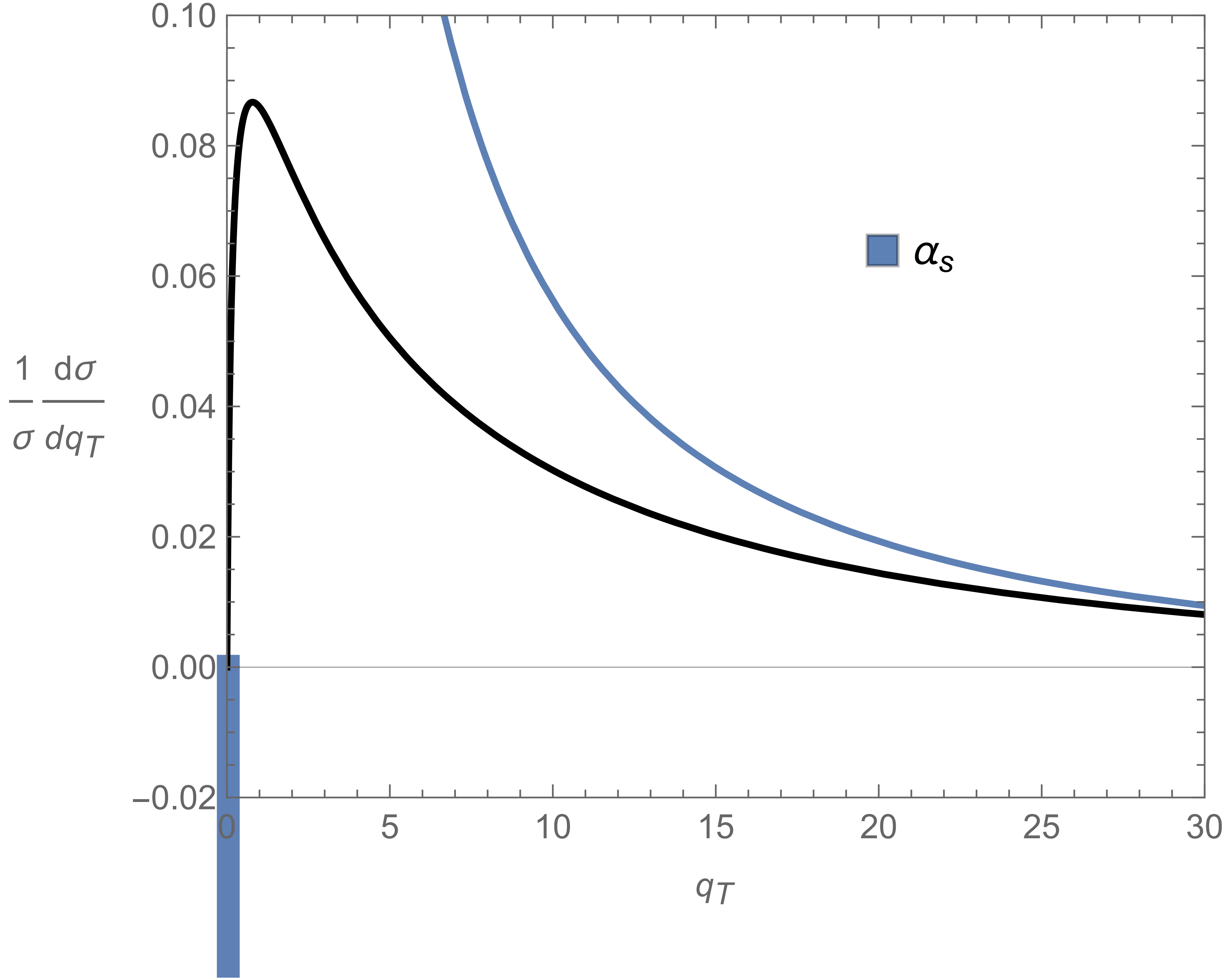
soft divergence   collinear divergence

The double logarithmic term $\alpha_s \ln^2(Q_T)$ is linked to a double divergence, where the emitted gluon is both soft and collinear

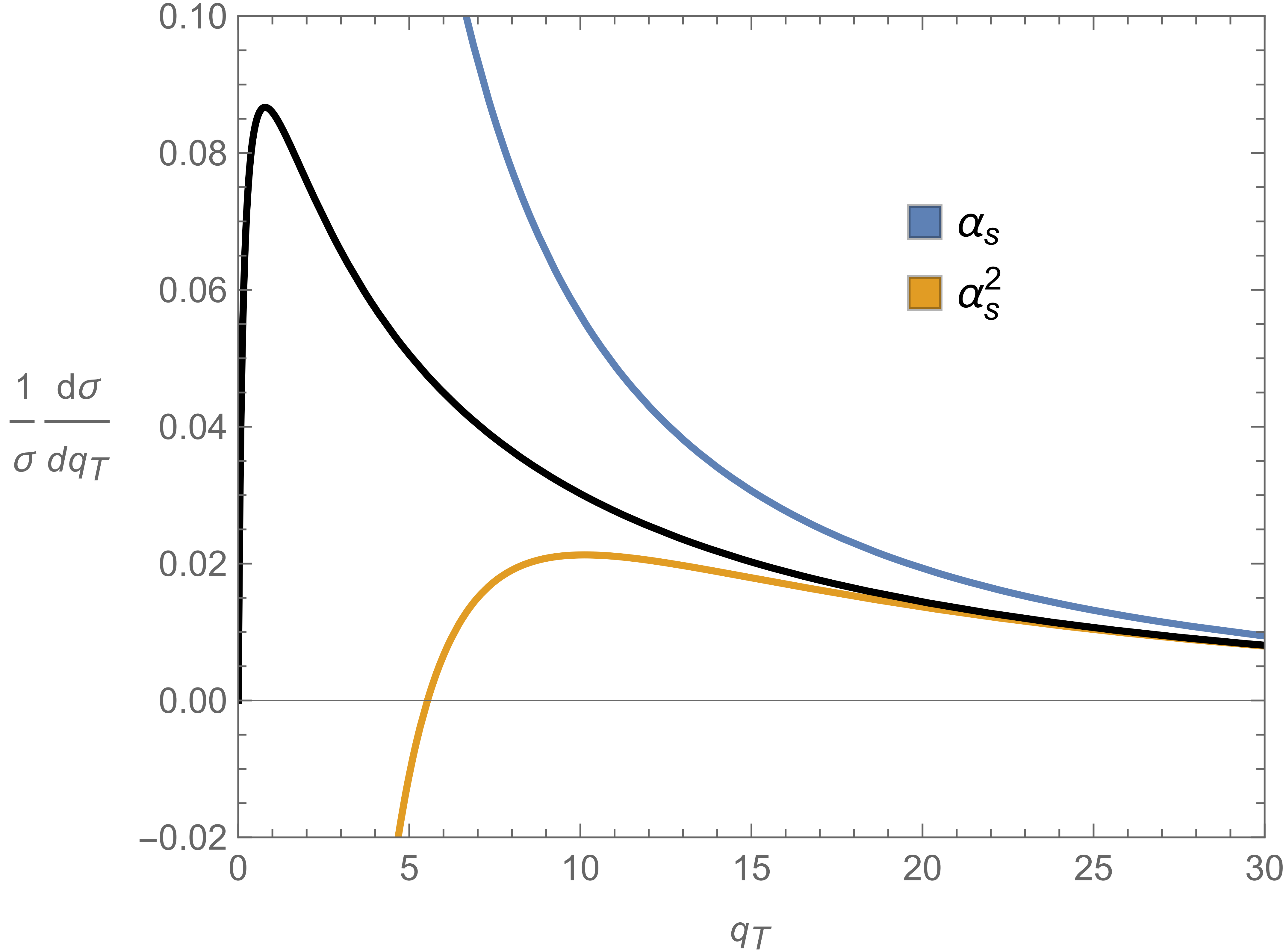
- squared amplitude is very simple in this region, also at higher orders, see later!

The divergence itself cancels against the virtual part

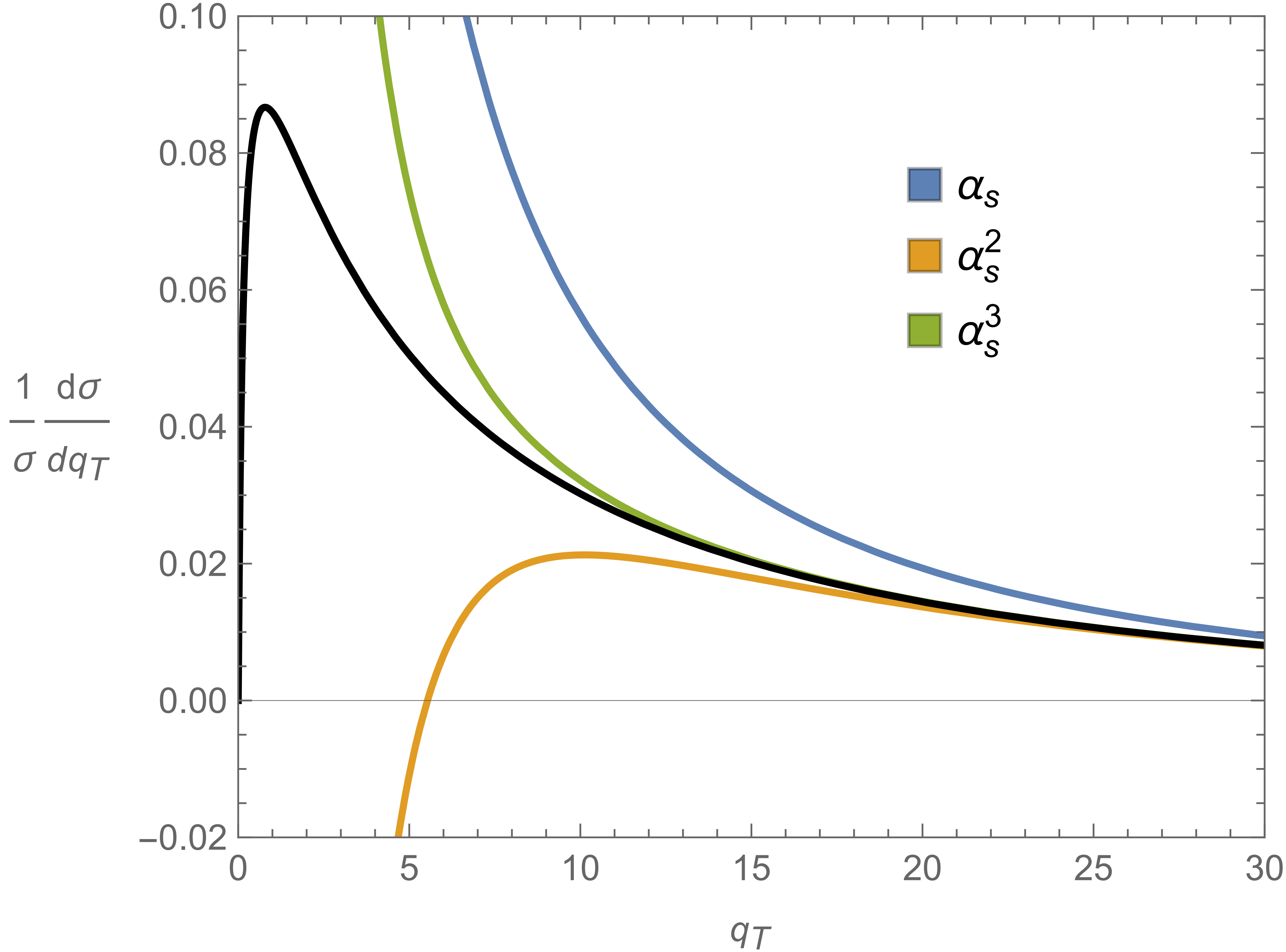
- can predict double logs purely from the divergences of loop diagrams.



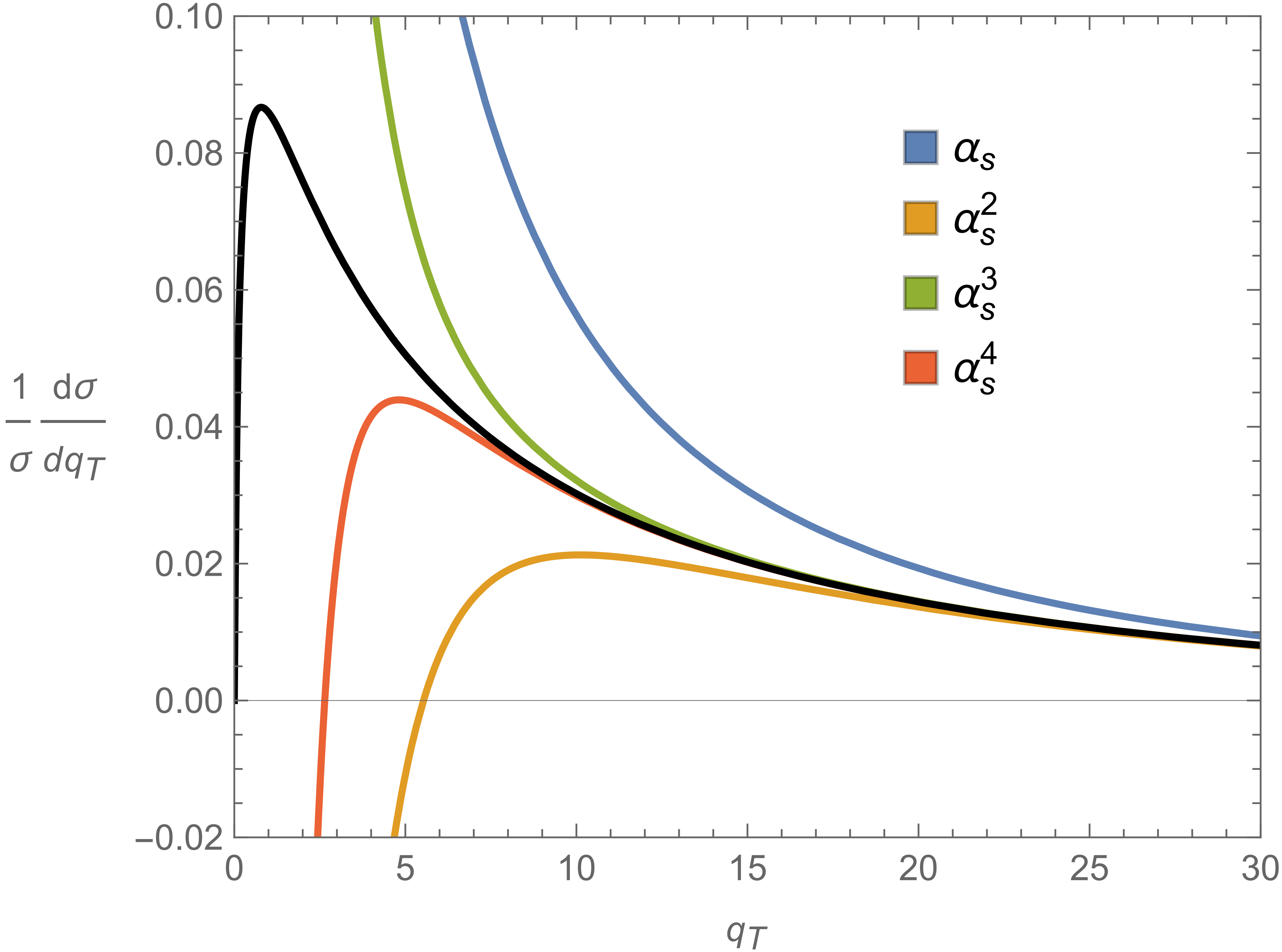
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
0
5
10
15
20
25
30
q_T
α_s



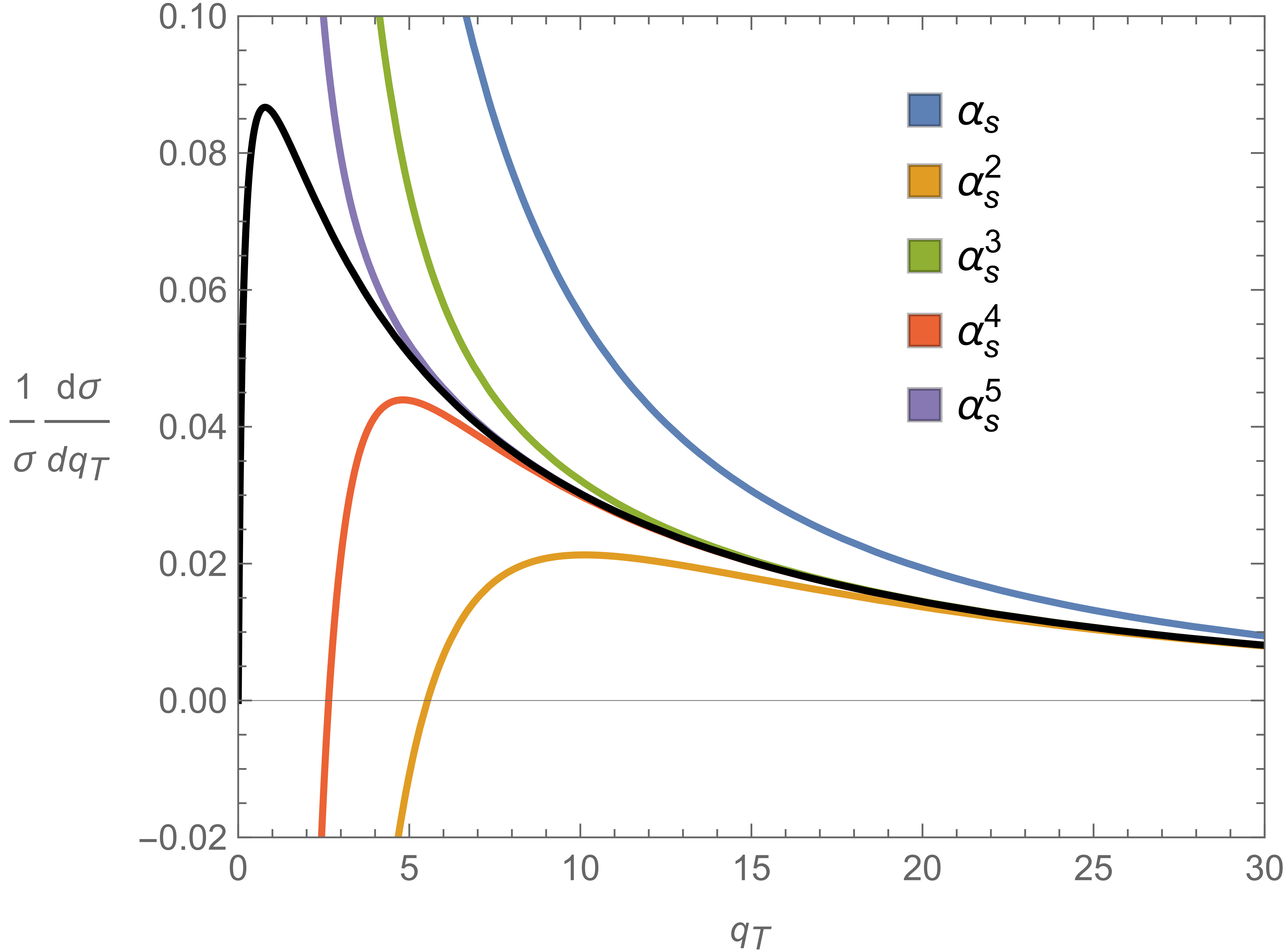
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
0
5
10
15
20
25
30
q_T
α_s
α_s^2



$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
0
5
10
15
20
25
30
q_T
α_s
α_s^2
α_s^3



0.10
0.08
0.06
0.04
0.02
0.00
−0.02
0
5
10
15
20
25
30
$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
q_T
α_s
α_s^2
α_s^3
α_s^4



0.10
0.08
0.06
0.04
0.02
0.00
−0.02
$\frac{1}{\sigma}\frac{d\sigma}{dq_T}$
0
5
10
15
20
25
30
q_T
α_s
α_s^2
α_s^3
α_s^4
α_s^5

Leading logarithmic (LL) result

The plots in the previous slides were obtained by defining

$$\boxed{\Sigma(q_T) = \int_0^{q_T} dq'_T \frac{1}{\sigma} \frac{d\sigma}{dq'_T}}$$

and using the approximation

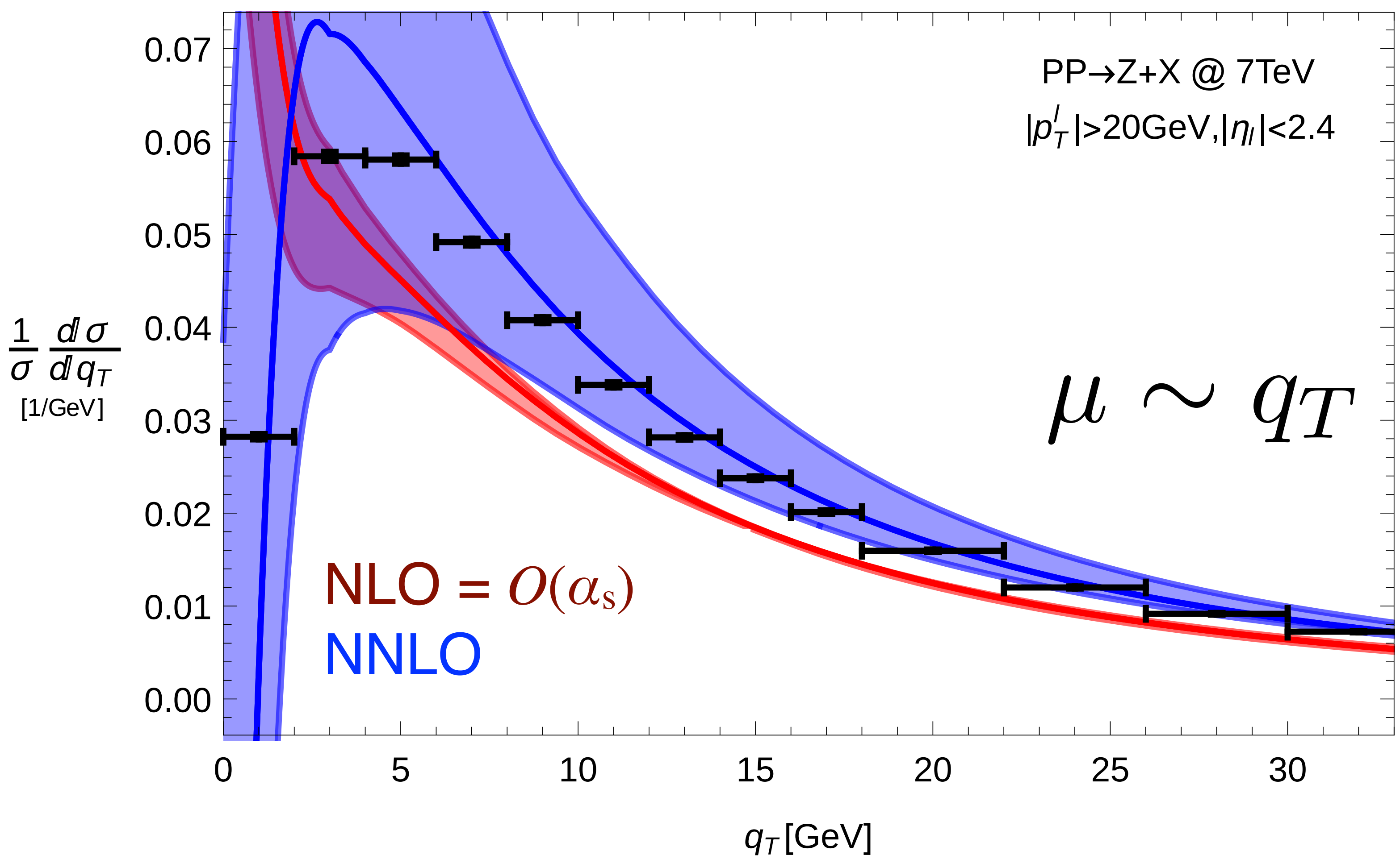
$$\boxed{\Sigma(q_T) = \exp(-a_s L^2)}$$

with

$$L = \ln \frac{M_Z^2}{q_T^2} \qquad a_s = \frac{C_F}{2\pi} \alpha_s(\mu)$$

[$C_F = 4/3,\ \alpha_s(M_Z) \approx 0.1$]

Full fixed-order result



- Qualitatively similar to the simple double-log approximation
- Bands from varying scale in strong coupling by factor 2 around default. Large scale uncertainty for low q_T

Sudakov logarithms

The integrated cross section

$$\Sigma(q_T) = \int_0^{q_T} dq'_T \, \frac{1}{\sigma} \frac{d\sigma}{dq'_T}$$

has for low q_T an expansion of the form ($L = \ln \frac{M_Z^2}{q_T^2}$)

$$\Sigma(q_T) = 1 + \alpha_s \left(c_2 L^2 + c_1 L + c_0 \right) + \alpha_s^2 \left(c_4 L^4 + c_3 L^3 + \ldots \right) + \alpha_s^3 \left(c_6 L^6 + \ldots \right) + \ldots$$

leading logarithms

next-to-leading logarithms

Counting logarithms

Enhanced higher-order corrections at small q_T because the logarithm L becomes large and overwhelms the α_s suppression. Natural counting is

$$L \sim \frac{1}{\alpha_s}$$

This is compatible with the running of the coupling:

$$\frac{\beta_0}{4\pi} L = \frac{1}{\alpha_s(M_Z)} - \frac{1}{\alpha_s(q_T)} + O(\alpha_s^0)$$

Counting implies that

$$\alpha_s L^2 \sim \frac{1}{\alpha_s}, \qquad \alpha_s^2 L^4 \sim \frac{1}{\alpha_s^2}, \qquad \alpha_s^3 L^6 \sim \frac{1}{\alpha_s^3}, \qquad \ldots$$

Counting in the cross section is not meaningful!
Higher log terms become more and more important.

Need full control over these enhanced terms to get meaningful results.

Solution: can show that (for many observables) the double-log terms exponentiate.

Exponentiation

One can show that cross section has the form

$$\Sigma(p_T) = \exp\left(L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \dots\right)$$

Nontrivial, crucial feature: **only one *L* per order** in the exponent!

Accuracy:

- LL: g_1; NLL: g_1, g_2; NNLL: g_1, g_2, g_3

Expand in α_s but count $\alpha_s L$ as $O(1)$

Exponentiation

One can show that cross section has the form

$$\Sigma(p_T) = \exp\left(L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \dots\right)$$

$\sim 1/\alpha_s$ $\quad\sim 1$ $\quad\sim \alpha_s$ $\quad\sim \alpha_s^2$

Accuracy:

- LL: g_1; NLL: g_1, g_2; NNLL: g_1, g_2, g_3

Expand in α_s but count $\alpha_s L$ as $O(1)$

Size of corrections

Correction to	L ~ 1	L ~ 1/α_s
LO	α_s	$\alpha_s{}^n L^{2n}$!
NLO	$\alpha_s{}^2$	$\alpha_s{}^n L^{2n}$!
LL		$\alpha_s{}^n L^n \sim 1$
NLL		α_s
NNLL		$\alpha_s{}^2$

$$\exp\Big(L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \ldots\Big)$$

Uncertainty estimate

Let us discuss how the estimate uncertainties in resummed results, using our LL result

$$\boxed{\Sigma(q_T) = \exp(-a_s L^2)}$$

$$L = \ln \frac{q_T^2}{M_Z^2}$$

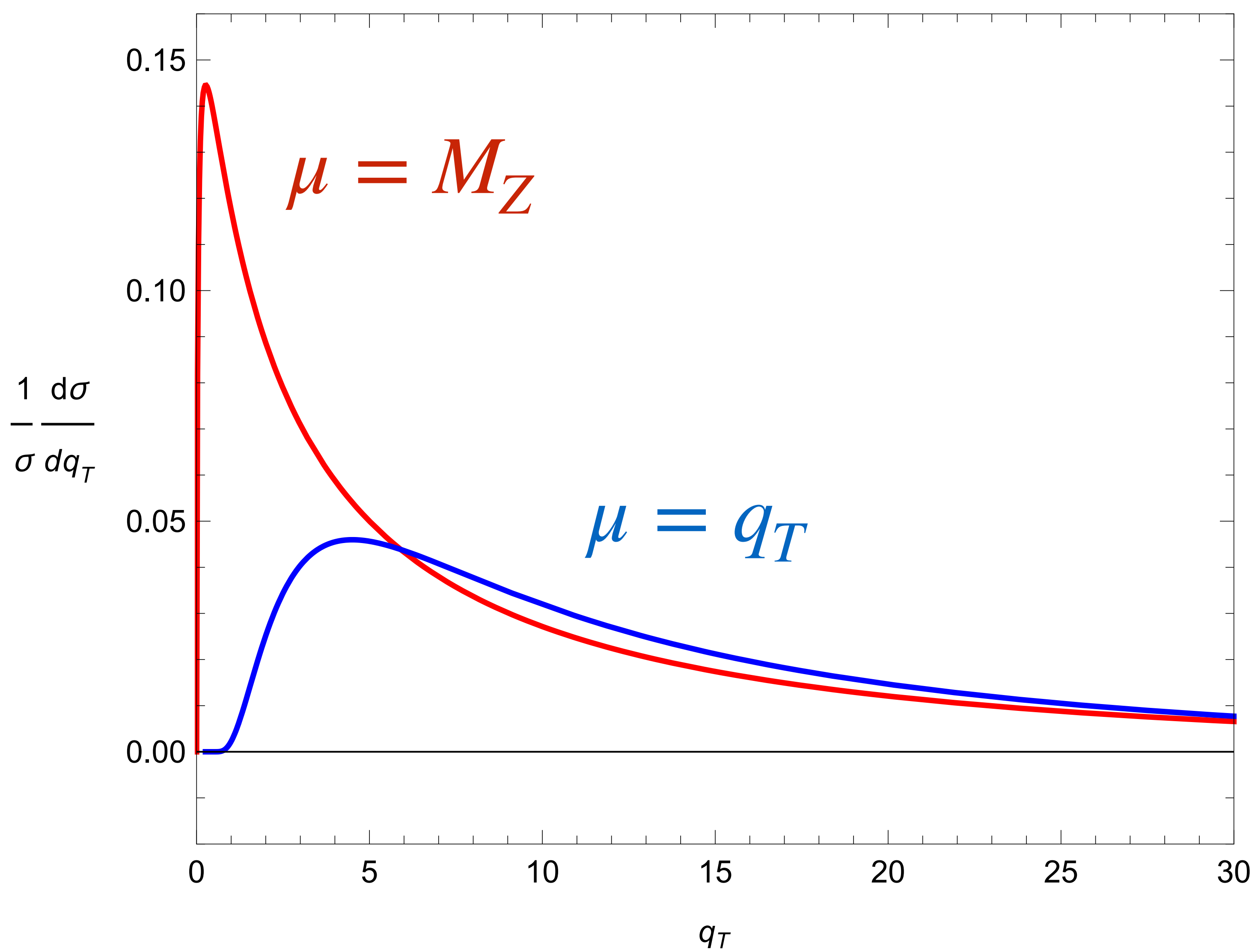
$$a_s = \frac{C_F}{2\pi} \alpha_s(\mu)$$

LL is not unique:

1. Can choose a value of μ in the coupling.
2. Can modify $L \to \tilde{L} = \ln \frac{q_T^2}{Q^2}$ with $Q \sim M_Z$ the ``resummation scale''. Vary this scale e.g. by a factor of two

Scale in coupling

In our exponentiation we have neglected running coupling effects, but these make a big difference and need to be included.



Including running effects

Proper expression for the double logarithmic part which takes into account the running coupling is

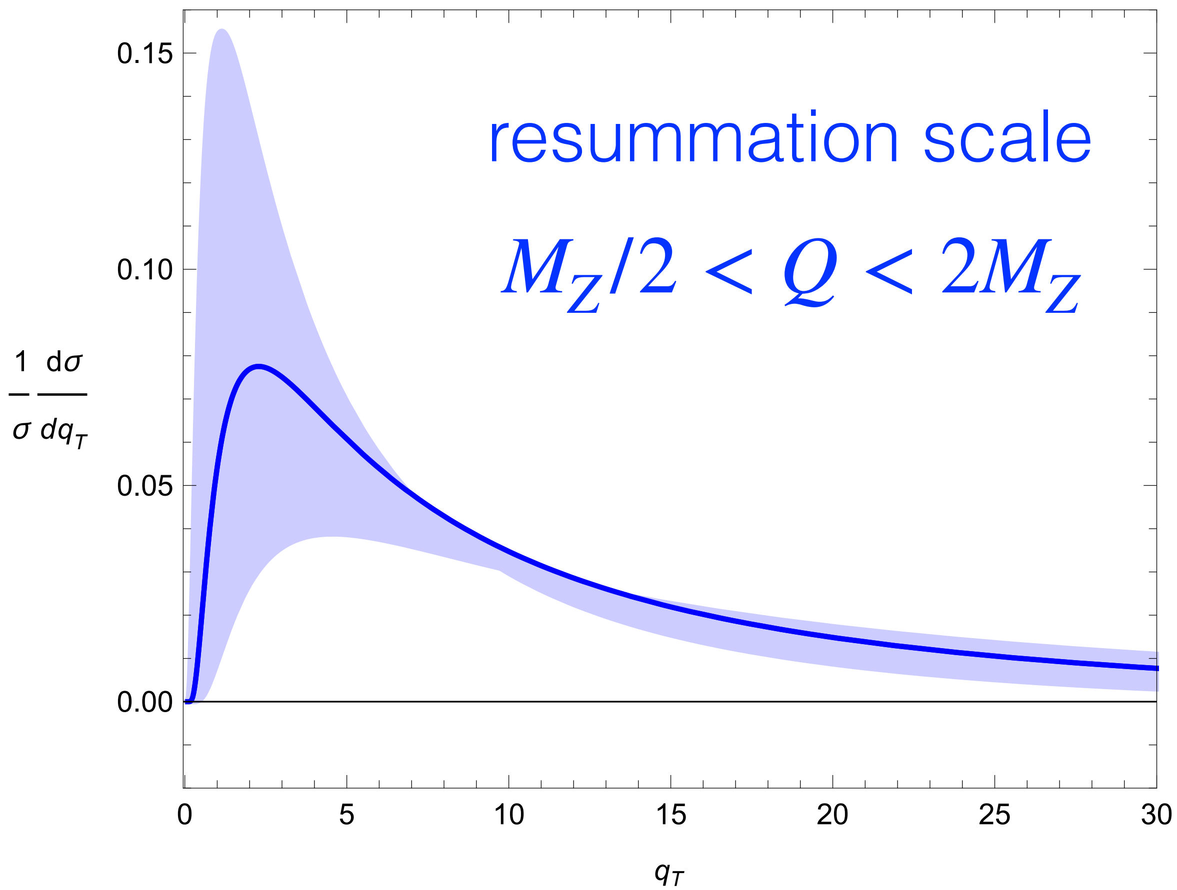
$$L\, g_1(\alpha_s L) = -\int_{q_T}^{M_Z} \frac{d\mu}{\mu} \frac{4C_F\, \alpha_s(\mu)}{\pi} \ln \frac{M_Z^2}{\mu^2}$$

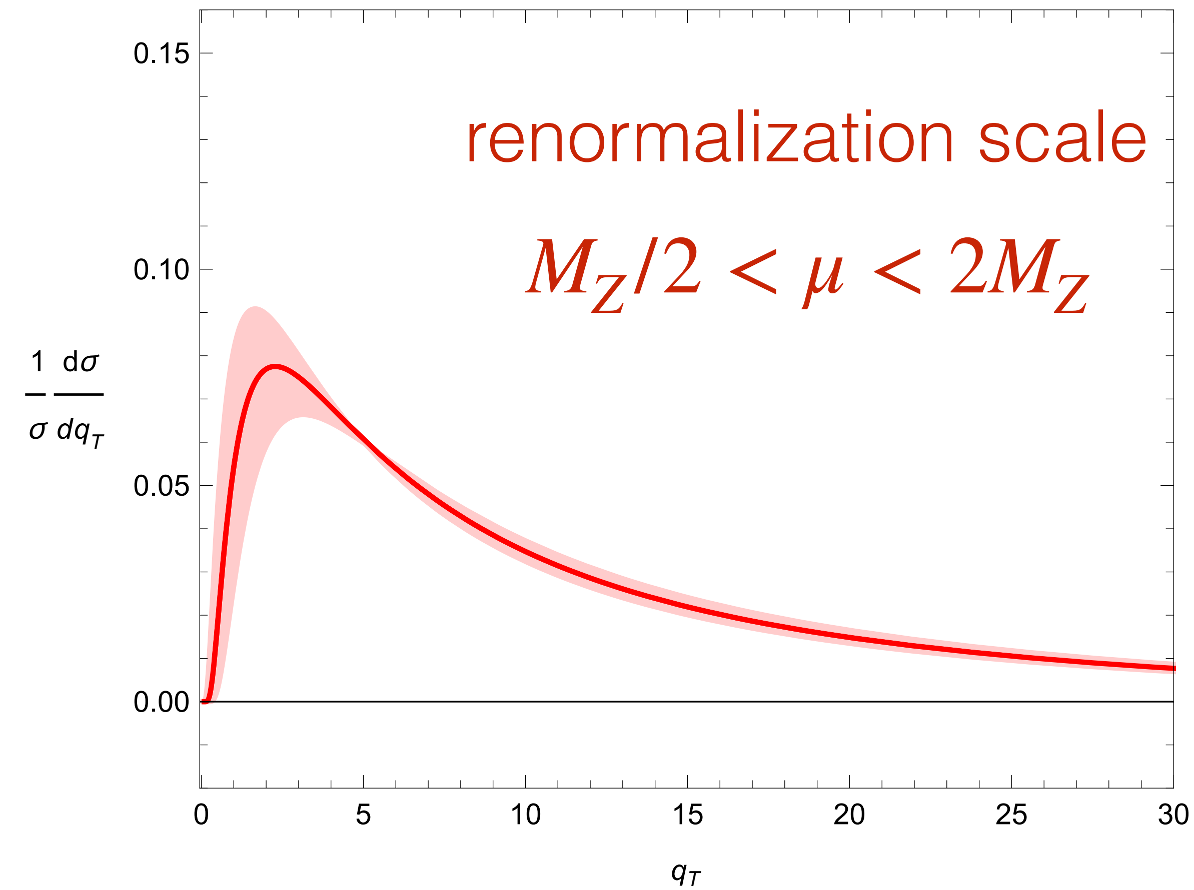
One can rewrite the entire integral as an integral over the running coupling. Remember that

$$\frac{\beta_0}{4\pi} L = \frac{1}{\alpha_s(M_Z)} - \frac{1}{\alpha_s(q_T)} + O(\alpha_s^0)$$

This is done in Soft-Collinear Effective Theory (SCET). To estimate uncertainties one then varies the scale in the high and low coupling.

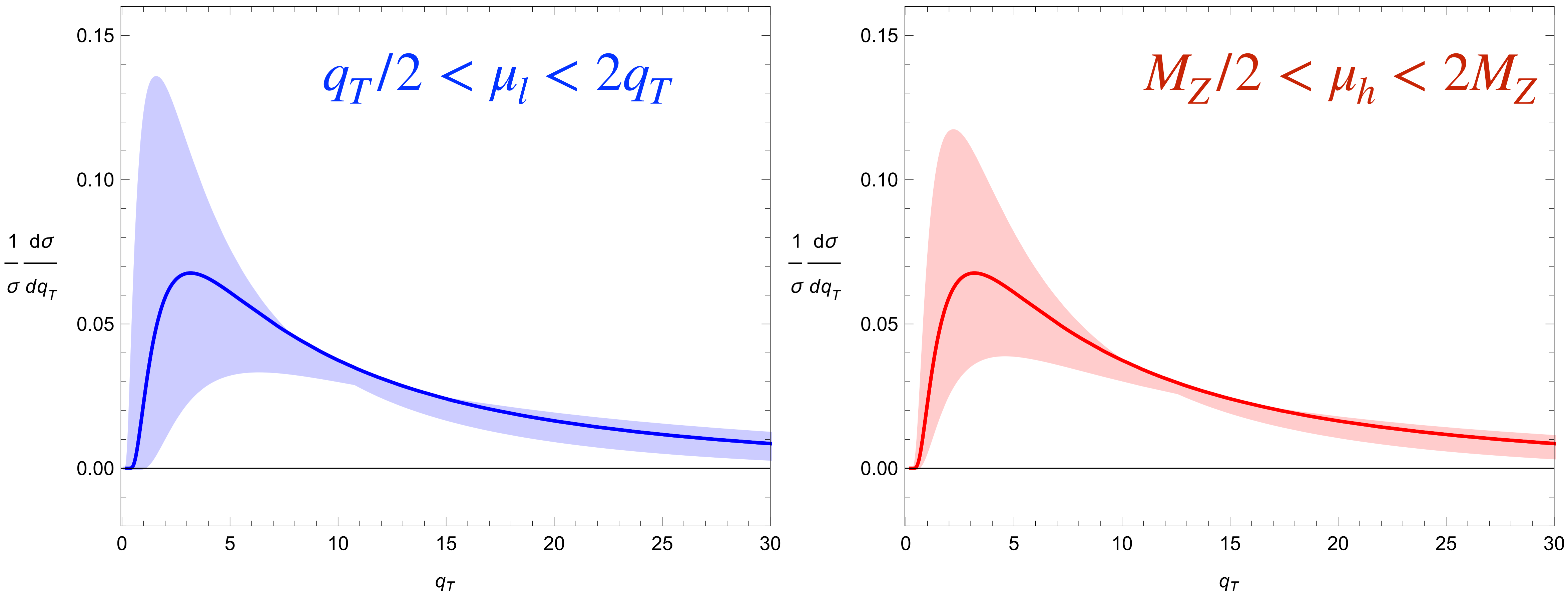
Uncertainty estimate in traditional resummation





Significant uncertainty: expected since missing NLL is an O(1) effect.

Uncertainty estimate à la SCET

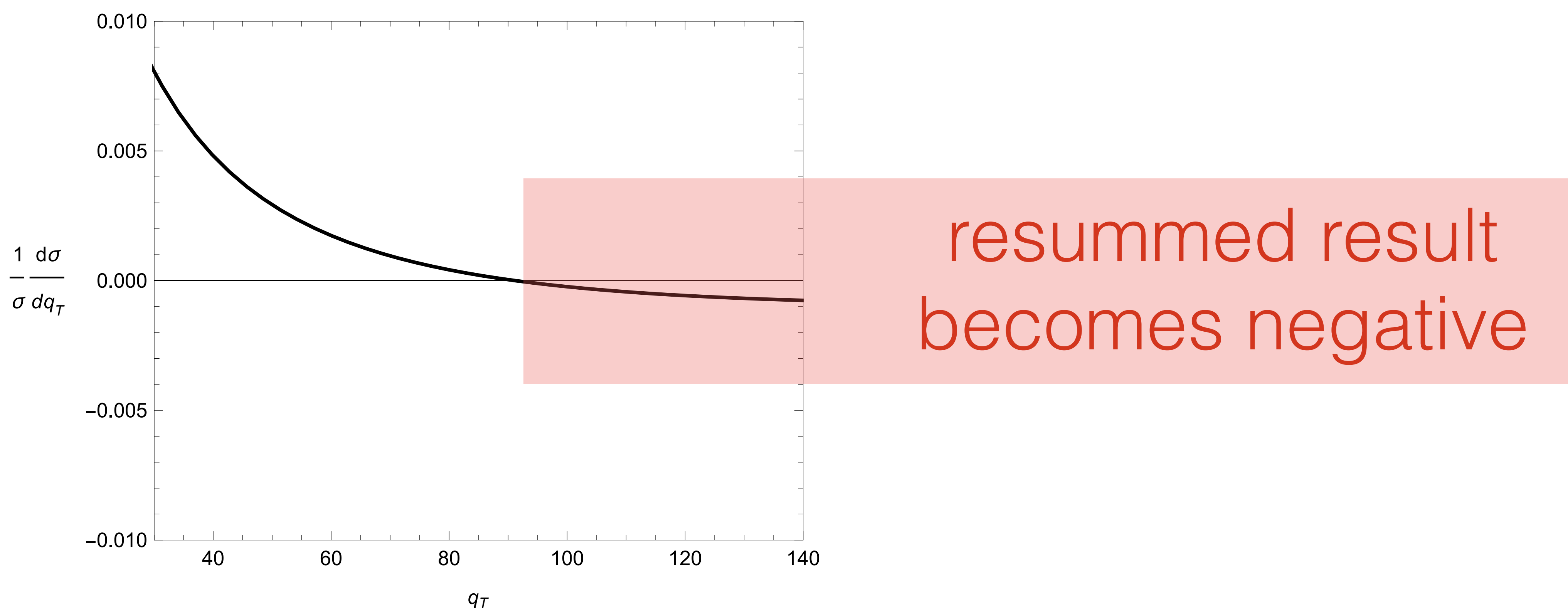


SCET is based on RG: logs are eliminated in factor of coupling constants at high and low scale.

- Similarly large (but not identical) to trad. approach

Switching off resummation

The resummed result is based on an expansion of the cross section for $q_T \to 0$. At high transverse momentum (hard emissions!) these results become unphysical!



Should switch off resummation at larger q_T and transition to standard fixed-order result.

Different ways to switch resummation off and match to fixed order

- Traditional approach: modify argument of logarithms L so that they switch themselves off at higher q_T.

- SCET: profile functions which modify scales so that $\mu_l(q_T) \rightarrow \mu_h$ for high q_T.

- Transition function $t(x)$, with $x = q_T^2/M_Z^2$ which smoothly transitions from resummed to fixed-order result, $t(x) \rightarrow 0$ for $x \rightarrow 1$.

All these methods are currently used.

Matching

Resummation is based on expansion at small q_T, but we can add back the power suppressed terms

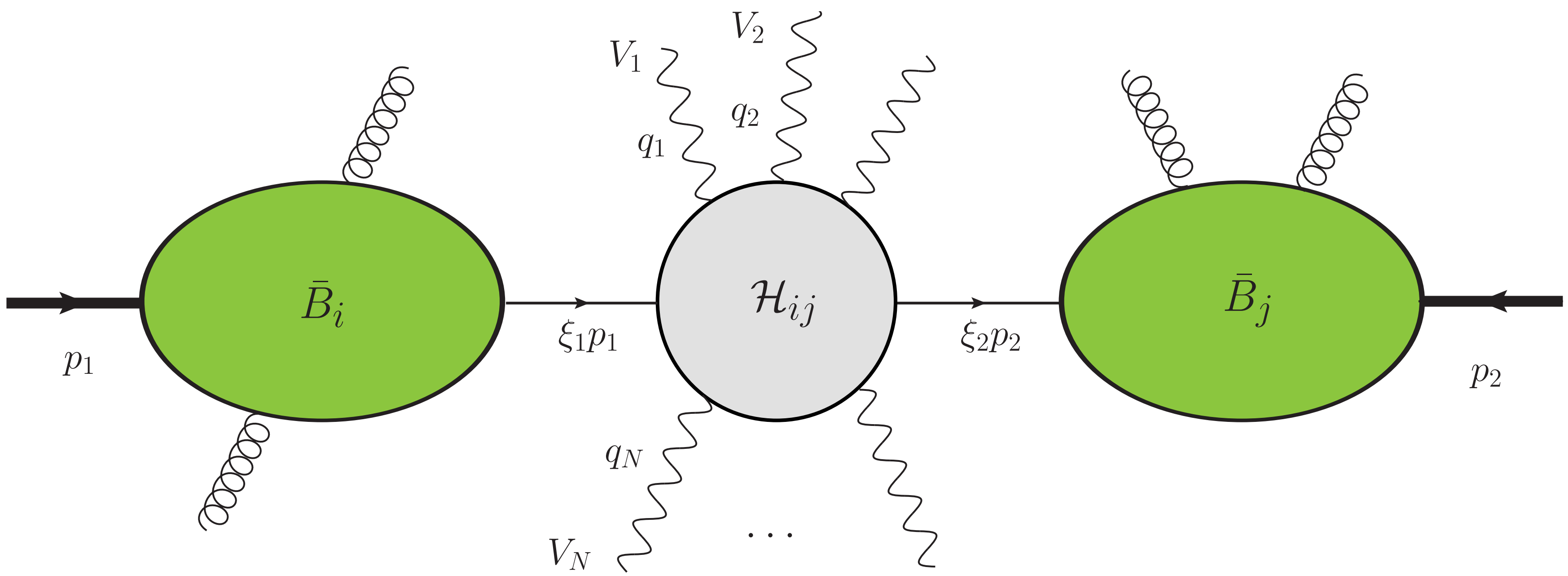
$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{naively matched to NNLO}} = \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \underbrace{\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T} - \left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{exp. to NNLO}}}_{\text{matching correction } \Delta\sigma}$$

Can combine matching and transition function $t(x)$

$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{matched to NNLO}} = t(x)\left(\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \Delta\sigma\big|_{q_T>q_0}\right) + (1-t(x))\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T}$$

All-order factorization

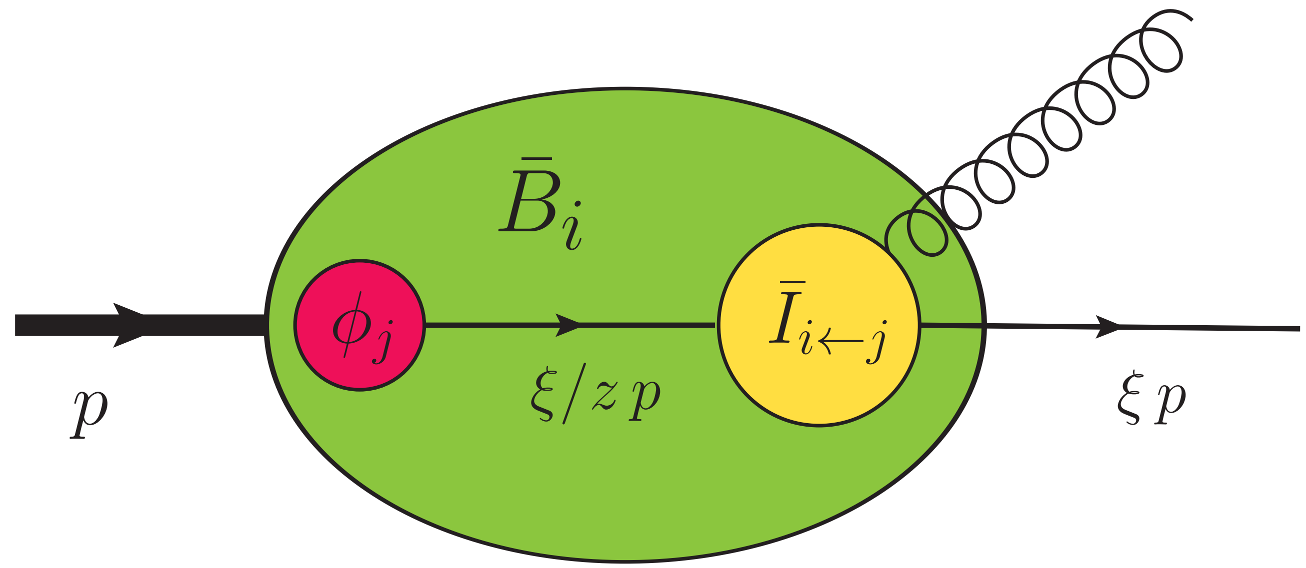
Collins, Soper, Sterman ’84. In SCET: TB Neubert ‘09



Beam function
soft + collinear emission

hard function
Born + virtual corrections

Factorization theorem originates from soft and collinear factorization, as we’ll discuss later.



Beam functions factorize further into parton distribution functions (PDFs) ϕ_q, ϕ_g, … and perturbatively calculable kernels $\bar{I}_{q\to g}$.

Fourier convolution

hard function: Born + virtual

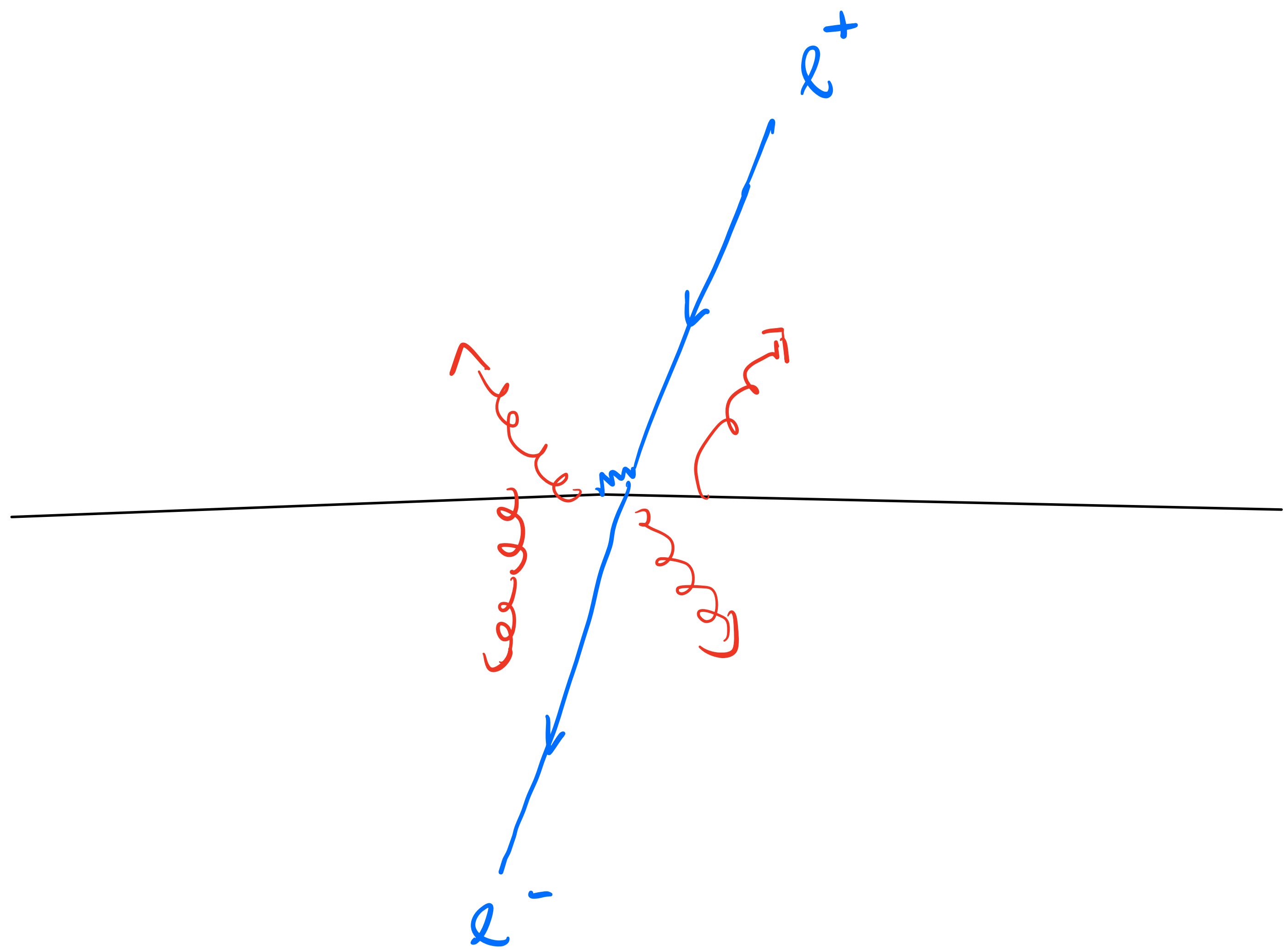
$$\mathrm{d}\sigma_{ij}(p_1,p_2,\{\underline{q}\}) = \int_0^1 \mathrm{d}\xi_1 \int_0^1 \mathrm{d}\xi_2\, \mathrm{d}\sigma^0_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\})\, \mathcal{H}_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\}, \mu)\,\cdot$$

$$\frac{1}{4\pi}\int \mathrm{d}^2 x_\perp\, e^{-iq_\perp x_\perp} \left(\frac{x_T^2 Q^2}{b_0^2}\right)^{-F_{ij}(x_\perp,\mu)} B_i(\xi_1, x_\perp, \mu)\cdot B_j(\xi_2, x_\perp, \mu)$$

collinear anomaly

beam functions

- Factorization takes place in **transverse position space**. Cross section is a Fourier integral!
- Product of beam functions has Q dependence from rapidity divergences (collinear anomaly).
- Setting $\mu = q_T$ including only NLL term in the exponent F_{ij} leads to a divergence in the cross section at low q_T. Frixione, Nason, Ridolfi '99



At low transverse momentum QCD emissions start recoiling against each other instead of the Z-boson. Relevant scale μ is higher than q_T of the boson.

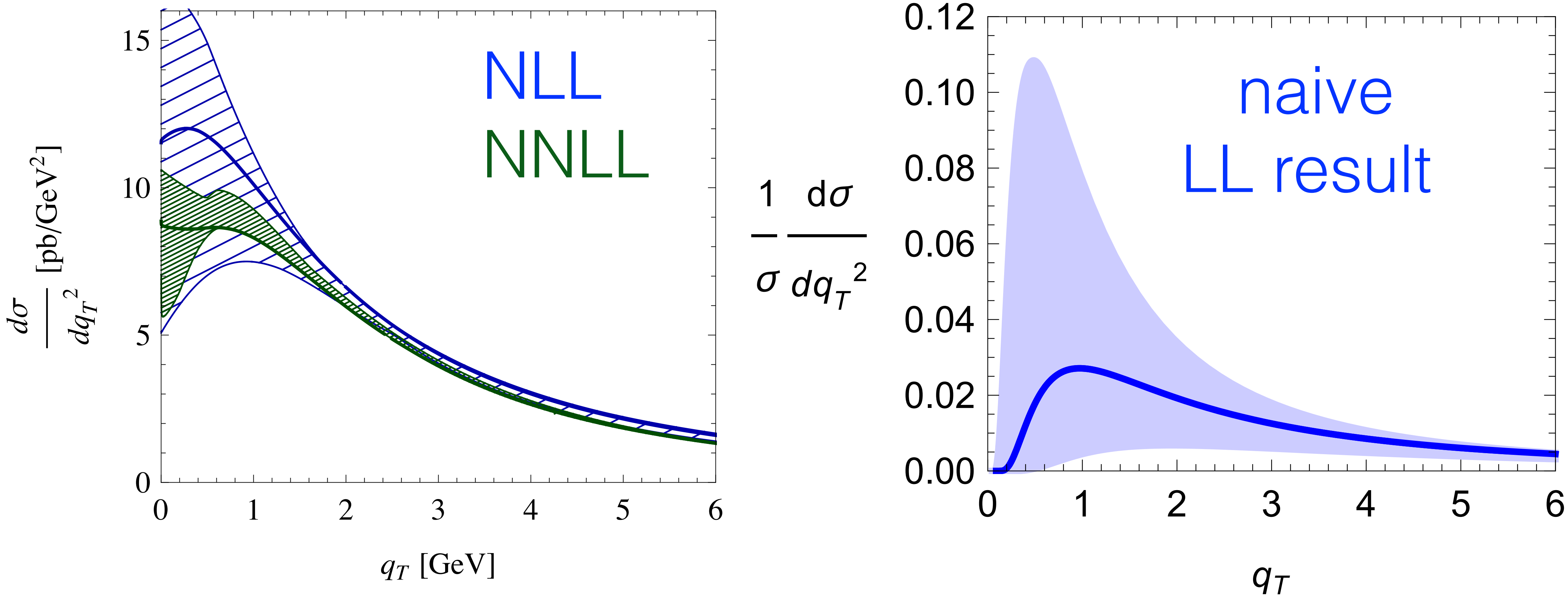
Solutions

- Set scale in position space, but introduce cutoff prescription (b* prescription) to avoid NP region
Collins, Soper, Sterman ’84

- Dedicated analysis of Fourier integral at low q_T reveals that one needs to systematically include some additional terms in exponent F and that $\mu \to q_*$ at low q_T, where $q_* \approx 2\,\mathrm{GeV}$ for Z-production
TB, Neubert ’11

- Set scale to transverse momentum of softest emission instead of q_T. Monni, Re, Torielli ’16

- Distributional scale setting Ebert, Tackmann ‘16



- Proper treatment has a dramatic effect at low q_T: $d\sigma/dq_T^2$ has non-zero intercept at $q_T = 0$

- It would be interesting to measure this intercept!

Ingredients

hard function: Born + virtual

$$\mathrm{d}\sigma_{ij}(p_1,p_2,\{\underline{q}\}) = \int_0^1 \mathrm{d}\xi_1 \int_0^1 \mathrm{d}\xi_2\, \mathrm{d}\sigma^0_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\})\, \mathcal{H}_{ij}(\xi_1 p_1, \xi_2 p_2, \{\underline{q}\}, \mu)\,\cdot$$

$$\frac{1}{4\pi}\int \mathrm{d}^2 x_\perp\, e^{-iq_\perp x_\perp} \left(\frac{x_T^2 Q^2}{b_0^2}\right)^{-F_{ij}(x_\perp,\mu)} B_i(\xi_1, x_\perp, \mu)\cdot B_j(\xi_2, x_\perp, \mu)$$

collinear anomaly

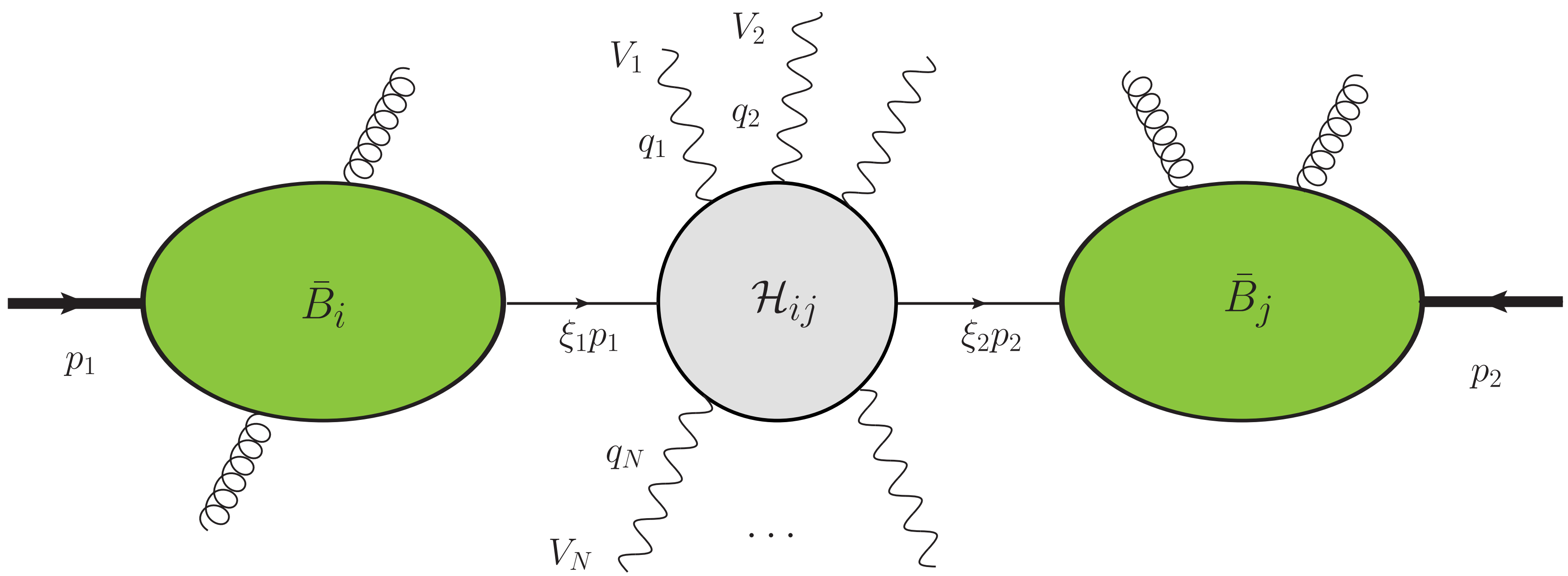
beam functions

- Ingredients known to high accuracy
  - three-loop beam functions Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu and Zhu '20
  - three-loop hard functions for Z/W/γ (new: singlet contributions Gehrmann, Primo '21 with top mass Chen, Czakon, Niggetiedt '21), two-loop for diboson processes
  - new: four-loop anomalous dimensions and anomaly exponent F_{ij}
  - Matching is also known to α_s^3 (MCFM, NNLOJet)

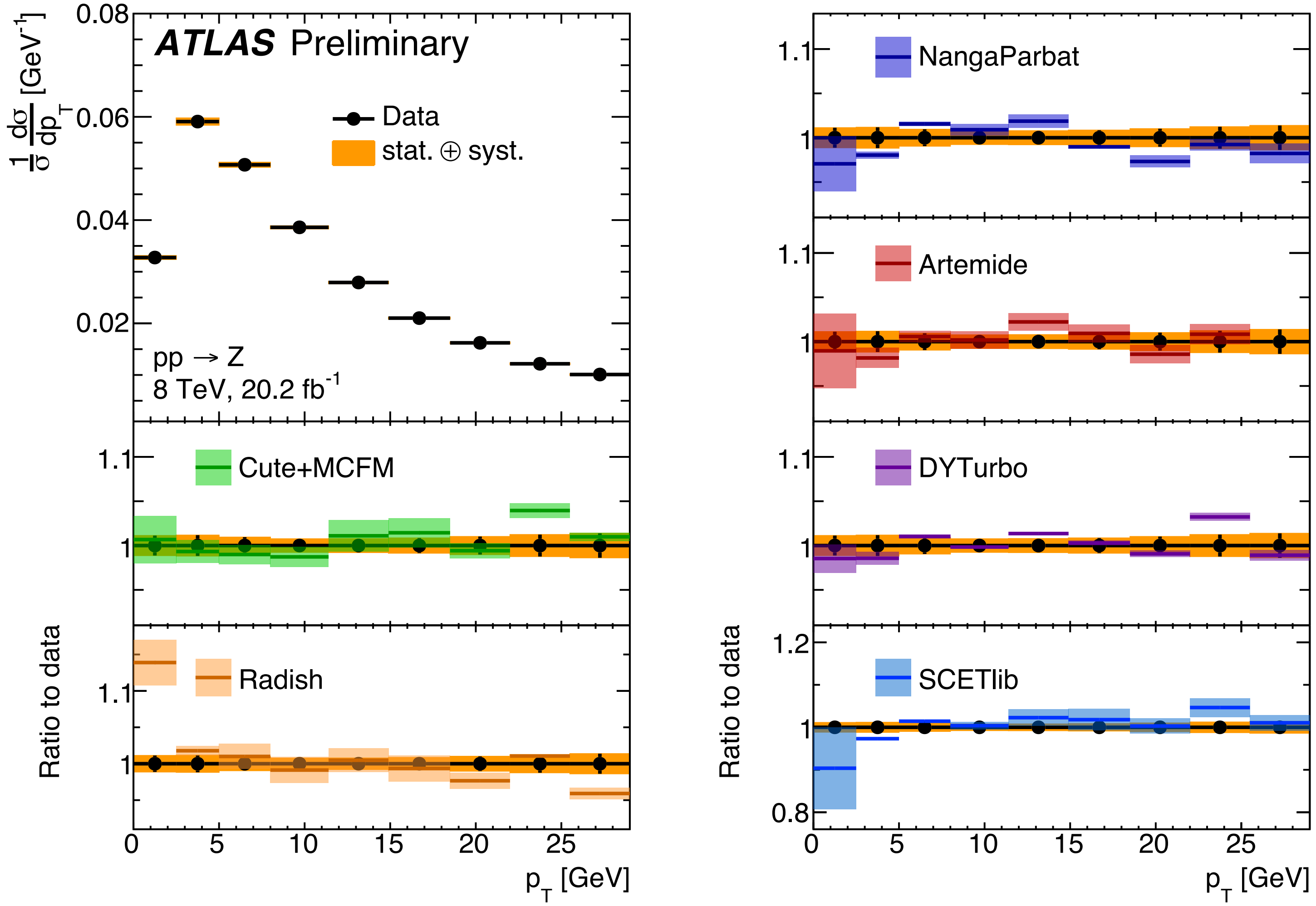
4-loop anomalous dimensions

- Anomaly exponent aka rapidity anomalous dimension can be extracted from regular 4-loop soft anomalous dimension obtained in Das, Moch, Vogt '19, Duhr, Mistlberger Vita, '22 through conformal mapping at $\beta(\varepsilon^*) = 0$ Vladimirov '16.
    - Independent extractions by Duhr, Mistlberger, Vita '22 and Moult, Zhu, Zhu '22
- four-loop hard anomalous dimensions Manteuffel, Panzer, and Schabinger '20; and full quark and gluon form factors Lee, Manteuffel, Schabinger, Smirnov, Smirnov, and M. Steinhauser '22.
- four-loop cusp Henn, Korchemsky, Mistlberger '19; Manteuffel, Panzer, and Schabinger '20 + … 5-loop cusp is missing, estimated to have very small effect.

Implementation



- Structure of resummation is the same as born-level + virtual in fixed-order computation
  - Resummation can piggyback on existing fixed-order codes MATRIX+RadISH Kallweit, Re, Rottoli, Wiesemann '20, CuTe-MCFM TB, Neumann '20, to get resummed fiducial cross sections.
  - Same for jet-veto cross section MadGraph5_aMC@NLO TB, Frederix, Neubert Rothen '14; MCFM-RE Arpino, Banfi, Jäger, Kauer '19; MCFM Campbell, Ellis, Neumann, Seth '23



ATLAS-CONF-2023-013

- aN^{4}LL resummations from several groups with different formalisms (public N^{4}LL: CuTe-MCFM Campbell, Neumann '22, DYTurbo Camarda, Cieri, Ferrera '23; ARTEMIDE Scimemi, Vladimirov '23)
- All results (except ARTEMIDE) include $\alpha_s{}^3$ fixed order matching from MCFM

Comparison and uncertainties

As we have seen, resummed computations are performed in a variety of (equivalent) formalisms and with different of scheme choices

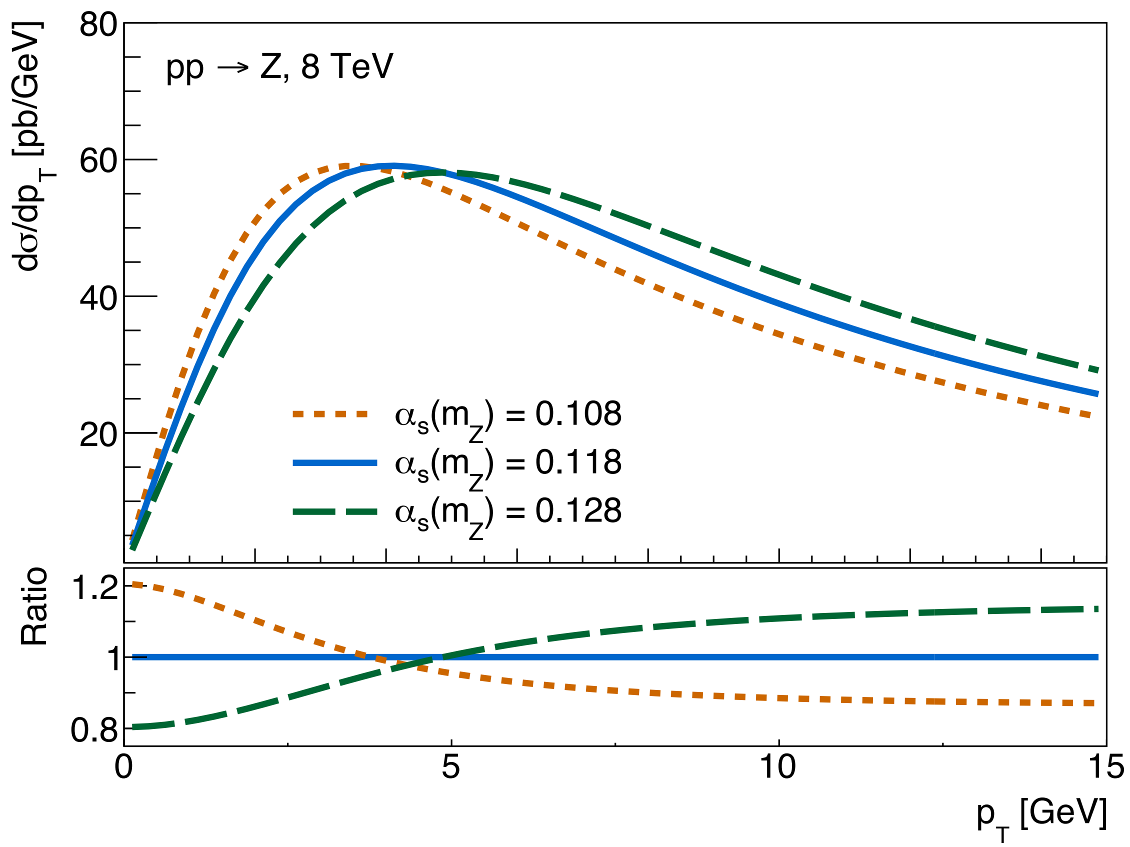
- Scale setting in momentum space (CuTe, Radish) versus impact parameter space (everyone else)
- Different formalisms for rapidity logs (CSS, collinear anomaly, RRG) and associated uncertainty
- Different matching schemes / transition to fixed order

Uncertainty estimates are much less standardized than for fixed-order computations!

- Ongoing comparison/benchmark efforts by LHC EW sub-group
- Workshop at CERN next Monday

ATLAS α_s extraction

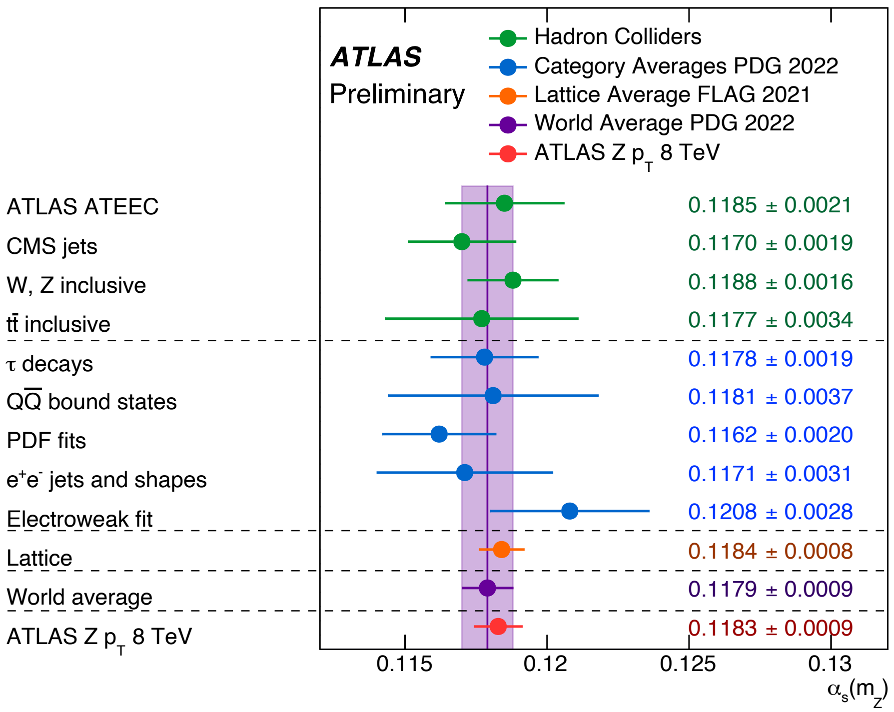
2309.12986



- Reconstruct inclusive spectrum rate from angular coefficients
- α_s from fit to DYTurbo
- MSHT20 approximate N^{3}LO PDFs
  - cross checks with NNLO sets
- Non-perturbative effects based on two-parameter ansatz by Collins Rogers '14

ATLAS α_s extraction

2309.12986



Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

One of the most precise determinations of α_s !

With these high-precision resummed and matched computations, we have entered a new regime of precision collider calculations.

Unprecedented precision, but also difficult to be sure the uncertainties are reliably estimated… we have no previous experience with 1% precision at hadron colliders!

QCD made simpler: the physics of soft and collinear emissions

QCD made simple(r)

There are two limits where the perturbative expressions for the scattering of quarks and gluons simplify considerably

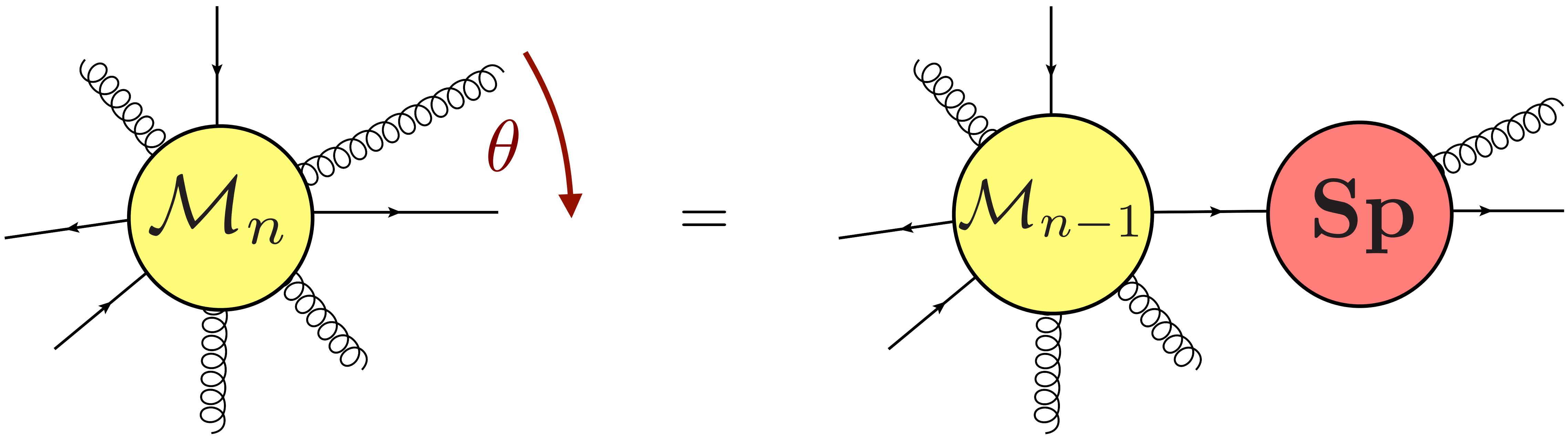
- Collinear limit, where multiple particles move in a similar direction.

- Soft limit, in which particles with small energy and momentum are emitted.

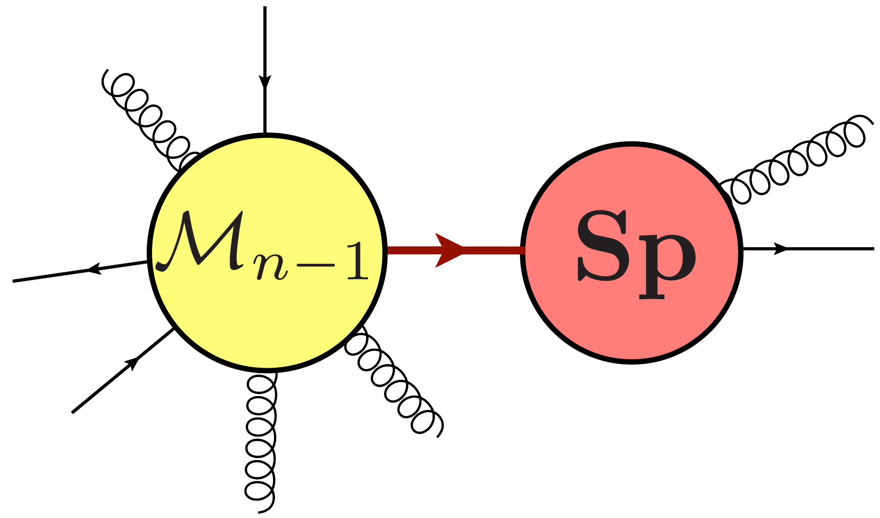
At the same time the cross sections are enhanced in these regions (→ large logarithms!).

- If these regions are relevant, we need to resum these contributions to all orders to get reliable predictions!

Collinear limit



In the limit $\theta \to 0$, where the partons become collinear, the n-parton amplitude factorizes into a product of an (n-1)-parton amplitude times a splitting amplitude $\mathbf{Sp}$.



The splitting amplitude diverges as $\theta \to 0$ and the factorization holds up to regular terms

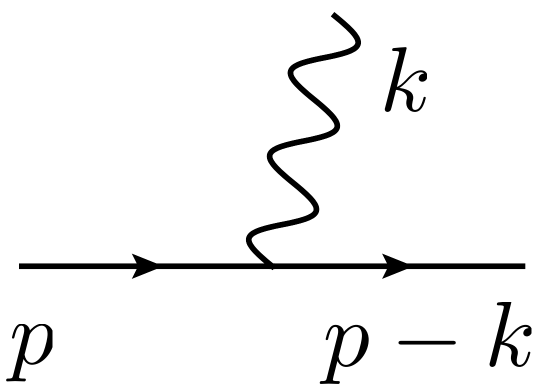
For the cross section, one finds

$$\mathrm{d}\sigma_n \sim d\sigma_{n-1} \frac{\mathrm{d}\theta}{\theta} \frac{dE_g}{E_g} d\phi$$

Logarithmic enhancements at small angle, and also at small gluon energy. No interference!

Soft limit

Also when particles with small energy and momentum are emitted, the amplitudes simplify:



$$\cdots \frac{\not{p} - \not{k} + m}{(p-k)^2 - m^2} \gamma_\mu \, u(p)$$

$$\approx \; \ldots \; u(p) \boxed{\frac{p_\mu}{p \cdot k}}$$

Soft emission factors from the rest of the amplitude.

$p \cdot k = E\,\omega\,(1 - \cos\theta)$ in denominator leads to logarithmic enhancements at small energy and small angle.

Since the emission of soft gluons is in the direction of the particle, the cross section for the emission of one gluon is

$$d\sigma_{n+1}^{\mathrm{soft}} = \frac{\alpha_s}{2\pi}\frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\sigma_n \sum_{i,j=1}^{n} C_{ij}\frac{\omega^2\, p_i \cdot p_j}{p_i \cdot k\, p_j \cdot k}$$

color factor ~ $\boldsymbol{T}_i \cdot \boldsymbol{T}_j$

So for massless particles soft emission is a *pure interference effect*, in marked contrast to collinear emissions!

Wilson lines

Soft emissions are only sensitive to the total charge of the object they radiate off. Also, the emission of soft quarks is suppressed compared to gluon emission.

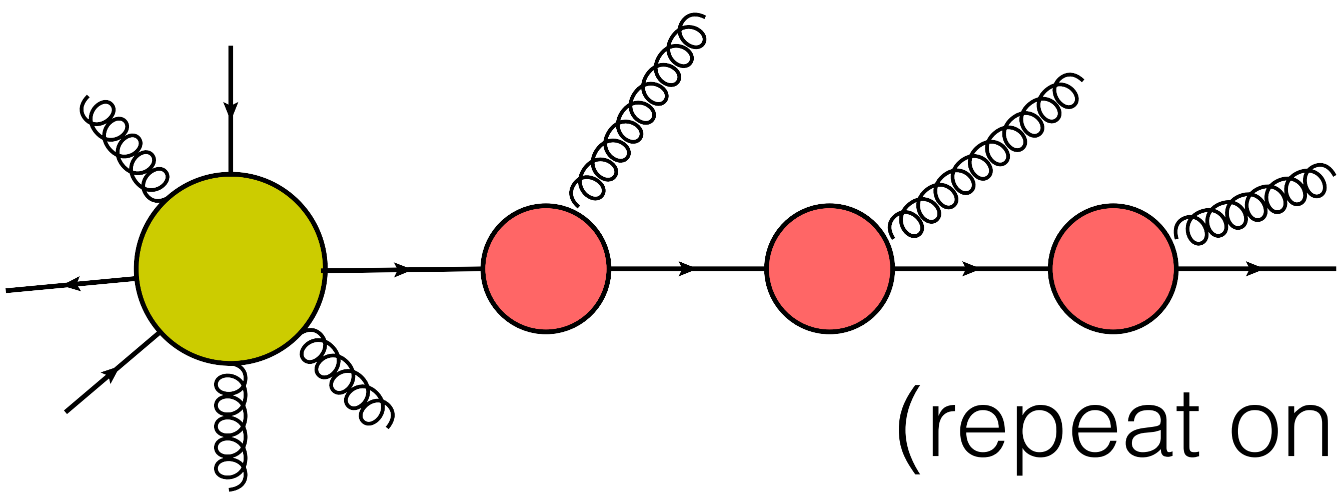
Interactions can be represented as

$$S_i = \mathbf{P} \exp\left[ig \int_0^\infty ds\, n_i \cdot A_s^a(sn_i)\boldsymbol{T}_i^a\right]$$

$n_i^\mu = p_i^\mu / E$ is a vector in the direction of the energetic particle, and $\boldsymbol{T}_i^a$ is its color charge

Link to parton shower

The parton shower generates multiple collinear emissions iteratively



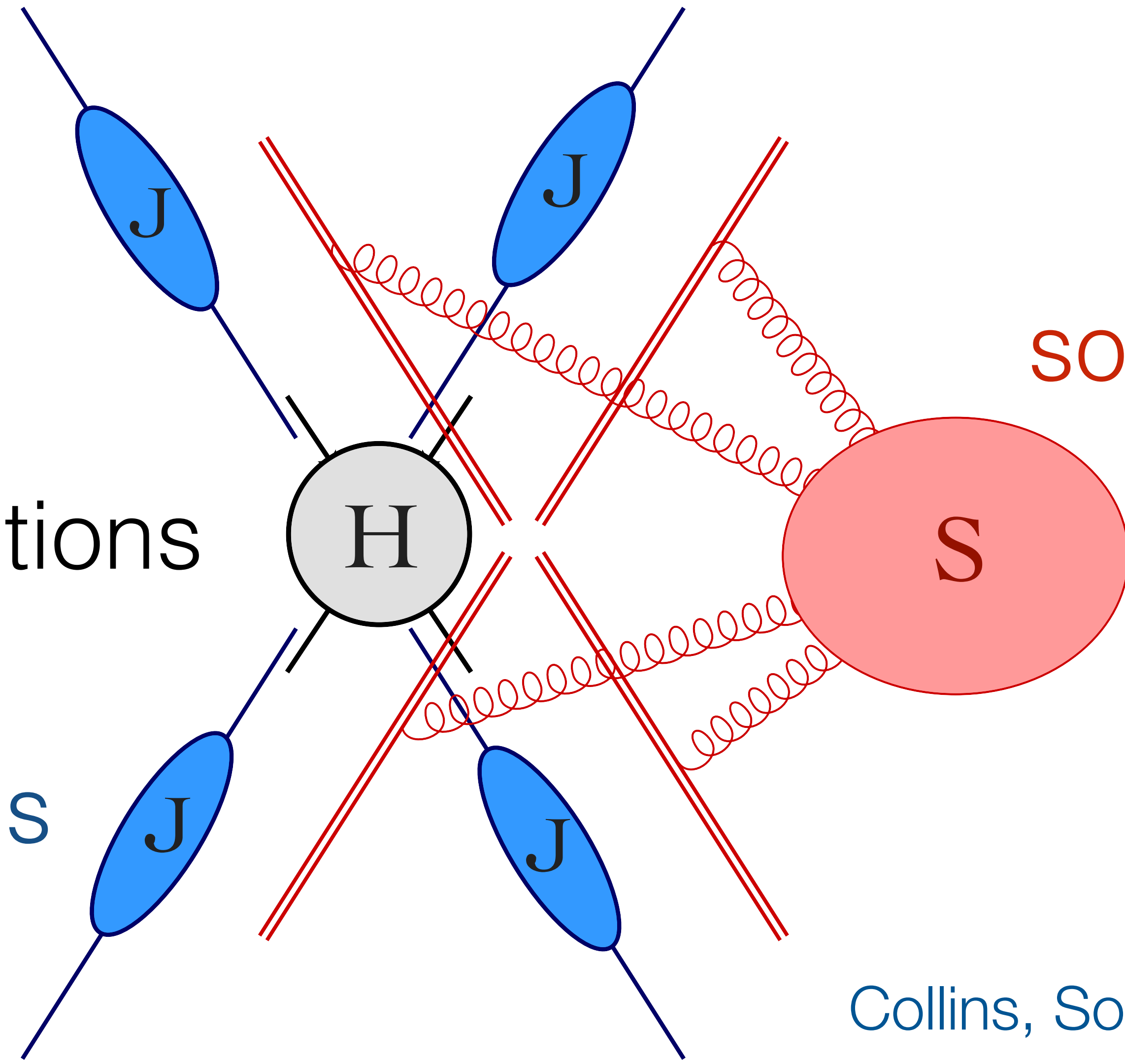
(repeat on each leg)

Without care, the shower will give the wrong result, even at LL, because it does not contain soft interference.

- Angle ordering disentangles soft radiation interference

see e.g. “QCD and Collider Physics”, by Ellis, Sterling and Webber

Soft-collinear factorization



Basis for higher-log resummation. More complicated than structure than what's implemented in a parton shower:

- Interference, color structure, spin, loop corrections.

Soft-Collinear Effective Theory

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke, Diehl et al. 2002; ...

Implements interplay between soft and collinear into effective field theory

Hard } **high-energy**

Collinear

Soft

} **low-energy part**

Correspondingly, EFT for such processes has two low-energy modes:

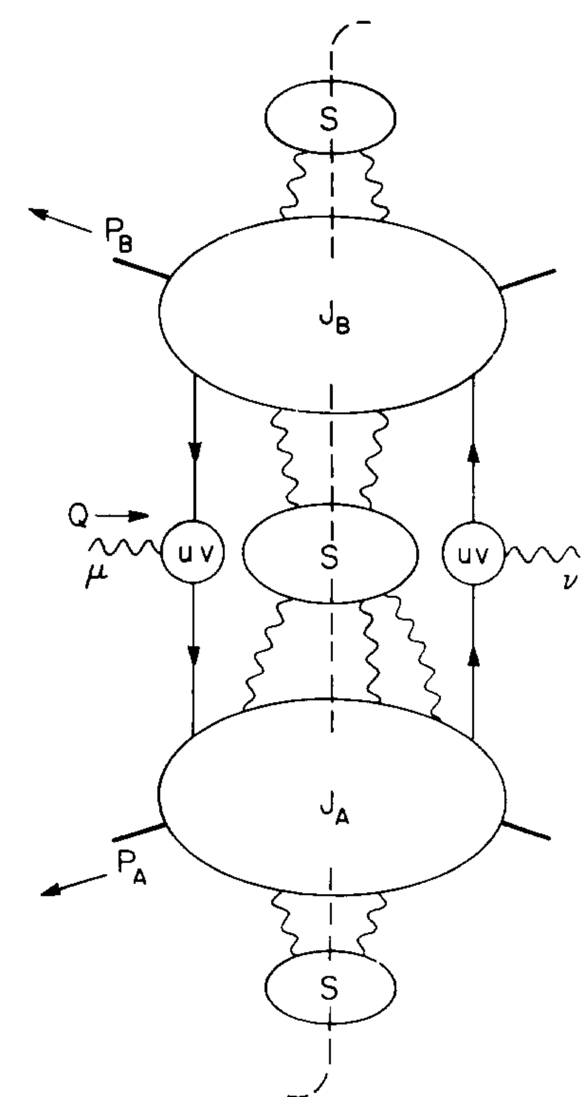
collinear fields describing the energetic partons propagating in each direction of large energy, and

soft fields which mediate long range interactions among them.

Diagrammatic Factorization

The simple structure of soft and collinear emissions forms the basis of the classic factorization proofs, which were obtained by analyzing Feynman diagrams.

Collins, Soper, Sterman 80's ...



Collins and Soper '81

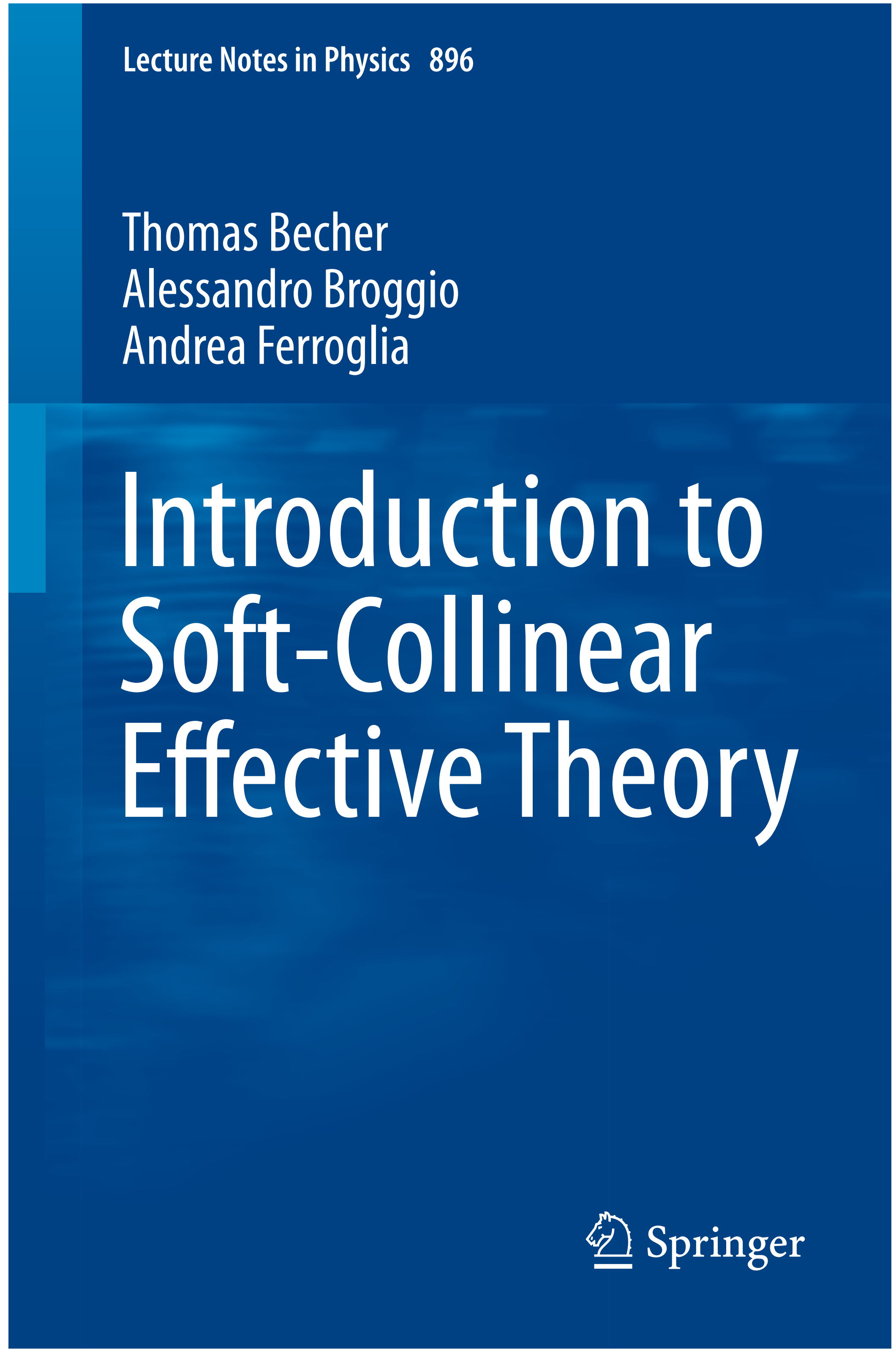
Advantages of the the SCET approach:

Simpler to exploit gauge invariance on the Lagrangian level

Operator definitions for the soft and collinear contributions

Resummation with renormalization group

Can include power corrections



arXiv:1410.1892